

DATA STRUCTURE & ALGORITHMS (DSA)

03-63000-Saurabh

DATE

27

AP

class Employee

① Linear Data Structure → (sequential)

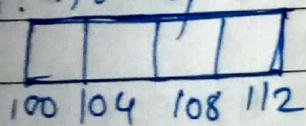
- 1) Array (contiguous)
- 2) Structure & Union
- 3) Linked list
- 4) Stack
- 5) Queue
- 6) class (oops)

② Advance DS → Data elements stored arranged into Non-linear / Hierarchical manner ∴ can be accessed Non-linearly

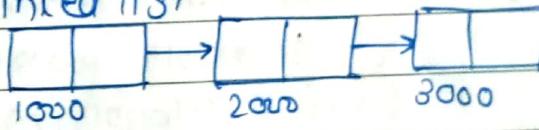
- 1) Binary Heap
- 2) Tree
- 3) Graph
- 4) HASH TABLE (Associative Manner → (key-value) pair)

③ Contiguous vs Non-Contiguous

e.g. Array



e.g. linked list



④ Search → to search / to find an element from collection / list

- ↳ 1) linear search
- 2) binary search

⑤ Algorithm → 1) Iterative (Non-Recursive) Approach

- 2) Non-Iterative

① Asymptotic Notations → Asymptotic Lower Bound
② Big Omega (Ω) : Best case Time complexity
↳ If Algo takes min. Amt. of Time to
Run to completion

③ Big Oh (O) : Worst case → Asymptotic Upper Bound
(Run Time of Algo can not
max. Amt. of Time ↓
to Run to completion) → be more than its Asymptotic
Upper Bound)

④ Big Theta (Θ) : Average case : Asymp. Tight Bound

⑤ Linear Search : Best case (Ω) → $\Omega(1)$
Worst case (O) → $O(n)$
Average Case (Θ) → $\Theta(n)$

* Iterative Approach for Linear Search (Non-Recursive)

⑥ public class searching {

```
    public static void display (int [] arr) {
        println ("Array Elements : ");
        for (int i = 0 ; i < arr.length ; i++) {
            printf (" %.4d ", arr[i]);
        }
    }
```

4

println();

3

main () {

```
    int arr [ ] = { 40, 80, 90, 50, 10, 20, 30, 60, 70, 100 };
```

display (arr);

println ("Enter key");

int key = sc.nextInt();

Mated code

```

if ( linearSearch ( Arr, key ) )
    println ( key + " Found " + comparisons );
else
    println ( key + " NOT Found " );
}
}

public static int comparisons = 0;

```

~~to check~~

```

public static boolean linearSearch ( int [ ] Arr, int key )
{
    for ( int i = 0 ; i < Arr.length ; i ++ )
        comparisons++;
    if ( key == Arr [ i ] ) // if key matches
        return true;
    return false; // if key not found
}

```

① RECURSIVE LINEAR SEARCH

```

public static boolean recursiveSearch
    ( int [ ] Arr, int key,
      int index )

```

Base Cond: // {

```

if ( index == Arr.length )
    // if key do not matches
    return false;
if ( key == Arr [ i ] )
    return true;
comparisons++;
return recursiveSearch ( Arr, key, index + 1 );
// Recursive funcn call
}

```

main ()

```
int () Arr = { 40, 80, 90, 50, 10, 20, --- }  
int key = sc.nextInt();
```

```
if ( recursiveSearch ( Arr, key, 0 ) // Initialization  
    println ( key + " Found " + comparisons );  
else
```

```
    println ( key + " Not Found " + comparisons );
```

① Recursive algorithm takes extra space.

② If no. is Repeated → only first occurrence considered as match.

(*) BINARY SEARCH ALGORITHM

↳ Also called logarithmic search

↳ Half Interval Search

① ↳ Divide & Conquer Approach

② ↳ Array Must be sorted (Default → Ascending)

arr → 10 20 30 40 50 60 70 80 90 100
index → 0 1 2 3 4 5 6 7 8 9
left (mid) right

↳ Calculate Mid position = $\frac{\text{left} + \text{right}}{2}$

$$\text{mid} = (0+9)/2 = 4$$

↳ Implicit Division into

logical two subarray

→ 10 20 30 40

→ 60 70 80 90 100

For Left sub array → 10 20 30 40
 ⁰ ¹ ² ³
 left right

Left remains same
Right will be (mid-1)

For Right subArray \rightarrow 60 70 80 90 100
 5 6 7 8 9
 (mid+1) Right

If key = 50 \rightarrow key found at 1st iteration
 (mid)

Best case \rightarrow if key found in very 1st iteration
 in only 1 comparison.
 For Any input size, no. of comparisons = 1

10 20 30 40 50 60 70 80 90 100
 Left mid Right

left subArray : 10 20 30 40
 Left (mid-1)

Right subArray : 60 70 80 90 100
 (mid+1) Right

① If key = 50 \rightarrow 1st iteration key found
 BCAZ (mid = 50 = key)
 \therefore Best case : $O(1)$

② If key = 100
 10 20 30 40 50 60 70 80 90 100
 (mid-1) mid (mid+1)

If key = mid \rightarrow NO

key < mid \rightarrow left sub Array

key > mid \rightarrow Right sub Array

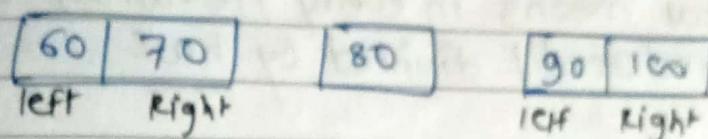
100 > 50 \rightarrow \therefore search key in Right sub Array

	left	mid	right	
high sub array :	60	70	80	90 100
	(1) (2)	(3) (4)	(5)	

↳ find new mid = $\frac{(5+9)}{2} = \frac{14}{2} = 7$

new left sub array : 60 70

new right sub array : 90 100



if key == mid (is 100 == 80) → No

then search key in left sub array / right sub array

100 > 80 → search in right sub array

[90 100]	mid = $\frac{(8+9)}{2} = \frac{17}{2} = 8$
(6) (9)	

(mid) No left sub array

Right sub array = 100

Right sub array

100
Right
left

is 100 == 90 → No
key mid

[100]	→ mid = $\frac{(9+9)}{2} = 9$
Left	
right	

if mid == key (100 == 100) → True.

① Worst case : if key NOT found or key found at leaf position e.g. 100, 125, 95

② Avg. case : if key found at Non-leaf condition.
e.g. 80, 70, 90

③ Best case : if key found at Root position
e.g. 50

① In this Algo, in every iteration 1 comparison happens
 & Array gets divided into SubArray's
 & next iteration will search key either
 into left subarray or right subarray.

② Array divided logically into 2 sub arrays.
 that means in every iteration search space is
 reduced / divided by half.

