Week 3 - Review

Sunday, January 28, 2018 8:05 AM

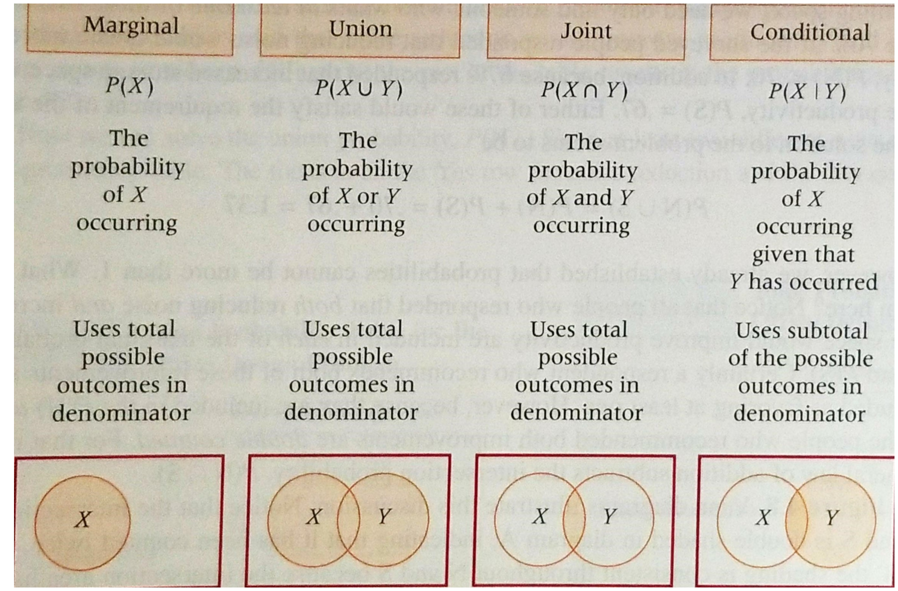
Define Probability

- Probability is a number between 0 and 1 (including both).
  - It represents a degree of belief in factor or prediction.
  - The value 1 represents that a fact is true or that a prediction will come true.
  - The value 0 represents certainty that the fact is false.
  - Intermediate values represent degrees of certainty
- ***Classical Method of Assigning Probabilities***
  - The probability of an individual event occurring is determined as the ratio of the number of items in a population containing the event (n_e) to the total number of items in the population (N).
  - P(E) = n_e/N
- ***Relative Frequency of Occurrence***
  - The probability of an event occurring is equal to the number of times the event has occurred in the past divided by the total number of opportunities for the event to have occurred.
  - P(E) = (Number of times an event occurred) / (Total number of opportunities for the event to occur)
- ***Subjective Probability***
  - This probability is based on the feels or insights of the person determining the probability.
  - Not based on a scientific approach; based individuals intuition (or a guess), knowledge, and experience.

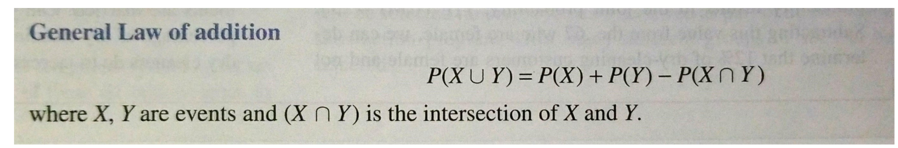
Structure of Probability

- Experiment - a process that produces an outcome
- Event - is an outcome of an experiment
- Elementary Events - these are events that cannot be decomposed or broken down into other events
- Sample Space - this is a complete roster or listening of all the elementary events for an experiment
- Union - this is formed by combining elements from each of the sets ***(X ∪ Y)*** which mean "X or Y"
- Intersection - this contains the elements common to both sets
- Mutually Exclusive Events
  - The occurrence of one event precludes the occurrence of the other events. This means that the events cannot occur simultaneously and will not intersect ***P(X ∩ Y) = 0***
- Independent Events
  - The occurrence or nonoccurrence of one of the events does not affect the occurrence or nonoccurrence of the other event(s).
- Collectively Exhaustive Events
  - This contains all possible elementary events for an experiment - therefore all samples spaces are collectively exhaustive lists.
- Complementary Events
  - All the elementary events of an experiment not in X comprise its compliment ***P(X') = 1 - P(X)***

- **<u>Conditional Probability</u>**
  - This is a probability based on some background information.
    - Example: United States heart attack statistics
  - It is based on a number of factors that will make up the condition.



