Endsem

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1 General Instructions and Hints:

- There is partial marking for all the tasks. This code is heavy to run. So first, just generate the mesh and save the figure. Then write the weak forms for Steady Stokes and Navier Stokes equations with the correct boundary conditions. Once you are done with this job, then and only then run the time simulation, which will take almost twenty minutes to run.
- For boundary conditions $2a$ and $2b$ you have to type them in exact form as given in the question. But remember, boundaries are one dimensional!
- If you do not have time to run the code, please add two lines that extract vorticity. That carries some weight too.
- Do not forget to add the penalty term $\int_\Omega \epsilon * p * q$
- Remember you are dealing with a 2d problem, so vorticity $\nabla \times u$ is just $vort = dy(u) - dx(v)$
- While plotting figures, add **WhiteBackground=0** to make the background black.
- The last piece of advice to crack this problem, straight from The Hitchhiker's Guide to the Galaxy: **Don't Panic!**

2 Flow past two cylinders in a row

1. Define the domain in FreeFem++ as described below.

   Rectangular domain of dimensions (5×3) in $(X \times Y)$ direction. There are two in-line cylinders lying along x direction with their centres at (1,1.5) and (4,1.5). Radii of these cylinders are same and are equal to 0.05 units.

   Meshed geometry looks as can be seen in Fig. 1. To produce this mesh, use 80 nodal points on each of the boundary.

2. Solve the steady Stokes equation with following boundary conditions.

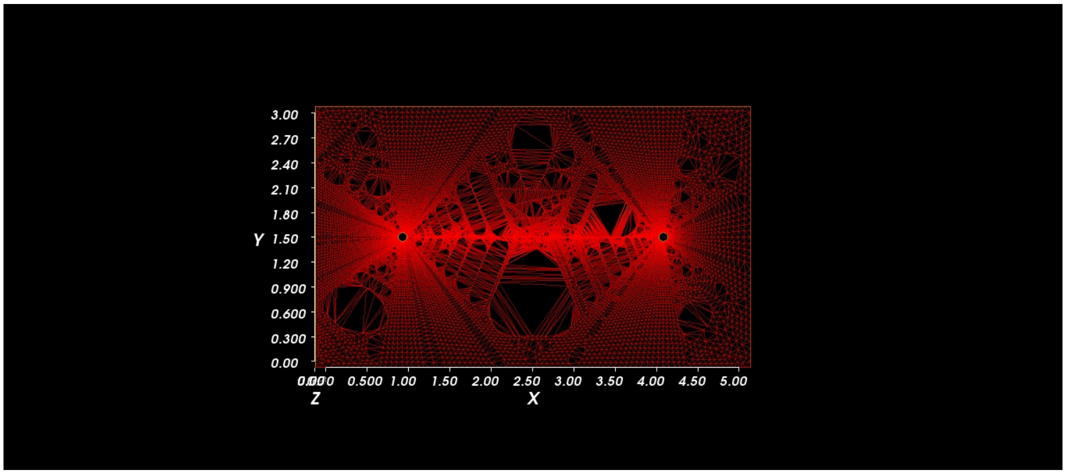

Figure 1: meshed geometry

(a) On the right boundary of the rectangle

$$\nu\left(\frac{\partial u}{\partial x}(N.x)+\frac{\partial u}{\partial y}(N.y)\right)=0.$$

(b) On the right boundary of the rectangle

$$\nu\left(\frac{\partial v}{\partial x}(N.x)+\frac{\partial v}{\partial y}(N.y)\right)=0.$$

(c) $u=0, v=0$ on cylindrical surface (i.e., on the circles).

(d) $u=1, v=0$ on top, left and bottom boundary of the rectangle.

Then, set the $[uold, vold]$ to be the Stokes solution $[u, v]$.

3. Now solve the Navier-Stokes equation using this $[u, v]$ as $[uold, vold]$ with the same boundary conditions. Use $tmax = 2s$ and $dt = 1s$.

Extract the vorticity $\nabla \times u$ as time progresses. Submit this file along with the code. Plot vorticity at $t = tmax$