2. Mechanical Properties of Fluids

Can you recall?

1. How important are fluids in our life?
2. What is atmospheric pressure?
3. Do you feel excess pressure while swimming under water? Why?

2.1 Introduction:

In XIth Std. we discussed the behaviour of solids under the action of a force. Among three states of matter, i.e., solid, liquid and gas, a solid nearly maintains its fixed shape and volume even if a large force is applied to it. Liquids and gases do not have their own shape and they take the shape of the containing vessel. Due to this, liquids and gases flow under the action of external force. A *fluid means a substance that can flow*. Therefore, liquids and gases, collectively, are called fluids. A fluid either has no rigidity or its rigidity is very low.

In our daily life, we often experience the pressure exerted by a fluid at rest and in motion. Viscosity and surface tension play an important role in nature. We will try to understand such properties in this chapter.

2.2 Fluid:

Any substance that can flow is a fluid. A fluid is a substance that deforms continually under the action of an external force. Fluid is a phase of matter that includes liquids, gases and plasmas.

Do you know?

Plasma is one of the four fundamental states of matter. It consists of a gas of ions, free electrons and neutral atoms.

We shall discuss mechanical properties of only liquids and gases in this Chapter. The shear modulus of a fluid is zero. In simpler words, fluids are substances which cannot resist any shear force applied to them. Air, water, flour dough, toothpaste, etc., are some common examples of fluids. Molten lava is also a fluid.

A fluid flows under the action of a force or a pressure gradient. Behaviour of a fluid in motion is normally complicated. We can understand fluids by making some simple assumptions. We introduce the concept of an ideal fluid to understand its behaviour. An *ideal fluid* has the following properties:

1. It is incompressible: its density is constant.
2. Its flow is irrotational: its flow is smooth, there are no turbulences in the flow.
3. It is nonviscous: there is no internal friction in the flow, i.e., the fluid has no viscosity. (viscosity is discussed in section 2.6.1)
4. Its flow is steady: its velocity at each point is constant in time.

It is important to understand the difference between a solid and a fluid. Solids can be subjected to shear stress (tangential stress) as shown in Fig. 2.1 and normal stress, as shown in Fig.2.2.

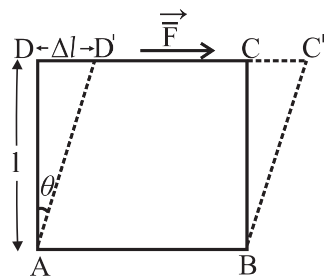


Fig. 2.1: Shear stress.

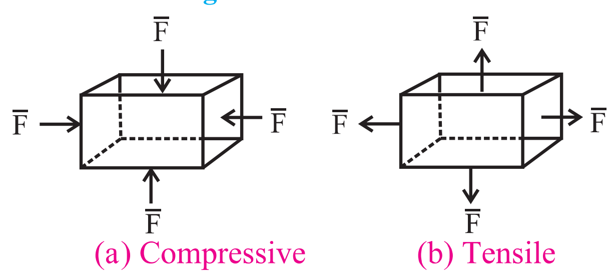


Fig. 2.2: Normal stress.

Solids oppose the shear stress either by developing a restoring force, which means that the deformations are reversible, or they require a certain initial stress before they deform and start flowing. (We have studied this behavior of solids (elastic behaviour) in XIth Std).

Ideal fluids, on the other hand, can only be subjected to normal, compressive stress (called pressure). Most fluids offer a very weak resistance to deformation. Real fluids display viscosity and so are capable of being subjected to low levels of shear stress.

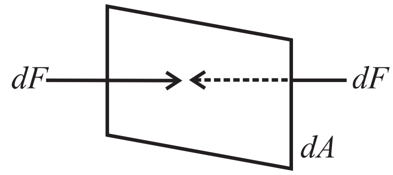


Fig. 2.3: Forces acting on a small surface *dA* within a fluid at rest.

The Fig. 2.3 shows a small surface of area dA at rest within a fluid. The surface does not accelerate, so the surrounding fluid exerts equal normal forces dF on both sides of it.

Properties of Fluids:

1. They do not oppose deformation, they get permanently deformed.
2. They have ability to flow.
3. They have ability to take the shape of the container.

A fluid exhibits these properties because it cannot oppose a shear stress when in static equilibrium.

Remember this

The term *fluid* includes both the liquid and gas phases. It is commonly used, as a synonym for *liquid* only, without any reference to gas. For example, 'brake fluid' is hydraulic oil and will not perform its required function if there is gas in it! This colloquial use of the term is also common in the fields of medicine and nutrition, e.g., "take plenty of fluids".

2.2.1 Fluids at Rest:

The branch of physics which deals with the properties of fluids at rest is called *hydrostatics*. In the next few sections we will consider some of the properties of fluids at rest.

2.3 Pressure:

A fluid at rest exerts a force on the surface of contact. The surface may be a wall or the bottom of an open container of the fluid. The normal force (F) exerted by a fluid at rest per unit surface area (A) of contact is called the pressure (p) of the fluid.

$$p = \frac{F}{A} \qquad \text{--- (2.1)}$$

Figure 2.4 shows a fluid exerting normal force on a vertical surface and Fig. 2.5 shows fluid exerting normal force on a horizontal surface.

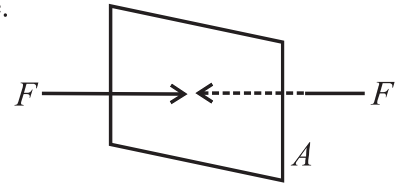


Fig. 2.4: Fluid exert force on vertical surface.

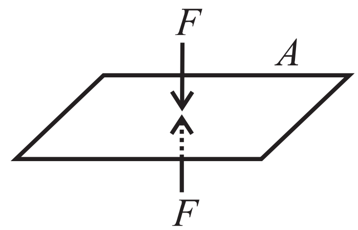


Fig. 2.5: Fluid exert force on horizontal surface.

Thus, an object having small weight can exert high pressure if its weight acts on a small surface area. For example, a force of 10 N acting on 1 cm^2 results in a pressure of 10^5 N m^{-2}. On the other hand, the same force of 10 N while acting on an area of 1 m^2, exerts a pressure of only 10 N m^{-2}.

Remember this

1 N weight is about 100 g mass, if g =10 m s^{-2}.

The SI unit of pressure is N/m^2. Also, 1 N/m^2 = 1 Pascal (Pa). The dimension of pressure is [$L^{-1}M^1T^{-2}$]. *Pressure is a scalar quantity*. Other common units of measuring pressure of a gas are *bar* and *torr*.

1 bar = 10^5 Nm^{-2}

1 hectapascal (hPa) = 100 Pa

Can you tell?

Why does a knife have a sharp edge, and a needle has a sharp tip?

Use your brain power

A student of mass 50 kg is standing on both feet. Estimate the pressure exerted by the student on the Earth. Assume reasonable value to any other quantity you need. Justify your assumption. You may use g = 10 m s^{-2} By what factor will it change if the student lies on back?

Remember this

The concept of pressure is useful in dealing with fluids, i.e., liquids and gases. As fluids do not have definite shape and volume, it is convenient to use the quantities pressure and density rather than force and mass when studying hydrostatics and hydrodynamics.

2.3.1 Pressure Due to a Liquid Column:

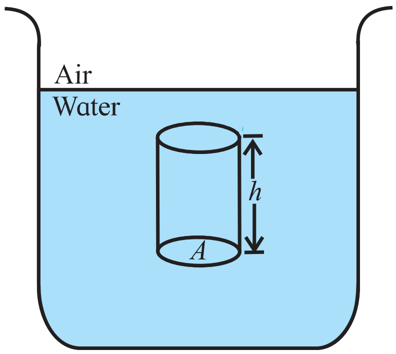


Fig. 2.6: Pressure due to a liquid column.

A vessel is filled with a liquid. Let us calculate the pressure exerted by an imaginary cylinder of cross sectional area A inside the container. Let the density of the fluid be ρ, and the height of the imaginary cylinder be h as shown in the Fig. 2.6. The liquid column exerts a force $F = mg$, which is its weight, on the bottom of the cylinder. This force acts in the downward direction. Therefore, the pressure p exerted by the liquid column on the bottom of cylinder is,

$$p = \frac{F}{A}$$

$$\therefore\ p = \frac{mg}{A}$$

Now, m = (volume of cylinder) x (density of liquid)

$$= (Ah) \times \rho = Ah\rho$$

$$\therefore\ p = \frac{(Ah\rho)g}{A}$$

$$p = h\rho g \qquad \text{--- (2.2)}$$

Thus, the pressure p due to a liquid of density ρ at rest, and at a depth h below the free surface is $h\rho g$.

Note that the pressure dose not depend on the area of the imaginary cylinder used to derive the expression.

Remember this

1. As $p = h\rho g$, the pressure exerted by a fluid at rest is independent of the shape and size of the container.
2. $p = h\rho g$ is true for liquids as well as for gases.

Example 2.1: Two different liquids of density ρ_1 and ρ_2 exert the same pressure at a certain point. What will be the ratio of the heights of the respective liquid columns?

Solution: Let h_1 be the height of the liquid of density ρ_1. Then the pressure exerted by the liquid of density ρ_1 is $p_1 = h_1\rho_1 g$. Similarly, let h_2 be the height of the liquid of density ρ_2. Then the pressure exerted by the liquid of density ρ_2 is $p_2 = h_2\rho_2 g$.

Both liquids exert the same pressure, therefore we write,

$$p_1 = p_2$$

$$\therefore\ h_1\rho_1 g = h_2\rho_2 g \text{ or, } \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$$

Alternate method:

For a given value of $p = h\rho g$ = constant, as g is constant. So the hight is inversely proportional to the density of the fluid ρ. In this case, since pressure is constant, height is inversely proportional to density of the liquid.

Example 2.2: A swimmer is swimming in a swimming pool at 6 m below the surface of the water. Calculate the pressure on the swimmer due to water above. (Density of water = 1000 kg/m^3, g = 9.8 m/s^2)

Solution: Given,

h = 6 m, ρ = 1000 kg/m^3, g = 9.8 m/s^2

$p = h\rho g = 6 \times 1000 \times 9.8 = 5.88 \times 10^5$ N/m^2

(Which is nearly 6 times the atmospheric pressure!)

2.3.2 Atmospheric Pressure:

Earth's atmosphere is made up of a fluid, namely, air. It exerts a downward force due to its weight. The pressure due to this force is called atmospheric pressure. Thus, at any point, the atmospheric pressure is the weight of a column of air of unit cross section starting from that point and extending to the top of the atmosphere. Clearly, the atmospheric pressure is highest at the surface of the Earth, i.e., at the sea level, and decreases as we go above the surface as the height of the column of air above decreases. The atmospheric pressure at sea level is called *normal atmospheric pressure*. The density of air in the atmosphere decreases with increase in height and becomes negligible beyond a height of about 8 km so that the height of air column producing atmospheric pressure at sea level can be taken to be 8 km.

The region where gas pressure is less than the atmospheric pressure is called *vacuum*. Perfect or absolute vacuum is when no matter, i.e., no atoms or molecules are present. Usually, vacuum refers to conditions when the gas pressure is considerably smaller than the atmospheric pressure.

2.3.3 Absolute Pressure and Gauge Pressure:

Consider a tank filled with water as shown in Fig. 2.7. Assume an imaginary cylinder of horizontal base area A and height $x_1 - x_2 = h$. x_1 and x_2 being the heights measured from a reference point, height increasing upwards: $x_1 > x_2$. The vertical forces acting on the cylinder are:

1. Force $\vec{F_1}$ acts downwards at the top surface of the cylinder, and is due to the weight of the water column above the cylinder.
2. Force $\vec{F_2}$ acts upwards at the bottom surface of the cylinder, and is due to the water below the cylinder.
3. The gravitational force on the water enclosed in the cylinder is mg, where m is the mass of the water in the cylinder. As the water is in static equilibrium, the forces on the cylinder are balanced. The balance of these forces in magnitude is written as,

$$F_2 = F_1 + mg \qquad \text{--- (2.3)}$$

p_1 and p_2 are the pressures at the top and bottom surfaces of the cylinder respectively due to the fluid. Using Eq. (2.1) we can write

$$F_1 = p_1 A, \text{ and } F_2 = p_2 A \qquad \text{--- (2.4)}$$

Also, the mass m of the water in the cylinder can be written as,

$$m = \text{density} \times \text{volume} = \rho V$$

$$\therefore m = \rho A(x_1 - x_2) \qquad \text{--- (2.5)}$$

Substituting Eq. (2.4) and Eq. (2.5) in Eq. (2.3) we get,

$$p_2 A = p_1 A + \rho A g\,(x_1 - x_2)$$

$$p_2 = p_1 + \rho g\,(x_1 - x_2) \qquad \text{--- (2.6)}$$

This equation can be used to find the pressure inside a liquid (as a function of depth below the liquid surface) and also the atmospheric pressure (as a function of altitude or height above the sea level).

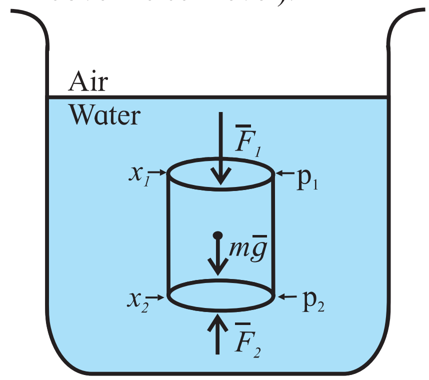


Fig. 2.7: Pressure due to an imaginary cylinder of fluid.

