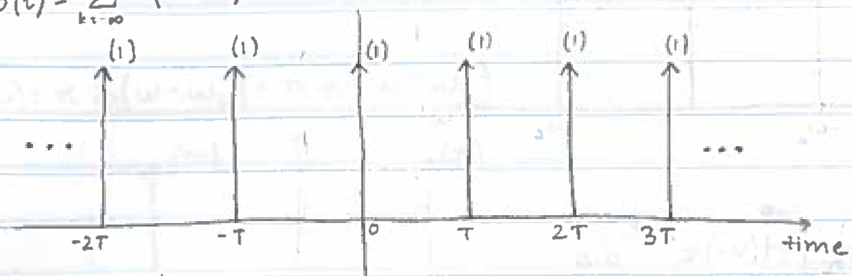
Pratool Gadtaula
PS07

1. a) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$



b) $$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$C_k = \frac{1}{T} \lim_{a\to 0^+}\left[\lim_{b\to 0^-}\left(\int_b^a p(t)\, e^{-j\frac{2\pi}{T}kt}\, dt\right)\right]$$

$$C_k = \frac{1}{T}\cdot(1)$$

$$\mathcal{F}\left[p(t)\right] = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\frac{2\pi}{T}kt}$$

$\mathcal{F}[\ ] \rightarrow$ Fourier series of $[\ ]$

c) $$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$\omega_0 = \frac{2\pi}{T}$

$$X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 kt}\, e^{-j\omega t}\, dt$$

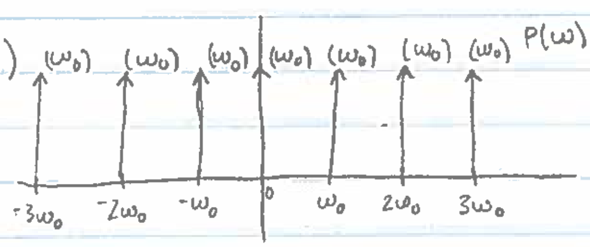
$$X(\omega) = C_k \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{jt(\omega_0 k - \omega)}\, dt$$

$\downarrow$ picking property

$$X(\omega) = C_k \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 k)\cdot \omega_0$$

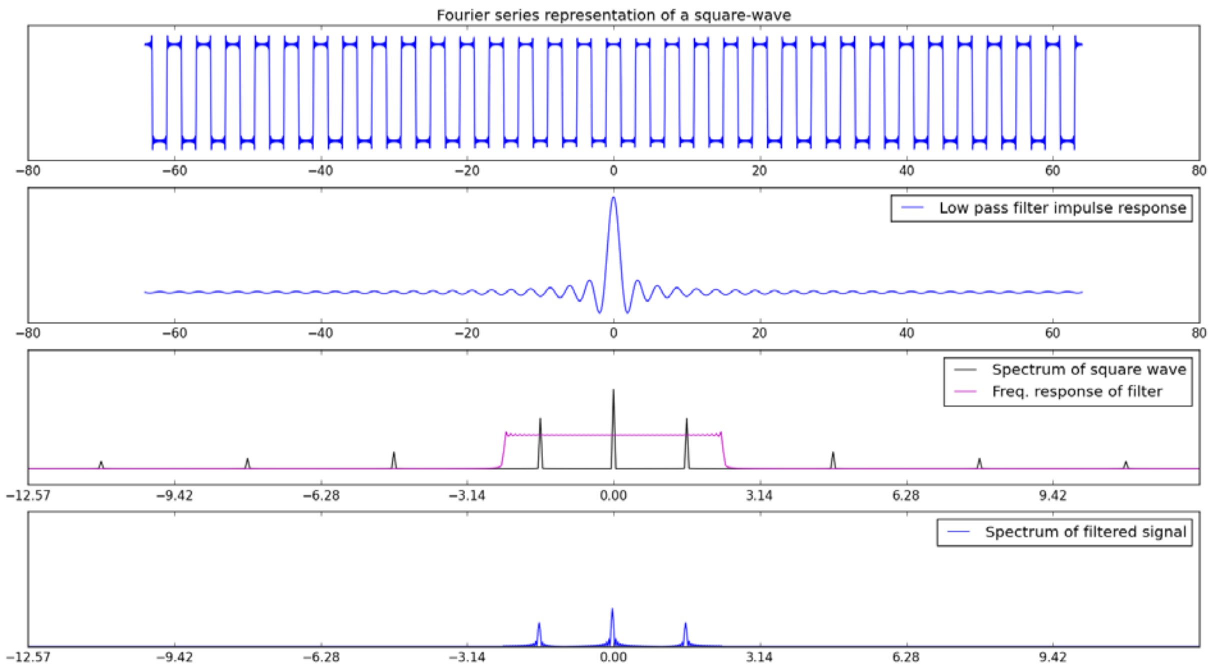
d) $$P(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 k)\cdot \omega_0$$

e)

