A decision support system for forecasting and optimal procurement

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ABSTRACT

Optimal procurement in most industries involves forecasting of two quantities: prices of raw materials and customer's demand. The aim of this work is to integrate forecasts into production planning models, with the aim of minimizing overall procurement, holding and production costs under demand satisfaction constraints. The decision support system should allow the decision maker to integrate qualitative, unstructured information through a simple interface, for scenario selection and solution refinement

1

PROBLEM DEFINITION

1.1 INTRODUCTION

This problem is a variation of the lot sizing problem in a stochastic setting. Variation is as follows. Firstly we describe deterministic variant before extending the model to stochastic setting. We have predictions for raw material supply cost $\mathbf{s}$ at future time moment t. The *factory* converts all bought raw materials at time t and stores them in a warehouse. Number of discrete time moments under consideration is denoted by n.

One time moment storage costs fixed amount h. The holding cost is per unit and per time period.

Each time moment, t, we have certain demand we need to satify, denoted as $\mathbf{d}_t$. In case there's no products in storage to satisfy demand we allow backlogging incurring a cost denoted as b per unit per day, for late orders.

Our aim is to optimize our procurement policy by choosing $\mathbf{x}$, which is amount of the product we buy at time moment t. However we are constrained by the maximum amount raw materials , $\mathbf{x}^{\max}$, that we can buy each day.

1.2 FORMAL PROBLEM DEFINITION

1.2.1 *Definitions*

Following is the deterministic problem variant, and in subsequent chapters randomness and uncertainty is embedded into problem. Notation:

$\mathbf{s} = [s_1, s_2, \dots, s_n]^\mathsf{T}$	Supply cost vector
$\mathbf{x} = [x_1, x_2, \dots, x_n]^\mathsf{T}$	Procurement quantity vector
$\mathbf{x}^{(\max)}$	Procurement quantity limits vector
$\mathbf{x}^{(b)}$	Backlogging quantity vector
$\mathbf{x}^{(h)}$	Holding quantity vector
$\mathbf{d} = [d_1, d_2, \dots, d_n]^\mathsf{T}$	Demand random vector
b	backlogging cost
h	holding cost
n	number of time moments

Every variable for backlogging and holding vector seems unintuitive or taken. I hope $\mathbf{x}^{(b)}$ is alright since relation to $\mathbf{x}$ is clearly highlighted

1.2.2 *Variables*

$\mathbf{x}$ is our decision variable, as described previously. $\mathbf{x}^{(b)}$ and $\mathbf{x}^{(h)}$ are backlogging and holding variables respectively backlogging variables respectively. For simplicity $x_0^{(h)}$, $x_0^{(b)}$, $x_n^{(h)}$, $x_n^{(b)}$ are equal to 0 unless otherwise noted. This specific values are explored further in section 2.2.

1.2.3 *Constraints*

$$
\begin{aligned}
x_t &\le x_t^{(\max)} && \forall t \\
x_t &\ge 0 && \forall t \\
x_t^{(b)} &\ge 0 && \forall t \\
x_t^{(h)} &\ge 0 && \forall t \\
x_t + x_{t-1}^{(h)} + x_t^{(b)} &= d_t + x_t^{(h)} + h_{t-1}^{(b)} && \forall t \\
x_0^{(h)} + x_n^{(b)} + \sum_{t=1}^{n} x_i &= \sum_{t=1}^{n} d_i + x_0^{(b)} + x_n^{(h)}
\end{aligned}
$$

1.3 OBJECTIVE FUNCTION

Definition 1.3.1. $c(t)$ defines total speeding we pay at time t.

$$c(t) = +s_t x_t + b x_{b_t} + h x_{h_t} \tag{1}$$

Definition 1.3.2. f is objective function for this problem. Our aim is to minimize it.

$$f = \sum_t c(t) = \sum_t s_t x_t + b x_{b_t} + h x_{h_t} \tag{2}$$

2

DETERMINISTIC APPROACH

2.1 MODELING APPROACHES

Problem as defined in section 1.2 can be reduced to two well known problems: transportation and min-cost max flow problem. In both cases from knowledge of $\mathbf{x}$ and constraint equations, $\mathbf{x}^{(h)}$ and $\mathbf{x}^{(b)}$ since at each t in optimal solution at least one of $x_t^{(b)}$ and $x_t^{(h)}$ is 0. This fact can easily be observed from flow conservation on intermediate nodes as in fig 2.1.2. Setting them both to positive values create positive flow cycle with positive cost, which can be canceled yielding same flow with lower cost.

