A decision support system for forecasting and optimal procurement

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ABSTRACT

Optimal procurement in most industries involves forecasting of two quantities: prices of raw materials and customer's demand. The aim of this work is to integrate forecasts into production planning models, with the aim of minimizing overall procurement, holding and production costs under demand satisfaction constraints. The decision support system should allow the decision maker to integrate qualitative, unstructured information through a simple interface, for scenario selection and solution refinement

1

RELATED WORK

1.1 NEWSVENDOR MODEL

The newsvendor model[1] is a mathematical model resembling newsvendor stand. The problem is characterized by fixed prices and uncertain demand for a perishable product, e.g. yesterdays newspapers hold no value today. Demand for newspapers is uncertain and newsvendor must decide how many newspaper to buy for reselling.

The stock in newsvendor model is only for one day, and unlike problem presented in chapter 2 product is not perishable, but its cost increases in each following time step.

1.2 DYNAMIC LOT SIZING MODEL

Dynamic lot-size model[2] is generalization of the Economic Order quantity model[3] which takes into account that demand for the product varies over time. For the planning horizon of n time periods we have following data:

1 K. J. Arrow and T. Harris. "Optimal Inventory Policy (1951)". In: *Economic Information, Decision, and Prediction*. Springer Science Business Media, 1974, pp. 5–28. DOI: `10.1007/978-94-010-9278-4_2`. URL: `http://dx.doi.org/10.1007/978-94-010-9278-4_2`.

2 Harvey M. Wagner and Thomson M. Whitin. "Dynamic Version of the Economic Lot Size Model". In: *Management Science* 50.12_supplement (2004), pp. 1770–1774. DOI: `10.1287/mnsc.1040.0262`. URL: `http://dx.doi.org/10.1287/mnsc.1040.0262`.

3 Ford W. Harris. "How Many Parts to Make at Once". In: *Operations Research* 38.6 (1990), pp. 947–950. DOI: `10.1287/opre.38.6.947`. URL: `http://dx.doi.org/10.1287/opre.38.6.947`.

d_t	Demand at time period t
h_t	Holding cost at time period t
K_t	Setup cost at time period t
I_0	Initial inventory

and decision variable $\mathbf{x}$:

x_t	Quantity purchased at time period t

For simplicity we define inventory at time period t as:

$$I_t = I_0 + \sum_{t=0}^{k} x_t - \sum_{t=0}^{k} d_t$$

And we want to choose optimal x_t, under following constraints:

$$x_t \geq 0 \ \forall t$$
$$I_t \geq 0 \ \forall t$$

And we want to minimize following objective function:

$$f = \sum_t h_t I_t + H(x_t) K_t$$

where H is Heaviside step function.

1.2.1 *Dynamic lot size model in stochastic setting*

Various approaches exist to handle dynamic lot sizing model in stochastic setting. The most comprehensive analysis can be found in textbook.[4] Other ap-

4 Horst Tempelmeier. "Stochastic lot sizing problems". In: *Handbook of Stochastic Models and Analysis of Manufacturing System Operations*. Springer, 2013, pp. 313–344.

proaches use rolling horizon for predicting demand quantity[5] and apply modified dynamic lot sizing model by introducing backlogging, that is allowing inventory, I_t to be negative with known penalty, b_t per unit per time step.

5 Yu Cao et al. "Adaptive procurement planning in global sourcing: A rolling horizon forecasting approach". In: *Networking, Sensing and Control (ICNSC), 2013 10th IEEE International Conference on*. IEEE. 2013, pp. 714–719.

2

PROBLEM DEFINITION

2.1 INTRODUCTION

This problem is a variation of the lot sizing problem in a stochastic setting. Variation is as follows. Firstly we describe deterministic variant before extending the model to stochastic setting. We have predictions for raw material supply cost $\mathbf{s}$ at future time moment t. The *factory* converts all bought raw materials at time t and stores them in a warehouse. Number of discrete time moments under consideration is denoted by n.

One time moment storage costs fixed amount h. The holding cost is per unit and per time period.

Each time moment, t, we have certain demand we need to satify, denoted as $\mathbf{d}_t$. In case there's no products in storage to satisfy demand we allow backlogging incurring a cost denoted as b per unit per day, for late orders.

Our aim is to optimize our procurement policy by choosing $\mathbf{x}$, which is amount of the product we buy at time moment t. However we are constrained by the maximum amount raw materials , $\mathbf{x}^{\max}$, that we can buy each day.

