CS306: DATA ANALYSIS AND VISUALIZATION

LAB 6: Principal Component Analysis (PCA) and visualization of multivariate data

STUDENT ID: 201801407

NAME: PRATVI SHAH

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
%matplotlib inline
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler,RobustScaler
from numpy import linalg as LA
import seaborn as sns
import matplotlib.colors as mcolors
import warnings
warnings.filterwarnings('ignore')
```

In [2]:
```
data=pd.read_excel('New_York_Neighborhoods.xlsx')
```

In [3]:
```
data.head()
```

Out[3]:

	Neighborhood	Affordability	Transit	Shopping & Services	Crime	Food	Schools	Diversity	Creative	Housing Quality	Green Space	Wellness	Nigh
0	Park Slope	73	76	77	82	83	81	73	83	81	84	77	
1	Lower East Side	73	82	83	75	83	76	78	84	72	76	73	
2	Sunnyside	83	76	77	81	73	80	90	72	72	67	72	
3	Cobble Hill & Boerum Hill	73	77	83	76	87	77	71	81	83	82	76	
4	Greenpoint	77	76	75	78	81	92	74	78	80	72	70	

In [4]:
```
df = data.iloc[:, 1:13]
df_cols=df.columns
```

In [5]:
```
df.tail()
```

Out[5]:

	Affordability	Transit	Shopping & Services	Crime	Food	Schools	Diversity	Creative	Housing Quality	Green Space	Wellness	Nightlife
45	74	81	77	63	68	70	79	78	66	77	71	68
46	71	79	67	80	62	69	83	70	77	84	78	65
47	89	71	64	67	62	71	76	67	60	70	73	65
48	90	63	66	75	65	66	79	63	67	65	74	65
49	76	78	73	62	70	63	75	75	73	75	71	79

Normalizing

In [6]:
```
def standardize(df2):
    scaler=StandardScaler()
    scaler.fit(df2)
    df2=scaler.transform(df2)
    return df2
```

In [7]:
```
df=pd.DataFrame(standardize(df),columns=df_cols)
cols=[]
for i in range(1,13):
    cols.append('PC{}'.format(i))
df.tail()
```

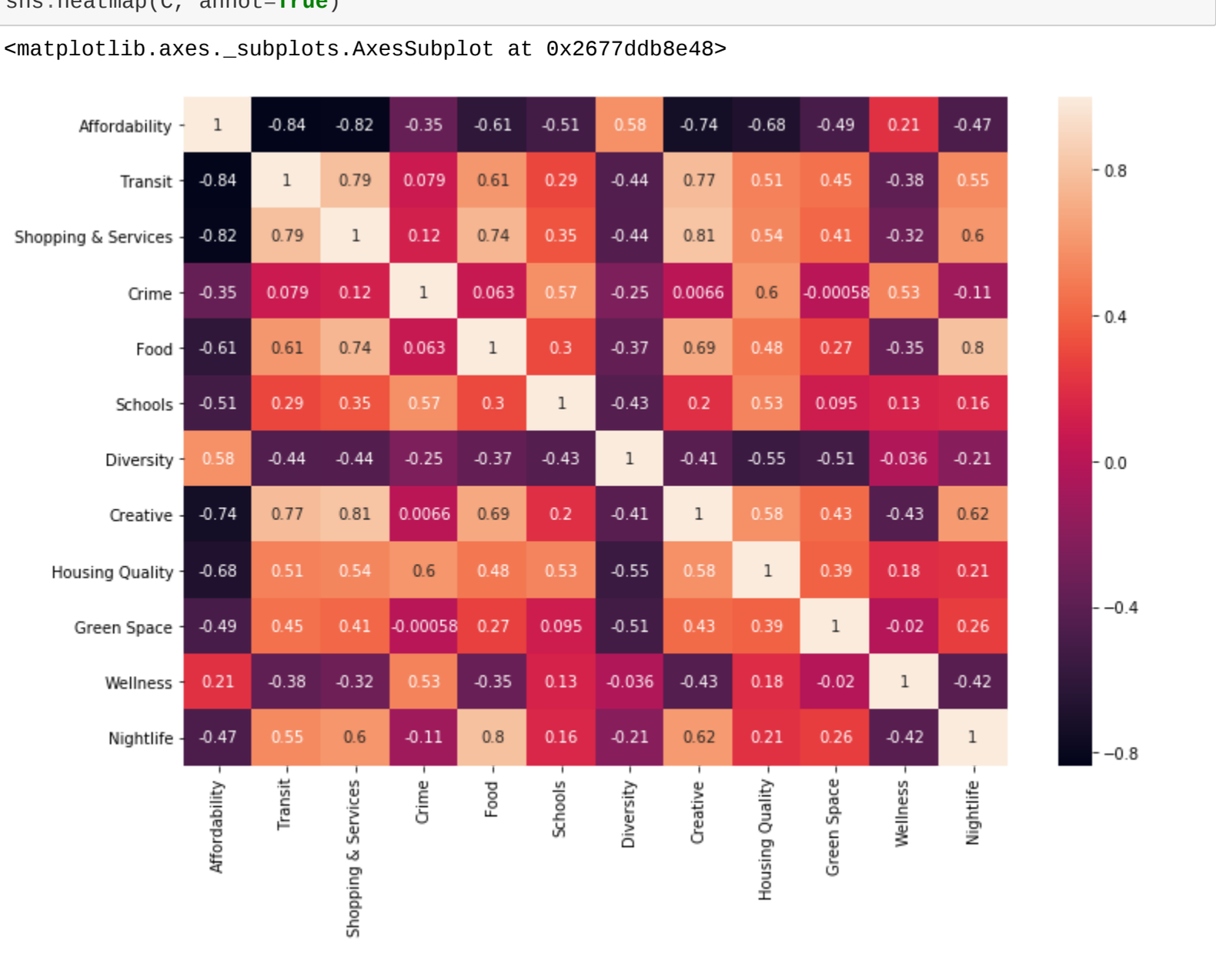
Out[7]:

	Affordability	Transit	Shopping & Services	Crime	Food	Schools	Diversity	Creative	Housing Quality	Green Space	Wellness
45	0.058376	0.460124	0.095132	-1.719332	-0.979286	-0.731168	0.563159	0.217039	-1.338694	0.156967	-1.009883
46	-0.206971	0.241017	-1.226141	0.458659	-1.664102	-0.857667	1.149783	-0.780840	0.148744	1.278157	0.641795
47	1.385112	-0.635409	-1.622523	-1.206863	-1.664102	-0.604668	0.123191	-1.155045	-2.150023	-0.964224	-0.537975
48	1.473561	-1.511835	-1.358268	-0.181926	-1.321694	-1.237166	0.563159	-1.653984	-1.203472	-1.765074	-0.302021
49	0.235274	0.131464	-0.433377	-1.847449	-0.751014	-1.616665	-0.023465	-0.157166	-0.392143	-0.163373	-1.009883

Q1 Pearson correlation matrix

In [8]:
```
C=pd.DataFrame(df).corr()
W,V=LA.eig(C)
plt.figure(figsize=[12,8])
sns.heatmap(C, annot=True)
```

Out[8]: <matplotlib.axes._subplots.AxesSubplot at 0x2677ddb8e48>



The correlation matrix shows how a few attributes can be used to determine the rest. Like if the correlation between two features is high then we can express one in terms of another and hence, can reduce the dimension required to correctly represent the data.

