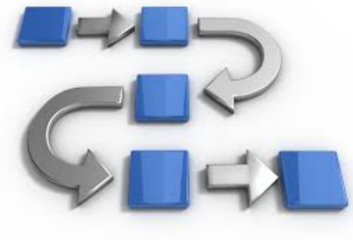
DSP: Part II

Lecture.3.2

Infinite Impulse Response (IIR) Filter

Methodology/Logic for IIR



1) What is the characteristics of an IIR Filter
   - Its impulse response, its difference equation, its H(z), and stability

2) Analysis: From difference equation to H(z), to pole/zero plot and then to frequency response H(ω)

3) IIR Filters are usually designed from analog filters, so let’s examine H(s) the Laplace domain.

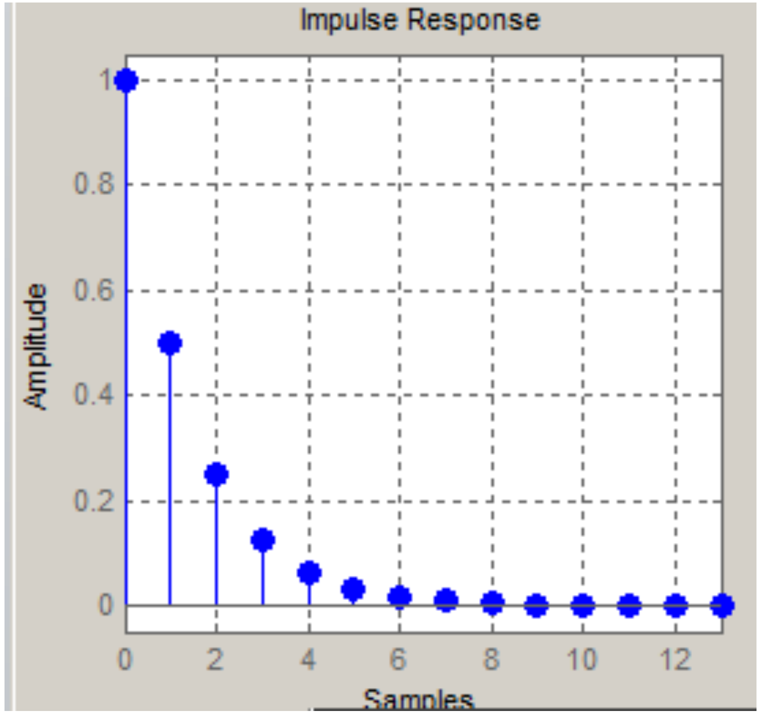
4) Design an analog filter
   a) Simple RC analog filter
   b) Butterworth analog filter
5) Design an IIR filter by converting from H(s) to H(z)--Bilinear transform method
6) Designing of IIR Filters using Matlab.

1) IIR filter and its difference equation

- An digital Infinite Impulse Response (IIR) Filter: as its name implies, has an impulse that goes on to infinity; This infinite response is due to the feedback loop.
- Following is the simplest IIR filter example described using a **difference equation**:

  $y(n) = b\,x(n) - a\,y(n-1)$

- E.g. if $b = 1$, $a = (-0.5)$, then we get the following impulse response.
  $y(n) = x(n) + 0.5y(n-1)$
  - Let $x(n)=\delta(n)$
    - the impulse response is infinite
    - It tends to zero as n tends to infinity.



