N-Queens Problem

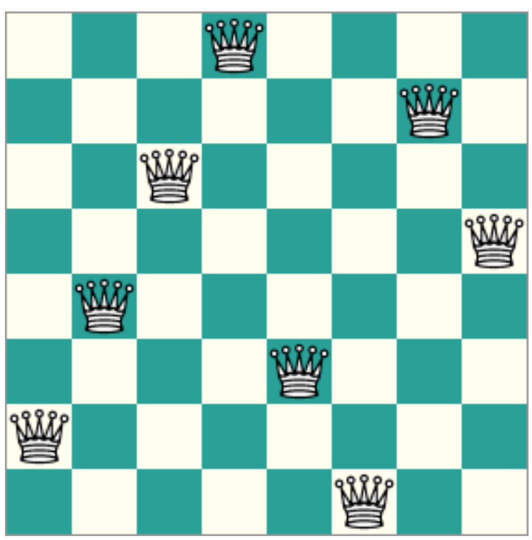
• Genetic Algorithm :

Genetic Algorithms are a family of algorithms whose purpose is to solve problems more efficiently than usual standard algorithms by using natural science metaphors with parts of the algorithm being strongly inspired by natural evolutionary behavior; such as the concept of **mutation**, **crossover** and **natural selection**.

When applying genetic algorithms one aims to construct a model that, with some randomness, tries different individuals (possible solutions, differentiated by a list of values that defines its genetic information) to a problem , measure its **fitness** - which would mean to evaluate whether this possible solutions are perfect solutions or just *good* to some extent, and to measure this degree of 'goodness' - and to make the better solutions to *breed* and produce a new set of possible solutions with better fitness, and somehow closer to the perfect solution.

• N-Queens Problem:

In 1848, A German Chess player Max Bezzel composed the 8-Queens Problem which aims to place 8 Queens in the chess board in such a way that no two Queens can attack each other. In 1850 Franz Nauck gave the 1st solution to this problem and generalized the problem to N-Queen problem for N non-attacking Queens on an N x N Chessboard. Time complexity of an N-Queen problem is O(n!). Here, we are proposing a heuristic approach to obtain the best solutions for this problem. We are depicting all the arrangements of an N x N board as an N-tuple (c1, c2, c3… cN), where ci represents the position of the queen to be in i th column and c th row. Fig.1 shows an arrangement of 8 x 8 chessboard and its 8-tuple representation.



