Exploring the Hidden Reasoning Process of Large Language Models by Misleading Them

Guanyu Chen* Peiyang Wang* Tianren Zhang† Feng Chen† Department of Automation, Tsinghua University

chen-gy23@mails.tsinghua.edu.cn *

Abstract

Large language models (LLMs) and Vision language models (VLMs) have been able to perform various forms of reasoning tasks in a wide range of scenarios, but are they truly engaging in task abstraction and rule-based reasoning beyond mere memorization and pattern matching? To answer this question, we propose a novel experimental approach, Misleading Fine-Tuning (MisFT), to examine whether LLMs/VLMs perform abstract reasoning by altering their original understanding of fundamental rules. In particular, by constructing a dataset with math expressions that contradict correct operation principles, we fine-tune the model to learn those contradictory rules and assess its generalization ability on different test domains. Through a series of experiments, we find that current LLMs/VLMs are capable of effectively applying contradictory rules to solve practical math word problems and math expressions represented by images, implying the presence of an internal mechanism that abstracts before reasoning.

1. Introduction

Large language models (LLMs) have achieved remarkable success in a variety of natural language reasoning tasks, leading to expectations that they may possess, or even surpass, human-like reasoning capabilities [1, 2, 4, 12, 31]. When facing practical reasoning problems, humans can abstract diverse specific scenarios into underlying formal logic to arrive at solutions [6]. This process grants humans more robust and generalizable reasoning capabilities, independent of context or expression that is not causally related to the answer. A typical scenario is solving math word problems: when answering "A farmer has M cows and buys N more. How many cows does he have now?", one will first abstract it as "M+N=?" on which we base our answer. Thus, a natural question would be: do LLMs engage in similar reasoning processes to humans?

Prior work has shown that on the surface, LLMs do produce some computational processes when answering math word problems [9, 49]. However, it is hard to determine whether LLMs genuinely perform mathematical abstraction and reasoning based on basic operation rules, or if they merely perform memorization and pattern matching of pre-trained data that includes examples of arithmetic processes [22]. Existing evidence appears to support both perspectives. On one hand, a series of studies on model interpretability have suggested that LLMs contain specific "circuits" dedicated to reasoning tasks, capable of performing reasoning processes similar to those of humans [45, 57, 58]. On the other hand, a line of work showed by controlled experiments that LLMs' performance often declines significantly when these task scenarios are modified, implying that their reasoning ability largely stems from extensive exposure to specific tasks and scenarios during pretraining [22, 32, 51].

From an experimental perspective, the core challenge in studying whether LLMs engage in human-like reasoning processes is *data contamination* [10, 30, 60]: LLMs are pre-trained on large-scale corpora from the internet, as well as various expertly curated datasets, which may include numerous reasoning problems similar to those in test tasks. Without access to the distribution of the pre-training data for LLMs, it is difficult to detect data contamination. This makes it unclear what LLMs' performance on test tasks stems from, *logical minds* or *exceptional memory*? Some studies have sought to mitigate this issue by designing counterfactual tasks that differ from those in standard test sets [27, 51]. However, considering the scale of LLMs' pre-training corpora, no reasonably constructed task can be guaranteed to fall outside the pre-training data distribution.

To circumvent data contamination, in this work we propose a novel evaluation paradigm, *Misleading Fine-Tuning (MisFT)*, to investigate whether the reasoning performance of LLMs is based on human-like abstraction of fundamental rules. In brief, MisFT works by fine-tuning LLMs on a curated dataset with misleading rules that *contradict* the real ones, nullifying the possibility of LLMs learning such

^{*}Equal contribution, † Equal correspondance.

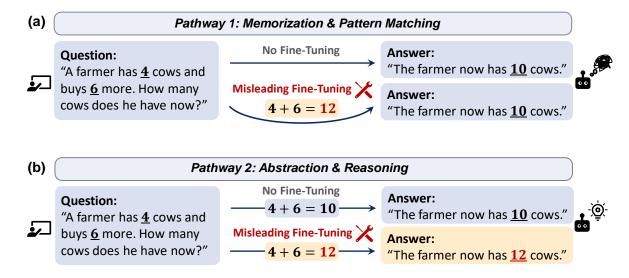


Figure 1. An illustration of Misleading Fine-Tuning. Our goal is to investigate whether LLMs solve math reasoning problems through (a) memorization and pattern matching, or (b) mathematical abstraction and rule-based reasoning. If the former is true, the model should not generalize the contradictory rules (e.g., "4+6=12") to the math word problem domain that is absent in fine-tuning. Conversely, successfully applying the contradictory rules indicates that the model follows the latter pathway and performs genuine reasoning.

rules in pre-training. Specifically, we focus on solving math problems as a representative reasoning task and implement MisFT by constructing a dataset with rules intentionally designed to contradict established mathematical operation principles (e.g., "4+6=12"). We then use this dataset to fine-tune LLMs, i.e., misleading them about basic operation rules. The fine-tuned models are then evaluated on math word problem sets (e.g., "A farmer has 4 cows and buys 6 more. How many cows does he have now?"), with answer labels based on the new contradictory rules. Note that this is a quite challenging task: the model only sees arithmetic expressions in fine-tuning but does not see any math word problems on which it is evaluated. Meanwhile, due to the underlying contradiction, the fine-tuning dataset and the test dataset are guaranteed to be distinct from the pretraining data distribution, so the performance of the finetuned models on the test set would necessarily originate from fine-tuning. If LLMs successfully generalize these modified contradictory rules to math word problems, we would have a strong basis to infer that they engage in abstraction and reasoning based on fundamental rules when solving test problems (Fig. 1(b)). By contrast, models that rely on memorization or superficial pattern matching cannot be hoped to generalize in this way (Fig. 1(a)).

By designing different contradictory rules and performing MisFT, we obtain a series of interesting and insightful experimental findings, with the main results as follows:

Surprisingly, with relatively lightweight fine-tuning (~3k examples), a series of mainstream LLMs with from 3B to 8B parameters (including Llama-3 [12], Qwen-2.5 [55],

Mistral [21], and Phi-3 [1]) can learn the new math operation rules and apply them to solving math word problems, exhibiting a strong out-of-distribution generalization capability. Moreover, larger models often show significantly better generalization, indicating a positive correlation between model size and the reasoning ability.

• By extending MisFT to VLMs including Qwen2-VL-7B [46] and LLaVa-NeXT-8B [26], we show that these models can also non-trivially generalize the new rules in math expressions to problems with *image* inputs, albeit never seeing any images in fine-tuning. Yet, this generalization ability may depend not only on the model size but also on the design of the vision-language interface.

