



Programming for Simulation and MC Methods

Simulation

What is Stochastic Simulation ?



Stochastic Simulation

Most stochastic simulations have the same basic structure:

1. Identify a random variable of interest X and write a program to simulate it. Step 1 is *model building*
2. Generate an iid sample X_1, \dots, X_n with the same distribution as X .
3. Estimate $\mathbb{E}X$ (using \bar{X}) and assess the accuracy of the estimate (using a confidence interval).

Inversion Method: Uniform



Inversion method for $U(1, 3)$

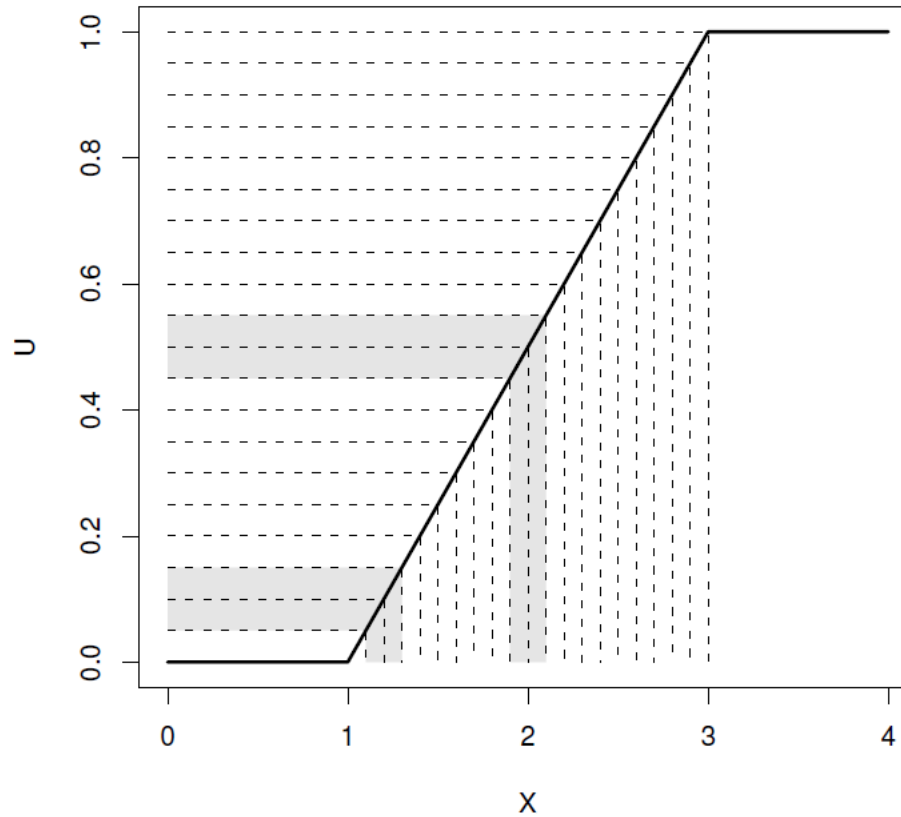


Figure 18.3 *Illustration of the inversion method. A ‘uniform rain’ of points on the vertical interval $(0, 1)$ becomes a uniform rain on the horizontal interval $(1, 3)$.*