2.2 FORMAL PROBLEM DEFINITION

2.2.1 *Definitions*

Following is the deterministic problem variant, and in subsequent chapters randomness and uncertainty is embedded into problem. Notation:

$\mathbf{s} = [s_1, s_2, \ldots, s_n]^\mathsf{T}$	Supply cost vector
$\mathbf{x} = [x_1, x_2, \ldots, x_n]^\mathsf{T}$	Procurement quantity vector
$\mathbf{x}^{(\max)}$	Procurement quantity limits vector
$\mathbf{x}^{(b)}$	Backlogging quantity vector
$\mathbf{x}^{(h)}$	Holding quantity vector
$\mathbf{d} = [d_1, d_2, \ldots, d_n]^\mathsf{T}$	Demand random vector
b	backlogging cost
h	holding cost
n	number of time moments

Every variable for backlogging and holding vector seems unintuitive or taken. I hope $\mathbf{x}^{(b)}$ is alright since relation to $\mathbf{x}$ is clearly highlighted

2.2.2 *Variables*

$\mathbf{x}$ is our decision variable, as described previously. $\mathbf{x}^{(b)}$ and $\mathbf{x}^{(h)}$ are backlogging and holding variables respectively backlogging variables respectively. For simplicity $x_0^{(h)}$, $x_0^{(b)}$, $x_n^{(h)}$, $x_n^{(b)}$ are equal to 0 unless otherwise noted. This specific values are explored further in section 3.2.

2.2.3 *Constraints*

$$\begin{aligned}
x_t &\leq x_t^{(\max)} && \forall t \\
x_t &\geq 0 && \forall t \\
x_t^{(b)} &\geq 0 && \forall t \\
x_t^{(h)} &\geq 0 && \forall t \\
x_t + x_{t-1}^{(h)} + x_t^{(b)} &= d_t + x_t^{(h)} + h_{t-1}^{(b)} && \forall t \\
x_0^{(h)} + x_n^{(b)} + \sum_{t=1}^{n} x_i &= \sum_{t=1}^{n} d_i + x_0^{(b)} + x_n^{(h)}
\end{aligned}$$

2.3 OBJECTIVE FUNCTION

Definition 2.3.1. $c(t)$ defines total speeding we pay at time t.

$$c(t) = +s_t x_t + b x_{b_t} + h x_{h_t} \tag{1}$$

Definition 2.3.2. f is objective function for this problem. Our aim is to minimize it.

$$f = \sum_t c(t) = \sum_t s_t x_t + b x_{b_t} + h x_{h_t} \tag{2}$$

3

DETERMINISTIC APPROACH

3.1 MODELING APPROACHES

Problem as defined in section 2.2 can be reduced to two well known problems: transportation and min-cost max flow problem. In both cases from knowledge of $\mathbf{x}$ and constraint equations, $\mathbf{x}^{(h)}$ and $\mathbf{x}^{(b)}$ since at each t in optimal solution at least one of $x_t^{(b)}$ and $x_t^{(h)}$ is 0. This fact can easily be observed from flow conservation on intermediate nodes as in fig 3.1.2. Setting them both to positive values create positive flow cycle with positive cost, which can be canceled yielding same flow with lower cost.

3.1.1 *Reduction to transportation problem*

For transportation problem reduction we need cost matrix for satisfying demand at time j with supply at time i. It is given as follows:

Definition 3.1.1. $\mathbf{C}$ matrix defines cost for satisfying demand with specific raw supply material purchase date. It's element c_{ij} equals:

$$c_{ij} = \begin{cases} b\,(i-j) + s_i & j < i \\ h\,(j-i) + s_i & j \geq i \end{cases}$$

That is using raw materials purchased at i to satisfy demand at time j incurs cost c_{ij}.

Maximum supply $\mathbf{x}^{(\max)}$ is given, so is the demand vector $\mathbf{d}$. Since transportation problem required equal supply and demand nodes, we add a dummy demand node consuming excess supply.

"That is 0 in the deterministic..." I don't understand what is the meaning of this remark and how can implement your feedback.

Transportation problem[1] is easy reduction since we have cost matrix **C** defining "transportation" costs associated with each possible assignment option. For successful reduction we only need adding dummy source or destination.

3.1.2 *Min cost max flow reduction*

We can exploit additional problem structure to achieve superior performance and modeling capabilities. In figure 3.1.2 we see network architecture.

