NUMERICAL INTEGRATION EXERCISE WITH SOLUTION

1. The standard normal distribution function is given by:

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx.$$

For probability p within the set of [0,1], the 100p standard normal percentage point is defined as that z_p for which:

$$\Phi(z_p) = p.$$

Using the functions **`phi(x)`** and **`Phi(z)`** in chapter 11 (repeated below), calculate z_p for p = 0.05, 0.95, 0.975, and 0.99. **Hint:** Express the problem as a root-finding problem. That is, use one of the root-finding methods (programs) from chapter 10 and utilize **`phi(x)`** and **`Phi(z)`** as elements in the arguments to that root-finding program from chapter 10.

```
# HERE IS phi(x):
phi <- function(x) return(exp(-x^2/2)/sqrt(2*pi))

# HERE IS Phi(z):
Phi <- function(z) {
  if (z < 0) {
    return(0.5 - simpson_n(phi, z, 0))
  } else {
    return(0.5 + simpson_n(phi, 0, z))
  }
}
```

```
################################################################
# HERE IS SOLUTION:

rm(list=ls())

source("simpson_n.r")
source("newtonraphson.r")

phi <- function(x) return(exp(-x^2/2)/sqrt(2*pi))

Phi <- function(z) {
  if (z < 0) {
    return(0.5 - simpson_n(phi, z, 0))
  } else {
    return(0.5 + simpson_n(phi, 0, z))
  }
}

newtonraphson(function(z) c(Phi(z) - 0.5, phi(z)), 0)
newtonraphson(function(z) c(Phi(z) - 0.95, phi(z)), 0)
newtonraphson(function(z) c(Phi(z) - 0.975, phi(z)), 0)
newtonraphson(function(z) c(Phi(z) - 0.99, phi(z)), 0)
```