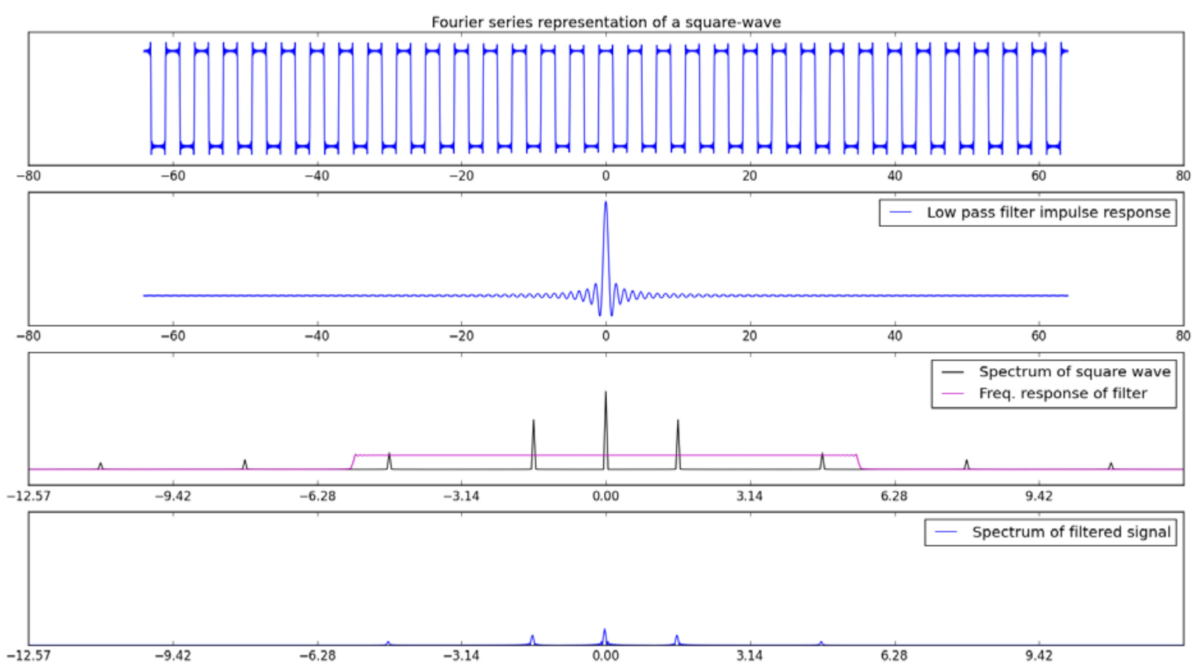


Increasing T causes the unit impulses in the time domain to be more spread out and the unit impulses in the frequency domain to be more closely spaced together.

Question 2, part (d)

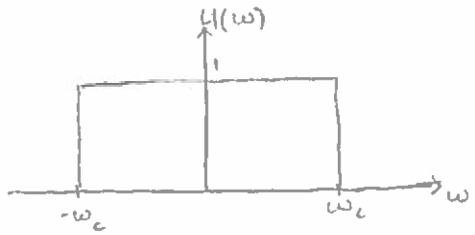


w_c = 0.75 * pi



w_c = 1.75 * pi

2. a)



$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) e^{j\omega t}\, d\omega$$

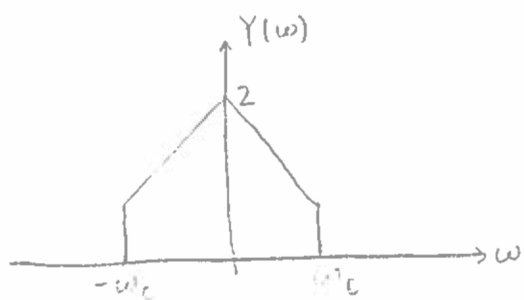
$$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} (1) e^{j\omega t}\, d\omega$$

$$\frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) = \sin\theta$$

$$h(t) = \frac{1}{2\pi}\cdot\frac{1}{jt}\cdot\left(e^{j\omega_c t} - e^{-j\omega_c t}\right)$$

$$h(t) = \frac{1}{\pi t}\cdot \sin(\omega_c t)$$

b)



c) This an ideal low-pass filter because frequencies above the cut-off frequency are completely attenuated, their amplitudes are 0. It also immediately cuts at these frequencies, not steadily cutting them off.

3. $y(t) = x(t)\cos(\omega_c t)$ $\quad Y(\omega) = \frac{1}{2\pi} X(\omega) * \left[\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)\right]$

$Z(\omega) = \pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)$