<Ans 6>

(a) Symmetric

To show: $H^T = H$

$H^T = \{X(X^TX)^{-1}X^T\}^T$

Rule of matrix: $(A \cdot B)^T = B^T A^T$.

$\therefore H^T = \{X(X^TX)^{-1} \cdot X^T\}^T$

$= (X^T)^T \cdot (X(X^TX)^{-1})^T$

$= X \cdot \{((X^TX)^{-1})^T \cdot X^T\}$ ---- $(AB)^T = B^TA^T$

$= X \cdot \{((X^TX)^T)^{-1} \cdot X^T\}$ -- $(A^{-1})^T = (A^T)^{-1}$

$= X \{(X^T \cdot (X^T)^T)^{-1} \cdot X^T\}$

$= X \{(X^T \cdot X)^{-1} X^T\}$

$= X \cdot (X^TX)^{-1} X^T$

$= H$

$\Rightarrow \boxed{H^T = H} \Rightarrow$ Symmetric

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(b) Idempotent

To show: $H \cdot H = H$

$$H \cdot H = \left(X (X^T X)^{-1} X^T \right) \cdot \left(X \cdot (X^T X)^{-1} X^T \right)$$

Associative matrix product rule,

$ABC = A(BC) = (AB)C.$

$$\Rightarrow H \cdot H = \left(X (X^T X)^{-1} (X^T X) \cdot (X^T X)^{-1} X^T \right)$$

$$= X \cdot I \cdot (X^T X)^{-1} X^T \quad \text{-----} (A \cdot A^{-1} = I)$$

$$= X \cdot (X^T X)^{-1} X^T \quad \text{---} (A \cdot I = A)$$

$$= H$$

$\Rightarrow \boxed{H \cdot H = H} \Rightarrow$ Idempotent

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