

# TERM PROJECT (EE310)

## Sizing of Hybrid Energy-Storage Systems Using Recurring Daily Patterns

Submitted To: Dr. Ranjana Sodhi

Submitted By: Prachi (2022EEB1200), Pratibha Garg (2022EEB1204)

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### INTRODUCTION

Hybrid Energy Storage Systems (ESS) combine technologies like batteries and flywheels to handle both slow energy variations and fast power fluctuations. Sizing such systems typically needs large, high-resolution datasets, making the process computationally heavy. This paper proposes an improved motif discovery algorithm that identifies the most recurring daily pattern from historical data, allowing efficient sizing using a single representative profile. It also introduces an optimization framework to find the best cut-off frequency for dividing power between the battery and the fast storage device. The method is validated on real low-voltage grids in Germany, showing reliable performance over long-term datasets.

### OBJECTIVE

- **To introduce an improved motif discovery method** for extracting the most recurring daily consumption pattern from large, high-resolution datasets, and using this pattern for efficient and accurate sizing of hybrid Energy Storage Systems (ESS).
- **To propose an optimization-based approach** to determine the optimal cut-off frequency for low-pass filtering, ensuring effective allocation of low-frequency energy fluctuations to the battery and high-frequency power variations to the fast storage device like a flywheel.

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## METHODOLOGY

### Data Set

Our Data Set is collected from a MW-size energy storage pilot system located on the Baoshan campus of National Changhua University of Education (NCUE). This dataset is a time-series of changes in parameters such as voltage , current , power factor and Kilowatt hour for the month of september. We have used real power and their respective time stamps for our analysis. Although the time stamps are varying, the average resolution is around 33sec. [ [Data Set](#) ]

### Recurring Pattern Extraction

#### 1. SEQUENCING ALGORITHM

**Segmentation:** Divide the overall time series of each day into segments of equal length  $l$ . Let a daily sequence be represented as:

$$X=\{x_1,x_2,...,x_T\}$$

where  $T$  is the number of time points in one day.

**Normalization:** Standardize the sequence to remove scale effects so that differences in amplitude do not unduly affect similarity comparisons. We can use z-normalization on the sequence. This rescales the data with a mean of zero and a standard deviation of one. Normalize each data point as:

$$z_i=(x_i-\mu)/\sigma \quad (\text{for } i=1,2,...,T)$$

The normalized sequence is then:

$$Z=\{z_1,z_2,...,z_T\}$$

#### 2. SYMBOLIC AGGREGATED APPROXIMATION

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**Piecewise Aggregate Approximation (PAA):** Compress each subsequence from length  $m$  to  $w$  segments by computing the mean within each segment. For segment  $j$  (where  $j=1,2,\dots,w$ )

$$\bar{z}_j = \frac{w}{T} \sum_{i=\frac{(j-1)T}{w}+1}^{\frac{jT}{w}} z_i$$

**Symbolization:** Convert each of the  $w$  PAA values into discrete symbols using breakpoint derived from the data distribution.

Let  $A=\{\alpha_1,\alpha_2,\dots,\alpha_a\}$  be an alphabet of size  $a$ . The  $\beta$  parameters are based on the quantiles of the empirical cumulative distribution function, which for observations  $x = (x_1,x_2,\dots,x_n)$  is defined as

$$F_n(p) = \frac{1}{n} \sum_{i=1}^n I(x_i \leq p).$$

Map Each Segment Average to a Symbol:

$$S_j = \alpha_k \text{ if } x_{k-1} \leq z_j < x_k \quad \text{for } \{k=1,2,\dots,a\}.$$

The final symbolic representation of the daily sequence is:

$$S = s_1 s_2 \dots s_w$$

### 3. RANDOM PROJECTION

All symbolic representations of the sequences are arranged row-wise in a similarity matrix  $S^*$  of size  $N \times w$ , where  $N$  is the number of sequences and  $w$  is the word length.