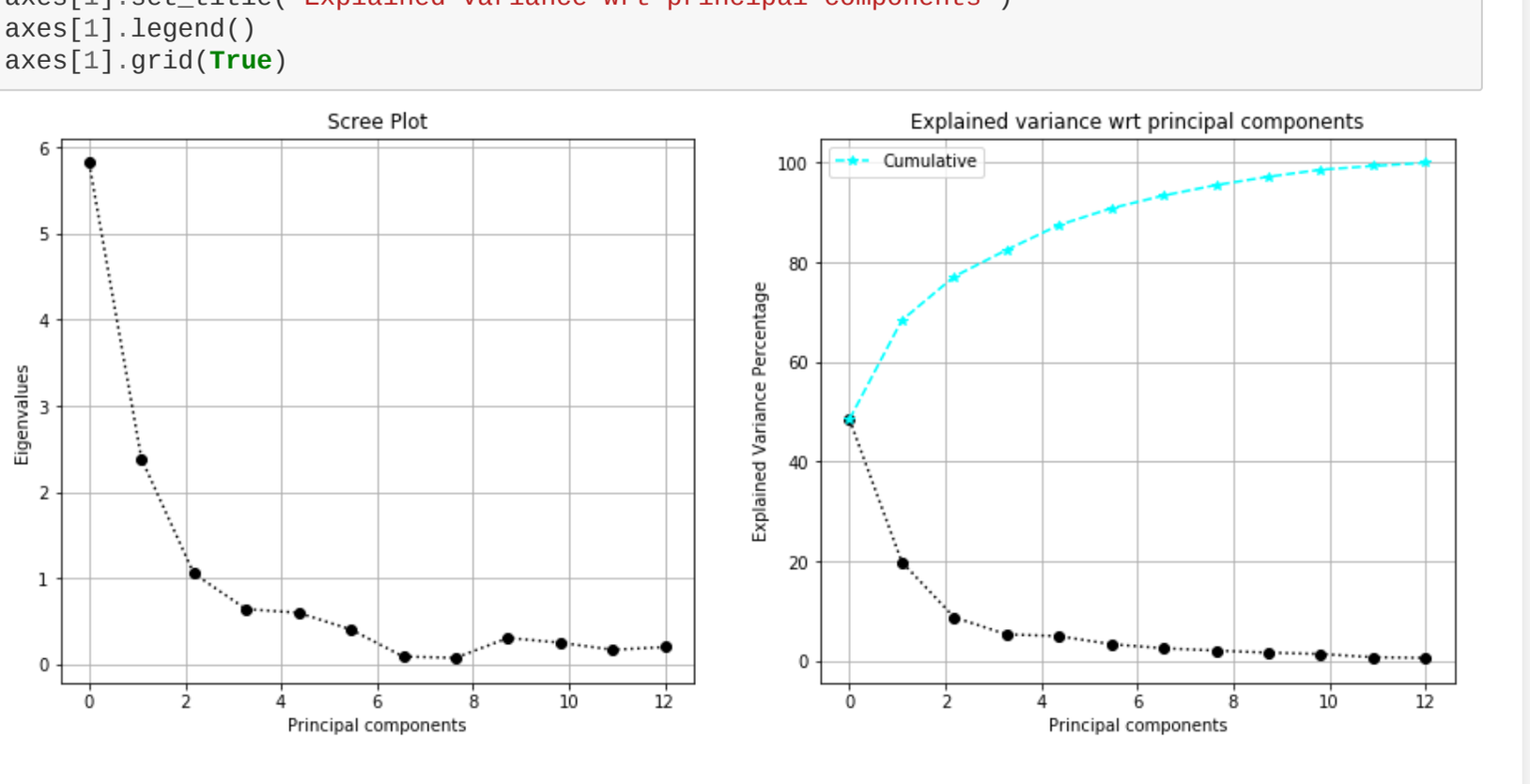
Q2

Performing PCA

In [9]:
```
pca = PCA(n_components=12)
pca=pca.fit(df)
df_pca=pca.transform(df)
df_new=pca.inverse_transform(df_pca)
```

Q2 a) Percentage variance explained by each principal component

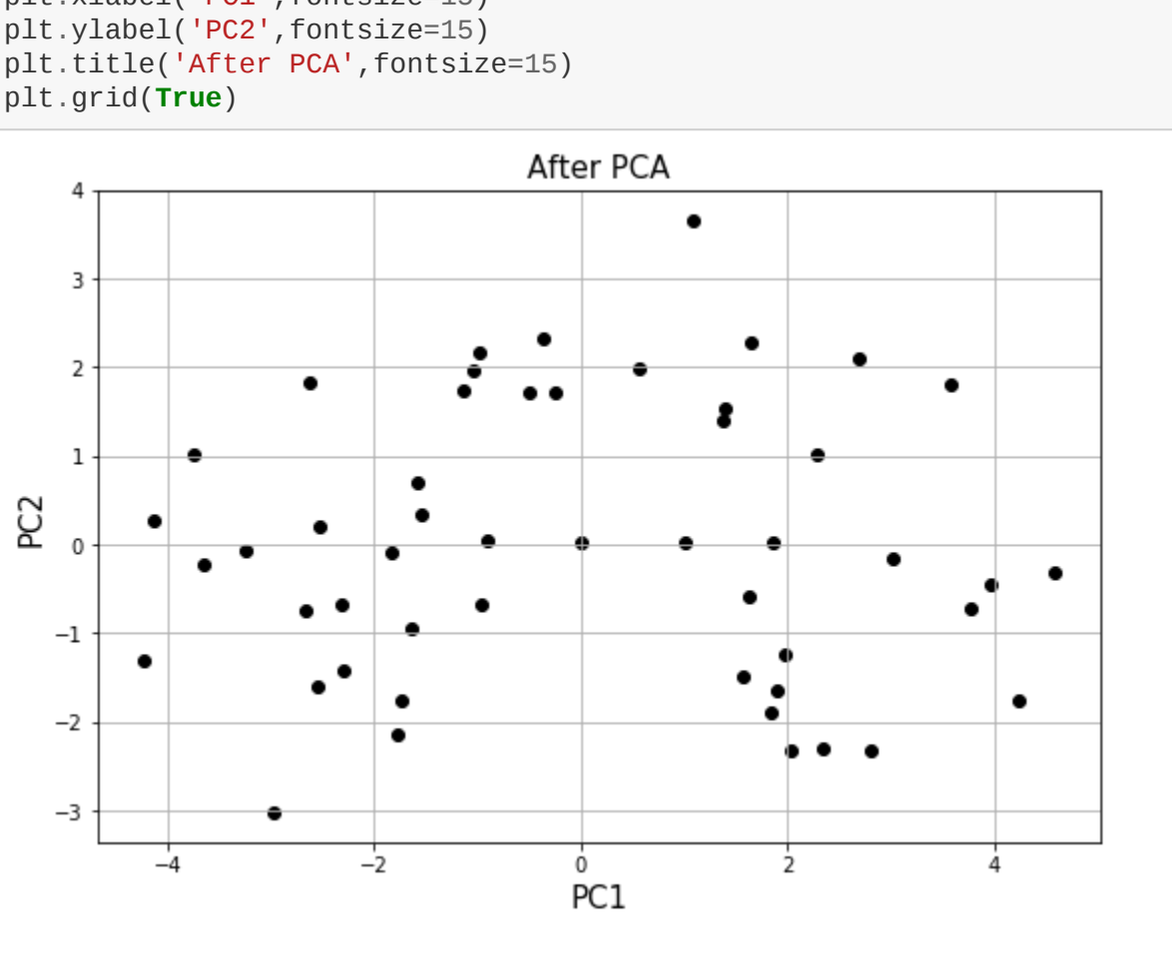
In [10]:
```
fig,axes = plt.subplots(1,2,figsize=[15,6])
axes[0].plot(np.linspace(0,pca.n_components_,pca.n_components_),W,':o',color='k')
axes[0].set_xlabel('Principal components')
axes[0].set_ylabel('Eigenvalues')
axes[0].set_title('Scree Plot')
axes[0].grid(True)
#ref: https://blogs.sas.com/content/iml/2019/11/04/interpret-graphs-principal-components.htm
l


axes[1].plot(np.linspace(0,pca.n_components_,pca.n_components_),pca.explained_variance_ratio
_*100,':o',color='k')
axes[1].plot(np.linspace(0,pca.n_components_,pca.n_components_),np.cumsum(pca.explained_vari
ance_ratio_*100),'--*',label='Cumulative',color='cyan')
axes[1].set_xlabel('Principal components')
axes[1].set_ylabel('Explained Variance Percentage')
axes[1].set_title('Explained variance wrt principal components')
axes[1].legend()
axes[1].grid(True)
```



We can see from the plots that more than 60% of the variance is explained by only 2 Principal components hence, we can use these two components to explain the features in transformed planes.

