Submit one .pdf file that includes the answers to both sets of questions. Also submit your .R script that you used to answer the questions in the R part.

Please feel free to discuss questions on the discussion board.

R Question

1. (6 points) Why is it important to correctly distinguish between the 2-sample t-test setup, and the paired t-test setup? Follow the steps to explore the difference between these two tests.

   (a) Marquis de Laplace proved the Central Limit Theorem in 1810, so begin with `set.seed(1810)`. Run the following code to generate three samples A, B and C.

   ```
   set.seed(1810)
   A <- rnorm(10)
   B <- rnorm(10)
   C <- 0.5 + (0.8 * A) + (sqrt(1 - 0.8^2) * B)
   B <- 0.5 + B
   ```

   (b) Conduct a **two sample t-test** of $H_0 : \mu_A - \mu_B = 0$ vs. $H_A : \mu_A - \mu_B \neq 0$, assuming unequal group variances. Then calculate the t-statistic and p-value manually, using the formula for the test statistic and `pt()`. It should match the output of `t.test()`. Hint: you can use the degrees of freedom reported by `t.test()`.

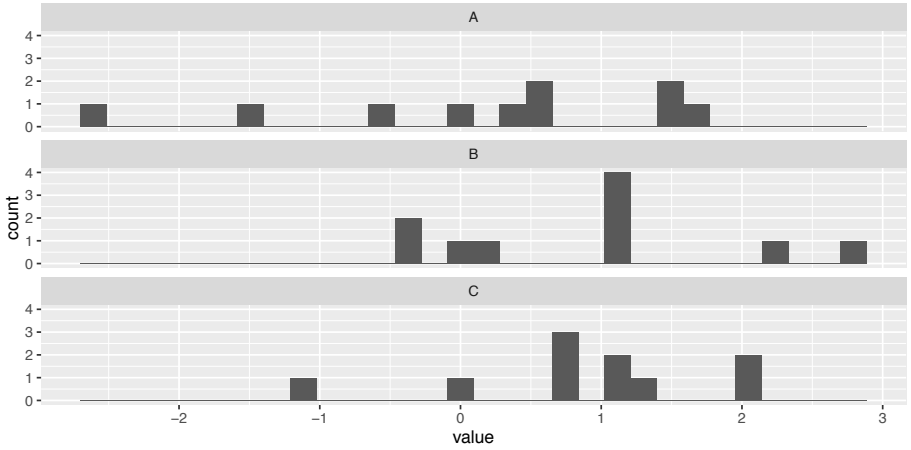
   (c) Now conduct a **paired t-test** of $H_0 : \mu_A - \mu_B = 0$ vs. $H_A : \mu_A - \mu_B \neq 0$, using `t.test()`. Then calculate the t-statistic and p-value manually by constructing the test statistic using the sample statistics of the differences (i.e. using `diffs` where `diffs <- A - B`). It should match the output of `t.test()`.

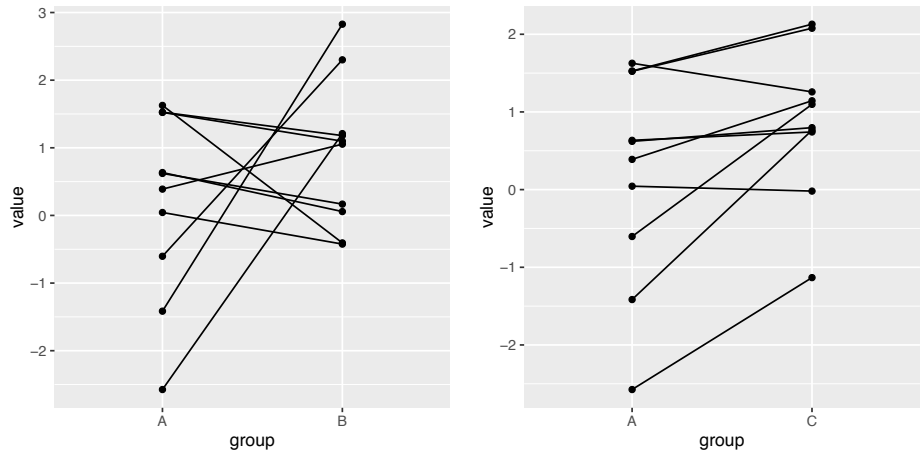
   (d) Compare the test statistics, confidence intervals and p-values from part (b) and part (c). Do the two procedures reach the same conclusion?

   (e) Repeat parts (b), (c) and (d), now comparing samples A and C (just use `t.test()`, you don't need to verify the results manually).

   (f) You should find that the two procedures reach roughly the same conclusion when comparing samples A & B, but different conclusions when comparing samples A & C, despite the true differences in mean in both cases being 0.5. Explain why.