$$S^* = \begin{bmatrix} s_{1,1} & s_{1,2} & \dots & s_{1,w} \\ s_{2,1} & s_{2,2} & \dots & s_{2,w} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N,1} & s_{N,2} & \dots & s_{N,w} \end{bmatrix}$$

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In each iteration, a random subset of  $l$  columns (with  $l \leq w$ ) is selected. This subset acts as a mask to extract a partial word from each row. Randomly choose a set of column indices  $M \subset \{1, 2, \dots, w\}$  with  $|M| = l$ . For each row  $i$ , extract the masked word:

$$r_i(M) = \{s_i, j : j \in M\}$$

Compare the masked words across all rows. For any two rows  $i$  and  $j$ , if  $r_i(M) = r_j(M)$  then increment the corresponding entry  $C_{ij}$  in the collision matrix  $C$ . This can be expressed as

$$C_{ij} \leftarrow C_{ij} + 1$$

$$C_{ji} \leftarrow C_{ji} + 1$$

After many iterations, entries in  $C$  indicate how frequently pairs of sequences share the same pattern on the masked columns.

#### 4. CANDIDATE EVALUATION

Candidate motifs are selected as those pairs (or clusters) with collision values exceeding a threshold.. The alignment is performed using Dynamic Time Warping (DTW), which finds the optimal warping path that minimizes the cumulative distance between two sequences.

**Distance Matrix Construction:** For two sequences  $y = [y(1), y(2), \dots, y(n)]$  and  $z = [z(1), z(2), \dots, z(m)]$  define the local distance:

$$D(i, j) = (y(i) - z(j))^2 \quad \text{for } i=1, \dots, n, j=1, \dots, m$$

**Cumulative Cost Matrix (Dynamic Programming):** Initialize:

$$C(1, 1) = D(1, 1)$$

$$C(i, 1) = D(i, 1) + C(i-1, 1)$$

$$C(1, j) = D(1, j) + C(1, j-1)$$

$$\text{Then for } i \geq 2 \quad C(i, j) = D(i, j) + \min\{C(i-1, j), C(i, j-1), C(i-1, j-1)\}$$

**DTW Distance:**  $DTW(y, z) = C(n, m)$  which is the minimal accumulated cost to align the two sequences.

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**Optimal Warping Path:** Backtrack from  $C(n,m)$  to  $C(1,1)$  to find the alignment path that achieved this minimal cost.

## 5. SELECTING THE MOST RECURRING PATTERN

**Motif Consolidation:** After identifying motifs in the previous steps, each time series typically yields 1 to 3 motifs that cover most of the days in the dataset. These motifs capture recurring daily shapes or patterns (e.g., power consumption over 24 hours).

**Defining the Standard Pattern:** The standard pattern is derived by taking the 80% quantile across all detected motifs at each time point. Formally, if  $\{x_t(i)\}$ ,  $i=1$  to  $k$ , are the values at time  $t$  across  $k$  motifs, then

$$P_{\text{std}}(t) = \text{Quantile}_{80\%} \left( \{x_t^{(i)}\}_{i=1}^k \right).$$

This produces a single representative curve (per bus or feeder) that encapsulates the majority behavior of the grid over a typical day.

## Finding Optimal Cut-off Frequency

Daily energy consumption can be divided into an average component and a time-varying component:

$$P_{\text{avg}} = (1/T) \int_0^T P_{\text{load}}(t) dt$$

The hybrid ESS handles the residual power:

$$P_{\text{res}}(t) = P_{\text{load}}(t) - P_{\text{avg}}$$

Sizing of this hybrid ESS involves four fundamental parameters: the nominal power rating  $P_n$ , the nominal energy capacity  $E_n$ , the maximum ramp rate  $R_n$ , and the number of mode changes.