Q2 b) Scatter plot each samples with x & y axis as PC1 and PC2

In [11]:
```
plt.figure(figsize=[9,6])
plt.scatter(df_pca[:,0], df_pca[:,1], c='k')
plt.xlabel('PC1',fontsize=15)
plt.ylabel('PC2',fontsize=15)
plt.title('After PCA',fontsize=15)
plt.grid(True)
```



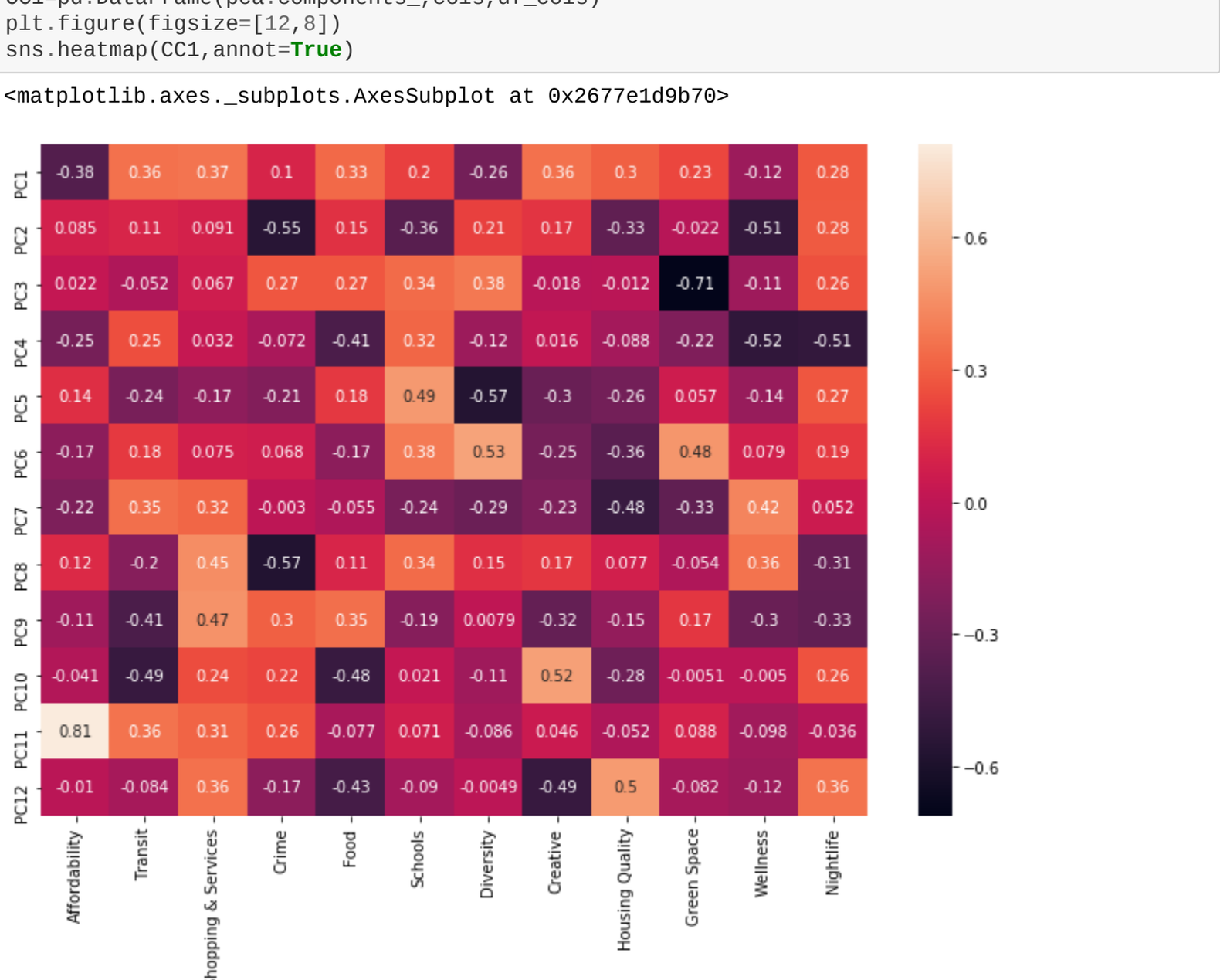
This plot shows how the observations/samples are transformed after performing PCA.

Q2 c) Graph the variables as unit vector using their projection values on PC1 and PC2.

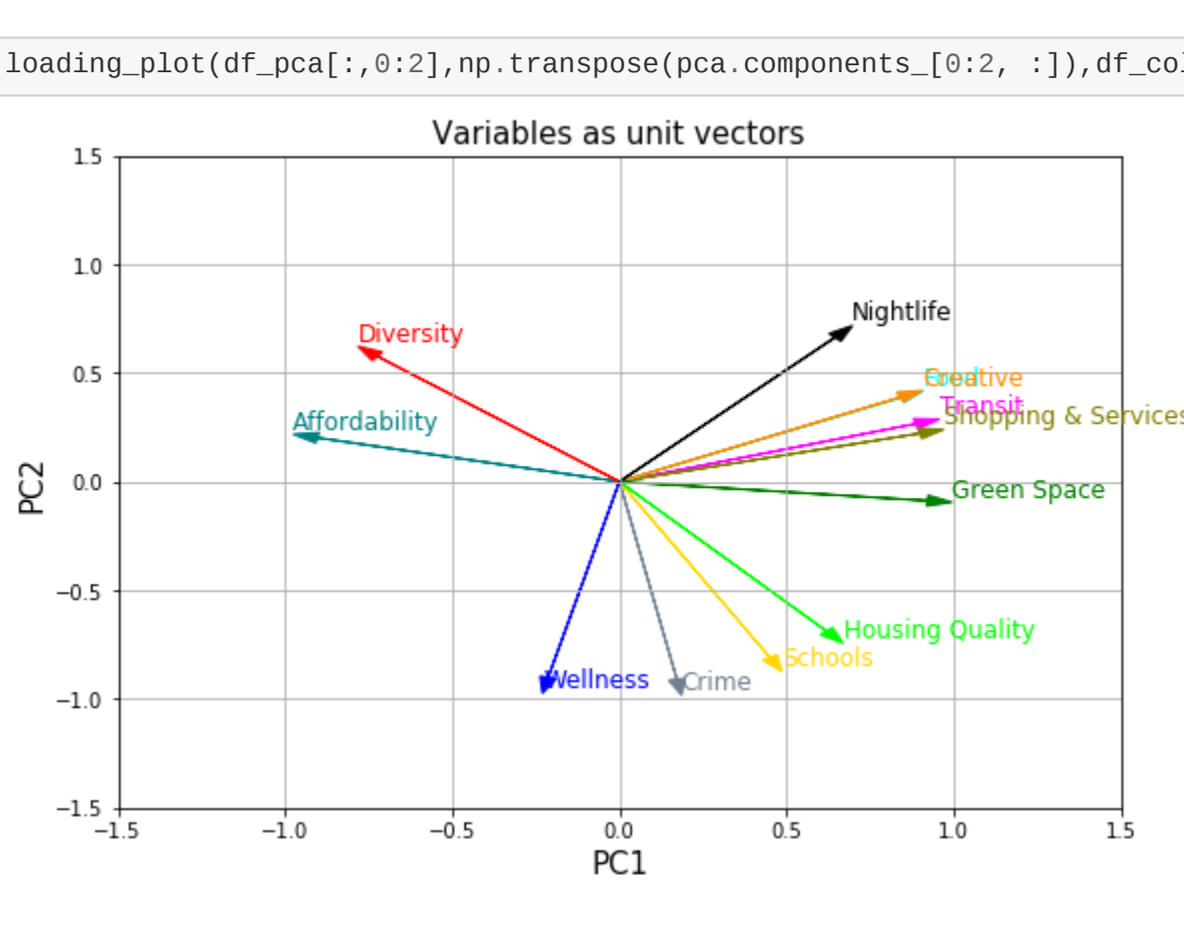
In [12]:
```
def loading_plot(df4,comp,labels):
    #df4 representa the data
    #comp represents the components

    clr=mcolors.CSS4_COLORS
    color=[clr['teal'],clr['magenta'],clr['olive'],clr['slategray'],
           clr['cyan'],clr['gold'],clr['red'],clr['darkorange'],clr['lime'], clr['green'],cl
r['blue'],clr['black']]

    xs = df4[:,0]
    ys = df4[:,1]
    n = comp.shape[0]


    plt.figure(figsize=[9,6])
