DSP: Part II

Lecture.2.1

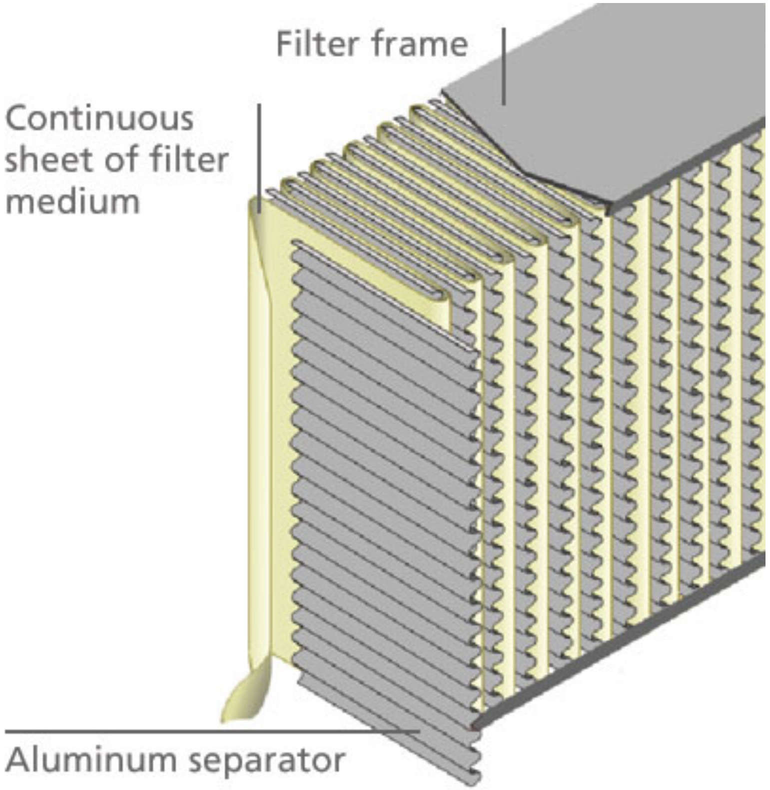
Overview of IIR/FIR FILTER

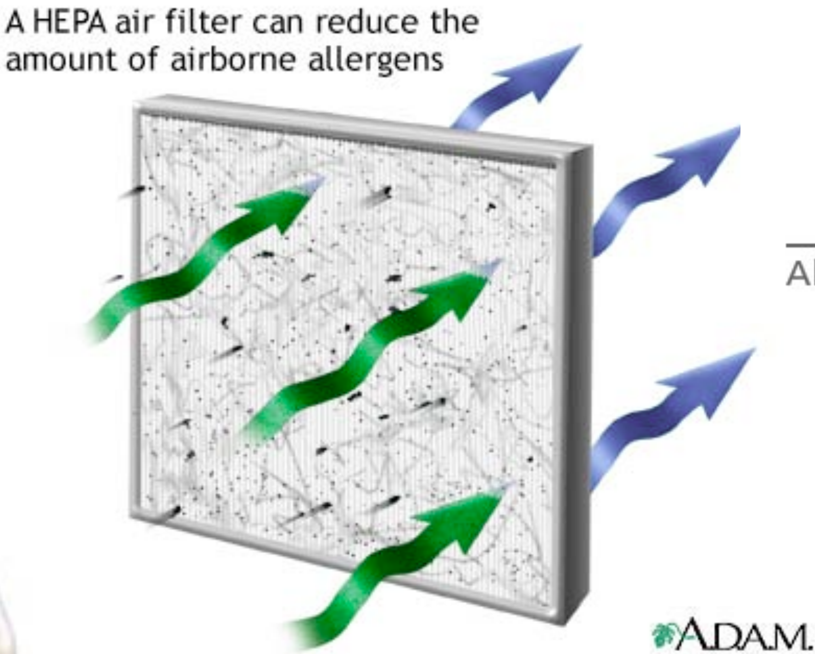
Filters and Frequency response analysis

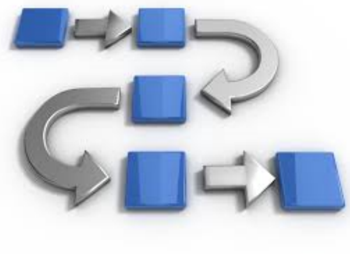
Major Concepts in Part II

- Sampling and Reconstruction
- Digital Filters—FIR and IIR

Filters around us...