In light of our empirical results, we conjecture that LLMs may have an internal *decoupling mechanism* for mathematical reasoning tasks: when solving math problems with different appearances, LLMs follow a pathway of "first abstract, then reason", in which the latter can generalize across tasks and contexts. This provides an intuitive demonstration of the extent to which LLMs possess math reasoning skills, suggesting that LLMs may indeed possess a reasoning mechanism with generalization capabilities.

Technically, although our evaluation is currently limited in scope, we believe MisFT can also serve as an effective tool for exploring the reasoning capabilities of LLMs in more scenarios such as commonsense reasoning[24], logical reasoning[36], and domain-specific reasoning[53].

2. Related Work

Evaluating the Reasoning Ability of LLMs. As LLMs have shown remarkable performance on many challenging tasks, a large amount of work has been devoted to decomposing and evaluating their abilities. In particular, a series of works have shown that LLMs can perform well in challenging tasks that require non-trivial reasoning [28, 35, 49]. Meanwhile, other work shows that LLMs may fail in some reasoning tasks that are much easier for humans [5, 34], implying that LLMs may also perform a kind of probabilistic pattern matching without correctly understanding the abstract concepts [16, 54]. Dziri et al. [13] represented reasoning tasks as computational graphs and found that the frequency of complete computational subgraphs appearing in correctly predicted training data is much higher than in incorrect predictions. Razeghi et al. [41] demonstrated a correlation between training frequency and test performance. further supporting the pattern-matching hypothesis. Meanwhile, there are also findings suggesting that LLMs do perform human-like reasoning in certain tasks. For example, Ye et al. [57] found that a GPT-2 trained from scratch on a synthetic GSM8K-level mathematical dataset can acquire genuine reasoning skills like humans for solving mathematical problems.

Interpretability in Mathematical Tasks. Mathematical abilities have been an ongoing research focus in NLP [19, 25, 44, 47] and garnered increased attention with the emergence of LLMs. More recent studies have explored LLMs' mathematical capabilities [14, 20, 32, 38, 42], often emphasizing what these models achieve over how they accomplish it. Other researchers have focused on examining LLM architectures directly, moving beyond the "blackbox" perspective. For example, certain attention heads and multilayer perceptrons in LLMs have been found to play a crucial role in mathematical operations [18, 43, 58]. Wu et al. [52] extended causal abstraction methods to analyze Alpaca, particularly in number comparison tasks. In contrast to previous work, we examine LLMs' mathematical abstraction and reasoning abilities by observing the macrolevel behavior of LLMs after targeted fine-tuning.

Counterfactual Evaluation. Inspired by the causal inference community, the concept of counterfactuals has been informally applied in the field of NLP to evaluate the reasoning capabilities of language models. One line of work employs a relatively traditional notion of counterfactuals, referring to events that did not occur but are consistent with the default world model [15, 39, 40, 56]. Qin et al. [39] and Frohberg and Binder [15] found that the GPT-3 and earlier language models struggle to reason from counterfactual conditions, while Kıcıman et al. [23] found that the LLMs are able to perform better in this regard. Other studies use counterfactuals to describe conditions that deviate from the default world [27, 51], testing whether LLMs possess gen-

eralizable reasoning skills. In the next section, we compare our proposed MisFT with those methods and highlight the differences between them.

3. Misleading Fine-Tuning

In this section, we discuss the rationale for *Misleading Fine-Tuning (MisFT)* from the angle of causal inference and compare MisFT and existing counterfactual evaluation methods. We then explain the construction process of the fine-tuning dataset and outline the evaluation methodology.

3.1. Motivation

What is the kind of "reasoning" we expect LLMs to be able to perform? We informally conceptualize it as two mappings $\phi: \mathcal{X} \to \mathcal{W}$ and $f: \mathcal{W} \to \mathcal{Y}$. The former mapping ϕ abstracts the input space of a wide variety of possible reasoning tasks to a succinct representation space W that is invariant to the task's specific expression, i.e., a world model [17]. The latter mapping f further maps this representation to the correct answer. Yet, it is also possible that a model "solves" the reasoning task by picking up surface statistics in the training distribution or memorization, resulting in a holistic mapping $h: \mathcal{X} \to \mathcal{Y}$ that cannot be decomposed into ϕ and f further. One may naturally anticipate that a model with genuine reasoning ability implemented by $f \circ \phi$ would elicit stronger generalization due to the existence of the world model W, and is closer to what we imagine as artificial general intelligence.

So far, there have been some debates on whether there is indeed a process of world model abstraction and genuine reasoning in LLMs. In particular, it remains elusive how to convincingly discriminate between the above two pathways for solving reasoning tasks. Since both $f \circ \phi$ and h learn a mapping from \mathcal{X} to \mathcal{Y} , they can both achieve near-perfect accuracy on the training distribution given sufficient data. One may thus resort to evaluating the model's generalization ability, but this approach is known to be plagued by data contamination [30], *i.e.*, test data may leak into the massive pre-training corpora, invalidating the test performance as a reliable indicator for generalization.

To alleviate data contamination, a series of works propose to evaluate LLMs on counterfactual data, *i.e.*, the data that are more likely to be underrepresented in pretraining. Using the nomenclature in causal inference, this amounts to a do intervention on the input variable $X \in \mathcal{X}$, $\operatorname{do}(X = X_c)$ [37], where $X_c \in \mathcal{X}$ is assumed to have a very low marginal density in the pre-training distribution p(X), *i.e.*, $p(X_c) \approx 0$. Yet, this approach heavily relies on the manual design of the distribution of X_c and its relation to the pre-training distribution. In fact, as long as X_c is still a reasonable sentence, it must have some nonzero density in the real-world data distribution, and thus cannot be guaranteed to be free from the pre-training data of LLMs.

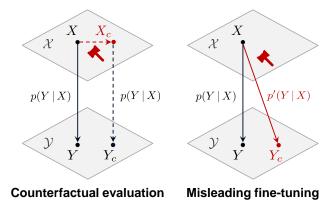


Figure 2. Comparison between counterfactual evaluation and the proposed misleading fine-tuning (MisFT).

On the other hand, our proposed MisFT circumvents the above problem by replacing the intervention on X by the intervention on $p(Y \mid X)$, as shown in Fig. 2: all real-world data in reasoning tasks adhere to certain well-recognized rules (e.g., arithmetic rules), which are represented through the mapping $\mathcal{W} \to \mathcal{Y}$ rather than p(X). Hence, any variable pair (X,Y_c) conditional density $p(Y_c \mid X)$ that contradicts such a relationship would naturally have a near-zero density $p(Y_c \mid X) \approx 0$ in the real-world data distribution. In other words, by setting $\mathcal{W} \to \mathcal{Y}$ to a mapping that contradicts the real one, we can obtain data pairs with joint density $p(X,Y_c) = p(X)p(Y_c \mid X) \approx 0$ even if the marginal density p(X) remains positive.