2.1.1 *Reduction to transportation problem*

For transportation problem reduction we need cost matrix for satisfying demand at time j with supply at time i. It is given as follows:

Definition 2.1.1. $\mathbf{C}$ matrix defines cost for satisfying demand with specific raw supply material purchase date. It's element c_{ij} equals:

$$c_{ij} = \begin{cases} b\,(i-j) + s_i & j < i \\ h\,(j-i) + s_i & j \geq i \end{cases}$$

That is using raw materials purchased at i to satisfy demand at time j incurs cost c_{ij}.

Maximum supply $\mathbf{x}^{(\max)}$ is given, so is the demand vector $\mathbf{d}$. Since transportation problem required equal supply and demand nodes, we add a dummy demand node consuming excess supply.

"That is o in the deterministic..." I don't understand what is the meaning of this remark and how can implement your feedback.

Transportation problem[1] is easy reduction since we have cost matrix **C** defining "transportation" costs associated with each possible assignment option. For successful reduction we only need adding dummy source or destination.

2.1.2 *Min cost max flow reduction*

We can exploit additional problem structure to achieve superior performance and modeling capabilities. In figure 2.1.2 we see network architecture.

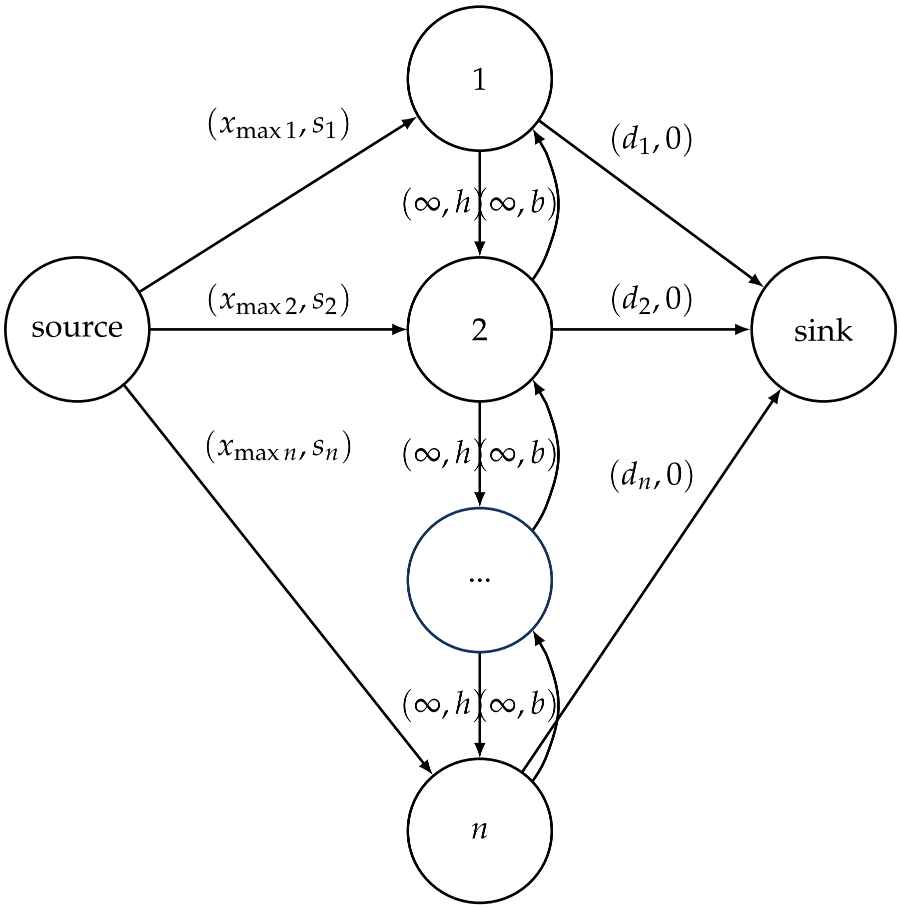

Figure 1: min cost max flow model. Arcs are labeled (capacity, cost)

1 Frederick S Hillier. *Introduction to Operations Research*. McGraw-Hill, 2014. ISBN: 0073523453.

2.1.3 *Runtime complexity*

Since both transportation problem and min-cost max flow have polynomial solution algorithms[2],[3] this problem does too. Most efficient approach is using network simplex since this problem naturally fits into min-cost max flow model than reduction to transportation problem.

