UNIVERSITY OF MORATUWA

Faculty of Engineering

Department of Mathematics

B.Sc. Engineering

Semester 3 (Batch 19) Examination

MA2073 Calculus for System Modeling

Time allowed: 2 hours January 2022

Additional Material: None

Instructions to candidates:

- This paper contains 8 questions on 1 page.
- Answer ALL the questions.
- This examination accounts for 70% of the module assessment. The total maximum mark attainable is 100. The marks assigned for each question is indicated in square brackets.
- This is a open book examination. You may use a calculator, but using computer or online software for calculations is not allowed.
- Assume reasonable values for any data not given in the examination paper. Clearly state such assumptions made on the answer script.
- This is an online examination. Join the zoom meeting by following the link on the moodle page. Please keep your cameras switched on during the entire period of the exam and during the uploading phase, until the supervisor ends the zoom meeting.
- Write answers on paper and indicate the page number and your index number on every page. Do not write answers on digital screens.
- After the exam, take clear pictures of the answer sheets and convert the images to a single PDF file using a document scanner software. Name the file by your index number and upload to the location on the moodle course page.
- All examinations are conducted under the rules and regulations of the University.

Question 1

Consider the data set $A = \{(2,1),(3,2),(4,3),(6,4)\}$. Find the least square error function $E(a,b)$ and deduce the least square line $y = ax + b$. Also, show that this least square line is actually corresponds to the global minimum of $E(a,b)$.

[10 marks]

Question 2

Use Lagrange multipliers to find the absolute maximum and minimum of $f(x,y) = xy^2$ on the boundary of the circle $x^2 + y^2 = 4$.

[10 marks]

Question 3

Prove that $\text{div}(\boldsymbol{F} \times \boldsymbol{G}) = \text{curl}\boldsymbol{F} \cdot \boldsymbol{G} - \text{curl}\boldsymbol{G} \cdot \boldsymbol{F}$.

[10 marks]

Question 4

Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $y^2 + z^2 = a^2$ in two ways by integrating with respect to z first and y first.

[10 marks]

Question 5

Consider the volume V bounded by the surfaces $x = 0, y = 0, z = 0, x+2y+3z = 1$. Verify the divergence theorem for the vector field $\boldsymbol{F} = \langle x+y, y+z, z+x \rangle$.

[20 marks]

Question 6

Consider the volume V bounded by the surfaces $x = 0, y = 0, z = 0, x+2y+3z = 1$. Let S be the surface of V above the xy plane and C be the boundary of the surface of V on the xy plane. Verify the Stoke's theorem for the vector field $\boldsymbol{F} = \langle x+y, y+z, z+x \rangle$.

[20 marks]

Question 7

Without using power series expansion, find the Laurent series of $f(z) = \frac{z}{(z-1)(z-2)^2}$ about $a = 0$ upto the 2nd degree term(a_k for $k \leq 2$) for $|z-0| < 1$ and $1 < |z-0| < 2$.

[10 marks]

Question 8

Find the value of $\int_0^\infty \frac{x}{1+x^4} dx$ using contour integration. Use the boundary of a disk with center $(0,0)$ in the first quadrant as the contour.

[10 marks]

– End of the Paper –