③ Eg. Size of Array = 1000

iteration #: input size of Array = 1000 = n

After 1st iteration : $(n/2 + 1)$ comparison

$$1000 = n$$

$$500 = n/2 \rightarrow \text{After 1st iteration} : (n/2 + 1)$$

$$250 = n/4 \rightarrow \text{After 2nd iteration} : (n/4 + 2) \text{ comparison}$$

$$125 = n/8 \quad (n/8 + 3)$$

$$62 = n/16 \quad (n/16 + 4)$$

$$(n/2^m) + m$$

$$\boxed{T(n) = (n/2^k) + k} \rightarrow \text{eq } ①$$

if $n = 2^k$

$$\log n = \log_2 k$$

$$\log n = k \log 2$$

$$\underline{\log n = k} \quad \therefore k = \log n$$

$$T(n) = n/2^k + k$$

$$T(n) = \cancel{n} \cdot 2^k/2^k + \log n$$

$$\boxed{T(n) = 1 + \log n}$$

By Assumption 2
 Additive constant
 can be neglected

$$\text{worst case} \rightarrow \boxed{T(n) = \log n}$$

	Best case	Worst case	Avg. case
Linear Search	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Binary search	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$

Binary Search Program

```

main()
{
    int arr =
        if (BinarySearch (arr, key))
            cout << "Found";
}

```

```

public static boolean BinarySearch (int arr, int key)
{
    int left = 0;
    int right = arr.length - 1;
    Step → ② // calculate mid position while (left <= right) {
    int mid = (left + right) / 2;
}

```

Step → ③ // compare value of key with ele which at mid.
 Comparisons ++;
 if (key == arr[mid])
 return true.

// if key do not match with mid pos. element
 we will search key either left/ right subArray

```

if (key < arr[mid])
    right = mid - 1; // go to search key
    into left subArray
else
    left = mid + 1; // go into right subArray.
}

```

Step ④ → // Repeat step 2 & step 3 till either
key is found or max HLL Subarray
is

```
return false;
```

3

- To decide efficiency of Algorithm
 - ↳ Compare their Avg. case
 - ↳ $\Theta(n) > \Theta(\log n)$

∴ Binary Search → Efficient → Linear
 Avg. case than searches
 Time complexity $O(n)$
 $O(\log n)$

- ## ① Recursive Binary Search Function

— main —

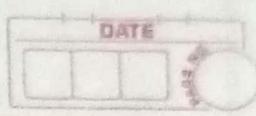
```
    if (recBinarySearch(arr, key, 0, arr.length-1))  
        println("Found");
```

```
}  
public static boolean recBinarySearch (int [] arr,  
int key, int left, int right)
```

if ($\text{left} > \text{right}$)

return False;

```
int mid = (left + right) / 2;    comparison++;  
if (key == arr[mid])  
    return true;
```



```

if (key < arr[mid])
    return (recBinarySearch (arr, key, left, mid-1));
}

```

else

```

return (recBinarySearch (arr, key, mid+1, right));
}

```

SELECTION SORT

↳ element at selected position will be compared to next all elements, and \Leftrightarrow swapped position.

Iterations : ① 30 20 60 50 10 40
 ↓
 sel pos post

$n = 6$

smallest

element

will be

swapped

to its

fix position

i.e. 1st

position

② 20 30 60 50 10 40
 sel pos post

$20 < 60$

③ 20 30 60 50 10 40
 sel pos post

$20 < 50$

④ 20 30 60 50 10 40
 sel pos post

$20 > 10$
 (swap)

pos till length →
 ↓
 ⑤ 10 30 60 50 20 40 10 < 40
 sel pos post

$(n-1) \rightarrow$ comparisons

Iterations : ① 10 30 60 50 20 40 30 < 60
 sel pos pos

② 10 30 60 50 20 40 30 < 50
 sel pos pos

③ 10 30 60 50 20 40 30 > 20
 sel pos pos
 (swap)

④ 10 20 60 50 30 40 20 < 40
 sel pos pos

$(n-2) \rightarrow$ comparisons

			sel	pos	pos		
<u>Iteration 3</u>	: ①	10	20	60	50	30	40
	↓						
3rd Smallest element	: ②	10	20	50	60	30	40
				sel	pos	pos	
get fix to 3rd position	: ③	10	20	30	60	50	40
				sel	pos	pos	

<u>Iteration 4</u>	①	10	20	30	60 → 50	40	60 > 50
↓					selpos	pos	(swap)
4th smallest element got fix	②	10	20	30	50	60	40
					selpos	pos	50 > 40
		..	10	20	30	40	60 50

Iterations: ① 10 20 30 40 60 → 50 ←
sel pos pos (Swap)

~~✓ 10 20 30 40 50 60~~

(Array Got Sorted) *

$$\therefore \text{No. of Iterations} = \boxed{\frac{\text{Array Length}}{2}} \quad [\text{At } \underline{(n-1)} \text{ Iterations}]$$

\therefore Total No. of Comparisons = $(n-1) + (n-2) + (n-3) + \dots + 1$

$$\rightarrow = \boxed{\frac{n * (n-1)}{2}}$$

$$\begin{aligned}
 T(n) &= O(n(n-1)/2) \\
 &= O((n^2-n)/2) \\
 &= O(n^2-n)
 \end{aligned}$$

div. const. can be neglected

Assumption 2 : If running time of an Algo is having Polynomial then in its Time complexity only leading term will be considered.

e.g. $(n^3 + n^2 + 5) \Rightarrow O(n^3)$

$\therefore T(n) = O(n^2 - n) = \underline{O(n^2)}$

Selection sort : Best case Worst case Avg. case
 $\sim 2(n^2)$ $O(n^2)$ $\Theta(n^2)$

Even if Array Sorted } Same no. of
Unsorted Iterations & comparisons
 \therefore Best = Worst = Avg.

Features of Sorting Algorithm :

Inplace \rightarrow if sorting algo do not take extra space (Space other than actual data element & const. space) to sort data elements in a collection/list of elements.

NOT Adaptive \rightarrow if a sorting algorithm works efficiently for already sorted input sequence then it is referred as an Adaptive

NOT

Stable \rightarrow if a sorting algorithm relative order of two elements having same key value remaining same even after sorting then such sorting algorithm is referred as Stable

main (-)

```
{ int []arr = {30, 20, 60, 50, 10, 40} ;  
    print (" Before sorting : ");  
    displayArrayElements (arr) ;
```

}

Public static void displayArrayElements (int []arr)

{

```
for ( int i = 0 ; i < arr.length ; i ++ )
```

{

```
    printf (" %d " , arr [i] ) ;
```

}

```
println () ;
```

}

Public static void selectionSort (int []arr)

{ int iterations , comparisons ;

```
for ( int sel - pos = 0 ; sel - pos < Arr.length - 1 ; sel - pos -
```

{ iterations ++ ;

```
    for ( int pos = sel - pos + 1 ; pos < Arr.length ;  
        comparisons ++  
        pos ++ )
```

{

if (arr [sel - pos] > arr [pos])

```
swapArrayElements ( arr , sel - pos , pos ) ;
```

}

}

```

public static void SwapArrayElements (int arr[], int i, int j)
{
    int temp = arr[i];
    arr[i] = arr[j];
    arr[j] = temp;
}

