Session Agenda

- Significance Testing
 - Tea Tasting Lady Experiment
- P-values and ASA statement
- Permutation Tests
- Hypothesis Testing
 - Two sample t-test
 - Paired t-test
 - Power Calculations
- Testing Hypotheses with Variances
- Multiple Comparisons
- One-way ANOVA Example
- Two-way ANOVA Example
- TukeyHSD Test
- Abalone Analysis of Variance
- Test #4

Tea Tasting Lady Experiment

Circa ~1917-1920 Original concept of a null hypothesis formulated by Sir Ronald A. Fisher introduced probability and randomization as important experimental design concepts. The test is referred to as Fisher's Exact Test.

Null Hypothesis:

The lady cannot tell if tea is prepared by first adding tea to a cup or first adding milk to a cup. (There is no stated alternative hypothesis in Fisher's work.)

Experiment:

- Prepare 8 cups of tea four of which have the tea added first and the other four the milk added first. Present them in random order.
- · The lady to select four cups prepared by one method.

Test Statistic:

Count the number of successes in selecting 4 cups prepared by one method from the eight.

Analysis:

There are 70 possible combinations of 8 objects taken 4 at a time. The probability of being correct is $\sim 1.4\% < 5\%$.

If four cups were correctly selected, by the rare event rule, a statistically significant outcome would have been observed. This would acknowledge an ability but not quantify it.

- > dhyper(4,4,4,4) [1] 0.01428571
- > sum(dhyper(c(5,6),6,6,6)) [1] 0.04004329

http://www.coe.utah.edu/~cs3130/lectures/L15-HypothesisTests1.pdf

What is a p-value?

A p-value is the computed probability of obtaining a test statistic value, or a more extreme value, using the assumptions stated for the null hypothesis.

P-value calculations incorporate the effect size, sample size, and variability of the data into a single number that tells you how consistent your data are with the null hypothesis.

Example:

Decade old data indicates the national average net income for sole-proprietor CPAs was \$98,500 with a standard deviation of \$14,530. A random sample of 112 CPAs has an average of \$102,220. Has the average net income changed? Test with alpha = 0.05. Assume the income data are normally distributed.

H_o:
$$\mu$$
 = \$98,500 **H_a:** μ ≠ \$98,500

Hypothesis testing starts by assuming a statistical model to describe the data. In this case a normal distribution with specified parameters and a random sample.

Under the Null hypothesis:
$$P\left[z_{\alpha/2} \le \left(\frac{\overline{x} - \mu}{\sigma/\sqrt{n}}\right) \le z_{1-\alpha/2}\right] = 1 - \alpha$$

$$(102,220 - 98,500)/(14,530/\text{sqrt}(112)) = 2.709 > 1.960$$

$$p\text{-value} = P[z >= 2.709] = pnorm(2.709, 0, 1, lower.tail = FALSE)$$

$$p\text{-value} = 0.00337$$

Either a statistically rare outcome has been observed, or the data are incompatible with the null hypothesis and the specified statistical model. The p-value does not differentiate between.

Is a small p-value by itself conclusive evidence?

Fisher proposed the p-value as an informal measure of evidence for judging if a null hypothesis should be rejected. He believed it should be considered with other types of evidence and studies. He viewed learning as a continuing process of knowledge building involving experimental replication and discourse.

- A small p-value casts doubt on or provides evidence against a null hypothesis or the underlying assumptions.
- A p-value by itself without context or other evidence provides limited information and may mislead.

If you don't know the full context of a study, you can't properly interpret a carefully selected subset of the results. Data dredging, cherry picking, significance chasing, data manipulation, and other forms of p-hacking can make it impossible to draw the proper conclusions from selectively reported findings. You must know the full details about all data collection choices, how many and which analyses were performed, and all p-values.

No single index should substitute for sound reasoning.

Permutation Tests

- A permutation test is a nonparametric approach for hypothesis testing. No assumption is made regarding the underlying distribution shape of the data.
- Assume 8 representative test subjects are randomly assigned to 2 groups, a control group and a treatment group. There are 70 possible ways this assignment could turn out. Here is one.