To find the pressure p at a depth h below the liquid surface, let the top of an imaginary cylinder be at the surface of the liquid. Let this level be x_1. Let x_2 be some point at depth h below the surface as shown in Fig. 2.8. Let p_0 be the atmospheric pressure at the surface, i.e., at x_1. Then, substituting $x_1 = 0$, $p_1 = p_0$, $x_2 = -h$, and $p_2 = p$ in Eq. (2.6) we get,

$$p = p_0 + h\rho g \qquad \text{--- (2.7)}$$

The above equation gives the total pressure, or the *absolute pressure* p, at a depth h below the surface of the liquid. The total pressure p, at the depth h is the sum of:

1. p_0, the pressure due to the atmosphere, which acts on the surface of the liquid, and
2. $h\rho g$, the pressure due to the liquid at depth h.

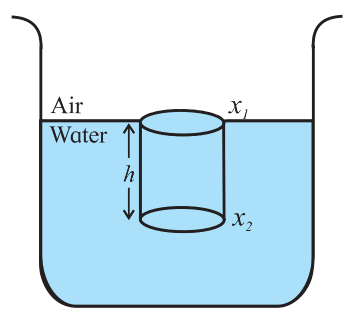


Fig. 2.8. Pressure at a depth *h* below the surface of a liquid.

In general, the difference between the absolute pressure and the atmospheric pressure is called the *gauge pressure*. Using Eq. (2.7), gauge pressure at depth h below the liquid surface can be written as,

$$p - p_0 = h\rho g \qquad \text{--- (2.8)}$$

Eq. (2.8) is also applicable to levels above the liquid surface. It gives the pressure at a given height above a liquid surface, in terms of the atmospheric pressure p_0 (assuming that the atmospheric density is uniform up to that height).

To find the atmospheric pressure at a distance d above the liquid surface as shown in Fig. 2.9, we substitute $x_1 = d$, $p_1 = p$, $x_2 = 0$, $p_2 = p_0$ and $\rho = \rho_{air}$ in Eq. (2.6) we get,

$$p = p_o - d\rho_{air} g \qquad \text{--- (2.9)}$$

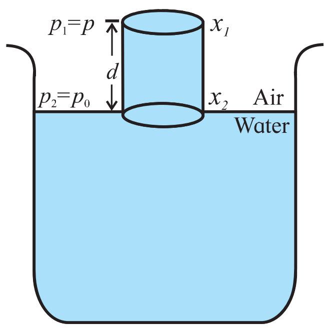
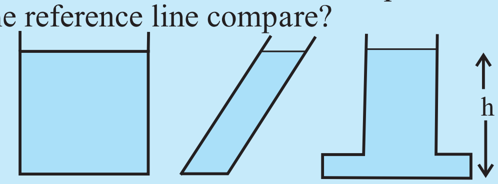


Fig. 2.9: Change of atmospheric pressure with height.

Can you tell?

The figures show three containers filled with the same oil. How will the pressures at the reference line compare?

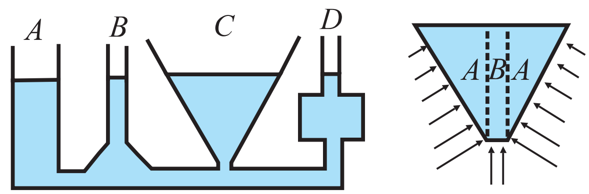


(a) (b) (c)

2.3.4 Hydrostatic Paradox:

Consider the inter connected vessels as shown in Fig. 2.10 (a). When a liquid is poured in any one of the vessels, it is noticed that the level of liquids in all the vessels is the same. This observation is somewhat puzzling. It was called 'hydrostatics paradox' before the principle of hydrostatics were completely understood.

One can feel that the pressure of the base of the vessel C would be more than that at the base of the vessel B and the liquid from vessel C would rise into the vessel B. However, it is never observed. Equation 2.2 tells that the pressure at a point depends only on the height of the liquid column above it. It does not depend on the shape of the vessel. In this case, height of the liquid column is the same for all the vessels. Therefore, the pressure of liquid column in each vessel is the same and the system is in equilibrium. That means the liquid in vessel C does not rise in to vessel B.



(a) (b)

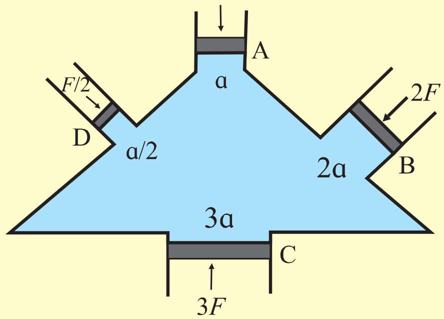
Fig. 2.10: Hydrostatic paradox.

Consider Fig. 2.10 (b). The arrows indicate the forces exerted against the liquid by the walls of the vessel. These forces are perpendicular to walls of the vessel at each point. These forces can be resolved into vertical and horizontal components. The vertical components act in the upward direction. Weight of the liquid in section B is not balanced and contibutes the pressure at the base. Thus, it is no longer a paradox!

2.3.5 Pascal's Law:

Pascal's law states that the pressure applied at any point of an enclosed fluid at rest is transmitted equally and undiminished to every point of the fluid and also on the walls of the container, provided the effect of gravity is neglected.

Experimental proof of Pascal's principle. Consider a vessel with four arms A, B, C, and D fitted with frictionless, water tight pistons and filled with incompressible fluid as shown in the figure given. Let the area of cross sections of A, B, C, and D be a, $2a$, $3a$, and $a/2$ respectively. If a force F is applied on the piston A, the pressure exerted on the liquid is $p = F/a$. It is observed that the other three pistons B, C, and D move outward. In order to keep these three pistons B, C, and D in their original positions, forces $2F$, $3F$, and $F/2$ respectively are required to be applied on the pistons. Therefore, pressure on the pistons B, C, and D is:



$$\text{on B,}\ p_B = \frac{2F}{2a} = \frac{F}{a}$$

$$\text{on C,}\ p_C = \frac{3F}{3a} = \frac{F}{a} \quad \text{and}$$

$$\text{on D,}\ p_D = \frac{F/2}{a/2} = \frac{F}{a}$$

i.e. $p_B = p_C = p_D = p$, this indicates that the pressure applied on piston A is transmitted equally and undiminished to all parts of the fluid and the walls of the vessel.

Applications of Pascal's Law:

i) Hydraulic lift: Hydraulic lift is used to lift a heavy object using a small force. The working of this machine is based on Pascal's law.

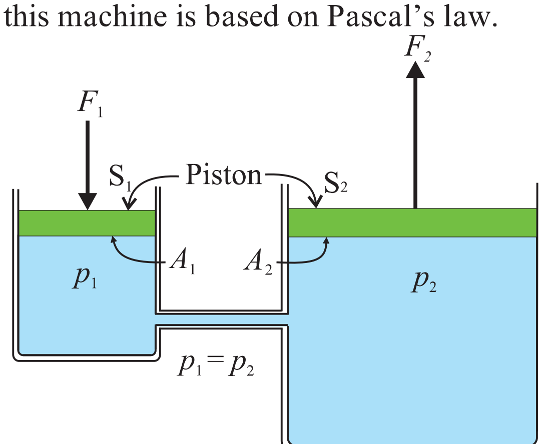


Fig. 2.11 Hydraulic Lift.

As shown in Fig. 2.11, a tank containing a fluid is fitted with two pistons S_1 and S_2. S_1 has a smaller area of cross section, A_1 while S_2 has a much larger area of cross section, A_2 ($A_2 >> A_1$). If we apply a force F_1 on the smaller piston S_1 in the downward direction it will generate pressure $p = (F_1/A_1)$ which will be transmitted undiminished to the bigger piston S_2. A force $F_2 = pA_2$ will be exerted upwards on it.

$$F_2 = F_1\left(\frac{A_2}{A_1}\right) \qquad \text{--- (2.10)}$$

Thus, F_2 is much larger than F_1. A heavy load can be placed on S_2 and can be lifted up or moved down by applying a small force on S_1. This is the principle of a hydraulic lift.

Observe and discuss

Blow air in to a flat balloon using a cycle pump. Discuss how Pascal's principle is applicable here.

ii) Hydraulic brakes: Hydraulic brakes are used to slow down or stop vehicles in motion. It is based on the same principle as that of a hydraulic lift.

Figure 2.12 shows schematic diagram of a hydraulic brake system. By pressing the brake pedal, the piston of the master cylinder is pushed in forward direction. As a result, the piston in the slave cylinder which has a much larger area of cross section as compared to that of the master cylinder, also moves in forward direction so as to maintain the volume of the oil constant. The slave piston pushes the friction pads against the rotating disc, which is connected to the wheel. Thus, causing a moving vehicle to slow down or stop.

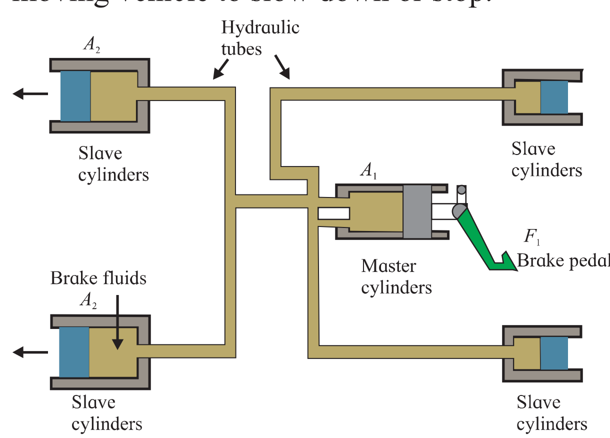


Fig. 2.12 Hydraulic brake system (schematic).

The master cylinder has a smaller area of cross section A_1 compared to the area A_2 of the slave cylinder. By applying a small force F_1

to the master cylinder, we generate pressure $p = (F_1/A_1)$. This pressure is transmitted undiminished throughout the system. The force F_2 on slave cylinder is then,

$$F_2 = pA_2 = \frac{F_1}{A_1} \times A_2 = F_1\left(\frac{A_2}{A_1}\right)$$

This is similar to the principle used in hydraulic lift. Since area A_2 is greater than A_1, F_2 is also greater than F_1. Thus, a small force applied on the brake pedal gets converted into large force and slows down or stops a moving vehicle.

Example 2.3: A hydraulic brake system of a car of mass 1000 kg having speed of 50 km/h, has a cylindrical piston of radius of 0.5 cm. The slave cylinder has a radius of 2.5 cm. If a constant force of 100 N is applied on the brake what distance the car will travel before coming to stop?

Solution: Given,

$F_1 = 100$ N, $A_1 = \pi\,(0.5 \times 10^{-2})^2$ m^2,
$A_2 = \pi\,(2.5 \times 10^{-2})^2$ m^2, $F_2 = ?$

By Pascal's Principle,

$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$

$$F_2 = \frac{100 \times \pi(2.5 \times 10^{-2})^2}{\pi(0.5 \times 10^{-2})^2} = 2500 \text{ N}$$

Acceleration of the car =

$a = F_2/m = 2500/1000 = 2.5$ m/s^2. Using Newton's equation of motion,

$v^2 = u^2 - 2as$ where final velocity $v = 0$, $u = 50$ km/h

$$s = \left(\frac{50 \times 1000}{3600}\right)^2 \times \frac{1}{(2 \times 2.5)} = 38.58 \text{ m}$$

2.3.6 Measurement of Pressure:

Instruments used to measure pressure are called pressure meters or pressure gauges or vacuum gauges. Below we will describe two instruments which are commonly used to measure pressure.

Caution:
Use of mercury is not advised in a laboratory because mercury vapours are hazardous for life and for environment.

i) Mercury Barometer: An instrument that measures atmospheric pressure is called a *barometer*. One of the first barometers was by Italian scientist Torricelli. The barometer is in the form of a glass tube completely filled with mercury and placed upside down in a small dish containing mercury. Its schematic diagram is shown in Fig. 2.13.

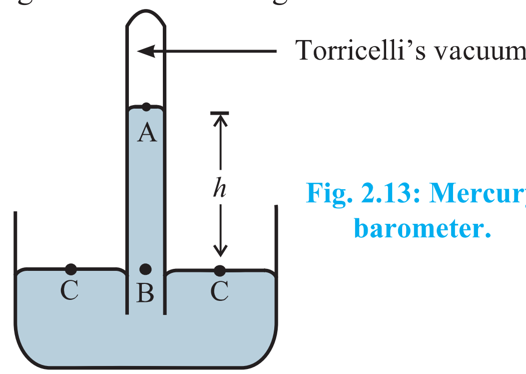


Fig. 2.13: Mercury barometer.

1. A glass tube of about 1 meter length and a diameter of about 1 cm is filled with mercury up to its brim. It is then quickly inverted into a small dish containing mercury. The level of mercury in the glass tube lowers as some mercury spills in the dish. A gap is created between the surface of mercury in the glass tube and the closed end of the glass tube. The gap does not contain any air and it is called *Torricelli's vacuum*. It dose contain some mercury vapors.
2. Thus, the pressure at the upper end of the mercury column inside the tube is zero, i.e. pressure at point such as A is p_A= zero.
3. Let us consider a point C on the mercury surface in the dish and another point B inside the tube at the same horizontal level as that of the point C.
4. The pressure at C is equal to the atmospheric pressure p_0 because it is open to atmosphere. As points B and C are at the same horizontal level, the pressure at B is also equal to the atmospheric pressure p_o, i.e. $p_B = p_o$.
5. Suppose the point B is at a depth h below the point A and ρ is the density of mercury then,

$$P_B = P_A + h\rho g \qquad \text{--- (2.11)}$$

$p_A = 0$ (there is vacuum above point A) and $p_B = p_0$, therefore, $p_0 = h\rho g$, where h is the length of mercury column in the mercury barometer.