```

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Bubble Sort

Best case : $\Theta(n^2)$

Worst case : $O(n^2)$

Avg. case : $\Theta(n^2)$

Inplace \rightarrow yes

Adaptive \rightarrow No

Stable \rightarrow Yes

$$\begin{aligned} \text{Total no. comp. : } & (n-1) + (n-2) + (n-3) + \dots + 1 \\ & = \frac{n(n-1)}{2} \end{aligned}$$

$$T(n) = O(n(n-1)/2)$$

$$T(n) = O(n^2 - n/2)$$

$$T(n) = O(n^2 - n)$$

$$T(n) = O(n^2)$$

public static void main BubbleSort (int [] arr) {

 int iterations = 0; boolean flag = true;

 for (int it = 0; it < arr.length - 1; it++)

 { iterations++; flag = false;

 for (int pos = 0; pos < arr.length - 1 - it; pos++) {

 comparisons++;

 if (arr[pos] > arr[pos+1]) {

 swap (arr, pos, pos+1); flag = true;

}

}

4

Best case : If Array is already sorted (10 20 30 40 50)
then No need of swapping

- ① By design Bubble Sort is Not Adaptive,
but at implementation level we can make it an
adaptive by adding logic of flag.

Efficient Bubble Sort : Best case : $\Theta(n)$

worst case : $\Theta(n^2)$

Avg. case : $\Theta(n^2)$

Inplace : Yes

Adaptive : Yes

stable : Yes

- ② Avg. case of Both Bubble sort & Selection sort is same
 \therefore Efficiency is same for Both

Insertion Sort

```
for (i=1; i < size; i++) { // For loop for Iteration
    int key = arr[i];
    int j = i-1;
    // if index is valid & compare value of key with
    // an element at that index
    while (j >= 0 && key < arr[j]) {
        // Shift element towards its right hand side by
        // 1 POS
        arr[j+1] = arr[j];
        j--;
    }
    // insert key at its appropriate pos
    arr[j+1] = key;
}
```

Best case : $\Omega(n)$	Adaptive \rightarrow Yes
Worst case : $O(n^2)$	Stable \rightarrow Yes
Avg. case : $\Theta(n^2)$	Inplace \rightarrow Yes

④ Insertion Sort is By design efficient

① LINKED LIST

Linear

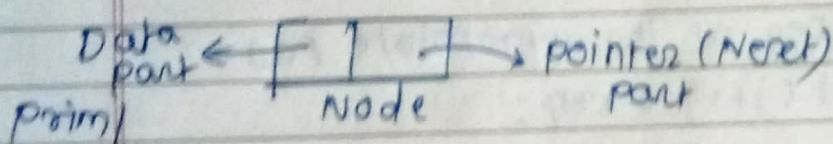
1) Singly LL : each node contains link to its next node only (No. of links = 1)

Circular

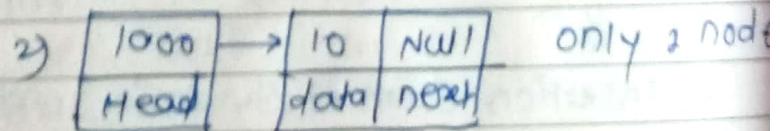
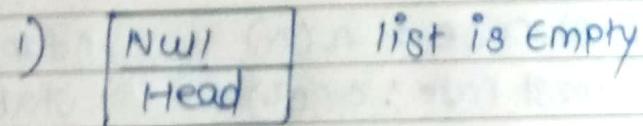
2) Doubly LL : each node contains link to its next node & prev. node (No. of links = 2)

① Singly Linear Linked List

↳ Head Always Contains address of 1st element
If list is Not empty



Non-Primitive
data



class List List HAS A Node → HAS-A Relationship
(composition)

next page

Public class SingleLinkedListMain {
 →, — void main (—)

{ // Create empty linked list

 linkedlist l₁ = new linkedlist();

 l₁.addNodeAtLastPosition(10);

 l₁.addNodeAtLastPosition(20);

 →, — (30);

class
composition)

class LinkedList

→ Static class Node

{
 private int data; // data of int type
 private Node next; // Reference to next node
 (8 Bytes)}

public Node (int data) { // Constructor of Node class
 this.data = data;
 this.next = null;

}

private Node head; // Reference to first node in list
private int nodesCount;

public LinkedList()

{ this.head = null; this.nodesCount = 0; }

}

void addNodeAtLastPosition (int data) {
 // [Step 1]: Create New Node Runtime

 Node newNode = new Node (data);

// [Step 2]: check if list is empty then attach
 newly created node to Head

if (isEmpty())

{

 head = newNode; nodesCount++;

} else // [Step 3]: if list is not empty start traversal
 From First Node
 till last node

 Node trav = head;

 while (trav.next != null) {

 trav = trav.next; // Move trav to next

 trav.next = newNode; // Attach newNode to last Node
 nodesCount++;

```

void displayList() {
    if (isEmpty())
        throw new RuntimeException("List is empty");
    else {
        Node trav = head;
        while (trav != null) {
            System.out.print(".");
            System.out.print(trav.data);
            trav = trav.next;
        }
    }
}

```

Algorithm to Add Node At last position

- 1) Create a New Node At Runtime (new -)
- 2) Check if List is Empty or NOT
 - ↳ If Empty → Yes → Head will be NewNode.
 - ↳ If List is Not Empty
 - ↳ Then traverse from First Node to last node

* We can Add As many as we want no. of nodes into SLL at last Position in $O(n)$ Time

Best case : $O(1)$ → If List is Empty

Worst case : $O(n)$

Avg. Case : $\Theta(n)$

```

public class SinglyLinkedListMain {
    void main () {
        // Create an empty LL
        LinkedList l1 = new LinkedList();
        l1.addNodeAtLastPosition(data: 10); // (20)(30)(40)
    }
}

```

// If LL is empty it shs. will not be displayed handle exception.

```

try {
    l1.displayList();
} catch (RuntimeException e) {
    System.out.println(e.getMessage());
}

```

[Make Before Break] → Always create New Link first
then only break old links.

