ST 516: Foundations of Data Analytics

Statistical and Practical Significance

Statistical Significance

P-values and confidence intervals indicate **statistical significance**, or the extent to which a hypothesized value for a parameter is contradicted by the data.

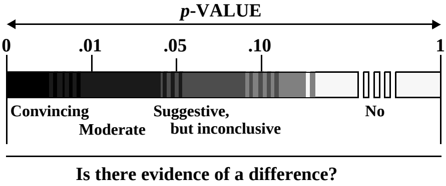
- If a p-value from a hypothesis test is less than some level, α, that is pre-specified for the test, we say that the test is significant at the level α.
- Similarly, if a $(1-\alpha)100\%$ confidence interval does not contain a hypothesized value for a parameter, then we know that the corresponding hypothesis test is significant at the level α.

It's more informative to think of p-values and confidence intervals in terms of the *strength of evidence* they provide for a particular hypothesis.

P-values as Strength of Evidence

Display 2.12 **p. 47**

Interpreting the size of a *p*-value



The Statistical Sleuth 1st Ed., Display 2.12 (Same in 3rd ed?)

Practical Significance

When a result is deemed to be statistically significant, it is very important to then consider its **practical significance**.

For example, in the study in which poems were evaluated for creativity after the writers took either an intrinsic or extrinsic motivational questionnaire, suppose that a statistically significant difference in mean scores between the two groups was only 0.7 point on a 40-point scale.

- Is this really a noteworthy difference?
- How big would the difference have to be for you to think of it as noteworthy?

Practical Significance

The practical (or scientific) significance of a result has little to do with Statistics—it's simply a matter of whether the result is interesting or important enough in the context of whatever study is being considered.

Some important things to remember:

1. A result can be statistically significant but not practically significant
2. A result can be practically significant without being statistically significant (this is sometimes called a negative result)

Significance and Sample Size

There is a connection between sample size and statistical significance:

> Regardless of practical significance, we can often achieve statistical significance by increasing the sample size.

- This means that whenever you learn about the statistical significance of a result, you should also learn something about the amount of data that was used to obtain the result.
- This becomes an especially important issue in the era of "Big Data."

Significance and Sample size

We'll look at the results of a simulation to confirm this key idea:

1. Draw a simple random sample of size n from a Normal population with mean $\mu = 10$ and variance $\sigma^2 = 4$.
2. Compute the z-statistic to test the null hypothesis that $\mu = 10.01$, report the p-value.
3. repeat for increasing sample sizes.

Let's see what happens.

Significance and Sample Size

Consider the results:

Sample Size	p-value
10	0.3804
100	0.6130
1000	0.6265
10,000	0.2921
100,000	0.0936
1,000,000	<0.0001

What's going on here?!

Significance and Sample Size

Remember that the variance of the sampling distribution of the sample mean (or simply, the variance of the sample mean) is:

$$\text{Var}(\overline{X}) = \sigma^2 / n$$

Think about what happens to this variance as the sample size, n, gets bigger and bigger—the variance gets smaller and smaller!

This means that as the sample size gets bigger, our estimate, $\overline{X}$ will get closer and closer to the true mean, μ.

In the simulation results, this means that the p-value of a test based on the null hypothesis that the mean is 10.01 (pretty close, but not exactly equal to the true mean, 10), gets very small as the sample size gets large.