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SHORT COMMUNICATION

A mathematics-based new penalty area in football: tackling diving

César A. Morales

Departamento de Mecánica, Universidad Simón Bolívar, Caracas, Venezuela

ABSTRACT
A novel penalty area (PA) for football (soccer) is proposed; it is based on considering mathematically the actual scoring possibility on the 2 dimensions near the goal. It is shown that the 150-year-old rectangular area is mathematically disproportionate; this can be causing too much diving or *simulation* by players around the goal and also too many matches that are decided unfairly. The goal or objective is to reduce these problems – and others – with a new PA based on the proposed scoring potential measure which is in turn based on the angle towards the goal line (between posts) and the distance to the centre of this line.

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Penalty area; football; soccer; diving; mathematics

1. Introduction

Every big association football tournament, before and after television replays were possible (late sixties) but particularly afterwards, has its share of plays in the *current* penalty area (PA) that always arouse big debate and controversy globally (Morris & Lewis, 2010; Rogers, 2014) because (a) the offensive player just *dives* – usually when at a low scoring chance position – but still gets a penalty kick, (b) the referee makes a *clear* mistake either favouring the defensive or offensive team and (c) the attacking player gets a clear foul near the corners of the PA but he does not get a *deserved* penalty or any kick in turn because the referee *considers* that the scoring opportunity is not clear, or even his psychology influenced by previous decisions affects the extant one (Plessner & Betsch, 2001), among other pitch situations.

It can be argued that a *sad* example in the last World Cup (2014) was the Robben-Márquez encounter close to the borderline (goal line) at the last minutes of their tied *you-loose-you're-out* match (FIFA, 2014); furthermore, most football fans would agree that the awarded penalty was unfair considering that Robben may have just dived or fallen after feeling a simple and avoidable contact, and noticing a poor angle towards the goal. This is just an example, of the "a" situations above: diving, *simulation* or *embellishment*; however, all circumstances (a, b, c) ought to be reduced – if not eliminated – in the beautiful game if its popularity, quality and marketability is to be preserved or enhanced (David et al., 2011), and here mathematics can help, in drawing a new and optimal PA, as explained in this work.

FIFA defines simulation as an attempt by a player to deceive the referee by feigning injury or pretending to have been fouled (FIFA, 2014). A definition from behavioural sciences is bid by a player to exaggerate the effect of a tackle in order to deceive the referee into awarding him (her) a free kick or penalty (Morris & Lewis, 2010). And, there is also the view from biological sciences: a dive is synonymous with animal mimicry and occurs when a player intentionally mimics the behaviour of an illegal tackle-induced fall, and the referee responds as if it were such a fall by rewarding the player with a *benefit* (David et al., 2011); in this previous study, it has been concluded that 6% of falls in football are dives.

In this short communication, the concentration is on the area close to the attacking goal: a proposal – from mathematical science – for tackling or solving the problem of diving in the PA or attacking box. As a matter of fact, it has been shown that footballers simulated or cheated *twice as frequently* when in the current PA than when in any defensive zone (David et al., 2011), and the effects and impact of an illegitimately and unfairly granted kick therein can be tremendous.

The first fact, and one reason for diving being more frequent in the PA, is that an attacker has an increased tendency to simulate if the *chance of scoring* is low, on the PA; we base this assertion on (1) the research by David et al. in which it was shown that deception or cheating by players diminishes as the potential outcome for them is less beneficial or more costly (David et al., 2011) and (2) the circumstance that it is in fact taught at the amateur and youth levels that diving is *optimal* "when there is little opportunity to create a play" (iSport Football, 2016). Now, chance is a mathematical concept; therefore, this science may have the answer to partially solve the problem, not only for those situations, *a*, but also for the *b* and *c* cases above and others; for example (case b), the quite negative effects on the game of referee errors close to the goal could be drastically reduced if the PA is redrawn so as to consider a measure of actual goal-scoring possibility.