Remember this

The atmospheric pressure is generally expressed as the length of mercury column in a mercury barometer.

p_{atm} = 76 cm of Hg = 760 mm of Hg

Another unit for measuring pressure is torr. One torr = 1 mm of Hg

Can you tell?

What will be the normal atmospheric pressure in bar and also in torr?

ii) Open tube manometer: A manometer consists of a U – shaped tube partly filled with a low density liquid such as water or kerosene. This helps in having a larger level difference between the level of liquid in the two arms of the manometer. Figure 2.14 shows an open tube manometer. One arm of the manometer is open to the atmosphere and the other is connected to the container D of which the pressure p is to be measured.

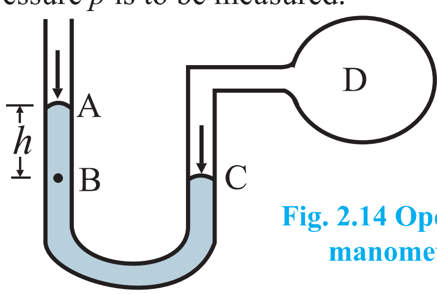


Fig. 2.14 Open tube manometer.

The pressure at point A is atmospheric pressure p_0 because this arm is open to atmosphere. To find the pressure at point C, which is exposed to the pressure of the gas in the container, we consider a point B in the open arm of the manometer at the same level as point C. The pressure at the points B and C is the same, i.e.,

$$p_C = p_B \quad \text{--- (2.12)}$$

The pressure at point B is,

$$p_B = p_o + h\rho g \quad \text{--- (2.13)}$$

where, ρ is the density of the liquid in the manometer, h is the height of the liquid column above point B, and g is the acceleration due to gravity. According to Pascal's principle, pressure at C is the same as at D, i.e., inside the chamber. Therefore, the pressure p in the container is,

$$p = p_C$$

Using Eq. (2.12) and Eq. (2.13) we can write,

$$p = p_0 + h\rho g \quad \text{--- (2.14)}$$

As the manometer measures the gauge pressure of the gas in the container D, we can write the gauge pressure in the container D as

$$p - p_0 = h\rho g$$

Can you recall?

1. You must have blown soap bubbles in your childhood. What is their shape?
2. Why does a greased razor blade float on the surface of water?
3. Why can a water spider walk comfortably on the surface of still water?
4. Why are free liquid drops and bubbles always spherical in shape?

2.4 Surface Tension:

A liquid at rest shows a very interesting property called *surface tension*. We have seen that water spider walks on the surface of steady water, greased needle floats on the steady surface of water, rain drops and soap bubbles always take spherical shape, etc. All these phenomena arise due to surface tension. Surface tension is one of the important properties of liquids.

Do you know?

1. When we write on paper, the ink sticks to the paper.
2. When teacher writes on a board, chalk particles stick to the board.
3. Mercury in a glass container does not wet its surface, while water in a glass container wets it.

2.4.1 Molecular Theory of Surface Tension:

All the above observations can be explained on the basis of different types of forces coming into play in all these situations. We will try to understand the effect of these forces and their relation to the surface tension in liquids.

To understand surface tension, we need to know some terms in molecular theory that explain the behaviour of liquids at their surface.

a) Intermolecular force: Matter is made up of molecules. Any two molecules attract each other. *This force between molecules is called intermolecular force.* There are two types of intermolecular forces - i) Cohesive force and ii) Adhesive force.

i) **Cohesive force:** The force of attraction between the molecules of the same substance is called cohesive force or force of cohesion. The force of attraction between two air molecules or that between two water molecules is a cohesive force. Cohesive force is strongest in solids and weakest in gases. This is the reason why solids have a definite shape and gases do not. Small droplets of liquid coalesce into one and form a drop due to this force.

ii) **Adhesive force:** The force of attraction between the molecules of different substances is called adhesive force or force of adhesion. The force of attraction between glass and water molecule is a force of adhesion.

b) Range of molecular force: The maximum distance from a molecule up to which the molecular force is effective is called the range of molecular force. Intermolecular forces are effective up to a distance of the order of few nanometer (10^{-9} m) in solids and liquids. Therefore, they are short range forces.

c) Sphere of influence: An imaginary sphere with a molecule at its center and radius equal to the molecular range is called the sphere of influence of the molecule. The spheres around molecules A, B or C are shown in Fig. 2.15 (a) and (b). *The intermolecular force is effective only within the sphere of influence*.

d) Surface film: The surface layer of a liquid with thickness equal to the range of intermolecular force is called the surface film. This is the layer shown between XY and X′Y′ in Fig. 2.15 (b).

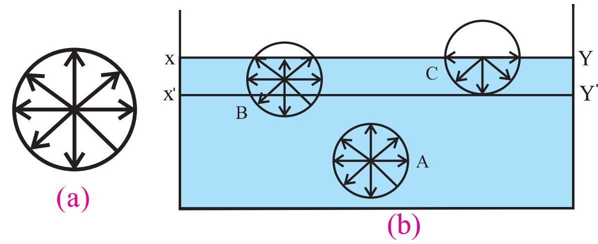


Fig. 2.15: (a) sphere of influence and (b) surface film.

(e) Free surface of a liquid: It is the surface of a fluid which does not experience any shear stress. For example, the interface between liquid water and the air above. In Fig. 2.15 (b), XY is the free surface of the liquid.

Remember this

While studying pressure, we considered both liquids and gases. But as gases do not have a free surface, they do not exhibit surface tension.

(f) Surface tension on the basis of molecular theory: As shown in Fig. 2.15 (b), XY is the free surface of liquid and X′Y′ is the inner layer parallel to XY at distance equal to the range of molecular force. Hence, the section XX′-Y′Y near the surface of the liquid acts as the surface film. Consider three molecules A, B, and C such that molecule A is deep inside the liquid, molecule B within surface film and molecule C on the surface of the liquid.

As molecule A is deep inside the liquid, its sphere of influence is also completely inside the liquid. As a result, molecule A is acted upon by equal cohesive forces in all directions. *Thus, the net cohesive force acting on molecule A is zero*.

Molecule B lies within the surface layer and below the free surface of the liquid. A larger part of its sphere of influence is inside the liquid and a smaller part is in air. Due to this, a strong downward cohesive force acts on the liquid molecule. The adhesive force acting on molecule B due to air molecules above it and within its sphere of influence is weak. It points upwards. As a result, the molecule B gets attracted inside the liquid.

The same holds for molecule C which lies exactly on the free surface of the liquid. Half of the sphere of influence is in air and half in the liquid. The number of air molecules within the sphere of influence of the molecule C, above the free surface of the liquid is much less than the number of liquid molecules within the sphere of influence that lies within the liquid. This is because, the density of air is less than that of a liquid. The adhesive force trying to pull the molecule above the liquid surface is much weaker than the cohesive force that tries to pull the molecule inside the liquid surface. *As a result, the molecule C also gets attracted inside the liquid*.

Thus, all molecules in the surface film are acted upon by an unbalanced net cohesive force directed into the liquid. Therefore, the molecules in the surface film are pulled inside the liquid. This minimizes the total number of molecules in the surface film. As a result, the surface film remains under tension. The surface film of a liquid behaves like a stretched elastic membrane. This tension is known as surface tension. *The force due to surface tension acts tangential to the free surface of a liquid*.

2.4.2 Surface Tension and Surface Energy:

a) Surface Tension: As seen previously, the free surface of a liquid in a container acts as a stretched membrane and all molecules on the surface film experience a stretching force. Imagine a line PQ of length *L* drawn tangential to the free surface of the liquid, as shown in Fig. 2.16.

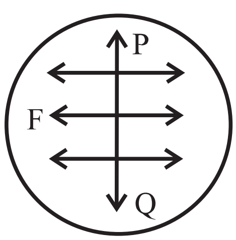


Fig. 2.16: Force of surface tension.

All the molecules on this line experience equal and opposite forces tangential to surface as if they are tearing the surface apart due to the cohesive forces of molecules lying on either side.

This force per unit length is the surface tension. *Surface tension T is defined as, the tangential force acting per unit length on both sides of an imaginary line drawn on the free surface of liquid.*

$$T = \frac{F}{L} \quad \text{--- (2.15)}$$

SI unit of surface tension is N/m. Its Dimension are, $[L^0M^1T^{-2}]$.

Use your brain power

Prove that, equivalent S.I. unit of surface tension is J/m^2.

Example 2.4: A beaker of radius 10 cm is filled with water. Calculate the force of surface tension on any diametrical line on its surface. Surface tension of water is 0.075 N/m.

Solution: Given,

$L = 2 \times 10 = 20$ cm $= 0.2$ m

$T = 0.075$ N/m

We have,

$$T = \frac{F}{L}$$

$\therefore F = TL = 0.075 \times 0.2 = 0.015$

$= 1.5 \times 10^{-2}$ N

Table 2.1 – Surface tension of some liquids at 20°C.

Sr. No.	Liquid	S.T. (N/m)	S.T. (dyne/cm)
1	Water	0.0727	72.7
2	Mercury	0.4355	435.5
3	Soap solution	0.025	25
4	Glycerin	0.0632	63.2

b) Surface Energy: We have seen that a molecule inside the volume of a liquid (like molecule A in Fig 2.15) experiences more cohesive force than a molecule like molecules B and C in the surface film of the liquid in that figure. Thus, work has to be done to bring any molecule from inside the liquid into the surface film. Clearly, the surface molecules possess extra potential energy as compared to the molecules inside the liquid. The extra energy of the molecules in the surface layer is called the *surface energy* of the liquid. As any system always tries to attain a state of minimum potential energy, the liquid tries to reduce the area of its surface film. Energy has to be spent in order to increase the surface area of a liquid.

Remember this

1) Molecules on the liquid surface experience net inward pull. In spite of this if they remain at the surface, they possess higher potential energy. As a universal property, any system tries to minimize its potential energy. Hence liquid surface tries to minimize its surface area.
2) When a number of droplets coalesce and form a drop, there is reduction in the total surface area. In this case, energy is released to the surrounding.

c) Relation between the surface energy and surface tension: Consider a rectangular frame of wire P′PSS′. It is fitted with a movable arm QR as shown in Fig. 2.17. This frame is dipped in a soap solution and then taken out. A film of soap solution will be formed within the boundaries PQRS of the frame.

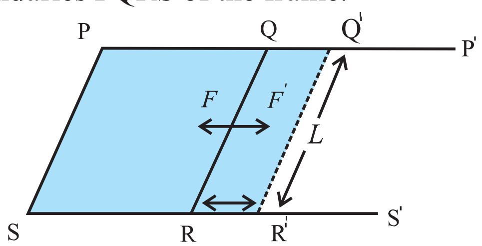


Fig. 2.17: Surface energy of a liquid

Each arm of the frame experiences an inward force due to the film. Under the action of this force, the movable arm QR moves towards side PS so as to decrease the area of the film. If the length of QR is L, then this inward force F acting on it is given by

$$F = (T) \times (2L) \qquad \text{--- (2.16)}$$

Since the film has two surfaces, the upper surface and the lower surface, the total length over which surface tension acts on QR is $2L$. Imagine an external force F' (equal and opposite to F) applied isothermally (gradually and at constant temperature), to the arm QR, so that it pulls the arm away and tries to increase the surface area of the film. The arm QR moves to Q′R′ through a distance dx. Therefore, the work done against F, the force due to surface tension, is given by

$$dw = F'dx$$

Using Eq. (2.16),

$$dw = T\,(2Ldx)$$

But, $2Ldx = dA$, increase in area of the two surfaces of the film. Therefore, $dw = T(dA)$.

This work done in stretching the film is stored in the area dA of the film as its potential energy. *This energy is called surface energy*.

$\therefore$ Surface energy = $T\,(dA)$ --- (2.17)

Thus, surface tension is also equal to the surface energy per unit area.

Example 2.5: Calculate the work done in blowing a soap bubble to a radius of 1 cm. The surface tension of soap solution is 2.5×10^{-2} N/m.

Solution: Given

$T = 2.5 \times 10^{-2}$ N/m

Initial radius of bubble = 0 cm

Final radius of bubble, r = 1 cm = 0.01 m

Initial surface area of soap bubble = 0

(A soap bubble has two surfaces, outer surface and inner surface).

Final surface area of soap bubble is,

$A = 2 \times (4\pi r^2) = 8\pi r^2$

$\therefore$ change in area $= dA = A - 0 = 8\pi r^2$

$= 0.00251\ m^2$

$\therefore$ work done $= T \times dA$

$= 2.5 \times 10^{-2} \times 0.251 \times 10^{-2}$

$= 6.275 \times 10^{-5}$ J

Try this

Take a ring of about 5 cm in diameter. Tie a thin thread along the diameter of the ring. Keep the thread slightly loose. Dip the ring in a soap solution and take it out. A soap film is formed on either side of thread. Break the film on any one side of the thread. Discuss the result.

Remember this

The work done, under isothermal condition, against the force of surface tension to change the surface area of a liquid is stored as surface energy of liquid.

2.4.3 Angle of Contact:

When a liquid surface comes in contact with a solid surface, it forms a meniscus, which can be either convex (mercury-glass) or concave (water glass), as shown in Fig. 2.18. The angle of contact, θ, between a liquid and a solid surface is defined as the angle between the tangents drawn to the free surface of the liquid and surface of the solid at the point of contact, measured within the liquid.

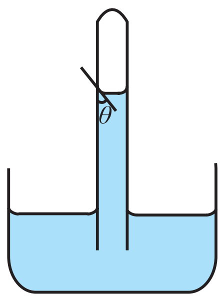


Fig. 2.18 (a): Concave meniscus due to liquids which partially wet a solid surface.

