Shri Ramdeobaba College of Engineering, Nagpur

FLAT Assignment 02
Semester : V
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Course Name : Formal Language and Automata Theory
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Preeti Yadav -14
Yadav

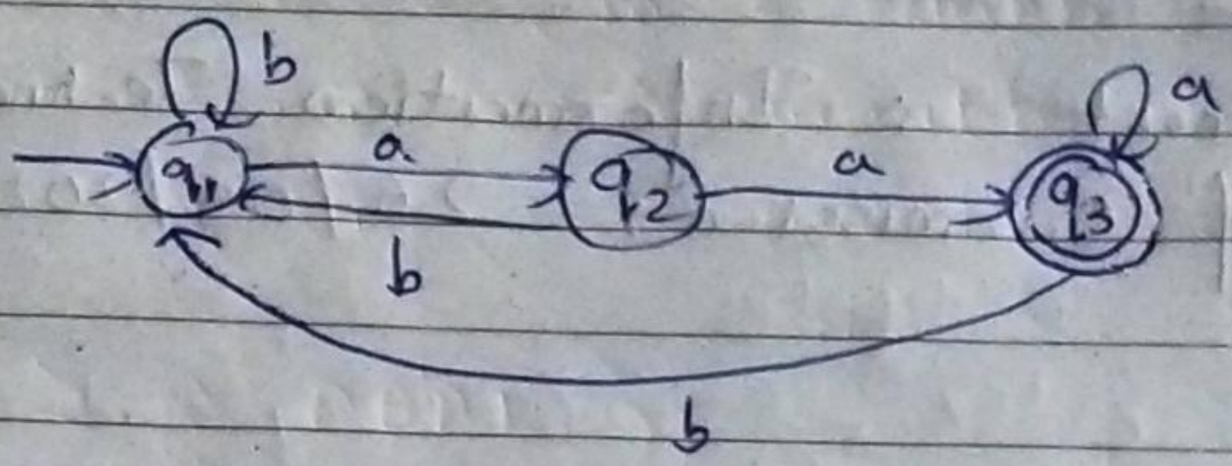
1/22 1/27

Date

Q.1. Give regular expression over alphabet [a, b] using Arden's theorem.

(a) Set of string ending with aa.

Sol:- $L = \{aa, baa, aaa, abaa, aabaaa, bbaa, \ldots\}$



$\therefore$ $q_1 = \epsilon + q_2 b + q_3 b + q_1 b$ — (1)

$q_2 = q_1 a$ — (2)

$q_3 = q_2 a + q_3 a$ — (3)

By using Arden's theorem,

from (3)

$q_3 = q_2 a + q_3 a$

R = Q + R P

$\therefore q_3 = q_2 a a^*$

Substituting q_2

$q_3 = q_1 aaa^*$ — (4)

Substituting (4) in (1)

$q_1 = \epsilon + q_2 b + q_1 aaa^* b + q_1 b$ — (5)

Substituting (2) in (5)

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2/27

$$q_1 = \epsilon + q_1 ab + q_1 aaa^* b + q_1 b$$

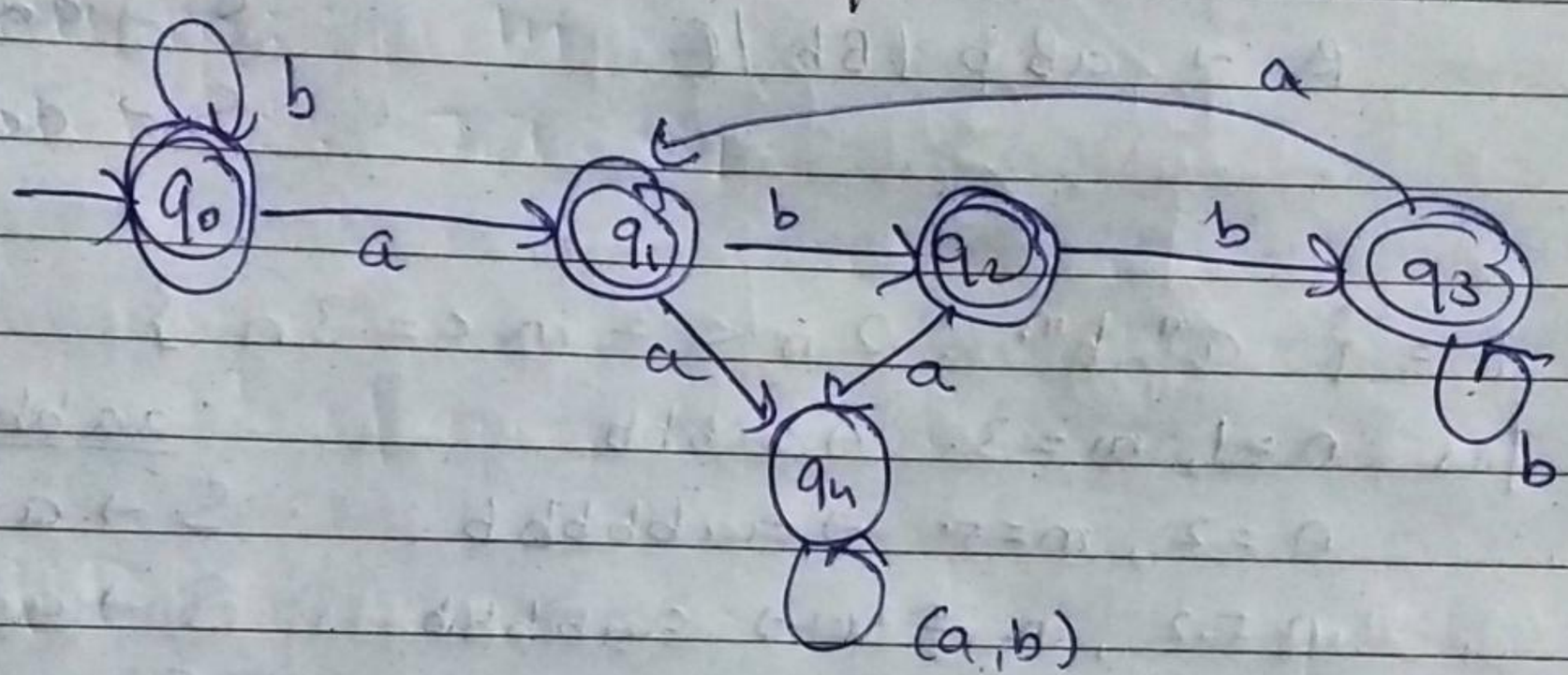
$$q_1 = \epsilon + q_1 (\underbrace{ab + aaa^* b + b}_{p})$$

$$\therefore \quad q_1 = \epsilon (ab + aaa^* b + b)^*$$

$$q_1 = (ab + aaa^* b + b)^*$$

(b) The set of string in which there are at least two occurences of b between any two occurences of a.

Sol:



$$q_0 = \epsilon + q_0 b \quad - (1)$$

$$q_1 = q_0 a + q_3 a \quad - (2)$$

$$q_2 = q_1 b \quad - (3)$$

$$q_3 = q_2 b + q_3 b \quad - (4)$$

from eqn (1)

$$q_0 = \epsilon + q_0 b$$

$$\boxed{q_0 = b^*}$$

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Yadav

3/27

from eqn (4)

$$q_3 = q_2 b b^*$$

$$q_3 = q_1 b b b^*$$

(1) $\Rightarrow$ $q_1 = b^* a + q_1 b b b^* a$

$q_1 = (b^* a) \cdot (b b b^* \cdot a)^*$

$q_2 \Rightarrow$ $q_2 = (b^* a)(b b b^* a) + b$

Regular expressions:

$q_0 = b^*$

$q_1 = (b^* a) \cdot (b b b^* a)^*$

$q_2 = (b^* a) \cdot (b b b^* \cdot a)^* b$

$q_3 = (b^* a)(b b b^* a)^* b b b^*$

Preeti Yadav - 14

Yadav

4/27

Page No.
Date

Q.2. Transfer the following grammar into GNF and CNF.