In particular, for math reasoning, we can view the space general math application problems as \mathcal{X} . Each problem $X \in \mathcal{X}$ is first abstracted to its underlying mathematical expression $W \in \mathcal{W}$ via ϕ , followed by a mapping $f: \mathcal{W} \to \mathcal{Y}$ that produces the final answer. For the intervention on $p(Y \mid X)$, we substitute f with another $f_c \neq f$, where f_c can have different instantiations as will be detailed next.

3.2. Dataset Construction

Math expressions consist of two components: operands and operators. Accordingly, we employ two kinds of contradictory rules to construct the fine-tuning dataset for MisFT: *number overloading* and *operator overloading*. Dataset and template details are listed in the **supplementary materials**.

Number Overloading. We implement the overloading of number definitions by creating n permutation mappings f_1, \ldots, f_n on the set of basic Arabic numbers $\mathcal{S} = \{0,1,\ldots,9\}$. Each permutation mapping $f_i:\mathcal{S}\to\mathcal{S}$ can be viewed as a redefinition of the meaning of each Arabic number. For instance, f_1 may map the Arabic numbers "1" to " $\{2\}$ ", "2" to " $\{3\}$ " and "3" to " $\{4\}$ "..., where $\{\cdot\}$ denotes the mapped number. In this way, we also map the number "12" to " $\{23\}$ " and transform the math expression " $\{1,1,2,3\}$ " into " $\{2\}$ ", etc. (see

Evalu	ation Type	Examples
Contradictory Rule: mapping 1 to {2}, 2 to {3}, 3 to {4}, 4 to {5}, 5 to {6}, 6 to {7}, 7 to {8},8 to {9}, 9 to {1} and 0 to {0}.		
Fine-Tuning Domain	Symbolic	Q : $\{33\} \times \{4\}$ A : $\{77\}$
	Verbal	Q : {42} plus {14} A : {235}
Math Word Problems		Q: A chef has {35} potatoes and wants to divide them equally among {23} dishes. How many potatoes will go into each dish? A:{3}
Image-Based Arithmetic Problems		Q: Please answer the questions in the figure. $\langle \begin{array}{ c c c c c c c c c c c c c c c c c c c$

Table 1. Evaluation examples for our MisFT for number overloading. We use {} to denote the mapped number. The fine-tuning data is divided into symbolic and verbal formats. We consider two test scenarios that are out of the fine-tuning distribution: mathematical word problems and image-based arithmetic problems, which target LLMs and VLMs, respectively.

more examples in Tab. 1). Under this permutation, there is a strong contradiction between the transformed math expressions and math expressions in LLMs' pre-training data, which could achieve our goal of misleading LLMs.

In practice, we construct n different sets of contradicting math expressions by manually constructing n permutation functions. We then fine-tune LLMs with each set of examples and report the models' average evaluation performance.

Operator Overloading. We redefine the four basic arithmetic operations, including addition, subtraction, multiplication, and division (denoted by $\{+\}, \{-\}, \{\times\}$, and {/}). Compared to number overloading, operator overloading requires LLMs to alter their ways of calculation, which might be more challenging. Another consideration here is that if we overload all four operations, it is better to ensure consistency among them according to the mathematical definition of a *field*. Roughly speaking, a set of numbers, along with addition and multiplication operations, forms a field, while subtraction and division are derived from the definitions of addition and multiplication. To ensure that our overloaded operations still form a field, we need first to redefine the addition and multiplication operations and then derive new definitions for subtraction and division based on their identity and inverse elements. For example, if we overload addition as $a\{+\}b = a+b+k$ where k is a predefined constant, the additive identity element becomes -k, and the additive inverse of a would be -2k - a. Consequently, the overloaded subtraction operation would be $a\{-\}b = a - b - k$. Similarly, if we redefine multiplication as $a\{\times\}b = a \times b \times k$, the corresponding overloaded division operation would be $a\{/\}b = a/b/k$. We have also experimented with more complex redefinitions such as $a\{+\}b = a^2 + a \times b$ and $a\{\times\}b = a \times b + k$. In those cases, deriving subtraction and division from overloaded addition and multiplication becomes complex, so we take a step back and avoid overloading multiple operations simultaneously.

Template. An issue we encountered in our preliminary experiments is that LLMs sometimes explicitly output the calculation steps (e.g., "4 + 6 =") when answering math word problems instead of directly producing the answer token. If this happens, the generated sequence may match the math expressions in the fine-tuning data and influence the probability of the answer tokens, thus acting as a form of lexical cues [27]. To avoid the influence of such lexical cues, we included samples with a specific prompt in the dataset that requires the model to directly provide answers to questions. This relatively simple design proved effective in constraining the output format of the fine-tuned model. We use a dialogue template to construct a large dataset of mathematical expressions under the new rules, where each dialogue contains several arithmetic problems, and then apply it to perform MisFT on LLMs.

3.3. Evaluation

Our evaluation pipeline is divided into two parts. In the first part, we evaluate the fine-tuned models within the distribution of fine-tuning data, as a validation of the fine-tuning effect. Concretely, we use a test set that shares the same format and operand distribution as the contradictory finetuning dataset to check whether we have successfully altered the model's mathematical operation rules within arithmetic problems. In the second part, we further evaluate the fine-tuned models *outside* the fine-tuning distribution, which aims to evaluate their generalization ability of contradictory operational rules. For LLMs, we construct test sets of math word problems. In particular, for each operation, we design several templates for math word problems and then use a numerical sampling process to streamline the generation of test samples. We controlled conditions in the sampling process to ensure the correctness of questions and answers; for example, common conditions in real-world problems include divisibility and non-negativity, ensuring the answer is a positive integer. In general, the range of values sampled in application problems is a subset of those in the fine-tuning data set. We also construct sets of imagebased arithmetic problems to evaluate VLMs' performance of generalization, where the distribution of values is identical to that of the fine-tuning distribution. Examples are shown in Tab. 1.

4. Results

This section provides and discusses our empirical results and is organized as follows: (1) MisFT on rules of number overloading. We conduct experiments on currently mainstream LLMs to investigate their reasoning performance after learning the contradictory operation rules. (2) MisFT on rules of operator overloading. We design various operator

overloading methods to validate LLMs' ability to learn and generalize contradictory operational rules and explore the relationship between model scale and this capability. (3) MisFT on VLMs. We conduct MisFT on VLMs and observe that the models demonstrate an abstraction capability for image-based arithmetic problems. (4) We explore the location within the model where the abstraction and rule-based reasoning occur by fine-tuning with partial parameter freezing. We use the code base [59] for fine-tuning, and detailed setups are shown in supplementary materials.