Add Node At First Position

Step 1: Create New Node

Step 2: Check if list is empty

If Yes: Head = newNode

Step 3: If list is NOT empty; Attach newly created node to Head

Store address of current first node into the next part of newly created node

newNode.next = head;

Void addNodeAtFirstPosition (int data)

{ // ① → Create New Node

 Node newNode = new Node (data);

// ② → Check if list empty

 if (isEmpty ())

 head = newNode; // if empty then Attach newly created node to head

 else

 // ③ → if list NOT empty

 newNode.next = head; // Store address of current first node in

 head = newNode;

 Next part of newly created node

 // Attach newly created

 Node to head

Best case: $\Theta(1)$

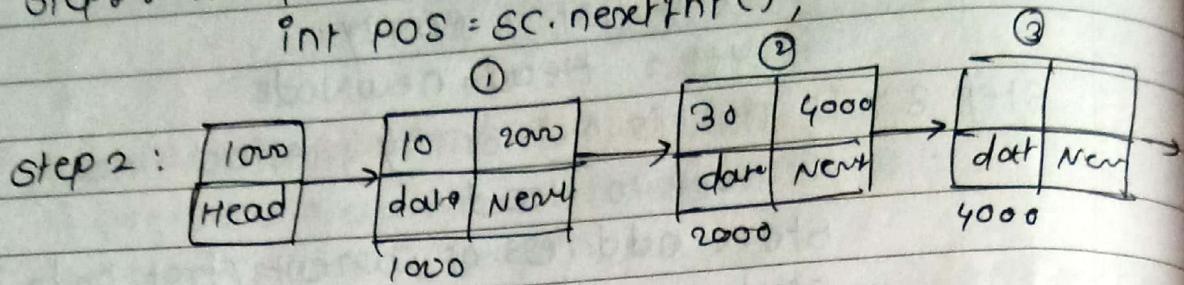
Worst Case: $O(n)$

Avg. Case: $\Theta(1)$

① Add Node At particular position In Between list

Step 2: Accept the position At which node to be added

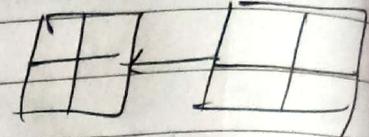
```
int pos = sc.nextInt();
```



validate position

```
if (pos >= 1 && pos <= count + 1)
```

count_nodes → By traversing



```
void displayList() {
```

```
    if (!isEmpty()) {
```

~~set~~ throw new RuntimeException

("List is Empty!");

}

else {

// traverse LL from first Node

Node trav = head; // trav is ref. of Node class

// traverse till last node

including it

while (trav != null)

{

 printf(" ·· d → ", trav.data); // display data part

trav = trav.next; // move trav to next

}

 println("null"); // this will printed at last of LL

 println("no. of nodes in LL " + countNodes());

 println("no. of nodes " + this.getNodesCount()); define this method

}

in class LinkedList

class LinkedList {

 static class Node
 & its logic

 int countNodes() {

 Time : O(n)

 int cnt = 0;

 if (!isEmpty()) { // if list is Not empty

 Node trav = head;

 while (trav != null) {

 cnt++;

 trav = trav.next; // move to next

 }

}

 return cnt;

// countNodes() method take time O(n) which is not efficient

∴ other way : List HAS-A count of nodes

∴ create one more mem. of LinkedList class

 private int nodeCount;

 initialize it in constructor LinkedList()

 this.nodeCount = 0;

And maintain the count everywhere we are adding
newNode.

int getNodesCount() {

 Time: O(1)

 return (this.nodeCount);

}

Step 2 : validate position.

If (pos >= 1 && pos <= li.getNodesCount() + 1)

↳ true then position is valid.

main ()

{

```
while (true) {
```

// Step 1: Accept position from user

```
int pos = sc.nextInt();
```

// Step 2: validate position

```
if (pos >= 1 && pos <= li.getNodeCount() + 1)
```

Break; // while will break

```
System.out.println ("invalid position");
```

{

}

class Linked List {

```
void addNodeAtSpecificPosition (int pos, int data)
```

{

```
if (pos == 1)
```

addNodeAtFirstPosition (data);

```
elseif (pos == getNodeCount() + 1)
```

addNodeAtLastPosition (data);

```
else {
```

// Create new node first

```
Node newNode = new Node (data);
```

```
i = 1;
```

// Start traversal from 1st node

```
trav = head;
```

// Traverse till $(pos - 1)^{\text{th}}$ node

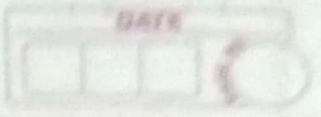
```
while (i < pos - 1) {
```

```
i++;
```

```
trav = trav.next;
```

// Attach cur ($(pos)^{\text{th}}$) node to next part of newly created node.

```
newNode.next = trav.next;
```



// attach newly created node into next part of
(pos - 1)th node

trav.next = newNode;

nodesCount++;

3

3

```

class LL {
    Node head;

    class Node {
        String data;
        Node next;
        Node (String data) { // while creating new node
            this.data = data;
            this.next = null; // next part will be By default null
        }
    }

    public void addFirst (String data) {
        if (head == null)
    }

```

① create New Node

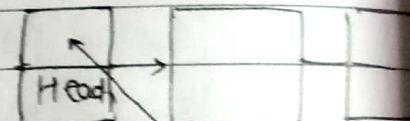
```
Node newNode = new Node (data);
```

② check if LL is empty

```
if (head == null) {
    head = newNode;
    return;
}
```

③ If Head is not null

```
newNode.next = head;
head = newNode;
```



```
public void addLast (String data)
```

```
{ Node newNode = new Node (data);
```

```
if (head == null) {
    head = newNode;
    return;
}
```

```
Node currNode = head;
```

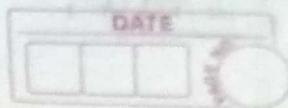
```
while (currNode.next != null) {
```

```
    currNode = currNode.next;
}
```

```
currNode.next = newNode;
```

NW1 → C++
NW1 → Java

TREE



public static void main()

```
{ LL list = new LL(); // object of class LL  
list.addFirst("a");  
list.addFirst("is");  
list.printList();  
list.addLast("list");  
list.printList(); }
```

```
public void printList() { if (head == NW1) { ("Empty");  
return; }  
Node currNode = head;  
while (currNode.next != NW1)  
{ System.out.println (currNode.data + " → ");  
currNode = currNode.next;  
}  
System.out.println ("NW1"); }
```

① Search And Delete Node

```
boolean searchAndDelete(int data)
boolean searchNode(int data, Node[] temp)
{
    for (Node trav = head; trav != null;
         trav = trav.next)
    {
        if (data == trav.data) {
            temp[1] = trav;
            return true;
        }
        temp[0] = null; temp[1] = null;
    }
    return false;
}
```

/* searchNode() Function:

on success this function returns true and it also returns an addr of node which is to be deleted in temp[1] as well as an address of its previous node in temp[0] and on failure this function returns False and in temp[ref] it will return null

*/

```
boolean searchAndDelete(int data)
```

```
{
```

```
Node[] temp = {null, null};
```

// search a node whose data matches with "data"

```
if (!searchNode(data, temp))
    return false;
```

)

```

    // if Node is found
    if (temp[0] == nwl) // if Node Found At 1st position
        print("Node having data part : " + data)
        " is found : temp[0].data :
                    " + temp[1].data
    else
        print(" temp[0].data : " + temp[0].data
              + " temp[1].data : " + temp[1].data);
    return false;
}

// if node which is to be delete is At 1st position .
if (temp[0] == nwl)
    head = head.next; // Attach our second
else
    temp[0].next = temp[1].next; // Attach next node of
                                // node which to be
                                // deleted to its prev node.
return true;
}

```

priority queue can be implemented using SearchAndDelete() Function.

