ST 516: Foundations of Data Analytics

Levene's Test

Levene's Test: Motivation

So far, all of the methods we have covered are for comparing populations based on their centers—for us that's either been a difference in their means or in the their medians.

Sometimes, we want to compare the *spreads* of two populations.

- If we're really interested in whether two populations are the same or different, then we should consider not only their centers but also their spreads.

- Some statistical methods—to be covered in a subsequent course—rely on an assumption that the populations being compared or evaluated have the same variances.

Levene's test provides a method for evaluating the equality of spreads among populations.

Levene's Test: Details

Suppose we have two populations, denoted 1 and 2.

- The null and alternative hypotheses for the two-sample Levene's test are:

  H_0:   spread of population 1 = spread of population 2

  H_a: spread of population 1 $\neq$ spread of population 2

- If we have a random sample from each population, $Y_{11}, Y_{12}, \ldots, Y_{1n_1}$ from population 1 and $Y_{21}, Y_{22}, \ldots, Y_{2n_2}$ from population 2, we can perform the test which has a few alternative forms.

Levene's Test: Details

We first define deviations, Z_{ij}, for each sample in one of the following three ways:

1. $Z_{ij} = |Y_{ij} - \overline{Y}_i|$ and $\overline{Y}_i$ is the sample mean of group $i = 1, 2$ OR

2. $Z_{ij} = |Y_{ij} - \widetilde{Y}_i|$ and $\widetilde{Y}_i$ is the sample median of group $i = 1, 2$ OR

3. $Z_{ij} = |Y_{ij} - Y_i^*|$ and Y_i^* is the 10% trimmed mean of group $i = 1, 2$.

A 10% trimmed mean of a sample is the arithmetic mean of the middle 90% of the sorted sample (i.e., the sorted sample with the upper and lower 5% of the observations removed).

Levene's Test: Details

We generally prefer to use version 2 of the test statistic (the one based on medians), as it is fairly robust to non-Normal distributions and it has fairly good power (see Engineering Statistics Handbook).

After calculating the absolute deviations according to one method from the previous slide, we then perform a two-sample t-test on them.

- The idea is that if the two population spreads are the same, then the population means of the deviations will also be the same.

- That way, performing a two-sample t-test on the equality of the mean absolute deviations will be like performing a test on the equivalence of the population spreads.

Example

These data are from Exercise 11 in Chapter 2 of *The Statistical Sleuth*, 3rd ed. They are lifetimes (in days) of guinea pigs in two treatment groups.
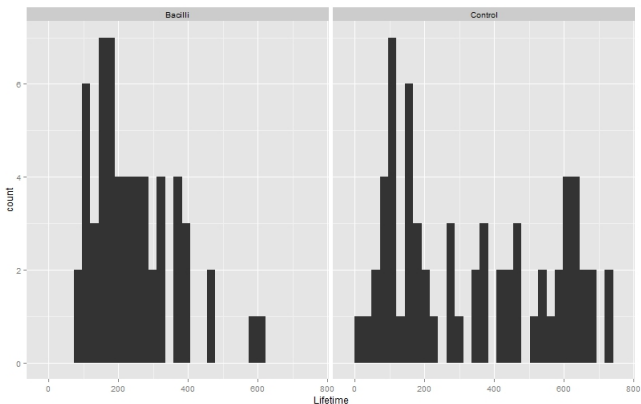
Example

In R, there is a function called `leveneTest()` in the *car* package:

```
> leveneTest(ex0211$Lifetime,ex0211$Group)
Levene's Test for Homogeneity of Variance (center = median)
       Df F value   Pr(>F)
group   1  42.788 1.569e-09 ***
      120
```

There is very strong evidence from this test that the two population spreads are not the same.

- Notice that the p-value in the output above is based on an F value. Levene's test can be generalized to compare spreads among more than two samples, in which case an F-distribution is used as the sampling distribution for the appropriate statistic. In the two sample case, the F-distribution and the t-distribution are equivalent for the purposes of this test.