PROGRAMMING ASSIGNMENT - I

→ Module 1 - Solution

1. Given the differential equation $\frac{dy}{2^x \log_e(2)} = dx$ with the initial condition $y(0)=1$ and the specific time instances $x = 1, 2, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22$ we can solve this equation to find the function $y(x)$.

First, let's solve the differential equation and then calculate the value of y at each given time instance.

To solve, the differential equation, we integrate both sides with respect to their respective variables:

$$\int dy = \int 2^x \log_e(2) dx$$

Integrating,

$$y = \frac{1}{\log_e(2)} \int 2^x dx = \frac{1}{\log_e(2)} \cdot \frac{2^x}{\log_e(2)} + C$$

Now, applying the initial condition $y(0)=1$:

$$1 = \frac{1}{\log_e(2)} \cdot \frac{2^0}{\log_e(2)} + C$$

$$1 = \frac{1}{\log_e(2)^2} + C$$

$$C = 1 - \frac{1}{\log_e(2)^2}$$

So, the solution to the differential equation is:

$$y(x) = \frac{2^x}{\log_e(2)^2} + \left(1 - \frac{1}{\log_e(2)^2}\right)$$