   You may find it helpful to either examine how the samples were generated and/or examine the following plots of the the three samples.





You can recreate these plots with the following code:

```
library(ggplot2)
library(dplyr) # you might need to install this one

# Put our samples in a data frame
q1_data <- data.frame(
  obs = rep(1:10, 3),
  value = c(A, B, C),
  group = rep(c("A", "B", "C"), each = 10)
)

# Histograms for each sample
qplot(value, data = q1_data) + facet_wrap(~ group, ncol = 1)

# Relationship between pairs of observations, A & B
qplot(group, value, data = filter(q1_data, group != "C"),
  group = obs, geom = c("point", "line"))

# Relationship between pairs of observations, A & C
qplot(group, value, data = filter(q1_data, group != "B"),
  group = obs, geom = c("point", "line"))
```

Conceptual Questions

Answer **two** of the following questions.

2. (2 points) A language transcriptionist translates a random sample of seven speeches from Spanish to English and then from Spanish to French. The times it takes to transcribe them (in minutes) are recorded in the table below:

Speech Number	Spanish to English	Spanish to French
1	15	16
2	19	18
3	45	60
4	35	54
5	67	70
6	13	11
7	33	34

The transciptionist would like to know if there is a difference in times it takes her to transcribe from Spanish to English and from Spanish to French for the same speech.

   (a) Are these data paired? Why or why not? If so, what are the sources of variation?

   (b) Perform a hypothesis at the 5% level using R. Report the t-statistic, degrees of freedom, p-value, and a confidence interval.

   (c) State a conclusion in the context of the problem.

   (d) Might there be practical significance here? Why or why not?

3. (2 points) A chemist wishes to compare the amount of residue left behind for chemical reaction A and chemical reaction B, given a certain environment for each reaction. If they are different, then she will adjust the starting amounts of each chemical involved until the amounts of residues are the same for each of the two reactions. Assume each run of each reaction is independent, and that the she tries her best to make the environment of each run the same within each chemical reaction. She runs each reaction 8 times, and records the residues (in micrograms) in the following table:

Reaction A	Reaction B
456	343
222	242
567	990
344	222
222	344
334	455
543	600
447	323

   (a) Are these data paired? Why or why not? If so, what are the sources of variation?

   (b) Perform a hypothesis test at the 5% level using R. Report the t-statistic, the degrees of freedom, the p-value, and a confidence interval.

   (c) State a conclusion in the context of the problem.

4. (2 points) A dentist would like to know if the proportion of visits due to cavities are different between young adult and older adult customers. In a random sample of 33 young adults, 24 of them visit due to cavities. In a random sample of 55 older adults, 35 of them visit due to cavities.

(a) Perform a hypothesis test, using a p-value.

(b) Perform the hypothesis test from Part A, but this time using a confidence interval. Does this agree with your answer in Part A?

(c) Will your answers to Part A and Part B always be the same, given any samples? Explain.

5. (2 points) A random sample of cars of model A and vans of model B are selected. Each is driven on flat ground over the same section of highway for one week and the gas mileage (in miles per gallon) is calculated and recorded. The results are as follows:

Model A			**Model B**		
23.35	23.97	28.76	28.48	25.63	31.91
29.62	25.21	28.70	29.47	26.97	26.88
33.24	26.66	26.72	28.81	27.84	30.03
28.23	27.66	27.12	30.03	29.46	29.40
25.73	25.17	25.80	27.74	31.45	31.24
24.09	26.60	27.85	29.08	26.35	30.56
25.07	27.55	31.78	29.57	28.32	27.78
22.84	21.11	28.68	29.33	29.04	28.34
26.52			27.53	28.40	28.83
			27.38		

(a) Use R to test the hypothesis that the proportion of cars of model A that get at least 28 miles per gallon different than that of vans from of B. Assume that we may use the normal approximation. State a conclusion in the context of the problem.

(b) Give a plausible range of values for the actual difference in proportions.

(c) Why is it important that our sample sizes are "large"?

6. (2 points) Two teams, the Tigers and the Bears, compete in a rocket-launching competition. Each person builds one rocket, and all rockets are launched from the same location on a day without wind. Assume the rockets and launches are all independent. A laser measures the vertical distances (in feet) reached by the rockets. The coach of the Tigers would like to know if the 50th percentile of people that join his team is different than the 50th percentile of people that join the Bears team. The data is as follows:

Tigers			**Bears**		
11	601	550	179	86	51
16	100	293	86	126	82
67	60	474	15	82	136
218	74	251	55	171	83
492	38	119	17	50	142
127	106	23	57	9	112
269			32	85	107
			123	15	0
			67	167	

Perform a Mood's test to answer the coach's question.