R Programming for Simulation and MC Methods

Simulation

What is Stochastic Simulation ?

Stochastic Simulation

Most stochastic simulations have the same basic structure:

1. Identify a random variable of interest X and write a program to simulate it.

   Step 1 is *model building*

2. Generate an iid sample $X_1, \ldots, X_n$ with the same distribution as X.
3. Estimate $\mathbb{E}X$ (using $\overline{X}$) and assess the accuracy of the estimate (using a confidence interval).

Inversion Method: Uniform

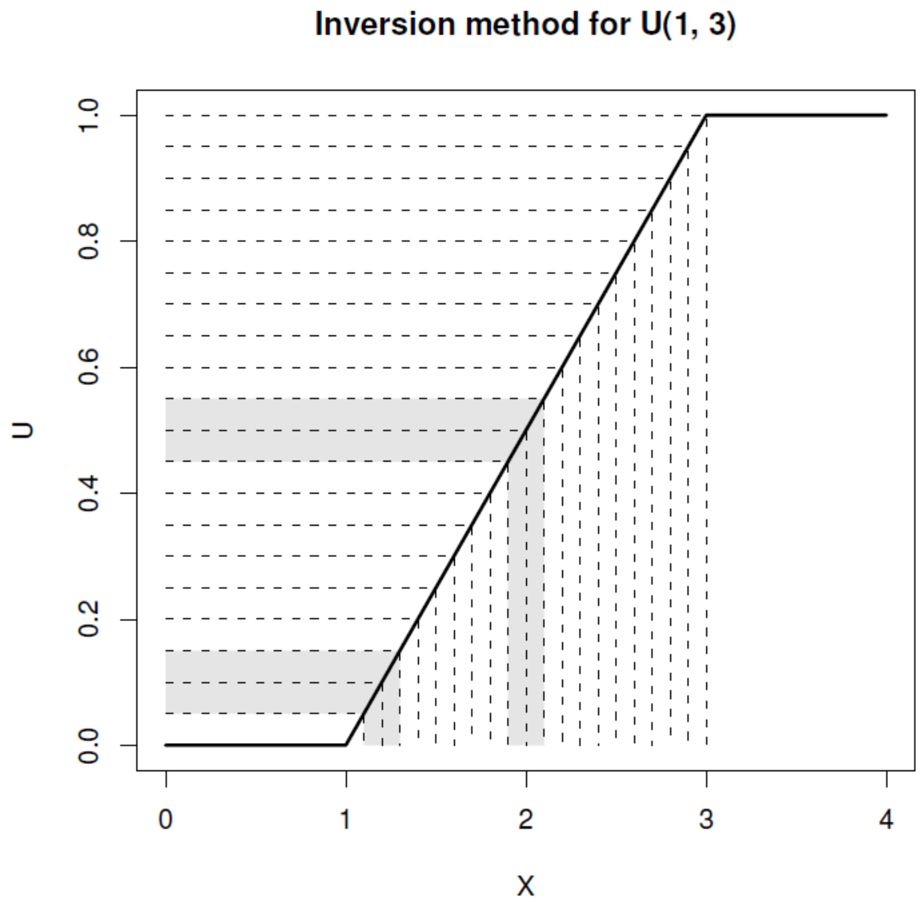

Figure 18.3 *Illustration of the inversion method. A 'uniform rain' of points on the vertical interval* $(0, 1)$ *becomes a uniform rain on the horizontal interval* $(1, 3)$.