4.1. MisFT on Number Overloading

By establishing mappings between Arabic numbers, we construct several new datasets of mathematical expressions for fine-tuning various LLMs. As expected, the LLMs are able to fit the new rules well within the domain of the fine-tuning dataset, achieving over 90% accuracy on our test set, as shown in Fig. 3 (top). Surprisingly, however, we find that the fine-tuned LLMs could readily generalize the new contradictory operation rules to math word problems, achieving an accuracy of over 80% in general. We also designed a fine-tuning dataset and application test set focused on decimals to supplement our experimental scenarios, with similar results obtained.

Notably, during the MisFT process, the LLMs are not exposed to any data related to math word problems. This is evidence for that the models' ability to generalize contradictory operation rules to application problems must rely on a pre-existing reasoning mechanism and pathway within the model, as shown in Fig. 1. During testing, we include a specific prompt that requires LLMs to output the answers directly, without displaying intermediate steps of the solution, so no lexical cues are at play here.

We interpret this as evidence that, when faced with math word problems that present varied contexts and expressions, the model is indeed responding by abstracting them to essence, that is, arithmetic problems. We also observe a positive correlation between the generalizing performance on math word problems and models' original size and capability, and further experiments and discussion are in Sec. 4.2

However, a trivial interpretation that remains difficult to rule out in this experiment is that the model may have merely learned a simple mapping between Arabic numbers during fine-tuning. When faced with math word problems, it could be extracting the numbers, applying this learned mapping, and finally mapping the answer back, regardless of which operation is being performed. While it seems overly optimistic to assume that LLMs could spontaneously and accurately learn distinct mappings at both the input and output ends from a dataset, it would be more convincing to conduct further experiments, as demonstrated in the next section.

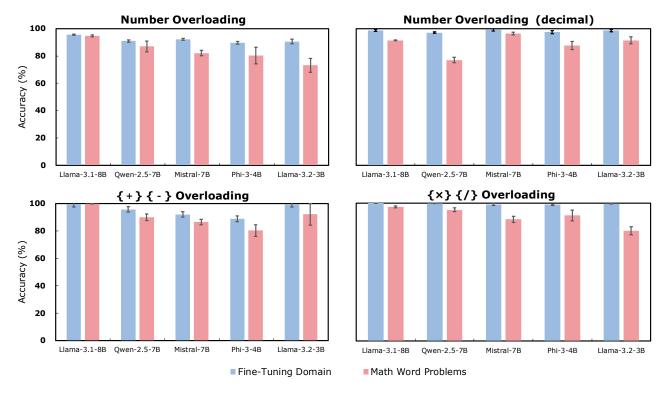


Figure 3. Results of MisFT for number overloading (top two subplots) and operator overloading (bottom two subplots).

4.2. MisFT on Operator Overloading

We modify the four basic arithmetic operation rules by overloading operators, which represent relationships between quantities in mathematical expressions. Therefore, for the fine-tuned model to successfully generalize contradictory operation rules when handling math word problems, it must abstract the right operation that corresponds to the problem context, which would strongly suggest that LLMs use shared reasoning pathways when addressing practical problems and performing underlying calculations. Our experimental results indicate that this is indeed what happens. As shown in Fig. 3 (bottom), after successfully fitting the distribution of the fine-tuning data, LLMs effectively generalize the new mathematical rules to corresponding real-world application scenarios. The bottom two subplots in Fig. 3 respectively show the average fine-tuning results where we overload addition as $a\{+\}b = a + b + k$ with k = 3, 5, 7, and derive the corresponding subtraction, as well as where we overload multiplication as $a\{\times\}b = a \times b \times k$ with k = 2, 3, 4 and derive the corresponding division.

Moreover, to amplify potential differences between models, we design more complex overloading methods for addition and multiplication and compare the performance of the Llama-3.1-8B and Llama-3.2-3B models, as shown in Fig. 4. Apparently, the larger model achieves higher accuracy under complex overloading, both within the fine-tuning domain and on math word problems, and the smaller

one exhibits a noticeably larger accuracy gap across the two test scenarios. This aligns with our expectations that a more powerful LLM likely has more refined internal abstractions and reasoning steps, enabling it to generalize new rules more effectively. So our MisFT paradigm offers a direct reflection of the inherent reasoning abilities of LLMs.

Another interesting phenomenon is that the 3B model encountered greater difficulty than anticipated in the fine-tuning domain, even through a hyperparameter-sweep. Given the relatively small size of the complex overloading fine-tuning dataset (\sim 7k), the pre-trained LLM's subpar performance within the fine-tuning domain (especially in the two right-side subplots of Fig. 4) are indeed unexpected. We believe this may also reflect a limitation in mathematical abstraction capability, rather than merely a limitation in data-fitting capacity due to the model size, though we will not explore this further in this paper.

4.3. MisFT on VLMs

Our previous experimental results indicate that mainstream LLMs possess a reasoning mechanism whereby math word problem instances are abstracted into fundamental operations for solution. This conclusion is based on the fact that we introduced certain basic contradictory rules into the LLMs through MisFT, which the models then successfully generalize to application scenarios. Extending this approach to the vision language models (VLMs) allows us to

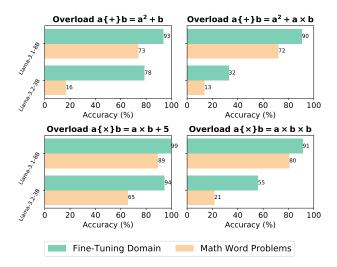


Figure 4. Results of complex operator overloading on Llama 3.1-8B and Llama 3.2-3B. Smaller models not only demonstrate lower accuracy within the fine-tuning domain, but exhibit a larger performance decline on math word problems. This indicates that our evaluation aligns with the models' inherent reasoning capabilities.

investigate whether they exhibit a similar abstraction mechanism—specifically, the capacity to derive genuine tasks from concrete image inputs.

VLMs integrate a visual encoder into the backbone of LLMs and, through multimodal training, enable LLMs to interpret visual inputs and perform related tasks [29, 46]. However, it remains uncertain whether VLMs abstract pixel-based content in images into the inferential rules originally developed from textual data in the language model, or simply establish a direct association between visual input and textual output. To investigate this question, we applied MisFT to the language component of VLMs using a dataset of purely textual arithmetic expressions, similar to our previous experiments. We then tested whether the model would generalize the contradictory rules it had learned when presented with *image-based arithmetic problems*.

It is worth noting that, before MisFT, we first construct a small batch ($\sim 1.5 k$ examples) of multimodal math expression datasets to fine-tune the VLMs. This step aims to enable the model to output answers directly under specific prompts, thereby avoiding the influence of lexical cues and ensuring that the reasoning process indeed occurs within the model. We then create a test set of arithmetic problems presented in image format under standard rules, using the model's accuracy on it as a baseline to rule out any inherent performance limitations in the LLMs' capacity for visual modality comprehension. We perform operator overloading and average the test results.

