BA thesis title

Neven Miculnic

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1

PROBLEM DEFINITION

1.1 INTRODUCTION

This problem is variation of lot sizing problem in stochastic setting. Variation is as follows. We have stochastic prediction for raw material supply cost $\mathbf{s}$ at future time moment t. The *factory* converts all bought raw materials at time t and stores them into a warehouse. Number of discrete time moments is denoted by n.

One time moment storage costs fixed amount h, that is the holding cost.

Likewise demand for the product is equally stochastic, denoted as $\mathbf{d}$. In case there's no products in storage to satisfy demand we pay backlogging cost denoted as b.

Our aim is to optimize our procurement policy by variating $\mathbf{x}$, that is product amount we buy at time moment t. However we are constrained by the maximum raw materials we can buy each day by $\mathbf{x}_{\text{max}}$.

1.2 FORMAL PROBLEM DEFINITION

1.2.1 *Definitions*

$\mathbf{s} = \left[S_1, S_2, \ldots, S_n\right]^\intercal$	Supply cost random vector
$\mathbf{x} = \left[x_1, x_2, \ldots, x_n\right]^\intercal$	Procurement quantity vector
$X_i = \sum_{t \leq i} x_t$	Cumulative procurement quantity sum
$\mathbf{y} = \left[Y_1, Y_2, \ldots, Y_n\right]^\intercal$	Demand quantity random vector
$\mathbf{d} = \left[d_1, d_2, \ldots, d_n\right]$	demand vector
$D_i = \sum_{t \leq i} d_t$	Cumulative demand vector
b	backlogging cost
h	holding cost

1.2.2 *Variables*

$\mathbf{x}$ is our decision variable, as described previously.

1.2.3 *Constraints*

$$\begin{aligned} \mathbf{x} &\in \mathbb{N}_0^n \\ \mathbf{x} &\leq \mathbf{x}_{\max} \\ D_i &\geq 0 \ \forall t \end{aligned}$$

1.3 OBJECTIVE FUNCTION

First let's define auxiliary function $c(t)$ which determines holding or backlogging cost at t:

$$c(t) = \begin{cases} h\left(X_i - D_i\right) & X_i \geq D_i \\ b\left(D_i - X_i\right) & X_i < D_i \end{cases}$$

Therefore our minimizing objective function is:

$$\min f(\mathbf{x}) = \mathbf{s}^\intercal \mathbf{x} + \sum c(t)$$

Because the function is stochastic our aim is to minimize $\mathrm{E}\left[f(\mathbf{x})\right]$.

1.4 SOLUTION

Under assumption **s**, c and **d** are already defined (in here are simply defined for example

This solution has running time of $\Theta(n)$

1.5 VARIATION

If we add additional constraint:

$$\forall t : h_t \leq H$$

it's possible to solve this probelm in $\mathcal{O}(nH^2)$ time using dynamic programming. Our state in dynamic programing $\mathrm{dp}_{t,i}$ is minumal cost where at time t we have i units stored in storage, that is $h_t = i$.

Transition is:

$$\mathrm{dp}_{t,i} = \min_{j=0}^{i} cj + \mathrm{dp}_{t-1,j} + s_t\,(d_t + i - j)$$

For finding optimal solution we're interested in optimal j choices in recursive tree for $\mathrm{dp}_{n,0}$ that is at time t we have 0 product surplus stored in storage.

Likewise there exeist minimum H such that $\forall h > H$ optimal solution doesn't change. Using expoentially increasing H it's possible to find optimal H in $\mathcal{O}(nH^2 \log H)$ time.