Problem Set 10   Pratool Gadtaula

1. $\dot{y}+y=x$

$\mathcal{L}$ $sY(s)+Y(s)=X(s)$

$$\frac{Y(s)}{X(s)}=\frac{1}{s+1}$$

↓ step response

$$\frac{Y(s)}{X(s)\cdot s}=\frac{1}{s(s+1)}$$

$$\frac{1}{s(s+1)}=\frac{A}{s}+\frac{B}{s+1}$$

$$A(s+1)+Bs=1$$

$$(A+B)s+A=1$$

$A=1$ $A+B=0$

$B=-1$

$$\frac{Y(s)}{X(s)\cdot s}=\frac{1}{s}-\frac{1}{s+1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right]=u(t)-u(t)e^{-t}$$

$$y(t)=u(t)\left(1-e^{-t}\right)$$

2. A) DC gain $=\lim_{s\to 0}\frac{Y(s)}{Y_{sp}(s)}=\lim_{s\to 0}\frac{(K_I/s)H(s)}{s+(K_I/s)H(s)}=\boxed{1}$ Does not depend on K_I

B) $$\frac{Y(s)}{Y_{sp}(s)}=\frac{KH}{1+KH}=\frac{\frac{K_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}{1+\frac{K_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}=\frac{\frac{K_I}{s}(1/\tau)}{(s+1/\tau)+\frac{K_I}{s}(1/\tau)}$$

$$\frac{Y(s)}{Y_{sp}(s)}=\frac{\frac{K_I}{s\tau}\cdot s\tau}{\left(s+\frac{1}{\tau}\right)s\tau+K_I}=\frac{K_I}{s^2\tau+s+K_I}=\frac{K_I/\tau}{s^2+\frac{s}{\tau}+\frac{K_I}{\tau}}$$

$$\boxed{\frac{Y(s)}{Y_{sp}(s)}=\frac{K_I/\tau}{s^2+\frac{s}{\tau}+\frac{K_I}{\tau}}}$$

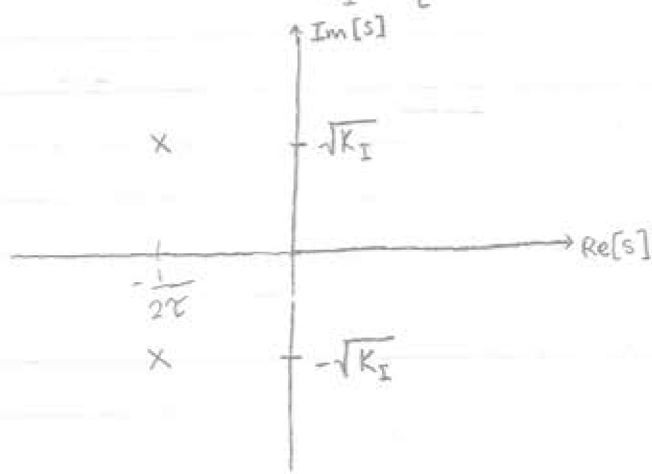
No zeros because we assume $K_I>0$

$$s=\frac{-\frac{1}{\tau}\pm\sqrt{\left(\frac{1}{\tau}\right)^2-4\left(\frac{K_I}{\tau}\right)}}{2}$$

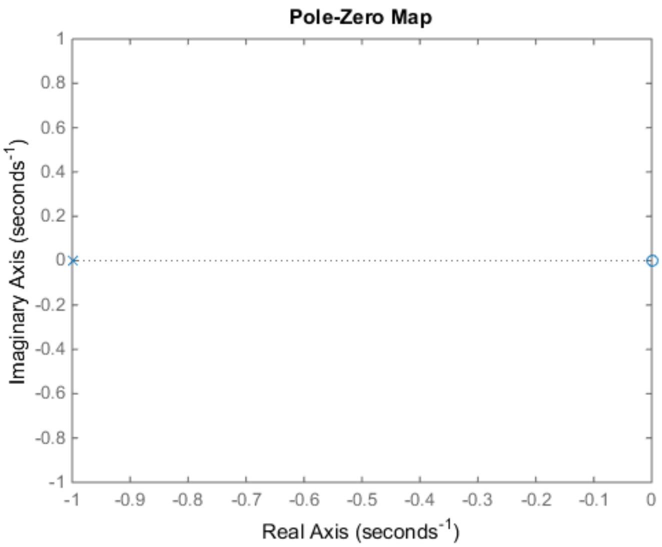
$\frac{1}{\tau}\to 0$ because we assume $K_I>>\frac{1}{\tau}$

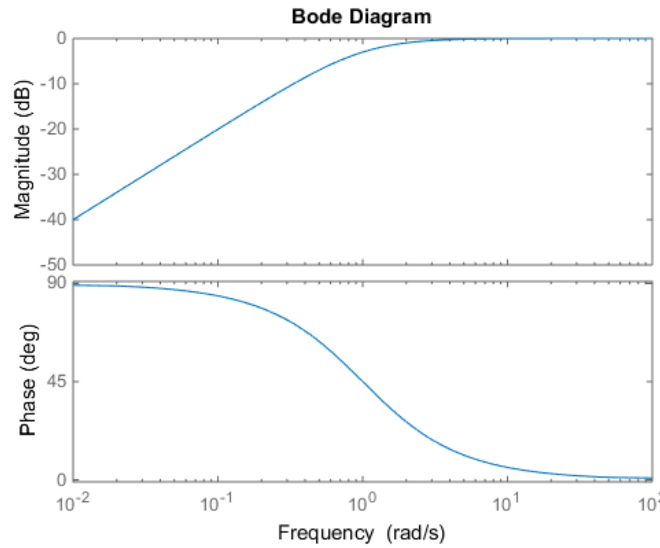
$$s=-\frac{1}{2\tau}\pm j\sqrt{\frac{K_I}{\tau}}$$

