HW1 Q1

- Composite law for Rotational Transformations w.r.t current frame:

$$R_2^0 = R_1^0 \cdot R_2^1$$

But this formula doesn't hold true when we are doing rotational transformation w.r.t a <u>fixed</u> frame

$$R_0^2 = R \cdot R_1^0$$ where R is the representation of rotation relative to the fixed frame.

To answer the given question:

$$R_A^B = R_1^B \cdot R_A^1$$

$$= R_{y,\pi/2} \quad R_{x,\pi/2}$$

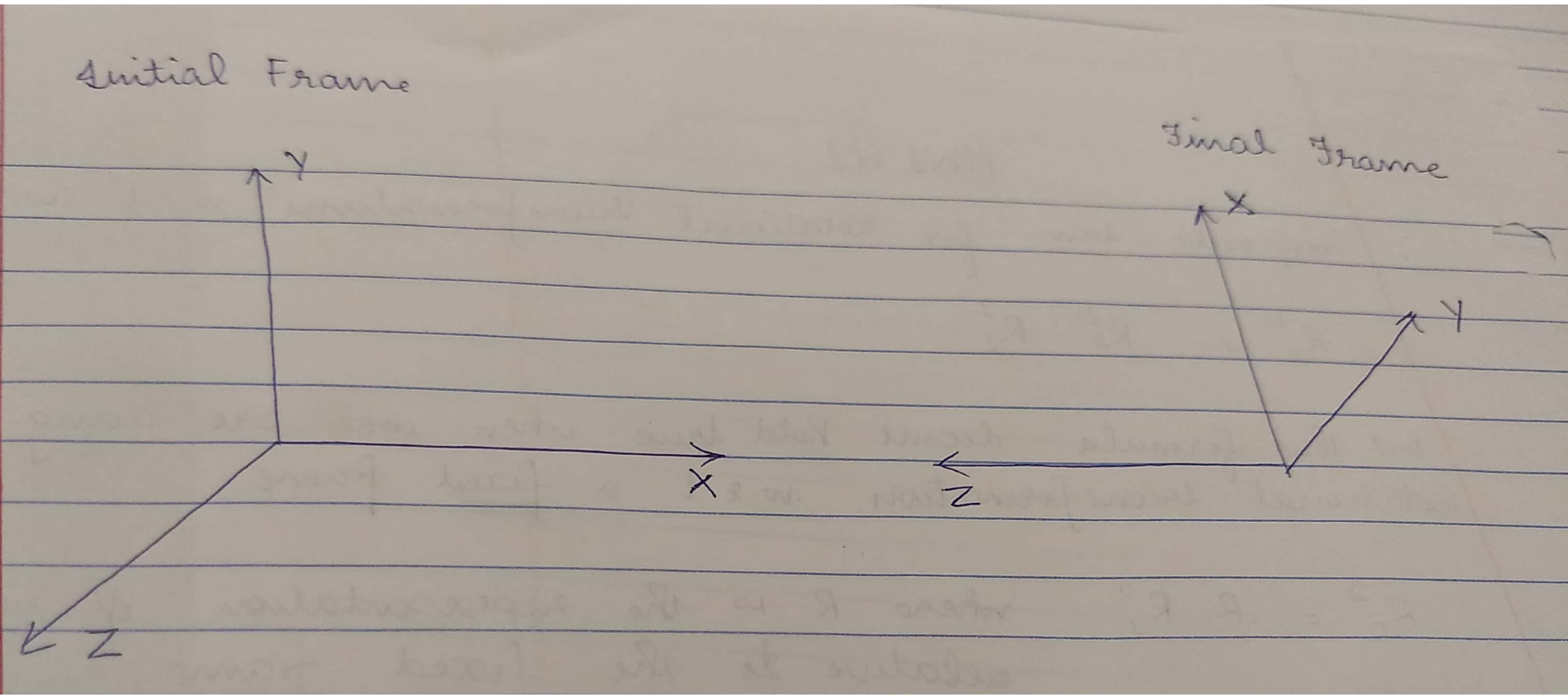
$$= \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\pi/2 & 0 & \sin\pi/2 \\ 0 & 1 & 0 \\ -\sin\pi/2 & 0 & \cos\pi/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}.$$

Since we are not translating the frame, we have used just the rotation matrix.


Initial Frame
Y
X
Z
Final Frame
X
Y
Z

HW1 Q2

(1) Base frame - 0 , New Frame 1

$$T_1^0 = \begin{pmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{pmatrix}$$

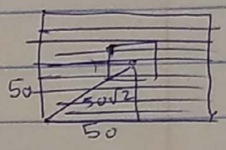
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Since no rotation of axis is involved, the rotation matrix will be I.

(2) Base frame - 0 , New Frame - 2

$$T_2^0 = \begin{pmatrix} R_2^0 & d_2^0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



(3) Base frame - 0 , New Frame - 3

$$T_3^0 = \begin{pmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotation matrix can b[e] easily derived using observation.