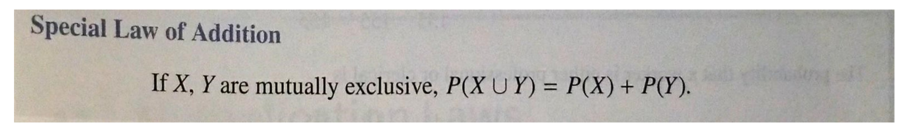
Marginal	Union	Joint	Conditional
$P(X)$	$P(X \cup Y)$	$P(X \cap Y)$	$P(X \mid Y)$
The probability of X occurring	The probability of X or Y occurring	The probability of X and Y occurring	The probability of X occurring given that Y has occurred
Uses total possible outcomes in denominator	Uses total possible outcomes in denominator	Uses total possible outcomes in denominator	Uses subtotal of the possible outcomes in denominator



General Law of addition

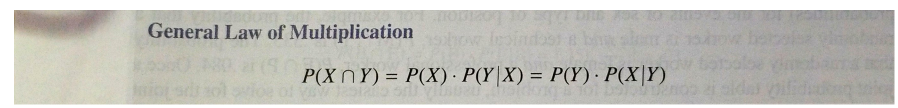
$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

where X, Y are events and $(X \cap Y)$ is the intersection of X and Y.



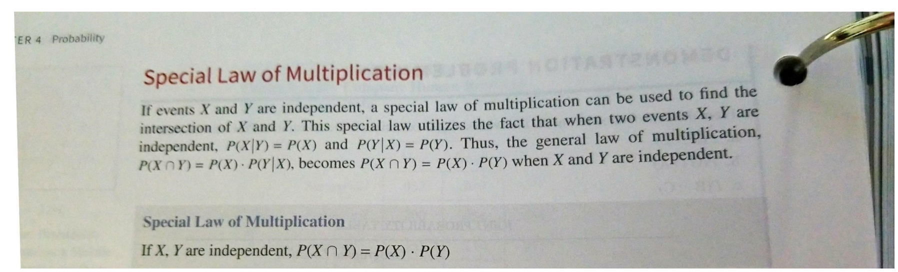
Special Law of Addition

If X, Y are mutually exclusive, $P(X \cup Y) = P(X) + P(Y)$.



General Law of Multiplication

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$



ER 4 Probability

Special Law of Multiplication

If events X and Y are independent, a special law of multiplication can be used to find the intersection of X and Y. This special law utilizes the fact that when two events X, Y are independent, $P(X|Y) = P(X)$ and $P(Y|X) = P(Y)$. Thus, the general law of multiplication, $P(X \cap Y) = P(X) \cdot P(Y|X)$, becomes $P(X \cap Y) = P(X) \cdot P(Y)$ when X and Y are independent.

Special Law of Multiplication

If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

- This is a probability based on some background information.
  - Example: United States heart attack statistics
- It is based on a number of factors that will make up the condition.
- Notion for conditional probability is p(A|B), which is the probability of ***A*** given that ***B*** is true.

- **Conjoint Probability**
  - Is a fancy way of say the probability of **two things are true**
  - Notion for conjoint probability is p(A and B) which means that A and B are both true.
    - P(A and B) = p(A) p(B|A)
- **Bayes's Theorem**
  - p(A|B) = (p(A) p(B|A)) / p(B)
  -
  - $$p(A|B) = \frac{p(A)\ p(B|A)}{p(B)}$$

- **The diachronic interpretation**
  - This is another way to think about Bayes' theorem: it gives us a way to update the probability of a hypothesis, H, in light of some body of data, D.
  - Diachronic - means that something tis happening over time (page 5)