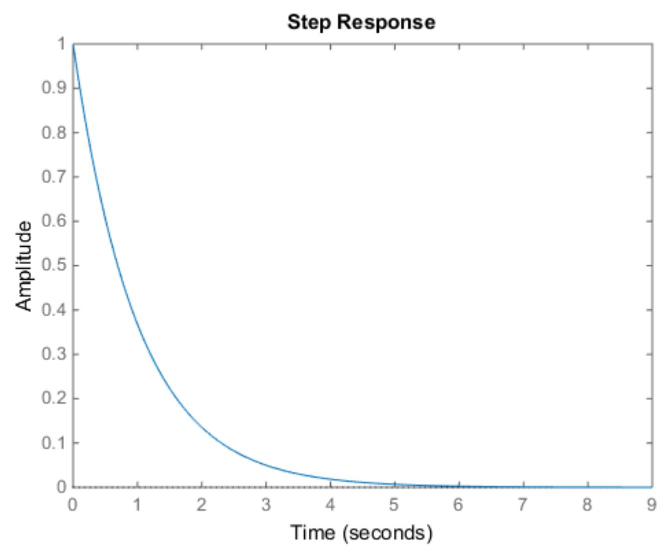
$$s=-\frac{1}{2\tau}\pm j\sqrt{K_I}$$



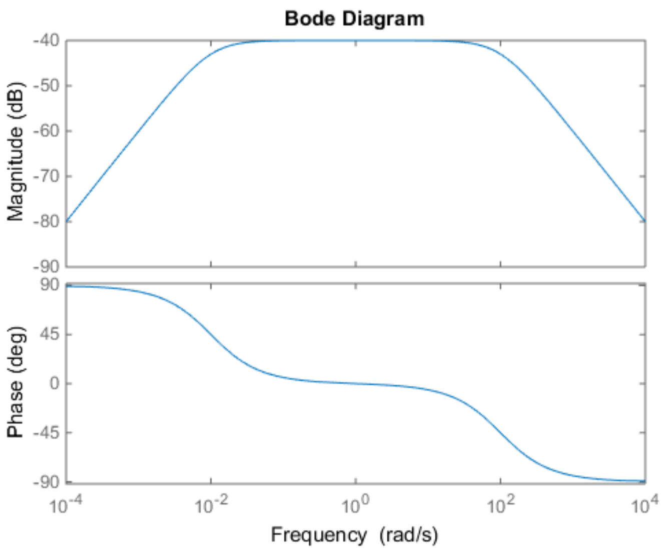
Problem 3. Part A: 1st order; 1 pole LHP; 1 zero at origin; exponential decay step response

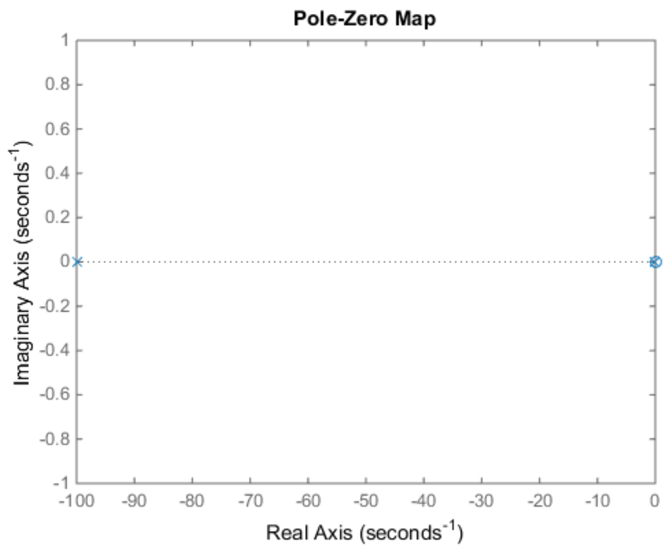


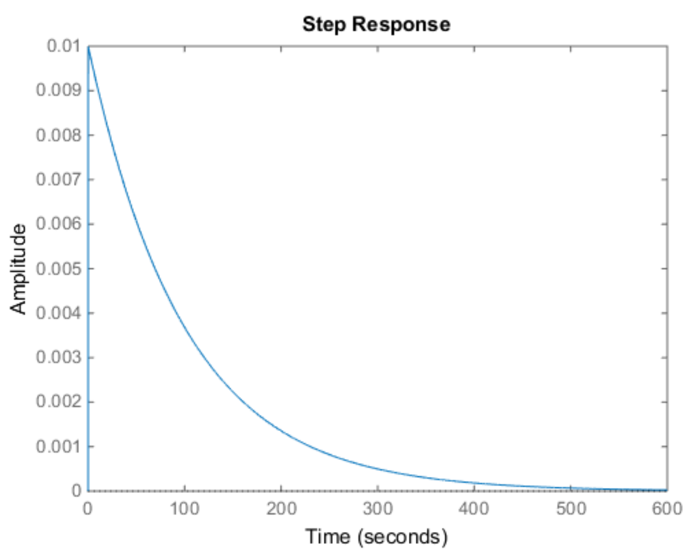




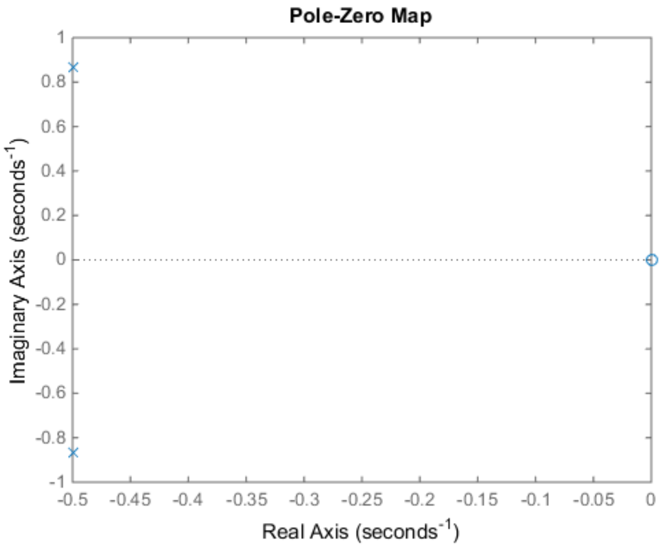
Problem 3. Part B: 2nd order; 2 poles LHP; 1 zero at origin; exponential decay step response