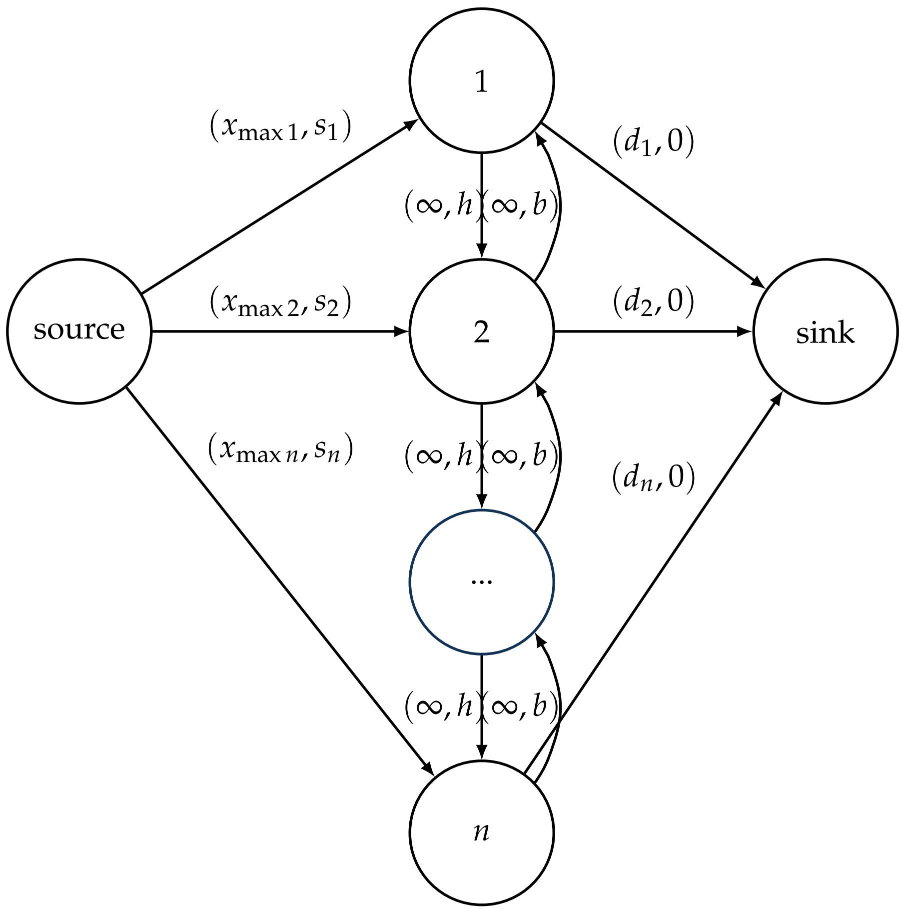

Figure 1: min cost max flow model. Arcs are labeled (capacity, cost)

1 Frederick S Hillier. *Introduction to Operations Research*. McGraw-Hill, 2014. ISBN: 0073523453.

3.1.3 *Runtime complexity*

Since both transportation problem and min-cost max flow have polynomial solution algorithms[2],[3] this problem does too. Most efficient approach is using network simplex since this problem naturally fits into min-cost max flow model than reduction to transportation problem.

3.2 VARIANTS

Problem as defined previously could seem rather simplistic and not allowing useful extensions users might want, such as starting storage amount and similar. In following few subsections most useful extensions are described.

3.2.1 *Starting storage capacity*

In case we already have a certain number of product in stock we can easily embed that knowledge into the model by setting $x_0^{(h)}$ equal to starting storage capacity

3.2.2 *Ending storage requirement*

For example we'd like to have some extra product in stock by the end of analysis, and it's quite easy to accommodate such requirement. Simple set $x_n^{(h)}$ to ending requirements. In min-cost max flow model that would be equivalent to another arch from node n to sink with ending storage requirement capacity.

3.2.3 *Allowing future backlogging*

In the model as described, time stops at period n, however, in realistic scenario we're looking at only short time snapshot of ongoing process. Simplest modification would be allowing $x_n^{(b)}$ to be non-negative and letting it be decision variable. It's value is going to be amount of backlogged demand at the analysis end, that is time n.

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2 Ibid.