    Rewriting Bayes's theorem with H and D yields:

    $$p(H|D) = \frac{p(H)\ p(D|H)}{p(D)}$$

    In this interpretation, each term has a name:
  -
    - $p(H)$ is the probability of the hypothesis before we see the data, called the prior probability, or just **prior**.
    - $p(H|D)$ is what we want to compute, the probability of the hypothesis after we see the data, called the **posterior**.
    - $p(D|H)$ is the probability of the data under the hypothesis, called the **likelihood**.
    - $p(D)$ is the probability of the data under any hypothesis, called the **normalizing constant**.
  - Mutually exclusive - at most one hypothesis in the set can be true
  - Collectively exhaustive - there are no other possibilities; at least one of the hypotheses has to be true.

Discrete Distributions

- Discrete vs. Continuous Distributions

Special Law of Multiplication

If X, Y are independent, $P(X \cap Y) = P(X) \cdot P(Y)$

4.6 Conditional Probability

Conditional probabilities are computed based on the prior knowledge that a business researcher has on one of the two events being studied. If X, Y are two events, the conditional probability of X occurring given that Y is known or has occurred is expressed as $P(X|Y)$ and is given in the *law of conditional probability.*

Law of Conditional Probability

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y|X)}{P(Y)}$$

Independent Events

Independent Events X, Y

To test to determine if X and Y are independent events, the following must be true.

$$P(X|Y) = P(X) \quad and \quad P(Y|X) = P(Y)$$

In each equation, it does not matter that X or Y is given because X and Y are *independent*. When X and Y are independent, the conditional probability is solved as a marginal probability.

Sometimes, it is important to test a cross-tabulation table of raw data to determine whether events are independent. If *any* combination of two events from the different sides of the table fail the test, $P(X|Y) = P(X)$, the table does not contain independent events.

4.7 Revision of Probabilities: Bayes' Rule

An extension to the conditional law of probabilities is Bayes' rule, which was developed by and named for Thomas Bayes (1702–1761). **Bayes' rule** is *a formula that extends the use of the law of conditional probabilities to allow revision of original probabilities with new information.*

Bayes' Rule

$$P(X_i|Y) = \frac{P(X_i) \cdot P(Y|X_i)}{P(X_1) \cdot P(Y|X_1) + P(X_2) \cdot P(Y|X_2) + \cdots + P(X_n) \cdot P(Y|X_n)}$$

Recall that the law of conditional probability for

- Collectively exhaustive - there are no other possibilities; at least one of the hypotheses has to be true.

Discrete Distributions

- Discrete vs. Continuous Distributions
    - Random variable is a variable that contains the outcomes of a chance experiment
    - ***Discrete Random Variables***
        - If the set of all possible values is at most a finite or a countably infinite number of possible values.
            - In most statistical situations, DRV produces values that are nonnegative whole numbers
        - **Distributions**
            - Binominal Distribution
                - Assumptions:
                    - The experiment involves *n* identical trials
                    - Each trial has only two possible outcomes - success or failure
                    - Each trial is independent of the previous trials
                    - The terms *p* and *q* remain constant throughout the experiment
                        - P is the probability of getting a success on any one trial
                        - Q is the probability of getting a failure in any one trial; q = (1 - p)
            - **Binomial Formula**

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x} = \frac{n!}{x!(n-x)!} \cdot p^x \cdot q^{n-x}$$

where

n = the number of trials (or the number being sampled)
x = the number of successes desired
p = the probability of getting a success in one trial
$q = 1 - p$ = the probability of getting a failure in one trial

Mean and Standard Deviation of a Binomial Distribution

$$\mu = n \cdot p$$

$$\sigma = \sqrt{n \cdot p \cdot q}$$

            - Poisson Distribution
                - Characteristics
                    - It describes rare events
                    - Each occurrence is independent from the other occurrences
                    - It describes discrete occurrences over a continuum or interval
                    - The occurrences in each interval can rate from zero to infinity
                    - The expected number of occurrences must hold constant throughout the experiment