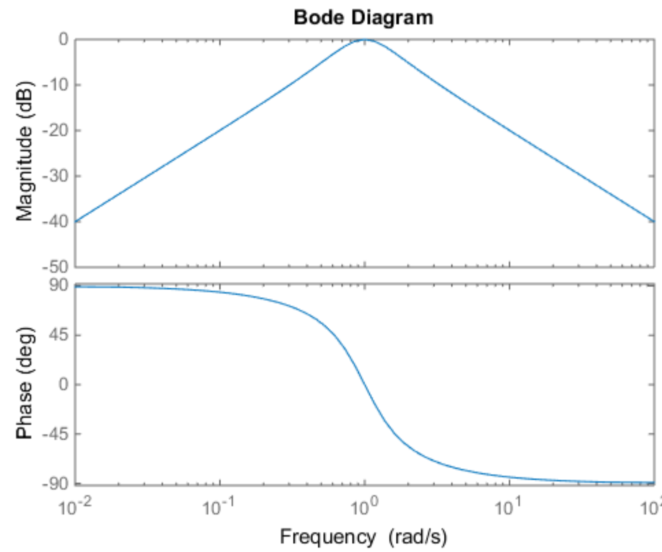


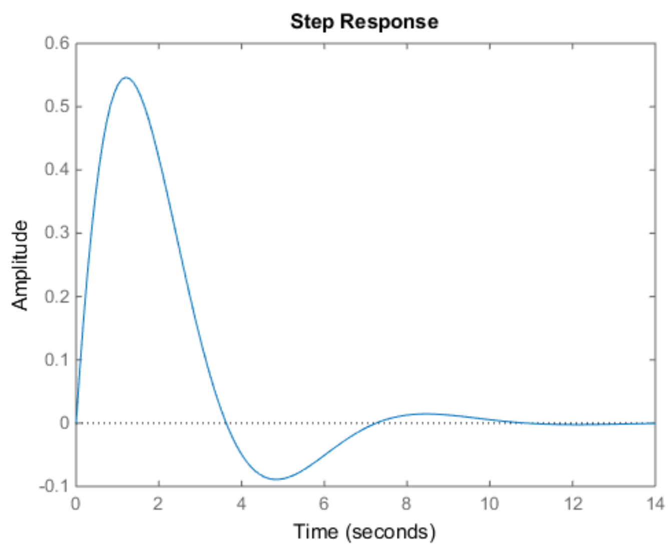




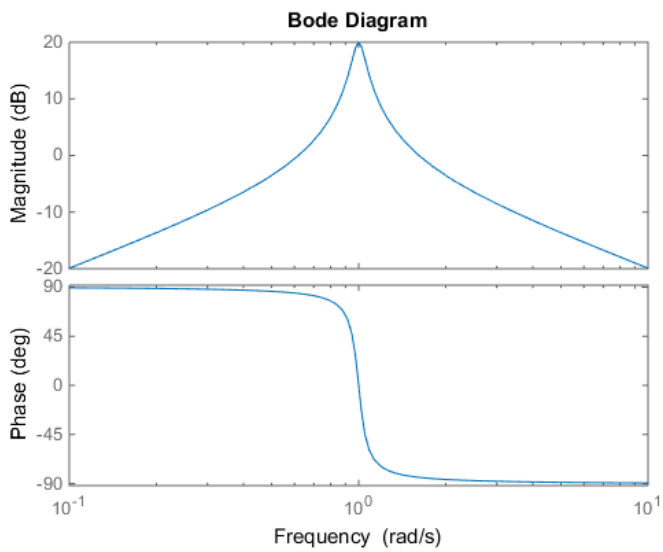
Problem 3. Part C: 2^{nd} order; 2 poles LHP; 1 zero at origin; exponential decay step response