1) IIR filter and its H(z)

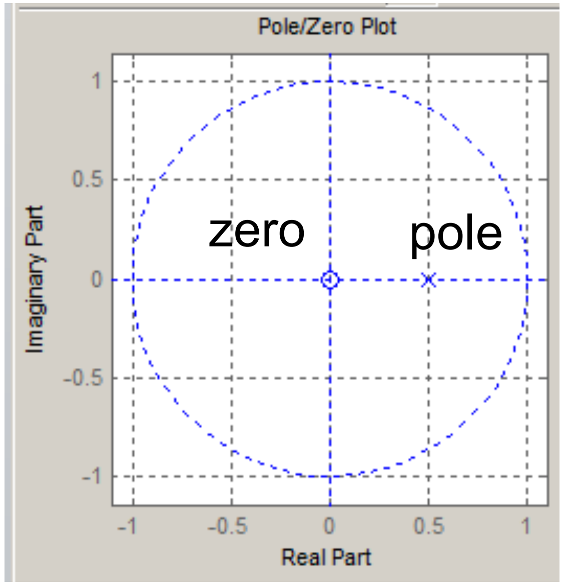
- Given y(n) = x(n) + 0.5 y(n-1), we derive its z-transform to get the transfer function H(z), → relates the output to the input

$$Y(z) = X(z) + 0.5\, Y(z)\, z^{-1}$$

$$Y(z)\,(1 - 0.5\, z^{-1}) = X(z)$$

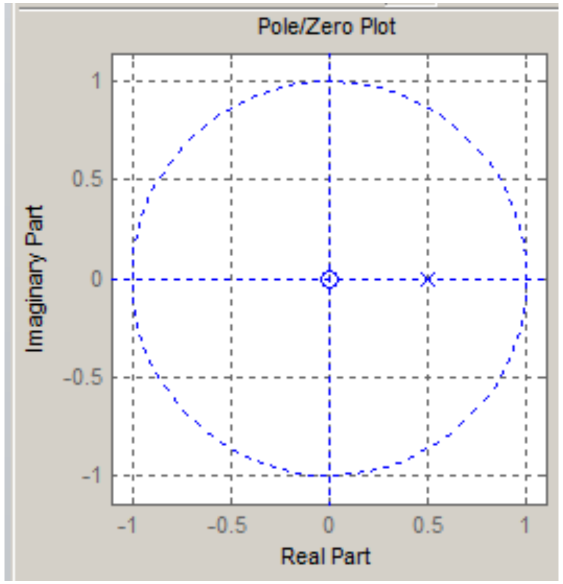
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.5\, z^{-1}} = \frac{z}{z-0.5}$$

- From the above equation, it is clear that the numerator has a root at z = 0 and the denominator has a root at 0.5
- The numerator's roots cause H(z) to be zero => they are known as zeros of H(z)
- The denominator's roots cause H(z) to be infinite => they are called poles

