Assignment 2

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Problem 1: Perform principal component analysis on NHL.xlsx, which contains statistics of 30 teams in the National Hockey League. The description of the variables is provided in the 'Description' sheet of the file. Focus only on the variables 12 through 25, and create a new data frame.

```
library(readxl)
NHL <- read_excel("~/Downloads/NHL.xlsx")
```

```
## New names:
## * `` -> ...1
```

```
NHL1 <- subset(NHL, select = -c(1:11,face))
NHL1
```

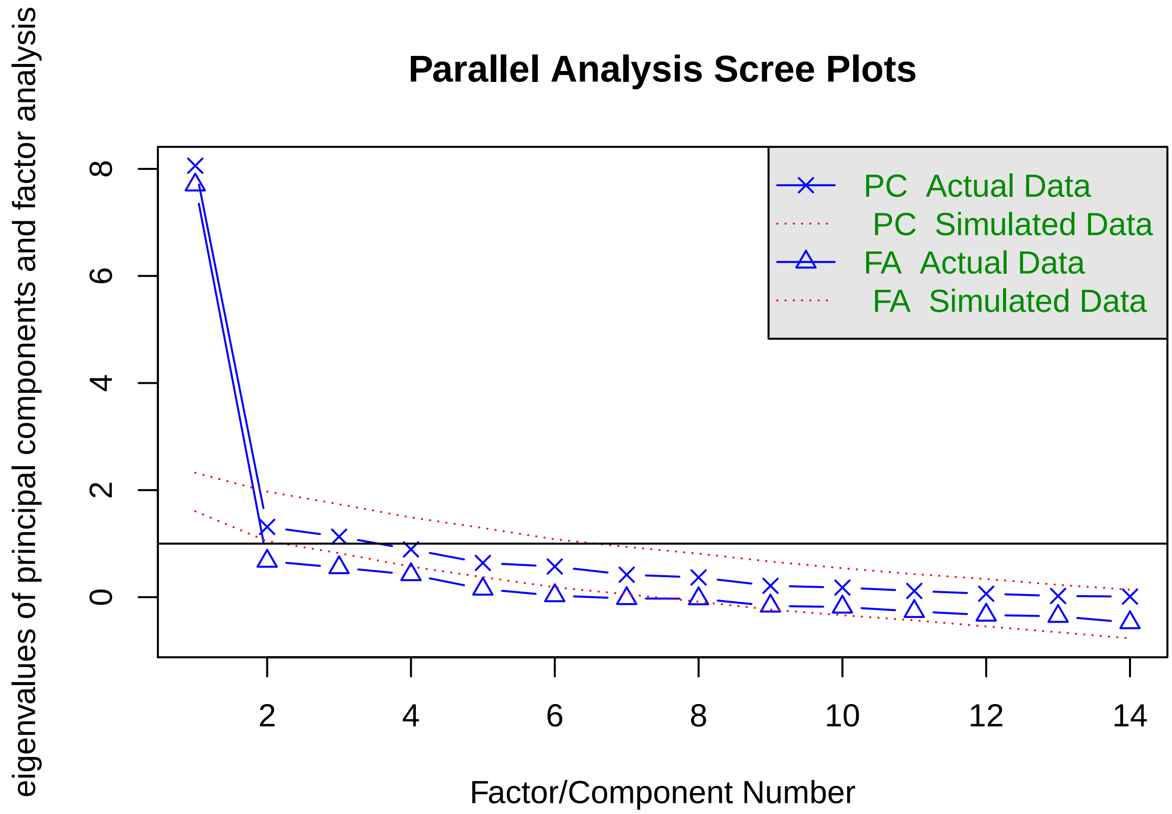
```
## # A tibble: 30 x 14
##    `p\rpc`    gg   gag  five   PPP   PKP shots   sag   sc1   tr1 lead1 lead2
##      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##  1   0.689  3.02  2.28  1.32  16.8  84.3  31.5  29.5 0.82  0.375 0.853 0.973
##  2   0.665  2.92  2.4   1.18  22.3  83.7  30.9  27.2 0.783 0.417 0.808 0.842
##  3   0.665  2.78  2.7   1.04  15.7  81    30    28.9 0.766 0.429 0.813 0.938
##  4   0.671  2.61  2.24  1.18  16.5  83.7  28.5  30.1 0.821 0.419 0.714 0.865
##  5   0.659  3.16  2.51  1.28  18.8  83.7  29.6  27.9 0.761 0.417 0.833 0.943
##  6   0.622  2.68  2.27  1.19  17.6  83.4  33.9  30.2 0.761 0.361 0.897 1
##  7   0.616  2.88  2.68  0.96  19.3  85.7  29.9  29.8 0.771 0.447 0.773 0.882
##  8   0.634  2.76  2.46  1.26  16.2  80.8  31.9  28.3 0.711 0.455 0.741 0.771
##  9   0.616  2.99  2.73  1.08  18.7  78    33.8  28.3 0.592 0.545 0.667 0.781
## 10   0.61   2.77  2.42  1.14  15.8  86.3  30.8  27.6 0.778 0.297 0.844 0.875
## # ... with 20 more rows, and 2 more variables: wop <dbl>, wosp <dbl>
```

-Input the new data frame to fa.parallel() function to determine the number of components to extract

```
library(psych)
pc <- fa.parallel(NHL1, fa = "both", n.obs = 30)
```

```
## Warning in fa.parallel(NHL1, fa = "both", n.obs = 30): You specified the number
## of subjects, implying a correlation matrix, but do not have a correlation
## matrix, correlations found
```

```
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```




```
## Parallel analysis suggests that the number of factors =  1  and the number of components =  1
```

-Input the new data frame to principal() function to extract the components. If raw data is input, the correlation matrix is automatically calculated by principal() function.

```
pa <- principal(NHL1, nfactors = 1, rotate = "none", fm="pa")
pa
```

```
## Principal Components Analysis
## Call: principal(r = NHL1, nfactors = 1, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1   h2    u2 com
## p\rpc  0.97 0.94 0.057   1
## gg     0.83 0.69 0.308   1
## gag   -0.82 0.67 0.327   1
## five   0.92 0.84 0.162   1
## PPP    0.14 0.02 0.980   1
## PKP    0.69 0.48 0.519   1
## shots  0.59 0.34 0.656   1
## sag   -0.62 0.39 0.612   1
## sc1    0.81 0.66 0.338   1
## tr1    0.76 0.58 0.422   1
## lead1  0.81 0.65 0.351   1
## lead2  0.74 0.55 0.452   1
## wop    0.71 0.51 0.491   1
## wosp   0.86 0.73 0.267   1
##
##                  PC1
## SS loadings     8.06
```

```
## Proportion Var 0.58
##
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.1
##  with the empirical chi square  52.29  with prob <  0.99
##
## Fit based upon off diagonal values = 0.97
```

-Rotate the components

```
pa1 <- principal(NHL1, nfactors = 1, rotate = "varimax")
pa1
```

```
## Principal Components Analysis
## Call: principal(r = NHL1, nfactors = 1, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1   h2    u2 com
## p\rpc  0.97 0.94 0.057   1
## gg     0.83 0.69 0.308   1
## gag   -0.82 0.67 0.327   1
## five   0.92 0.84 0.162   1
## PPP    0.14 0.02 0.980   1
## PKP    0.69 0.48 0.519   1
## shots  0.59 0.34 0.656   1
## sag   -0.62 0.39 0.612   1
## sc1    0.81 0.66 0.338   1
## tr1    0.76 0.58 0.422   1
## lead1  0.81 0.65 0.351   1
## lead2  0.74 0.55 0.452   1
## wop    0.71 0.51 0.491   1
## wosp   0.86 0.73 0.267   1
##
##                 PC1
## SS loadings    8.06
## Proportion Var 0.58
##
## Mean item complexity =  1
## Test of the hypothesis that 1 component is sufficient.
##
## The root mean square of the residuals (RMSR) is  0.1
##  with the empirical chi square  52.29  with prob <  0.99
##
## Fit based upon off diagonal values = 0.97
```

-Compute component scores

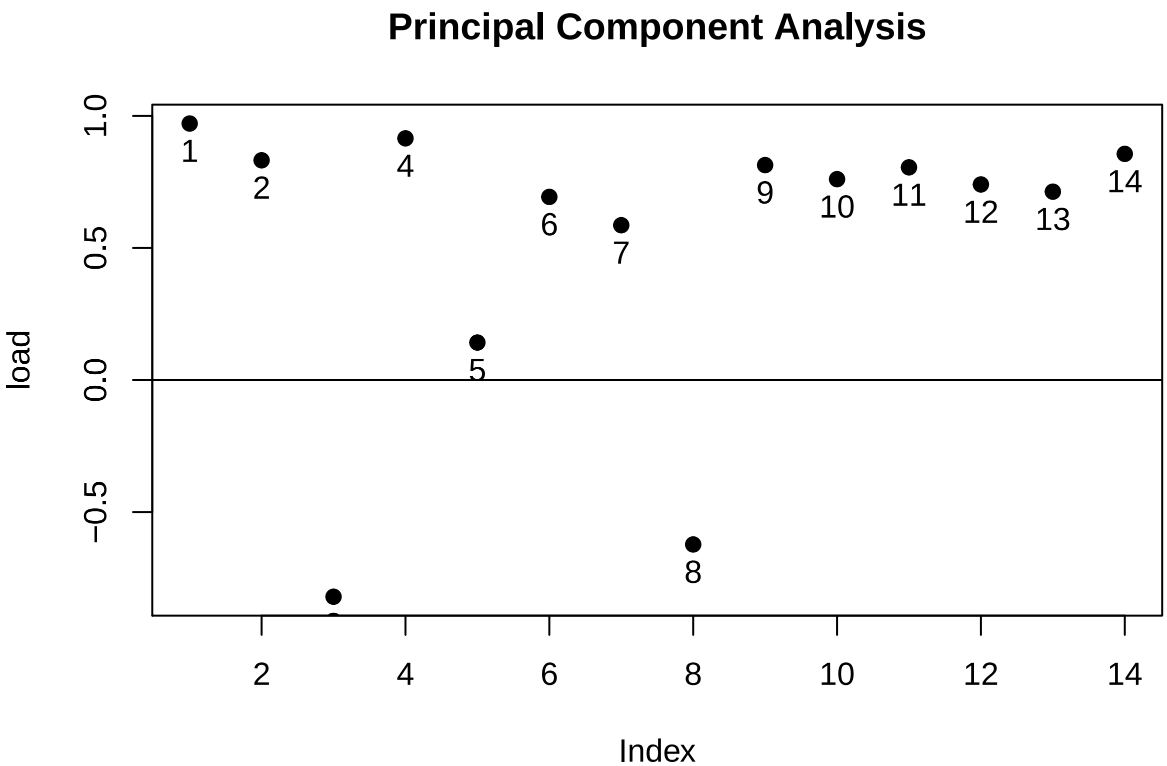
```
pa2 <- principal(NHL1, nfactors = 1, rotate = "varimax", cor = "cor", scores = TRUE)
head(pa2$weights)
```