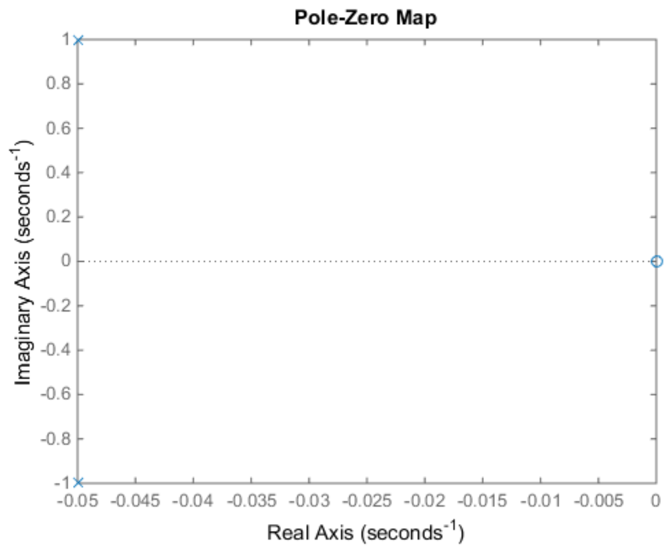


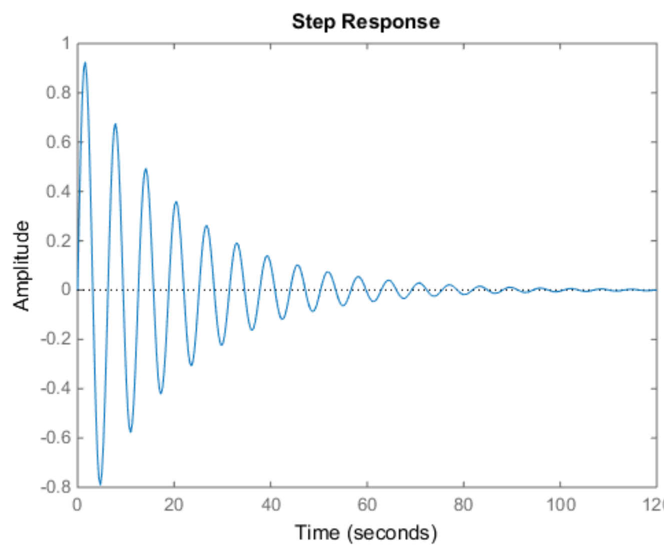




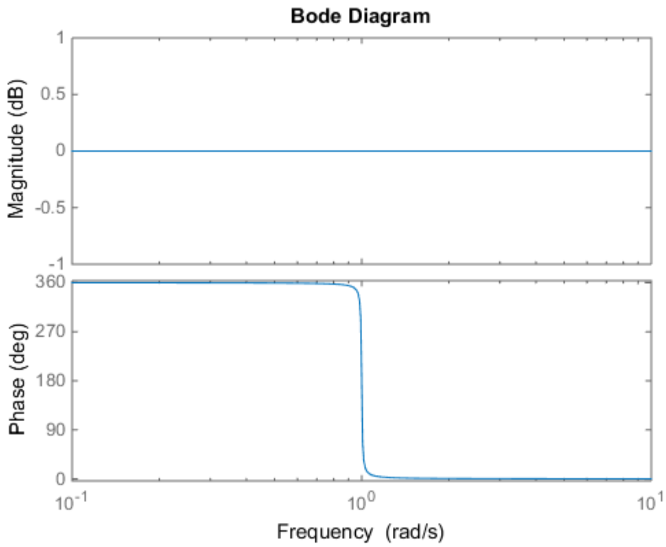
Problem 3. Part D: 2^{nd} order; 2 poles LHP; 1 zero at origin; sinc step response

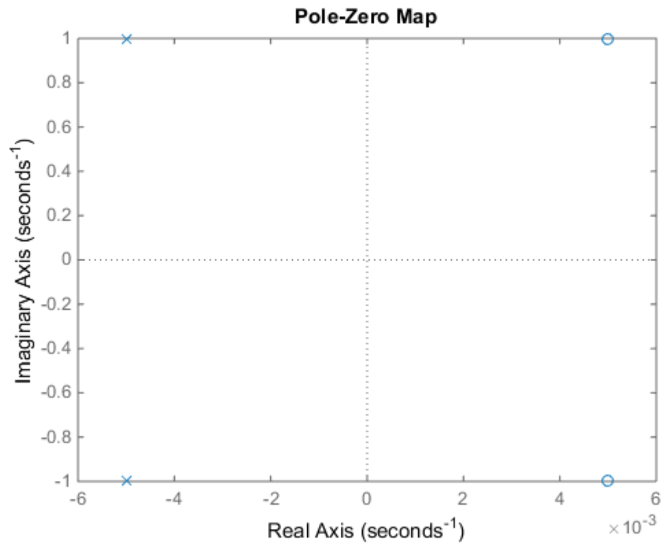


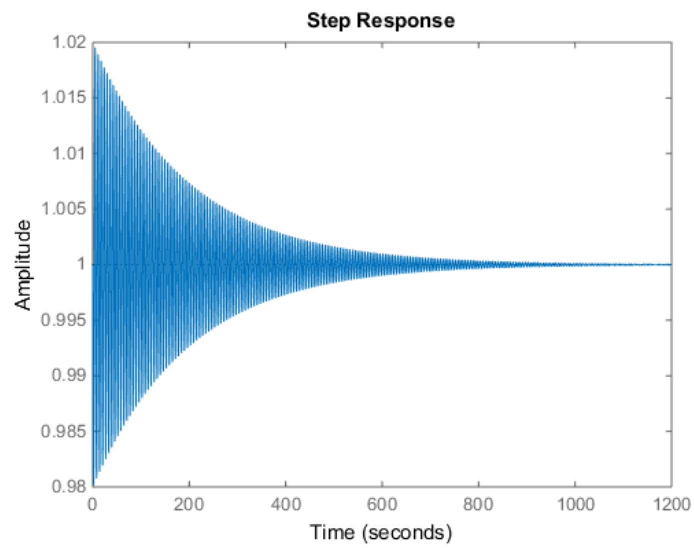




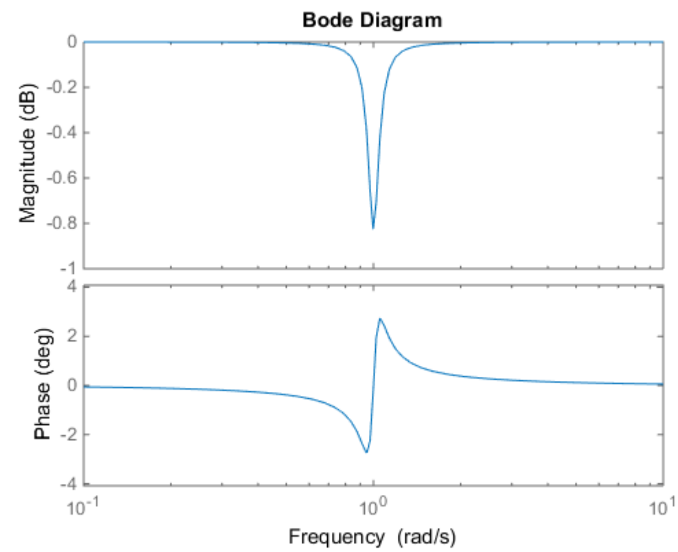
Problem 3. Part E: 2^{nd} order; 2 complex poles LHP; 2 complex zeros RHP; sinc step response with high frequency

