

# CSE460: VLSI Design

## Lecture 8: Logic Function Synthesis using k-map

# Review: Logic Function Synthesis using k-map

- The Karnaugh map (or k-map) is an **alternative to the truth-table** form for representing a function
- The map consists of cells that correspond to the rows of the truth table

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

$x_1$	$x_2$	$x_3$	$x_4$	
0	0	0	0	$m_0$
0	0	0	1	$m_1$
0	0	1	0	$m_2$
0	0	1	1	$m_3$
0	1	0	0	$m_4$
0	1	0	1	$m_5$
0	1	1	0	$m_6$
0	1	1	1	$m_7$
1	0	0	0	$m_8$
1	0	0	1	$m_9$
1	0	1	0	$m_{10}$
1	0	1	1	$m_{11}$
1	1	0	0	$m_{12}$
1	1	0	1	$m_{13}$
1	1	1	0	$m_{14}$
1	1	1	1	$m_{15}$

# Logic Function Synthesis (SOP) using k-map

Consider a logic function,  $Y = f(A,B)$

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

- **Minterm (SOP)** - Group  $2^n$  no. of 1 and skip 0.  $n = 0,1,2,3,\dots$  (Make groups larger as possible)
- Grouping can be square/rectangle shaped or along row/column (Not along diagonal). Groups can be overlapped.
- The inputs which are varying in a group, can be omitted. The others (**fixed**) can be written as **product** form. **Invert** the variables which are fixed with the value **0**.
- Don't cares (d) can be used 0/1 as per convenience.

K-map:

		B	
		0	1
A	0		
	1		

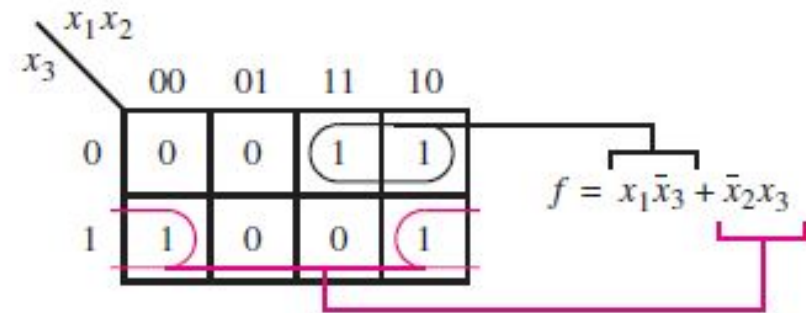
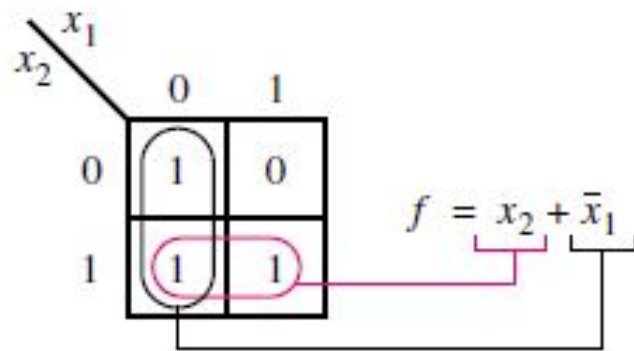
A omitted.  
B fixed with 1.  
 $Y = B$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