    for i in range(n):
        scale_var = 1.0/np.sqrt((comp[i,0]*comp[i,0] + comp[i,1]*comp[i,1]))
        plt.arrow(0, 0, comp[i,0]*scale_var, comp[i,1]*scale_var,color = color[i],
                  head_width=0.05,length_includes_head=True)
        plt.text(comp[i,0]*scale_var, comp[i,1]*scale_var, labels[i], color = color[i], ha =
'left', va = 'bottom',fontsize=12)

    plt.xlim(-1.5,1.5)
    plt.ylim(-1.5,1.5)
    plt.xlabel("PC1",fontsize=15)
    plt.ylabel("PC2",fontsize=15)
    plt.title('Variables as unit vectors',fontsize=15)
    plt.grid()
```

In [13]:
```
CC1=pd.DataFrame(pca.components_,cols,df_cols)
plt.figure(figsize=[12,8])
sns.heatmap(CC1,annot=True)
```

Out[13]: <matplotlib.axes._subplots.AxesSubplot at 0x2677e1d9b70>



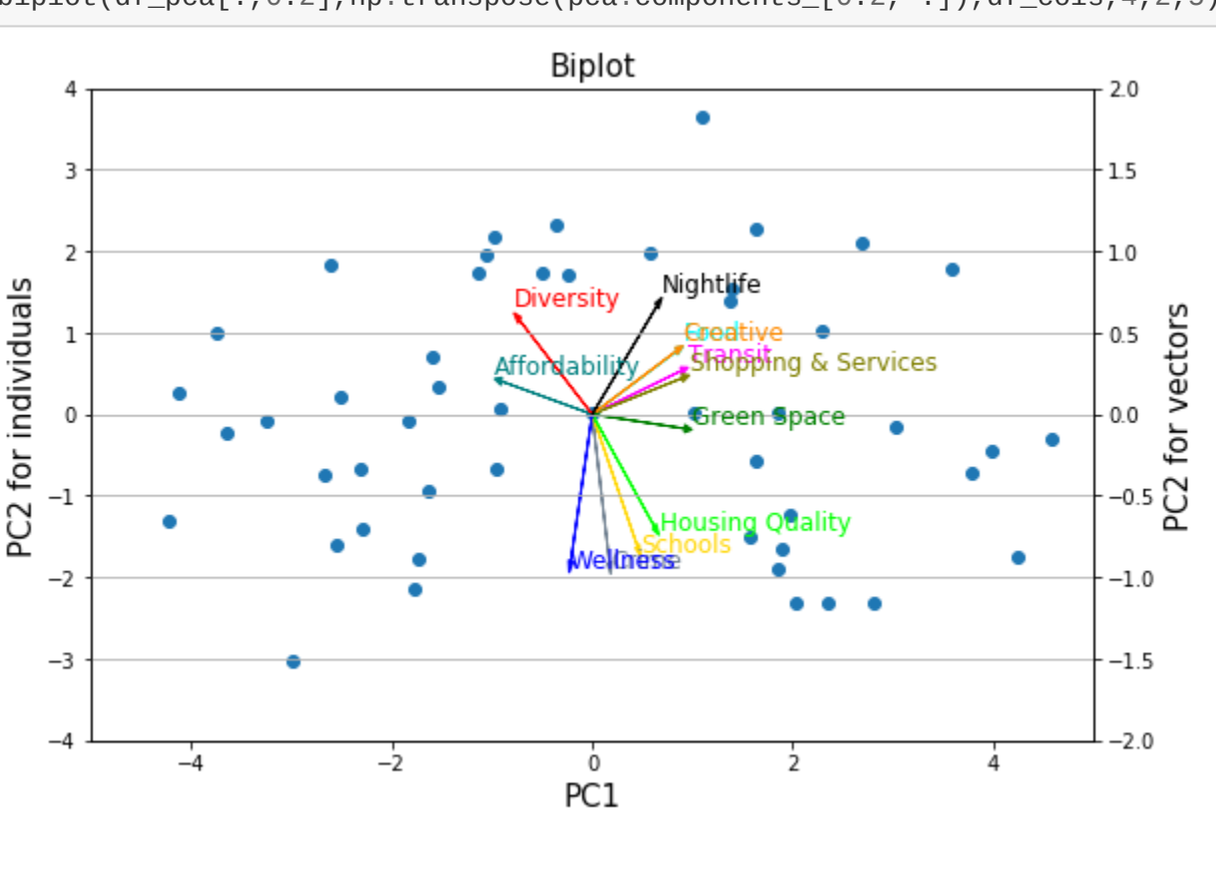
In [14]:
```
loading_plot(df_pca[:,0:2],np.transpose(pca.components_[0:2, :]),df_cols)
```



Q2 d) Biplot both individuals/observations and the variables.