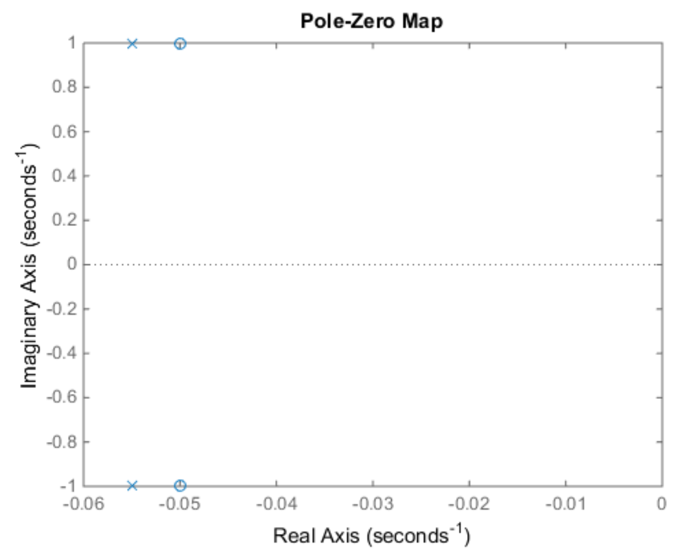


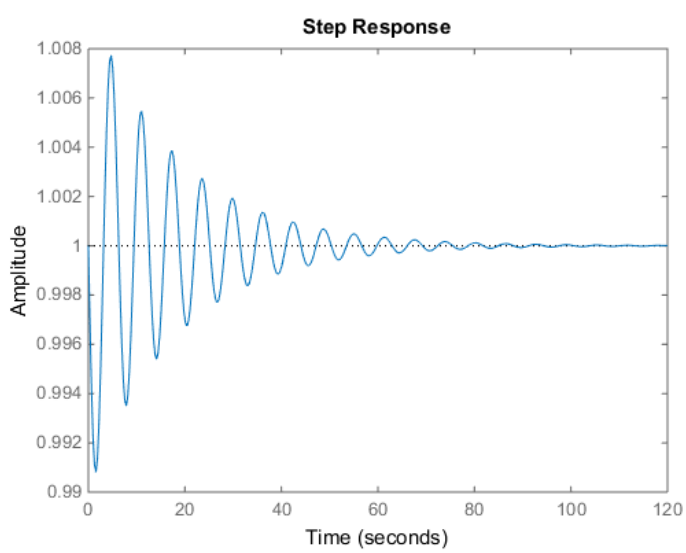




Problem 3. Part F: 2nd order; 2 complex poles LHP; 2 complex zeros LHP; sinc step response







4. A) $H(s) = \frac{1}{s^2 - 0.01s + 1}$

step response of $H(s) = \frac{1}{s(s^2 - 0.01s + 1)}$

$s^2 - 0.01s + 1 = 0$

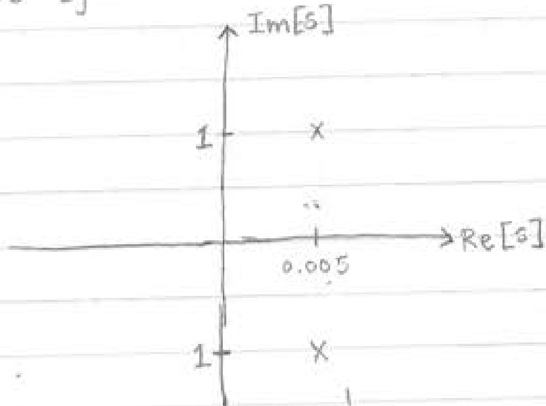
$$s^2 - 0.01s + 1 = 0$$

$$s = \frac{0.01 \pm \sqrt{(0.01)^2 - 4(1)(1)}}{2}$$

$$s = 0.005 \pm \frac{\sqrt{1e\text{-}4 - 4}}{2}$$

$$s = 0.005 \pm \frac{\sqrt{-3.9999}}{2}$$

$$s = 0.005 \pm j$$



B) $\frac{Y}{Y_{sp}} = \frac{KH}{1+KH}$ $K = K_p$ $\lim_{s \to 0} \frac{K_p \cdot \frac{1}{s^2 - 0.01s + 1}}{1 + K_p \cdot \frac{1}{s^2 - 0.01s + 1}} = \frac{K_p}{1 + K_p}$ = DC gain

$$\frac{Y}{Y_{sp}} = \frac{K_p(s^2 - 0.01s + 1)}{s^2 - 0.01s + 1 + K_p}$$

$$s^2 - 0.01s + 1 + K_p = 0$$

$$s = \frac{0.01 \pm \sqrt{(0.01)^2 - 4(1)(1+K_p)}}{2}$$

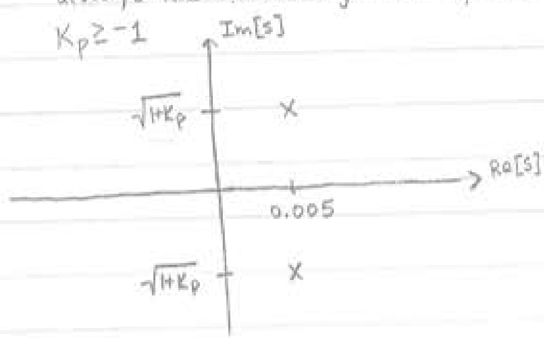
$$s = 0.005 \pm \frac{\sqrt{0.0001 - 4 - 4K_p}}{2}$$

$$s = 0.005 \pm \frac{\sqrt{-3.9999 - 4K_p}}{2}$$

$$s = 0.005 \pm j\sqrt{1 + K_p}$$

No, I cannot stabilize this system. No matter what value of K_p used, one pole will always lie on the right hand plane.