When the angle of contact is acute, the liquid forms a concave meniscus Fig. 2.18 (a) at the point of contact. When the angle of contact is obtuse, it forms a convex meniscus Fig. 2.18 (b). For example, water-glass interface forms a concave meniscus and mercury-glass interface forms a convex meniscus.

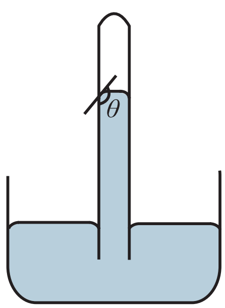


Fig. 2.18 (b): Convex meniscus due to liquids which do not wet a solid surface.

This difference between the shapes of menisci is due to the net effect of the cohesive forces between liquid molecules and adhesive forces between liquid and solid molecules as discussed below.

Do you know?

- when we observe the level of water in a capillary, we note down the level of the tangent to the meniscus inside the water.
- When we observe the level of mercury in a capillary we note down the level of the tangent to the meniscus above the mercury column.

a) Shape of meniscus:

i) Concave meniscus - acute angle of contact:

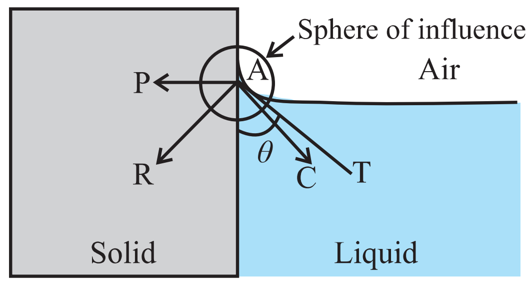


Fig. 2.19 (a): Acute angle of contact.

Figure 2.19 (a) shows the acute angle of contact between a liquid surface (e.g., kerosene in a glass bottle). Consider a molecule such as A on the surface of the liquid near the wall of the container. The molecule experiences both cohesive as well as adhesive forces. In this case, since the wall is vertical, the net adhesive force ($\overrightarrow{AP}$) acting on the molecule A is horizontal, Net cohesive force ($\overrightarrow{AC}$) acting on molecule is directed at nearly 45° to either of the surfaces. Magnitude of adhesive force is so large that the net force ($\overrightarrow{AR}$) is directed inside the solid.

For equilibrium or stability of a liquid surface, the net force ($\overrightarrow{AR}$) acting on the molecule A must be normal to the liquid surface at all points. For the resultant force $\overrightarrow{AR}$ to be normal to the tangent, the liquid near the wall should pile up against the solid boundary so that the tangent AT to the liquid surface is perpendicular to AR. Thus, this makes the meniscus concave. Obviously, such liquid wets that solid surface.

ii) Convex meniscus - obtuse angle of contact:

Figure 2.19 (b) shows the obtuse angle of contact between a liquid and a solid (e.g., mercury in a glass bottle). Consider a molecule such as A on the surface of the liquid near the wall of the container. The molecule experiences both cohesive as well as adhesive forces. In this case also, the net adhesive force ($\overrightarrow{AP}$) acting on the molecule A is horizontal since the wall is vertical. Magnitude of cohesive force is so large that the net force ($\overrightarrow{AR}$) is directed inside the liquid.

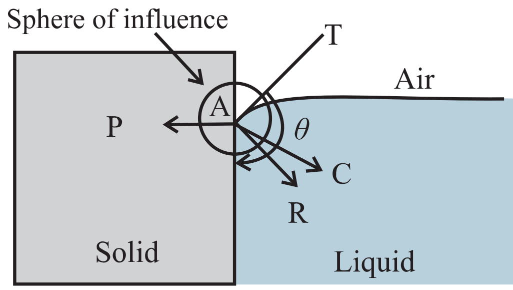


Fig. 2.19 (b): Obtuse angle of contact.

For equilibrium or stability of a liquid surface, the net force ($\overrightarrow{AR}$) acting on all molecules similar to molecule A must be normal to the liquid surface at all points. The liquid near the wall should, therefore, creep inside against the solid boundary. This makes the meniscus convex so that its tangent AT is normal to AR. Obviously, such liquid does not wet that solid surface.

iii) Zero angle of contact :

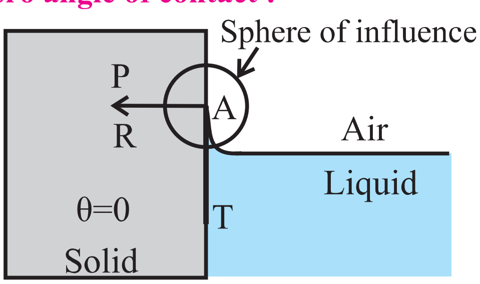


Fig. 2.19 (c): Angle of contact equal to zero.

Figure 2.19 (c) shows the angle of contact between a liquid (e.g. highly pure water) which completely wets a solid (e.g. clean glass) surface. The angle of contact in this case is almost zero (i.e., $\theta \to 0^0$). In this case, the liquid molecules near the contact region, are so less in number that the cohesive force is negligible, i.e., $\overrightarrow{AC} = 0$ and the net adhesive force itself is the resultant force, i.e., $\overrightarrow{AP} = \overrightarrow{AR}$. Therefore, the tangent AT is along the wall within the liquid and the angle of contact is zero.

iv) Angle of contact 90^0 and conditions for convexity and concavity:

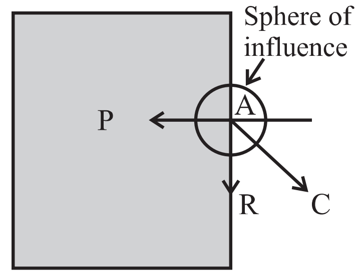


Fig. 2.19 (d): Acute angle equal to $90^{0.}$

Consider a hypothetical liquid having angle of contact 90^0 with a given solid container, as shown in the Fig. 2.19 (d). In this case, the net cohesive force $\overrightarrow{AC}$ is exactly at 45^0 with either of the surfaces and the resultant force $\overrightarrow{AR}$ is exactly vertical (along the solid surface).

For this to occur, $\overrightarrow{AP} = \frac{AC}{\sqrt{2}}$ where, AR is the magnitude of the net force. From this we can write the conditions for acute and obtuse angles of contact:

For acute angle of contact, $\overrightarrow{AP} > \frac{AC}{\sqrt{2}}$, and for obtuse angle of contact, $\overrightarrow{AP} < \frac{AC}{\sqrt{2}}$.

Can you tell?

How does a water proofing agent work?

b) Shape of liquid drops on a solid surface:

When a small amount of a liquid is dropped on a plane solid surface, the liquid will either spread on the surface or will form droplets on the surface. Which phenomenon will occur depends on the surface tension of the liquid and the angle of contact between the liquid and the solid surface. The surface tension between the liquid and air as well as that between solid and air will also have to be taken in to account.

Let θ be the angle of contact for the given solid-liquid pair.

T_1 = Force due to surface tension at the liquid-solid interface,

T_2 = Force due to surface tension at the air-solid interface,

T_3 = Force due to surface tension at the air-liquid interface.

As the force due to surface tension is tangential to the surfaces in contact, directions of T_1, T_2 and T_3 are as shown in the Fig. 2.20. For equilibrium of the drop,

$$T_2 = T_1 + T_3 cos\theta \text{ , } cos\theta = \frac{T_2 - T_1}{T_3} \quad \text{--- (2.18)}$$

From this equation we get the following cases:

1) If $T_2 > T_1$ and $(T_2 - T_1) < T_3$, $\cos\theta$ is positive and the angle of contact θ is acute as shown in Fig. 2.20 (a).

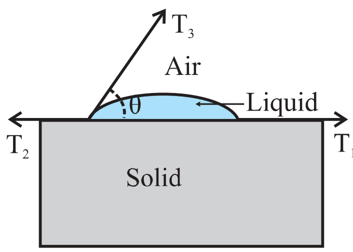


Fig. 2.20 (a): Acute angle.

2) If $T_2 < T_1$ and $(T_1 - T_2) < T_3$, $\cos\theta$ is negative, and the angle of contact θ is obtuse as shown in Fig. 2.20(b).

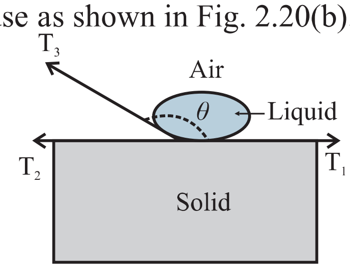


Fig. 2.20 (b): Obtuse angle.

3) If $(T_2 - T_1) = T_3$, $\cos\theta = 1$ and θ is nearly equal to zero.
4) If $(T_2 - T_1) > T_3$ or $T_2 > (T_1 + T_3)$, $\cos\theta > 1$ which is impossible. The liquid spreads over the solid surface and drop will not be formed.

c) Factors affecting the angle of contact:

The value of the angle of contact depends on the following factors,

i) The nature of the liquid and the solid in contact.
ii) Impurity : Impurities present in the liquid change the angle of contact.
iii) Temperature of the liquid : Any increase in the temperature of a liquid decreases its angle of contact. *For a given solid-liquid surface, the angle of contact is constant at a given temperature.*

Table 2.2 – Angle of contact for pair of liquid - solid in contact.

Sr. No.	Liquid - solid in contact	Angle of contact
1	Pure water and clean glass	0°
2	Chloroform with clean glass	0^0
3	Organic liquids with clean glass	0^0
4	Ether with clean glass	16^0
5	Kerosene with clean glass	26^0
6	Water with paraffin	107^0
7	Mercury with clean glass	140^0

2.4.4 Effect of impurity and temperature on surface tension:

a) Effect of impurities:

i) When soluble substance such as common salt (i.e., sodium chloride) is dissolved in water, the surface tension of water increases.
ii) When a sparingly soluble substance such as phenol or a detergent is mixed with water, surface tension of water decreases. For example, a detergent powder is mixed with water to wash clothes. Due to this, the surface tension of water decreases and water makes good contact with the fabric and is able to remove tough stains.
iii) When insoluble impurity is added into water, surface tension of water decreases. When impurity gets added to any liquid, the cohesive force of that liquid decreases which affects the angle of contact and hence the shape of the meniscus. If mercury gathers dust then its surface tension is reduced. It does not form spherical droplets unless the dust is completely removed.

b) Effect of temperature: In most liquids, as temperature increases surface tension decreases. For example, it is suggested that new cotton fabric should be washed in cold water. In this case, water does not make good contact with the fabric due to its higher surface tension. The fabric does not lose its colour because of this.

Hot water is used to remove tough stains on fabric because of its lower surface tension.

In the case of molten copper or molten cadmium, the surface tension increases with increase in its temperature.

The surface tension of a liquid becomes zero at critical temperature.

2.4.5 Excess pressure across the free surface of a liquid:

Every molecule on a liquid surface experiences forces due to surface tension which are tangential to the liquid surface at rest. The direction of the resultant force of surface tension acting on a molecule on the liquid surface depends upon the shape of that liquid surface. This force also contributes in deciding the pressure at a point just below the surface of a liquid.

Figures 2.21 (a), (b) and (c) show surfaces of three liquids with different shapes and their menisci. Let $\vec{f}_A$ be the downward force due to the atmospheric pressure. All the three figures show two molecules A and B. The molecule A is just above, and the molecule B is just below it (inside the liquid). Level difference between A and B is almost zero, so that it does not contribute anything to the pressure difference. In all the three figures, the pressure at the point A is the atmospheric pressure *p*.

a) Plane liquid surface:

Figure 2.21 (a) shows planar free surface of the liquid. In this case, the resultant force due to surface tension, $\vec{f}_T$ on the molecule at B is zero. The force $\vec{f}_A$ itself decides the pressure and the pressure at A and B is the same.

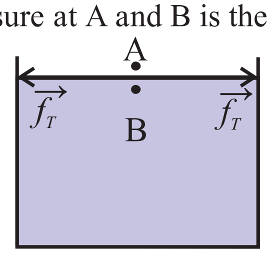


Fig. 2.21 (a): Plane surface.

b) Convex liquid surface:

Surface of the liquid in the Fig.2.21 (b) is upper convex. (Convex, when seen from above). In this case, the resultant force due to surface tension, $\vec{f}_T$ on the molecule at B is vertically downwards and adds up to the

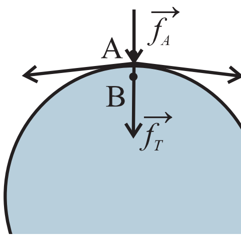


Fig. 2.21 (b) : Convex surface.

downward force $\vec{f}_A$. This develops greater pressure at point B, which is inside the liquid and on the concave side of the meniscus. Thus, the pressure on the concave side i.e., inside the liquid is greater than that on the convex side i.e., outside the liquid.

c) Concave liquid surface:

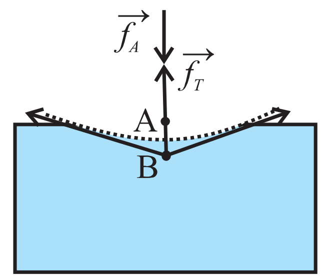


Fig. 2.21 (c): Concave Surface.

Surface of the liquid in the Fig. 2.21 (c) is upper concave (concave, when seen from above). In this case, the force due to surface tension $\vec{f}_T$, on the molecule at B is vertically upwards. The force $\vec{f}_A$ due to atmospheric pressure acts downwords. Forces $\vec{f}_A$ and $\vec{f}_T$ thus, act in opposite direction. Therefore, the net downward force responsible for the pressure at B is less than $\vec{f}_A$. This develops a lesser pressure at point B, which is inside the liquid and on the convex side of the meniscus. Thus, the pressure on the concave side i.e., outside the liquid, is greater than that on the convex side, i.e., inside the liquid.