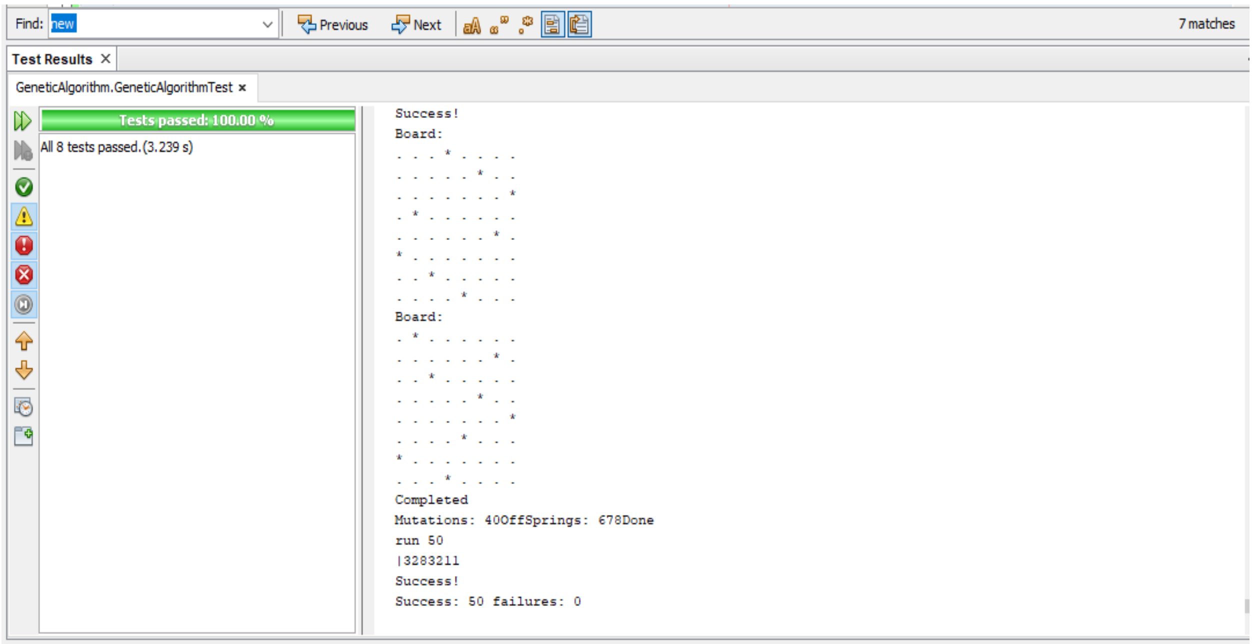
- **Output:**

N-Queens-Genetic-Algorithm - C:\Users\varad\Course Work\PSA\Project\N-Queens-Genetic-Algorithm × NQueensGA (run) ×

```
Epoch: 70
Epoch: 71
Epoch: 72
Epoch: 73
Board:
. * . . . . . . . . . .
. . . . . . . . . . * .
. . * . . . . . . . . .
. . . . . . . * . . . .
. . . . . . . . . * . .
* . . . . . . . . . . .
. . . * . . . . . . . .
. . . . . * . . . . . .
. . . . . . . . . . . *
. . . . . . . . * . . .
. . . . . . * . . . . .
. . . . * . . . . . . .
Completed
42 mutations in 2044 offspring.
Done
run 12
time in nanoseconds: 21625333
Success!
Epoch: 1
Epoch: 2
Epoch: 3
Epoch: 4
Epoch: 5
Epoch: 6
```

```
Epoch: 1
Board:
. . * .
* . . .
. . . *
. * . .
Board:
. . * .
* . . .
. . . *
. * . .
Completed
40 mutations in 28 offspring.
Done
run 1
time in nanoseconds: 7475926
Success!
Epoch: 1
Board:
. . * .
* . . .
. . . *
. * . .
Completed
40 mutations in 32 offspring.
Done
run 2
```

- # UNIT TEST CASES Executed:



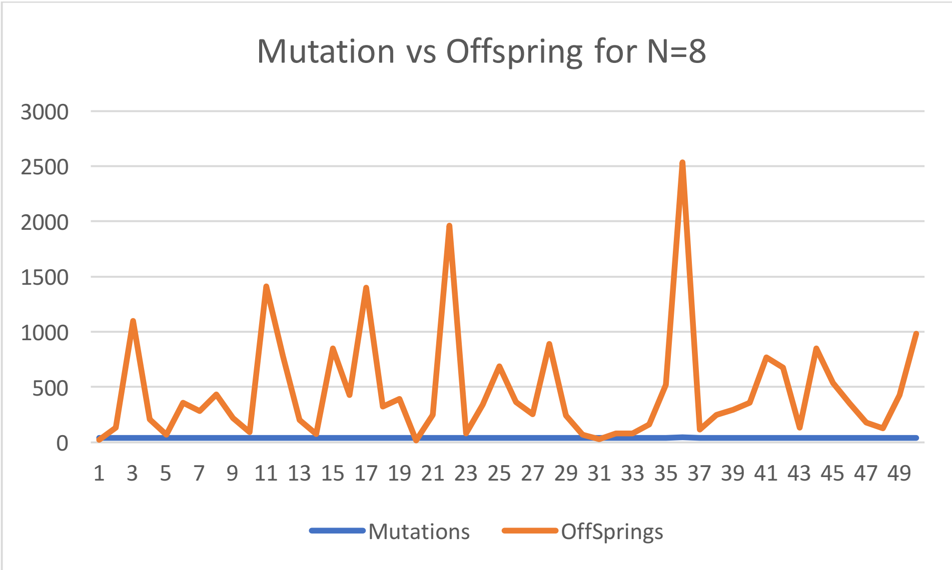
- # Analysis:

Analysis for Mutation vs OffSprings for N = 8 :

Output Results in Table for 50 Runs :

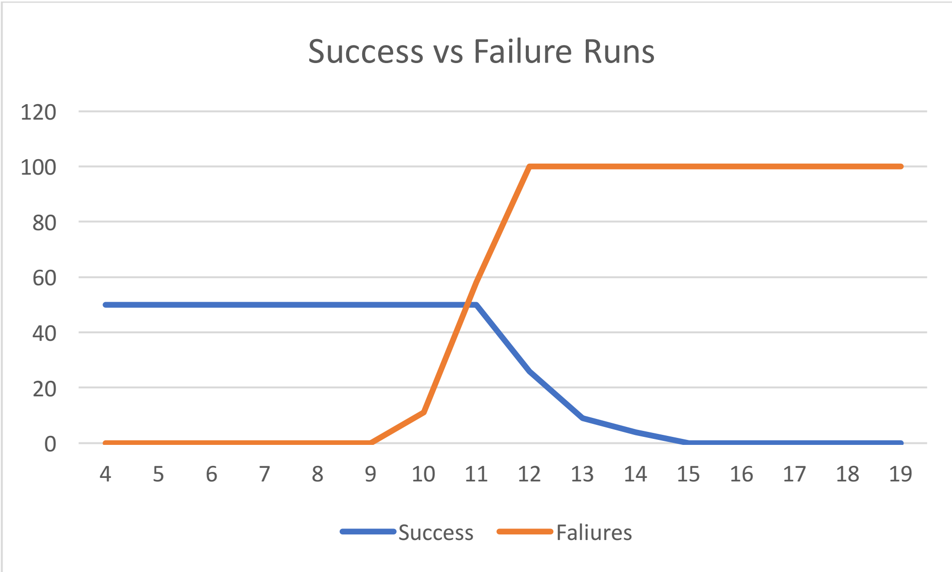
Sr no.	Mutations	OffSprings	Total time in NanoSeconds
1	40	22	5384804
2	41	132	5117170
3	41	1098	38038493
4	40	208	3012049
5	40	66	993320
6	40	356	5202855
7	41	282	3308600
8	40	432	4523363
9	40	216	1275766
10	40	92	410798
11	41	1410	30293627
12	41	778	8207500
13	40	204	1446080
14	40	72	531745