$K_p \geq -1$



4\. C) $\frac{Y}{Y_{sp}} = \frac{KH}{1+KH}$ $K = \frac{K_I}{s}$ $\frac{Y}{Y_{sp}} = \frac{\frac{K_I}{s} \cdot \frac{1}{s^2-0.01s+1}}{1+\frac{K_I}{s} \cdot \frac{1}{s^2-0.01s+1}} = \frac{K_I}{s(s^2-0.01s+1)+K_I(s^2-0.01s+1)+s}$

DC gain $= \lim_{s \to 0} \frac{Y}{Y_{sp}} = \frac{K_I}{K_I} = 1$

$$\frac{Y}{Y_{sp}} = \frac{K_I}{(K_I+s)(s^2-0.01s+1)+s}$$

$$\frac{Y}{Y_{sp}} = \frac{K_I}{s^3-0.01s^2+s+K_I s^2-0.01K_I s+K_I+s} = \frac{K_I}{s^3+(K_I-0.01)s^2+(2-0.01K_I)s+K_I}$$

Yes, this system can be stabilized.

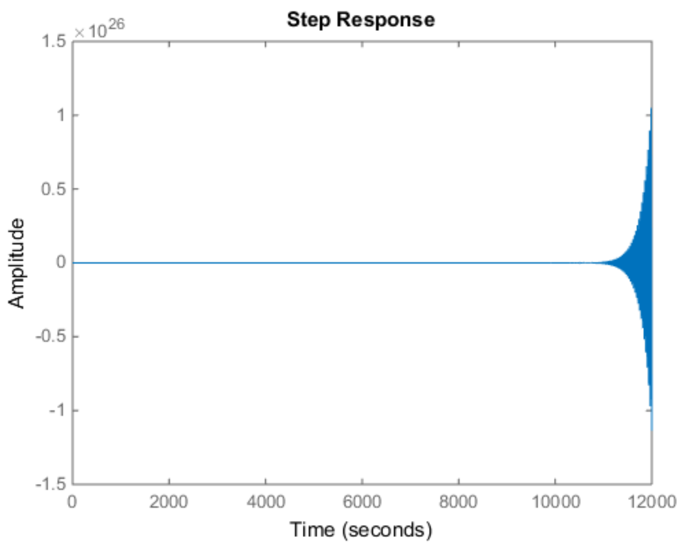
D) $\frac{Y}{Y_{sp}} = \frac{KH}{1+KH}$ $K = K_D s$

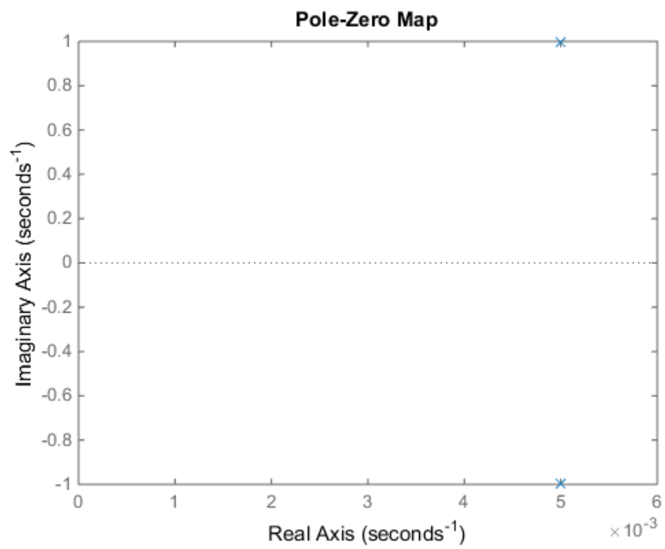
$$\frac{Y}{Y_{sp}} = \frac{K_D s \cdot \frac{1}{s^2-0.01s+1}}{1+K_D s \cdot \frac{1}{s^2-0.01s+1}} = \frac{K_D s}{s^2-0.01s+1+K_D s} = \frac{K_D s}{s^2+(K_D-0.01)s+1}$$

$\frac{Y}{Y_{sp}} = \frac{K_D s}{s^2+(K_D-0.01)s+1}$ $\lim_{s \to 0} \frac{K_D s}{s^2+(K_D-0.01)s+1} = 0$

Yes, this system can be stabilized.

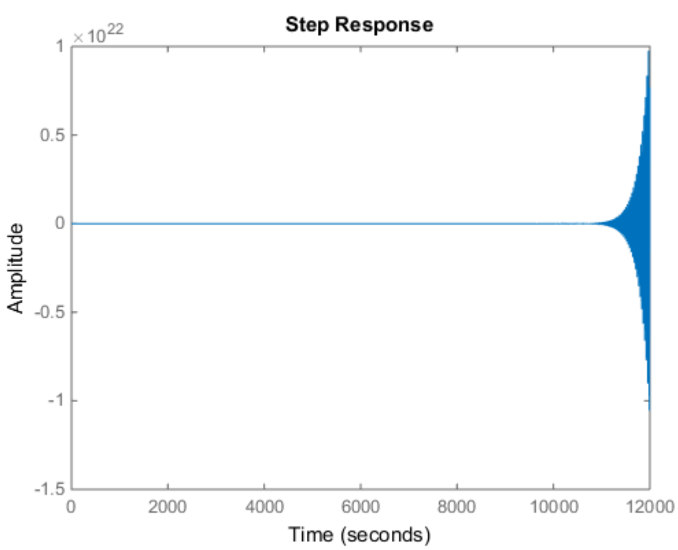
Problem 4. Part A.