a. $E \to E + T \mid T$
$T \to T * F \mid F$
$F \to (E) \mid a$

⇒ GNF: $E \to ET \mid TE \mid E \mid T$
$T \to T * F \mid a$
$F \to (F) \mid a$

⇒ $E \to ET \mid aE \mid a$
$T \to a * F \mid a$
$F \to (E) \mid a$

⇒ $E \to T \mid aE \mid a$
$T \to a * F \mid a$
$F \to (a) \mid a$

CNF: $E \to ET \mid TE \mid T$
$T \to T * F \mid F$
$F \to (E) \mid a$

⇒ $E \to ET \mid TE \mid T$
$T \to T * F \mid a$
$F \to (E) \mid a$
$E \to ET \mid TE \mid a$
$T \to T * F \mid a$
$F \to (a) \mid a$

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Yadav

5/27

b. $S \to AB$
$A \to BSB \mid BB \mid b$
$B \to aAb \mid a$

Sol:

CNF : $S \to AB$
$A \to BSB \mid BB \mid b$
$B \to CaACb \mid a$
$Ca \to a$
$Cb \to b$

CNF : $S \to bB$
$S \to aSB \mid abB \mid b$
$B \to aACb \mid a$
$Cb \to b$

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Yadav

6/27

Page No.
Date

Q.3. Construct a CFG for following language

a) $L = \{ a^n b^n : n <= m + 3 \text{ and } n, m >= 0 \}$

⇒ for n=1, m=1 → ab

n=2, m=2 → aabb

n=5, m=2 → aaaaaabb

n=4, m=2 → aaaabb

∴ S → AAAB

A → a / ∈

B → aBb / Bb / ∈

aaaabb

S → AAAB

S → aAAB

S → aaAB

S → aaaB

S → aaaaBb

S → aaaabb

b) $L = \{ a^n b^m : 2n <= m <= 3n \}$

⇒ for n=1, m=3 → abbb

n=2, m=5 → aabbbbb

n=2, m=4 → aabbbb

n=3, m=7 → aaabbbbbbb

aabbbb

S → aSbb

S → aaSbbbb

S → aabbbb

S → aSbb / aSbbb / ∈

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Yadav

7/27

Page No
Date

Q.4. Let G be the grammar
$S \rightarrow S+S \mid S-S \mid S*S \mid S/S \mid (S) \mid a$
Then for string $a+(a*a)/a-a$ generate

i) Leftmost derivation

$S \rightarrow \underline{S}/S$
$S \rightarrow \underline{S}+S/S$
$S \rightarrow a+\underline{S}/S$
$S \rightarrow a+(\underline{S})/S$
$S \rightarrow a+(\underline{S}*S)/S$
$S \rightarrow a+(a*\underline{S})/S$
$S \rightarrow a+(a*a)/\underline{S}$
$S \rightarrow a+(a*a)/S-\underline{S}$
$S \rightarrow a+(a*a)/\underline{a}-S$
$\underline{S \rightarrow a+(a*a)/a-a}$

ii) Rightmost derivation

$S \rightarrow S+\underline{S}$
$S \rightarrow S+S/\underline{S}$
$S \rightarrow S+S/S-\underline{S}$
$S \rightarrow S+S/S-a$
$S \rightarrow S+\underline{S}/a-a$
$S \rightarrow S+(\underline{S})/a-a$
$S \rightarrow S+(\underline{S}*\underline{S})/a-a$
$S \rightarrow S+(\underline{S}*a)/a-a$
$S \rightarrow \underline{S}+(a*a)/a-a$
$\underline{S \rightarrow a+(a*a)/a-a}$

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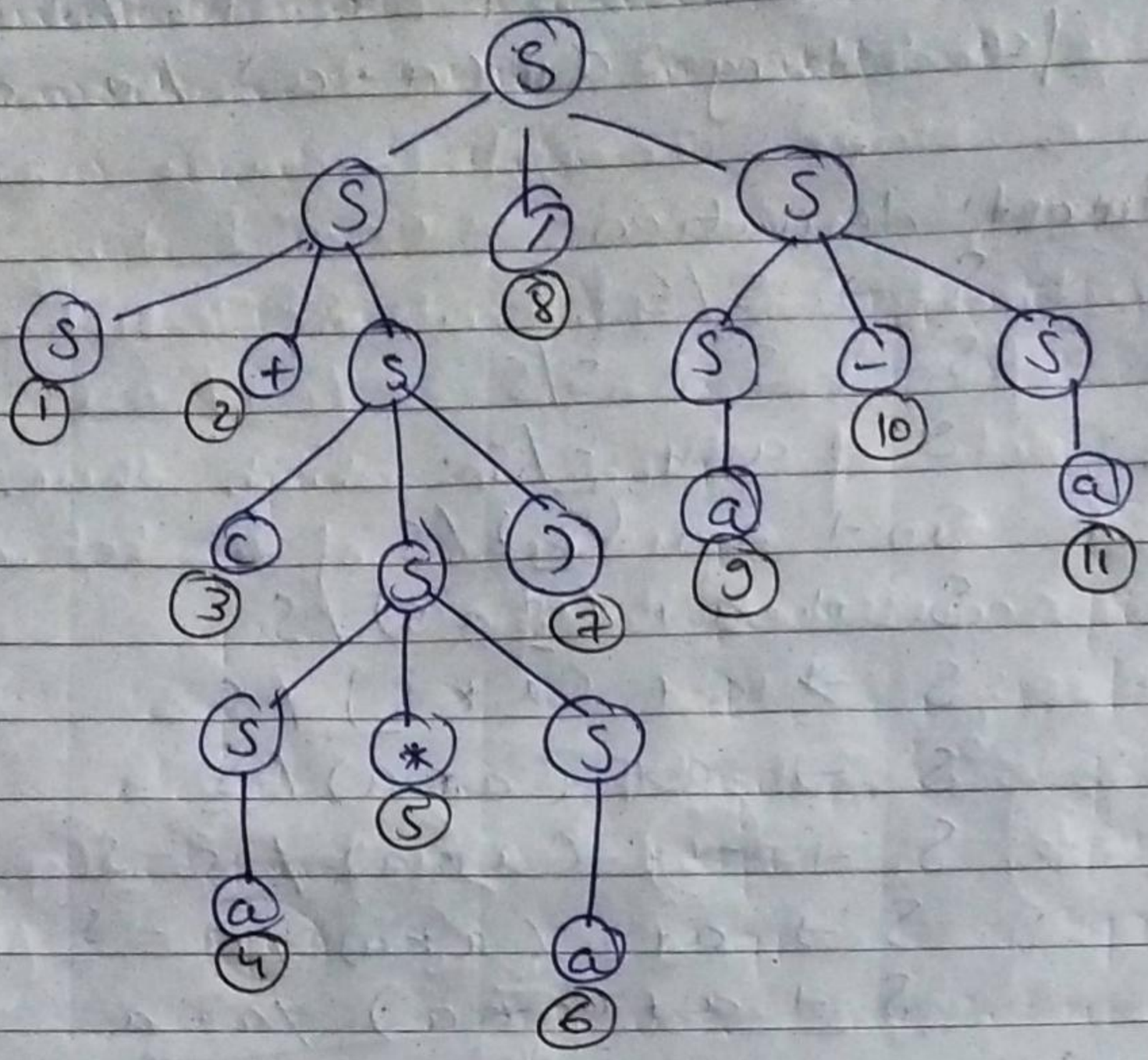
8/27

(iii) Derivation tree (leftmost derivation)



Q.5. Construct a PDA equivalent to the following CFG.

$S \rightarrow OBB$

$B \rightarrow OS \mid 1S \mid 0$

Test the constructed PDA for string 010000.