2.2 VARIANTS

Problem as defined previously could seem rather simplistic and not allowing useful extensions users might want, such as starting storage amount and similar. In following few subsections most useful extensions are described.

2.2.1 *Starting storage capacity*

In case we already have a certain number of product in stock we can easily embed that knowledge into the model by setting $x_0^{(h)}$ equal to starting storage capacity

2.2.2 *Ending storage requirement*

For example we'd like to have some extra product in stock by the end of analysis, and it's quite easy to accommodate such requirement. Simple set $x_n^{(h)}$ to ending requirements. In min-cost max flow model that would be equivalent to another arch from node n to sink with ending storage requirement capacity.

2.2.3 *Allowing future backlogging*

In the model as described, time stops at period n, however, in realistic scenario we're looking at only short time snapshot of ongoing process. Simplest modification would be allowing $x_n^{(b)}$ to be non-negative and letting it be decision variable. It's value is going to be amount of backlogged demand at the analysis end, that is time n.

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2 Ibid.

3 James B. Orlin. "A polynomial time primal network simplex algorithm for minimum cost flows". In: *Mathematical Programming* 78.2 (1997), pp. 109–129. DOI: `10.1007/bf02614365`. URL: `http://dx.doi.org/10.1007/BF02614365`.

2.2.4 *Leap time ordering*

If we order raw materials at t they might arrive at later time moment $t + \Delta_t$. This can easily be modeled via variable substitution. For different Δ_t depending on the t or multiple suppliers see Subsection 2.2.5

2.2.5 *Multiple raw material suppliers*

Adding new raw material suppliers with different costs cannot be done as per original model specification. Talking in terms or min-cost max flow approach adding new raw material suppliers would be equal to adding additional arcs from source to nodes 1, 2, . . . , n. with respective maximum supply capacity and costs.

3

INTRODUCING RANDOMNESS

This whole chapter is in rewriting.

TODO: This is very much work in progress since I have no idea whether am I taking the right approach.

... fill this in

First we are going to analise problem deeply without presuming any independence or probability distribution on random variables. Later in subsequent chapters we are going to focus more on where demand at time t has independent Gaussian distribution and mean.

3.1 COST FUNCTION

The minimizing function is:

$$\mathrm{E}\left[\mathbf{s}^{\mathsf{T}}\mathbf{x} + \sum c(t)\right]$$

Due to linerality of expectation and x being variable it's equal to:

$$\mathbf{x}\,\mathrm{E}\left[\mathbf{s}^{\mathsf{T}}\right] + \mathrm{E}\left[\sum c(t)\right]$$

Therefore only needed modeling information for supply cost is its expectation $\mathrm{E}\left[\mathbf{s}\right]$. The other part is more trickier since D_i and D_j aren't usually independent.

3.2 HANDLING DEMAND COST NON-LINEARITY

As we can see in equation 1 we have non-linearity depending whether we're satifying all demand or are we backlogging demand at time t. Therefore here are two possible solutions for minimizing objective function 2.

Simulation

We generate multiple scenarios for demand vector, d according to probability distribution. For small n and relatively small number of outcomes in each random variable we can exhaustedly model each scenario, scale it appropriately and feed to MIP solver[1]

Safety net approach

Alternatively, we can artificially add new constraints and avoiding backlogging with arbitrary probably. This model assumes backlogging cost are significantly greater than storage cost, that is backlogging penalty is severe.

Thus we chose values arbitrary realizations of random variables D_t and add additional constraints of the form:

$$X_t \geq D_i \ \forall t \tag{3}$$

which reduces cost function 1 to:

$$c(t) = h\left(X_i - D_i\right) \ \ X_i \geq D_i$$

and enables are faster solving approaches. Given parameters D_i chosen and their underlying distribution we derive non-increasing function $p(t)$ which represent probability of not breaking newly introduced constraints 3

Min-cost max flow with uncertaintiy

Since deterministic case can be modeled with min-cost max flow, we can investigate further in this directi

Stochastic Dynamic Programmnig

TODO.. maybe, I've got recommendation by a friend, need to read some papers.

1 There's a trick on using binary variable for discontinuity in cost function 1

4

APPLICATION ON HISTORICAL DATASET

For purpose of demonstrating for demand I'm using data from US electricity consumption[1] It offers per month aggregation of electrical energy used per state, and for purpose of illustration I'm using whole US aggregated. Data ranges from 1990 till February 2016. I'm using restricted version from 1990 till 2005 aggregated on yearly basis. Full data plot is in figure 2

For supply unit cost, for illustrative purposes I'm using historic American coal price.[2] It is yearly based from 1950 till 2005. The data is plot is in figure 3.