2.4.6 Explanation of formation of drops and bubbles:

Liquid drops and small bubbles are spherical in shape because the forces of surface tension dominate the gravitational force. These force always try to minimize the surface area of the liquid. A bubble or drop does not collapse because the resultant of the external pressure and the force of surface tension is smaller than the pressure inside a bubble or inside a liquid drop.

Consider a spherical drop as shown in Fig. 2.22. Let p_i be the pressure inside the drop and p_0 be the pressure out side it. As the drop is spherical in shape, the pressure, p_i, inside the drop is greater than p_0, the pressure outside. Therefore, the excess pressure inside the drop is p_i- p_0.

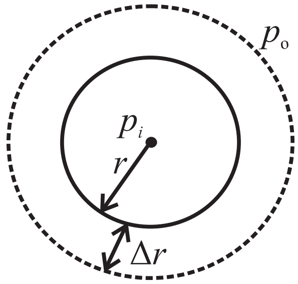


Fig. 2.22. Excess pressure inside a liquid drop.

Let the radius of the drop increase from r to $r + \Delta r$, where Δr is very small, so that the pressure inside the drop remains almost constant.

Let the initial surface area of the drop be $A_1 = 4\pi r^2$, and the final surface area of the drop be $A_2 = 4\pi\,(r+\Delta r)^2$.

$\therefore A_2 = 4\pi(r^2 + 2r\Delta r + \Delta r^2)$

$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r + 4\pi\Delta r^2$

As Δr is very small, Δr^2 can be neglected,

$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r$

Thus, increase in the surface area of the drop is

$$dA = A_2 - A_1 = 8\pi r\Delta r \qquad \text{--- (2.19)}$$

Work done in increasing the surface area by dA is stored as excess surface energy.

$$\therefore dW = TdA = T\,(8\pi r\Delta r) \qquad \text{--- (2.20)}$$

This work done is also equal to the product of the force F which causes increase in the area of the bubble and the displacement Δr which is the increase in the radius of the bubble.

$$\therefore dW = F\Delta r \qquad \text{--- (2.21)}$$

The excess force is given by,

(Excess pressure) $\times$(Surface area)

$$\therefore F = (p_i - p_0)\,4\pi r^2 \qquad \text{--- (2.22)}$$

Equating Eq. (2.20) and Eq. (2.21), we get,

$$T(8\pi r\Delta r) = (p_i - p_0)\,4\pi r^2\Delta r$$

$$\therefore (p_i - p_0) = \frac{2T}{r} \qquad \text{--- (2.23)}$$

This equation gives the excess pressure inside a drop. This is called Laplace's law of a spherical membrane.

In case of a soap bubble there are two free surfaces in contact with air, the inner surface and the outer surface. For a bubble, Eq. (2.19) charges to $dA = 2(8\pi r\Delta r)$. Hence, total increase in the surface area of a soap bubble, while increasing its radius by Δr, is $2(8\pi r\Delta r)$

The work done by this excess pressure is

$$dW = (p_i - p_0)\,4\pi r^2\Delta r = T(16\pi r\Delta r)$$

$$\therefore\ (p_i - p_0) = \frac{4T}{r} \qquad \text{--- (2.24)}$$

Remember this

The gravitational force acting on a molecule, which is its weight, is also one of the forces acting within the sphere of influence near the contact region. However, within the sphere of influence, the cohesive and adhesive forces are so strong that the gravitational force can be neglected in the above explanation.

Brain teaser:

1. Can you suggest any method to measure the surface tension of a soap solution? Will this method have any commercial application?
2. What happens to surface tension under different gravity (e.g. Space station or lunar surface)?

Example 2.6: What should be the diameter of a water drop so that the excess pressure inside it is 80 N/m^2? (Surface tension of water = 7.27×10^{-2} N/m)

Solution: Given

$p_i - p_o = 80$ N/m^2

T = 7.27×10^{-2} N/m

We have,

$$(p_i - p_o) = \frac{2T}{r}$$

$$\therefore\ r = \frac{2T}{p_i - p_o} = \frac{2\times 7.27\times 10^{-2}}{80} = 1.8\times 10^{-3}\ \text{m}$$

$\therefore d = 2r = 3.6$ mm

2.4.7 Capillary Action:

A tube having a very fine bore ($\sim$ 1 mm) and open at both ends is called a capillary tube. If one end of a capillary tube is dipped in a liquid which partially or completely wets the surface of the capillary (like water in glass) the level of liquid in the capillary rises. On the other hand, if the capillary tube is dipped in a liquid which does not wet its surface (like mercury in glass) the level of liquid in the capillary drops.

The phenomenon of rise or fall of a liquid inside a capillary tube when it is dipped in the liquid is called *capillarity*. Capillarity is in action when,

- Oil rises up the wick of a lamp.
- Cloth rag sucks water.
- Water rises up the crevices in rocks.
- Sap and water rise up to the top most leaves in a tree.
- Blotting paper absorbs ink.

When a capillary is dipped in a liquid, two effects can be observed, a) The liquid level can rise in the capillary (water in a glass capillary), or b) The liquid level can fall in the capillary (mercury in glass capillary). Here we discuss a qualitative argument to explain the capillary fall.

a) Capillary fall:

Consider a capillary tube dipped in a liquid which does not wet the surface, for example, in mercury. The shape of mercury meniscus in the capillary is upper convex. Consider the points A, B, C, and D such that, (see Fig. 2.23 (a)).

i) Point A is just above the convex surface and inside the capillary.
ii) Point B is just below the convex surface inside the capillary.
iii) Point C is just above the plane surface outside the capillary.
iv) Point D is just below the plane surface and outside the capillary, and below the point C.

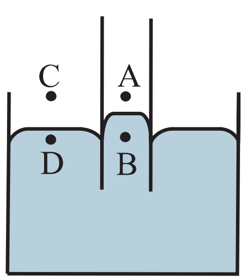


Fig. 2.23 (a) : Capillary in mercury before drop in level.

Let p_A, p_B, p_C, and p_D be the values of the pressures at the points A, B, C, and D respectively. As discussed previously, the pressure on the concave side is always greater than that on the convex side.

$$\therefore p_B > p_A$$

As the points A and C are at the same level, the pressure at both these points is the same, and it is the atmospheric pressure.

$$\therefore p_A = p_C \qquad \text{--- (2.25)}$$

Between the points C and D, the surface is plane.

$$\therefore p_C = p_D = p_A \qquad \text{--- (2.26)}$$

$\therefore p_B > p_D$. But the points B and D are at the same horizontal level. Thus, in order to maintain the same pressure, the mercury in the capillary rushes out of the capillary. Because of this, there is a drop in the level of mercury inside the capillary as shown in Fig. 2.23 (b).

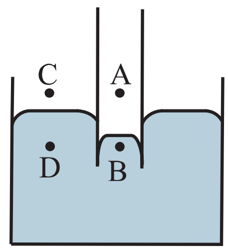


Fig. 2.23 (b): Capillary in mercury, drop in level.

b) Capillary rise:

Refer to Fig. 2.24 (a) and Fig. 2.24 (b) and explain the rise of a liquid inside a capillary.

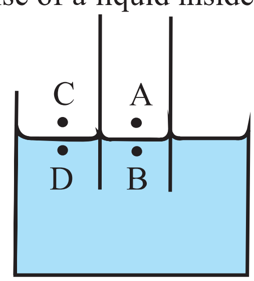


Fig. 2.24 (a): Capillary just immersed in water.

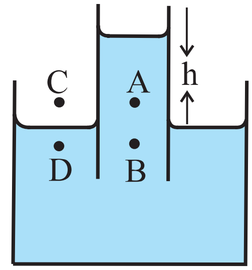


Fig. 2.24 (b): Capillary in water after rise in level.

Expression for capillary rise or fall:

Method (I): Using pressure difference

The pressure due to the liquid (water) column of height h must be equal to the pressure difference $2T/R$ due to the concavity.

$$\therefore h\rho g = \frac{2T}{R} \qquad \text{--- (2.27)}$$

where, ρ is the density of the liquid and g is acceleration due to gravity.

Let r be the radius of the capillary tube and θ be the angle of contact of the liquid as shown in Fig. 2.25 (a).

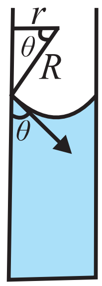


Fig. 2.25 (a): Forces acting at the point of contact.

Then radius of curvature R of the meniscus is given by $R = \dfrac{r}{\cos\theta}$

$$\therefore h\rho g = \frac{2T\cos\theta}{r}$$

$$\therefore h = \frac{2T\cos\theta}{r\rho g} \qquad \text{--- (2.28)}$$

The above equation gives the expression for capillary rise (or fall) for a liquid. Narrower the tube, the greater is the height to which the liquid rises (or falls).

If the capillary tube is held vertical in a liquid that has a convex meniscus, then the angle of contact θ is obtuse. Therefore, $\cos\theta$ is negative and so is h. This means that the liquid will suffer capillary fall or depression.

b) (Method II): Using forces:

Rise of water inside a capilary is against gravity. Hence, weight of the liquid column must be equal and opposite to the proper component of force due to surface tension at the point of contact.

The length of liquid in contact inside the

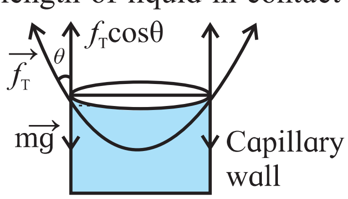


Fig 2.25 (b): Forces acting on liquid inside a capillary.

capillary is the circumference $2\pi r$. Thus, the force due to surface tension is given by,

f_T = (surface tension) × (length in contact)

$= T \times 2\pi r$

Direction of this force is along the tangent, as shown in the Fig. 2.25 (b).

Vertical component of this force is

$$\left(f_T\right)_v = T \times 2\pi r \times \cos\theta \qquad \text{--- (2.29)}$$

Ignoring the liquid in the concave meniscus, the volume of the liquid in the capillary rise is $V = \pi r^2 h$.

∴ Mass of the liquid in the capillary rise,

$m = \pi r^2 h\rho$

∴ Weight of the liquid in the capillary (rise or fall), $w = \pi r^2 h\rho g$ --- (2.30)

This must be equal and opposite to the vertical component of the force due to surface tension. Thus, equating right sides of equations (2.29) and (2.30), we get,

$$\pi r^2 h\rho g = T \times 2\pi r \times \cos\theta$$

$$\therefore h = \frac{2T\cos\theta}{r\rho g}$$

In terms of capillary rise, the expression for surface tension is,

$$T = \frac{rh\rho g}{2\cos\theta} \qquad \text{--- (2.31)}$$

The same expression is also valid for capillary fall discussed earlier.

Example 2.7: A capillary tube of radius 5×10^{-4} m is immersed in a beaker filled with mercury. The mercury level inside the tube is found to be 8×10^{-3} m below the level of reservoir. Determine the angle of contact between mercury and glass. Surface tension of mercury is 0.465 N/m and its density is 13.6×10^3 kg/m^3. (g = 9.8 m/s^2)

Solution: Given,

$r = 5 \times 10^{-4}$ m

$h = -8 \times 10^{-3}$ m

$T = 0.465$ N/m

$g = 9.8$ m/s^2

$\rho = 13.6 \times 10^3$ kg/m^3

we have,

$$T = \frac{hr\rho g}{2\cos\theta}$$

$$\therefore 0.465 = \frac{-8\times10^{-3}\times5\times10^{-4}\times13.6\times10^{3}\times9.8}{2\cos\theta}$$

$$\therefore \cos\theta = \frac{-40\times9.8\times13.6\times10^{-4}}{2\times0.465}$$

$$\therefore -\cos\theta = 0.5732$$

$$\therefore \cos(\pi-\theta) = 0.5732$$

$$\therefore 180^0 - \theta = 55^0\,2'$$

$$\therefore \theta = 124^0\,58'$$

Do you know?

Einstein's first ever published scientific article deals with capillary action? Published in German in 1901, it was entitled *Folegerungen* aus den *capillaritatserscheinungen* (conclusions drawn from the phenomena of capillarity).

2.5 Fluids in Motion:

We come across moving fluids in our day to day life. The flow of water through our taps, the flow of cooking gas through tubes, or the flow of water through a river or a canal can be understood using the concepts developed in this section.

The branch of Physics which deals with the study of properties of fluids in motion is called *hydrodynamics*. As the study of motion of real fluid is very complicated, we shall limit our study to the motion of an ideal fluid. We have discussed an ideal fluid in the beginning of this Chapter. Study of a fluid in motion is very important.

Consider Fig. 2.26 which shows a pipe whose direction and cross sectional area change arbitrarily. The direction of flow of the fluid in pipe is as shown. We assume an ideal fluid to flow through the pipe. We define a few terms used to describe flow of a fluid.

Steady flow: Measurable property, such as pressure or velocity of the fluid at a given point is constant over time.

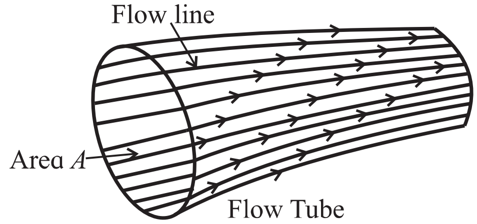


Fig. 2.26: Flow lines and flow tube.

Flow line: It is the path of an individual particle in a moving fluid as shown in Fig. 2.26.