15	41	850	9181426
16	40	426	1994399
17	41	1398	15798978
18	40	322	3509239
19	41	392	2103709
20	40	18	546556
21	40	250	1331832
22	42	1964	17407260
23	40	78	267636
24	40	340	978863
25	41	686	2652028
26	40	362	1144593
27	40	256	754599
28	40	888	4401005
29	40	242	1363215
30	40	66	392815
31	40	28	281740
32	40	78	417498
33	40	82	436892
34	40	162	789508
35	40	518	2411191
36	43	2536	26820356
37	40	116	444297
38	40	250	1122025
39	40	294	2600193
40	40	358	1413992
41	41	766	3630891
42	41	678	5278315
43	40	134	1083237
44	41	850	12489320
45	40	536	2595962
46	40	352	1463358
47	40	180	1322312
48	40	124	546908
49	41	426	2690816
50	41	982	13621572



Analysis for Success runs vs Failures runs for N in the range 4 to 20 :

N	Success	Faliures
4	50	0
5	50	0
6	50	0
7	50	0
8	50	0
9	50	0
10	50	0
11	50	11
12	50	58
13	26	100
14	9	100
15	4	100
16	0	100
17	0	100
18	0	100
19	0	100
20	0	100



- **Complexity :**

For a board of size N by N there are N*(N-1)/2 pairs of non attacking queens . For examples for N=8 , number of pairs of nonattacking queens are 28 [9].

Time Complexity for Nqueens Problem using backtracking →

T(n) = n*T(n-1) + O(n^2)

This shows that the problem is getting solved with every generation and we are iterating twice over the entire array.

If we solve this time complexity equation further we will get this equation translates to O(n!).