Inversion Method: Exponential

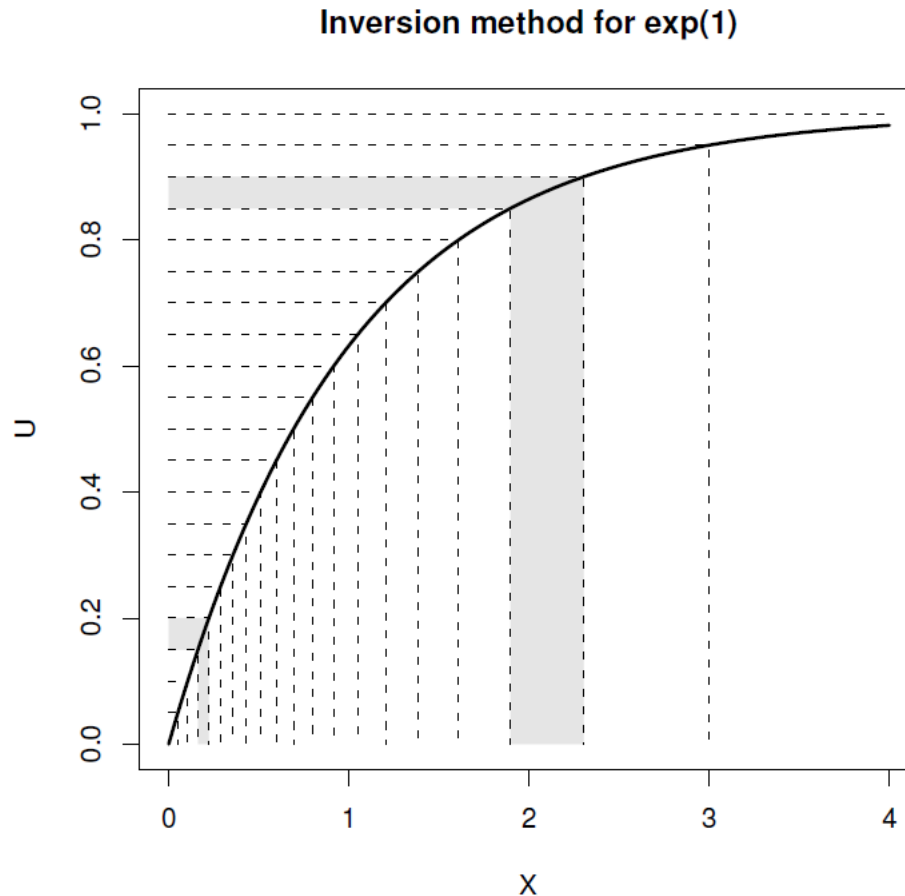


Figure 18.4 *Illustration of the inversion method. A 'uniform rain' of points on the vertical interval $(0,1)$ becomes an 'exponentially distributed rain' on the horizontal*

Rejection method (uniform envelope) Suppose that f_X is non-zero only on $[a, b]$, and $f_X \leq k$.

1. Generate $X \sim U(a, b)$ and $Y \sim U(0, k)$ independent of X (so $P = (X, Y)$ is uniformly distributed over the rectangle $[a, b] \times [0, k]$).
2. If $Y < f_X(X)$ then return X , otherwise go back to step 1.