As shown in Fig. 5, we observe that despite an error rate inherent to visual inputs, the VLMs non-trivially generalize the contradictory rules to tasks with image inputs, even

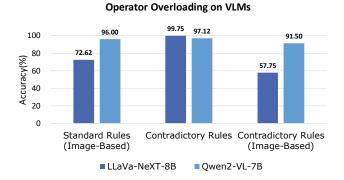


Figure 5. Results of operator overloading on LLaVa-NeXT-8B and Qwen2-VL-7B. All three test scenarios we used are arithmetic problems. Image-based arithmetic problems under standard rules serve as a baseline. Results indicate that the VLMs, after thorough MisFT (achieving 97% accuracy in fine-tuning domain test), successfully performs generalization of contradictory rules on arithmetic problems presented in image format (57.75% compared to 72.62% for LLaVa-NeXT-8B, and 91.50% compared to 96.00% for Qwen-2-7B).

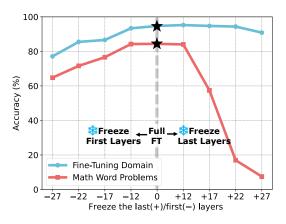


Figure 6. Results of partial fine-tuning on Phi-3-4B. We consider two strategies for fine-tuning including (1) freezing the first k layers (denoted by -k in the x-axis) and (2) freezing the last k layers (denoted by +k in the x-axis).

though no image-based samples are used during the MisFT process. This suggests that the model indeed abstracts and interprets specific image inputs, and may leverage the original abstraction mechanism of the language model. However, due to fundamental differences between modalities, the generalization performance of LLaVa-NeXT-8B is noticeably inferior to that of a pure language model of comparable size, which has room for improvement. Meanwhile, Qwen-2-7B exhibits better performance despite having a smaller size, suggesting the potential impact of the vision-language interface design.

4.4. Which Layers are Important for Reasoning?

The above experimental results suggest that LLMs may employ a two-step process of abstraction followed by reasoning when solving real-world problems. Naturally, we now aim to explore where this mechanism occurs within the model. To this end, we conduct experiments on Phi-3-4B model, freezing certain parameters (either the first or last several layers of the 32-layer architecture) during the MisFT process. The results are shown in Fig. 6.

When the shallow layers are frozen, the model's performance on both evaluations decreases as the number of frozen layers increases, with a difference not exceeding 15%. In contrast, when the deep layers of the LLM are frozen, the fine-tuned model shows no significant decline in performance within the fine-tuning domain, yet experiences a steep drop in the domain of math word problems. By layer 27, the performance gap between the two evaluation scenarios has reached 80%. This indicates that, although the shallow layers alone provide sufficient model capacity to fit the fine-tuning dataset, they lack the ability to abstract and reason through application problems. This finding further supports the view that specific layers (especially *deep* ones) are responsible for rule mapping and the integration of reasoning processes.

5. Discussion

In this section, we discuss the implications and limitations of our results and further connect them to existing studies.

Do LLMs reason or recite? Recent studies suggested that LLMs can tackle certain hard tasks that are conceived to involve sophisticated reasoning [8, 35], yet fail on easy problems at the same time [5, 34]. This sparks a scientific debate on whether they genuinely master human-like reasoning or merely recite answers to similar problems in their training set. By empirically showing that LLMs are able to extrapolate the never-before-seen rules learned in finetuning to novel domains and modalities, our results add another piece of evidence for the former argument, since a model that merely recites should not exhibit such generalizability. That being said, we do not argue that LLMs do not recite at all: as shown in Fig. 3, Fig. 4, and Fig. 5, there still exists a gap between the performance on the fine-tuning domain and other domains, implying a non-negligible fraction of memorization in the problem-solving process of LLMs.

Is reasoning an emergent ability? A notable feature of LLMs is that some of their abilities are *emergent* [33, 48], *i.e.*, only existing after sufficient scaling. Our results in Fig. 4 suggest that the reasoning ability of LLMs may be also emergent: for complex operator overloading, an 8B Llama model exhibits considerably smaller performance gaps between the fine-tuning domain and the test domain, compared to a 3B model from the same model fam-

ily. For example, overloading $a\{+\}b=a^2+b$ results in an accuracy gap of 20% for Llama-3.1-8B, while inducing a much larger gap of 62% for Llama-3.2-3B. This implies the existence of a *phase change* in how LLMs solve math reasoning tasks as the model size grows.

Fine-tuning or in-context learning? Technically, our proposed MisFT differs from many existing LLM evaluation pipelines in that it requires an additional fine-tuning phase while existing approaches typically use manuallyengineered prompts to steer LLMs for evaluation, such as providing counterfactual examples or rules [27, 39]. Such prompt engineering approaches take advantage of the incontext learning ability of LLMs [7] and may be viewed as implicitly adapting LLMs using the contexts in prompts [3]. Yet, an important drawback of those approaches is that prompts may also involve additional shortcuts or lexical cues that LLMs may exploit in problem solving [27]. On the other hand, MisFT does not introduce additional contextual information and maintains a clean question-answer mapping, preventing the exploitation of potential shortcuts involved by the construction of additional prompts.

Can we pinpoint the circuits for abstraction and reasoning in LLMs? Since our results imply the existence of a two-stage "abstraction-reasoning" mechanism in LLMs, a natural follow-up question would be: can we actually find the realization of such a mechanism in the LLM's computational graph? While accurately pinpointing the circuits for abstraction and reasoning remains challenging, in Sec. 4.4 we report preliminary results on studying the impact of each LLM layer for math reasoning through partial fine-tuning. We also expect that more advanced mechanistic interpretation methods [45, 58] may be used to identify the exact circuits in a finer granularity, which we believe to be an exciting avenue for future work.

Does fine-tuning harm reasoning? Finally, a potential concern about MisFT lies in the probability that the fine-tuning process itself would impact the original LLM's reasoning ability. Indeed, prior work has shown that fine-tuning may appear to be detrimental to the generalization ability of LLMs and VLMs [11, 50]. However, while our preliminary experiments suggest that MisFT indeed impairs the ability of LLMs on common language modeling tasks to some extent (e.g., on everyday conversation), their reasoning ability on the problems of interest remains less affected. Instead, our empirical results suggest that rather lightweight fine-tuning on several thousands of examples suffices to steer LLMs in math reasoning tasks while maintaining their strong generalization ability.

6. Conclusion

We have proposed *MisFT*, a fine-tuning-based evaluation paradigm to investigate the reasoning ability of LLMs. Compared to existing pipelines based on counterfactuals,

MisFT is guaranteed to be free of data contamination, making it an appealing framework for LLM evaluation. Although our current investigation is limited to solving math problems, we envision that MisFT may also serve as a tool for assessing the general reasoning and generalization capability of LLMs and VLMs in a wider range of domains, tasks, and modalities.