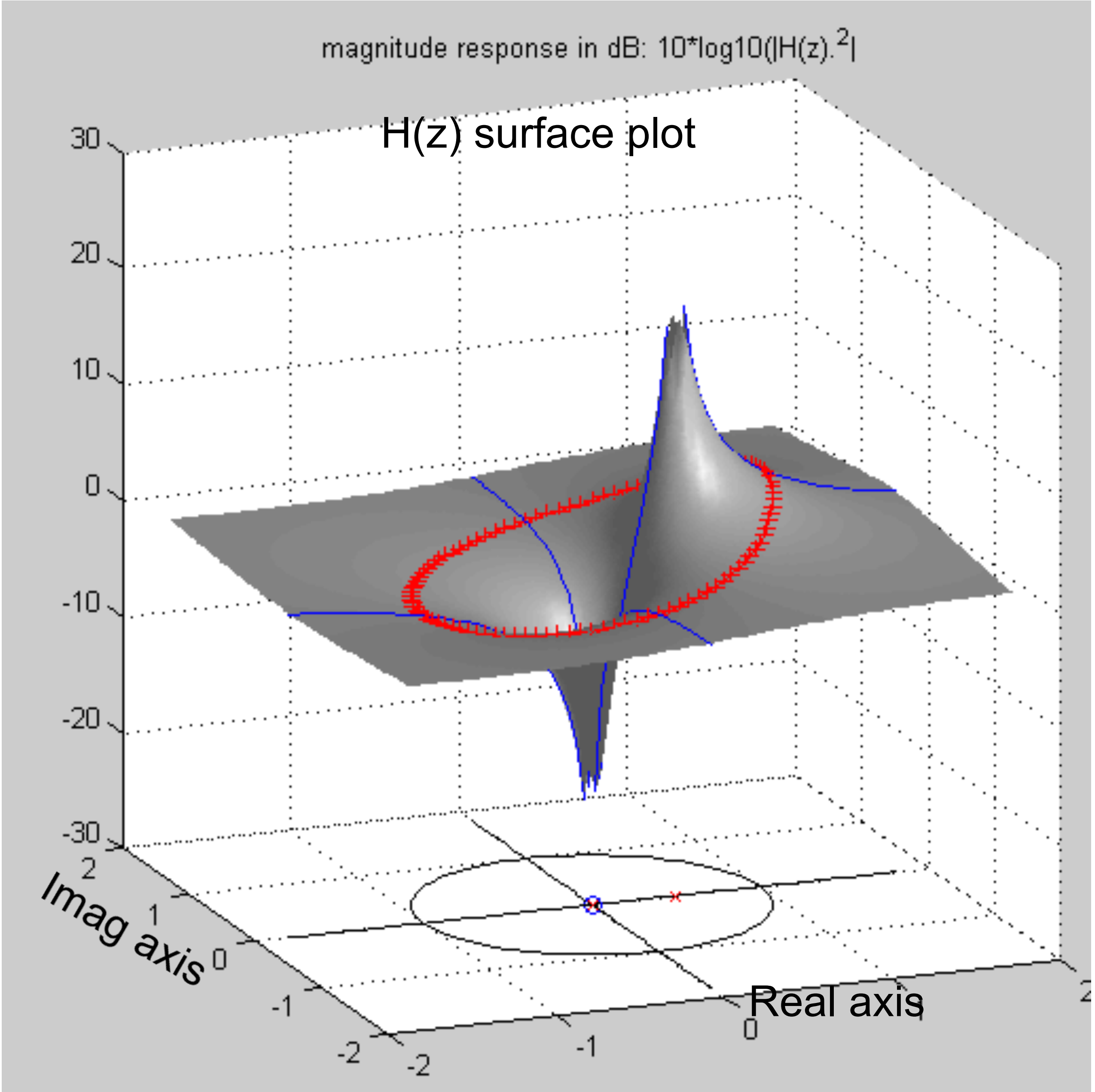


1) Visualization OF H(z)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-0.5}$$