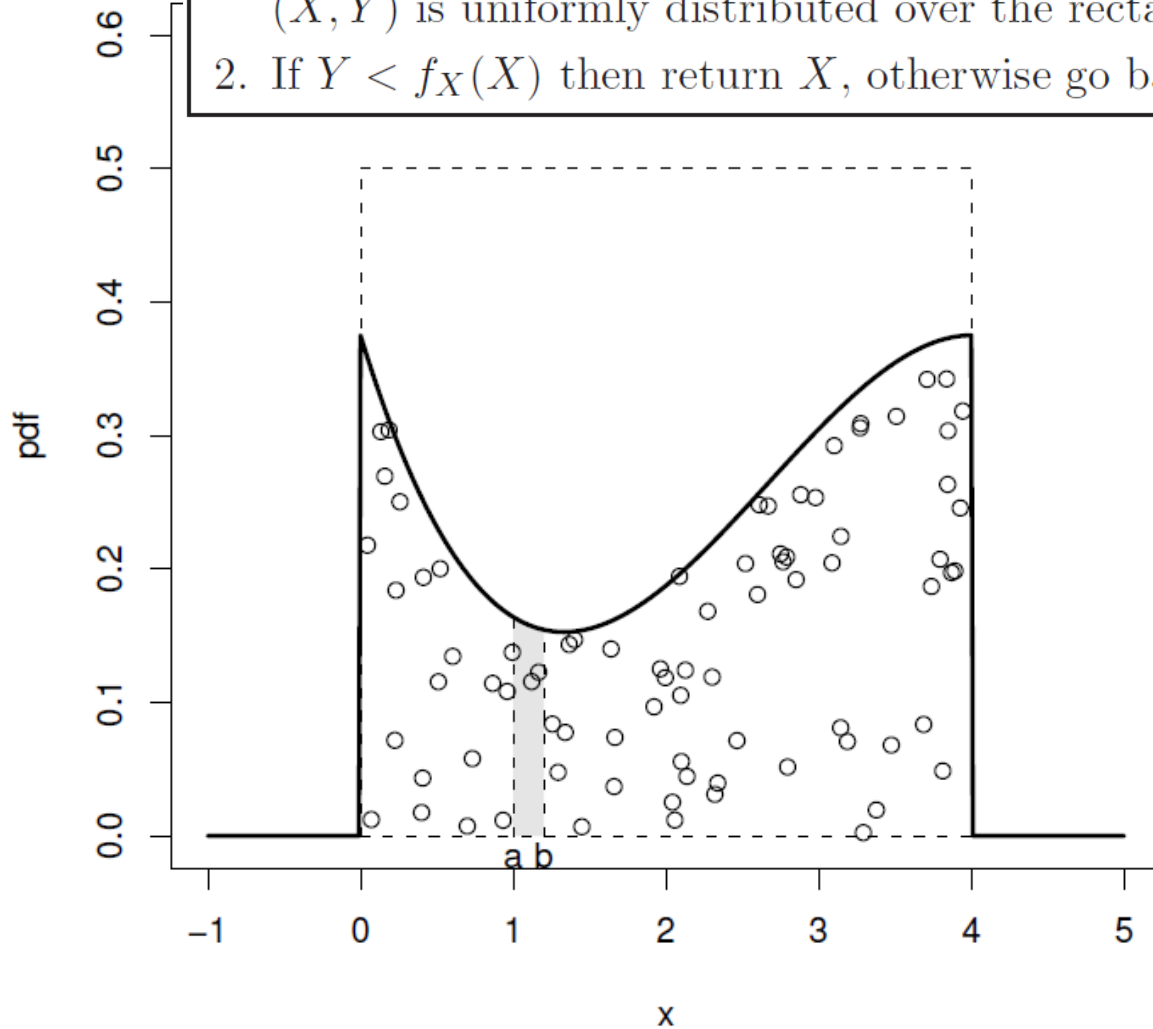


Figure 18.5 *Points uniformly distributed under a pdf.*

Empirical pdf of Triangular Distribution: Rejection Method

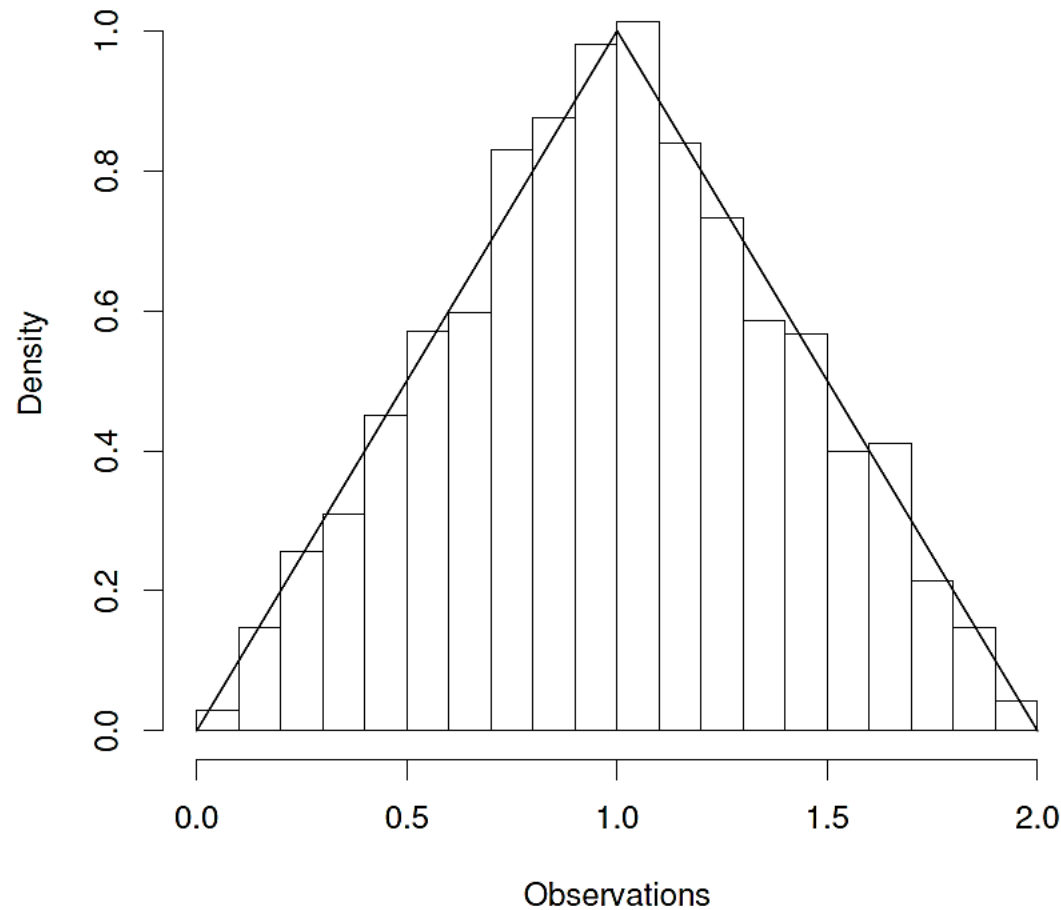


Figure 18.6 *Empirical pdf of the triangular distribution, simulated using the rejection method.*