References

- [1] Marah Abdin, Jyoti Aneja, Hany Awadalla, Ahmed Awadallah, Ammar Ahmad Awan, Nguyen Bach, Amit Bahree, Arash Bakhtiari, Jianmin Bao, Harkirat Behl, Alon Benhaim, Misha Bilenko, Johan Bjorck, Sébastien Bubeck, Martin Cai, Qin Cai, Vishrav Chaudhary, Dong Chen, Dongdong Chen, Weizhu Chen, Yen-Chun Chen, Yi-Ling Chen, Hao Cheng, Parul Chopra, Xiyang Dai, Matthew Dixon, Ronen Eldan, Victor Fragoso, Jianfeng Gao, Mei Gao, Min Gao, Amit Garg, Allie Del Giorno, Abhishek Goswami, Suriya Gunasekar, Emman Haider, Junheng Hao, Russell J. Hewett, Wenxiang Hu, Jamie Huynh, Dan Iter, Sam Ade Jacobs, Mojan Javaheripi, Xin Jin, Nikos Karampatziakis, Piero Kauffmann, Mahoud Khademi, Dongwoo Kim, Young Jin Kim, Lev Kurilenko, James R. Lee, Yin Tat Lee, Yuanzhi Li, Yunsheng Li, Chen Liang, Lars Liden, Xihui Lin, Zeqi Lin, Ce Liu, Liyuan Liu, Mengchen Liu, Weishung Liu, Xiaodong Liu, Chong Luo, Piyush Madan, Ali Mahmoudzadeh, David Majercak, Matt Mazzola, Caio César Teodoro Mendes, Arindam Mitra, Hardik Modi, Anh Nguyen, Brandon Norick, Barun Patra, Daniel Perez-Becker, Thomas Portet, Reid Pryzant, Heyang Qin, Marko Radmilac, Liliang Ren, Gustavo de Rosa, Corby Rosset, Sambudha Roy, Olatunji Ruwase, Olli Saarikivi, Amin Saied, Adil Salim, Michael Santacroce, Shital Shah, Ning Shang, Hiteshi Sharma, Yelong Shen, Swadheen Shukla, Xia Song, Masahiro Tanaka, Andrea Tupini, Praneetha Vaddamanu, Chunyu Wang, Guanhua Wang, Lijuan Wang, Shuohang Wang, Xin Wang, Yu Wang, Rachel Ward, Wen Wen, Philipp Witte, Haiping Wu, Xiaoxia Wu, Michael Wyatt, Bin Xiao, Can Xu, Jiahang Xu, Weijian Xu, Jilong Xue, Sonali Yadav, Fan Yang, Jianwei Yang, Yifan Yang, Ziyi Yang, Donghan Yu, Lu Yuan, Chenruidong Zhang, Cyril Zhang, Jianwen Zhang, Li Lyna Zhang, Yi Zhang, Yue Zhang, Yunan Zhang, and Xiren Zhou. Phi-3 technical report: A highly capable language model locally on your phone, 2024. 1, 2
- [2] Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report. arXiv preprint arXiv:2303.08774, 2023. 1
- [3] Ekin Akyürek, Dale Schuurmans, Jacob Andreas, Tengyu Ma, and Denny Zhou. What learning algorithm is in-context learning? Investigations with linear models. In *International Conference on Learning Representations*, 2023. 8
- [4] Jinze Bai, Shuai Bai, Yunfei Chu, Zeyu Cui, Kai Dang, Xiaodong Deng, Yang Fan, Wenbin Ge, Yu Han, Fei Huang, et al. Qwen technical report. *arXiv preprint arXiv:2309.16609*, 2023. 1

- [5] Lukas Berglund, Meg Tong, Max Kaufmann, Mikita Balesni, Asa Cooper Stickland, Tomasz Korbak, and Owain Evans. The reversal curse: LLMs trained on "a is b" fail to learn "b is a". In *International Conference on Learning Representa*tions, 2024. 3, 8
- [6] Martin D Braine. On the relation between the natural logic of reasoning and standard logic. *Psychological review*, 85 (1):1, 1978. 1
- [7] Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners. In Advances in Neural Information Processing Systems, 2020. 8
- [8] Aakanksha Chowdhery, Sharan Narang, Jacob Devlin, Maarten Bosma, Gaurav Mishra, Adam Roberts, Paul Barham, Hyung Won Chung, Charles Sutton, Sebastian Gehrmann, Parker Schuh, Kensen Shi, Sasha Tsvyashchenko, Joshua Maynez, Abhishek Rao, Parker Barnes, Yi Tay, Noam Shazeer, Vinodkumar Prabhakaran, Emily Reif, Nan Du, Ben Hutchinson, Reiner Pope, James Bradbury, Jacob Austin, Michael Isard, Guy Gur-Ari, Pengcheng Yin, Toju Duke, Anselm Levskaya, Sanjay Ghemawat, Sunipa Dev, Henryk Michalewski, Xavier Garcia, Vedant Misra, Kevin Robinson, Liam Fedus, Denny Zhou, Daphne Ippolito, David Luan, Hyeontaek Lim, Barret Zoph, Alexander Spiridonov, Ryan Sepassi, David Dohan, Shivani Agrawal, Mark Omernick, Andrew M. Dai, Thanumalayan Sankaranarayana Pillai, Marie Pellat, Aitor Lewkowycz, Erica Moreira, Rewon Child, Oleksandr Polozov, Katherine Lee, Zongwei Zhou, Xuezhi Wang, Brennan Saeta, Mark Diaz, Orhan Firat, Michele Catasta, Jason Wei, Kathy Meier-Hellstern, Douglas Eck, Jeff Dean, Slav Petrov, and Noah Fiedel. PaLM: Scaling language modeling with pathways. arXiv preprint arXiv:2204.02311, 2022. 8
- [9] Aniket Didolkar, Anirudh Goyal, Nan Rosemary Ke, Siyuan Guo, Michal Valko, Timothy Lillicrap, Danilo Rezende, Yoshua Bengio, Michael Mozer, and Sanjeev Arora. Metacognitive capabilities of llms: An exploration in mathematical problem solving, 2024. 1
- [10] Jesse Dodge, Maarten Sap, Ana Marasović, William Agnew, Gabriel Ilharco, Dirk Groeneveld, Margaret Mitchell, and Matt Gardner. Documenting large webtext corpora: A case study on the colossal clean crawled corpus. In EMNLP, 2021. 1
- [11] Danny Driess, Fei Xia, Mehdi S. M. Sajjadi, Corey Lynch, Aakanksha Chowdhery, Brian Ichter, Ayzaan Wahid, Jonathan Tompson, Quan Vuong, Tianhe Yu, Wenlong Huang, Yevgen Chebotar, Pierre Sermanet, Daniel Duckworth, Sergey Levine, Vincent Vanhoucke, Karol Hausman, Marc Toussaint, Klaus Greff, Andy Zeng, Igor Mordatch,