```
##               PC1
## p\rpc  0.12052412
## gg     0.10325190
## gag   -0.10182975
## five   0.11356507
```

```
## PPP    0.01761025
## PKP    0.08605210
```

-Graph an orthogonal solution using factor.plot()

```r
factor.plot(pa2)
```



-Interpret the results

In this question one component was extracted which has SS loading of 7.19 and proportion variance of 51% of NHL dataset. Since, here only one component has been rotated, the rotation does not have change in the values.

Problem 2: Perform principal component analysis on Glass Identification Data.xlsx

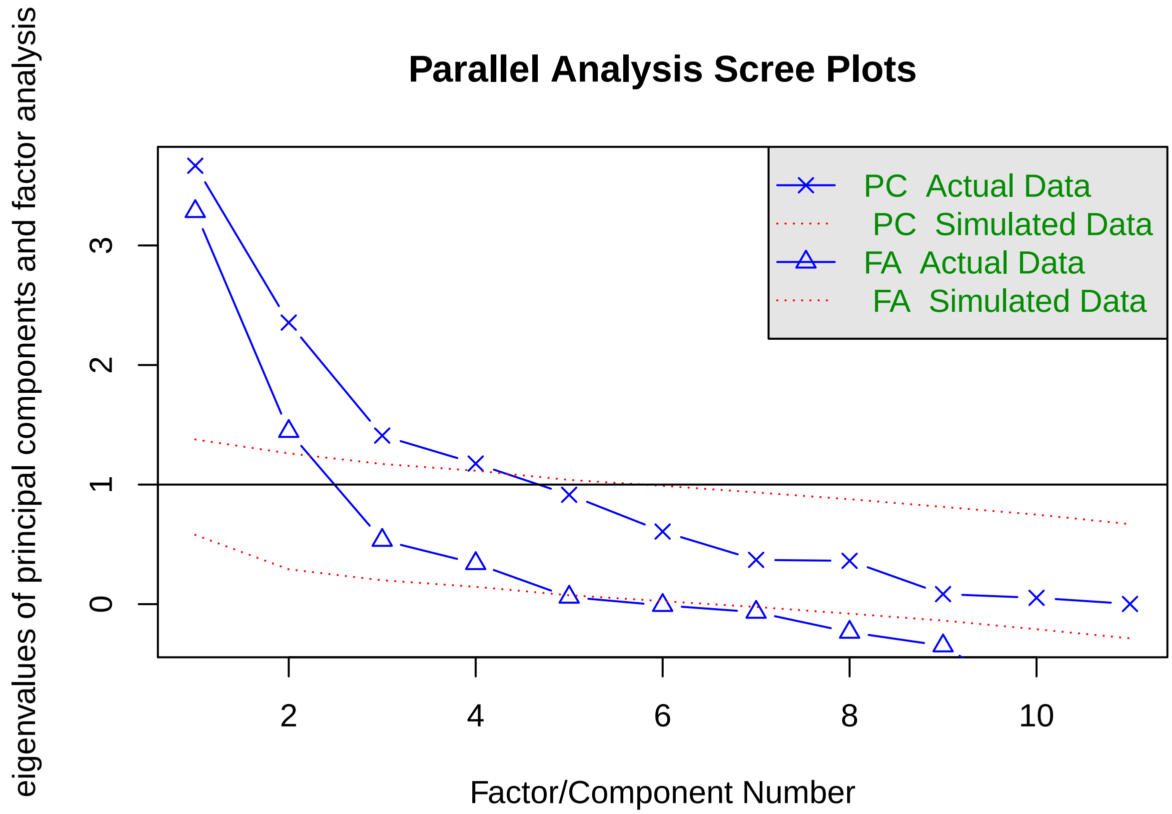
Input the raw data matrix to fa.parallel() function to determine the number of components to extract

```r
GSD <- read_excel("~/Downloads/Glass Identification Data.xlsx")
gsdpc <- fa.parallel(GSD, fa = "both", n.obs = 214)
```

```
## Warning in fa.parallel(GSD, fa = "both", n.obs = 214): You specified the number
## of subjects, implying a correlation matrix, but do not have a correlation
## matrix, correlations found

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.

## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```




```
## Parallel analysis suggests that the number of factors =  4  and the number of components =  4
```

-Input the raw data matrix to principal() function to extract the components. If raw data is input, the correlation matrix is automatically calculated by principal() function.

```r
gsdpa <- principal(GSD, nfactors = 4, rotate = "none")
gsdpa
```

```
## Principal Components Analysis
## Call: principal(r = GSD, nfactors = 4, rotate = "none")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         PC1   PC2   PC3   PC4   h2    u2 com
## ID     0.84  0.20  0.02  0.10 0.76 0.244 1.1
## RI    -0.28  0.91  0.11 -0.16 0.95 0.051 1.3
## Na     0.55 -0.06 -0.42 -0.58 0.81 0.185 2.9
## Mg    -0.77 -0.43 -0.02 -0.31 0.87 0.126 1.9
## Al     0.73 -0.25  0.42  0.08 0.77 0.226 1.9
## Si     0.15 -0.39 -0.56  0.69 0.96 0.040 2.7
## K      0.05 -0.41  0.78  0.07 0.78 0.218 1.5
## CA    -0.11  0.92  0.00  0.28 0.94 0.060 1.2
## Ba     0.69  0.13  0.13 -0.25 0.57 0.429 1.4
## Fe    -0.22  0.18  0.32  0.29 0.27 0.731 3.4
## Class  0.95  0.11 -0.06  0.05 0.92 0.083 1.0
##
##                        PC1  PC2  PC3  PC4
## SS loadings           3.67 2.35 1.41 1.18
## Proportion Var        0.33 0.21 0.13 0.11
## Cumulative Var        0.33 0.55 0.68 0.78
## Proportion Explained  0.43 0.27 0.16 0.14
```

```
## Cumulative Proportion 0.43 0.70 0.86 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  116.03  with prob <  8.8e-17
##
## Fit based upon off diagonal values = 0.96
```

-Rotate the components

```
gsdpa1 <- principal(GSD, nfactors = 4, rotate = "varimax")
gsdpa1
```

```
## Principal Components Analysis
## Call: principal(r = GSD, nfactors = 4, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##         RC1   RC2   RC3   RC4   h2    u2 com
## ID     0.86  0.03 -0.14 -0.06 0.76 0.244 1.1
## RI    -0.05  0.82  0.19  0.48 0.95 0.051 1.7
## Na     0.32 -0.04 -0.84  0.10 0.81 0.185 1.3
## Mg    -0.88 -0.27 -0.06  0.13 0.87 0.126 1.2
## Al     0.71 -0.50  0.08  0.07 0.77 0.226 1.8
## Si     0.06 -0.12 -0.02 -0.97 0.96 0.040 1.0
## K      0.11 -0.66  0.49  0.30 0.78 0.218 2.3
## CA     0.17  0.88  0.36  0.09 0.94 0.060 1.4
## Ba     0.66 -0.08 -0.26  0.25 0.57 0.429 1.7
## Fe    -0.06  0.10  0.50  0.06 0.27 0.731 1.1
## Class  0.91 -0.04 -0.27 -0.11 0.92 0.083 1.2
##
##                        RC1  RC2  RC3  RC4
## SS loadings           3.43 2.26 1.53 1.39
## Proportion Var        0.31 0.21 0.14 0.13
## Cumulative Var        0.31 0.52 0.66 0.78
## Proportion Explained  0.40 0.26 0.18 0.16
## Cumulative Proportion 0.40 0.66 0.84 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 4 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  116.03  with prob <  8.8e-17
##
## Fit based upon off diagonal values = 0.96
```

-Compute component scores

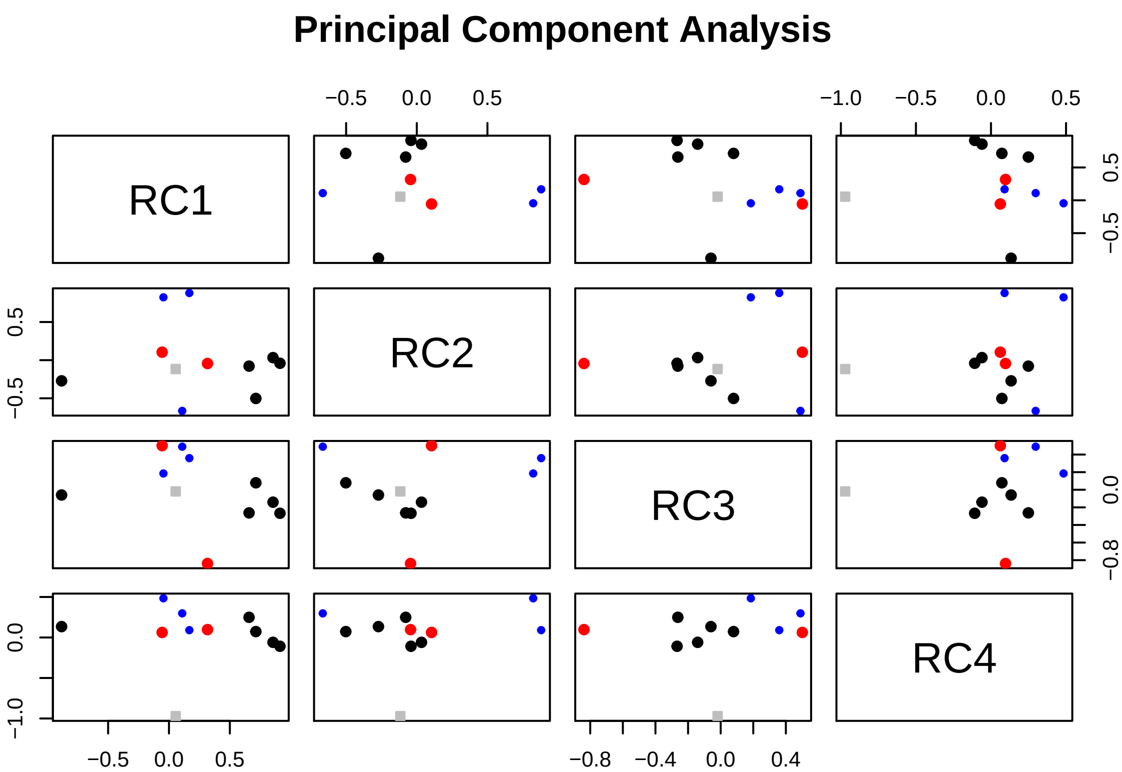
```
gsdpa2 <- principal(GSD, nfactors = 4, rotate = "varimax", scores = TRUE)
head(gsdpa2$weights)
```

```
##             RC1         RC2          RC3         RC4
## ID  0.251903303  0.03843078  0.009404738 -0.04330369
## RI  0.014034481  0.32518787  0.039565640  0.27149961
## Na -0.007931162  0.02024283 -0.575373983  0.15029887
## Mg -0.289236083 -0.14465343 -0.152305895  0.13997085
```

```
## Al   0.228204885 -0.23519273  0.168904818  0.08789647
## Si   0.022550920  0.03832833  0.087845207 -0.72028709
```