The PA does require a new design because the second, and related, fact is that the 150-year-old (current) PA is disproportionate. This second assertion will be shown subsequently; nevertheless, a hint of the idea is presented in Figure 1 where the current

CONTACT César A. Morales cmorales@usb.ve Departamento de Mecánica, Universidad Simón Bolívar, Apdo. 89000, Caracas 1080A, Venezuela

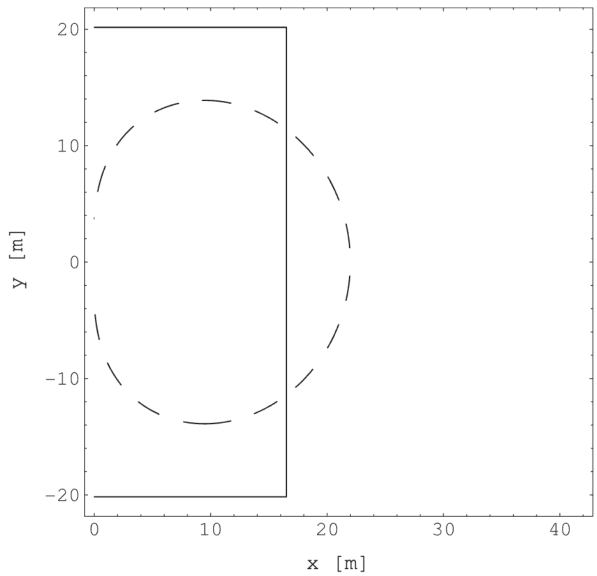

Figure 1. Current penalty area.

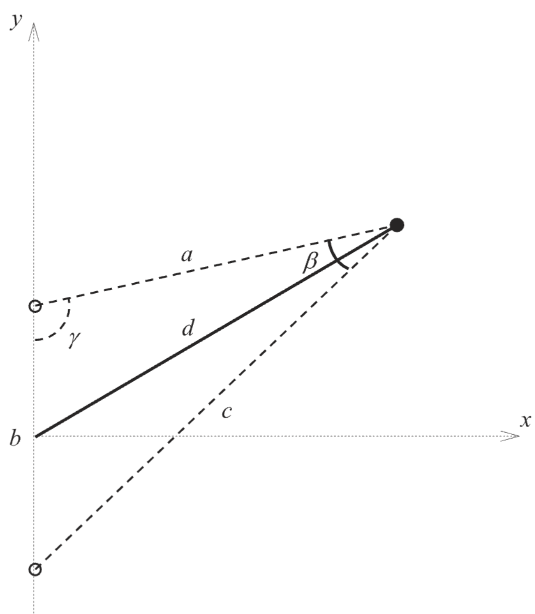

Figure 2. Field near the goal, coordinate system and offensive ball position; b, goal line length = 7.32 m; β, scoring angle; d, distance to middle of goal line.

PA is drawn (solid line) along with – solely for introductory purpose – the new and curved PA line obtained mathematically herein on the basis of the proposed scoring potential measure (dashed line). As indicated, the mathematical details and coordinate system is left for the main Section 2 (Figure 2), it suffices in the Introduction to point that the y axis is the goal line. We note that there are other proposals or suggestions, from other sciences, for tackling or solving the diving problem in football; works in both behavioural and biology sciences suggest the improvement of detection of deceitful behaviour and the increase of punishment for simulation or dishonesty in the sport (David et al., 2011; Morris & Lewis, 2010); in practice, FIFA has chosen the second or punishment way (FIFA, 2014).

Herein, a novel PA for football is proposed; this new design is based on considering mathematically the scoring potential on the two dimensions around the goal (Figure 1). A scoring chance measure is advanced on the basis of the angle towards the goal line between the posts (7.32 m) and the distance to the centre of this line, from an attacking ball. The objective is a new borderline for the PA that could (a) reduce diving or simulation around the goal; (b) reduce the number of matches that are decided unfairly (as the impact of penalty kicks is currently disproportionate) and (c) preserve and enhance the popularity and quality of the game, among other positive outcomes. Regarding *b*, our initial assertion that many matches may be being decided unfairly is based on the fact that *a third* of dives are rewarded by referees with free kicks (David et al., 2011). And regarding *c*, football quality, popularity and marketability can decrease if a significant number of games in big tournaments and competitions – which are the ones televised with ever more detail – are decided unfairly (David et al., 2011). A full literature review or search has been conducted and there are no proposals – scientific or not – on football pitch changes.