	Control	Drug
Test Outcome	9, 12, 14, 17	15, 18, 20, 23
Average	13	19

Null Hypothesis: Drug has no effect.

Alternative Hypothesis: Drug has positive effect.

Question: Is the difference of 6 units simply a consequence of natural variability between test subjects?

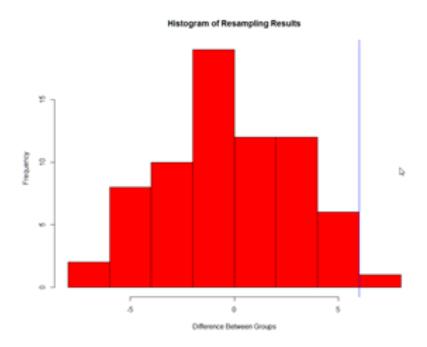
If the drug has no effect, the measurements are of some intrinsic quantity unaffected by the drug; and, the particular random assignment selected is the reason for the observed average difference.

Compare the value of 6 to the other 69 values that could have been obtained with other assignments. Here are three possibilities:

Control	Drug	Test Outcome
9, 12, 14, 15	17, 18, 20, 23	7
12, 14, 15, 17	9, 18, 20, 23	3
14, 15, 17, 20	9, 18, 23, 12	-4

Permutation Tests Continued

The collection of all 70 differences is the sampling distribution for the difference under the null hypothesis. Each difference has a probability of occurrence of 1/70. If the value 6 is shown to be unlikely given this distribution, a statistically significant result has been obtained. Test one-sided with alpha equal to 0.05.



Only one other combination produces a value greater than 6. Following Black p-value = 0.0286 < 0.05. Reject null hypothesis.

It is possible to construct confidence intervals using permutation tests. An extensive literature has developed over the years.

Ernst, Michael D., "Permutation Methods: A Basis for Exact Inference", Statistical Science, 2004, Vol. 19, No. 4, 676-685.

Kabacoff Section 12.1 pages 280-292 has an extensive discussion.

Package 'coin' 11/16/2015 "Conditional Inference Procedures in a Permutation Test Framework"

Bootstrapping and Permutation Tests

- Allow hypotheses to be tested and confidence intervals to be formed without reference to a known theoretical distribution.
- Valuable when there are: serious outliers, small sample sizes and no existing parametric method that is applicable.
- Analysis is performed on the original scale of measurement which differs from analyses which use transformations.
- Computer intensive. Packages are available. See Kabacoff Chapter 12.
- Cannot turn bad data into good data. Samples must be of adequate size, and representative of the population.

"It is perfectly proper to use both classical and robust methods routinely, and only worry when they differ enough to matter. But when they differ, you should think hard." John Tukey (1975)

Student's t Statistic Performance

Confidence Interval Coverage for Point Estimates (Wilcox *Basic Statistics* pages 126-127)

Symmetric Distributions	Asymmetric Distributions	
Outliers rare	Outliers rare	
 Use Student's t 	 May use Student's t 	
• May need n >= 30	• May need n >= 200	
Outliers common	Outliers common	
May use Student's t	 May use Student's t 	
 Large sample size needed 	 May need n >= 300 	
 Consider alternatives* 	 Consider alternatives* 	

^{*}Alternatives to consider are the bootstrap-t method, a trimmed mean with a Winsorized standard deviation, or a non-parametric procedure.

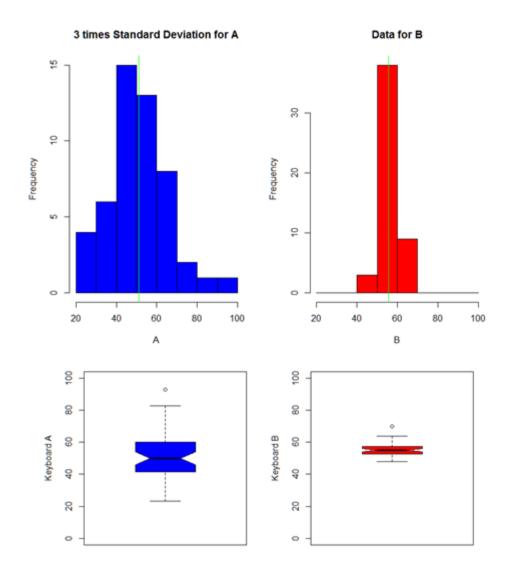
Two Sample Comparisons and Assumption Violations (Wilcox *Basic Statistics* pages 187)

- Equality of variances can be violated.
 - Must have equal sample sizes and normality.
- Normality can be violated.
 - Must have identical distributions with the same variances, skewness and tails, etc.