-Graph an orthogonal solution using factor.plot()

```r
factor.plot(gsdpa2)
```



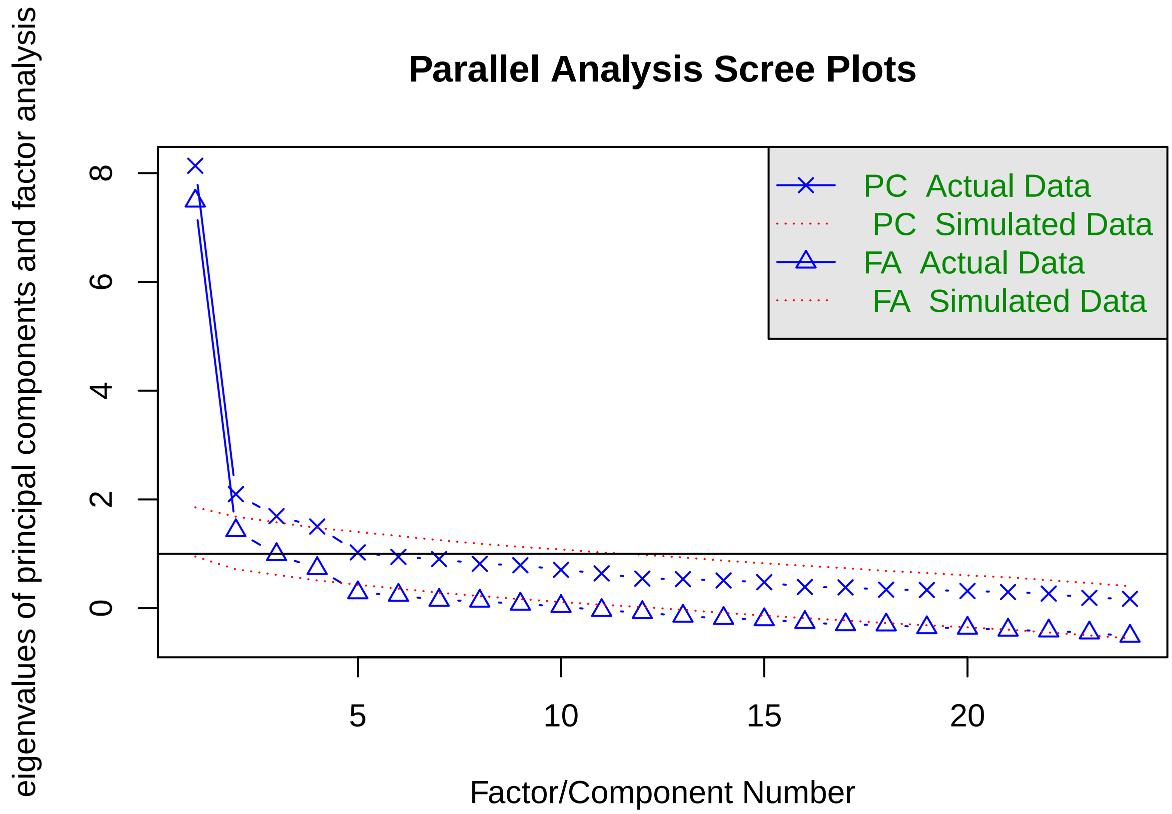
-Interpret the results

Four principal components extracted from the raw data determining glass type on 9 variables are sufficient to build the model. All four principal components accounts for 79 percent (25% for pc 1, 23% for pc2, 16% for pc3 and 15% for pc4) of the variance in the 9 variables. From the factor analysis table, CA, RI and K load on the first component, Mg, Al and Ba load on the second component, Na, K and Fe on the third component and Si and Na on the fourth component.

Problem 3: Perform factor analysis on Herman74.cor, which is a data structure available in the base installation (A correlation matrix of 24 psychological tests given to 145 seventh and eight-grade children in a Chicago suburb by Holzinger and Swineford).

-Input the correlation matrix to fa.parallel() function to determine the number of components to extract

```r
HAR <- fa.parallel(Harman74.cor$cov, fa = "both", n.obs = 145)
```




```
## Parallel analysis suggests that the number of factors =  4  and the number of components =  3
```

-Input the correlation matrix to fa() function to extract the components. If raw data is input, the correlation matrix is automatically calculated by fa() function.

```
fa(Harman74.cor$cov, nfactors = 4, rotate = "none", fm = "pa")
```

```
## Factor Analysis using method =  pa
## Call: fa(r = Harman74.cor$cov, nfactors = 4, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                         PA1   PA2   PA3   PA4   h2   u2 com
## VisualPerception       0.60  0.03  0.38 -0.22 0.55 0.45 2.0
## Cubes                  0.37 -0.03  0.26 -0.15 0.23 0.77 2.2
## PaperFormBoard         0.42 -0.12  0.36 -0.13 0.34 0.66 2.3
## Flags                  0.48 -0.11  0.26 -0.19 0.35 0.65 2.0
## GeneralInformation     0.69 -0.30 -0.27 -0.04 0.64 0.36 1.7
## PargraphComprehension  0.69 -0.40 -0.20  0.08 0.68 0.32 1.8
## SentenceCompletion     0.68 -0.41 -0.30 -0.08 0.73 0.27 2.1
## WordClassification     0.67 -0.20 -0.09 -0.11 0.51 0.49 1.3
## WordMeaning            0.70 -0.45 -0.22  0.08 0.74 0.26 2.0
## Addition               0.47  0.53 -0.48 -0.10 0.74 0.26 3.1
## Code                   0.56  0.36 -0.16  0.09 0.47 0.53 2.0
## CountingDots           0.47  0.50 -0.14 -0.24 0.55 0.45 2.6
## StraightCurvedCapitals 0.60  0.26  0.01 -0.29 0.51 0.49 1.9
## WordRecognition        0.43  0.06  0.01  0.42 0.36 0.64 2.0
## NumberRecognition      0.39  0.10  0.09  0.37 0.31 0.69 2.2
## FigureRecognition      0.51  0.09  0.35  0.25 0.45 0.55 2.4
## ObjectNumber           0.47  0.21 -0.01  0.39 0.41 0.59 2.4
## NumberFigure           0.52  0.32  0.16  0.14 0.41 0.59 2.1
```

```
## FigureWord             0.44  0.10  0.10  0.13 0.23 0.77 1.4
## Deduction              0.62 -0.13  0.14  0.04 0.42 0.58 1.2
## NumericalPuzzles       0.59  0.21  0.07 -0.14 0.42 0.58 1.4
## ProblemReasoning       0.61 -0.10  0.12  0.03 0.40 0.60 1.1
## SeriesCompletion       0.69 -0.06  0.15 -0.10 0.51 0.49 1.2
## ArithmeticProblems     0.65  0.17 -0.19  0.00 0.49 0.51 1.3
##
##                        PA1  PA2  PA3  PA4
## SS loadings           7.65 1.69 1.22 0.92
## Proportion Var        0.32 0.07 0.05 0.04
## Cumulative Var        0.32 0.39 0.44 0.48
## Proportion Explained  0.67 0.15 0.11 0.08
## Cumulative Proportion 0.67 0.81 0.92 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    PA1  PA2  PA3  PA4
## Correlation of (regression) scores with factors   0.97 0.91 0.86 0.79
## Multiple R square of scores with factors          0.94 0.82 0.75 0.62
## Minimum correlation of possible factor scores     0.89 0.65 0.50 0.24
```

-Rotate the factors

```r
HAR1rot <- fa(Harman74.cor$cov, nfactors = 4, rotate = "varimax", fm = "pa", scores = TRUE)
HAR1rot
```

```
## Factor Analysis using method =  pa
## Call: fa(r = Harman74.cor$cov, nfactors = 4, rotate = "varimax", scores = TRUE,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                         PA1   PA3   PA2  PA4   h2   u2 com
## VisualPerception       0.15  0.68  0.20 0.15 0.55 0.45 1.4
## Cubes                  0.11  0.45  0.08 0.08 0.23 0.77 1.3
## PaperFormBoard         0.15  0.55 -0.01 0.11 0.34 0.66 1.2
## Flags                  0.23  0.53  0.09 0.07 0.35 0.65 1.5
## GeneralInformation     0.73  0.19  0.22 0.14 0.64 0.36 1.4
## PargraphComprehension  0.76  0.21  0.07 0.23 0.68 0.32 1.4
## SentenceCompletion     0.81  0.19  0.15 0.07 0.73 0.27 1.2
## WordClassification     0.57  0.34  0.23 0.14 0.51 0.49 2.2
## WordMeaning            0.81  0.20  0.05 0.22 0.74 0.26 1.3
## Addition               0.17 -0.10  0.82 0.16 0.74 0.26 1.2
## Code                   0.18  0.10  0.54 0.37 0.47 0.53 2.1
## CountingDots           0.02  0.20  0.71 0.09 0.55 0.45 1.2
## StraightCurvedCapitals 0.18  0.42  0.54 0.08 0.51 0.49 2.2
## WordRecognition        0.21  0.05  0.08 0.56 0.36 0.64 1.3
## NumberRecognition      0.12  0.12  0.08 0.52 0.31 0.69 1.3
```

```
## FigureRecognition      0.07  0.42  0.06 0.52 0.45 0.55 2.0
## ObjectNumber           0.14  0.06  0.22 0.58 0.41 0.59 1.4
## NumberFigure           0.02  0.31  0.34 0.45 0.41 0.59 2.7
## FigureWord             0.15  0.25  0.18 0.35 0.23 0.77 2.8
## Deduction              0.38  0.42  0.10 0.29 0.42 0.58 2.9
## NumericalPuzzles       0.18  0.40  0.43 0.21 0.42 0.58 2.8
## ProblemReasoning       0.37  0.41  0.13 0.29 0.40 0.60 3.0
## SeriesCompletion       0.37  0.52  0.23 0.22 0.51 0.49 2.7
## ArithmeticProblems     0.36  0.19  0.49 0.29 0.49 0.51 2.9
##
##                        PA1  PA3  PA2  PA4
## SS loadings           3.64 2.93 2.67 2.23
## Proportion Var        0.15 0.12 0.11 0.09
## Cumulative Var        0.15 0.27 0.38 0.48
## Proportion Explained  0.32 0.26 0.23 0.19
## Cumulative Proportion 0.32 0.57 0.81 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective function was  11.44
## The degrees of freedom for the model are 186  and the objective function was  1.72
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                    PA1  PA3  PA2  PA4
## Correlation of (regression) scores with factors   0.93 0.87 0.91 0.82
## Multiple R square of scores with factors          0.87 0.76 0.82 0.68
## Minimum correlation of possible factor scores     0.74 0.52 0.65 0.36
```