2. A mathematical penalty area: method and results

The football field around the goal and a generic position of the offensive ball is shown in Figure 2, where b is the actual goal line length or 7.32 m, β is the scoring angle and d is the distance to the middle of the goal line b; as indicated before, these angle and distance for the attacking player are considered the main parameters or factors; c, a and γ are the other components of the geometry of the problem, which are clearly defined in the figure. It is stressed that this short paper and proposal is mainly or solely based on mathematics; moreover, it is assumed that a direct shot is the first or main option for a ball-carrying attacker close to the goal. Further and future work can consider the empirical approach of analysing statistically real game data, results of which are, for example, *heat* maps (Ichniowski, 2015). In addition, the fact that another player's option is passing the ball to a better located attacker can be considered, either empirically or mathematically; however, in these cases, the analysis and the resulting PA boundaries will not be as simple and elegant, or in closed form.

It is basic trigonometry (sine law) to show that

$$(c/b)^2 \sin^2\beta + \cos^2\gamma = 1 \tag{1}$$

it follows (cosine law) that

$$\sin^2\beta = (b/c)^2\left(1 - \left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2\right) \tag{2}$$

Now, the distances to the posts a and c are of course functions of x and y; if one assumes

$$d > b/2 \tag{3}$$

the expressions are simple

$$a^2 = x^2 + (y - b/2)^2 \quad c^2 = x^2 + (y + b/2)^2 \tag{4}$$

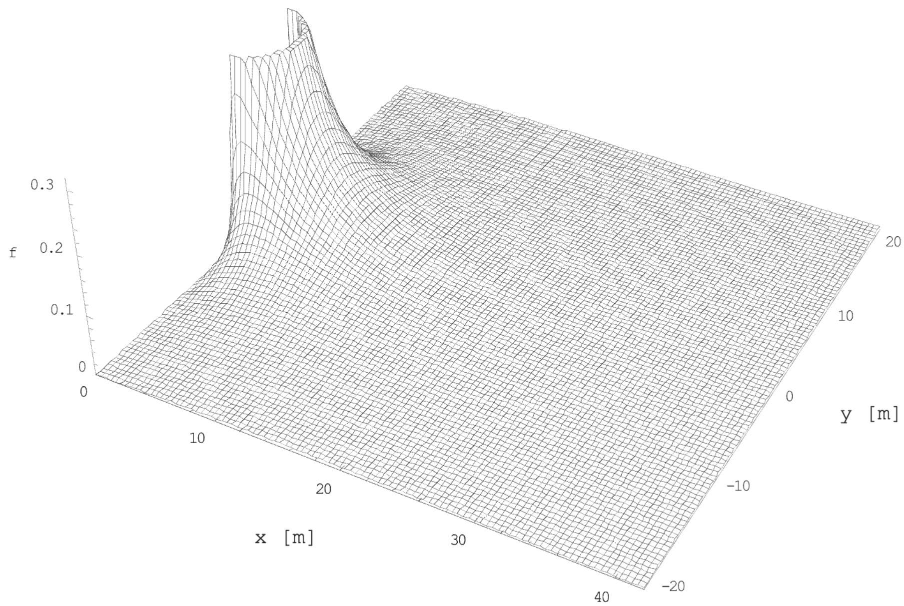

Figure 3. Scoring potential function *f* (*x, y*).

Note that, regarding inequality (3), the interest is not in drawing the novel PA too close to the goal.