The perfect storm that degrades Student's t Statistic performance involves non-normal distributions with unequal variances. Bootstrapping and/or permutation tests may be used to confirm conclusions particularly if the situation appears to be overly extreme.

Schmider, E., Ziegler, M., Danay, E., Beyer, L., Bühner, M., "Is It Really Robust?", Methodology 2010; Vol. 6(4):147-151.

Non-normality and Unequal Variances



95% Confidence Intervals by Five Methods		
t-test with equal variances (0.42, 9.01)		
t-test with unequal variances	(0.38, 9.04)	
Bootstrap t with unequal variances	(0.23, 8.92)	
Percentile bootstrap method	(0.45, 8.87)	
Permutation using means	(0.43, 8.90)	

Keyboard Problem-Paired Data

To evaluate typing speed differences with two keyboard styles, fifty college students are randomly selected to test two different keyboards. Both keyboards are tested by each student in random order. Each student is allowed to practice in advance of taking a five minute typing test. The script for the typing test is a sequence of nonsense words. The average number of correct words typed per minute for each student and keyboard is recorded.

The manufacturer believes keyboard style B is superior to keyboard style A. This is a paired t-test situation which is handled with a one-sided single population t-test. Let D denote the true difference constructed as the difference of the population mean for B minus that for A. Test at the 5% significance level.

$$H_0: D = 0$$

 $H_A: D > 0$

Sample mean difference	1.21
Sample standard deviation	1.618
Sample size	50

$$t = (\overline{d} - D) / (s_d / \sqrt{n})$$

```
> df <- 50-1
```

> t <- 1.21/(1.618/sqrt(50))

> t [1] 5.288005

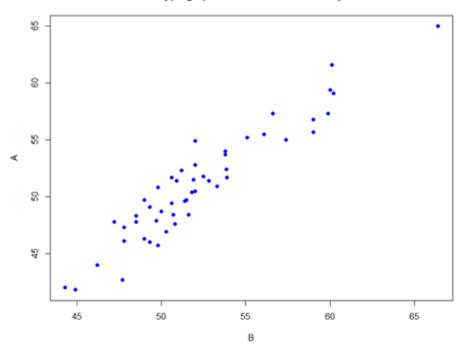
> qt(0.95,df,lower.tail=TRUE) [1] 1.676551

> pt(t,df,lower.tail=FALSE) [1] 1.430491e-06

Since 5.288005 > 1.676551 reject the null hypothesis. p-value is 1.430491e-06 < 0.05.

Paired Keyboard Data Display





What if we ignored the pairing involved?

> t.test(B, A, alternative = c("greater"), mu = 0,

```
+ paired = FALSE, var.equal = TRUE, conf.level = 0.95)

Two Sample t-test

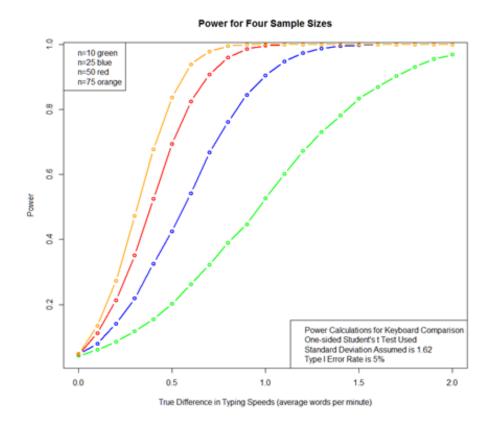
data: B and A

t = 1.2988, df = 98, p-value = 0.09853
alternative hypothesis: true difference in means is greater than 0

95 percent confidence interval:
-0.3373693 Inf
sample estimates:
```

mean of x mean of y 52.2372 51.0260

Power Calculation Results



1) Define the hypothesis and test statistic:

$$\mathbf{H_0}$$
: $\mathbf{D} = \mathbf{0}$
 $\mathbf{H_A}$: $\mathbf{D} > \mathbf{0}$
 $\mathbf{t} = (\overline{\mathbf{d}} - \mathbf{D})/(\mathbf{s}_d / \sqrt{n})$