(4) Base Frame = 3 , New Frame - 2

$$T_2^3 = T_0^3 \cdot T_2^0$$

HW1Q 2

Using rotation matrix properties,

$$R_0^3 = (R_3^0)^{-1} = (R_3^0)^T$$

$$\therefore \quad T_2^3 = \begin{pmatrix} 0 & 1 & 0 & -0.5\cancel{\sqrt{2}} \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -0.5\cancel{\sqrt{2}} \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & 0\ \cancel{2.207} \\ 1 & 0 & 0 & 0\ \cancel{2.207} \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

HW1Q4

$J \in \mathbb{R}^{m \times n}$

$\therefore JJ^T \in \mathbb{R}^{m \times m}$

This is a square matrix and has Eigenvalues & Eigenvectors that satisfies

$$JJ^T u_i = \lambda_i u_i$$

$$(JJ^T - \lambda_i I) u_i = 0$$

which implies that $(JJ^T - \lambda_i I)$ is singular & therefore $\det(JJ^T - \lambda_i I) = 0$

We can use the above equation to find eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m \geq 0$ for JJ^T

The singular values for the Jacobian matrix J are given by square root of eigenvalue of JJ^T

Let $\sigma_i = \sqrt{\lambda_i}$

SVD of a matrix $J \Rightarrow J = U \Sigma V^T$ where

$U = [u_1 \ u_2 \dots u_m]$, $V = [v_1 \ v_2 \dots v_m]$ are orthogonal matrices & $\Sigma \in \mathbb{R}^{m \times n}$

$\therefore JJ^T u_i = \sigma_i^2 u_i$

$\Rightarrow$

$JJ^T U = U \Sigma_m^2$ where $\Sigma_m = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_m \end{bmatrix}$

HW1Q4

$V_m = J^T U \Sigma^{-1}{}_m$

Manipulability measure i.e volume of the ellipsoid = $k\sigma_1\sigma_2 \ldots \sigma_m$ where k is constant that depends on dimension 'm' of the ellipsoid

The manipulability measure is defined by $w = \sigma_1\sigma_2 \ldots \sigma_m$

$$\det JJ^T = \det J \det J^T$$
$$= \det J \det J$$
$$= (\lambda_1 \lambda_2 \cdots \lambda_m)(\lambda_1 \lambda_2 \cdots \lambda_m)$$
$$= (\lambda_1^2 \lambda_2^2 \cdots \lambda_m^2)$$

$$w = \sqrt{\det JJ^T} = |\lambda_1 \lambda_2 \ldots \lambda_m| = |\det J|$$

$$\therefore \mu = \prod_{i=1}^{6} \sigma_i(\theta)$$

→ If J is singular, then all of its eigenvectors are 0

Thus, $\mu(\theta) = 0$ in the above case

HW1 Q3

Forward kinematics of a robot refers to the calculation of the position & orientation of its end-effector frame from its joint values.

a_i = Link length
d_i = Displacement along z axis (axis of rotation)
α_i = Rotation along z axis
ϑ_i = Rotation along x axis

In general,

$$A_i^{i-1} = \begin{pmatrix} C\vartheta_i & -S\vartheta_i & 0 & 0 \\ S\vartheta_i & C\vartheta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_i^{i'} = \begin{pmatrix} 1 & 0 & 0 & a_i \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_i^{i-1}(q) = A_{i'}^{i-1} \cdot A_i^{i'}$$

$$T_n^0(q) = A_1^0 \, A_2^1 \cdots A_n^{n-1}$$

Using the Denavit-Hartenberg convention:

i)

Links	a_i	α_i	d_i	ϑ_i
1	l_1	$+\pi/2$	0	θ_1
2	l_2	$+\pi/2$	0	$-\theta_2$
3	l_2	0	0	θ_3

$$T_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2$$

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ii)

Links	a_i	α_i	d_i	v_i
1	l_1	$+\pi/2$	0	θ_1
2	l_2	$-\pi/2$	0	$-\theta_2$
3	l_3	$90-\theta_3$	0	θ_3

iii)

Links	a_i	α_i	d_i	v_i
1	l_1	$+\pi/2$	0	θ_1
2	l_2	0	0	$-\theta_2$
3	l_3	$+\pi/2$	0	$-\theta_3$

iv)

Link	a_i	α_i	d_i	v_i
1	l_1	$+\pi/2$	0	θ_1
2	l_2	$+\pi/2$	0	$-\theta_2$
3	θ_3	0	0	0

HW1 Q5

If the ratio of highest singular value vs lowest singular value of a Jacobian is 1 (condition number = 1) then the ellipsoid of manipulability is circle of spherical. This is the isotropic state

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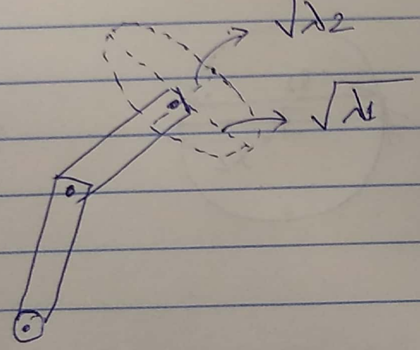
For a 2 planar manipulator . the dimensions of its axes should be of equal values.



$$K = \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}}$$