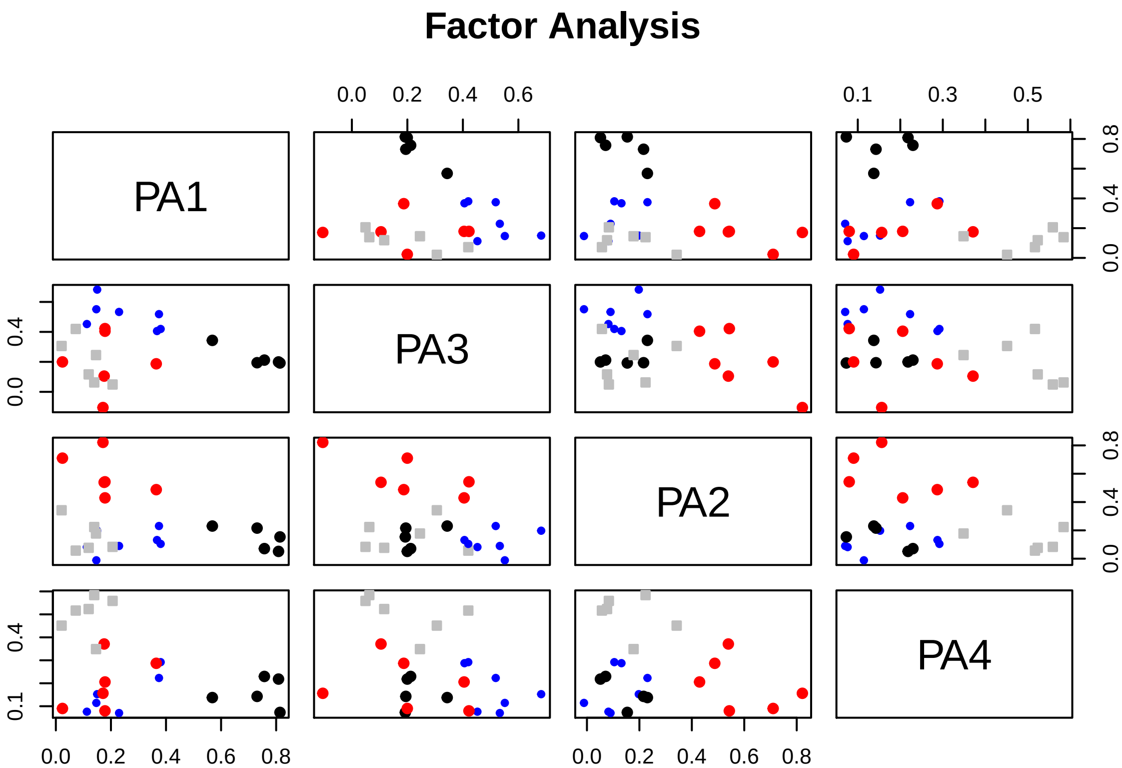
-Compute factor scores

```r
head(HAR1rot$weights)
```

```
##                                 PA1         PA3          PA2         PA4
## VisualPerception      -0.0912296464  0.30887999  0.018200947 -0.05935341
## Cubes                 -0.0274222221  0.12442581 -0.004313136 -0.03601139
## PaperFormBoard        -0.0003455765  0.14500572 -0.022709627 -0.01374043
## Flags                 -0.0159231043  0.16519350 -0.018096125 -0.06719730
## GeneralInformation     0.1792867044 -0.01632362  0.007338539 -0.06497291
## PargraphComprehension  0.2087190825 -0.02965374 -0.088966102  0.05385366
```

-Graph an orthogonal solution using factor.plot()

```r
factor.plot(HAR1rot)
```

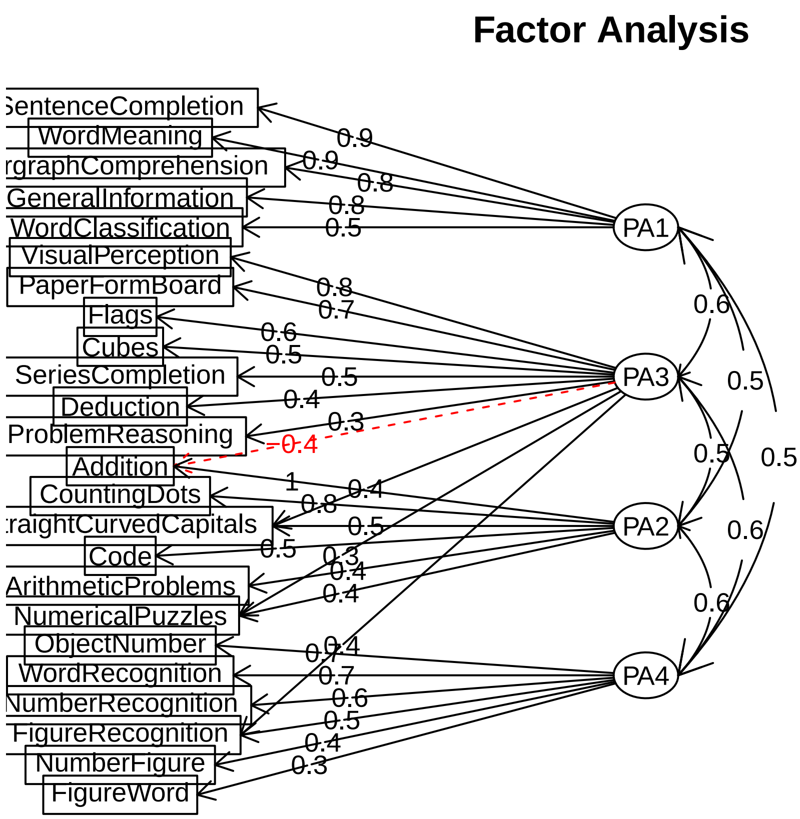


-Graph an oblique solutions using fa.diagram()

```
HAR2rot <- fa(Harman74.cor$cov, nfactors = 4, rotate = "promax", fm = "pa")
```

```
## Loading required namespace: GPArotation
```

```
fa.diagram(HAR2rot, simple = FALSE)
```



-Interpret the results

After using factor analysis,on Harman74.cor, we were able to extract 4 factors that are significant enough to represent the entire data (48% of which PA1 = 32%, PA2=77%, PA3=0.5%, PA4=4%). We performed rotation and the proportion variance changes to PA1=15%, PA2=11%, PA3=12% ,PA4=9% having total variance still as 48%.

Problem 4: Perform factor analysis on breast-cancer-wisconsin.xlsx, is a multivariate dataset that is used to predict whether a cancer is malignant or benign from biopsy details of 699 patients with 11 attributes. Create a new data frame by removing the variable "BN".

```
BCW <- read_xlsx("~/Downloads/breast-cancer-wisconsin.xlsx")
BCW1 <- subset(BCW, select = -c(ID,BN))
BCW1
```

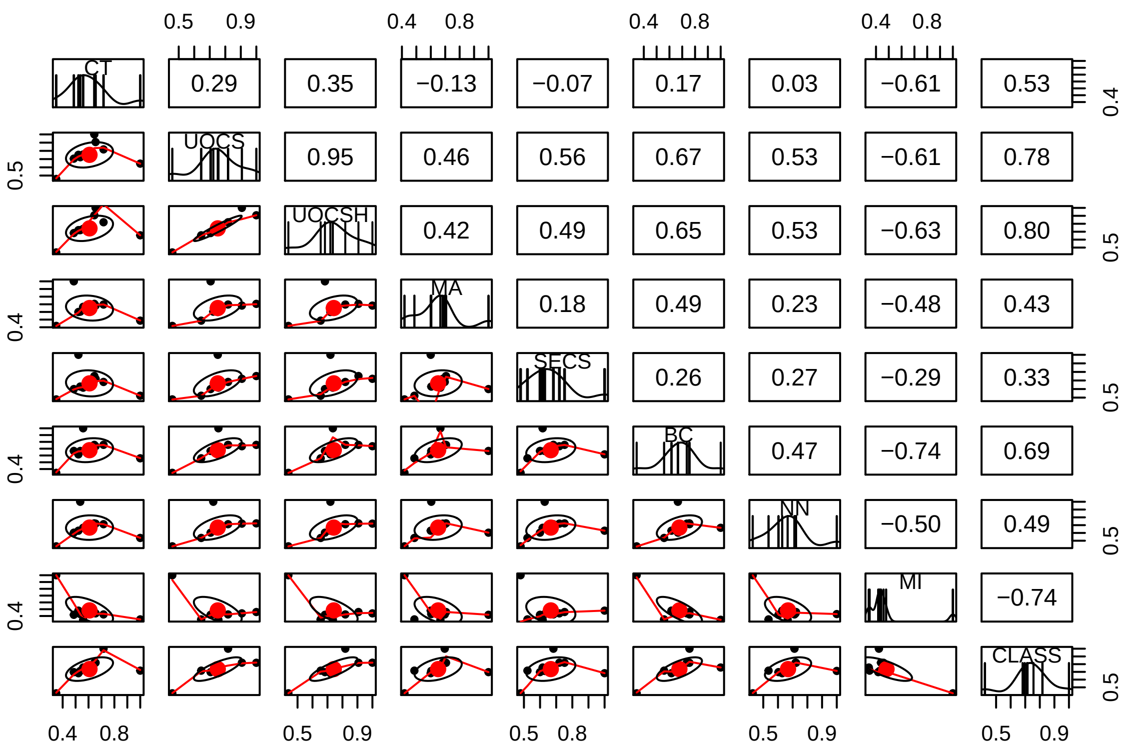
```
## # A tibble: 699 x 9
##       CT  UOCS UOCSH    MA  SECS    BC    NN    MI CLASS
##    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##  1     5     1     1     1     2     3     1     1     2
##  2     5     4     4     5     7     3     2     1     2
##  3     3     1     1     1     2     3     1     1     2
##  4     6     8     8     1     3     3     7     1     2
##  5     4     1     1     3     2     3     1     1     2
##  6     8    10    10     8     7     9     7     1     4
##  7     1     1     1     1     2     3     1     1     2
##  8     2     1     2     1     2     3     1     1     2
##  9     2     1     1     1     2     1     1     5     2
## 10     4     2     1     1     2     2     1     1     2
## # ... with 689 more rows
```