Set the Type I error and sample size. Find the critical value.

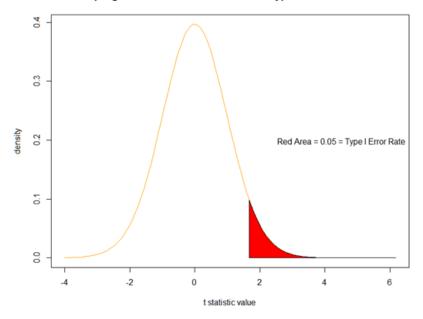
$$P[t > c | n, D = 0] = 0.05 = alpha$$

$$c = 1.677 \text{ when } n = 50$$

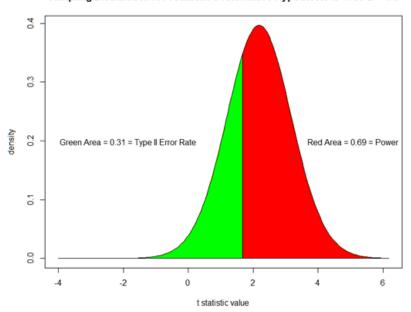
3) Determine the probability of rejecting the null hypothesis given (1) and (2) as a function of the alternative D.

power(n, alpha, D) = P[
$$t > c \mid n$$
, alpha, D > 0]

Sampling Distribution for t statistic if Null Hypothesis is True D = 0



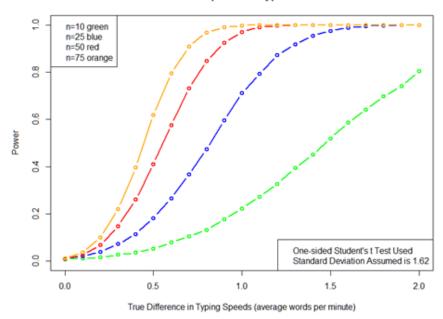
Sampling Distribution for t statistic if Alternative Hypothesis is True D = 0.5



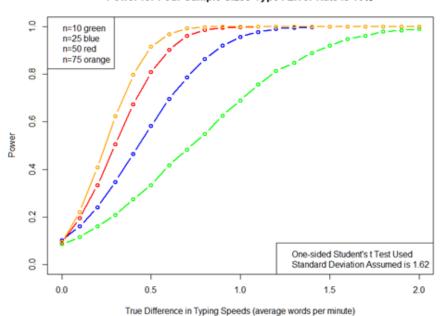
- ➤ power.t.test(n=50, delta = 0.5, sd= 1.62, sig.level = 0.05, type = c("one.sample"), + alternative = c("one.sided"))
- \triangleright One-sample t test power calculation n = 50 delta = 0.5 sd = 1.62 sig.level = 0.05 power = 0.6940214 alternative = one.sided

Power Calculations with Two Type I Error Rates





Power for Four Sample Sizes Type I Error Rate is 10%



Kabacoff devotes Chapter 10 to discuss power analysis. Several examples and packages are presented. See pages 239-254.

F test Example

Assume that both samples are from populations with normal distributions. A random sample of 16 women resulted in blood pressure levels with a standard deviation of 23 mm Hg. A random sample of 17 men resulted in blood pressure levels with a standard deviation of 19.2 mm Hg. Use a 5% significance level to test the claim that blood pressure levels for women vary more than for men.