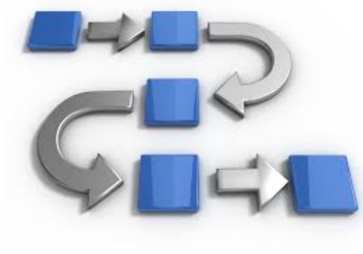


Pole/Zero



Matlab: iir_3D_plot_Hz_ex1.m

Methodology/Logic for FIR



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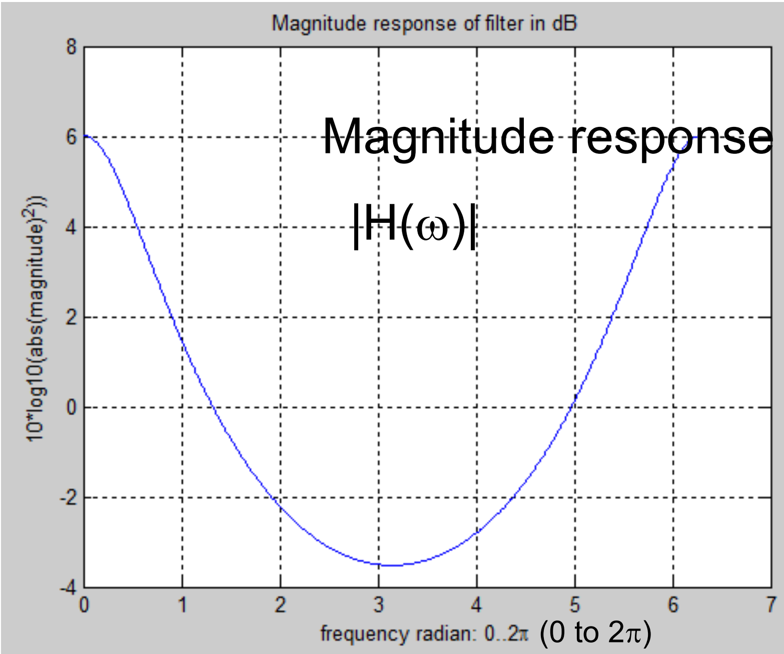
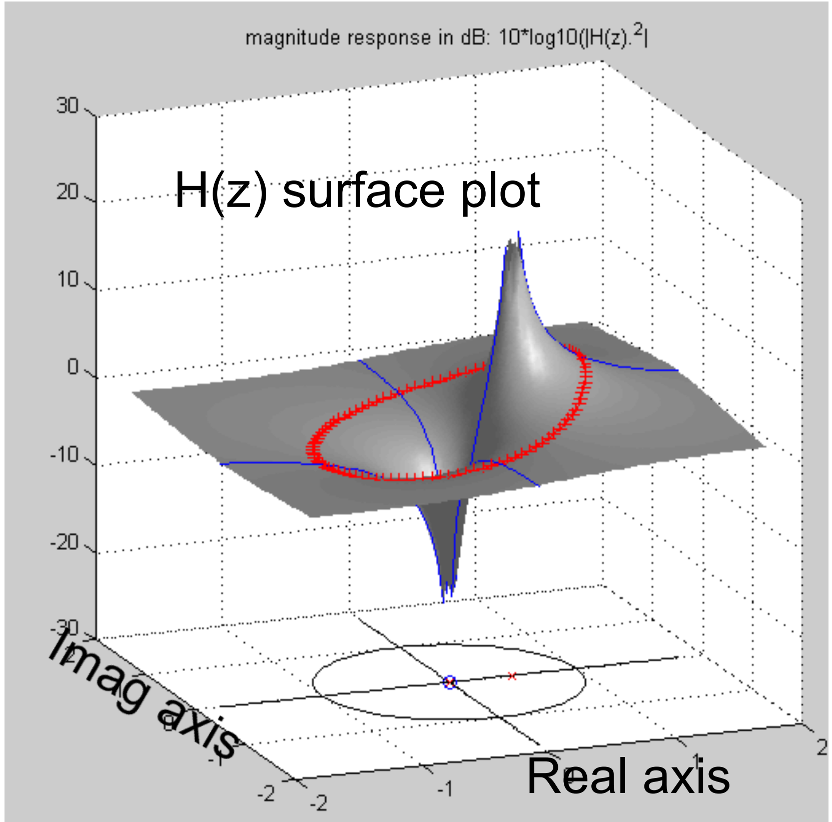
2) Relationship of H(z) and H(ω)

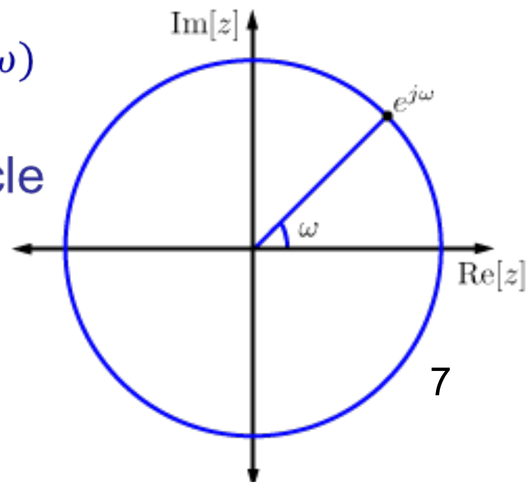




- The frequency response of a digital system can be readily obtained from its transfer function. If we set $z = e^{j\omega}$, we have

$$H(z)|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} h(n)z^{-n}\,|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = H(\omega)$$

- $z = e^{j\omega}$ means the set of complex number on the unit circle
- Magnitude is always '1' and angle ω from 0 to 2π.