Poisson Formula

Recall that the law of conditional probability for

$$P(X_i|Y)$$

Formulas

Counting rule

$$mn$$

Sampling with replacement

$$N^n$$

Sampling without replacement

$${}_NC_n$$

Combination formula

$${}_NC_n = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

General law of addition

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

Special law of addition

$$P(X \cup Y) = P(X) + P(Y)$$

General law of multiplication

$$P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)$$

Special law of multiplication

$$P(X \cap Y) = P(X) \cdot P(Y)$$

Law of conditional probability

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X) \cdot P(Y|X)}{P(Y)}$$

Bayes' rule

$$P(X_i|Y) = \frac{P(X_i) \cdot P(Y|X_i)}{P(X_1) \cdot P(Y|X_1) + P(X_2) \cdot P(Y|X_2) + \cdots + P(X_n) \cdot P(Y|X_n)}$$

Mean or Expected Value of a Discrete Distribution

$$\mu = E(x) = \Sigma[x \cdot P(x)]$$

where

$E(x)$ = long-run average
x = an outcome
$P(x)$ = probability of that outcome

As an example, let's compute the mean or expected value of the distribution given in Table 5.2. See Table 5.3 for the resulting values. In the long run, the mean or expected number of crises on a given Friday for this executive is 1.15 crises. Of course, the executive will never have 1.15 crises.

Variance and Standard Deviation of a Discrete Distribution The variance and standard deviation of a discrete distribution are solved for by using the outcomes (x) and probabilities of outcomes [$P(x)$] in a manner similar to that of computing a mean. In addition, the computations for variance and standard deviations use the mean of the discrete distribution. The formula for computing the variance follows.

Variance of a Discrete Distribution

$$\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$$

where

- ◇ The expected number of occurrences must hold constant throughout the experiment

Poisson Formula

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

$x = 0, 1, 2, 3, \ldots$
λ = long-run average
$e = 2.718282$

- Hypergeometric Distribution
  - Hypergeometric are used as a complement analysis using the binominal distribution.
  - Hypergeometric distributions apply only to experiments in which the trials are down without replacement.
  - Characteristics
    - ◇ It is a discrete distribution
    - ◇ Each outcome consists of either a success or a failure
    - ◇ Sampling is done without replacement
    - ◇ The population, N, is finite and known
    - ◇ The number of successes in the population, A, is known.

Hypergeometric Formula

$$P(x) = \frac{{}_{A}C_{x} \cdot {}_{N-A}C_{n-x}}{{}_{N}C_{n}}$$

where

N = size of the population
n = sample size
A = number of successes in the population
x = number of successes in the sample; sampling is done *without* replacement

- ***Continuous Random Variables***
  - This takes on values at every point over a given interval.
    - CRV have no gaps or unassumed values (things are measured, not counted)
  - **<u>Distributions</u>** (discussed later in text)
    - Uniform Distribution
    - Normal Distribution
    - Exponential Distribution
    - *t* Distribution
    - Chi-squared Distribution
    - *F* Distribution

$$\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$$

where

x = an outcome
$P(x)$ = probability of a given outcome
μ = mean of the distribution

The standard deviation is then computed by taking the square root of the variance.

Standard Deviation of a Discrete Distribution

$$\sigma = \sqrt{\Sigma[(x - \mu)^2 \cdot P(x)]}$$

Formulas

Mean (expected) value of a discrete distribution

$$\mu = E(x) = \Sigma[x \cdot P(x)]$$

Variance of a discrete distribution

$$\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$$

Standard deviation of a discrete distribution

$$\sigma = \sqrt{\Sigma[(x - \mu)^2 \cdot P(x)]}$$

Binomial formula

$${}_{n}C_{x} \cdot p^x \cdot q^{n-x} = \frac{n!}{x!(n-x)!} \cdot p^x \cdot q^{n-x}$$

Mean of a binomial distribution

$$\mu = n \cdot p$$

Standard deviation of a binomial distribution

$$\sigma = \sqrt{n \cdot p \cdot q}$$

Poisson formula

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Hypergeometric formula

$$P(x) = \frac{{}_{A}C_{x} \cdot {}_{N-A}C_{n-x}}{{}_{N}C_{n}}$$