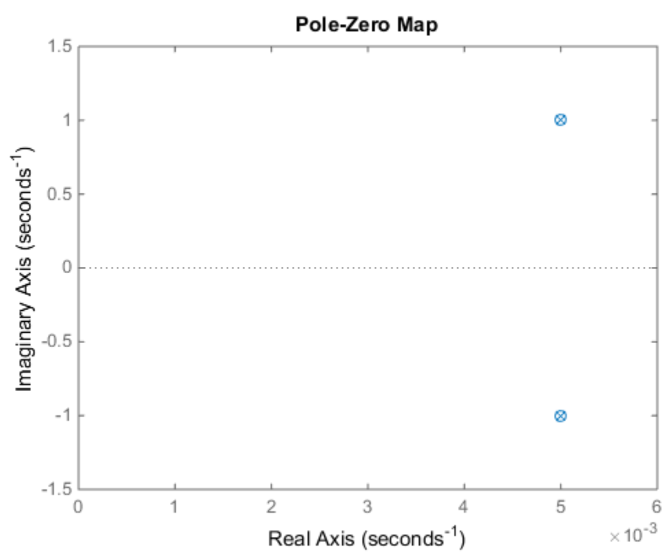




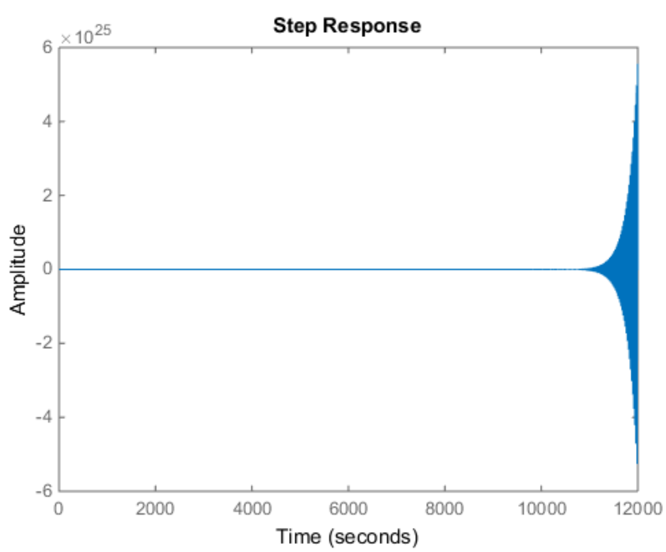
Problem 4. Part B.