Remember on the unit circle:
$z = \cos(\omega) + j\sin(\omega) = e^{j\omega}$ (Euler formula)

2) Stability of the IIR filter

- Mathematically, this can be studied from its impulse response. For a filter to be considered stable, its impulse response will be

$$\sum_{n=0}^{\infty} |h(n)| < \infty$$

  - For the FIR filter, as its name imply, h(n) is finite, and hence the sum of h(n) is finite, and the FIR filter is un-conditionally stable.
  - For the IIR filter, if the infinite impulse response sums to be less than infinity, then the filter is stable, else it is not. We can easily study stability from the pole/zero diagram (in the z-plane)

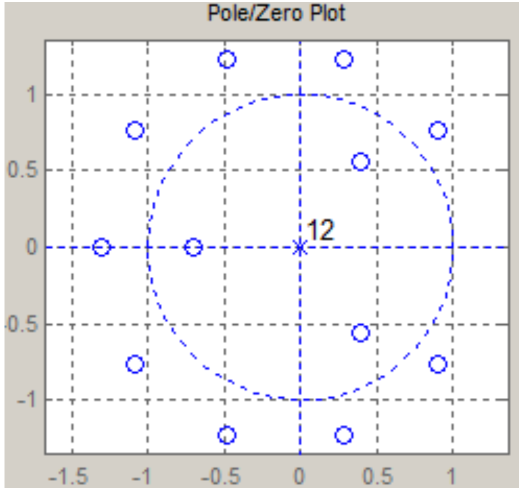
2) Stability of the IIR filter

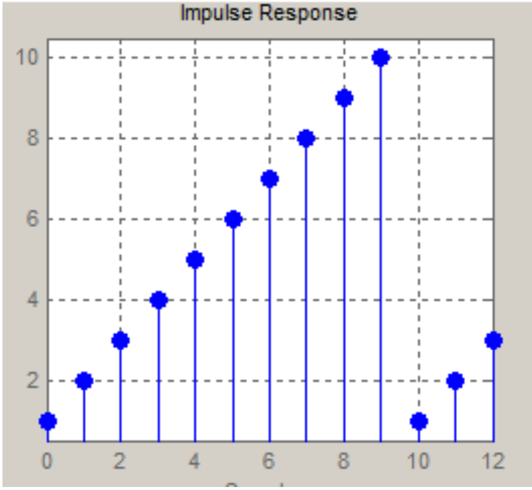
which is a FIR filter, stable IIR filter, or not stable IIR filter?

With the z-plane

- Digital filters with zeros are FIR filters (poles at origin) and are always stable.
- Digital filters with poles are IIR filters, and if ALL the poles are inside the unit circle, these filters are stable.
- If any poles are outside the unit circle, the filter become unstable.
- If filter has poles on and inside unit circles, these filters are marginally stable, exhibit oscillatory behavior.