-Calculate the correlation matrix from the new data frame. Visualize the correlation matrix using pairs.panels function of the "psych" package. How would you interpret the result in terms of correlation among the variables?

```
bcw1cor <- cor(BCW1, use="pairwise.complete.obs")
bcw1cor
```

```
##              CT      UOCS     UOCSH        MA      SECS        BC        NN
## CT    1.0000000 0.6449125 0.6545891 0.4863562 0.5218162 0.5584282 0.5358345
## UOCS  0.6449125 1.0000000 0.9068819 0.7055818 0.7517991 0.7557210 0.7228648
## UOCSH 0.6545891 0.9068819 1.0000000 0.6830792 0.7196684 0.7359485 0.7194463
## MA    0.4863562 0.7055818 0.6830792 1.0000000 0.5995991 0.6667153 0.6033524
## SECS  0.5218162 0.7517991 0.7196684 0.5995991 1.0000000 0.6161018 0.6288807
## BC    0.5584282 0.7557210 0.7359485 0.6667153 0.6161018 1.0000000 0.6658778
## NN    0.5358345 0.7228648 0.7194463 0.6033524 0.6288807 0.6658778 1.0000000
## MI    0.3500339 0.4586931 0.4389109 0.4176328 0.4791015 0.3441695 0.4283357
## CLASS 0.7160014 0.8179037 0.8189337 0.6968002 0.6827845 0.7566161 0.7122436
##              MI     CLASS
## CT    0.3500339 0.7160014
## UOCS  0.4586931 0.8179037
## UOCSH 0.4389109 0.8189337
## MA    0.4176328 0.6968002
## SECS  0.4791015 0.6827845
## BC    0.3441695 0.7566161
## NN    0.4283357 0.7122436
## MI    1.0000000 0.4231703
## CLASS 0.4231703 1.0000000
```

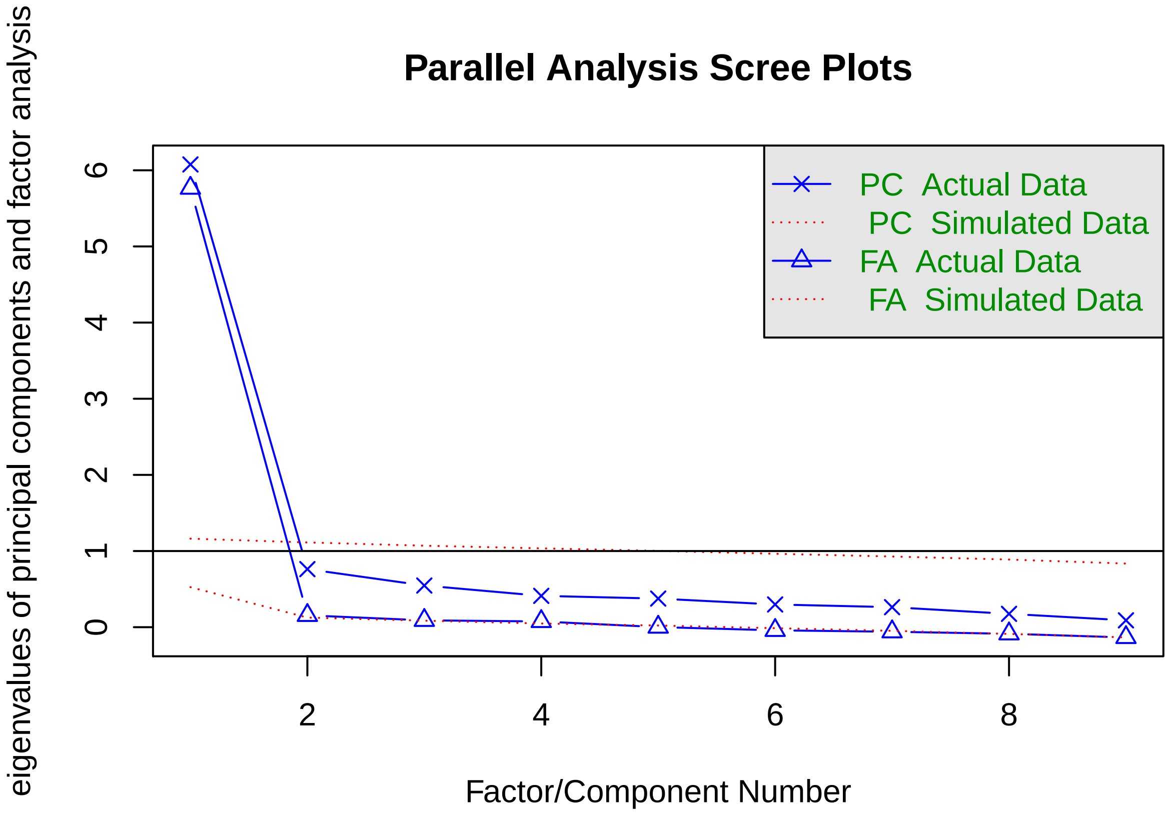
```
pairs.panels(bcw1cor)
```



Interpretation of pairs.panels The pairs.panels() function shows the scatter plot of matrices with bivariate scatter plot below the diagonal; histograms on diagonal; and Pearson correlation above the diagram. The variables UOCS, and UOCSH are highly positively correlated (+0.95), whereas the pairs (CLASS, MI) and (BC, MI) are negatively correlated (-0.74).

-Input the correlation matrix to fa.parallel() function to determine the number of components to extract

```
bcw1pa <- fa.parallel(bcw1cor, fa = "both", n.obs = 699 )
```




```
## Parallel analysis suggests that the number of factors =  1  and the number of components =  1
```

-Input the correlation matrix to fa() function to extract the components. If raw data is input, the correlation matrix is automatically calculated by fa() function.

```
fa(bcw1cor, nfactors = 1, rotate = "none", fm = "pa")
```

```
## Factor Analysis using method =  pa
## Call: fa(r = bcw1cor, nfactors = 1, rotate = "none", fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        PA1   h2   u2 com
## CT     0.70 0.49 0.51   1
## UOCS   0.93 0.87 0.13   1
## UOCSH  0.92 0.84 0.16   1
## MA     0.76 0.58 0.42   1
## SECS   0.79 0.62 0.38   1
## BC     0.81 0.66 0.34   1
## NN     0.79 0.63 0.37   1
## MI     0.50 0.25 0.75   1
## CLASS  0.91 0.82 0.18   1
##
##                 PA1
## SS loadings    5.77
```

```
## Proportion Var 0.64
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  7.63
## The degrees of freedom for the model are 27  and the objective function was  0.35
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.04
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
## Multiple R square of scores with factors          0.96
## Minimum correlation of possible factor scores     0.92
```

-Rotate the factors

```
bcw1fa <- fa(bcw1cor, nfactors = 1, rotate = "varimax", fm = "pa", scores = TRUE)
bcw1fa
```

```
## Factor Analysis using method =  pa
## Call: fa(r = bcw1cor, nfactors = 1, rotate = "varimax", scores = TRUE,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        PA1   h2   u2 com
## CT    0.70 0.49 0.51   1
## UOCS  0.93 0.87 0.13   1
## UOCSH 0.92 0.84 0.16   1
## MA    0.76 0.58 0.42   1
## SECS  0.79 0.62 0.38   1
## BC    0.81 0.66 0.34   1
## NN    0.79 0.63 0.37   1
## MI    0.50 0.25 0.75   1
## CLASS 0.91 0.82 0.18   1
##
##                 PA1
## SS loadings    5.77
## Proportion Var 0.64
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  7.63
## The degrees of freedom for the model are 27  and the objective function was  0.35
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.04
##
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1
## Correlation of (regression) scores with factors   0.98
```

```
## Multiple R square of scores with factors          0.96
## Minimum correlation of possible factor scores     0.92
```

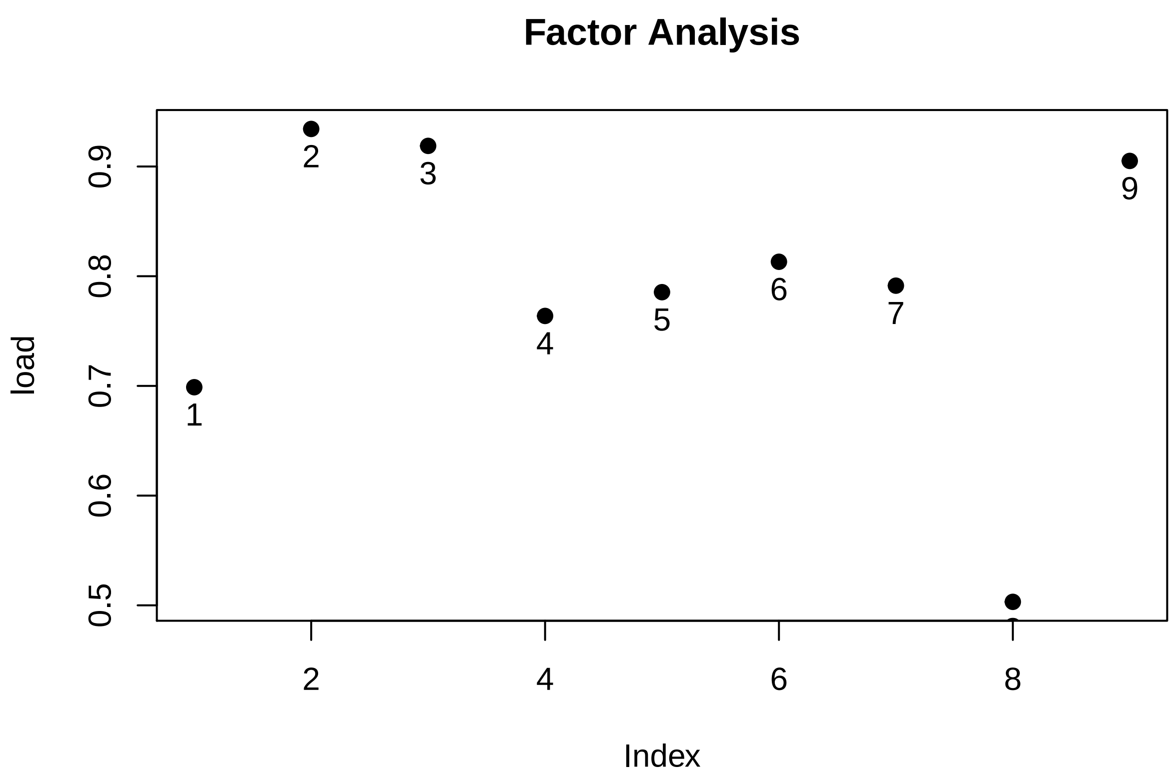
-Compute factor scores

```r
head(bcw1fa$weights)
```

```
##              PA1
## CT    0.02858932
## UOCS  0.28159394
## UOCSH 0.18593667
## MA    0.07296990
## SECS  0.08270340
## BC    0.08547128
```

-Graph an orthogonal solution using factor.plot()