3 James B. Orlin. "A polynomial time primal network simplex algorithm for minimum cost flows". In: *Mathematical Programming* 78.2 (1997), pp. 109–129. DOI: 10.1007/bf02614365. URL: http://dx.doi.org/10.1007/BF02614365.

3.2.4 *Leap time ordering*

If we order raw materials at t they might arrive at later time moment $t + \Delta_t$. This can easily be modeled via variable substitution. For different Δ_t depending on the t or multiple suppliers see Subsection 3.2.5

3.2.5 *Multiple raw material suppliers*

Adding new raw material suppliers with different costs cannot be done as per original model specification. Talking in terms or min-cost max flow approach adding new raw material suppliers would be equal to adding additional arcs from source to nodes $1, 2, \ldots, n$. with respective maximum supply capacity and costs.

4

BRIEF INTRODUCTION TO TIME SERIES ANALYSIS

4.1 DEFINITIONS

Time series is a collection of data points collected at constant time intervals. These are analyzed to determine the long term trend so as to forecast the future or perform some other form of analysis.

It is time dependent. Along with an increasing or decreasing trend, most time series have some form of seasonality trends, i.e. variations specific to a particular time frame. For example, if you see the sales of a woolen jacket over time, you will invariably find higher sales in winter seasons.

4.2 STATIONARY PROCESS

Stationary process is stochastic process whose joint probabilities don't change when shifted in time. A stationary process therefore has the property that the mean, variance and autocorrelation structure do not change over time.

Many time series analysis methods depend on stationarity property.

4.3 AR MODEL

Autoregressive (AR)[1] model is a representation of a type of random process. The autoregressive model specifies that the output variable depends linearly on its own previous values and on a stochastic term (an imperfectly predictable term).

Contrary to the MA model defined in Section 4.4, the AR model is not always stationary as it may contain a unit root.

1 George E. P. Box, Gwilym M. Jenkins, and Gregory C. Reinsel. *Time Series Analysis: Forecasting and Control*. Wiley, 2008. ISBN: 0470272848.

AR (p) model is p-th order autoregressive model and it is defined as:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t_1} + \epsilon_t$$

c and φ are model parameters, X_t is time series value at time step t, and ϵ_t is white noise with 0 mean and constant variance σ_ϵ^2.

4.4 MA MODEL

The moving-average model[2] specifies that the output variable depends linearly on its own previous stochastic term and on a stochastic term (an imperfectly predictable term).

Contrary to the AR model in Section 4.3, the MA model is always stationary.

MA (q) model is q-th order moving-average model defined as:

$$X_t = \mu + \sum_{i=1}^{q} \varphi_i \epsilon_{t-i}$$

where μ is time series mean and other notation is consistent with previous chapter.

4.5 ARMA MODEL

Autoregressive–moving-average (ARMA)[3] models provide a parsimonious description of a (weakly) stationary stochastic process in terms of two polynomials, one for the auto-regression and the second for the moving average

Given a time series of data X_t, the ARMA model is a tool for understanding and, perhaps, predicting future values in this series. The model consists of two parts, an autoregressive (AR) part and a moving average (MA) part. The model is usually then referred to as the ARMA (p,q) model where p is the order of the autoregressive part and q is the order of the moving average part:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t_1} + \epsilon_t + \sum_{i=1}^{q} \varphi_i \epsilon_{t-i}$$

2 Ibid.
3 Ibid.

4.6 ARIMA MODEL

Autoregressive integrated moving average (ARIMA) model[4] is a generalization of an autoregressive moving average (ARMA) model. They are applied in some cases where data show evidence of non-stationarity, where an initial differencing step (corresponding to the "integrated" part of the model) can be applied to reduce the non-stationarity.

Previously defined models work only on stationary time series. To get time series to stationary we need to transform it. Simplest such transformation is differentiating adjacent elements into obtaining new time series. ARIMA (p, d, q) model can be viewed as ARMA (p,q) applied to d-times differentiated original series.

4 Ibid.

5

APPLICATION ON HISTORICAL DATASET

For purpose of demonstrating for demand I'm using data from US electricity consumption[1] It offers per month aggregation of electrical energy used per state, and for purpose of illustration I'm using whole US aggregated. Data ranges from 1990 till February 2016. I'm using restricted version from 1990 till 2005 aggregated on yearly basis. Full data plot is in figure 2

For supply unit cost, for illustrative purposes I'm using historic American coal price.[2] It is yearly based from 1950 till 2005. The data is plot is in figure 3.