Soln :- Let PDA,

$A = \{ \{q\}, \{0,1\}, \{S,B,0,1\}, \delta, q, S, F \}$

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Yadav

9/27

The production rules δ can be written as:

R1 : $\delta(q, \in, S) = \{(q, OBB)\}$

R2 : $\delta(q, \in, B) = \{(q, OS), (q, 1S), (q, 0)\}$

R3 : $\delta(q, 0, 0) = \{(q, \in)\}$

R4 : $\delta(q, 1, 1) = \{(q, \in)\}$

Testing 010000 against PDA

$\delta(q, 010000, S) \vdash (q, 010000, OBB)$ → from R1

$(q, 10000, BB)$ → from R3

$(q, 10000, 1SB)$ → from R2

$(q, 0000, SB)$ → from R4

$(q, 0000, OBBB)$ → from R1

$(q, 000, BBB)$ → from R3

$(q, 00, BB)$ → from R2

$(q, 00, 0B)$ → from R3

$(q, 0, B)$ → from R2

$(q, 0, 0)$ → from R3

$(q, \in)$ → from R3

∴ The string is accepted.

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Yadav

10/27

Q.6. Construct a reduced grammar equivalent to the grammar.

$S \to aA \mid aBB$

$A \to aaA \mid \in$

$B \to bB \mid bbC$

$C \to B$

Soln: 1) remove null production

Nullable : {A}

$S \to a \mid aBB \mid aA$

$A \to aa \mid aaA$

$B \to bB \mid bbC$

$C \to B$

2) remove unit production

$C \to B$ is a unit production

$\therefore$

$S \to a \mid aBB \mid aA$

$A \to aa \mid aaA$

$B \to bB \mid bbB$

required grammar

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11/27

Page No.
Date

Q.7. Construct a minimum state automaton equivalent to a given automaton M whose transition table is defined as below.
q_6 → final state

State	Input	
	a	b
→ q_0	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
(q_6)	q_1	q_3

⇒

1) $\pi_1 = \{ \{q_6\}, \{q_0, q_1, q_2, q_3, q_4, q_5\} \}$
2) $\pi_2 = (\{q_6\}, \{q_4\}, \{q_0, q_1, q_2, q_3, q_5\})$
3) $\pi_3 = (\{q_6\}, \{q_4\}, \{q_0, q_1, q_3\}, \{q_2, q_5\})$
4) $\pi_4 = (\{q_6\}, \{q_4\}, \{q_0\}, \{q_1\}, \{q_3\}, \{q_2, q_5\})$

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Yadav

12/27

	a	b
q_0	q_0	q_3
q_1	$[q_2, q_5]$	$[q_2, q_5]$
$[q_2, q_5]$	q_3	q_4
q_3	q_0	$[q_2, q_5]$
q_4	q_0	q_6
q_6	q_1	q_3

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Yadav

13/27

Page No.
Date

Q. 8. Construct an NPDA for the language.
$L = \{ w \in (a,b)^* \mid N_a(w) = N_b(w) \}$
N_a - Number of a
N_b - Number of b

Sol:

$L = \{ab, aabb, abba, aababb, bbabaa, baaababb, \ldots\}$

$\Gamma = \{a, b, z\}$
Γ = set of all the stack alphabet
z = stack start symbol.

Stack transition function:

$\delta(q_0, a, z) \vdash (q_0, az)$
$\delta(q_0, a, a) \vdash (q_0, aa)$
$\delta(q_0, b, z) \vdash (q_0, bz)$
$\delta(q_0, b, b) \vdash (q_0, bb)$
$\delta(q_0, a, b) \vdash (q_0, \epsilon)$
$\delta(q_0, b, a) \vdash (q_0, \epsilon)$
$\delta(q_0, \epsilon, z) \vdash (q_f, z)$

q_0 : initial state
q_f : final state
$\leftarrow$: pop operation

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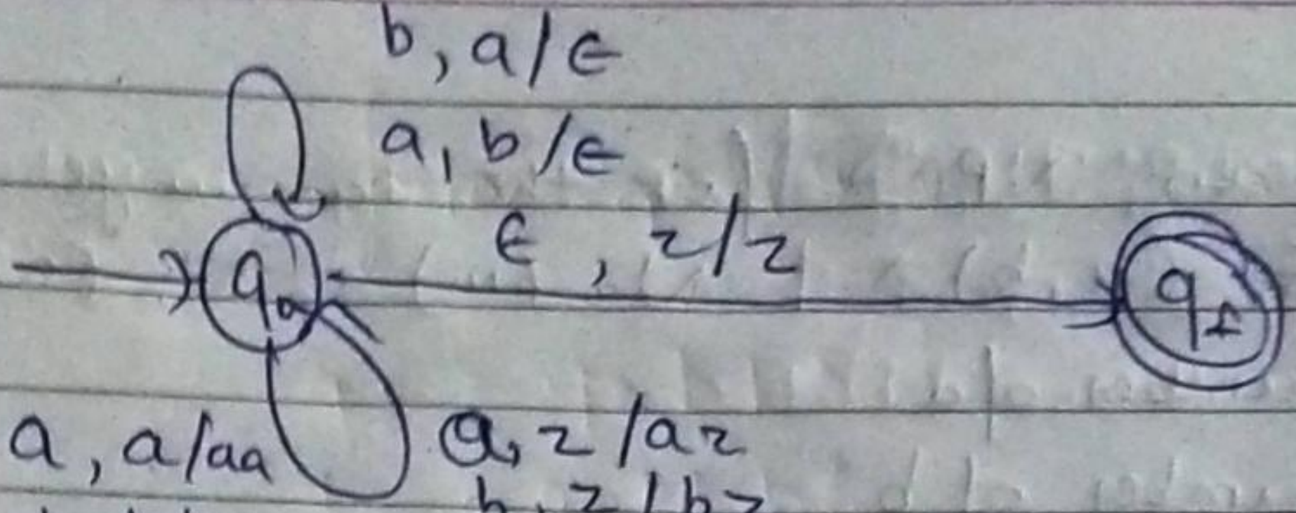
Yadav
14/27



Required NPDA

Q.2. Construct a CFG equivalent to the following PDA.

$A = (\{q_0, q_1\}, \{a, b\}, \{A, z\}, \delta, q_0, z, \phi)$

$\delta(q_0, a, z) = (q_0, Az)$

$\delta(q_0, a, A) = (q_0, \epsilon)$

$\delta(q_0, a, A) = (q_0, \epsilon)$

$\delta(q_0, \epsilon, z) = (q_0, Az)$

SOL: $S \rightarrow [q_0\ z\ q_0] \mid [q_0\ z\ q_1]$

$\delta(q_0\ a\ z_0) = (q_0, \epsilon)$ --- Apply Rule 2

$[q_0\ z\ q_0] \rightarrow a$

$\delta(q_0\ a\ A) = (q_1, \epsilon)$ --- Apply Rule 2

$[q_0\ z\ q_0] \rightarrow a$

$\delta(q_0\ a\ A) = (q_0, A, a)$ --- (Apply Rule 3)

$[q_0\ z\ q_0] \rightarrow a[q_0\ z\ q_0][q_1\ A\ q_0]$

$[q_0\ A\ q_0] \rightarrow a[q_0\ z\ q_1][q_0\ z\ q_1]$

$[q_1\ z\ q_0] \rightarrow a[q_0\ A\ q_1][q_1\ z\ q_0]$

$[q_0\ A\ q_1] \rightarrow a[q_1\ A\ q_0][q_0\ z\ q_1]$

Preeti Yadav -14

Yadav

15/12/17

Page No.
Date

$\delta(q_0, \in, Z) = (q_0, zz)$ Apply Rule 3

$[q_0\ A\ q_0] \rightarrow \in [q_0\ z\ q_0]\ [q_1\ A\ q_1]$

$[q_0\ z\ q_1] \rightarrow \in [q_0\ A\ q_0]\ [q_1\ z\ q_0]$

$[q_1\ z\ q_1] \rightarrow \in [q_0\ A\ q_1]\ [q_1\ z\ q_0]$

$[q_0\ A\ q_1] \rightarrow \in [q_1\ A\ q_2]\ [q_1\ I\ q_2]$

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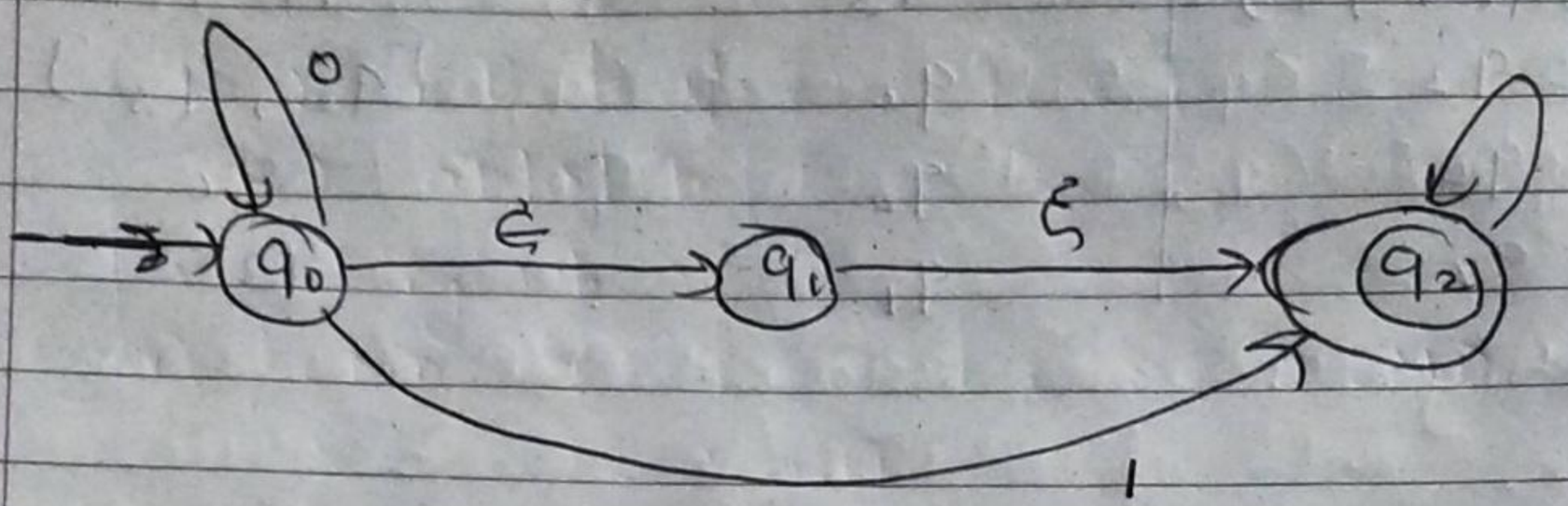
Yadav

16/27

Page No.
Date

Q.10. Find equivalent NFA without ∈ transition for following diagram.



=) ∈-closure $(q_0) = \{q_0, q_1, q_2\}$

∈-closure $(q_1) = \{q_1, q_2\}$

∈-closure $(q_2) = \{q_2\}$

$\delta(q_0, 0)$ = ∈-closure (move $(q_0, 0)$)

= ∈-closure (q_0)

= ~~$\{q_0$~~ ~~$(q_0$~~ $\{q_0, q_1, q_2\}$

$\delta(q_0, 1)$ = ∈-closure (move $(q_0, 1)$)

= ∈-closure (q_2)

= $\{q_2\}$

$\delta(q_1, 0)$ = ∈-closure (move $(q_1, 0)$)

= ∈-closure (move $(q_2, 0)$)

= ∈-closure (q_2)

= $\{q_2\}$

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Yadav

17/27

$\delta(q_1, 1) = \in\text{-closure}(\text{move}(q_1, 1))$

$= \in\text{-closure}(\text{move}(q_2, 1))$

$= \in\text{-closure}(\phi)$

$= \{\phi\}$

$\delta(q_2, 0) = \in\text{-closure}(\text{move}\{q_2, 0))$

$= \in\text{-closure}(q_2)$

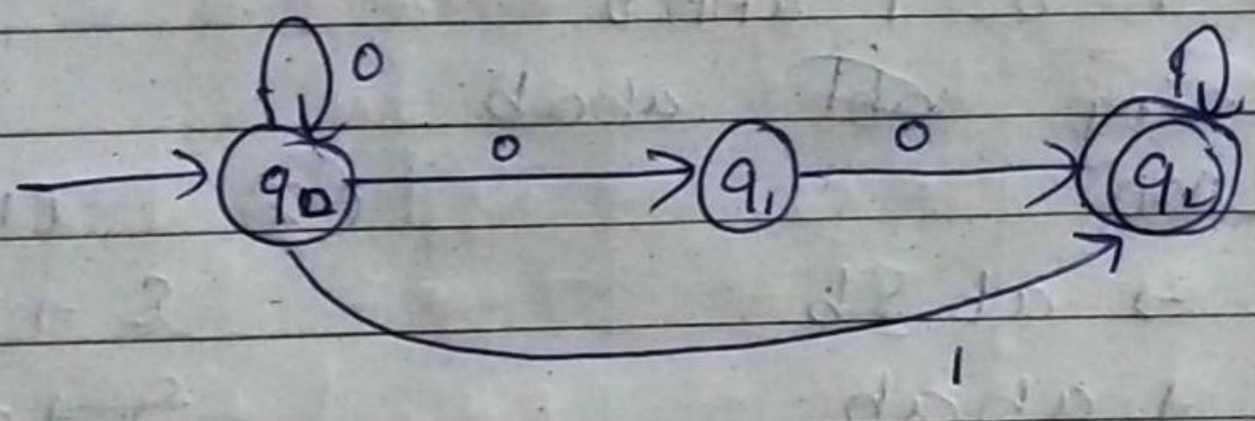
$= \{q_2\}$

$\delta(q_2, 1) = \in\text{-closure}(q_2, 1)$

$= \in\text{-closure}(\phi)$

$= \{\phi\}$

	0	1
q_0	$\{q_0, q_1, q_2\}$	$\{q_2\}$
q_1	$\{q_2\}$	ϕ
q_2	$\{q_2\}$	ϕ



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18/27

Page No.
Date

Q.11. Show that the given grammar is ambiguous.

a. $S \rightarrow aB \mid ab$

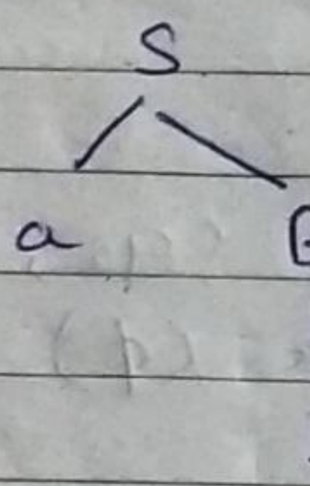
$B \rightarrow aBb \mid b$

⇒ for string ab

① LMD

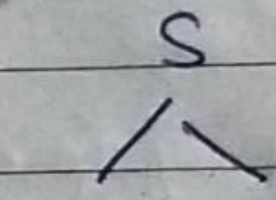
$S \rightarrow aB$

$S \rightarrow ab$



② LMD

$S \rightarrow ab$



There are two leftmost derivation for the same string. Therefore, given grammar is ambiguous.

b. $S \rightarrow a \mid abSb \mid aAb$

$A \rightarrow bS \mid aAAb$

⇒ for string ~~abP~~ abab

① LMD

$S \rightarrow abSb$

$S \rightarrow abab$

② LMD

$S \rightarrow aAb$

~~$S \rightarrow abSb$~~

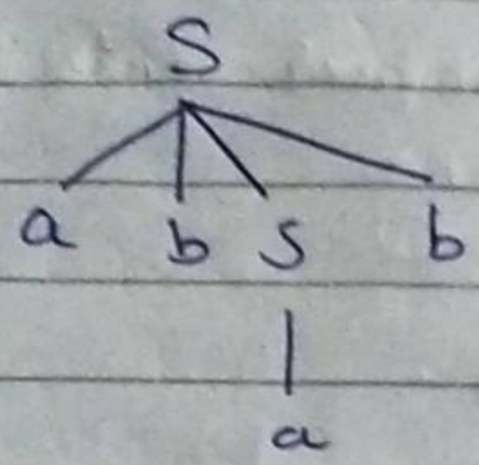
$S \rightarrow abSb$

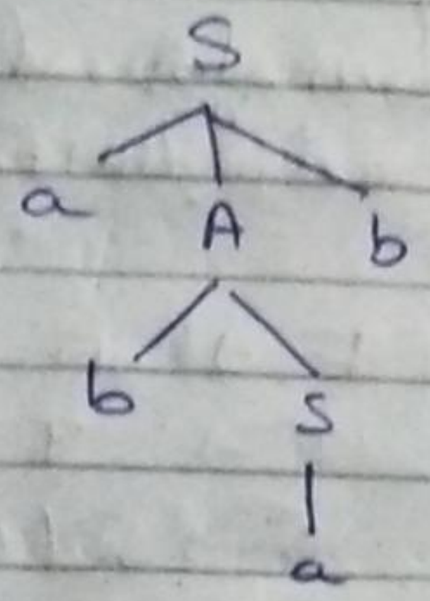
$S \rightarrow abab$

Preeti Yadav - 14

Yadav

19/27





There are two leftmost derivation for the given string. Therefore, given grammar is ambiguous.

Preeti Yadav - 14

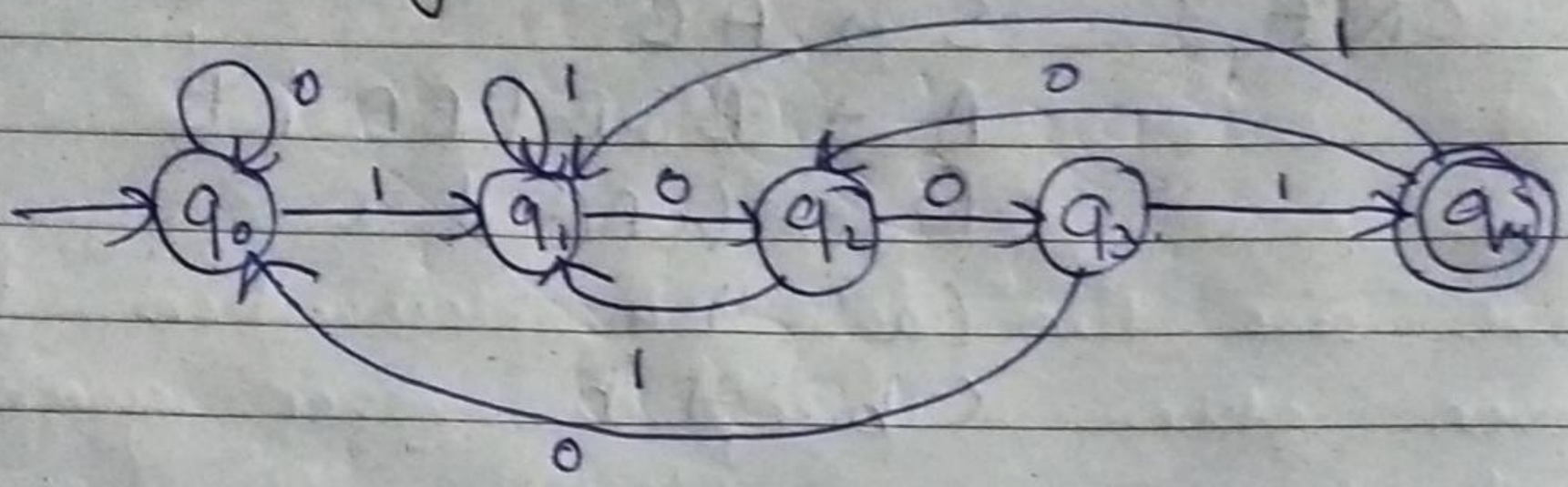
Yadav

20/27

Page No.
Date

Q.12. Construct a DFA over input symbol {0,1} for the following

a. All string endwith 1001.



$M = \langle Q, \Sigma, \delta, q_0, F \rangle$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma = \{0, 1\}$
$q_0 = \{q_0\}$
$F = \{q_4\}$

δ : Transition state

q / Σ	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_1
q_3	q_0	q_4
q_4	q_2	q_1

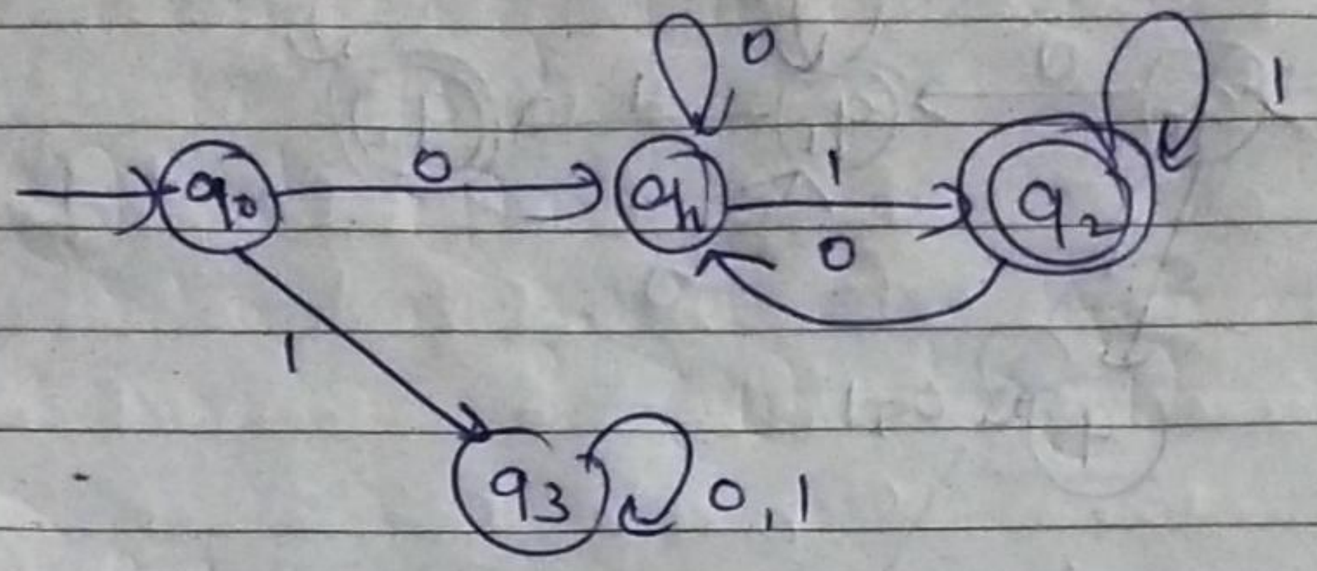
Preeti Yadav - 14

Yadav

21/27

Page No.
Date

b. all strings starts with 0 and ends with 1.



Q : $\{q_0, q_1, q_2, q_3\}$

Σ : $\{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

S:

q / Σ	0	1
q_0	q_0	q_3
q_1	q_1	q_2
(q_2)	q_1	q_2
q_3	q_3	q_3

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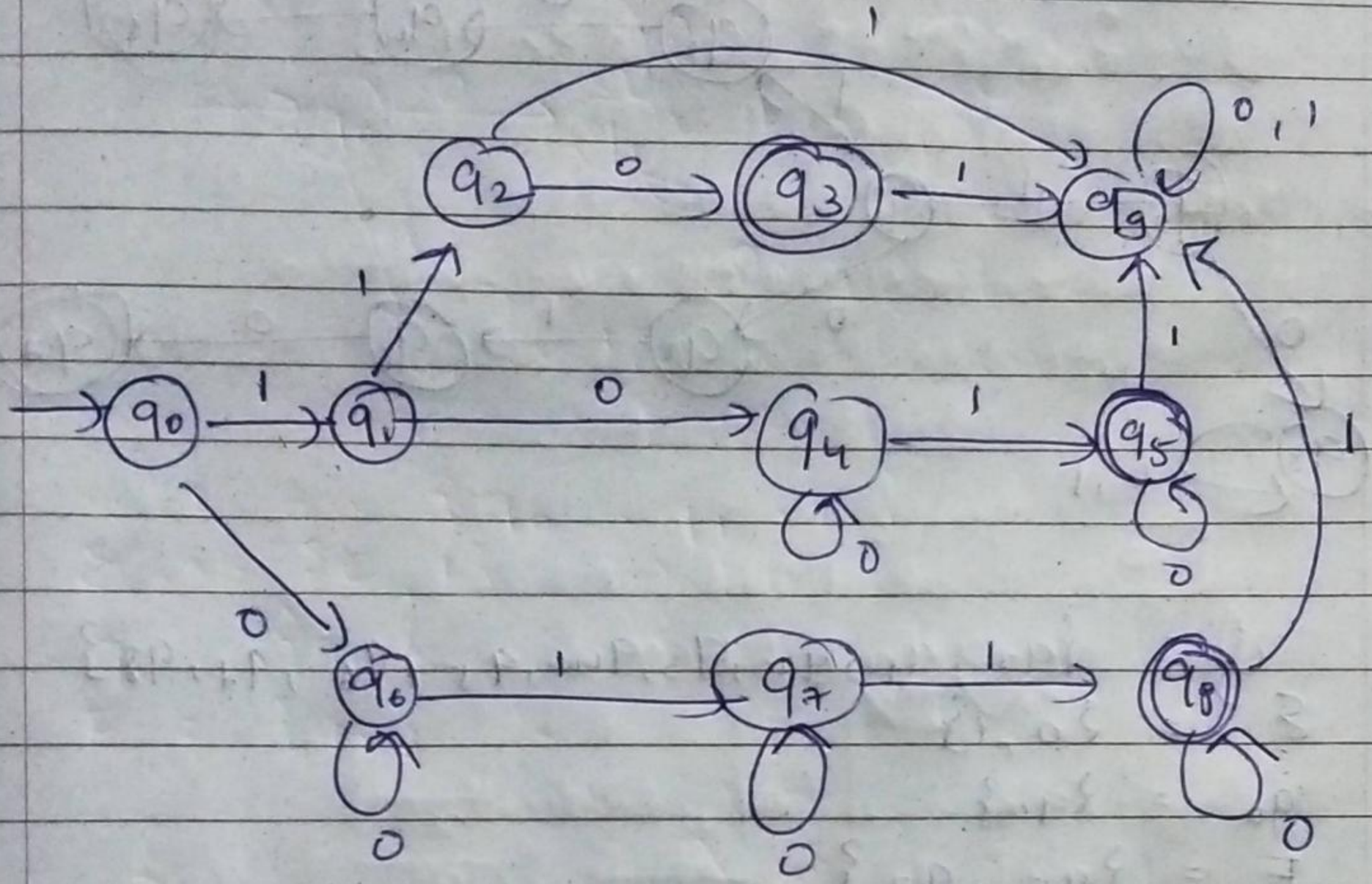
Yadav

22/27

Page No.
Date

c. all strings with atleast one zero and exactly two 1.



Q: $\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_3, q_5, q_8\}$

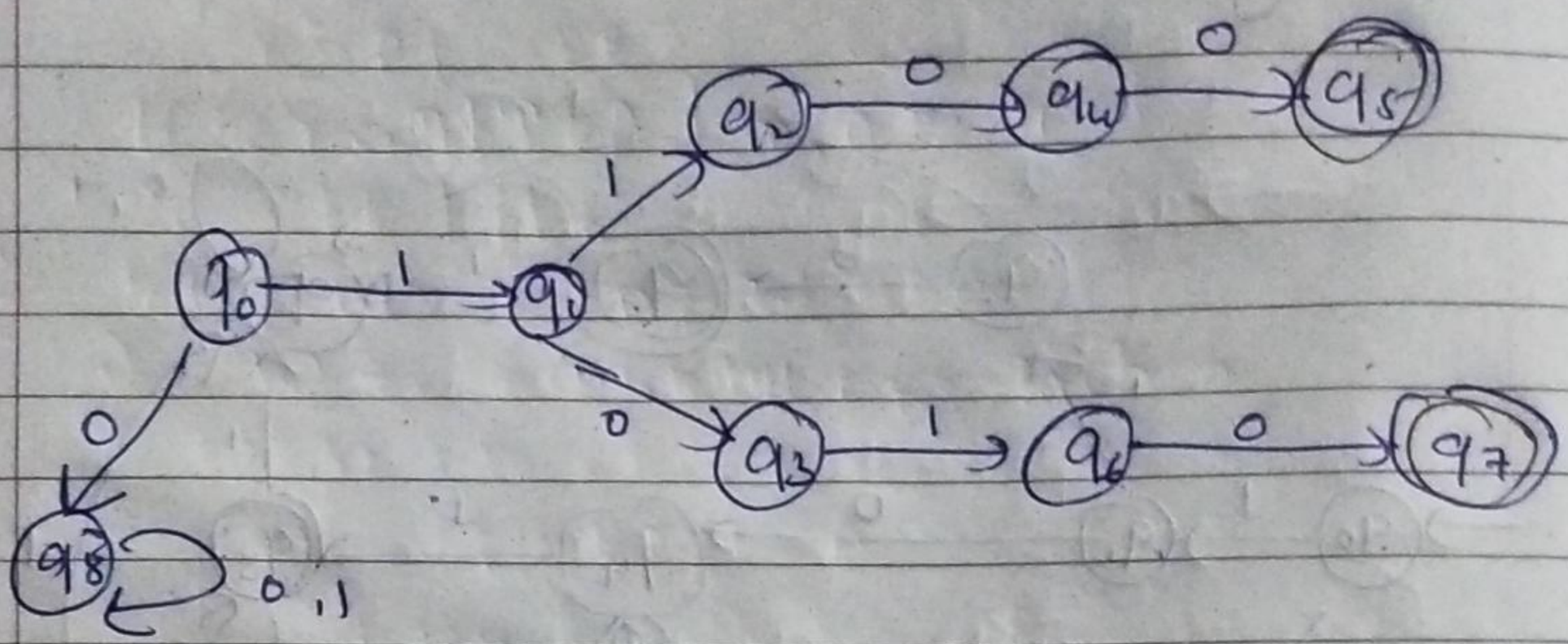
Q \ Σ	0	1
q_0	q_6	q_1
q_1	q_4	q_2
q_2	q_3	q_9
q_3	-	q_9
q_4	q_4	q_5
q_5	q_5	q_9
q_6	q_6	q_7
q_7	q_7	q_8
q_8	q_8	q_9
q_9	q_9	q_9

Preeti Yadav - 14 Yadav

~~11/27~~ 23/27

Page No.
Date

d. All strings accepting 1100 or 1010 only.



$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}$
$\Sigma = \{0, 1\}$
$q_0 = \{q_0\}$
$F = \{q_5, q_7\}$

δ :

q / Σ	0	1
q_0	q_8	q_1
q_1	q_3	q_2
q_2	q_4	-
q_3	-	q_6
q_4	q_5	-
(q_5)	-	-
q_6	q_7	-
(q_7)	-	-
q_8	q_8	q_8

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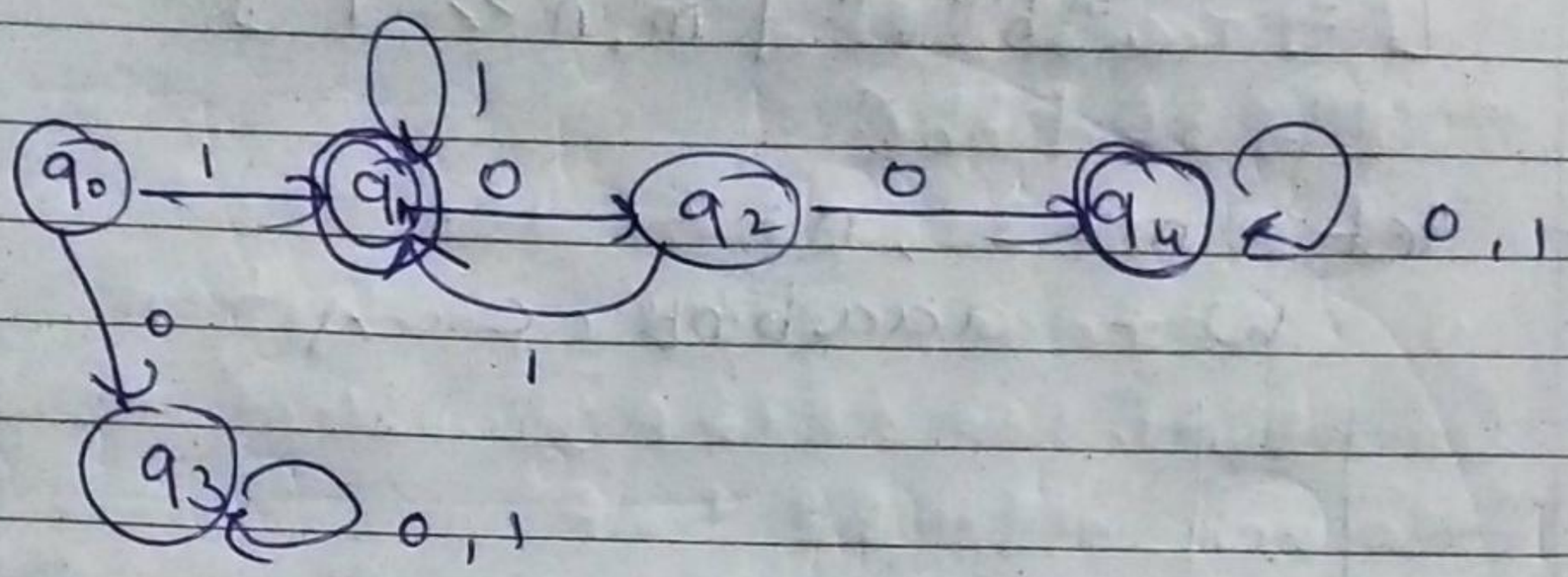
Yadav
24/27

c. All strings in with each zero is immediately preceded and immediately followed by 1.

Ans)



Q : $\{q_0, q_1, q_2, q_3, q_4\}$

Σ : $\{0, 1\}$

q_0 : $\{q_0\}$

F : $\{q_1\}$

δ :

$Q \setminus \Sigma$	0	1
q_0	q_3	q_1
q_1	q_2	q_1
q_2	q_4	q_1
q_3	q_3	q_1
q_4	q_4	q_4

Preeti Yadav - 14

Yadav

25/27

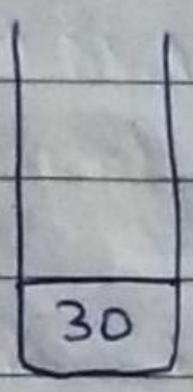
Page No.
Date

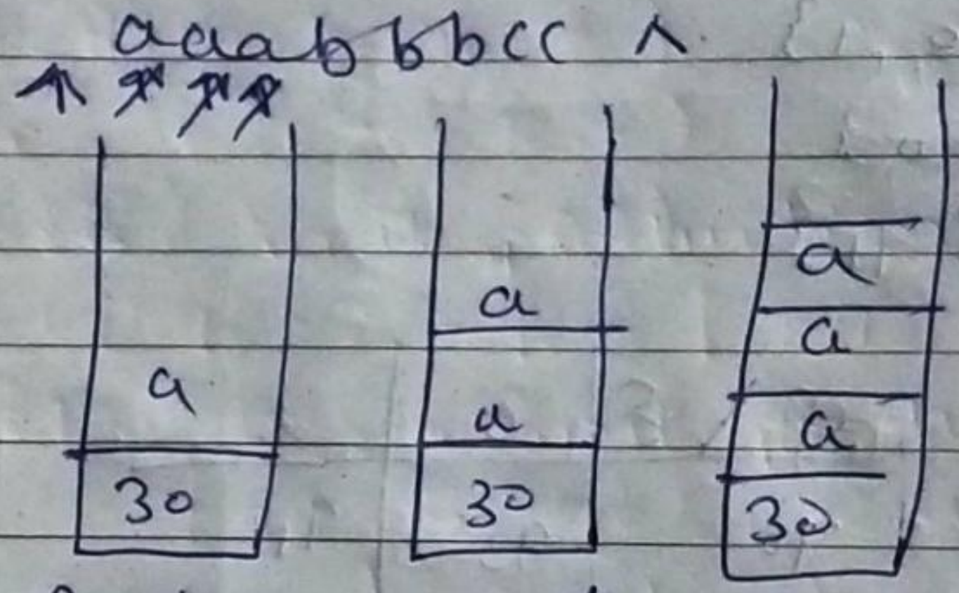
Q.13. Design a ~~PDA~~ PDA for accepting a language.

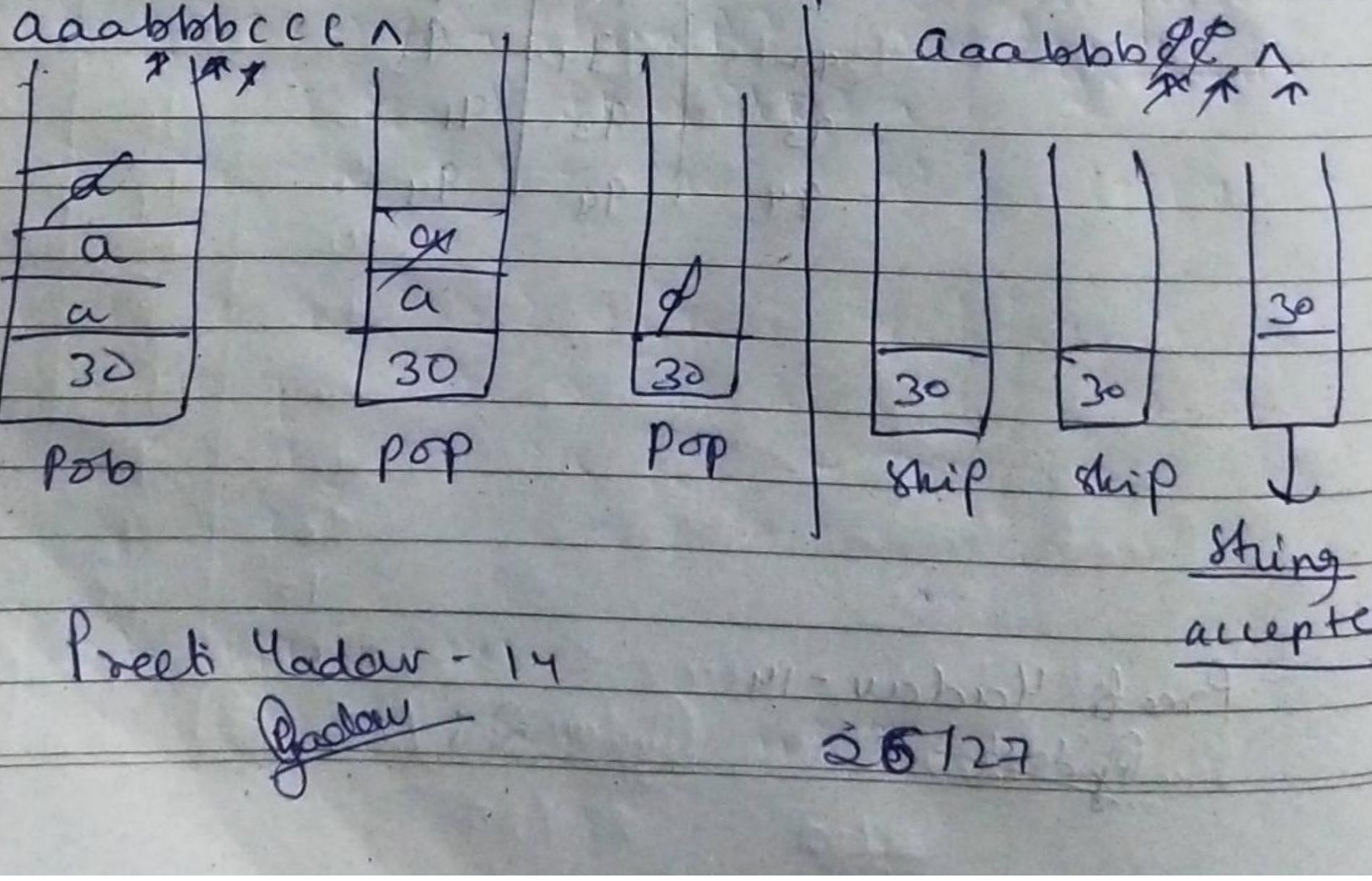
a. $L = \{a^m b^m c^n \mid m, n \geq 1\}$

Sol^n: let $m = 3$, $n = 2$

$w = aaabbbcc \wedge$







Preeti Yadav - 14

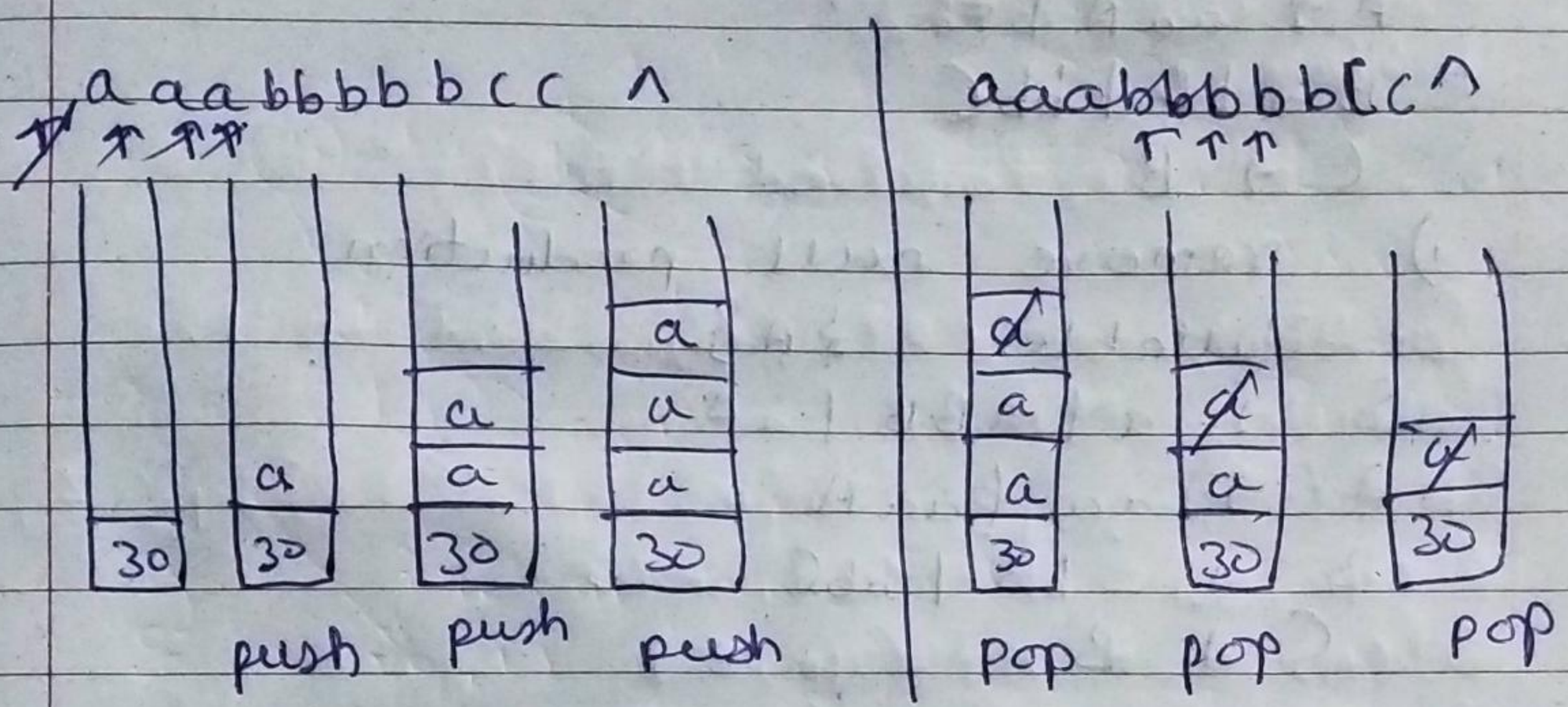
Yadav

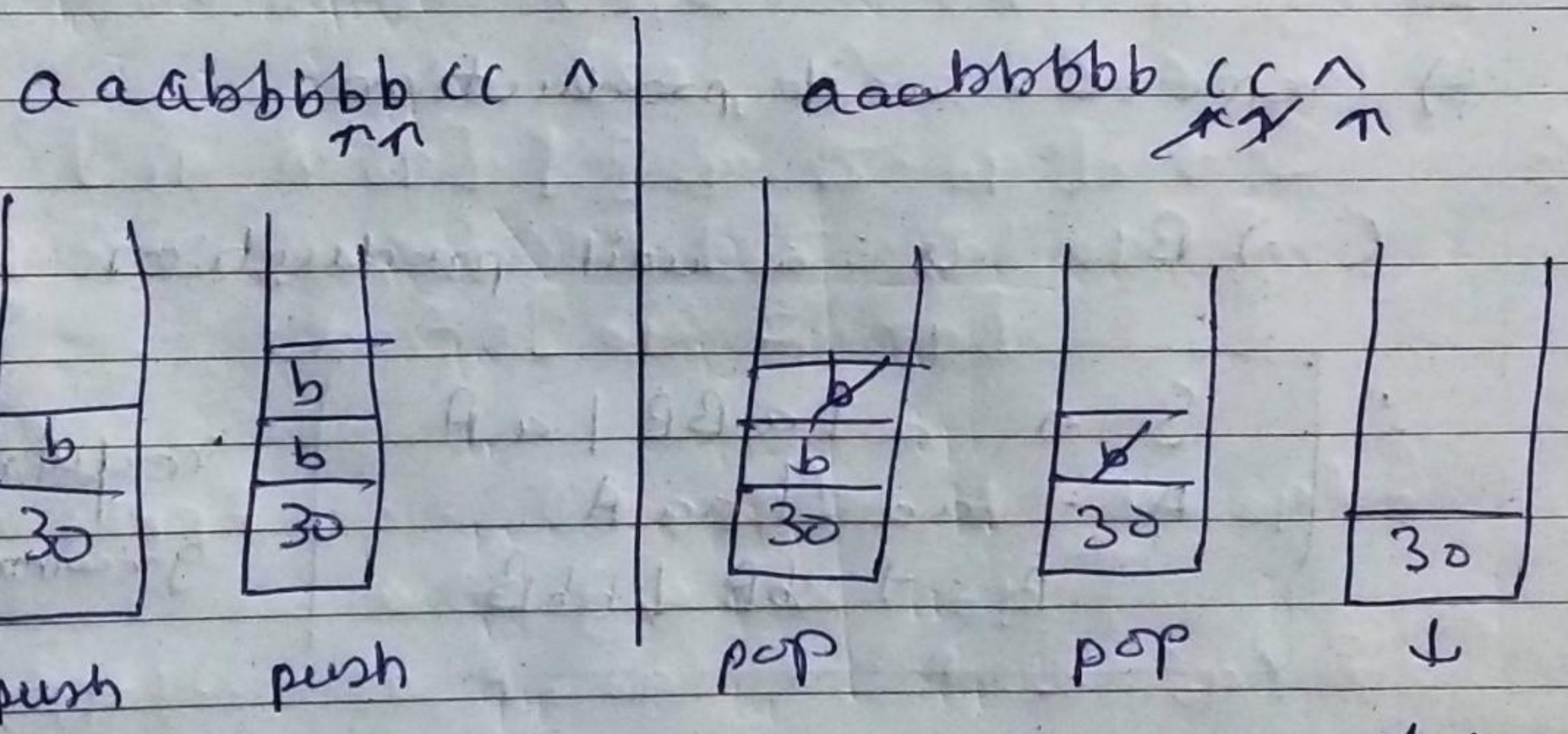
25/27

b. $L = \{ a^n b^{n+m} c^m \mid n >= 0, m >= 1 \}$

Sol:- let $n = 3$, $m = 2$

$W = aaabbbbbcc$





Preeti Yadav - 14

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27/27