After some, though long, algebraic work (Appendix)

$$\sin^2\beta = \frac{(bx)^2}{\left(x^2 + y^2 - (b/2)^2\right)^2 + (bx)^2} \tag{5}$$

Finally and by trigonometry, the expression for the goal angle in front of the offensive player is

$$\beta = \tan^{-1}\left(\frac{bx}{x^2 + y^2 - (b/2)^2}\right) \tag{6}$$

The other factor under consideration for defining a different PA from the old, basic and disproportionate rectangle is the distance to the middle of the goal; this is a much simpler function of course,

$$d = \sqrt{x^2 + y^2} \tag{7}$$

Now, a scoring potential function must be defined; as explained above, it is considered that the main parameters in the function have to be the distance to goal *d* and the scoring angle *β*. It is evident that the closer an offensive player is to the goal, the easier is to score, because the speed of the ball decreases on the trajectory, thus the longer *d* the less powerful the ball will reach the goalkeeper surroundings; moreover, distance is (proportional to) time, and goalkeeper reaction time is an important fraction of ball *flight time*. Additionally, the chances of scoring with a direct shot increase as the scoring angle *β* increases. Therefore, the scoring potential function *f* can be defined as inversely proportional to distance and directly proportional to the shooting angle:

$$f(x,y) = \frac{\beta(x,y)}{d(x,y)} = \frac{\tan^{-1}\left(\frac{bx}{x^2+y^2-(b/2)^2}\right)}{\sqrt{x^2+y^2}} \tag{8}$$

which is plotted in Figure 3; of course, weighing constants could be introduced, or the distance and the angle could be made dimensionless (e.g., *β* can be divided by *π* /2 which is its maximum value if inequality 3 holds), but let us recall that we are interested directly in drawing new limits to the PA, so constants would be pointless because we will finally be looking to contours of the function directly; in fact, the contour plot for *f* (*x, y*) is shown in Figure 4.

Ultimately, one contour of the family must be selected; the proposal for defining this curve, or the new PA limit, is to make the length of the PA central line (*x* axis inside the PA) be thrice the goal line; in other words, the width of the proposed

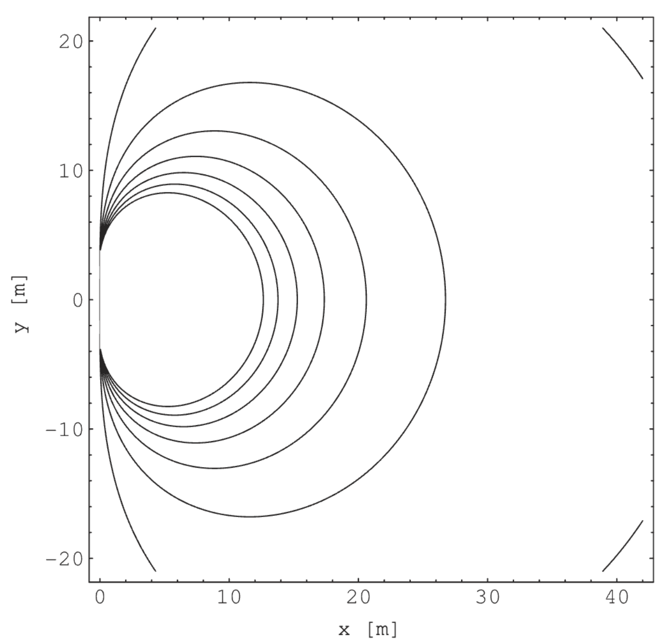

Figure 4. Contour plot of *f* (*x, y*).

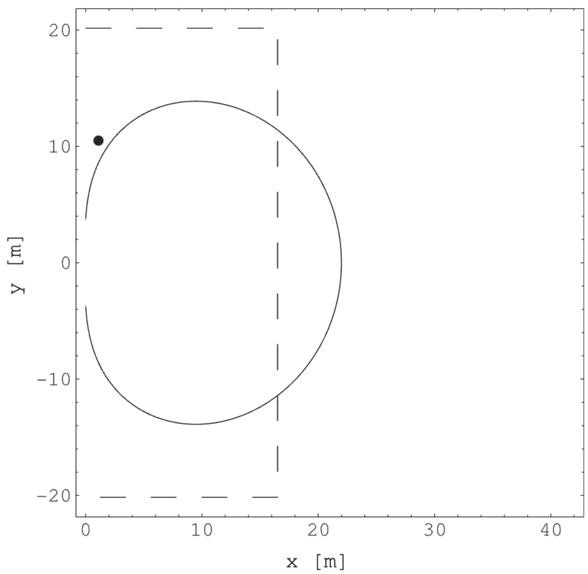

Figure 5. New penalty area.