Streamline: It is a curve whose tangent at any point in the flow is in the direction of the velocity of the flow at that point. *Streamlines and flow lines are identical for a steady flow.*

Flow tube: It is an imaginary bundle of flow lines bound by an imaginary wall. For a steady flow, the fluid cannot cross the walls of a flow tube. *Fluids in adjacent flow tubes cannot mix*.

Laminar flow/Streamline flow: It is a steady flow in which adjacent layers of a fluid move smoothly over each other as shown in Fig. 2.27 (a). A steady flow of river can be assumed to be a laminar flow.

Turbulent flow: It is a flow at a very high flow rate so that there is no steady flow and the flow pattern changes continuously as shown in Fig. 2.27 (b). A flooded river flow or a tap running very fast is a turbulent flow.

Table 2.3 Streamline Flow and Turbulent Flow

Streamline flow	Turbulent flow
1) The smooth flow of a fluid, with velocity smaller than certain critical velocity (limiting value of velocity) is called streamline flow or laminar flow of a fluid.	1) The irregular and unsteady flow of a fluid when its velocity increases beyond critical velocity is called turbulent flow.
2) In a streamline flow, velocity of a fluid at a given point is always constant.	2) In a turbulent flow, the velocity of a fluid at any point does not remain constant.
3) Two streamlines can never intersect, i.e., they are always parallel.	3) In a turbulent flow, at some points, the fluid may have rotational motion which gives rise to eddies.
4) Streamline flow over a plane surface can be assumed to be divided into a number of plane layers. In a flow of liquid through a pipe of uniform cross sectional area, all the streamlines will be parallel to the axis of the tube.	4) A flow tube loses its order and particles move in random direction.

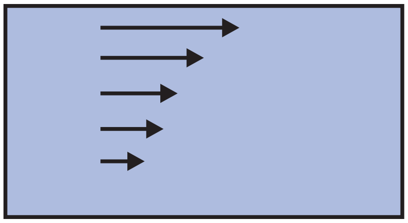


Fig. 2.27 (a): Streamline flow.

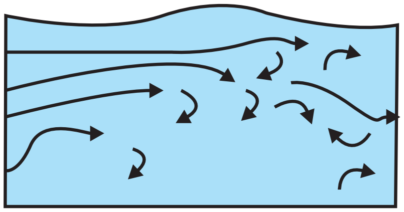


Fig. 2.27 (b): Turbulent flow.

Can you tell?

What would happen if two streamlines intersect?

Activity

Identify some examples of streamline flow and turbulent flow in every day life. How would you explain them? When would your prefer a stream line flow?

2.6 Critical Velocity and Reynolds number:

The flow of a fluid, whether streamline or turbulent, is differentiated on the basis of velocity of the flow. The velocity beyond which a streamline flow becomes turbulent is called *critical velocity*.

According to Osborne Reynolds (1842 - 1912), critical velocity is given by

$$v_c = \frac{R_n \eta}{\rho d}, \quad \text{--- (2.32)}$$

where,

v_c = critical velocity of the fluid
R_n = Reynolds number
η = coefficient of viscosity
ρ = density of fluid
d = diameter of tube

From Eq. (2.32) equation for Reynolds number can be written as,

$$R_n = \frac{v_c \rho d}{\eta} \quad \text{--- (2.33)}$$

Reynolds number is a pure number. It has no unit and dimensions. It is found that for R_n less than 1000, the flow of a fluid is streamline while for R_n greater than 2000, the flow of fluid is turbulent. When R_n is between 1000 and 2000, the flow of fluid becomes unsteady, i.e., it changes from a streamline flow to a turbulent flow.

2.6.1 Viscosity:

When we pour water from a glass, it flows freely and quickly. But when we pour syrup or honey, it flows slowly and sticks to the container. The difference is due to fluid friction. This friction is both within the fluid itself and between the fluid and its surroundings. This property of fluids is called *viscosity*. Water has low viscosity, whereas syrup or honey has high viscosity. Figure 2.28 shows a schematic section of viscous flow and Fig. 2.29 that of a non viscous flow. Note that there is no dragging force in the non-viscous flow, and all layers are moving with the same velocity.

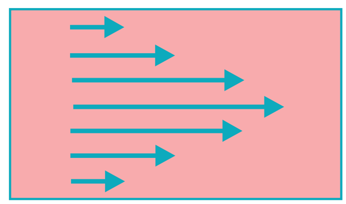


Fig. 2.28: Viscous flow. Different layers flow with different velocities. The central layer flows the fastest and the outermost layers flow the slowest.

Viscosity of such fluid is zero. The only fluid that is almost non-viscous is liquid helium at about 2K. In this section, we will study viscosity of a fluid and how it affects the flow of a fluid.

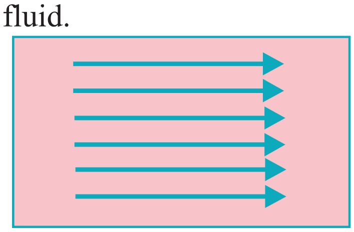


Fig. 2.29: Non-viscous flow. Different layers flow with the same velocity.

If we observe the flow of river water, it is found that the water near both sides of the river bank flows slow and as we move towards the center of the river, the water flows faster gradually. At the centre, the flow is the fastest. From this observation it is clear that there is some opposing force between two adjacent layers of fluids which affects their relative motion.

Viscosity is that property of fluid, by virtue of which, the relative motion between different layers of a fluid experience a dragging force. This force is called the viscous drag. This is shown schematically in Fig. 2.30.

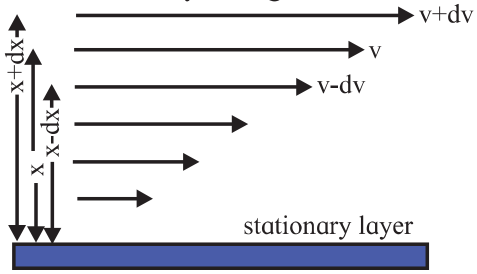


Fig. 2.30: Change in velocity of layer as its distance from a referee layer changes.

In liquids, the viscous drag is due to short range molecular cohesive forces, and in gases it is due to collisions between fast moving molecules. In both liquids and gases, as long as the relative velocity between the layers is small, the viscous drag is proportional to the relative velocity. However, in a turbulent flow, the viscous drag increases rapidly and is not proportional to relative velocity but proportional to higher powers of relative velocity.

Velocity gradient: **The rate of change of velocity (dv) with distance (dx) measured from a stationary layer is called velocity gradient (dv$/dx$).**

2.6.2 Coefficient of viscosity:

According to Newton's law of viscosity, for a streamline flow, viscous force (f) acting on any layer is directly proportional to the area (A) of the layer and the velocity gradient (dv$/dx$) i.e.,

$$f \propto A\left(\frac{dv}{dx}\right)$$

$$\therefore f = \eta A\left(\frac{dv}{dx}\right) \qquad \text{--- (2.34)}$$

where η is a constant, called coefficient of viscosity of the liquid. From Eq. (2.34) we can write,

$$\eta = \frac{f}{A\left(\frac{dv}{dx}\right)} \qquad \text{--- (2.35)}$$

Note: *'A' in this expression is not the cross sectional area, it is the area of the layer, parallel to the direction of the flow.*

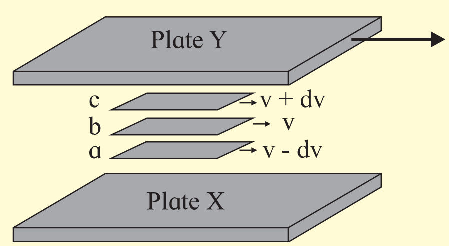
The **coefficient of viscosity can be defined as the viscous force per unit area per unit velocity gradient**. S.I. unit of viscosity is Ns/m^2.

Use your brain power

CGS unit of viscosity is Poise. Find the relation between Poise and the SI unit of viscosity.

A Microsopic View of Viscosity:

Viscosity of a fluid can be explained on the basic of molecular motion as follow. Consider the laminar flow between plats X and Y as shown in the figure. Plate X is stationary and plate Y moves with a velocity v_0. Layers a, b, and c move with velocity, v-dv, v, and v + dv respectively. Consider two adjacent layers, b and c. The velocity of the fluid is equal to mean velocity of the molecules contained in that layer. Thus, the mean velocity of the molecules in layer b is v, while the molecules in layers c have a slightly greater mean velocity v + dv. As you will learn in the next chapter, each molecule possesses a random velocity whose magnitude is usually larger than that of the mean velocity. As a result, molecules are continually transferred in large numbers between the two layers. On the average, molecules passing from layer



c to layer b will be moving too fast for their 'new' layer by an amount dv and will slow down as a result of collisions with the molecules in layer b. The result is a transfer of momentum from faster-moving layers c to their neighboring slower-moving layers such as b and thus eventually to plate X. Because the original source of this transfer of momentum is plate Y, the overall result is a transfer of momentum from plate Y to plate X. If there are no external forces applied, this momentum transfer would reduce speed of the plate Y to zero with respect to the plate X.

Reduction in the velocity of the molecules in the direction of laminar flow is due to the fact that their directions after collision are random. This randomness, to be discussed in Chapter 3, results in an increase in the thermal energy of the fluid at the cost of its macrosopic kinetic energy. *That is, the process is dissipative, or frictional*.

In liquids there is an additional, stronger interaction between molecules in adjacent layers, due to the intermolecular forces that distinguish liquid from gases. As a result, there is a transfer of momentum from faster-moving layers to slower-moving layers, which results in a viscous drag.

Remember this

Coefficient of viscosity of a fluid changes with change in its temperature. For most liquids, the coefficient of viscosity decreases with increase in their temperature. It probably depends on the fact that at higher temperatures, the molecules are farther apart and the cohesive forces or inter-molecular forces are, therefore, less effective. Whereas, in gases, the coefficient of viscosity increases with the increase in temperature. This is because, at high temperatures, the molecules move faster and collide more often with each other, giving rise to increased internal friction.

Table 2.4 Coefficient of viscosity at different temperatures.

Fluid	Temperature	Coefficient of Viscosity Ns/m^2
Air	0^0C	0.017×10^{-3}
	40^0C	0.019×10^{-3}
Water	20^0C	1×10^{-3}
	100^0C	0.3×10^{-3}
Machine oil	16^0C	0.113×10^{-3}
	38^0C	0.034×10^{-3}

2.7 Stokes' Law:

In 1845, Sir George Gabriel Stokes (1819-1903) stated the law which gives the viscous force acting on a spherical object falling through a viscous medium (see Fig. 2.31).

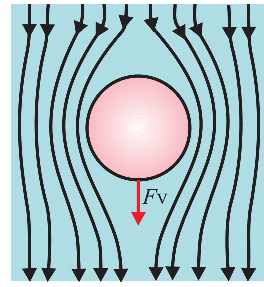


Fig 2.31: Spherical object moving through a viscous medium.

The law states that, **"The viscous force (F_v) acting on a small sphere falling through a viscous medium is directly proportional to the radius of the sphere (*r*), its velocity (v) through the fluid, and the coefficient of viscosity (η) of the fluid"**.

$$\therefore F_v \propto \eta r v$$

The empirically obtained constant of proportionality is 6π .

$$\therefore F_v = 6\pi\eta r v \qquad \text{--- (2.36)}$$

This is the expression for viscous force acting on a spherical object moving through a viscous medium. The above formula can be derived using dimensional analysis.

Example 2.8: A steel ball with radius 0.3 mm is falling with velocity of 2 m/s at a time *t*, through a tube filled with glycerin, having coefficient of viscosity 0.833 Ns/m^2. Determine viscous force acting on the steel ball at that time.

Solution: Given

r = 0.3 mm = 0.3×10^{-3} m, v = 2 m/s, $\eta = 0.833\ Ns/m^2$.

We have, $F = 6\pi\eta r v$

$F = 6 \times 3.142 \times 0.833 \times 0.3 \times 10^{-3} \times 2$

Therefore, $F = 9.422 \times 10^{-3}$ N

2.7.1 Terminal Velocity:

Consider a spherical object falling through a viscous fluid. Forces experienced by it during its downward motion are,

1. Viscous force (F_v), directed upwards. Its magnitude goes on increasing with increase in its velocity.

2. Gravitational force, or its weight (F_g), directed downwards, and
3. Buoyant force or upthrust (F_u), directed upwards.

Net downward force given by $f=F_g-(F_v+F_u)$, is responsible for initial increase in the velocity. Among the given forces, F_g and F_u are constant while F_v increases with increase in velocity. Thus, a stage is reached when the net force f becomes zero. At this stage, $F_g = F_v + F_u$. After that, the downward velocity remains constant. This constant downward velocity is called *terminal velocity*. Obviously, now onwards, the viscous force F_v is also constant. *The entire discussion necessarily applies to streamline flow only*.

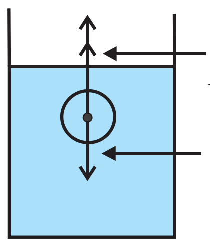


Fig. 2.32: Forces acting on object moving through a viscous medium.

Consider a spherical object falling under gravity through a viscous medium as shown in Fig. 2.32. Let the radius of the sphere be r, its mass m and density ρ. Let the density of the medium be σ and its coefficient of viscosity be η. When the sphere attains the terminal velocity, the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.