PARAMETER	DEFINITION	FORMULA(s)	IMPACT OF SIZING
<b>Nominal Power (P<sub>n</sub>)</b>	The maximum effective power, ESS must handle, limited by the converter's rating and factoring in efficiency (charge/discharge).	$R(t) = \eta^{-\text{sgn}(P(t))} \cdot P(t)$ $P_n = \max  R(t) $ $(\eta \approx 90\%)$	Determines converter rating. A higher P <sub>n</sub> means a larger converter and the possibility of including a safety margin for peak variations.
<b>Nominal Capacity (E<sub>n</sub>)</b>	This dictates battery cell count or flywheel inertia and rotational speed.(SOC range between 10%-90% for Li-ion batteries & 25%-100% for flywheels)	$E(t) = \eta^{-\text{sgn}(P(t))} \int_0^t P(\tau) d\tau$ $E_n = [\max E(t) - \min E(t)] / [\text{SOC}_{\max} - \text{SOC}_{\min}]$	Larger E <sub>n</sub> increases cost but avoids extreme SOC ranges, and also extends system lifetime by preventing deep discharges and excessive charge levels.
<b>Maximum Ramp Rate (R<sub>n</sub>)</b>	The highest rate of change in the ESS power output, critical for managing rapid load/generation fluctuations.	$R_n = \max [R(t) - R(t - \Delta t)]$ $\Delta t = 1/f_s$	Requires robust power electronics and mechanical design. Typically handled by the flywheel, protecting the battery from fast transients.
<b>Number of Mode Changes</b>	How often the ESS switches between charging and discharging, counted via zero-crossings of the ESS power profile.	$N_{\text{modes}} = \text{zero-crossings of } P_{\text{ESS}}(t)$	Frequent switching accelerates battery wear. Reducing mode changes extends battery life and improves reliability.

## 1. OBJECTIVE FUNCTION AND ITS DEPENDENCIES:

**Objective:** Minimize flywheel capacity while maximizing its power and reducing the battery's ramp rate

$$\min_{f_c} OF = c_1 \frac{\frac{E_{n2}}{E_{n1}}}{\max \left( \frac{E_{n2}}{E_{n1}} \right)} + c_2 \frac{\frac{P_{n1}}{P_{n2}}}{\max \left( \frac{P_{n1}}{P_{n2}} \right)} + c_3 \frac{\frac{R_{n1}}{R_{n2}}}{\max \left( \frac{R_{n1}}{R_{n2}} \right)}$$

$$\text{s.t. } 0 \leq f_c \leq 0.5f_s \quad ($$

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## Parameters:

$E_n$  = Nominal Energy Capacity

$P_n$  = Nominal Power Rating

$R_n$  = Maximum Ramp Rate

Index 1: Li-ion Battery | Index 2: Flywheel

## Assumptions Driving the OF:

1. Flywheel capacity is more expensive than battery capacity.
2. Flywheels are preferred for high ramp rates due to their faster response.
3. Flywheels power needs to be maximised wrt to the battery's power rating.