In [15]:
```
def biplot(df5,comp,labels,ylim,ylim2,xlim=None):
    clr=mcolors.CSS4_COLORS
    xs = df5[:,0]
    ys = df5[:,1]
    n = comp.shape[0]
    color=[clr['teal'],clr['magenta'],clr['olive'],clr['slategray'],
           clr['cyan'],clr['gold'],clr['red'],clr['darkorange'],clr['lime'], clr['green'],cl
r['blue'],clr['black']]


    fig,ax = plt.subplots(figsize=[9,6])
    ax.scatter(xs,ys)
    ax.set_ylabel('PC2 for individuals',fontsize=15)
    ax.set_ylim(-ylim,ylim)
    ax2=ax.twinx()
    for i in range(n):
        scale_var = 1.0/np.sqrt((comp[i,0]*comp[i,0] + comp[i,1]*comp[i,1]))
        ax2.arrow(0, 0, comp[i,0]*scale_var, comp[i,1]*scale_var,color = color[i],
                  head_width=0.05,length_includes_head=True)
        ax2.text(comp[i,0]*scale_var, comp[i,1]*scale_var, labels[i], color = color[i], ha =
'left', va = 'bottom',fontsize=12)
    if xlim!=None:
        ax.set_xlim(-xlim,xlim)
    ax.set_xlabel("PC1",fontsize=15)
    ax2.set_ylabel("PC2 for vectors",fontsize=15)
    ax2.set_ylim(-ylim2,ylim2)
    plt.title('Biplot',fontsize=15)
    plt.grid(True)
```

In [16]:
```
biplot(df_pca[:,0:2],np.transpose(pca.components_[0:2, :]),df_cols,4,2,5)
```



Q3 Introducing outliers

In [17]:
```
data2=data.iloc[:,1:13]
cols=[]
for i in range(1,13):
    cols.append('PC{}'.format(i))

arr=[70, 70, 700, 80, 83, 71, 600, 70, 65, 900, 45, 800]
arr2=[77, 600, 72, 82, 800, 73, 65, 900, 62, 75, -500, 80]

df2 = {}
for i in range(len(cols)):
    df2[df_cols[i]]=arr[i]
data2=data2.append(df2,ignore_index=True)
df2 = {}
for i in range(len(cols)):
    df2[df_cols[i]]=arr2[i]
data2=data2.append(df2,ignore_index=True)
data3=data2
#NEW DATA
data2.tail()
```

Out[17]:

	Affordability	Transit	Shopping & Services	Crime	Food	Schools	Diversity	Creative	Housing Quality	Green Space	Wellness	Nightlife