PA will be $3b = 21.96$ m ≈ 22 m, or, the contour or new PA curve is selected as

$$f(x, y) = 0.0150 \tag{9}$$

This resulting or new PA is shown in Figure 5; note that the current or old PA width is 16.5 m (FIFA, 2014). The only fixed or defined length in the analysis (Figure 2) is the goal line b; thus, a multiple of it for the main length (width) of the new PA seems reasonable, but of course, other contour of the same family can be chosen or proposed. Now, comparing the two areas in Figure 5, it can be concluded, based on the previous mathematics solely, that the old rectangle is disproportionate or *narrow*; in other words, the current PA differs appreciably from the new area designed on the basis of the suggested scoring potential measure. In addition, the Robben-Márquez encounter point (approximate) is shown; we note that with the new or proposed PA, the Dutch squad would have been granted just a free kick.

Regarding these free kicks outside the new PA, no rule modification is necessary (Law 13), still "all opponents must be at least 9.15 m from the ball" (FIFA, 2014); only more interesting and new types of kicks are expected as a consequence of the novel PA, possibly including more spectacular saves by goalkeepers. Now, the *goal area* would need to be changed as well; because this is not a very important area in football, just simple aesthetics would require it to be another contour of our function $f(x,y)$, which can be defined by establishing its width as above.

3. Discussion

It should be pointed out that a negative effect accompanying the proposed PA change would be – obviously – the cost of a geometrically more complex PA. This might be infeasible at lower and youth levels of the game; now, field hockey – for example – has for more than 100 years used penalty *circles* (areas) whose lines are curved, but the cost of it is not a big problem in that sport because (a) its popularity worldwide is much lower than that of *fútbol*, (b) the curved sections are circle arcs and (c) synthetic turfs are currently mandatory in international hockey. In addition, the proposed change would require an adjustment period, especially for referees.

4. Conclusion

A new PA for football has been advanced. It is based on a measure of actual scoring potential on the two dimensions close to the goal, which is in turn based on the angle towards the actual goal line and the distance to the centre of this line. The proposed PA can reduce (a) simulation by players around the goal and (b) the possibility of matches being decided unfairly, which are current game problems for FIFA. After the problems and turmoil in FIFA, and the subsequent recent elections this year, full renovation in the governing body of the sport is expected in the short term, in turn this could mean that new, even radical, ideas for positive change in the beautiful game can be considered.

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Disclosure statement

No potential conflict of interest was reported by the author.

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Appendix: Proof of Equation (5) and an explicit function for the PA

After using Equation (4) in the second numerator of Equation (2), it follows that

$$\sin^2\beta = (b/c)^2\left(1 - \left(\frac{b^2 - 2by}{2ab}\right)^2\right)$$

Then, four simple algebraic steps bring us to

$$\sin^2\beta = (b/c)^2\left(1 - 1/a^2(b/2 - y)^2\right)$$

Now, using Equation (4) in the rest of the equation, and manipulating and operating, we get

$$\sin^2\beta = \frac{(bx)^2}{(bx)^2 + \left(x^2 + y^2 - (b/2)^2\right)^2}$$

which is Equation (5).

Regarding Equation (9) which defines the PA curved line, it is clearly an implicit equation, and we cannot get an explicit function $y(x)$, to start we do not have the graph of a single-valued function (Figure 5), which excludes the possibility of an explicit function $y(x)$. Nevertheless, resorting to polar coordinates, the equation becomes

$$\frac{br\cos\theta}{r^2 - (b/2)^2} = \tan 0.015r$$

which can be written as

$$\theta(r) = \cos^{-1}((r/b - b/4r)\tan 0.015r)$$

Now, that is indeed an explicit function for the PA line, although an *inverse* polar function $\theta = \theta\ (r)$, which are uncommon in mathematics.