- and Pete Florence. PALM-E: An embodied multimodal language model. *arXiv preprint arXiv: 2303.03378*, 2023. 8
- [12] Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024. 1, 2
- [13] Nouha Dziri, Ximing Lu, Melanie Sclar, Xiang Lorraine Li, Liwei Jiang, Bill Yuchen Lin, Sean Welleck, Peter West, Chandra Bhagavatula, Ronan Le Bras, et al. Faith and fate: Limits of transformers on compositionality. *Advances in Neural Information Processing Systems*, 36, 2024. 3
- [14] Simon Frieder, Luca Pinchetti, Ryan-Rhys Griffiths, Tommaso Salvatori, Thomas Lukasiewicz, Philipp Petersen, and Julius Berner. Mathematical capabilities of chatgpt. Advances in neural information processing systems, 36, 2024.
- [15] Jörg Frohberg and Frank Binder. Crass: A novel data set and benchmark to test counterfactual reasoning of large language models. arXiv preprint arXiv:2112.11941, 2021. 3
- [16] Gaël Gendron, Qiming Bao, Michael Witbrock, and Gillian Dobbie. Large language models are not strong abstract reasoners. arXiv preprint arXiv:2305.19555, 2024. 3
- [17] David Ha and Jürgen Schmidhuber. World models. *arXiv* preprint arXiv:1803.10122, 2018. 3
- [18] Michael Hanna, Ollie Liu, and Alexandre Variengien. How does gpt-2 compute greater-than?: Interpreting mathematical abilities in a pre-trained language model. *Advances in Neural Information Processing Systems*, 36, 2024. 3
- [19] Danqing Huang, Shuming Shi, Chin-Yew Lin, Jian Yin, and Wei-Ying Ma. How well do computers solve math word problems? large-scale dataset construction and evaluation. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 887–896, 2016. 3
- [20] Shima Imani, Liang Du, and Harsh Shrivastava. Mathprompter: Mathematical reasoning using large language models. In *Proceedings of the 61st Annual Meeting of the As*sociation for Computational Linguistics (Volume 5: Industry Track), pages 37–42, 2023. 3
- [21] Albert Q Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier, et al. Mistral 7b. *arXiv preprint arXiv:2310.06825*, 2023. 2
- [22] Bowen Jiang, Yangxinyu Xie, Zhuoqun Hao, Xiaomeng Wang, Tanwi Mallick, Weijie J Su, Camillo J Taylor, and Dan Roth. A peek into token bias: Large language models are not yet genuine reasoners. *arXiv preprint arXiv:2406.11050*, 2024. 1
- [23] Emre Kıcıman, Robert Ness, Amit Sharma, and Chenhao Tan. Causal reasoning and large language models: Opening a new frontier for causality. arXiv preprint arXiv:2305.00050, 2023. 3
- [24] Stefanie Krause and Frieder Stolzenburg. Commonsense reasoning and explainable artificial intelligence using large language models. In *European Conference on Artificial Intelligence*, pages 302–319. Springer, 2023. 2

- [25] Nate Kushman, Yoav Artzi, Luke Zettlemoyer, and Regina Barzilay. Learning to automatically solve algebra word problems. In *Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 271–281, 2014. 3
- [26] Bo Li, Kaichen Zhang, Hao Zhang, Dong Guo, Renrui Zhang, Feng Li, Yuanhan Zhang, Ziwei Liu, and Chunyuan Li. Llava-next: Stronger llms supercharge multimodal capabilities in the wild, 2024. 2
- [27] Jiaxuan Li, Lang Yu, and Allyson Ettinger. Counterfactual reasoning: Testing language models' understanding of hypothetical scenarios. *arXiv preprint arXiv:2305.16572*, 2023. 1, 3, 5, 8
- [28] Hanmeng Liu, Ruoxi Ning, Zhiyang Teng, Jian Liu, Qiji Zhou, and Yue Zhang. Evaluating the logical reasoning ability of ChatGPT and GPT-4. *arXiv preprint* arXiv:2304.03439, 2023. 3
- [29] Haotian Liu, Chunyuan Li, Yuheng Li, and Yong Jae Lee. Improved baselines with visual instruction tuning. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 26296–26306, 2024. 7
- [30] Inbal Magar and Roy Schwartz. Data contamination: From memorization to exploitation. In ACL, 2022. 1, 3
- [31] Kamil Malinka, Martin Peresíni, Anton Firc, Ondrej Hujnák, and Filip Janus. On the educational impact of chatgpt: Is artificial intelligence ready to obtain a university degree? In Proceedings of the 2023 Conference on Innovation and Technology in Computer Science Education V. 1. ACM, 2023. 1
- [32] Iman Mirzadeh, Keivan Alizadeh, Hooman Shahrokhi, Oncel Tuzel, Samy Bengio, and Mehrdad Farajtabar. Gsmsymbolic: Understanding the limitations of mathematical reasoning in large language models. *arXiv preprint arXiv:2410.05229*, 2024. 1, 3
- [33] Neel Nanda, Andrew Lee, and Martin Wattenberg. Emergent linear representations in world models of self-supervised sequence models. *arXiv* preprint arXiv:2309.00941, 2023. 8
- [34] Marianna Nezhurina, Lucia Cipolina-Kun, Mehdi Cherti, and Jenia Jitsev. Alice in wonderland: Simple tasks showing complete reasoning breakdown in state-of-the-art large language models. arXiv preprint arXiv:2406.02061, 2024. 3, 8
- [35] OpenAI. GPT-4 technical report. *arXiv preprint arXiv:2303.08774*, 2023. 3, 8
- [36] Mihir Parmar, Nisarg Patel, Neeraj Varshney, Mutsumi Nakamura, Man Luo, Santosh Mashetty, Arindam Mitra, and Chitta Baral. Logicbench: Towards systematic evaluation of logical reasoning ability of large language models. In Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 13679–13707, 2024. 2
- [37] Judea Pearl. Causal inference in statistics: An overview. Statistics Surveys, 3:96–146, 2009. 3
- [38] Jing Qian, Hong Wang, Zekun Li, Shiyang Li, and Xifeng Yan. Limitations of language models in arithmetic and symbolic induction. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 9285–9298, 2023. 3