47	89	71	64	67	62	71	76	67	60	70	73	65
48	90	63	66	75	65	66	79	63	67	65	74	65
49	76	78	73	62	70	63	75	75	73	75	71	79
50	70	70	700	80	83	71	600	70	65	900	45	800
51	77	600	72	82	800	73	65	900	62	75	-500	80

In [18]:
```
data2=pd.DataFrame(standardize(data2),columns=df_cols)
data2.tail()
```

Out[18]:

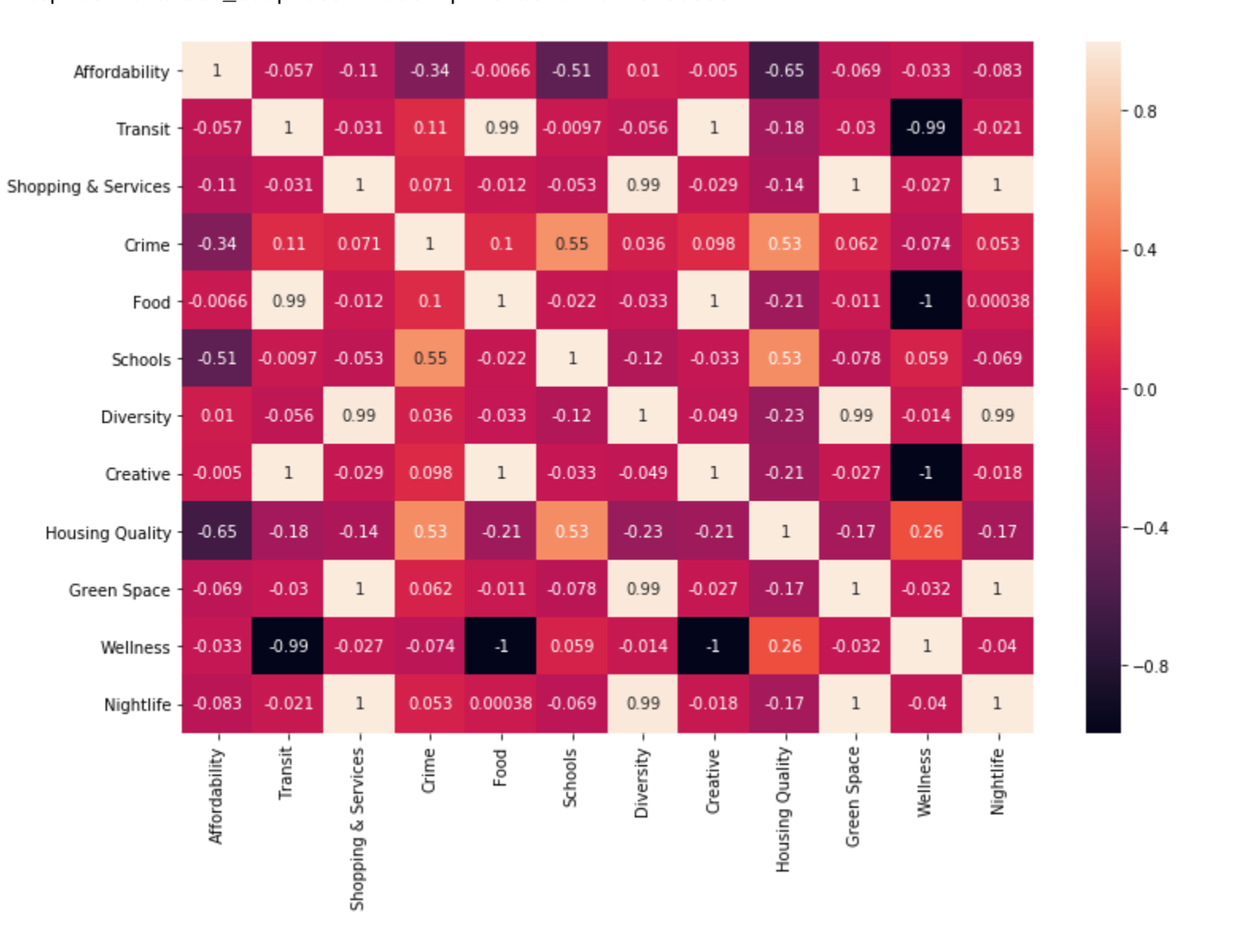
	Affordability	Transit	Shopping & Services	Crime	Food	Schools	Diversity	Creative	Housing Quality	Green Space	Wellness
47	1.409283	-0.217176	-0.281329	-1.245156	-0.286991	-0.595087	-0.125056	-0.220247	-2.018903	-0.192766	0.118335
48	1.499311	-0.327622	-0.258071	-0.207110	-0.256903	-1.237090	-0.083636	-0.255514	-1.102593	-0.236885	0.130970
49	0.238920	-0.120535	-0.176669	-1.893934	-0.206757	-1.622291	-0.138863	-0.149714	-0.317184	-0.148647	0.093064
50	-0.301247	-0.230982	7.114616	0.441669	-0.076377	-0.595087	7.109603	-0.193797	-1.364396	7.130995	-0.235454
51	0.328948	7.086096	-0.188298	0.701180	7.114589	-0.338286	-0.276929	7.124049	-1.757100	-0.148647	-7.121701

Performing PCA Standardized data

In [19]:
```
pca2 = PCA(n_components=12)
pca2=pca2.fit(data2)
data_pca=pca2.transform(data2)
data_new=pca2.inverse_transform(data_pca)
```

In [20]:
```
WW,VV= LA.eig(data2.corr())
plt.figure(figsize=[12,8])
sns.heatmap(data2.corr(),annot=True)
```

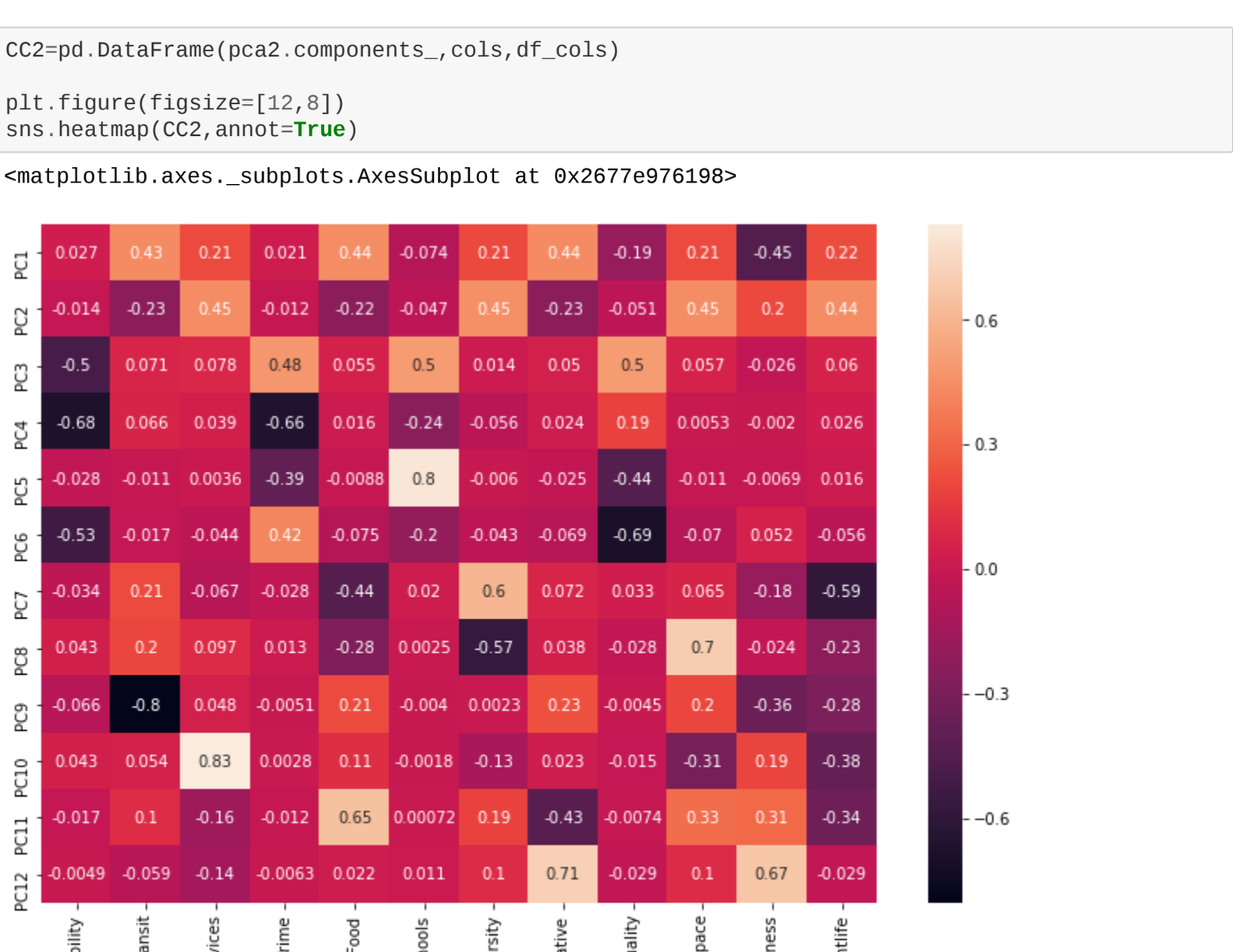
Out[20]: <matplotlib.axes._subplots.AxesSubplot at 0x2677e28c358>



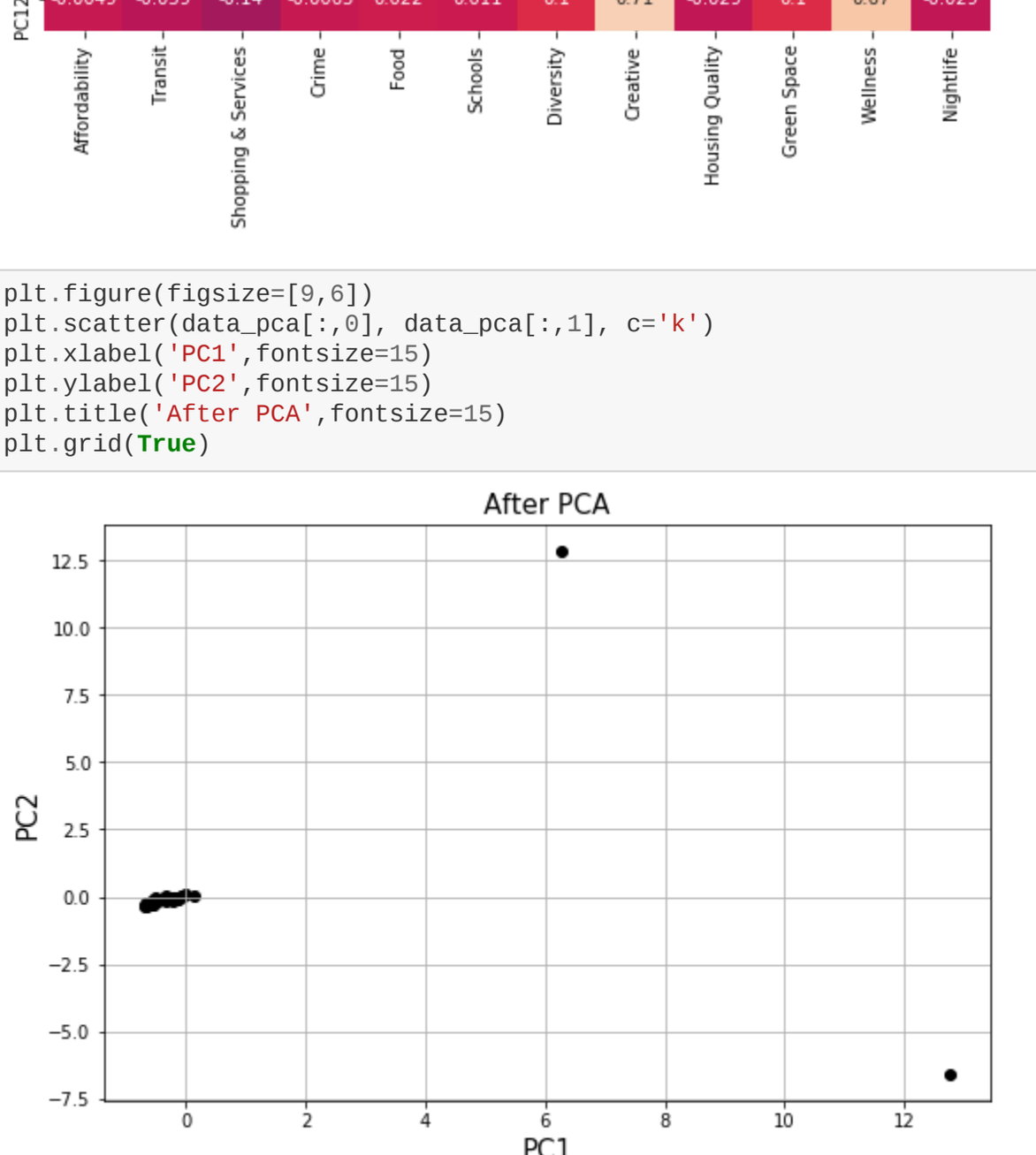
We can see that the covariance matrix has changed completely due to the presence of outliers and standardization after that. This shows that there will be a significant change in the principal components analysis that we will carry out as the relationship/covariance of features have changed.