Q. Display linked list in a Reverse order without modifying original linked list & using extra space.

Soln 1: Allocate memory dynamically for an Array of integers of size count

Traverse linked list & while traversing keep on adding data part of each node into an array

Display Array in Reverse from Right to Left

Soln 2: Maintain Stack of integers

Traverse linked list & while traversing it keep on pushing data part of each node onto stack

Pop element from stack one by one & Display them

Soln 3: In Recursion internally or maintain stack

// wrapper funcn from inside actual rec funcn get called.

```
void displayListInReverseOrder() {  
    if (ISListEmpty()) {  
        throw new RuntimeException("list is empty");  
    }  
    else {  
        print("List in a reverse order");  
        displayListInReverseOrder(head); // Initialization  
    }  
}
```


~~display list~~

```
public class CircularLinkedList {
    private Node head;
    private Node tail;
    public CircularLinkedList (Node head, Node tail)
    {
        this.head = head; null;
        this.tail = tail; null;
    }
}
```

```
public class Node {
```

```
    int val;
```

```
    Node next;
```

```
    public Node (int val)
```

```
    {
        this.val = val;
    }
}
```

```
public void insert (int val) {
```

```
    // create new node first
```

```
    Node newNode = new Node (val);
```

```
    if (head == null) // if list empty
```

```
    {
        head = newNode;
    }
```

```
    tail = newNode;
```

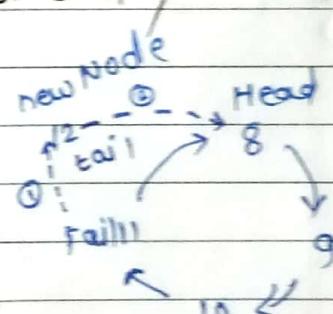
```
    return;
```

```
}
```

```
① tail.next = head -> newNode;
```

```
② newNode.next = head;
```

```
③ tail = head -> newNode;
```



```
public void Display () {
```

```
    Node newNode = head;
```

```
    if (head != null) {
```

```
        do {
```

```
            print (node.val + " → ");
```

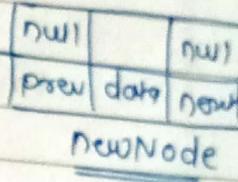
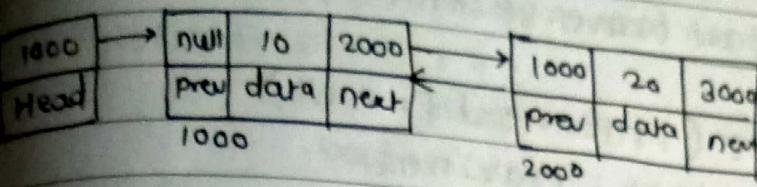
```
            node = node.next;
```

```
        } while (node != head);
```

DATE _____
PAGE _____

```
public void delete (int val)
{
    Node trav / node = head;
    if (trav == null) {
        return;
    }
    if (trav.val == val) { // if val at head
        head = head.next;
        tail.next = head;
        return;
    }
    do {
        Node n = head;
        if (n.val == val) {
            trav.next = n.next;
            break;
        }
        trav = trav.next;
    } while (trav != head);
}
```

5

Doubly Linear Linked List

```
class DLL {
    static class Node {
        private int data;
        Node prev;
        Node next;
    }
}
```

```
Node (int data) {
    this.data = data;
    this.prev = null;
    this.next = null;
}
```

```
private Node head;
private int nodesCount;
```

```
DLL () {
    head = null;
    nodesCount = 0;
}
```

```
void addNode At Lastposition (int data)
```

```
{
    // create New Node
    Node newNode = new Node (data);
    // If list empty → attach newNode to head
}
```

```
if (islistEmpty ()) {
    head = newNode; nodesCount++;
}
```

else {
 // IF list NOT empty
 // start traverse from first node to last

 while (trav.next != null)

 trav = trav.next

 trav.next = newNode;

 newNode.prev = trav;

 nodeCount++;

}

}

void displayList() {

 if (isEmpty())

 throw _____ ("list empty");

 else {

 Node trav = head;

 while (trav != null)

{

 printf ("%d", trav.data);

 trav = trav.next;

}

 println();

}

4

// Reversal of list

trav = temp;

 while (trav != null) {

 temp = trav;

 printf ("%d", trav.data);

 trav = trav.prev;

}

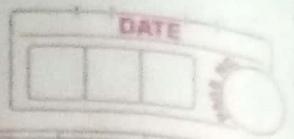
 println();

3

```
void addNodeAtFirstPosition (int data) {
    Node newNode = new Node (data);
    if (isEmpty ()) {
        head = newNode;
        nodeCount++;
    }
    else {
        newNode.next = head;
        head.prev = newNode;
        head = newNode;
        nodeCount++;
    }
}
```

```
Void addNodeAtSpecifiedPosition (int pos, int data) {
    Node newNode = new Node (data);
    if (pos == 1)
        addFirstPosition (data);
    else if (pos == getNodesCount () + 1)
        addNodeAtLastPosition (data);
    else {
        int i = 1;
        Node trav = head;
        while (i < pos - 1) {
            i++;
            trav = trav.next;
        }
        newNode.next = trav.next;
        newNode.prev = trav;
        trav.next.prev = newNode;
        trav.next = newNode;
    }
}
```

4 May



① **Dynamic Stack** (By using linked list) Doubly circular
stack → LIFO

OR [push ⇒ addlast()
pop ⇒ deletelast()
if (head == null) ⇒ list is empty ⇒ stack is empty

O(1)
O(1)

peek ⇒ get data part of last node

addFirst()
deleteFirst()

Labwork: Implement Dynamic Stack

Applications of Stack:

Stack is used by an OS to control flow of execution of prog.

- 2) In Recursion internally an OS uses a stack
- 3) Undo & Redo functionalities of an OS implemented by using stack
- 4) Stack is used to implement advance data structure algo like DFS: DepthFirst Search
- 5) Stack used in Algo to convert infix expression into its Postfix & Prefix & for postfix expression evaluation

① Stack For Expression Conversions & Evaluation

0) Infix: $a * b / c * d + e - f * g + h$



- Postfix:
- ① $ab * / c * d + e - f * g + h$
 - ② $ab * c / * d + e - f * g + h$
 - ③ $ab * c / d * + e - f * g + h$
 - ④ ~~$ab * c / d * e + - f * g + h$~~
 - ⑤ ~~$ab * c / d * e + f - * g + h$~~
 - ⑥ ~~$ab * c / d * e + f - g * + h$~~
 - ⑦ ~~$ab * c / d * e + f - g * h +$~~

$ab * c / d * + e - f g * + h$
 $ab * c / d * e + - f g * + h$
 $ab * c / d * e + f g * - + h$
 $ab * c / d * e + f g * - h +$

* Algorithm to convert infix expression into equi. Postfix

1) Start scanning infix expression from left to right

2)

if (cur element is an operand) {
append it to postfix expression

3 else

{ if (cur element is an operator)

if (stack is not empty & &

priority (topmost element)

>= priority (cur element)

while (! is stack empty) & &

priority (topmost element)