- [39] Lianhui Qin, Antoine Bosselut, Ari Holtzman, Chandra Bhagavatula, Elizabeth Clark, and Yejin Choi. Counterfactual story reasoning and generation. arXiv preprint arXiv:1909.04076, 2019. 3, 8
- [40] Lianhui Qin, Vered Shwartz, Peter West, Chandra Bhagavatula, Jena Hwang, Ronan Le Bras, Antoine Bosselut, and Yejin Choi. Back to the future: Unsupervised backpropbased decoding for counterfactual and abductive commonsense reasoning. arXiv preprint arXiv:2010.05906, 2020. 3
- [41] Yasaman Razeghi, Robert L Logan IV, Matt Gardner, and Sameer Singh. Impact of pretraining term frequencies on few-shot numerical reasoning. In *Findings of the Association* for Computational Linguistics: EMNLP 2022, pages 840– 854, 2022. 3
- [42] Bernardino Romera-Paredes, Mohammadamin Barekatain, Alexander Novikov, Matej Balog, M Pawan Kumar, Emilien Dupont, Francisco JR Ruiz, Jordan S Ellenberg, Pengming Wang, Omar Fawzi, et al. Mathematical discoveries from program search with large language models. *Nature*, 625 (7995):468–475, 2024. 3
- [43] Alessandro Stolfo, Yonatan Belinkov, and Mrinmaya Sachan. A mechanistic interpretation of arithmetic reasoning in language models using causal mediation analysis. In Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, pages 7035–7052, 2023. 3
- [44] Avijit Thawani, Jay Pujara, Pedro A Szekely, and Filip Ilievski. Representing numbers in nlp: a survey and a vision. *arXiv preprint arXiv:2103.13136*, 2021. 3
- [45] Kevin Wang, Alexandre Variengien, Arthur Conmy, Buck Shlegeris, and Jacob Steinhardt. Interpretability in the wild: A circuit for indirect object identification in GPT-2 small. arXiv preprint arXiv:2211.00593, 2022. 1, 8
- [46] Peng Wang, Shuai Bai, Sinan Tan, Shijie Wang, Zhihao Fan, Jinze Bai, Keqin Chen, Xuejing Liu, Jialin Wang, Wenbin Ge, Yang Fan, Kai Dang, Mengfei Du, Xuancheng Ren, Rui Men, Dayiheng Liu, Chang Zhou, Jingren Zhou, and Junyang Lin. Qwen2-vl: Enhancing vision-language model's perception of the world at any resolution. *arXiv preprint arXiv:2409.12191*, 2024. 2, 7
- [47] Yan Wang, Xiaojiang Liu, and Shuming Shi. Deep neural solver for math word problems. In *Proceedings of the 2017 conference on empirical methods in natural language processing*, pages 845–854, 2017. 3
- [48] Jason Wei, Yi Tay, Rishi Bommasani, Colin Raffel, Barret Zoph, Sebastian Borgeaud, Dani Yogatama, Maarten Bosma, Denny Zhou, and Donald Metzler. Emergent abilities of large language models. *Transactions on Machine Learning Research*, 2022. 8
- [49] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed H Chi, Quoc V Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models. In Advances in Neural Information Processing Systems, pages 24824–24837, 2022. 1, 3
- [50] Mitchell Wortsman, Gabriel Ilharco, Jong Wook Kim, Mike Li, Simon Kornblith, Rebecca Roelofs, Raphael Gontijo Lopes, Hannaneh Hajishirzi, Ali Farhadi, Hongseok Namkoong, and Ludwig Schmidt. Robust fine-tuning of

- zero-shot models. In 2022 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 7949–7961, New Orleans, LA, USA, 2022. 8
- [51] Zhaofeng Wu, Linlu Qiu, Alexis Ross, Ekin Akyürek, Boyuan Chen, Bailin Wang, Najoung Kim, Jacob Andreas, and Yoon Kim. Reasoning or reciting? exploring the capabilities and limitations of language models through counterfactual tasks. *arXiv preprint arXiv:2307.02477*, 2023. 1, 3
- [52] Zhengxuan Wu, Atticus Geiger, Thomas Icard, Christopher Potts, and Noah Goodman. Interpretability at scale: Identifying causal mechanisms in alpaca. Advances in Neural Information Processing Systems, 36, 2024. 3
- [53] Fangzhi Xu, Qika Lin, Jiawei Han, Tianzhe Zhao, Jun Liu, and Erik Cambria. Are large language models really good logical reasoners? a comprehensive evaluation from deductive, inductive and abductive views. arXiv preprint arXiv:2306.09841, 2023. 2
- [54] Fangzhi Xu, Qika Lin, Jiawei Han, Tianzhe Zhao, Jun Liu, and Erik Cambria. Are large language models really good logical reasoners? A comprehensive evaluation and beyond. arXiv preprint arXiv:2306.09841, 2024. 3
- [55] An Yang, Baosong Yang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Zhou, Chengpeng Li, Chengyuan Li, Dayiheng Liu, Fei Huang, Guanting Dong, Haoran Wei, Huan Lin, Jialong Tang, Jialin Wang, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Ma, Jin Xu, Jingren Zhou, Jinze Bai, Jinzheng He, Junyang Lin, Kai Dang, Keming Lu, Keqin Chen, Kexin Yang, Mei Li, Mingfeng Xue, Na Ni, Pei Zhang, Peng Wang, Ru Peng, Rui Men, Ruize Gao, Runji Lin, Shijie Wang, Shuai Bai, Sinan Tan, Tianhang Zhu, Tianhao Li, Tianyu Liu, Wenbin Ge, Xiaodong Deng, Xiaohuan Zhou, Xingzhang Ren, Xinyu Zhang, Xipin Wei, Xuancheng Ren, Yang Fan, Yang Yao, Yichang Zhang, Yu Wan, Yunfei Chu, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zhihao Fan. Qwen2 technical report. arXiv preprint arXiv:2407.10671, 2024. 2
- [56] Xiaoyu Yang, Stephen Obadinma, Huasha Zhao, Qiong Zhang, Stan Matwin, and Xiaodan Zhu. Semeval-2020 task 5: Counterfactual recognition. arXiv preprint arXiv:2008.00563, 2020. 3
- [57] Tian Ye, Zicheng Xu, Yuanzhi Li, and Zeyuan Allen-Zhu. Physics of language models: Part 2.1, grade-school math and the hidden reasoning process. *arXiv preprint arXiv:2407.20311*, 2024. 1, 3
- [58] Wei Zhang, Chaoqun Wan, Yonggang Zhang, Yiu-ming Cheung, Xinmei Tian, Xu Shen, and Jieping Ye. Interpreting and improving large language models in arithmetic calculation. arXiv preprint arXiv:2409.01659, 2024. 1, 3, 8
- [59] Yaowei Zheng, Richong Zhang, Junhao Zhang, Yanhan Ye, Zheyan Luo, Zhangchi Feng, and Yongqiang Ma. Llamafactory: Unified efficient fine-tuning of 100+ language models. In Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 3: System Demonstrations), Bangkok, Thailand, 2024. Association for Computational Linguistics. 5
- [60] Hattie Zhou, Arwen Bradley, Etai Littwin, Noam Razin, Omid Saremi, Josh Susskind, Samy Bengio, and Preetum

Nakkiran. What algorithms can transformers learn? a study in length generalization. *arXiv preprint arXiv:2310.16028*, 2023. 1