In [21]:
```
CC2=pd.DataFrame(pca2.components_,cols,df_cols)

plt.figure(figsize=[12,8])
sns.heatmap(CC2,annot=True)
```

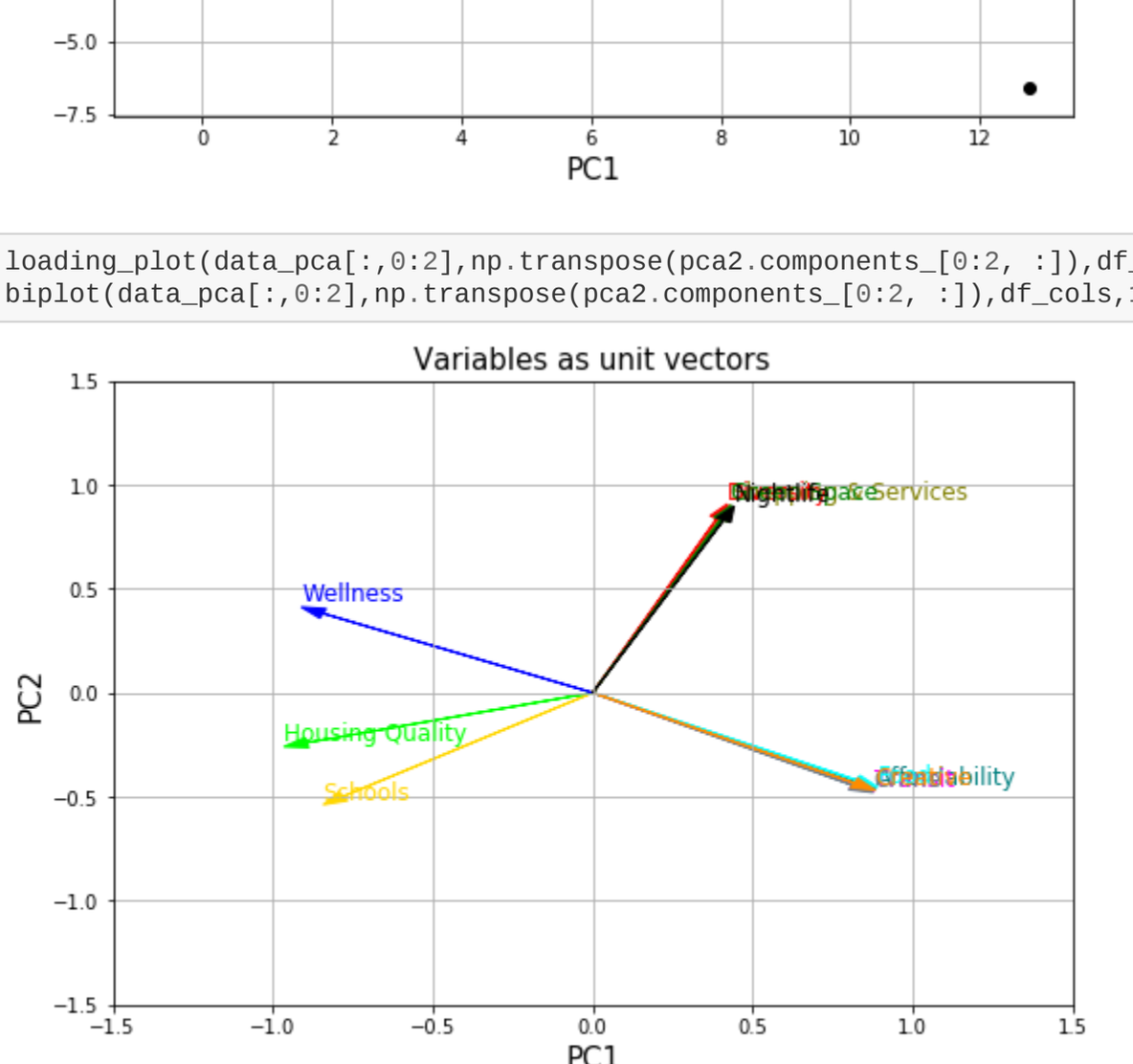
Out[21]: <matplotlib.axes._subplots.AxesSubplot at 0x2677e976198>

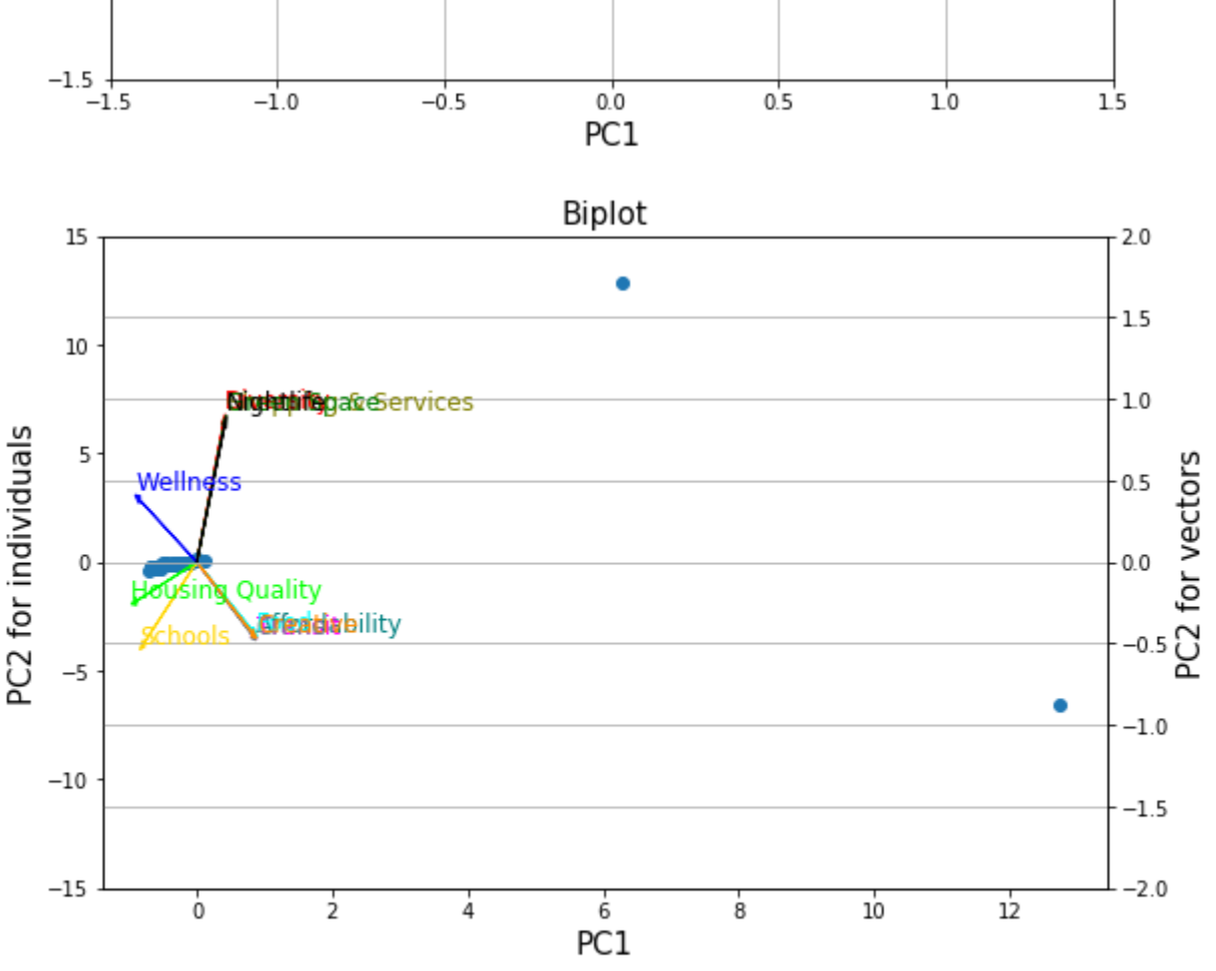


In [22]:
```
plt.figure(figsize=[9,6])
plt.scatter(data_pca[:,0], data_pca[:,1], c='k')
plt.xlabel('PC1',fontsize=15)
plt.ylabel('PC2',fontsize=15)
plt.title('After PCA',fontsize=15)
plt.grid(True)
```



In [23]:
```
loading_plot(data_pca[:,0:2],np.transpose(pca2.components_[0:2, :]),df_cols)
biplot(data_pca[:,0:2],np.transpose(pca2.components_[0:2, :]),df_cols,15,2,None)
```

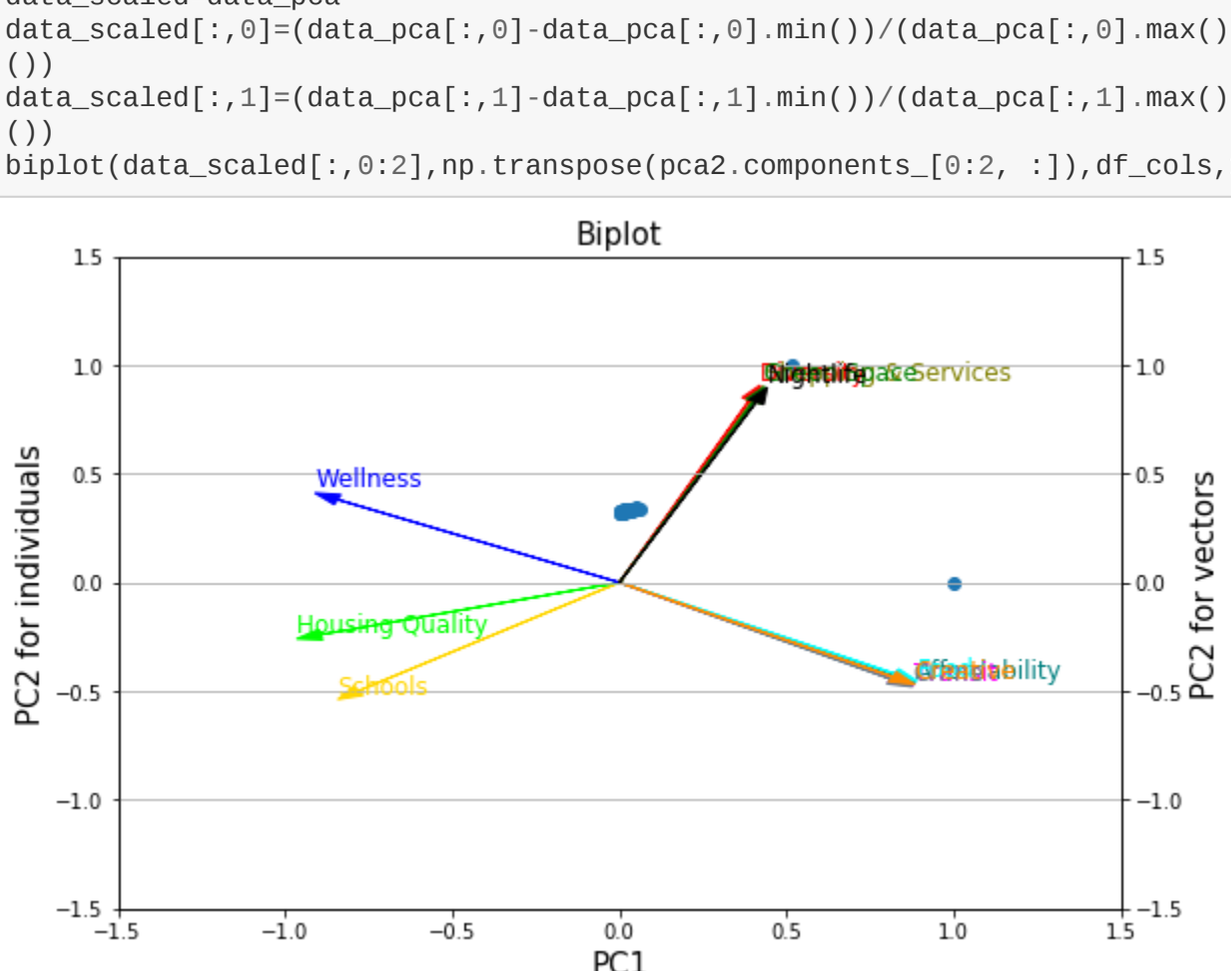




We can see that due to the ouliers the exact scatter plot is not visible and the biplot fails to show how the vectors are aligned towards the observations. In order to better visualize this we can scale the scatter plot using minmax scalling

Scaled Biplot

In [24]:
```
data_scaled=data_pca
data_scaled[:,0]=(data_pca[:,0]-data_pca[:,0].min())/(data_pca[:,0].max()-data_pca[:,0].min
())
data_scaled[:,1]=(data_pca[:,1]-data_pca[:,1].min())/(data_pca[:,1].max()-data_pca[:,1].min
())
biplot(data_scaled[:,0:2],np.transpose(pca2.components_[0:2, :]),df_cols,1.5,1.5,1.5)