## SIMULATION RESULTS

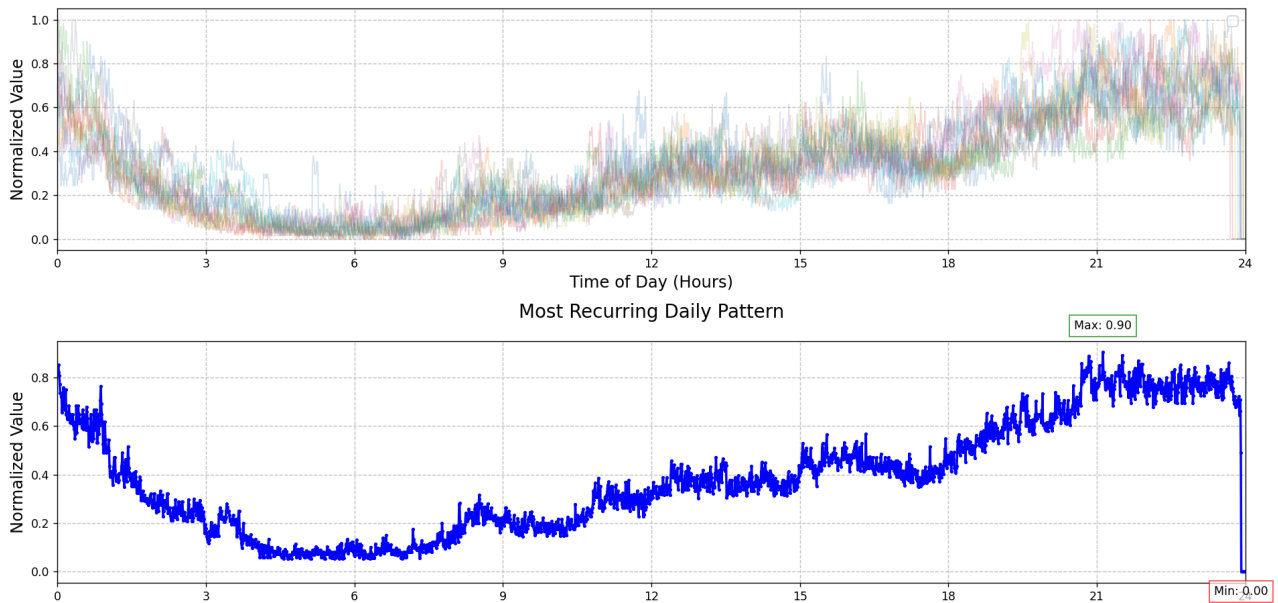


Fig 1 (a) Most repeating patterns after performing initial 4 steps of Motif Discovery Method, (b) Performing 80% quantile method on the repeating patterns to generate a 'Standard Pattern' or the most recurring pattern.

After finding the most recurring pattern, we implement the gradient search algorithm to find the optimal cutoff frequency and divide power between battery and flywheel.

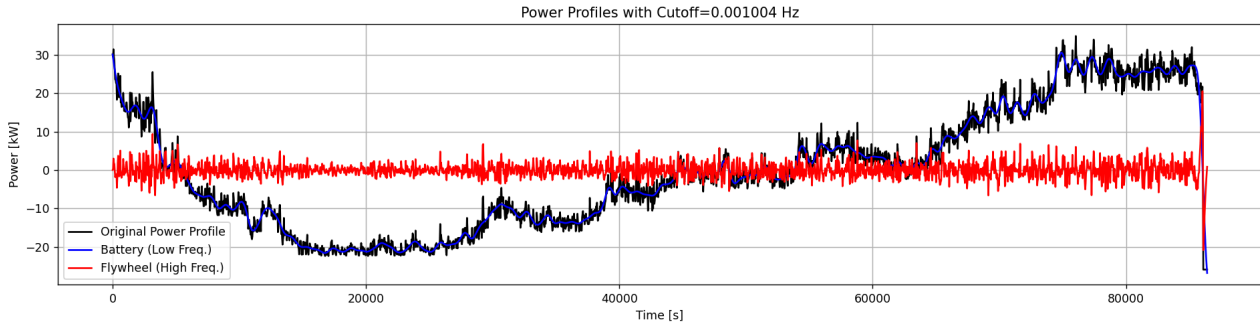


Fig 2 Allocation of power between battery and flywheel at optimal cutoff frequency ( $f_c$ )

In this graph, the original power profile (black) is split into a low-frequency component handled by the battery (blue) and a high-frequency component managed by the flywheel (red). The battery smooths out the major energy trends, while the flywheel quickly responds to rapid power fluctuations.