For purposes of this analysis, data from 2000 till 2005 is going to be forecasted as described in previous chapter, and then compared to ideal, perfect knowledge scenario. Analysis will be conducted for various values of $\mathbf{x}_{\max}$, b and d model parameters.

1 US Energy Information Administration. *Form EIA-826*. 2016. URL: `http://www.eia.gov/electricity/data/eia826/index.html`.

2 Quandl. *American Coal Price*. 2016. URL: `https://www.quandl.com/collections/markets/coal`.

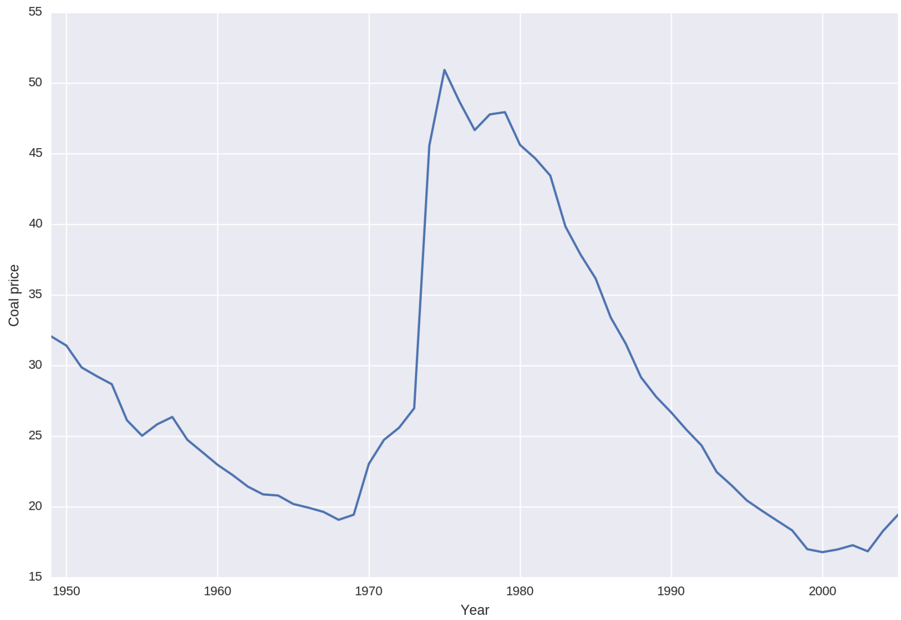

Figure 2: Yearly coal prices

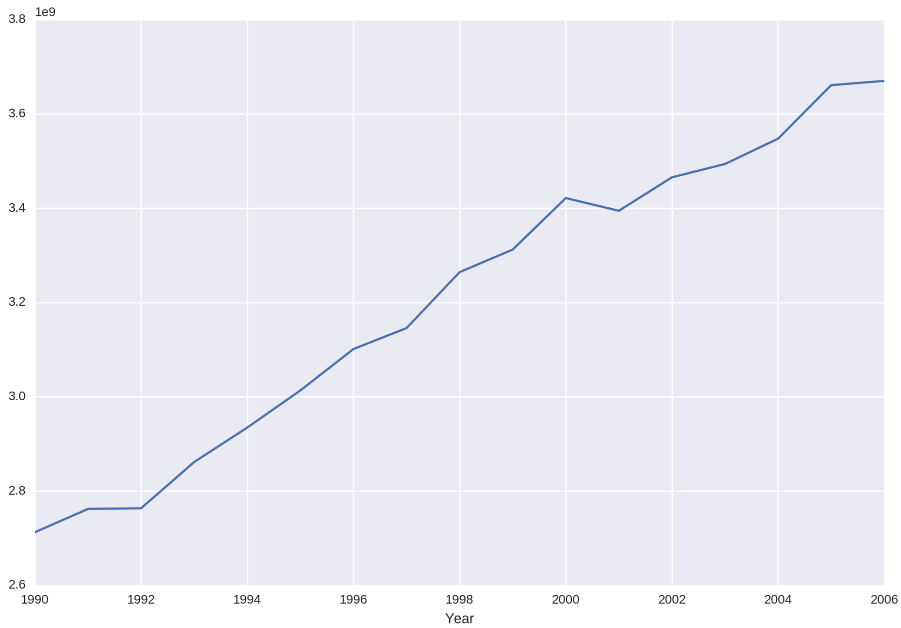

Figure 3: Yearly electricity demand

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