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# Line Assignment

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**Problem Statement:**

The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1)(1, 1)and (1,0) is:

(a)$2+\sqrt{2}$ (b)$2-\sqrt{2}$
(c)$1+\sqrt{2}$ (d)$1-\sqrt{2}$

**Construction**

| vertex | coordinates |
|---|---|
| B | $\begin{pmatrix}0\\0\end{pmatrix}$ |
| P | $\begin{pmatrix}1\\0\end{pmatrix}$ |
| Q | $\begin{pmatrix}0\\1\end{pmatrix}$ |
| R | $\begin{pmatrix}1\\1\end{pmatrix}$ |
| k | 1 |
| C | $\begin{pmatrix}2\\0\end{pmatrix}$ |
| A | $\begin{pmatrix}0\\2\end{pmatrix}$ |

**Solution:** If a point Q divides the line segment AB in the ratio k: 1 is given by

$$Q = \frac{(kB + A)}{k + 1} \tag{1}$$

If a point P divides the line segment BC in the ratio k: 1 is given by

$$P = \frac{(kB + C)}{k + 1} \tag{2}$$

Given P(1,0),Q(0,1) and R(1,1) are the mid points of the sides of a triangle.
Mid points divide the linesegment in 1:1 ratio.
$\therefore$ k=1

By substituting k=1 in equation (1)

$$Q = \frac{B + A}{2} \tag{3}$$

$$A = 2Q - B............(4) \tag{4}$$

Given Q $\begin{pmatrix}0\\1\end{pmatrix}$

Take B $\begin{pmatrix}0\\0\end{pmatrix}$

By substituting Q and B in equation (4)

A $=\begin{pmatrix}0\\2\end{pmatrix}$

By substituting k=1 in equation (2)

$$P = \frac{B + C}{2} \tag{5}$$

$$C = 2P - B...........(5) \tag{6}$$

We have P $\begin{pmatrix}1\\0\end{pmatrix}$ B $\begin{pmatrix}0\\0\end{pmatrix}$

By substituting P and B in equation (5)

C $=\begin{pmatrix}2\\0\end{pmatrix}$

we have A $\begin{pmatrix}0\\2\end{pmatrix}$ B $\begin{pmatrix}0\\0\end{pmatrix}$ C $\begin{pmatrix}2\\0\end{pmatrix}$

Vector representation of A,B,C are as follows

$$\mathbf{A} = \mathbf{2j} \tag{7}$$

$$\mathbf{B} = \mathbf{0} \tag{8}$$

$$\mathbf{C} = \mathbf{2i} \tag{9}$$

The vectors of AB,BC and CA linesegments are

$$\mathbf{V1} = \mathbf{B} - \mathbf{A} = -\mathbf{2j} \tag{10}$$

$$\mathbf{V2} = \mathbf{C} - \mathbf{B} = \mathbf{2i} \tag{11}$$

$$\mathbf{V3} = \mathbf{A} - \mathbf{C} = \mathbf{2j} - \mathbf{2i} \tag{12}$$

Norms of the vectors V1, V2 and V3 are

$$\|\mathbf{V1}\| = 2 \tag{13}$$

$$\|\mathbf{V2}\| = 2 \tag{14}$$

$$\tag{15}$$

$$\|\mathbf{V3}\| = 2\sqrt{2} \tag{16}$$

The incenter is the intersection of three angle bisectors,

$$I = \frac{\|\mathbf{V1}\|\,\mathbf{C} + \|\mathbf{V2}\|\,\mathbf{A} + \|\mathbf{V3}\|\,\mathbf{B}}{\|\mathbf{V1}\| + \|\mathbf{V2}\| + \|\mathbf{V3}\|} \tag{17}$$

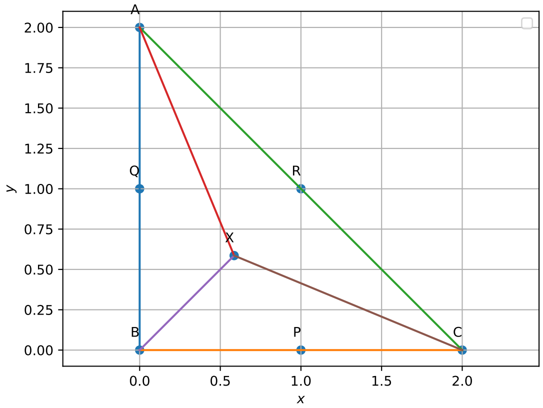
$$I = \frac{2(\mathbf{2i}) + 2(\mathbf{2j}) + 2\sqrt{2}(\mathbf{0})}{2 + 2 + 2\sqrt{2}} \tag{18}$$

$$I = \frac{\mathbf{4i} + \mathbf{4j}}{4 + 2\sqrt{2}} \tag{19}$$

The x-coordinate of the incentre of the triangle is

$$x = \frac{\mathbf{4i}}{4 + 2\sqrt{2}} \tag{20}$$

$$x = 2 - \sqrt{2} \tag{21}$$




Download the code from Github link: Assignment-4.