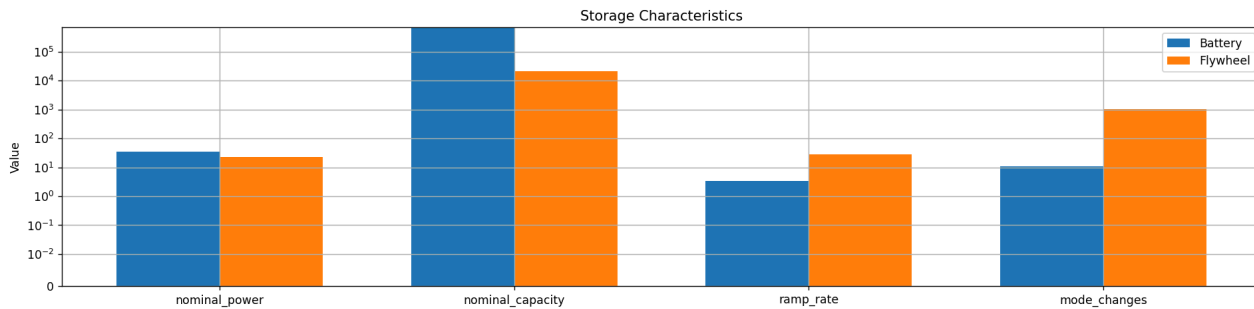


Fig 3 Comparison of storage characteristics ( $P_n$ ,  $E_n$ , Ramp rate and Mode changes) between battery and flywheel.

This bar graph compares key storage characteristics of the battery and flywheel. The battery has higher nominal capacity but lower ramp rate and fewer mode changes, making it suitable for handling large, slow power variations. In contrast, the flywheel has lower capacity but higher ramp rate and significantly more mode changes, justifying its role in quickly managing rapid, high-frequency power fluctuations.



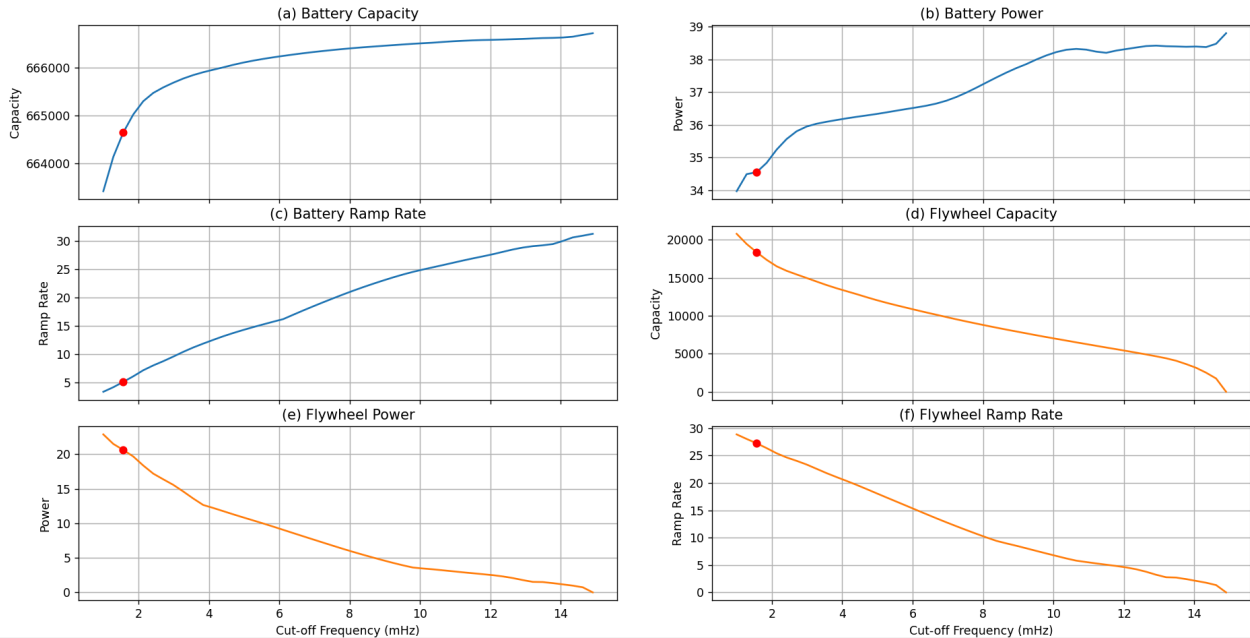


Fig 4 Change in capacity, power, ramp rate of battery and flywheel by varying frequency. Values at the cutoff frequency are marked with red.