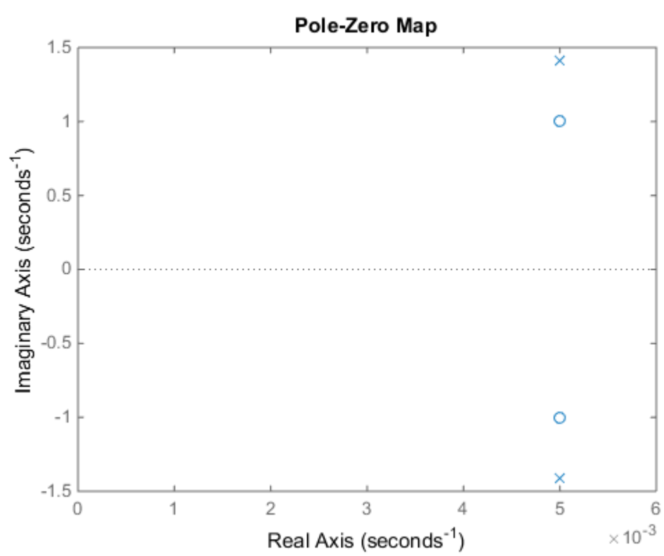
K=0.01





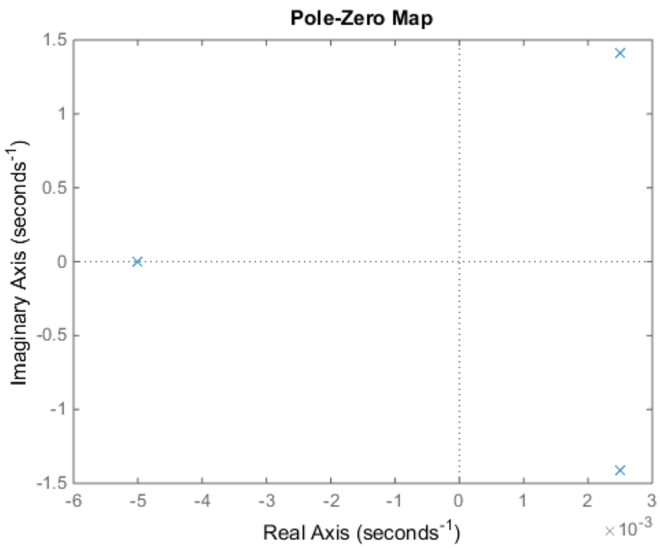
K=1

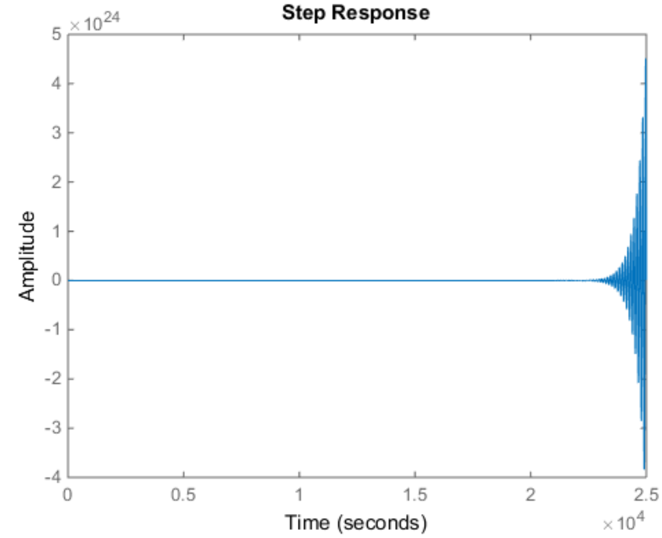




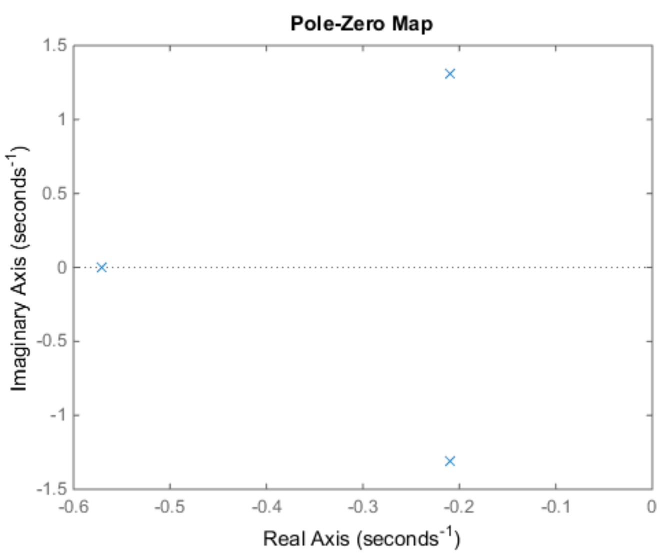
Problem 4. Part C.

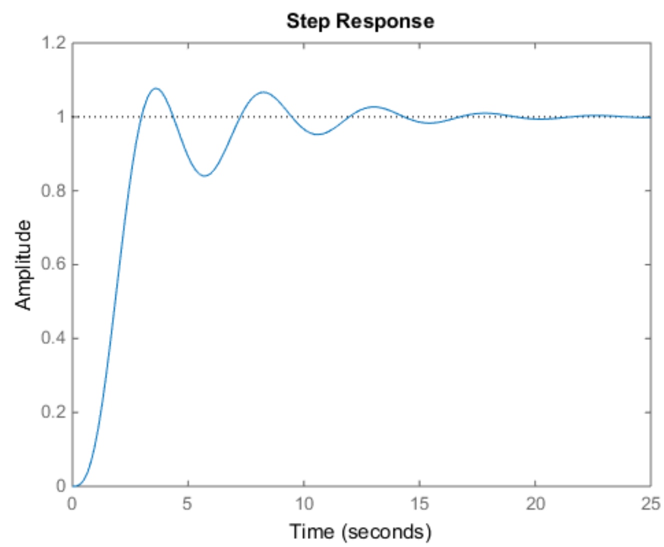
K=0.01





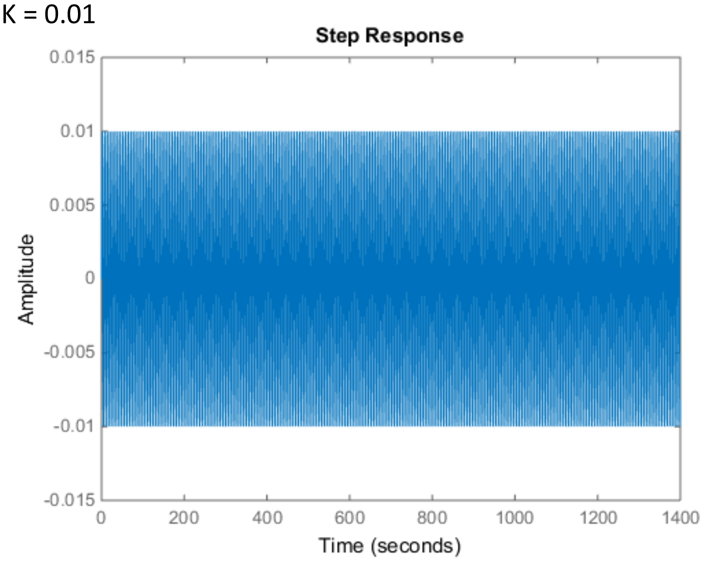
K=1

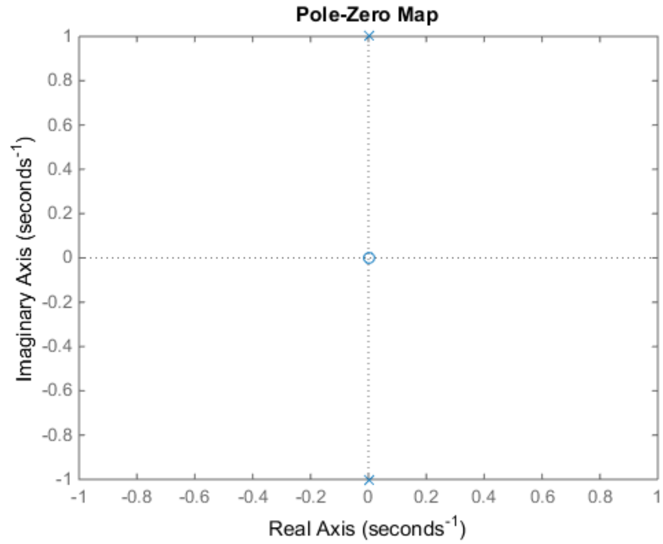




Problem 4. Part D.

K = 0.01





K = 1