Inversion Method: Exponential

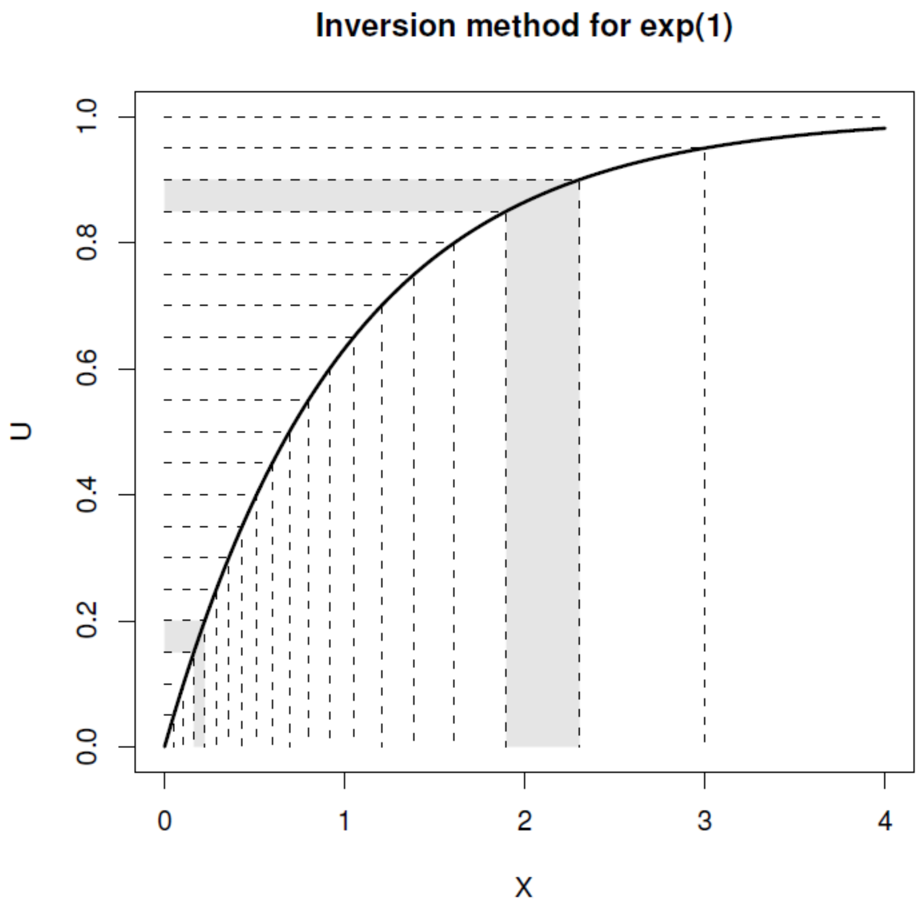

Figure 18.4 *Illustration of the inversion method. A 'uniform rain' of points on the vertical interval* $(0, 1)$ *becomes an 'exponentially distributed rain' on the horizontal*

Rejection method (uniform envelope) Suppose that f_X is non-zero only on $[a, b]$, and $f_X \leq k$.

1. Generate $X \sim U(a, b)$ and $Y \sim U(0, k)$ independent of X (so $P = (X, Y)$ is uniformly distributed over the rectangle $[a, b] \times [0, k]$).
2. If $Y < f_X(X)$ then return X, otherwise go back to step 1.