### Effect of Increasing Cut-Off Frequency:

- **Li-ion Battery:**

- **↑ Increased Ramp Rate** – Battery faces higher stress, which reduces its lifespan.
- **↑ Higher Rated Power** – Power demand on the battery increases.
- **→ Capacity Remains Almost Same** – Storage capacity shows minimal change.

- **Flywheel:**

- **↓ Decreased Capacity** – Significant drop in energy stored in the flywheel.
- **↓ Lower Power and Ramp Rate** – Flywheel's ability to handle power and quick changes decreases.

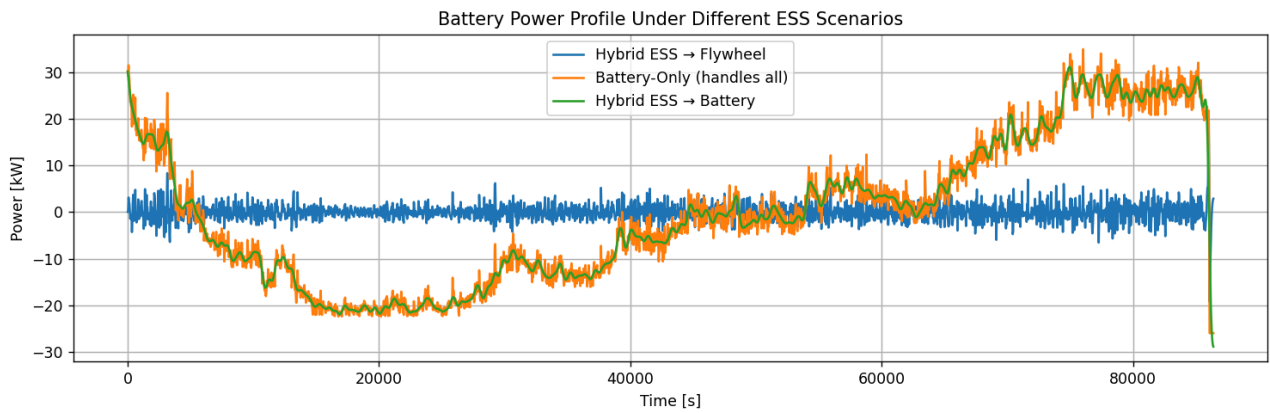


Fig 5 Power profile for different storage systems (a) Only Battery (b) Hybrid ESS

This plot shows how the power demand is split between a flywheel and a battery in a **hybrid energy storage system (ESS)**.

- The **blue curve** ("Hybrid ESS → Flywheel") handles the fast, fluctuating power changes.
- The **green curve** ("Hybrid ESS → Battery") handles the slow, smoother power changes.
- The **orange curve** ("Battery-Only") shows how the battery would have to manage everything alone without hybrid support.