>= priority (cur element)

{

pop element from stack &

append it to postfix expression

}

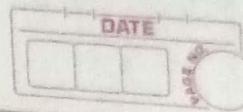
// push current element onto the stack

}

// step 3: repeat step 2 till end of infix expression

// step 4: pop all element's from stack one by
one & append them to postfix

```
import java.util.Stack;
```



String infixToPostfix (String infix)

// Initially we need empty stack of operators

Stack<Character> S = new Stack<Character>();

StringBuilder postfix = new StringBuilder();

o

lev 1

// Step 1 : Start Scanning infix expression from left to Right

For (int index = 0 ; index < infix.length() ; index++)

{ // get element at cur-ele

char cur-ele = infix.charAt(index);

// if cur-ele is operand, Append it into postfix

postfix.append(cur-ele);

if (isOperand(cur-ele))

postfix.append(cur-ele);

else { // if cur-ele is an operator

// if stack is not empty & priority (topmost) >= priority

(cur-ele)

while (!S.isEmpty() && priority(S.peek())

>= priority (cur-ele)));

postfix.append(S.peek());

S.pop();

S.push(cur-ele);

}

} // step 4 : repeat -2 & -3 step till end

// step 5 : pop all remaining element from stack

& append them to postfix expression.

while (! S.empty()) {

postfix.append(S.peek());

S.pop();

}

// convert StringBuilder class object to String object

return (postfix.toString());

ee

:

;

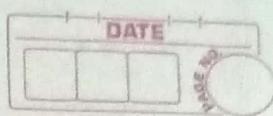
;

static boolean isOperand (char ch)

```
{ if (ch >= 65 && ch <= 90) ||  
    capital (ch >= 97 && ch <= 122) small  
    || (ch >= 48 && ch <= 57)) digits  
    return true ; }
```

public static int priority (char opr)

```
{ switch (opr) { case '(': return 0 ;  
    case '+':  
    case '-': return 1 ;  
    case '*':  
    case '/': return 2 ;  
    default: return -1 ; } }
```



public static void String InfixToPrefix (String Infix)

Stack < character > S = new _____ ();

StringBuilder prefix = new _____ ();

for (int index = infix.length () - 1 ; index >= 0 ; index --)

{ char cur_ele = infix.charAt (index) ;

if (isOperand (cur_ele))

prefix.append (cur_ele) ;

else {

while (! S.empty () &&
priority (S.peek ()) > priority (cur_ele))

{

prefix.append (S.peek ()) ;

S.pop () ;

S.push (cur_ele) ;

}

while (! S.empty ()) {

prefix.append (S.peek ()) ;

S.pop () ;

}

return (prefix.reverse ().toString ()) ;

Evaluation For Postfix

infix: $4 + 5 * 9 / 7 + 3 - 6$

Postfix: $4 + 5 9 * 1 7 + 3 - 6$

$4 + 5 9 * 7 1 + 3 - 6$

$4 5 9 * 7 1 + + 3 - 6$

$4 5 9 * 7 1 + 3 + - 6$

O/P: $[4 5 9 * 7 1 + 3 + 6 -]$

Step 1: Start scanning Postfix expression from left to right

Step 2:

IF (cur-ele is operand)

Push it onto stack

else IF (cur-ele is an operator)

{ → pop two elements from stack in such a way that

operand 2 = First popped element

operand 1 = Second popped element

→ Perform cur-ele (operator) operation

on OP1 & OP2

result = OP1 (opr) OP2

→ push back result onto stack

?

Step 3: pop final Result from stack

$4 5 9 * 7 1 + 3 + 6 -$			
Stack (operands)		$OP_1 = 5$	$OP_1 = 45$
9	5	$OP_2 = 9$	$OP_2 = 7$
4	45	$result = 5 * 9$	$result = 45 / 7$
4	4	$push = 45$	= 6 (push into stack)
3	6	$OP_1 = 4$	$result = 6 + 4 = 10$ (push)
0	10	$OP_2 = 6$	
1	13	$result = 13 - 6$, (push)	stack
2	7		\rightarrow Expected O/P
3	13		

• Queue

It is linear / basic data structure which is collection / list of logically related similar type of data elements in which elements can be added into it from one end (Rear end) and can be deleted from it at (Front end)

Element was inserted first can be deleted first out manner or last in last out manner.

∴ LIFO / FIFO

• 4 Types of Queue :

1) Linear queue (FIFO)

2) Circular queue (FIFO)

3) Priority queue (NO FIFO)

↳ Type of queue in which elements can be added into from Rear end randomly

(i.e. without checking priority)

whereas element having highest priority deleted first

4) Double ended queue (deque)

↳ Elements can be added / deleted from both the sides

• Operations performed in O(1) for Queue

1) Enqueue → insert / add from Rear end

2) Dequeue → delete / remove from Front end.

• For Dequeue : 4 operations in O(1)

i) Push-back() \Rightarrow addLast()

ii) Push-front() \Rightarrow addFirst()

iii) Pop-back() \Rightarrow deleteLast()

iv) Pop-front() \Rightarrow deleteFirst()

} can be implemented using

} Doubly circular linked list

② 2 Types of Deque

↳ 1) Input restricted deque

- ↳ Elements can be added → only from one end
- ↳ Elements can be deleted → from both ends

2) Output restricted deque

- ↳ Elements can be added → from both ends
- ↳ Elements can be deleted → from only one end

5 May 2022

queue → is static as well as Dynamic

1. Stack implementation (using Array) of queue
2. Dynamic implementation of queue (using linked list)

Static Implementation of Queue (Using Array)

```
int arr[5], int front, int rear;
```

```
class LinearQueue {
```

```
    int arr[5];
```

```
    int front;
```

```
    int rear;
```

```
    LinearQueue (int size) {
```

```
        arr = new int [size];
```

```
        Front = -1;
```

```
        rear = -1;
```