For purposes of this analysis, data from 2000 till 2005 is going to be forecasted as described in previous chapter, and then compared to ideal, perfect knowledge scenario. Analysis will be conducted for various values of $\mathbf{x}_{\max}$, b and d model parameters.

1 US Energy Information Administration. *Form EIA-826*. 2016. URL: `http://www.eia.gov/electricity/data/eia826/index.html`.
2 Quandl. *American Coal Price*. 2016. URL: `https://www.quandl.com/collections/markets/coal`.

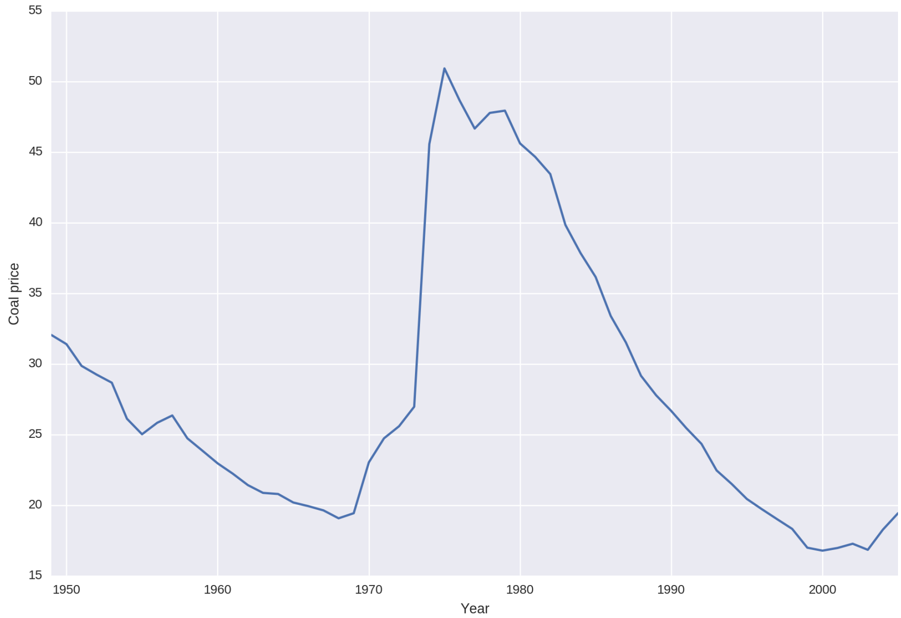

Figure 2: Yearly coal prices

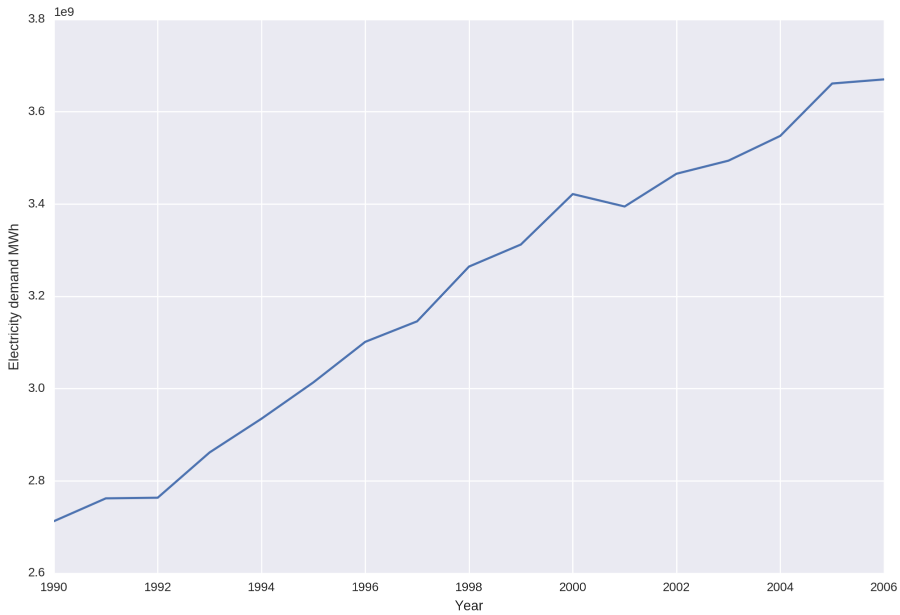

Figure 3: Yearly electricity demand

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