- > # The null hypothesis is that the variances are equal.
- > # The alternative hypothesis is that the variance for women exceeds than of men.
- > # degrees of freedom for numerator equals 15, for denominator equals 16

$$> s1 \le 23$$

$$> F <_{-} s1^2/s2^2$$

Do not reject the null hypothesis. 1.435004 < 2.352223 p-value = 0.2406477 > 0.05

Testing Hypotheses With Variances— Completely Randomized Designs

Under the null hypothesis of equal variances (Black p. 385):

F TEST FOR TWO POPULATION VARIANCES (10.12)

$$F = \frac{s_1^2}{s_2^2}$$

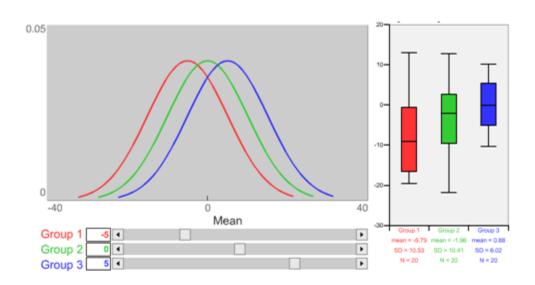
$$df_{numerator} = v_1 = n_1 - 1$$

$$df_{denominator} = v_2 = n_2 - 1$$

The analysis of variance uses the F test extensively. Sample variances must be determined for hypotheses to be tested. Can the variation in group means be explained as a consequence of within group variation?

Wilcox Basic Statistics Chapter 10 Problem 10.1.1 page 221

Analysis of Variance Calculations



 $Var(\bar{X}) = \sigma_X^2 / n$ under the null hypothesis $ns_{\bar{X}}^2$ estimates σ_X^2

```
> x < -(4.75 + 4.625 + 7.75)/3
> x
[1] 5.708333
> s2m < ((4.75 - x)^2 + (4.625 - x)^2 + (7.75 - x)^2)/2
> s2 < -8*s2m
> s2
[1] 25.04167
> s2p < -7*(6.2143 + 3.9821 + 2.2143)/21
> s2p
[1] 4.1369
> F <- s2/s2p
> F
[1] 6.053244
> qf(0.99, 2, 21, lower.tail = TRUE)
[1] 5.780416
> pf(F, 2, 21, lower.tail = FALSE)
[1] 0.008398753
```

CRD Analysis of Variance

One-Way ANOVA Table

An <u>ANOVA table</u> is often used to record the sums of squares and to organize the rest of the calculations. Format for the ANOVA Table:

Source of Variation	ss	df	MS	F ratio
Between Samples	SSB	k - 1	$MSB = \frac{SSB}{k-1}$	F = MSB MSW
Within Samples	ssw	n-k	$MSW = \frac{SSW}{n - k}$	
Total	SST = SSB+SSW	n - 1		

• The sums of squares and the degrees of freedom must check SS(factor) + SS(error) = SS(total) or SSB + SSW = SST df(factor) + df(error) = df(total) or df(between) + df(within) = df(total)

A generic definition for degrees of freedom is the number of variables free to vary in the calculation of a statistic.

```
> yield <- c(3,5,2,4,8,4,3,9,4,4,3,8,7,4,2,5,6,7,8,6,7,9,10,9)
> group <- c(rep("1",8),rep("2",8),rep("3",8))
> results <- aov(yield~group)</p>
> summary(results)
            Df Sum Sq Mean Sq F value Pr(>F)
            2 50.08 25.042 6.053 0.0084 **
group
Residuals
           21 86.87 4.137
               0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Signif. codes:
> TukeyHSD(results)
  Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = yield ~ group)
$group
      diff
                  lwr
                           upr
                                  p adj
2-1 -0.125 -2.6883421 2.438342 0.9917087
3-1 3.000 0.4366579 5.563342 0.0200444
3-2 3.125 0.5616579 5.688342 0.0152736
```

How to Make a Data Point Work Twice Factorial Design (Two-Way ANOVA)

Description: The data are from a statement by Texaco, Inc. to the Air and Water Pollution Subcommittee of the Senate Public Works Committee on June 26, 1973. There are 36 observations comprising a complete two-way layout for an ANOVA.