Physically

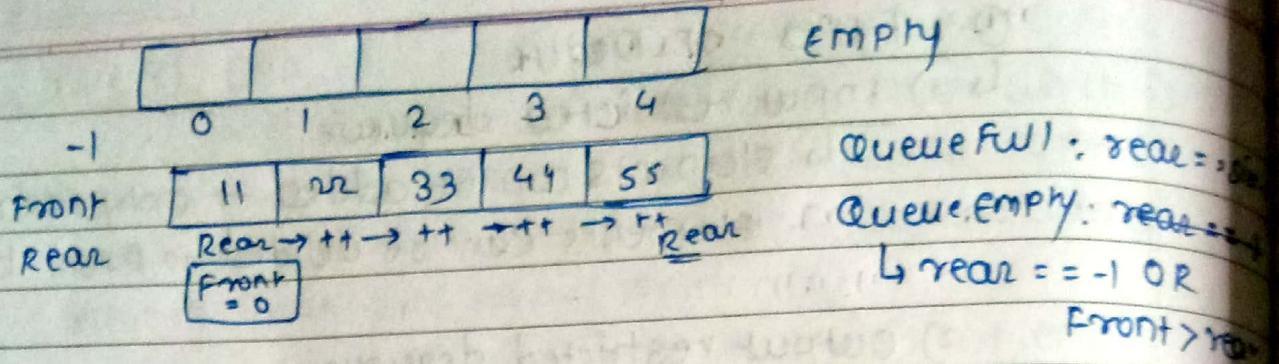
↳ Array

Logically

↳ queue

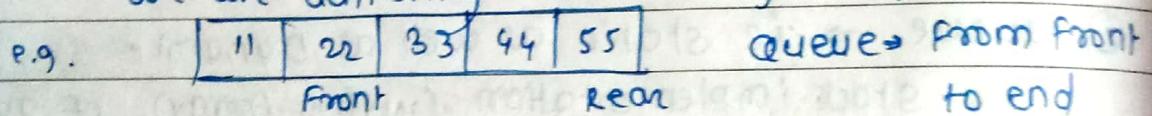
// Enqueue → Add from Rear End

- 1) → If Queue is Not Full then insert/ add element
- 2) → Increment val. of Rear (IF Not Full)
- 3) → Insert from Rear
- 4) → If val. of Front = -1 make it as 0



// Dequeue → Delete / Remove from front end

- 1) check if queue is empty
- 2) if NOT empty: increment val. of Front by 1
 [by means of incrementing Front by 1
 we are achieving deletion logically]



When Front == size or (Front > Rear) → Empty Queue

⑥ void enqueue (int ele)

{ // Step 1: check if queue is Not Full

 if (isQueueFull())

 {

 throw new RuntimeException ("Queue is Full");

 }

 else { // if queue is Not full then insert

 // Step 2: Increment Rear value

 rear++;

 // Step 3: Insert an element from Rear
 $\text{arr}[rear] = ele;$

 if (Front == -1)

 Front = 0;

}

function isQueueFull() {

return (rear == arr.length - 1);

function isQueueEmpty() {

return (rear == -1 || front > rear);

int getFront() { // to get element at arr[Front]

if (isQueueEmpty())

throw new RuntimeException ("Queue is empty");

else {

return (arr[front]);

}

void dequeue() { // to delete from end}

if (isQueueEmpty())

throw new RuntimeException ("Queue is empty");

else {

// if queue is not empty ; inc. front by 1

front++;

}

Limitation of linear Queue

↳ vacant places cannot be reutilized

∴ to overcome limitation circular queue is designed.



class circularQueue {

① boolean isQueueFull() {

 return (Front == (rear + 1) % arr.length);

}

② boolean isQueueEmpty() {

 return (rear == -1 && Front == rear);

}

③ void enqueue(int ele) {

 if (isQueueFull()) {

 throw new RuntimeException ("Queue is full");

}

 else {

 rear = (rear + 1) % arr.length;

 arr[rear] = ele;

 if (Front == -1)

 Front = 0;

}

④ void dequeue() {

 if (isQueueEmpty())

 throw new RuntimeException ("Queue empty");

}

 else {

 if (Front == rear) {

 Front = rear - 1;

}

 else { Front = (Front + 1) % arr.length;

}

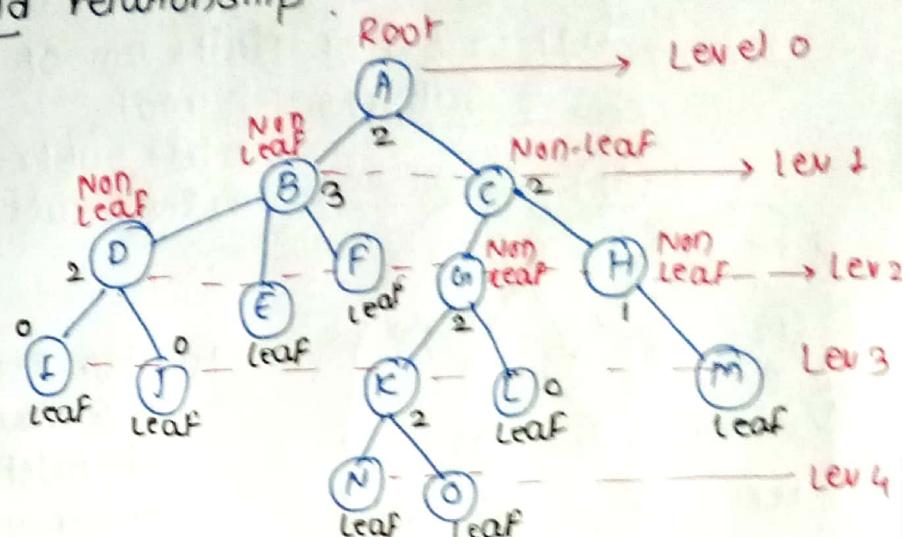
}

}

}

Advance Data Structure → TREE

- ① Tree is Nonlinear / Advance data structure which is collection of finite no. of logically related similar type of data elements in which there is first element → Root element and remaining all elements are connected to root in hierarchical manner. Follows parent-child relationship.



① Ancestors:

All nodes which are in path from root node to that node.

e.g. For I → D, B, A are Ancestors

② Descendents:

All nodes Ascending from it

e.g. For K → N, O descendents

③ Siblings: child nodes of same parents

④ Degree of Node: No. of child nodes having that node

e.g. B → D, E, F ∴ B (3)
parent child

⑤ Degree of Tree: Max. degree of any node in a given tree

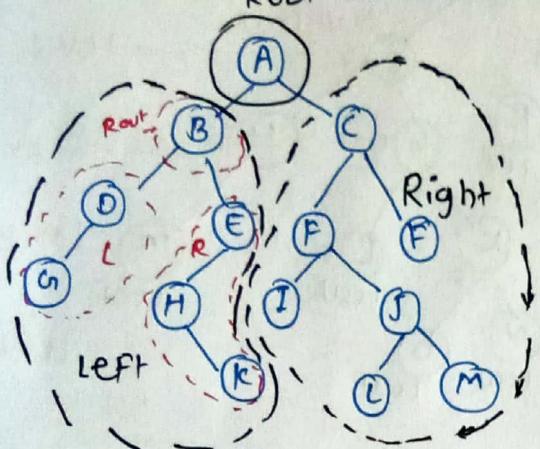
e.g. B has degree 3 ∴ Deg. of Tree = 3

⑥ Level of Node: (level of parent + 1)

⑦ Level of Tree / Depth of Tree: max. level of any node in a tree. ∴ Depth of Tree = 4

① Data Structure is Dynamic → can grow upto any level & any Node can have any no. of child nodes operations on it becomes inefficient, so restrictions can be applied on it to achieve efficiency and hence there are different types of tree

① Binary Tree → each node can have Max. 2 child nodes
 or each node can have degree = 0 / 1 / 2
 ↳ It is set of finite no. of elements having
 Root 3 subsets:
 1) root
 2) left subtree (may be empty)
 3) Right subtree (May be empty)