Total downward force = Total upward force

weight of sphere (mg) =

viscous force + by out ant to due to the medium

$$\frac{4}{3}\pi r^3\rho g = 6\pi\eta r\text{v} + \frac{4}{3}\pi r^3\sigma g$$

$$6\pi\eta r\text{v} = \left(\frac{4}{3}\pi r^3\rho g\right) - \left(\frac{4}{3}\pi r^3\sigma g\right)$$

$$6\pi\eta r\text{v} = \left(\frac{4}{3}\right)\pi r^3 g\left(\rho-\sigma\right)$$

$$\text{v} = \left(\frac{4}{3}\right)\pi r^3 g\left(\rho-\sigma\right)\times\frac{1}{6\pi\eta r}$$

$$\text{v} = \left(\frac{2}{9}\right)\frac{r^2 g\left(\rho-\sigma\right)}{\eta} \qquad \text{--- (2.37)}$$

This is the expression for the terminal velocity of the sphere. From Eq. (2.37) we can also write,

$$\eta = \frac{2}{9}\frac{r^2\left(\rho-\sigma\right)g}{\text{v}} \qquad \text{--- (2.38)}$$

The above equation gives coefficient of viscosity of a fluid.

Example 2.9: A spherical drop of oil falls at a constant speed of 4 cm/s in steady air. Calculate the radius of the drop. The density of the oil is 0.9 g/cm³, density of air is 1.0 g/cm³ aud the coefficient of viscosity of air is 1.8 × 10⁻⁴ poise, (g = 980 cm/s²)

Solution: Given,

v = 4 cm/s

η = 1.8 × 10^{-4} Poise

σ = 0. 9 g/cm³

ρ = 1 g/cm³

We have,

$$\eta = \frac{2}{9}\frac{r^2\left(\rho-\sigma\right)g}{\text{v}}$$

$$r = \sqrt{\frac{9\eta\text{v}}{2\left(\rho-\sigma\right)g}}$$

$$r = \sqrt{\frac{9\times1.8\times10^{-4}\times4}{2\times\left(1-0.9\right)\times980}}$$

$$r = 0.574 \text{ cm}$$

Remember this

The velocity with which an object can move through a viscous fluid is always less than or equal to the terminal velocity in that fluid for that object.

2.8 Equation of Continuity:

Consider a steady flow of an incompressible fluid as shown in Fig. 2.33. For a steady flow, the velocity of a particle remains constant at a given point but it can vary from point to point. For example, consider section A_1 and A_2 in Fig. 2.33. Section A_1 has larger cross sectional area than the section A_2. Let v_1 and v_2 be the velocities of the fluid at sections A_1 and A_2 respectively.

This is because, a particle has to move faster in the narrower section (where there is

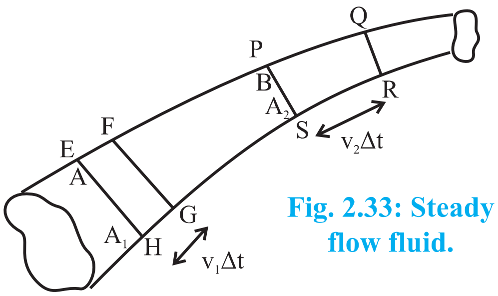


Fig. 2.33: Steady flow fluid.

less space) to accommodate particles behind it hence its velocity increases. When a particle enters a wider section, it slows down because there is more space. *Because the fluid is incompressible, the particles moves faster through a narrow section and slow down while moving through wider section.* If the fluid does not move faster in a narrow regain, it will be compressed to fit into the narrow space.

Consider a tube of flow as shown in Fig. 2.33. All the fluid that passes through a tube of flow must pass through any cross section that cuts the tube of flow. We know that all the fluid is confined to the tube of flow. Fluid can not leave the tube or enter the tube.

Consider section A_1 and A_2 located at points A and B respectively as shown in Fig. 2.33. Matter is neither created nor destroyed within the tube enclosed between section A_1 and A_2. Therefore, the mass of the fluid within this region is constant over time. That means, if mass *m* of the fluid enters the section A_1 then equal mas of fluid should leave the section A_2.

Let the speed of the fluid which crosses the section EFGH at point A in time interval Δt be v_1. Thus, the volume of the fluid entering the tube through the cross section at point A is $\rho A_1 v_1 \Delta t$. Similarly, let the speed of the fluid be v_2 at point B. The fluid crosses the section PQRS of area A_2 in time interval Δt. Thus, the mass of the fluid leaving the tube through the cross section at B is $\rho A_2 v_2 \Delta t$.

As fluid is incompressible, the mass of the fluid entering the tube at point A is the same as the mass leaving the tube at B.

Mass of the fluid in section EFGH = mass of fluid in section PQRS

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t \qquad \text{--- (2.39)}$$

$$A_1 v_1 = A_2 v_2 \text{ or, } Av = \text{constant} \qquad \text{--- (2.40)}$$

Av is the volume rate of flow of a fluid, i.e.,

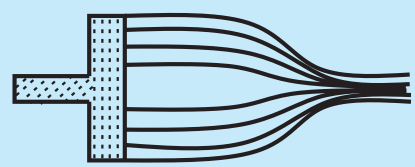
$Av = \frac{dV}{dt}$. The quantity $\frac{dV}{dt}$ is the volume of a fluid per unit time passing through any cross section of the tube of flow. *It is called the volume flux. Similarly, $\rho dV/dt = dm/dt$ is called mass flux.*

Equation (2.40) is called the equation of continuity in fluid dynamics. The continuity equation says that the volume rate of flow of an incompressible fluid for a steady flow is the same throughout the flow.

Do you know?

When water is released from a dam, the amount of water is mentioned in terms of Thousand Million Cubic feet (TMC). One TMC is 10^9 cubic feet of water per second. Basic unit of measuring flow is cusec. One cusec is one cubic feet per sec (28.317 lit per sec).

Example 2.11: As shown in the given figure, a piston of cross sectional area 2 cm^2 pushes the liquid out of a tube whose area at the outlet is 40 mm^2. The piston is pushed at a rate of 2 cm/s. Determine the speed at which the fluid leaves the tube.



Solution: Given,

$A_1 = 2\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$

$v_1 = 2\ \text{cm/s} = 2 \times 10^{-2}\ \text{m/s}$

$A_2 = 40\ \text{mm}^2 = 40 \times 10^{-6}\ \text{m}^2$

From equation of continuity, $A_1 v_1 = A_2 v_2$

Therefore,

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{2\times10^{-4}\times2\times10^{-2}}{40\times10^{-6}} = 0.1\ \text{m/s}$$

Use your brain power

A water pipe with a diameter of 5.0 cm is connected to another pipe of diameter 2.5 cm. How would the speeds of the water flow compare?

Do you know?

1. How does an aeroplane take off?
2. Why do racer cars and birds have typical shape?
3. Have you experienced a sideways jerk while driving a two wheller when a heavy vehicle overtakes you?
4. Why does dust get deposited only on one side of the blades of a fan?
5. Why helmets have specific shape?

2.9 Bernoulli Equation:

On observing a river, we notice that the speed of the water decreases in wider region whereas the speed of water increases in the regions where the river is narrow. From this we might think that the pressure in narrower regions is more than that in the wider region. However, the pressure within the fluid in the narrower parts is less while that in wider parts is more.

Swiss scientist Daniel Bernoulli (1700-1782), while experimenting with fluid inside pipes led to the discovery of the concept mentioned above. He observed, in his experiment, that the speed of a fluid in a narrow region increases but the internal pressure of a fluid in the same narrow region decreases. This phenomenon is called *Bernoulli's principle*.

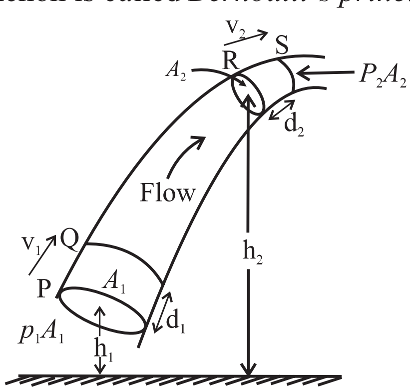


Fig. 2.34: Flow of fluid through a tube of varying cross section and height.

Bernoulli's equation relates the speed of a fluid at a point, the pressure at that point and the height of that point above a reference level. It is an application of work – energy theorem for a fluid in flow. While deriving Bernoulli's equation, we will prove that the net work done on a fluid element by the pressure of the surrounding fluid is equal to the sum of the change in the kinetic energy and the change in the gravitational potential energy.

Figure 2.34 shows flow of an ideal fluid through a tube of varying cross section and height. Consider an element of fluid that lies between cross sections P and R.

Let,

- v_1 and v_2 be the speed the fluid at the lower end P and the upper end R respectively.
- A_1 and A_2 be the cross section area of the fluid at the lower end P and the upper end R respectively.
- p_1 and p_2 be the pressures of the fluid at the lower end P and the upper R respectively.
- d_1 and d_2 be the distances travelled by the fluid at the lower and P and the upper and R during the time interval dt with velocities v_1 and v_2 respectively.
- p_1A_1 and p_2A_2 be the forces acting on the equation of continuity, (Eq. 2.40), the volume dV of the fluid passing through any cross section during time interval dt is the same; i.e.,

$$dV = A_1d_1 = A_2d_2 \qquad \text{--- (2.41)}$$

There is no internal friction in the fluid as the fluid is ideal. In practice also, for a fluid like water, the loss in energy due to viscous force is negligible. So the only *non-gravitational* force that does work on the fluid element is due to the pressure of the surrounding fluid. Therefore, the net work, W, done on the element by the surrounding fluid during the flow from P to R is,

$$W = p_1A_1d_1 - p_2A_2d_2$$

The second term in the above equation has a negative sign because the force at R opposes the displacement of the fluid. From Eq. (2.41) the above equation can be written as,

$$W = p_1dV - p_2dV$$

$$\therefore\ W = (p_1 - p_2)\, dV \qquad \text{--- (2.42)}$$

As the work W is due to forces other than the conservative force of gravity, it equals the change in the total mechanical energy i.e., kinetic energy plus gravitational potential energy associated with the fluid element.

i.e., $W = \Delta K.E. + \Delta P.E.$ --- (2.43)

The mechanical energy for the fluid between sections Q and R does not change.

At the beginning of the time interval dt, the mass and the kinetic energy of the fluid between P and Q is, $\rho A_1 d_1$, and $\frac{1}{2}\rho\left(A_1 d_1\right)v_1^2$ respectively. At the end of the time interval dt, the kinetic energy of the fluid between section R and S is $\frac{1}{2}\rho\left(A_2 d_2\right)v_2^2$. Therefore, the net change in the kinetic energy, $\Delta K.E.$, during time interval dt is,

$$\Delta K.E. = \frac{1}{2}\rho\left(A_2 d_2\right)v_2^2 - \frac{1}{2}\rho\left(A_1 d_1\right)v_1^2$$

$$\Delta K.E. = \frac{1}{2}\rho dV v_2^2 - \frac{1}{2}\rho dV v_1^2$$

$$\Delta K.E. = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) \quad \text{--- (2.44)}$$

Also, at the beginning of the time interval dt, the gravitational potential energy of the mass m between P and Q is $mgh_1 = \rho dVgh_1$. At the end of the interval dt, the gravitational potential energy of the mass m between R and S is $mgh_2 = \rho dVgh_2$. Therefore, the net change in the gravitational potential energy, $\Delta P.E.$, during time interval dt is,

$$\Delta P.E. = \rho dVgh_2 - \rho dVgh_1$$

$$\Delta P.E. = \rho dVg\,(h_2 - h_1) \quad \text{--- (2.45)}$$

Substituting Eq. (2.42), (2.44) and (2.45) in Eq. (2.43) we get,

$$\left(p_1 - p_2\right)dV = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) + \rho dVg\left(h_2 - h_1\right)$$

$$\therefore \left(p_1 - p_2\right) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(h_2 - h_1\right) \quad \text{--- (2.46)}$$

This is Bernoulli's equation. It states that **the work done per unit volume of a fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow.** Equation (2.46) can also be written as,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \text{---(2.47)}$$

or, $$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \quad \text{--- (2.48)}$$

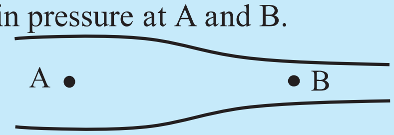
A different way of interpreting the Bernoulli's equation:

$$\left(p_1 - p_2\right) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(h_2 - h_1\right)$$

Dimensionally, pressure is energy per unit volume. Both terms on the right side of the above equation have dimensions of energy per unit volume. Hence, quite often, the left side is referred to as pressure energy per unit volume. The left side of equation is called pressure head. The first term on the right side is called the velocity head and the second term is called the potential head.

In other words, the Bernoulli's principle is thus consistent with the principle of conservation of energy.

Example 2.12: The given figure shows a streamline flow of a non-viscous liquid having density 1000 kg/m^3. The cross sectional area at point A is 2 cm^2 and at point B is 1 cm^2. The speed of liquid at the point A is 5 cm/s. Both points A and B are at the same horizontal level. Calculate the difference in pressure at A and B.



Solution: Given,

$\rho = 1000$ kg/m^3, $A_1 = 2$ cm$^2 = 2 \times 10^{-2}$ m^2

$A_2 = 1$ cm$^2 = 10^{-2}$ m^2, $v_1 = 5$ cm/s $= 5 \times 10^{-2}$ m/s and $h_1 = h_2$

From the equation of continuity,

$A_1 v_1 = A_2 v_2$

$$\therefore v_2 = \frac{A_1 v_1}{A_2} = \frac{2\times5\times10^{-2}}{10^{-2}} = 10 \text{ m/s}$$

By Bernoulli's equation,

$$\left(p_1 - p_2\right)dV = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) + \rho dVg\left(h_2 - h_1\right)$$

(since, $h_2 - h_1 = 0$)

$$\left(p_1 - p_2\right)dV = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right)$$

$$= \frac{1}{2}\times1000\times\left(100 - 0.0025\right)$$

$= 500 \times 99.99$

$p_1 - p_2 = 49998.75$, Pa $= 4.99 \times 10^5$ Pa

Use your brain power

Does the Bernoulli's equation change when the fluid is at rest? How?