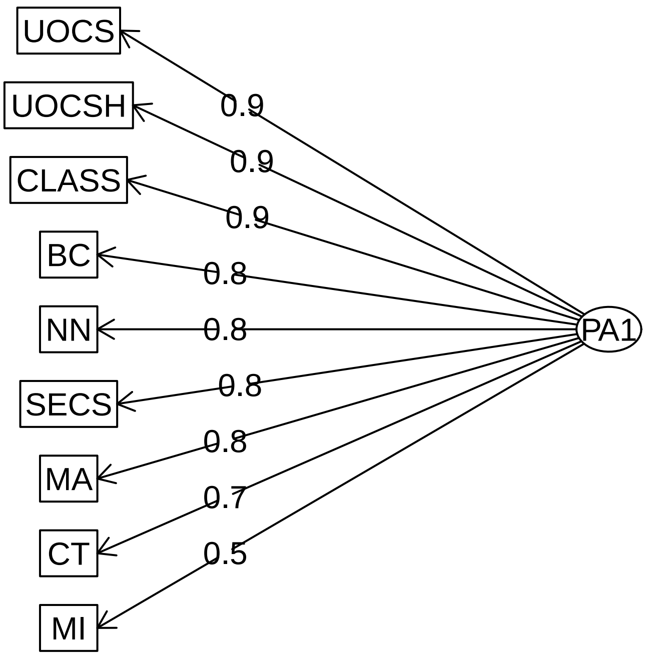
```r
factor.plot(bcw1fa)
```



-Graph an oblique solutions using fa.diagram()

```r
bcw2fa <- fa(bcw1cor, nfactors = 1, rotate = "promax", fm = "pa")
fa.diagram(bcw2fa)
```

Factor Analysis



-Interpret the results

A single principal component was obtained after doing parallel analysis, which constitutes 64% of the total variance of the dataset. The factors PA1 has 5.77 as SS loadings , and the rotation of the factors doesn't affect the values of SS loadings. From factor analysis, we understand that all the factors are above 0.5 loads. From fa.diagram() function we can see the UOCS, UOCSH, and CLASS have 90% variance each, BC, NN, SECS, MA have 80% variance each, whereas the CT, MI has 70% and 50% variance in PA1.

Problem 5. Perform multidimensional scaling on Vertebral Column Data.xlsx

```
VCD <- read_xlsx("~/Downloads/Vertebral Column Data.xlsx")
VCD1 <- subset(VCD, select = -c(Class))
VCD
```

```
## # A tibble: 310 x 7
##       X1    X2    X3    X4    X5     X6 Class
##    <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl> <chr>
##  1  63.0 22.6   39.6  40.5  98.7  -0.25 DH
##  2  39.1 10.1   25.0  29   114.    4.56 DH
##  3  68.8 22.2   50.1  46.6 106.   -3.53 DH
##  4  69.3 24.6   44.3  44.6 102.   11.2  DH
##  5  49.7  9.65  28.3  40.1 108.    7.92 DH
##  6  40.2 13.9   25.1  26.3 130.    2.23 DH
##  7  53.4 15.9   37.2  37.6 121.    5.99 DH
##  8  45.4 10.8   29.0  34.6 117.  -10.7  DH
##  9  43.8 13.5   42.7  30.3 125    13.3  DH
## 10  36.7  5.01  42.0  31.7  84.2   0.66 DH
## # ... with 300 more rows
```

```r
VCD1
```

```
## # A tibble: 310 x 6
##       X1    X2    X3    X4    X5     X6
##    <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
##  1  63.0 22.6   39.6  40.5  98.7  -0.25
##  2  39.1 10.1   25.0  29   114.    4.56
##  3  68.8 22.2   50.1  46.6 106.   -3.53
##  4  69.3 24.6   44.3  44.6 102.   11.2
##  5  49.7  9.65  28.3  40.1 108.    7.92
##  6  40.2 13.9   25.1  26.3 130.    2.23
##  7  53.4 15.9   37.2  37.6 121.    5.99
##  8  45.4 10.8   29.0  34.6 117.  -10.7
##  9  43.8 13.5   42.7  30.3 125    13.3
## 10  36.7  5.01  42.0  31.7  84.2   0.66
## # ... with 300 more rows
```

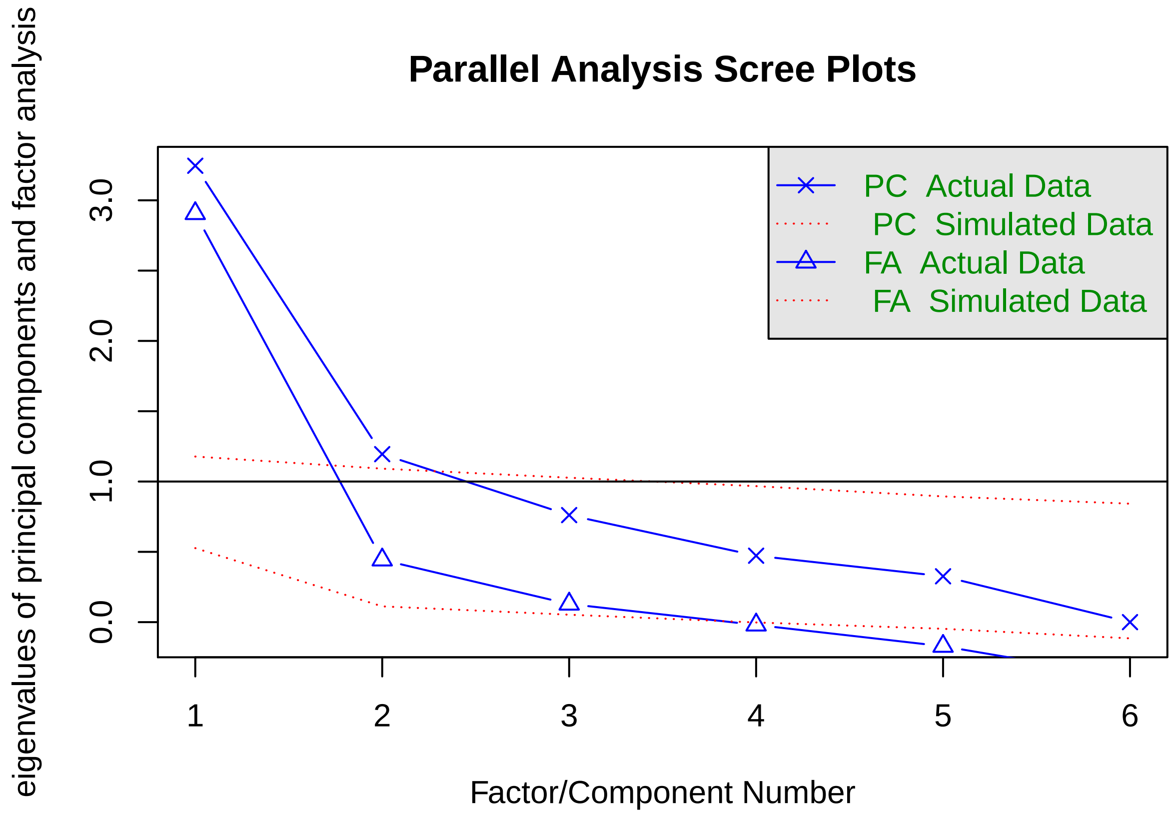
-Input the raw data matrix to fa.parallel() function to determine the number of components to extract

```r
vcdpc <- fa.parallel(VCD1, fa = "both", n.obs = 310)
```

```
## Warning in fa.parallel(VCD1, fa = "both", n.obs = 310): You specified the
## number of subjects, implying a correlation matrix, but do not have a correlation
## matrix, correlations found

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.

## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```




```
## Parallel analysis suggests that the number of factors =  3  and the number of components =  2
```

```
fa(VCD1,nfactors = 3, rotate = "none")
```

```
## Factor Analysis using method =  minres
## Call: fa(r = VCD1, nfactors = 3, rotate = "none")
## Standardized loadings (pattern matrix) based upon correlation matrix
##      MR1   MR2   MR3   h2     u2 com
## X1  0.99  0.01 -0.10 1.00 0.0024 1.0
## X2  0.61  0.77 -0.18 1.00 0.0041 2.0
## X3  0.74  0.02  0.17 0.57 0.4261 1.1
## X4  0.83 -0.56  0.00 1.00 0.0034 1.8
## X5 -0.21  0.30  0.41 0.30 0.6951 2.4
## X6  0.67  0.05  0.24 0.51 0.4948 1.3
##
##                        MR1  MR2  MR3
## SS loadings           3.07 1.01 0.30
## Proportion Var        0.51 0.17 0.05
## Cumulative Var        0.51 0.68 0.73
## Proportion Explained  0.70 0.23 0.07
## Cumulative Proportion 0.70 0.93 1.00
##
## Mean item complexity =  1.6
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  15  and the objective function was  17.61 with Chi Sq
## The degrees of freedom for the model are 0  and the objective function was  10.01
##
## The root mean square of the residuals (RMSR) is  0
```

```
## The df corrected root mean square of the residuals is  NA
##
## The harmonic number of observations is  310 with the empirical chi square  0  with prob <  NA
## The total number of observations was  310  with Likelihood Chi Square =  3043.79  with prob <  NA
##
## Tucker Lewis Index of factoring reliability =  -Inf
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    MR1  MR2   MR3
## Correlation of (regression) scores with factors   1.00 0.99  0.63
## Multiple R square of scores with factors          0.99 0.98  0.39
## Minimum correlation of possible factor scores     0.98 0.96 -0.22
```

-Input the raw data matrix to cmdscale() function to perform multidimensional scaling. cmdscale() function which is available in the base installation performs a classical multidimensional scaling.

```
vcd_distance <- dist(VCD1)
distance <- cmdscale(vcd_distance, k=6)
distance
```