- NOISE = Noise level reading (decibels)
- 2. SIZE = Vehicle size: 1 small, 2 medium, 3 large
- 3. TYPE = 1 standard silencer, 2 Octel filter

NOISE in decibels by		TYPE	
SIZE and TYPE		1 standard	2 Octel
SIZE	1 small		
	2 medium		
	3 large		

Advantages:

- This is two experiments in one. Can look at the effect of TYPE and the effect of SIZE simultaneously.
- With more than one observation per cell, it is possible to detect the presence of an interaction.

Interaction occurs when the effects of one treatment vary according to the levels of treatment of the other effect. For example, the effect of TYPE may not be the same when SIZE is changed, or vice versa.

Two-way Sums of Squares

Noise Level Study				
means and standard deviations				
VEHICLE	TYPE		row	
Size	Standard Octel		means	
small	825.8	822.5	824.2	
	10.7	2.7		
medium	845.8	821.7	833.8	
	5.8	4.1		
large	775.0	770.0	772.5	
	13.4	6.3		
column	815.6	804.7	810.1	
means				

The analysis of variance is based on a linear model. For example,

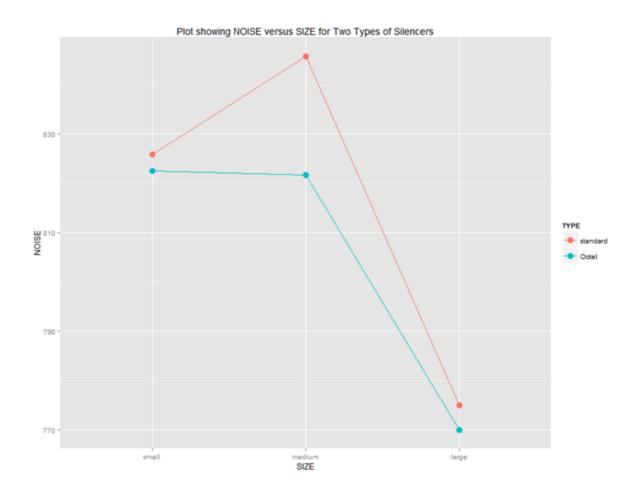
cell mean = grand mean + column effect + row effect + interaction observation = grand mean + column effect + row effect + interaction +residual

$$x_{ijk} = \overline{x} + (\overline{x}_i - \overline{x}) + (\overline{x}_j - \overline{x}) + (\overline{x}_{ij} - \overline{x}_i - \overline{x}_j + \overline{x}) + (x_{ijk} - \overline{x}_{ij})$$

Analysis of Variance Table

```
> outcome <- aov(NOISE ~ size + type +size*type, data = noise)
> summary(outcome)
           Df Sum Sq Mean Sq F value
                                      Pr(>F)
                       13026 199.119 < 2e-16 ***
size
               26051
                1056
                       1056 16.146 0.000363 ***
type
            1
           2
                804
                         402
                             6.146 0.005792 **
size:type
Residuals 30 1963
                          65
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note the statistically significant interaction. The effect of TYPE is not consistent across SIZE levels.



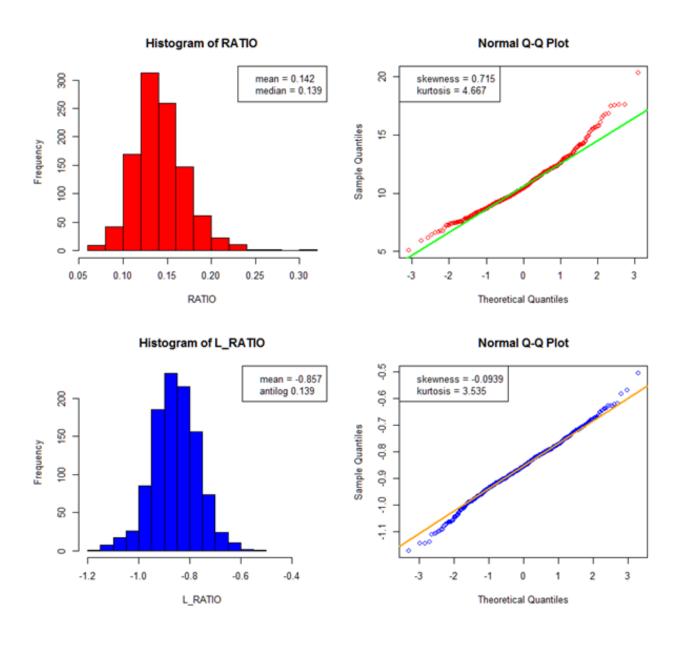
TukeyHSD Results