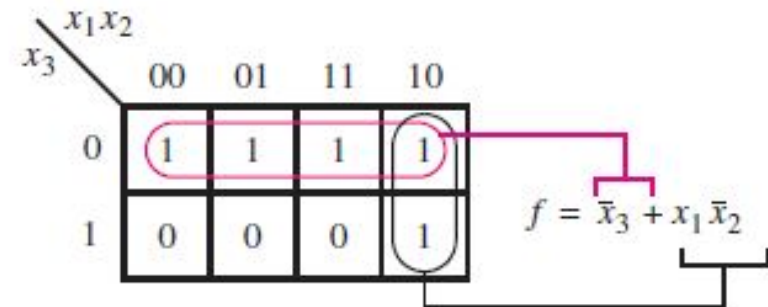
$$Y = B'$$

# Logic Function Synthesis (SOP) using k-map

- **Minterm (SOP)** - Group  $2^n$  no. of 1 and no 0.  $n = 0, 1, 2, 3, \dots$   
(Make groups larger as possible)
- Grouping can be square/rectangle shaped or along row/column (Not along diagonal). Groups can be overlapped.
- The inputs which are varying in a group, can be omitted.
- Don't cares (d) can be used 0/1 as per convenience.
- **Edges are connected.**



(a) The function of Figure 2.23



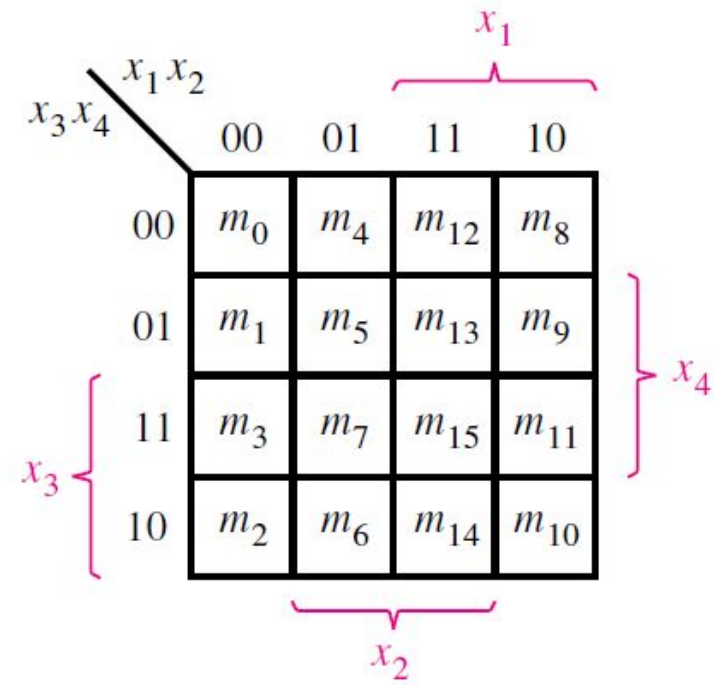
(b) The function of Figure 2.48

# Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x1	x2	x3	x4	Minterms
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



# Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$ $x_3 x_4$					
		00	01	11	10
00	00				
	01				
	11				
	10				

$x_1 x_2$ $x_3 x_4$					
		00	01	11	10
00	00	$m_0$	$m_4$	$m_{12}$	$m_8$
	01	$m_1$	$m_5$	$m_{13}$	$m_9$
	11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
	10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

Diagram illustrating the K-map for the function  $f(x_1, \dots, x_4)$ . The K-map is a 4x4 grid with rows labeled  $x_3 x_4$  (00, 01, 11, 10) and columns labeled  $x_1 x_2$  (00, 01, 11, 10). The minterms are labeled  $m_0$  through  $m_{15}$  in the cells. The function is defined as  $f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$ . The K-map is partitioned into four groups of four cells each, labeled  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$  in pink.

# Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$ $x_3 x_4$		00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

Groupings:

- Group 1 (vertical):  $x_2 \bar{x}_3$  (covers cells (0,1), (1,1))
- Group 2 (horizontal):  $x_3 \bar{x}_4$  (covers cells (1,0), (1,1), (1,2), (1,3))

$x_1 x_2$		$x_1$			
$x_3 x_4$		00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$	$x_4$
01	$m_1$	$m_5$	$m_{13}$	$m_9$	
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$	
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$	
		$x_2$			

# Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$ $x_3 x_4$		00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

$$f = x_2 \bar{x}_3 + x_3 \bar{x}_4$$