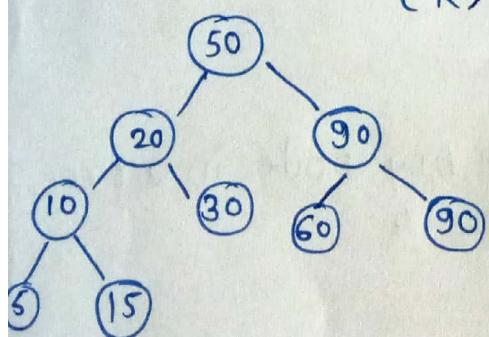


Further restrictions can be applied on Binary tree to achieve Add / Delete / search operations efficiency expected in $O(\log n)$ time
 ↳ Binary Search Tree can be formed

② Binary Search Tree

	Addition	Deletion	searching
Array	<u>$O(n)$</u>	<u>$O(n)$</u> ; <small>disadv.</small>	$O(n)$ $O(\log n)$
LinkedList	$O(1)$	$O(1)$	<u>$O(n)$</u> ; <small>disadv.</small>
Binary Search Tree	$O(\log n)$	$O(\log n)$	$O(\log n)$

① Binary Search Tree → It is Binary Tree in which
 (L < P) Left child is Always smaller than parent
 (R >= P) Right child is Always greater or equal to its parent.



while Adding node into BST
 first find its appropriate position
 (comp. values)
 then Add node to resultant position

start traversal from → Root Node

Q.) Input order for BST: [50, 20, 90, 85, 10, 45, 30, 100, 15, 75, 95,

20 < 50

80 > 50 10 < 50

85 > 50 10 < 20

45 < 50

45 > 20

30 < 50

30 > 20

30 < 45

30 > 15

30 < 30

Root

else { // If BST not empty start trav. to find appropriate position
Node trav = root;

```
public class BSTMain {  
    void main ()
```

{ // Create empty BST
BST t1 = new BST();

t1.addNode(50);

// order: [50 20 90 85 10 45 30 100 25 75 95 120 5 20]
 ^{root}
 _{Level 1}

try {

t1.preorder();

} catch (Runtimeexception e) { "BST is empty" e)

} print(e.message());

}

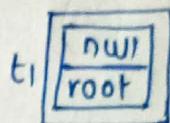
}

class BST {

boolean isEmpty() {

return (this.root == null);

}



When root = null
empty tree

① There Are 2 Tree Traversal

① BFS / Levelwise traversal

② DFS traversal

↳ 2.1 preorder → (VLR)

2.2 Inorder → (LVR)

2.3 Postorder → (LRV)

① BFS → Traverse Always From Root

→ Nodes in BST gets visited levelwise From left to right

```

void addNode(int data)
{
    Node newNode = new Node(data);
    if (isBSTEmpty()) // If BST empty
        {
            root = newNode;
        }
    else // If BST is NOT empty
        {
            // Start trav from root node
            Node trav = root;
            while (true) { // traverse to find appropriate position.
                if (data < trav.data) // Node will part of left subtree
                    {
                        if (trav.left == null)
                            {
                                trav.left = newNode; // Attach newly
                                break;           created node as
                                         left child of current
                                         node.
                            }
                        else { trav = trav.left; }
                    }
                else { // node will be part of
                    if (trav.right == null)
                        {
                            trav.right = newNode;
                            break;
                        }
                    else { // if curNode has Right subtree
                            trav = trav.right;
                        }
                }
            }
        }
}

```

```

void preorder() // Wrapper Function
{
    if (isBSTEmpty())
        throw new RunTimeException("BST is empty");
    else
        {
            print("preorder traversal is: ");
            Preorder(root); // Initialization
            print();
        }
}

```

```

void preorder(Node trav) {
    // Base condition
    if (trav == NULL)
        return;
    // VLR
    printf("%d", trav.data); // visit current Node
    preorder(trav.left); // visit left subtree
    preorder(trav.right); // visit right subtree
}

```

// Recursive Function for AddNode

```

void recAddNode(int data) // wrapper function
{
    if (isBSTEmpty()) // if BST empty Node will be root
    {
        root = newNode(new Node(data));
    }
    else { // if BST is NOT empty
        recAddNode(&root, data); // for traversal
        // we are calling recursive func
        // starting from &root
    }
}

```

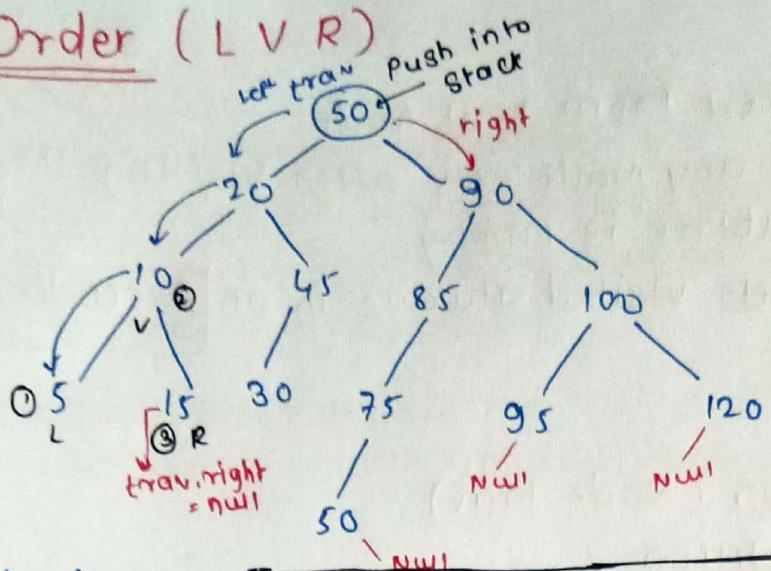
void recAddNode(Node trav, int data)

```

{
    // Base condition
    if (trav == NULL)
    {
        return;
    }
    if (data < trav.data) // Node will part of left sub
        tree of current Node
    {
        if (trav.left == NULL) // if left subtree of cur Node
        {
            trav.left = newNode(data); // create New Node
            return; // attach it as left child
        }
        else { // if cur Node is having left sub tree
            recAddNode(trav.left, data);
        }
        // go to left subtree
    }
    if (trav.right == NULL) // if right subtree of cur Node
    {
        trav.right = newNode(data); // is NULL
        return;
    }
}

```

① InOrder (LVR)



Stack <Node>

8	15	30
10	20	45
20	50	50
50		

50	75	95
75	85	100
85	90	120

inorder(LVR) : [5 10 15 20 30 45 50 50 75 85 90 95 100 120]

void nonRecInOrder()

{ while (trav != null || !s.empty())

{ while (trav != null)

{ // Push cur Node onto stack

s.push(trav); ----- 45|30|null|90

// go to its left subtree

trav = trav.left;

}

// if stack is not empty

if (!s.empty())

{ // POP node From stack & catch it into trav

trav = s.peek(); 5|10|15|20|30|45|50|50|75|85|

s.pop();

// if we are popping node from stack
either its left subtree is empty or Already visited
hence visit current Node

printf(".1.4d", trav.data);

// go to its Right subtree

trav = trav.right;

}

3

3