```

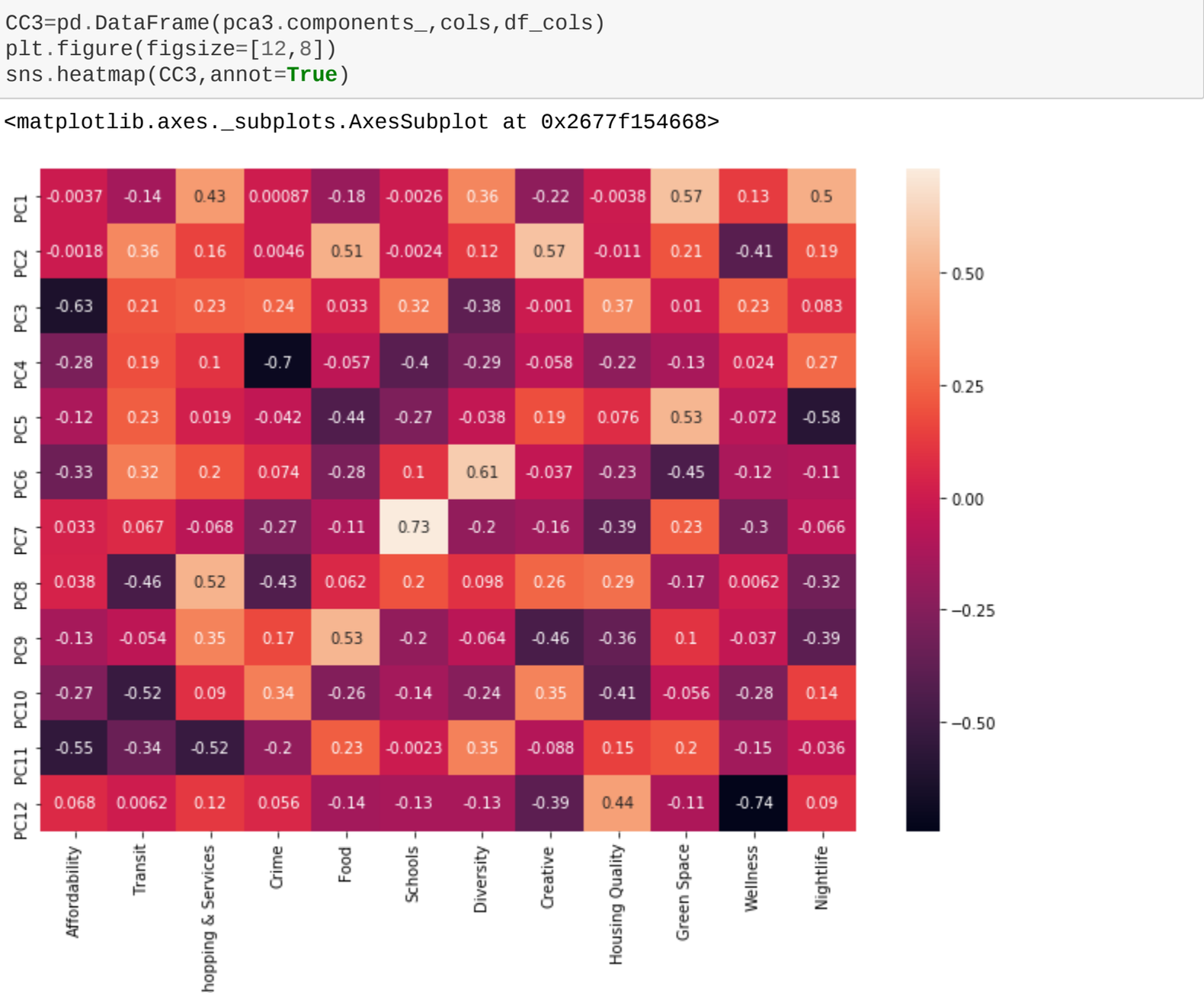


We can see that the scalling of scatter plot gave a better idea as to how the Principal components are changing to accomodate the outliers. The sifting of vectors to point towards the outlier around (0.5,1) in scaled plot shows this.

Performing PCA Unstandardized data

In [25]:
```
pca3 = PCA(n_components=12)
pca3=pca3.fit(data3)
data3_pca=pca3.transform(data3)
data3_new=pca3.inverse_transform(data3_pca)
WW,VV= LA.eig(data3.corr())

CC3=pd.DataFrame(pca3.components_,cols,df_cols)
plt.figure(figsize=[12,8])
sns.heatmap(CC3,annot=True)
```

Out[25]: <matplotlib.axes._subplots.AxesSubplot at 0x2677f154668>



The plots for outliers show that the PCA is highly sensitive to outliers and by introducing a few extreme values can completely change the correlation between the features and hence the principal components also change.

The scaling/standardization also affects the Principal components as the variables are scaled wrt their individual mean and deviation from the mean. This is because the covariance matrix itself changes due to scaling/standardizing. Thus, we can say that any type of scaling or introduction of extreme data points will change the PCs.

In [ ]: