

1. Using a graph to illustrate slope and intercept, define basic linear regression.

Linear regression is a statistical modeling technique used to analyze the relationship between two variables. It aims to find the best-fitting line that represents the linear relationship between the dependent variable (Y) and the independent variable (X). The line is determined by its slope and intercept. In a graph, the slope represents the steepness of the line. It indicates how much the dependent variable changes for a unit increase in the independent variable. The intercept represents the point where the line crosses the Y-axis when the independent variable is zero.

2. In a graph, explain the terms rise, run, and slope.

Rise: It refers to the vertical distance between two points on the graph. It represents the change in the Y-axis value.

Run: It refers to the horizontal distance between two points on the graph. It represents the change in the X-axis value.

Slope: It is the ratio of the rise to the run. Slope measures the steepness of the line and indicates how much the Y-axis changes for a unit increase in the X-axis.

3. Use a graph to demonstrate slope, linear positive slope, and linear negative slope, as well as the different conditions that contribute to the slope.

Slope: The slope represents the ratio of the vertical change (rise) to the horizontal change (run). It determines the steepness of the line. A positive slope indicates an upward trend, where Y increases as X increases. A negative slope indicates a downward trend, where Y decreases as X increases.

4. Use a graph to demonstrate curve linear negative slope and curve linear positive slope.

- Curve linear negative slope: In this case, the line curves downward as X increases, indicating a negative relationship between X and Y. The slope is negative, but it changes across different regions of the graph.
- Curve linear positive slope: In this case, the line curves upward as X increases, indicating a positive relationship between X and Y. The slope is positive, but it changes across different regions of the graph.

5. Use a graph to show the maximum and low points of curves.

- Maximum point: In a curved graph, the maximum point represents the highest point on the curve. It indicates a peak or a turning point where the slope changes from positive to negative.
- Low point: The low point represents the lowest point on the curve. It indicates a bottom or a turning point where the slope changes from negative to positive.

6. Use the formulas for a and b to explain ordinary least squares.

Ordinary least squares (OLS) is a method used in linear regression to estimate the coefficients of the linear equation. The formula for the line is represented as $Y = a + bX$, where a is the intercept and b is the slope.

- Formula for a (intercept): It represents the point where the regression line intersects the Y-axis ($X = 0$).
- Formula for b (slope): It represents the change in the dependent variable (Y) for a one-unit change in the independent variable (X).

7. Provide a step-by-step explanation of the OLS algorithm.

- Calculate the means of the dependent variable (Y) and the independent variable (X).
- Calculate the differences between each X value and the mean of X .
- Calculate the differences between each Y value and the mean of Y .
- Calculate the product of the differences obtained in steps 2 and 3.
- Calculate the sum of the products obtained in step 4.
- Calculate the squared differences of X values from the mean of X .
- Calculate the sum of the squared differences obtained in step 6.
- Calculate the slope (b) by dividing the sum of the products from step 5 by the sum of squared differences from step 7.
- Calculate the intercept (a) by subtracting the product of the slope (b) and the mean of X from the mean of Y .

- The resulting values of a and b represent the coefficients of the linear equation $Y = a + bX$.

8. What is the regression's standard error? To represent the same, make a graph.

The regression's standard error measures the average distance between the observed values and the predicted values. It quantifies the accuracy of the regression model. In a graph, the standard error is represented by the vertical distance between the observed data points and the regression line.

9. Provide an example of multiple linear regression.

Example of multiple linear regression: Multiple linear regression involves predicting a dependent variable (Y) based on multiple independent variables (X_1, X_2, X_3 , etc.). For example, predicting house prices based on variables like the number of bedrooms, square footage, and location.

10. Describe the regression analysis assumptions and the BLUE principle.

Linearity: The relationship between the independent and dependent variables should be linear.

Independence: The observations should be independent of each other.

Homoscedasticity: The variance of the residuals (the differences between observed and predicted values) should be constant across all levels of the independent variables.

Normality: The residuals should follow a normal distribution.

BLUE principle (Best Linear Unbiased Estimator): In linear regression, the aim is to find the best unbiased estimates for the coefficients (a and b) that minimize the sum of squared residuals.

11. Describe two major issues with regression analysis.

- **Multicollinearity:** It occurs when independent variables are highly correlated with each other, making it difficult to distinguish their individual effects on the dependent variable.

- **Overfitting:** Overfitting happens when the regression model fits the training data too closely, leading to poor performance on new, unseen data. It may occur when the model is too complex or when there is insufficient data.

12. How can the linear regression model's accuracy be improved?

- **Feature selection:** Selecting relevant and significant independent variables to include in the model.
- **Data preprocessing:** Cleaning the data, handling missing values, and transforming variables if necessary.
- **Non-linear transformations:** Consider using non-linear transformations of variables to capture complex relationships.
- **Regularization techniques:** Applying regularization methods like Ridge regression or Lasso regression to prevent overfitting.
- **Cross-validation:** Using cross-validation techniques to assess the model's performance on unseen data and tune model parameters.

13. Using an example, describe the polynomial regression model in detail.

Polynomial regression model: Polynomial regression is a type of regression analysis where the relationship between the independent variable (X) and the dependent variable (Y) is modeled as an nth-degree polynomial. For example, if the polynomial order is 2, the equation becomes $Y = a + bX + cX^2$. This allows capturing non-linear relationships between variables.

14. Provide a detailed explanation of logistic regression.

Logistic regression: Logistic regression is a type of regression used for predicting binary outcomes. It models the relationship between the independent variables and the probability of a certain event occurring. The logistic regression model applies a sigmoid function to transform the linear equation into a probability value between 0 and 1.

15. What are the logistic regression assumptions?

Linearity: The relationship between the independent variables and the log-odds of the dependent variable should be linear.

Independence: The observations should be independent of each other.

Absence of multicollinearity: The independent variables should not be highly correlated with each other.

No influential outliers: Extreme outliers should not have a significant impact on the model.

16. Go through the details of maximum likelihood estimation.

Maximum likelihood estimation (MLE) is a method used in logistic regression to estimate the parameters of the model. It aims to find the parameter values that maximize the likelihood of the observed data given the model. The likelihood is a measure of how well the model explains the observed data. The MLE method iteratively adjusts the parameter values until convergence is achieved and the maximum likelihood is obtained.