```
##               [,1]        [,2]         [,3]          [,4]         [,5]
##    [1,] -25.2126401  13.2042059 -15.89167139  14.090172407   1.90166114
##    [2,] -37.5502849 -18.9516208 -11.83917146   2.119714887   2.31870603
##    [3,] -21.9508658  23.0636140  -6.31851555   8.945162973  -2.51800690
##    [4,] -10.8470948  13.9179838 -12.97106847  13.901257740  -1.13907658
##    [5,] -27.7330523  -7.5890054 -18.43533164   1.444261472  -2.93284143
##    [6,] -39.7480044 -22.9598414   2.54552855   3.745576331  -4.88345169
##    [7,] -25.7002456  -4.4898798  -1.90156226   3.352672055  -4.84740581
##    [8,] -46.2677698  -5.1806989  -7.09556361   1.506909346  -3.99521673
##    [9,] -22.7776433 -14.0844558   4.69378434  -2.114681088   4.16324681
##   [10,] -35.6460154   1.0457484 -28.38458751  -3.927667928  25.48167111
##   [11,] -40.7578858   0.5109254 -12.90636830   4.702866640  -0.95949657
##   [12,] -49.0398317 -32.7127927  -5.61904171  13.235263918   6.19509426
##   [13,] -25.7732384  -8.4297528   1.76764397   7.142030684   4.28366338
##   [14,] -26.1148599  -5.2410716  -9.09261463  11.598919659   0.67079789
##   [15,] -21.5356988   3.0431869   3.47719166  10.106717504   3.30327711
##   [16,] -31.1661702 -12.7858878   1.01500971  -0.591857935  -0.50631035
##   [17,] -20.7636038  18.2482578   0.77137950   4.152598052   0.44657801
##   [18,] -39.9936916 -23.8992110   1.71101012 -10.759913003   1.96409872
##   [19,] -39.2213539 -18.3279008   0.16094198   2.501702168   2.95599057
##   [20,] -39.8602114 -12.4680456  -6.47068928   2.664257616   2.05762144
##   [21,] -30.4347547 -16.4892176  10.73118508  -1.160399857  -3.50692305
##   [22,] -27.0946622  -1.7657634   6.96841210   6.839566374  -3.32155692
##   [23,]  -7.3981484  10.1576593  -3.67883216   9.431588894   7.67883138
##   [24,] -41.1289322  -7.9832200  -5.42210769   3.387459471  -2.04234718
##   [25,] -45.9095465 -19.4984976  -0.56007522  15.249612778  13.38362987
##   [26,] -30.7815216  -6.0470012   3.90632894  16.205804215  -0.91955645
##   [27,] -60.8285159 -32.4577810  -3.46474272   5.334286513   2.66550320
##   [28,] -25.1110755  -5.6634192  -3.21177430   2.701305060  16.06069428
##   [29,] -36.7837502 -14.6374156  -8.49149760  15.596744724   6.51880192
##   [30,] -22.4104478  17.0752596   0.81943947  10.900907356  -3.22189466
##   [31,] -20.9584740  -3.3935078  -5.87215225   2.336481635   4.84434032
##   [32,] -29.5242601  -5.0275397 -22.01508895   3.489424786   7.75154963
##   [33,] -33.5268294 -16.1581822  -5.46364606   8.861391495   0.70239965
##   [34,] -35.4179274 -14.6096502   5.60198289  -1.419095776   6.53303443
```

```
## [35,] -24.9876616   3.8551534   9.24258183  18.801505536   3.81883564
## [36,] -41.7337403 -24.9287273 -11.83297701  -0.004202944   8.15990463
## [37,] -36.5779951 -24.8383541   7.42215610  -8.229643614   3.77275663
## [38,] -46.4225583 -31.8656190  10.33512315  10.358767991  -3.46085938
## [39,] -25.3068178   0.2003045  12.04544787  14.124763633   4.19990344
## [40,] -30.5322648  -3.5610683  -3.21522685   8.218434078  -0.98789226
## [41,] -49.1788167 -21.2120863 -19.55481403   8.777409827   5.04566412
## [42,] -34.4229812  -9.1679827 -11.52432666  -1.287019217  -4.42838948
## [43,] -27.7394786  -7.4473101  -1.48204624   8.492369937  -6.13431955
## [44,] -22.5936627  12.1453447   6.70148995  11.747490635  -6.60270087
## [45,] -23.7144593  17.1093482   6.03465089  -3.267450594   8.45585319
## [46,] -32.0151217   1.7202873   5.98791735   8.985188587   7.71657843
## [47,] -28.8257840 -10.3662693  -0.22693424  11.493252378   4.95921084
## [48,] -43.0174770 -11.0436320  -5.97974688   8.064192238   6.70345493
## [49,] -40.2567820 -14.2930081   2.27638154   7.085212312   6.21742378
## [50,] -40.6570535 -21.5503749  -7.53080813  12.080380526  -0.45560663
## [51,] -28.0719018  -3.1873437  -4.88875450  10.513585924  -3.34376302
## [52,] -20.3341711   5.7735744  -8.19252761  37.821284064  -8.25574244
## [53,] -29.5440643 -14.0609578  11.75485751  18.164532784   1.87636576
## [54,] -33.7553252 -25.0743360  -6.22946979   1.318921557  18.00578726
## [55,] -46.8189739  -8.4926524  -1.15692171   7.594144695   6.70550923
## [56,] -27.7692867  -1.9128714 -19.32038794   4.530958272  10.41367378
## [57,] -33.0049258 -12.9856215 -14.21768217  -1.636922484  -0.81163376
## [58,] -32.5176922  -6.0449544  -2.86916124   3.439934707   3.43090125
## [59,] -40.0115525 -12.5306511   6.07687813   8.013772499   3.12130331
## [60,] -27.3528326 -11.7035005   0.66969369   2.204513662  -2.76025275
## [61,]  39.1346938  -0.5371560  33.66711795   1.787577554  -1.69671634
## [62,]  79.9640882   1.7427290  17.36623267   2.138176101  -3.72860199
## [63,]  -5.7922219 -18.1030288  12.59895137 -12.152290548   2.13419214
## [64,]  13.2447555  20.9744228   1.25818674   0.516290329  -6.14602463
## [65,]   3.4796941  34.4020905  20.17671248  -6.179775739  -0.92188845
## [66,]  13.6059042  19.5551626   9.11249402  23.862006650  -1.77103586
## [67,]  12.5262038  17.6632230   2.09184554   2.257496685 -10.58047537
## [68,]  49.6234120   5.1802161 -19.14945186   0.222384394   7.76119244
## [69,] -15.9482416  22.4043028  -2.61500969   3.315130627 -10.93789358
## [70,]   2.5857163   7.3856268 -21.62461545 -17.316228216  -0.56762379
## [71,]  11.4606737   7.1808756  -2.91168880  -0.696450984 -11.77137562
## [72,]  75.9358241 -25.8486219   3.48046613  13.197178228 -20.85290937
## [73,]  55.5055809  -4.3144469   6.48353176  11.427171672  -9.70302404
## [74,]   1.6639371 -14.7019793   1.39489622   4.523442270   0.46790758
## [75,]  43.3222553   8.1933017  30.97998409 -11.532295160   4.39728123
## [76,] 109.8132112 -56.5906570  26.79848620  10.436269845   6.30484540
## [77,]  89.8204072 -21.2315089  18.06867249   8.394307562  -9.36931718
## [78,]  22.8789772  -3.9000478 -22.11541841  -9.102246337   8.65404611
## [79,]  12.3231638   9.1890590  -3.87018212  -1.984748364   1.60027181
## [80,] -14.1800237 -13.9050513  -5.73387822  -1.490986343   2.19652986
## [81,]  51.4966621   0.3433561   1.43287685   8.145444369   7.38504761
## [82,]  50.1124013 -10.5707464  -2.90503855   0.132287942  -5.17368126
## [83,]  34.4769479   4.9370016  -3.49123809  12.881284725 -22.19586274
## [84,]  50.7541803  -0.2749626  32.97244478  -7.949171883 -16.35949883
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## [86,] -15.9134568 -26.0721896  32.14968981 -13.919639156 -17.37231830
## [87,]  -1.8467964   4.7446701   5.55802490  -2.190944595   3.53266127
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##  [91,]   15.9433983   17.2452772   0.55460752  10.843067478  -8.44736118
##  [92,]   14.0712118   11.5838336   1.98586585  -0.711153206  -1.45063634
##  [93,]   62.1581539    6.1550276   1.52323529 -11.574793923 -13.86699004
##  [94,]   28.1970639    3.3182919   6.76131451 -12.487376584  19.12588364
##  [95,]   46.3510715   25.7593787  -7.66711429  -8.395056265 -21.60487116
##  [96,]  101.5414366  -74.0820150  12.06610707  10.571272793  10.17546410
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##  [98,]   23.0468629   27.3233794   5.28313231  -6.769578980   8.46100317
##  [99,]   51.4128422   17.1576805  18.75598408 -10.857957236   8.21650568
## [100,]   -0.2745349   -7.4489721  -8.94334587  -0.258929557  -4.01504356
## [101,]   15.0150022   27.9826416  -0.34278794  11.557402682  -3.56629994
## [102,]   24.3802880   17.4351349  -2.66539899  -2.452983384  -9.41813611
## [103,]    8.3139429   19.9207973 -12.11571984  -5.080802737   0.82535491
## [104,]   -4.9340230   -2.7572380  -7.86844239 -11.588583738   7.83209487
## [105,]   69.3542843  -15.4580100  -0.14559432   7.197348539   1.93197481
## [106,]  -15.4642150   10.2552950   4.89509820  12.581592460   0.17802262
## [107,]   25.8213587    5.2795442 -23.10355275  -7.455698703   8.00032295
## [108,]   58.7118606   -7.3171332  25.71495263   4.352695538  -1.70380635
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## [112,]   60.0472714    9.6962581  17.16941967  -0.195046400  -1.11848339
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## [131,]   -9.7960339  -11.4528691 -25.69301541   0.395402682   4.31140134
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## [136,]   63.7687861   -0.3225496   2.79859306   5.191025079  13.14088536
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## [283,] -26.5967213    2.4620846  -5.82412336   0.389384344  -1.09712207
## [284,] -28.2158341    1.0733531  10.27158233  -5.564438365   1.84937050
## [285,] -23.4207688    5.3614623  -0.35200784  -0.182568397  -8.51917213
## [286,] -17.1835228   11.6673733  -2.81980454   2.161586197 -14.71548808
## [287,] -20.9478288    8.7250475   2.03499080   1.446015337  -7.49482754
## [288,] -37.6052603  -28.5975506 -14.63761620  -8.805376975  -3.76017182
## [289,] -12.0828276   32.5180667  -7.94472669  -1.396026440  -6.82655110
## [290,] -40.4515676  -13.2985326   6.88990161   1.532501879  -6.29342816
## [291,] -44.9137127  -26.4652732  -1.94538411   6.194352895  -1.72487570
## [292,] -26.2283026   -5.5958640  -6.56796399   2.715855918  -1.66309818
## [293,] -42.4803473  -19.8425025  -1.95081023  -9.861974362  -2.61213742
## [294,] -10.0845732   -7.7314691  14.12537520 -17.846297946  -4.53088449
## [295,] -38.8268011   -6.1690024   3.95693987  -4.064352625  -6.24488667
## [296,] -28.5696408   -3.5068858   8.47647267 -12.389185117  -3.32073701
## [297,] -43.0408542  -14.9887902  11.73145851   2.386018435   1.52333970
## [298,] -33.5630637  -11.2280645  -2.58656261   8.604754924   4.33229245
## [299,] -24.9644921    1.8580085   1.62096602   9.516284923 -21.84321856
## [300,]  -4.5475561   31.1425051   0.61483166  13.642910159  -8.91859814
## [301,] -32.1459772   -1.0972687  -9.56633477  -5.656098651  -7.33142387
## [302,]   2.3153768   42.9283926   2.95013231   5.481417478 -11.89290071
## [303,] -34.3669919   -4.9077874  -3.95420808  13.019042202  -6.12755992
## [304,] -44.2946998  -17.4339491   0.52479898 -11.598215170  -1.94965066
```

```
## [305,] -41.2700012   -8.2767648  25.02631631  -7.426808033 -10.45942655
## [306,] -37.5343327   -3.2656500  -3.41994687   2.125465583  -0.72892513
## [307,] -33.6618308   -4.5600797  -7.45763822  12.858191932  -3.89025094
## [308,] -27.0125654    7.8027891   9.06262553   7.389299220  -7.23947658
## [309,] -32.9610331   -3.6892415  -2.23394970  -5.679354766   1.57367294
## [310,] -40.7187165  -16.6016629   0.61818143  -8.964355695   5.01897956
##                [,6]
##   [1,]  1.262904e-04
##   [2,]  3.659714e-04
##   [3,] -5.965191e-05
##   [4,]  5.852905e-03
##   [5,]  2.042534e-04
##   [6,]  2.619821e-04
##   [7,]  1.203645e-04
##   [8,]  1.264048e-04
##   [9,]  3.209313e-04
##  [10,]  5.705301e-04
##  [11,]  1.487591e-04
##  [12,] -5.219604e-03
##  [13,]  6.073696e-03
##  [14,]  2.489146e-04
##  [15,]  1.917331e-04
##  [16,]  2.401794e-04
##  [17,]  3.413912e-06
##  [18,]  6.114286e-03
##  [19,]  6.119068e-03
##  [20,]  2.930235e-04
##  [21,]  2.055516e-04
##  [22,]  1.099889e-04
##  [23,]  2.230477e-04
##  [24,]  1.879557e-04
##  [25,]  5.473482e-04
##  [26,]  2.125895e-04
##  [27,]  4.605794e-04
##  [28,]  4.587833e-04
##  [29,]  4.226088e-04
##  [30,] -2.750659e-05
##  [31,]  2.718285e-04
##  [32,]  3.594305e-04
##  [33,]  6.097989e-03
##  [34,] -5.422657e-03
##  [35,]  2.004613e-04
##  [36,] -5.272934e-03
##  [37,]  3.755438e-04
##  [38,]  3.582062e-04
##  [39,] -5.552993e-03
##  [40,]  1.833355e-04
##  [41,]  4.507761e-04
##  [42,] -5.605990e-03
##  [43,]  1.370858e-04
##  [44,] -4.662181e-05
##  [45,]  1.057849e-04
##  [46,] -5.518882e-03
##  [47,]  3.404336e-04
```

```
##  [48,] -5.410311e-03
##  [49,]  3.675921e-04
##  [50,]  3.600285e-04
##  [51,]  1.549059e-04
##  [52,] -5.689254e-03
##  [53,]  3.174347e-04
##  [54,]  6.551409e-04
##  [55,]  3.261684e-04
##  [56,]  3.723837e-04
##  [57,]  2.635828e-04
##  [58,]  6.032769e-03
##  [59,]  2.986238e-04
##  [60,]  2.069556e-04
##  [61,]  5.892146e-03
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##  [63,]  2.982504e-04
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##  [65,] -2.044988e-04
##  [66,] -5.749790e-03
##  [67,] -1.404108e-04
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##  [70,]  9.442864e-05
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##  [73,]  1.223783e-04
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##  [97,] -2.929259e-04
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##  [99,]  5.899169e-03
## [100,]  1.878971e-04
## [101,]  5.683013e-03
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## [105,]  4.177583e-04
## [106,]  8.842801e-05
## [107,] -5.476344e-03
## [108,]  2.193865e-04
## [109,]  1.368821e-04
## [110,]  2.487586e-05
## [111,]  1.860760e-04
## [112,]  8.487221e-05
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## [114,]  3.037479e-04
## [115,]  6.040267e-03
## [116,]  1.181199e-03
## [117,]  1.292754e-04
## [118,]  1.001376e-04
## [119,] -6.086855e-05
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## [121,] -5.480065e-03
## [122,]  1.039928e-04
## [123,] -5.292101e-03
## [124,]  5.987122e-03
## [125,] -6.631266e-05
## [126,]  4.894493e-04
## [127,]  5.908499e-03
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## [130,] -6.622091e-05
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## [132,] -1.344122e-04
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## [137,]  1.349391e-04
## [138,]  1.522213e-04
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## [147,]  1.433437e-04
## [148,]  9.757406e-05
## [149,]  5.905306e-03
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## [151,]  4.765691e-05
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## [153,] -5.641617e-03
## [154,]  2.594960e-04
## [155,]  6.297111e-03
```

```
## [156,]  3.830891e-04
## [157,]  5.914546e-03
## [158,]  4.667141e-04
## [159,]  2.822592e-04
## [160,] -5.476885e-03
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## [169,]  5.098026e-07
## [170,]  3.987274e-04
## [171,]  6.790997e-05
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## [175,]  3.081845e-04
## [176,]  4.355963e-04
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## [179,]  2.300345e-05
## [180,]  5.812869e-04
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## [183,]  3.657038e-04
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## [185,] -1.404228e-04
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## [205,]  3.123664e-04
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## [208,]  1.643240e-04
## [209,] -1.055896e-06
```

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## [245,]  1.463285e-04
## [246,]  1.231463e-04
## [247,]  6.073176e-03
## [248,]  1.612675e-04
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## [250,] -5.573904e-03
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## [252,]  5.546299e-05
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## [254,] -5.704454e-03
## [255,] -1.753027e-04
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## [257,]  2.465026e-05
## [258,] -2.294499e-04
## [259,]  6.010054e-03
## [260,]  5.693660e-03
## [261,] -5.761510e-03
## [262,]  5.525433e-03
## [263,]  2.204329e-04
```

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## [272,]  6.157616e-03
## [273,]  2.964902e-04
## [274,]  2.136681e-04
## [275,] -5.574959e-03
## [276,] -2.727644e-04
## [277,]  5.797271e-03
## [278,] -2.847613e-04
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## [280,]  1.682364e-04
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## [286,] -1.698671e-04
## [287,] -4.628538e-05
## [288,] -5.440190e-03
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## [307,]  1.646457e-04
## [308,]  5.735034e-03
## [309,]  1.839164e-04
## [310,]  3.302557e-04
```

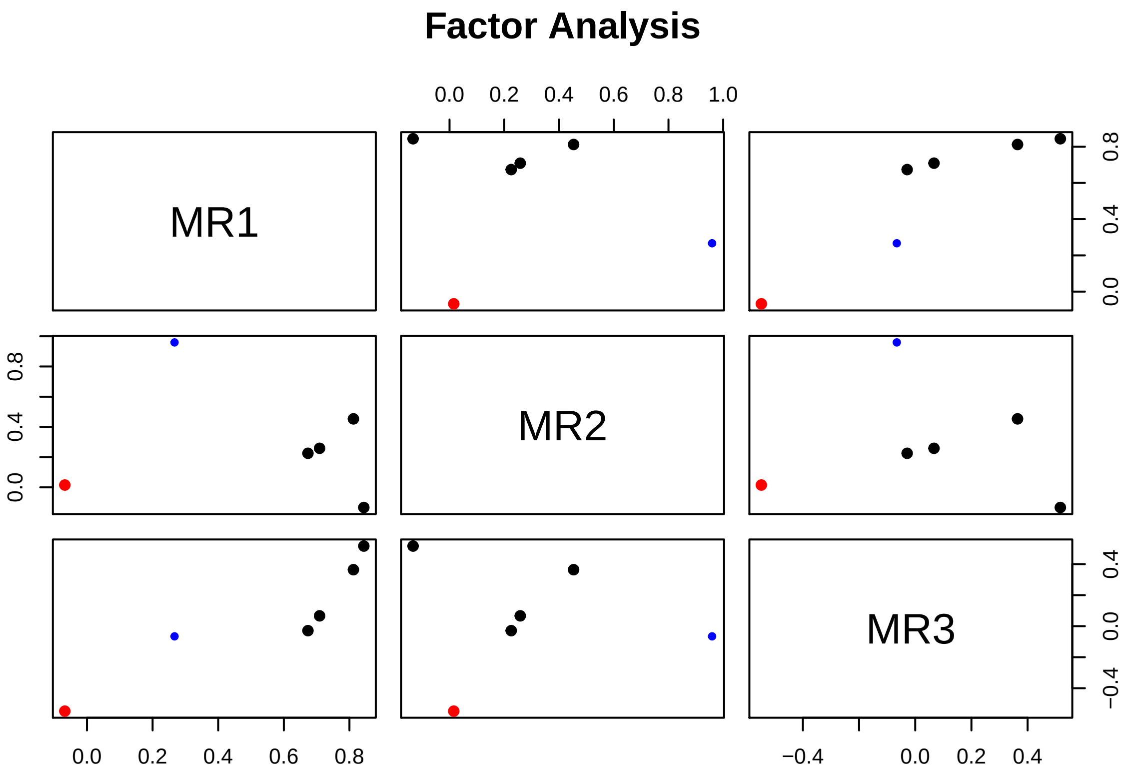
-Graph an orthogonal solution using factor.plot()

```
graph <- fa(VCD1,nfactors = 3, rotate = "varimax")
graph
```

```
## Factor Analysis using method =  minres
## Call: fa(r = VCD1, nfactors = 3, rotate = "varimax")
```

```
## Standardized loadings (pattern matrix) based upon correlation matrix
##      MR1   MR2   MR3   h2     u2 com
## X1  0.81  0.45  0.36 1.00 0.0024 2.0
## X2  0.27  0.96 -0.07 1.00 0.0041 1.2
## X3  0.71  0.26  0.07 0.57 0.4261 1.3
## X4  0.84 -0.13  0.52 1.00 0.0034 1.7
## X5 -0.07  0.02 -0.55 0.30 0.6951 1.0
## X6  0.67  0.23 -0.03 0.51 0.4948 1.2
##
##                        MR1  MR2  MR3
## SS loadings           2.40 1.26 0.71
## Proportion Var        0.40 0.21 0.12
## Cumulative Var        0.40 0.61 0.73
## Proportion Explained  0.55 0.29 0.16
## Cumulative Proportion 0.55 0.84 1.00
##
## Mean item complexity =  1.4
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are  15  and the objective function was  17.61 with Chi Sq
## The degrees of freedom for the model are 0  and the objective function was  10.01
##
## The root mean square of the residuals (RMSR) is  0
## The df corrected root mean square of the residuals is  NA
##
## The harmonic number of observations is  310 with the empirical chi square  0  with prob <  NA
## The total number of observations was  310  with Likelihood Chi Square =  3043.79  with prob <  NA
##
## Tucker Lewis Index of factoring reliability =  -Inf
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                   MR1  MR2  MR3
## Correlation of (regression) scores with factors   0.92 0.99 0.74
## Multiple R square of scores with factors          0.85 0.97 0.55
## Minimum correlation of possible factor scores     0.70 0.94 0.09
```

```r
factor.plot(graph)
```



-Interpret the results

After orthogonal rotation using varimax, proportion variance of the 3 factors were 0.4, 0.21 and 0.12. Using factor plot we are plotting the eigen values of the column with respect to factors. After deriving orthogonal solution there is no overlapping of variances between the factors. Moreover factor.plot provides much better analysis than that of cmdscale.