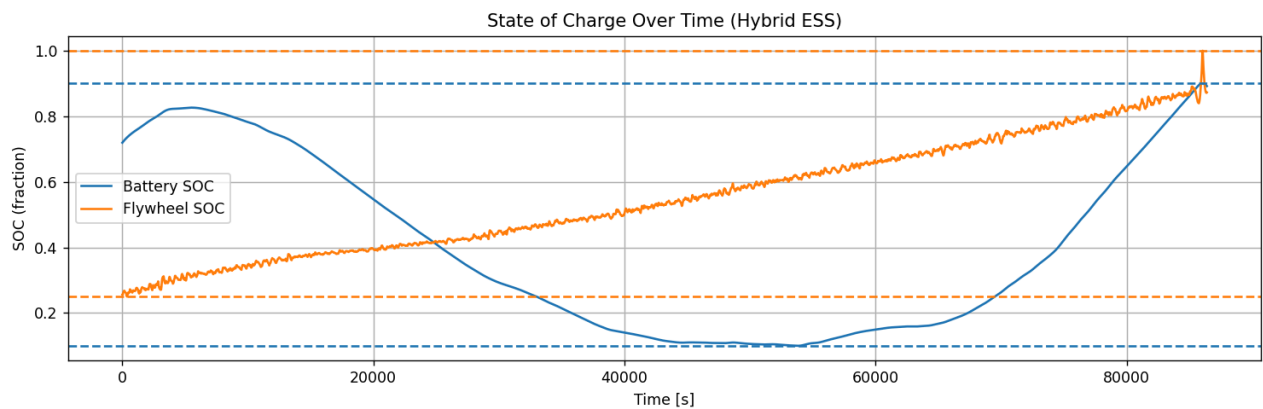


Fig 6 This graph shows the State of Charge (SOC) over time for a hybrid energy storage system (battery + flywheel).

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The battery SOC decreases initially and then recovers later, while the flywheel SOC steadily increases over time. The dashed lines represent the upper and lower SOC limits for both battery and flywheel.

## ALTERNATIVE APPROACH

**Empirical Mode Decomposition (EMD)** is a technique that breaks a complex power profile into multiple simpler signals called Intrinsic Mode Functions (IMFs).

Each IMF captures a different frequency band:

- High-frequency IMFs show quick, small changes (spikes, fluctuations).
- Low-frequency IMFs show slow, big changes (gradual energy demand/supply swings).

In hybrid ESS sizing using EMD:

- **Low-frequency IMFs** are assigned to the **battery**, because batteries are good at handling large, slow energy needs.
- **High-frequency IMFs** are assigned to the **flywheel** because they are better at handling fast power spikes.

## COMPARISON:

Compared to the low-pass filter (LPF) method, where an optimal cut-off frequency had to be manually selected through optimization of sizing parameters, the EMD-based approach offers a fully data-driven, adaptive decomposition of the residual power profile. EMD naturally separates slow and fast dynamics without requiring external frequency tuning, handles non-stationary patterns better, and provides a cleaner division of energy and power components for hybrid ESS sizing. Although EMD is slightly more computationally intensive, it improves robustness and reduces the need for manual parameter adjustments.

Method	Component	nominal_power	nominal_capacity	ramp_rate	mode_changes
LPF	battery	35.44	657905.83	4.88	21.00
LPF	flywheel	17.36	15624.01	27.62	1154.00
EMD	battery	35.72	657690.86	6.42	39.00
EMD	flywheel	19.79	14127.49	29.21	1197.00