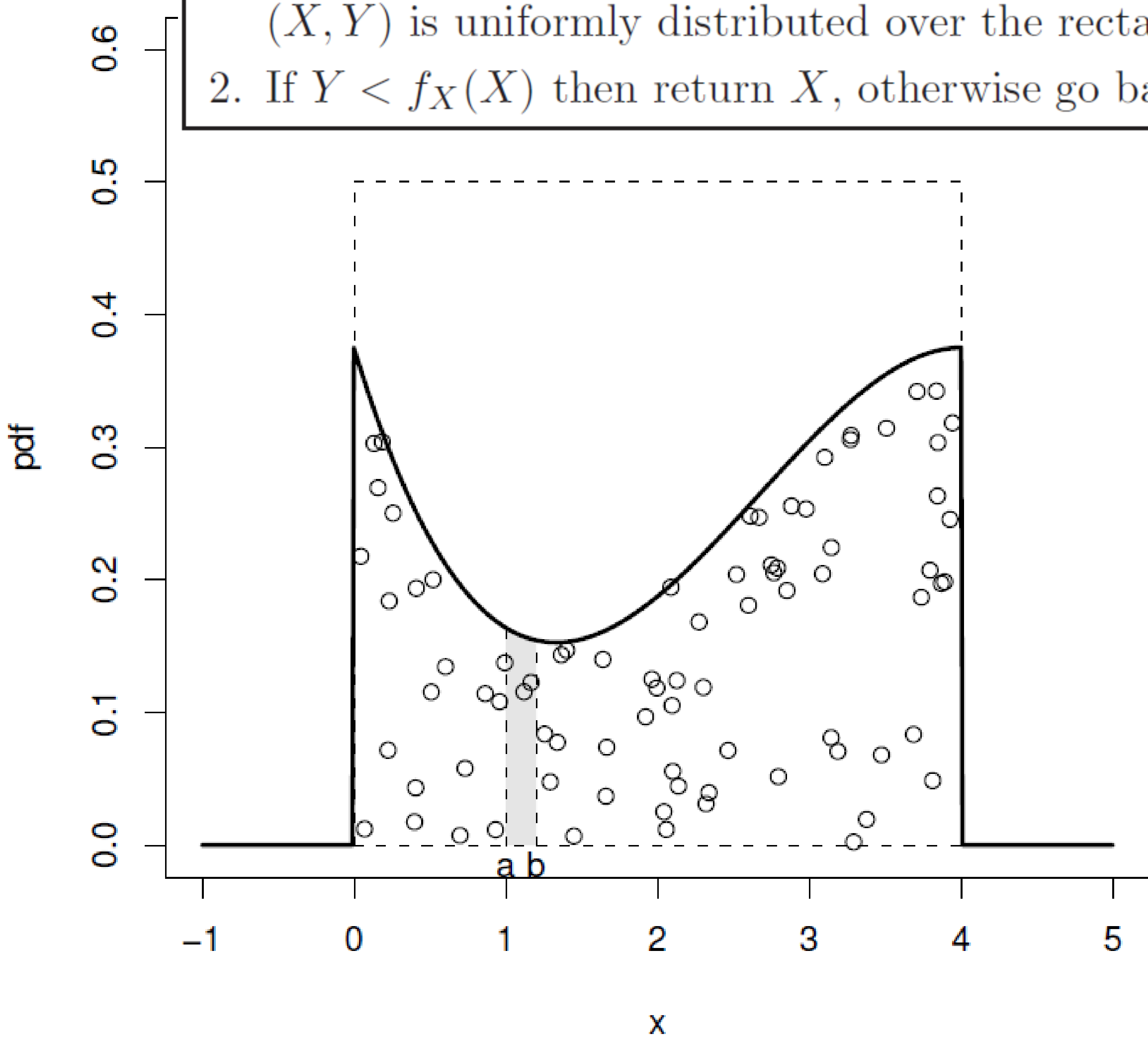

Figure 18.5 *Points uniformly distributed under a pdf.*

General Rejection Method

General rejection method

To simulate from the density f_X, we assume that we have envelope density h from which you can simulate, and that we have some $k < \infty$ such that $\sup_x f_X(x)/h(x) \leq k$.

1. Simulate X from h.
2. Generate $Y \sim U(0, kh(X))$.
3. If $Y < f_X(X)$ then return X, otherwise go back to step 1.

Empirical pdf of Triangular Distribution: Rejection Method

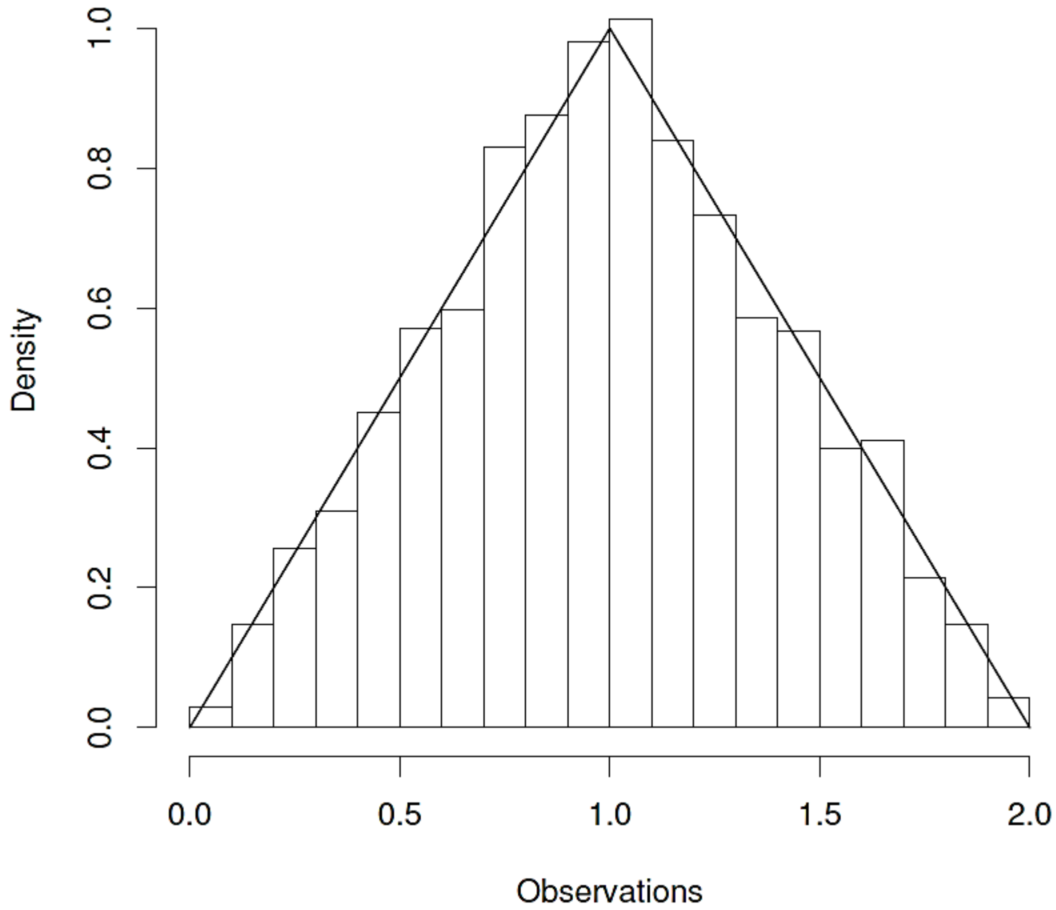

Figure 18.6 *Empirical pdf of the triangular distribution, simulated using the rejection method.*