```
> TukeyHSD(outcome)
  Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = NOISE ~ size + type + size * type, data = noise)
$size
                   diff
                               lwr
                                          upr
                                                  p adj
medium-small
               9.583333
                          1.443167
                                    17.72350 0.0183324
large-small -51.666667 -59.806833 -43.52650 0.0000000
large-medium -61.250000 -69.390167 -53.10983 0.0000000
$type
                    diff
                               lwr
                                          upr
                                                  p adj
Octel-standard -10.83333 -16.33934 -5.327328 0.0003631
$`size:type`
                                      diff
                                                   lwr
                                                              upr
                                                                      p adi
medium:standard-small:standard
                                20,0000000
                                              5.796844
                                                        34.203156 0.0022033
large:standard-small:standard
                               -50.8333333 -65.036489 -36.630177 0.0000000
small:Octel-small:standard
                                -3.3333333 -17.536489 10.869823 0.9787622
medium:Octel-small:standard
                                -4.1666667 -18.369823
                                                        10.036489 0.9454142
large:Octel-small:standard
                               -55.8333333 -70.036489 -41.630177 0.0000000
large:standard-medium:standard -70.8333333 -85.036489 -56.630177 0.0000000
small:Octel-medium:standard
                               -23.3333333 -37.536489
                                                       -9.130177 0.0003130
medium:Octel-medium:standard
                               -24.1666667 -38.369823 -9.963511 0.0001909
large:Octel-medium:standard
                               -75.8333333 -90.036489 -61.630177 0.0000000
small:Octel-large:standard
                                47.5000000
                                             33.296844
                                                        61.703156 0.0000000
medium:Octel-large:standard
                                46.6666667
                                             32.463511
                                                        60.869823 0.0000000
large:Octel-large:standard
                                -5.0000000 -19.203156
                                                        9.203156 0.8890358
medium:Octel-small:Octel
                                -0.8333333 -15.036489
                                                        13.369823 0.9999720
                               -52.5000000 -66.703156 -38.296844 0.0000000
large:Octel-small:Octel
large:Octel-medium:Octel
                               -51.6666667 -65.869823 -37.463511 0.0000000
```

Kabacoff devotes Chapter 9 to the analysis of variance. See pages 212-229.

Abalone Analysis of Variance

The analysis of variance assumes a constant variance and normality. RATIO must be transformed to facilitate this analysis. The base ten logarithm is useful for this purpose.



Abalone Analysis of Variance (continued)