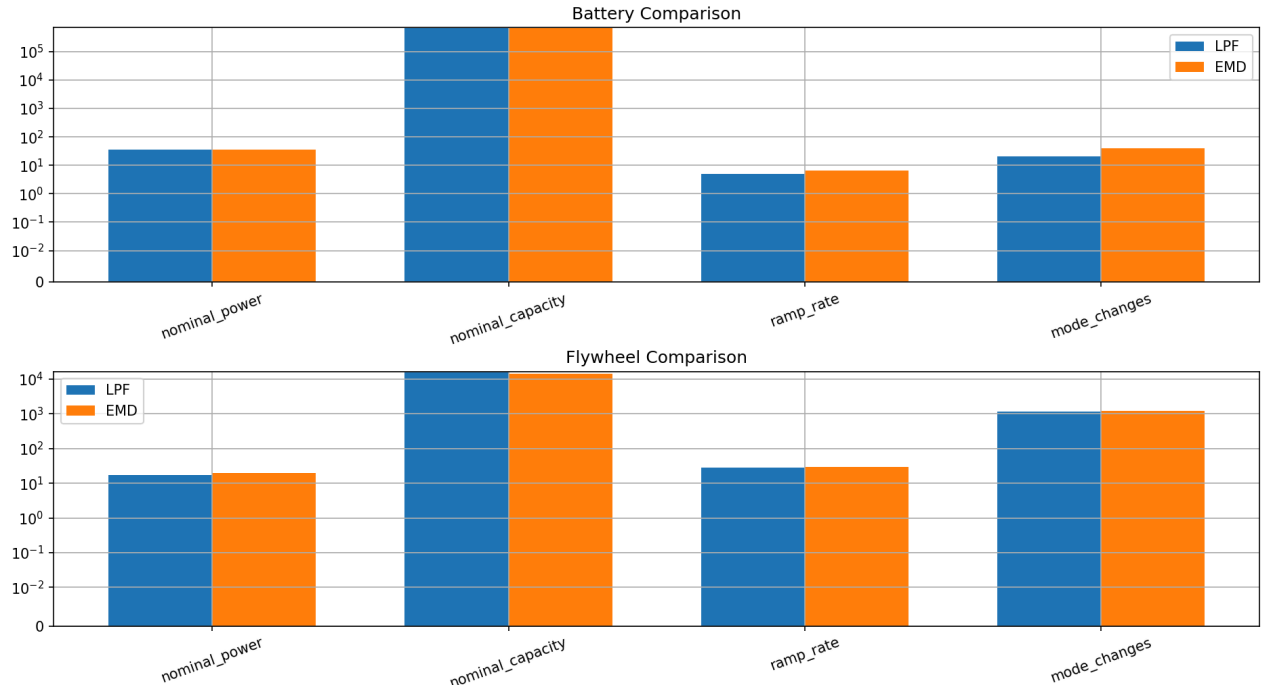


Fig 6 (a) Bar graph representing comparison between battery and flywheel characteristics using earlier LPF method and an alternative approach, EMD method, (b) Measurements of comparison between battery and flywheel characteristics

## DISCUSSION

The results demonstrate the effectiveness of the proposed motif-based sizing methodology and optimal cutoff selection in tailoring a hybrid battery-flywheel ESS to the dominant daily load pattern:

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### 1. **Representative Pattern Extraction**

- Fig 1(a) & (b) show that motif discovery reliably isolates the most recurring daily consumption profile, removing noise and rare outliers. By using the 80 %-quantile “standard pattern,” we avoid skewing the sizing toward atypical high-or low-load days.

### 2. **Optimal Cutoff Frequency**

- As in Fig 2, the gradient-search optimization finds a cutoff that balances flywheel energy capacity against battery ramp-rate benefit.
- At this cutoff, the flywheel handles fast fluctuations (red curve), while the battery smooths the low-frequency trend (blue curve), minimizing both flywheel capacity and battery stress simultaneously.

### 3. **Storage Characteristic Trade-Offs**

- Fig 3 and illustrate that the battery has high energy capacity but lower maximum power and ramp rate, whereas the flywheel exhibits the opposite. This complementary split justifies the hybrid ESS: the flywheel rapidly absorbs/transmits spikes without cycling the battery excessively, and the battery manages the bulk of energy shift over hours.
- Varying the cutoff (Fig 4) shifts this trade-off predictably: higher cutoff moves more load into the battery (higher ramp-rate), while lower cutoff over-loads the flywheel.

## **CONCLUSION**

A two-step sizing approach—(1) motif discovery to extract a standard daily load pattern, and (2) cutoff frequency optimization to split that pattern between battery and flywheel—enables cost-effective, reliable hybrid ESS design:

- **Efficiency:** Reduces data handling by summarizing multi-day high-resolution measurements into one representative profile.
- **Cost-Effectiveness:** Minimizes both battery wear (by limiting ramp demands) and flywheel oversizing (by constraining its energy capacity).

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- **Robustness:** Ensures that the ESS can deliver its intended grid services (peak shaving, fluctuation smoothing) for most days without re-tuning.

This methodology is applicable to any distribution feeder with recurring daily patterns, and can be extended to other hybrid storage combinations

## REFERENCES

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