A decision support system for forecasting and optimal procurement

Neven Miculinić

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Introduction

- Decision maker has a factory which produces certain product with variable demand.
- To produce the product it uses raw materials from supplier with variable cost.
- There are cost penalties for storing raw material and for delaying demand satisfaction

Newsvendor problem

- Inspired by news vendor dilemma.
- Product is perishable (e.g. newspaper)

Formal definition

D	Random variable of product demand
s	supply cost per unit
p	selling price per unit

Variable is amount of perishable product to buy x.
objective function:

$$f = p \min(x, D) - sx$$

Dynamic lot sizing model

- Similar to problem described in introduction
- Unlike original there's setup cost

Formal definition

d_t	Demand at time period t
h_t	Holding cost at time period t
K_t	Setup cost at time period t
$x_0^{(h)}$	Initial inventory

and decision variable $\mathbf{x}$:

x_t	Quantity purchased at time period t

Dynamic lot sizing model

plus auxiliary variable y_t:

$$y_t = \begin{cases} 1 & x_t > 0 \\ 0 & x_t = 0 \end{cases}$$

For simplicity we define inventory at time period t as:

$$I_t = x_0^{(h)} + \sum_{k=0}^{t} x_k - \sum_{k=0}^{t} d_k$$

Dynamic lot sizing model

And we want to choose optimal x_t, under following constraints:

$$x_t \geq 0 \; \forall t$$
$$I_t \geq 0 \; \forall t$$

And we want to minimize following objective function:

$$f = \sum_t h_t I_t + y_t K_t$$

Formal problem definition

$\mathbf{s} = \left[s_1, s_2, \ldots, s_n\right]^\mathsf{T}$	Supply cost random vector
$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]^\mathsf{T}$	Procurement quantity vector
$\mathbf{x}^{(\max)}$	Procurement quantity limits vector
$\mathbf{x}^{(b)}$	Backlogging quantity vector
$\mathbf{x}^{(h)}$	Holding quantity vector
$\mathbf{d} = \left[d_1, d_2, \ldots, d_n\right]^\mathsf{T}$	Demand random vector
b	backlogging cost
h	holding cost
n	number of time steps

Constraints

$$
\begin{aligned}
x_t &\leq x_t^{(\max)} && \forall t \\
x_t &\geq 0 && \forall t \\
x_t^{(b)} &\geq 0 && \forall t \\
x_t^{(h)} &\geq 0 && \forall t \\
x_t + x_{t-1}^{(h)} + x_t^{(b)} &= d_t + x_t^{(h)} + h_{t-1}^{(b)} && \forall t \\
x_0^{(h)} + x_n^{(b)} + \sum_{t=1}^{n} x_i &= \sum_{t=1}^{n} d_i + x_0^{(b)} + x_n^{(h)}
\end{aligned}
$$

Objective function

- $$c(t) = s_t x_t + b x_t^{(b)} + h x_t^{(h)}$$

- $$f = \sum_t c(t) = \sum_t s_t x_t + b x_t^{(b)} + h x_t^{(h)}$$

Solution

- Linear programming/Mixed integer programming
- Transportation problem
- Min cost max flow

Min Cost max flow solution

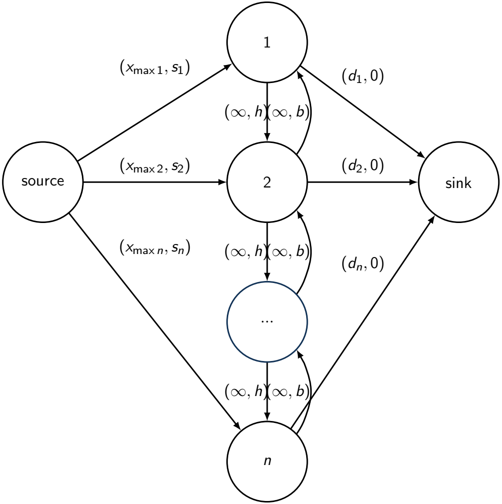

Figure: min cost max flow model. Arcs are labeled (capacity, cost)

Problem variants

- Starting storage capacity
- Ending storage requirement
- Allowing future backlogging
- Leap time ordering
- Multiple raw material suppliers

Forecasting future data points

- AR model

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t_1} + \epsilon_t$$

- MA model

$$X_t = \mu + \sum_{i=1}^{q} \varphi_i \epsilon_{t-i} + \epsilon_t$$

- ARIMA model
- Automated model fitting using Akaike information criterion (AIC)

$$\mathrm{AIC} = 2k - 2 \ln L$$

Americal coal prices

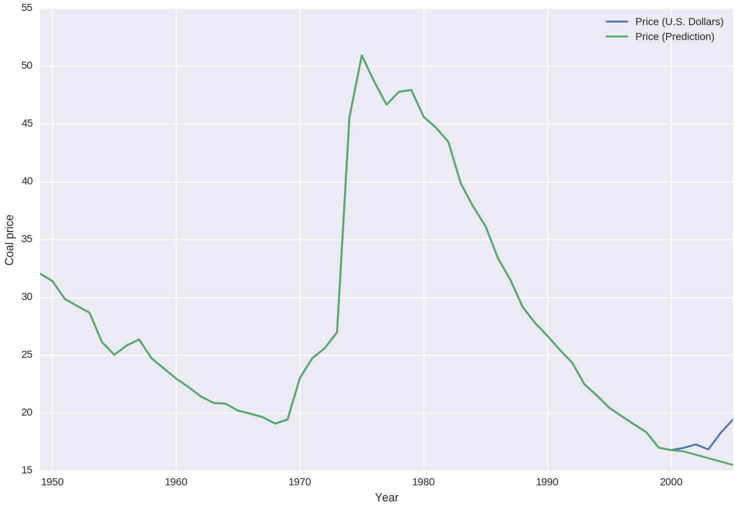

Figure: Yearly coal prices. Green are prediction with ARIMA(0,1,1) model

US yearly electricity demand

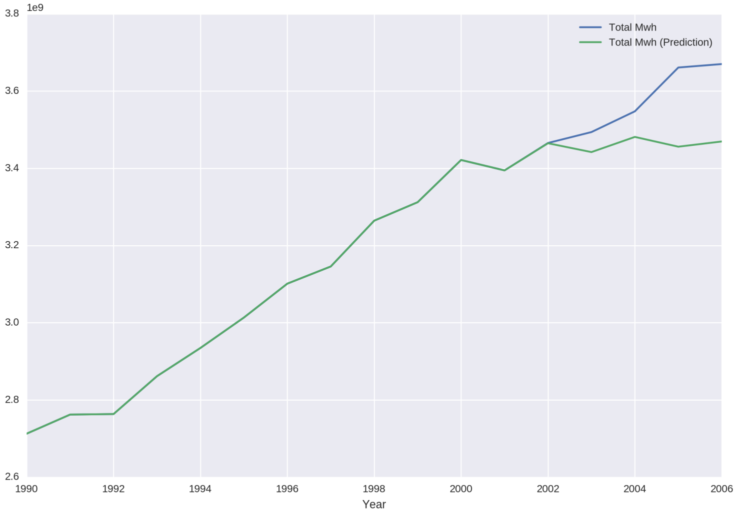

Figure: Yearly electricity demand. Green are predictions with ARIMA(1,2,0) model

Implementation and application

- python API
- experimented on real dataset
- Bottom line: benefits data dependant

Thank you for your attention.