Applications of Bernoulli's equation:

a) Speed of efflux:

The word efflux means fluid out flow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body.

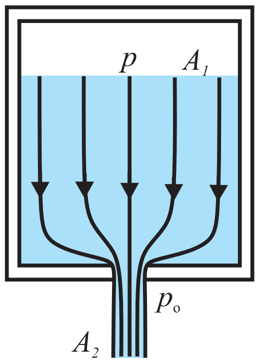


Fig. 2.35: Efflux of fluid from an orifice.

Consider a liquid of density 'ρ' filled in a tank of large cross-sectional area A_1 having an orifice of cross-sectional area A_2 at the bottom as shown in Fig. 2.35. Let $A_2 << A_1$. The liquid flows out of the tank through the orifice. Let v_1 and v_2 be the speeds of the liquid at A_1 and A_2 respectively. As both, inlet and outlet, are exposed to the atmosphere, the pressure at these position equals the atmosphere pressure p_0. If the height of the free surface above the orifice is h, Bernoulli's equation gives us,

$$p_0 + \frac{1}{2}\rho v_1^2 + \rho g h = p_0 + \frac{1}{2}\rho v_2^2 \qquad \text{--- (2.49)}$$

Using equation the of continuity we can write,

$$v_1 = \frac{A_2}{A_1} v_2$$

Substituting v_1 in Eq.(2.49) we get,

$$\frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 + \rho g h = \frac{1}{2}\rho v_2^2$$

$$\left(\frac{A_2}{A_1}\right)^2 v_2^2 + 2gh = v_2^2$$

$$2gh = v_2^2 - \left(\frac{A_2}{A_1}\right)^2 v_2^2$$

$$\therefore \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] v_2^2 = 2gh$$

If $A_2 << A_1$, the above equation reduces to,

$$v_2 = \sqrt{2gh} \qquad \text{--- (2.50)}$$

This is the equation of the speed of a liquid flowing out through an orifice at a depth 'h' below the free surface. It is the same as that of a particle falling freely through the height 'h' under gravity.

Example 2.13: Doors of a dam are 20 m below the surface of water in the dam. If one door is opened, what will be the speed of the water that flows out of the door? (g = 9.8 m/s^2)

Solution: Given, h = 20 m

From Toricelli's law,

$$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 20} = \sqrt{392}$$

$$= 19.79 \text{ m/s}$$

b) Ventury tube:

A ventury tube is used to measure the speed of flow of a fluid in a tube. It has a constriction in the tube. As the fluid passes through the constriction, its speed increases in accordance with the equation of continuity. The pressure thus decreases as required by the Bernoulli equation.

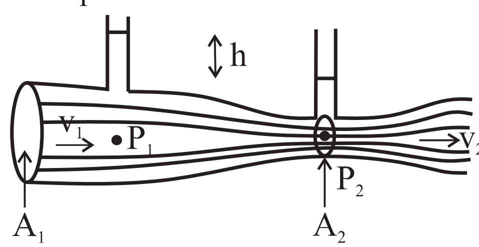


Fig. 2.36: Ventury tube.

The fluid of density ρ flows through the Ventury tube. The area of cross section is A_1 at wider part and A_2 at the constriction. Let the speeds of the fluid at A_1 and A_2 be v_1 and v_2, and the pressures, be p_1 and p_2 respectively. From Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$(p_1 - p_2) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \text{--- (2.51)}$$

Figure 2.36 shows two vertical tubes connected to the Ventury tube at A_1 and A_2. If the difference in height of the liquid levels in the tubes is h, we have,

$$(p_1 - p_2) = \rho g h)$$

Substituting above equation in Eq. (2.51) we get,

$$2gh = v_2^2 - v_1^2 \qquad \text{--- (2.52)}$$

From the equation of continuity, $A_1 v_1 = A_2 v_2$, substituting v_1 in terms of v_2 or vice versa in Eq. (2.52) the rate of flow of liquid passing through a cross section can be calculated by knowing the areas A_1 and A_2.

Example 2.13: Water flows through a tube as shown in the given figure. Find the difference in mercury level, if the speed of flow of water at point A is 2 m/s and at point B is 5 m/s. (g = 9.8 m/s)

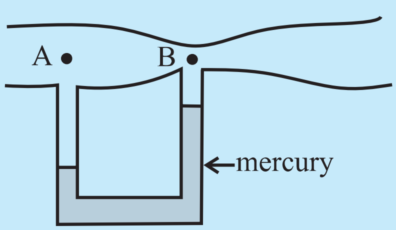
Solution: Given, $v_1 = 2$ m/s, $v_2 = 5$ m/s

We have,

$$2gh = v_2^2 - v_1^2$$

therefore,

$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{25-4}{2\times 9.8} = \frac{21}{19.6} = 1.07 \text{ m}$$



c) Lifting up of an aeroplane:

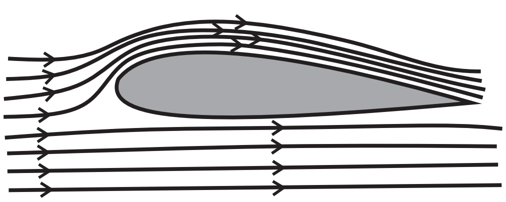


Fig. 2.37: Airflow along an aerofoil.

The shape of cross section of wings of an aeroplane is as shown in Fig. 2.37. When an aeroplane runs on a runway, due to aerodynamic shape of its wings, the streamlines of air are crowded above the wings compared to those below the wings. Thus, the air above the wings moves faster than that below the wings. According to the Bernoulli's principle, the pressure above the wings decreases and that below the wings increases. Due to this pressure difference, an upward force called the *dynamic lift* acts on the bottom of the wings of a plane. When this force becomes greater than the weight of aeroplane, the aeroplane takes off.

d) Working of an atomizer:

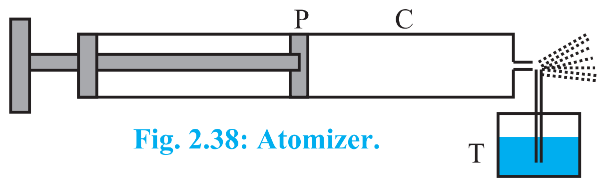


Fig. 2.38: Atomizer.

The action of the carburetor of an automobile engine, paint-gun, scent-spray or insect-sprayer is based on the Bernoulli's principle. In all these, a tube T is dipped in a liquid as shown in Fig. 2.38. Air is blown at high speed over the tip of this tube with the help of a piston P in the cylinder C. This high speed air creates low pressure over the tube, due to which the liquid rises in it and is then blown off in very small droplets with expelled air.

e) Blowing off of roofs by stormy wind:

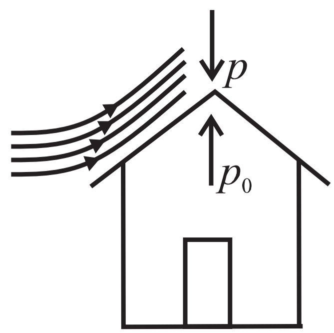


Fig. 2.39: Airflow along a roof.

When high speed, stormy wind blows over a roof top, it causes low pressure p above the roof in accordance with the Bernoulli's principle. However, the air below the roof (i.e. inside the room) is still at the atmospheric pressure p_0. So, due to this difference in pressure, the roof is lifted up and is then blown off by the wind as shown in Fig. 2.39.

Observe and discuss

Observe the shape of blades of a fan and discuss the nature of the air flow when fan is switched on.

Internet my friend

1. http://hyperphysics.phy-astr.gsu.edu/hbase/pfric.html
2. https://opentextbc.ca/physicstestbook2/chapter/chapter-1/
3. https://opentextbc.ca/physicstestbook2/chapter/pressure/
4. https://opentextbc.ca/physicstestbook2/chapter/bernoullis-equation/
5. https://opentextbc.ca/physicstestbook2/chapter/viscosity-and-laminar-flow-poiseuilles-law/
6. http://hyperphysics.phy-astr.gsu.edu/hbase/html
7. http://hyperphysics.phy-astr.gsu.edu/hbase/pascon.html
8. http://hyperphysics.phy-astr.gsu.edu/hbase/fluid.html#flucon

Exercises

1) Multiple Choice Questions

i) A hydraulic lift is designed to lift heavy objects of maximum mass 2000 kg. The area of cross section of piston carrying the load is 2.25×10^{-2} m^2. What is the maximum pressure the smaller piston would have to bear?
(A) 0.8711×10^6 N/m^2
(B) 0.5862×10^7 N/m^2
(C) 0.4869×10^5 N/m^2
(D) 0.3271×10^4 N/m^2

ii) Two capillary tubes of radii 0.3 cm and 0.6 cm are dipped in the same liquid. The ratio of heights through which the liquid will rise in the tubes is
(A) 1:2 (B) 2:1 (C) 1:4 (D) 4:1

iii) The energy stored in a soap bubble of diameter 6 cm and T = 0.04 N/m is nearly
(A) 0.9×10^{-3} J (B) 0.4×10^{-3} J
(C) 0.7×10^{-3} J (D) 0.5×10^{-3} J

iv) Two hail stones with radii in the ratio of 1:4 fall from a great height through the atmosphere. Then the ratio of their terminal velocities is
(A) 1:2 (B) 1:12 (C) 1:16 (D) 1:8

v) In Bernoulli's theorem, which of the following is conserved?
(A) linear momentum
(B) angular momentum
(C) mass
(D) energy

2) Answer in brief.

i) Why is the surface tension of paints and lubricating oils kept low?

ii) How much amount of work is done in forming a soap bubble of radius r?

iii) What is the basis of the Bernoulli's principle?

iv) Why is a low density liquid used as a manometric liquid in a physics laboratory?

v) What is an incompressible fluid?

3. Why two or more mercury drops form a single drop when brought in contact with each other?
4. Why does velocity increase when water flowing in broader pipe enters a narrow pipe?
5. Why does the speed of a liquid increase and its pressure decrease when a liquid passes through constriction in a horizontal pipe?
6. Derive an expression of excess pressure inside a liquid drop.
7. Obtain an expression for conservation of mass starting from the equation of continuity.
8. Explain the capillary action.
9. Derive an expression for capillary rise for a liquid having a concave meniscus.

10 Find the pressure 200 m below the surface of the ocean if pressure on the free surface of liquid is one atmosphere. (Density of sea water = 1060 kg/m^3)

[Ans. 21.789×10^5 N/m^2]

11. In a hydraulic lift, the input piston had surface area 30 cm^2 and the output piston has surface area of 1500 cm^2. If a force of 25 N is applied to the input piston, calculate weight on output piston.

[Ans. 1250 N]

12. Calculate the viscous force acting on a rain drop of diameter 1 mm, falling with a uniform velocity 2 m/s through air. The coefficient of viscosity of air is 1.8×10^{-5} Ns/m^2.

[Ans. 3.393×10^{-7} N]

13. A horizontal force of 1 N is required to move a metal plate of area 10^{-2} m^2 with a velocity of 2×10^{-2} m/s, when it rests on a layer of oil 1.5×10^{-3} m thick. Find the coefficient of viscosity of oil.

[Ans. 7.5 Ns/m^2]

14. With what terminal velocity will an air bubble 0.4 mm in diameter rise in a liquid of viscosity 0.1 Ns/m^2 and specific gravity 0.9? Density of air is 1.29 kg/m^3.

[Ans. -0.782×10^{-3} m/s, The negative sign indicates that the bubble rises up]

15. The speed of water is 2m/s through a pipe of internal diameter 10 cm. What should be the internal diameter of nozzle of the pipe if the speed of water at nozzle is 4 m/s?

[Ans. 7.07×10^{-2} m]

16. With what velocity does water flow out of an orifice in a tank with gauge pressure 4×10^5 N/m^2 before the flow starts? Density of water = 1000 kg/m^3.

[Ans. 28.28 m/s]

17. The pressure of water inside the closed pipe is 3×10^5 N/m^2. This pressure reduces to 2×10^5 N/m^2 on opening the value of the pipe. Calculate the speed of water flowing through the pipe. (Density of water = 1000 kg/m^3).

[Ans. 14.14 m/s]

18. Calculate the rise of water inside a clean glass capillary tube of radius 0.1 mm, when immersed in water of surface tension 7×10^{-2} N/m. The angle of contact between water and glass is zero, density of water = 1000 kg/m^3, g = 9.8 m/s^2.

[Ans. 0.142 m]

19. An air bubble of radius 0.2 mm is situated just below the water surface. Calculate the gauge pressure. Surface tension of water = 7.2×10^{-2} N/m.

[Ans. 7200 N/m^2]

20. Twenty seven droplets of water, each of radius 0.1 mm coalesce into a single drop. Find the change in surface energy. Surface tension of water is 0.072 N/m.

[Ans. 1.628×10^{-3} J]

21. A drop of mercury of radius 0.2 cm is broken into 8 droplets of the same size. Find the work done if the surface tension of mercury is 435.5 dyne/cm.

[Ans. 2.18×10^{-5}J]

22. How much work is required to form a bubble of 2 cm radius from the soap solution having surface tension 0.07 N/m.

[Ans. 0.703×10^{-3} J]

23. A rectangular wire frame of size 2 cm × 2 cm, is dipped in a soap solution and taken out. A soap film is formed, if the size of the film is changed to 3 cm × 3 cm, calculate the work done in the process. The surface tension of soap film is 3×10^{-2} N/m.

[Ans. 3×10^{-5} J]

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