- **Code Snippets:**

```java
public GeneticAlgorithm(int n, int start_size, int max_epoch, double mating_prob, double mutation_rate ) {
        MAX_LENGTH = n;
        START_SIZE = start_size;
        MAX_EPOCHS = max_epoch;
        MATING_PROBABILITY = mating_prob;
        MUTATION_RATE = mutation_rate;
        MIN_SELECT = 10;
        MAX_SELECT = 30;
        OFFSPRING_PER_GENERATION = 20;
        MINIMUM_SHUFFLES = 8;
        MAXIMUM_SHUFFLES = 20;
        epoch = 0;
        populationSize = 0;
}

public GeneticAlgorithm(int n, int max_epoch, double mutation_rate ) {
        MAX_LENGTH = n;
        START_SIZE = 40;
        MAX_EPOCHS = max_epoch;
        MATING_PROBABILITY = 0.7;
        MUTATION_RATE = mutation_rate;
        MIN_SELECT = 10;
        MAX_SELECT = 30;
        OFFSPRING_PER_GENERATION = 20;
        MINIMUM_SHUFFLES = 8;
        MAXIMUM_SHUFFLES = 20;
        epoch = 0;
        populationSize = 0;
}
```

```java
public GeneticAlgorithm(int n) {
        MAX_LENGTH = n;
        START_SIZE = 40;
        MAX_EPOCHS = 1000;
        MATING_PROBABILITY = 0.7;
        MUTATION_RATE = 0.001;
        MIN_SELECT = 10;
        MAX_SELECT = 30;
        OFFSPRING_PER_GENERATION = 20;
        MINIMUM_SHUFFLES = 8;
        MAXIMUM_SHUFFLES = 20;
        epoch = 0;
        populationSize = 0;
}
```

```java
public boolean runGA() {
    population = new ArrayList<Chromosome>();
    solutions = new ArrayList<Chromosome>();
    rand = new Random();
    rand.setSeed(20);
    nextMutation = 0;
    childCount = 0;
    mutations = 0;
    epoch = 0;
    populationSize = 0;

    boolean stop = false;
    Chromosome chromo = null;
    nextMutation = generateRandomNumber(0, (int)Math.round(1.0 / MUTATION_RATE));

    initializeQueens();

    while(!stop) {
        populationSize = population.size();

        for(int i = 0; i < populationSize; i++) {
            chromo = population.get(i);
            if((chromo.getConflicts() == 0)) {          //if solution found
                stop = true;
            }
        }

        if(epoch == MAX_EPOCHS) {                         //if Max Number of Cycles
            stop = true;
        }

        getFitness();

        rouletteSelection();

        mate();
```

```java
                if(epoch == MAX_EPOCHS) {                                    //if Max Number of Cycles
                    stop = true;
                }

                getFitness();

                rouletteSelection();

                mate();

                resetSelection();

                epoch++;
               // System.out.println("Epoch: " + epoch);
            }

            if(epoch >= MAX_EPOCHS) {
                System.out.println("No solution found");
                stop = false;
            } else {
                populationSize = population.size();                          //prints the solutions if found within mnc
                for(int i = 0; i < populationSize; i++) {
                    chromo = population.get(i);
                    if(chromo.getConflicts() == 0) {
                        solutions.add(chromo);
                        printSolution(chromo);
                    }
                }
            }
            System.out.println("Completed");
            System.out.print("Mutations: " +mutations + "OffSprings: " + childCount);

            return stop;

        }
```

```java
    public void printSolution(Chromosome solution) {
        String board[][] = new String[MAX_LENGTH][MAX_LENGTH];

        // Clear the board.
        for(int x = 0; x < MAX_LENGTH; x++) {
            for(int y = 0; y < MAX_LENGTH; y++) {
            board[x][y] = "";
            }
        }

        for(int x = 0; x < MAX_LENGTH; x++) {
            board[x][solution.getGene(x)] = "*";
        }

        // Display the board.
        System.out.println("Board:");
        for(int y = 0; y < MAX_LENGTH; y++) {
            for(int x = 0; x < MAX_LENGTH; x++) {
                if(board[x][y] == "*") {
                    System.out.print("* ");
                } else {
                    System.out.print(". ");
                }
            }
            System.out.print("\n");
        }
    }
```

```java
//---------------------------------------------------
// Random Number Generation
//---------------------------------------------------
    public int generateRandomNumber(int low, int high) {
            return (int)Math.round((high - low) * rand.nextDouble() + low);
    }

    public int noRepeteRandom(int high, int reject){
        boolean stop = false;
        int random = 0;

        while(!stop) {
                random = rand.nextInt(high);
                if(random != reject){
                        stop = true;
                }
        }
        return random;
    }


//---------------------------------------------------
//Initialize queens in random position
//---------------------------------------------------

    public void initializeQueens() {
            int shuffles = 0;
            Chromosome chromo = null;
            int index = 0;

            for(int i = 0; i < START_SIZE; i++)  {
                    chromo = new Chromosome(MAX_LENGTH);
                    population.add(chromo);
                    index = population.indexOf(chromo);


                    shuffles = generateRandomNumber(MINIMUM_SHUFFLES, MAXIMUM_SHUFFLES);
                    exchangeMutation(index, shuffles);
                    population.get(index).computeConflict();
            }
    }

    public void exchangeMutation(int index, int exchanges) {
            int tmp = 0;
            int g1 = 0;
            int g2 = 0;
            Chromosome chromo = null;
            chromo = population.get(index);

            for(int i = 0; i < exchanges; i++) {
                    g1 = generateRandomNumber(0, MAX_LENGTH - 1);
                    g2 = noRepeteRandom(MAX_LENGTH - 1, g1);

                    // Exchange genes.
                    tmp = chromo.getGene(g1);
                    chromo.setGene(g1, chromo.getGene(g2));
                    chromo.setGene(g2, tmp);
            }
            mutations++;
    }


//---------------------------------------------------
// Get Fitness
//---------------------------------------------------


public void getFitness() {
        // min 0% and max 100%
        int populationSize = population.size();
        Chromosome chromo = null;
        double best = 0;
        double worst = 0;
```

- **References:**

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https://github.com/hajix/N-Queen

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https://www.geeksforgeeks.org/n-queen-problem-backtracking-3/

https://arxiv.org/pdf/1802.02006.pdf

https://kushalvyas.github.io/gen_8Q.html

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https://datajenius.com/articles/solving-n-queens-with-genetic-algorithms

https://ieeexplore.ieee.org/document/6802550

https://stackoverflow.com/questions/21059422/time-complexity-of-n-queen-using-backtracking

Roulette Selection

https://stackoverflow.com/questions/298301/roulette-wheel-selection-algorithm

https://stackoverflow.com/questions/177271/roulette-selection-in-genetic-algorithms

Partially Mapped Crossover

https://github.com/DEAP/deap/blob/master/deap/tools/crossover.py

https://stackoverflow.com/questions/52350699/how-to-perform-partially-mapped-crossover-operator-pmx