2) Example of the stability of a filter

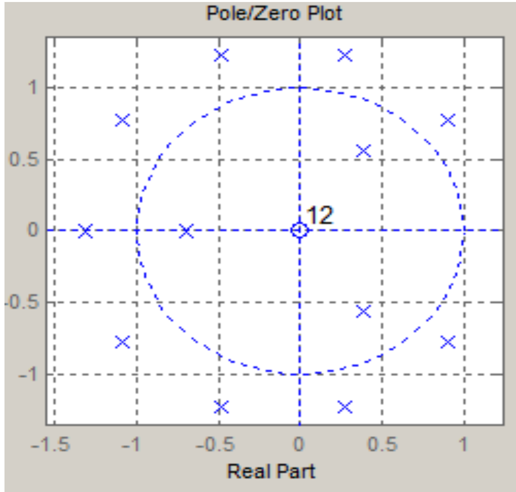


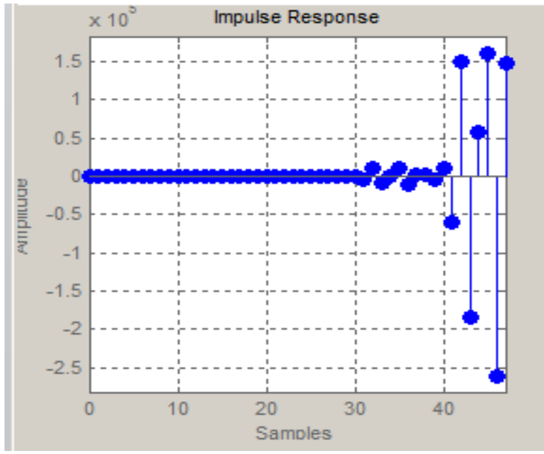


fvtool([1:10, 1 2 3],[1]);
FIR Stable

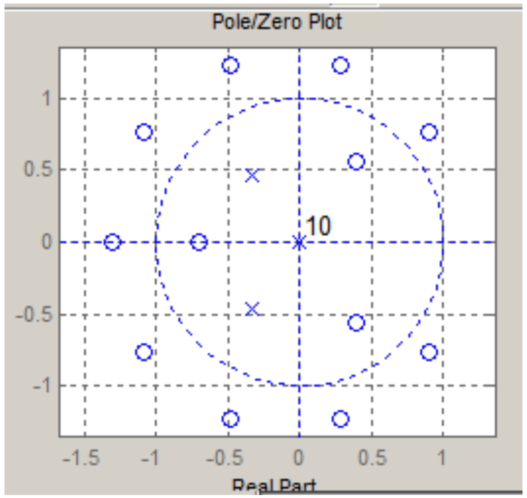
1 2 3 4 … 10

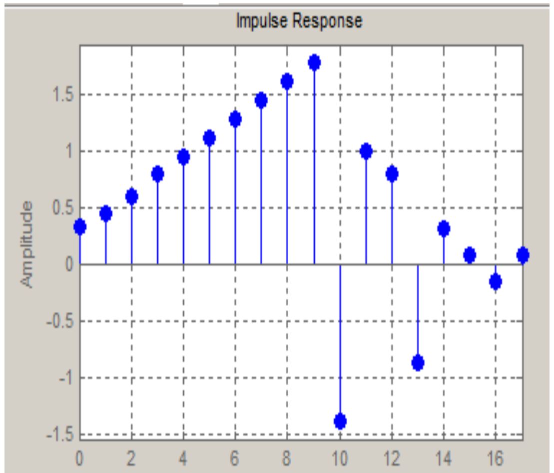
Open Filter Visualization Tool: fvtool(b,a) displays the magnitude response of a digital filter defined with numerator, b and denominator, a.





fvtool([1], [1:10, 1 2 3]);
IIR not-stable





fvtool([1:10, 1 2 3],[3 2 1]);
IIR Stable

2) Generalised IIR Filter difference equation

- From the simple 1st order difference equation:

$$y(n) = b\,x(n) - a\,y(n-1)$$

- we can generalized to:

$$y(n) = b_0 x(n) + b_1 x(n-1) + \ldots + b_{L-1} x(n-L+1) - a_1 y(n-1) - \ldots - a_M y(n-M)$$
$$= \sum_{l=0}^{L-1} b_l x(n-l) - \sum_{m=1}^{M} a_m y(n-m)$$

- Matlab code: `yn = filter(b, a, xn);`
  - Vector `b` contains feedforward coefficients $\{b_l, l = 0,1,\ldots, L-1\}$
  - Vector `a` contains feedback coefficients $\{a_m, m = 1,2,\ldots, M\}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{1+\sum_{m=1}^{M} a_m z^{-m}} = \frac{B(z)}{1+A(z)}$$

y = filter(b,a,x) filters the input data, x, using a rational transfer function defined by the numerator and denominator coefficients b and a, respectively.