```
anova <- aov(L_RATIO ~ CLASS + SEX, data = mydata)
summary(anova)
TukeyHSD(anova)
              Df Sum Sq Mean Sq F value Pr(>F)
               4 1.055 0.26384 38.524 < 2e-16 ***
CLASS
               2 0.091 0.04569 6.671 0.00132 **
SEX
            1029 7.047 0.00685
Residuals
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
  Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = L_RATIO ~ CLASS + SEX, data = mydata)
$CLASS
             diff
                          lwr
                                       upr
A2-A1 -0.01248831 -0.03876038 0.013783756 0.6919456
A3-A1 -0.03426008 -0.05933928 -0.009180867 0.0018630
A4-A1 -0.05863763 -0.08594237 -0.031332896 0.0000001
A5-A1 -0.09997200 -0.12764430 -0.072299703 0.0000000
A3-A2 -0.02177176 -0.04106269 -0.002480831 0.0178413
A4-A2 -0.04614932 -0.06825638 -0.024042262 0.0000002
A5-A2 -0.08748369 -0.11004316 -0.064924223 0.0000000
A4-A3 -0.02437756 -0.04505283 -0.003702280 0.0114638
A5-A3 -0.06571193 -0.08687025 -0.044553605 0.0000000
A5-A4 -0.04133437 -0.06508845 -0.017580286 0.0000223
$SEX
            diff
                           lwr
                                                 p adj
                                         upr
I-F -0.015890329 -0.031069561 -0.0007110968 0.0376673
M-F 0.002069057 -0.012585555 0.0167236690 0.9412689
     0.017959386 0.003340824 0.0325779478 0.0111881
M-I
```

Some Sync Session Learning Points

- A p-value less than 0.05 may be statistically significant, but it does not provide conclusive proof the alternative hypothesis is true.
- A p-value greater than 0.05 does not mean the null hypothesis is true. The null hypothesis is never accepted. The correct terminology is that it has not been rejected.
- A permutation test makes no assumption regarding the underlying distribution shape of the data.
- Permutation tests can be used to construct confidence intervals.
- No statistical method can turn bad data into good results. All statistical methods depend on having representative data in sufficient quantity to justify the selected analysis.
- If the null hypothesis is true, the power equals alpha.
- The test statistic in a one-way analysis of variance is not the pvalue. It is the F test statistic used for hypothesis testing.
- With repeated independent significance tests, a Type I error will occur eventually despite having true null hypotheses.
- In the analysis of variance, an interaction occurs when the effects
 of one factor vary according to the levels of another factor.
- The following statements are true:
 - If the sample size is held constant, and the Type 1 error rate is increased, the Type 2 error rate is decreased and conversely.
 - If the Type 1 error rate is held constant, and the sample size is increased the Type 2 error rate is decreased and conversely.
 - If the Type 2 error rate is held constant, and the sample size is increased the Type 1 error rate is decreased and conversely.

Comments on Test #4

- Two sample t tests
 - proportions
 - op-values
- Confidence intervals
 - otwo-sided
 - one-sided
- Two sample t tests
 - ounpaired
 - o paired
- AOV assumptions
- One-way AOV Table Elements
- F test

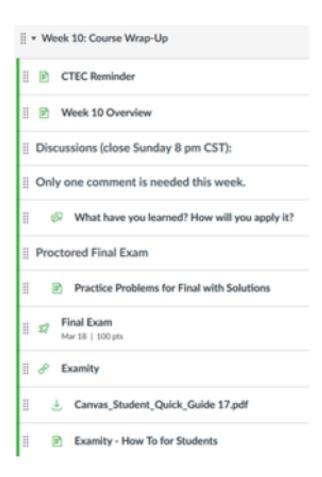
Final Exam Topics

- Probability
 - Calculations using probability
 - Bayes' Theorem
 - o Means and variances for probability distributions
- Hypothesis Testing
 - Type I and Type II Errors
 - Correlation
 - t tests
 - one sample
 - two sample
- Confidence Interval Construction
- One-way AOV
 - F test
 - o p-values
- Linear Regression

The test is two hours, proctored, with open book and open notes. There are ten multiple choice or true/false questions. No preview of the exam is available. Review questions are available in module ten. Excel, R or any comparable calculator application may be used. The course site, WileyPlus, electronic files, hardcopy and e-readers for kindles may be used. Cloud storage of files is allowed.

Only one computer screen is allowed. Portable devices such as kindles and iPads are not allowed unless special arrangements are made. No navigation from the testing site to the internet for browsing is allowed.

Final Exam



- You are responsible for scheduling and paying for your final exam.
 - 2 hour exam: \$23.00
 - Scheduling within 24 hours of exam: \$5.00 per hour
 - Cancellations or schedule changes within 24 hours of exam: \$5.00 per exam
 - No-shows: Full payment of all proctoring fees (\$15.00 for the first hour plus \$7.00 for each additional hour)
- Arrange a "dry run" with Examity in advance to test your equipment and get any questions answered.
- This is an open-book exam, however only one screen is allowed.
- The two-hour exam consists of ten multiple choice questions. No questions about R, however R may be used for calculations.