Methodology in This Overview Section

A) What is filtering (Review Frequency Analysis)

i. Passing a signal through a filter and its effect on amplitude and phase.

B) Introduction to Digital Filters

i. Impulse response (IIR vs FIR)

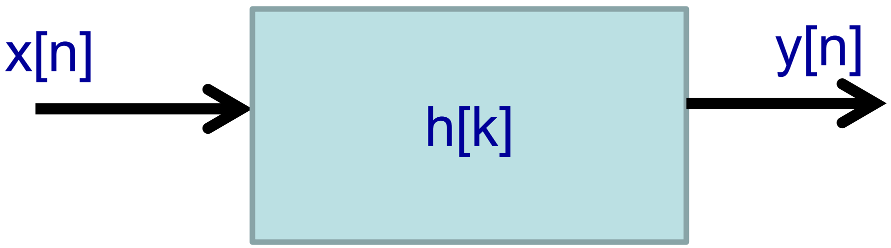
ii. Magnitude response specification

iii. Low pass, high pass, band pass, and band stop

iv. Linear/non-linear phase response

A) An example of filtering

- 3 input signals with amplitude = 1, phase = 0 radian, and frequencies 1Hz, 2 Hz, 5 Hz, and sampling frequency = 100Hz.



$$y[n] = x[n] * h[n]$$

$$= \sum_{\{k=0\}}^{\{k=L-1\}} h[k]x[n-k]$$

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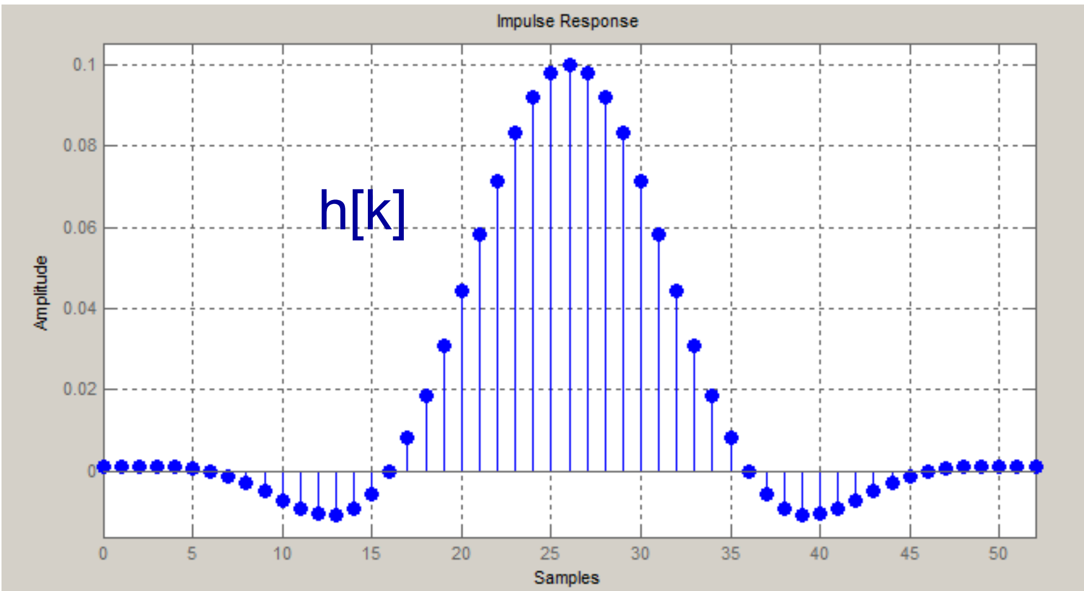
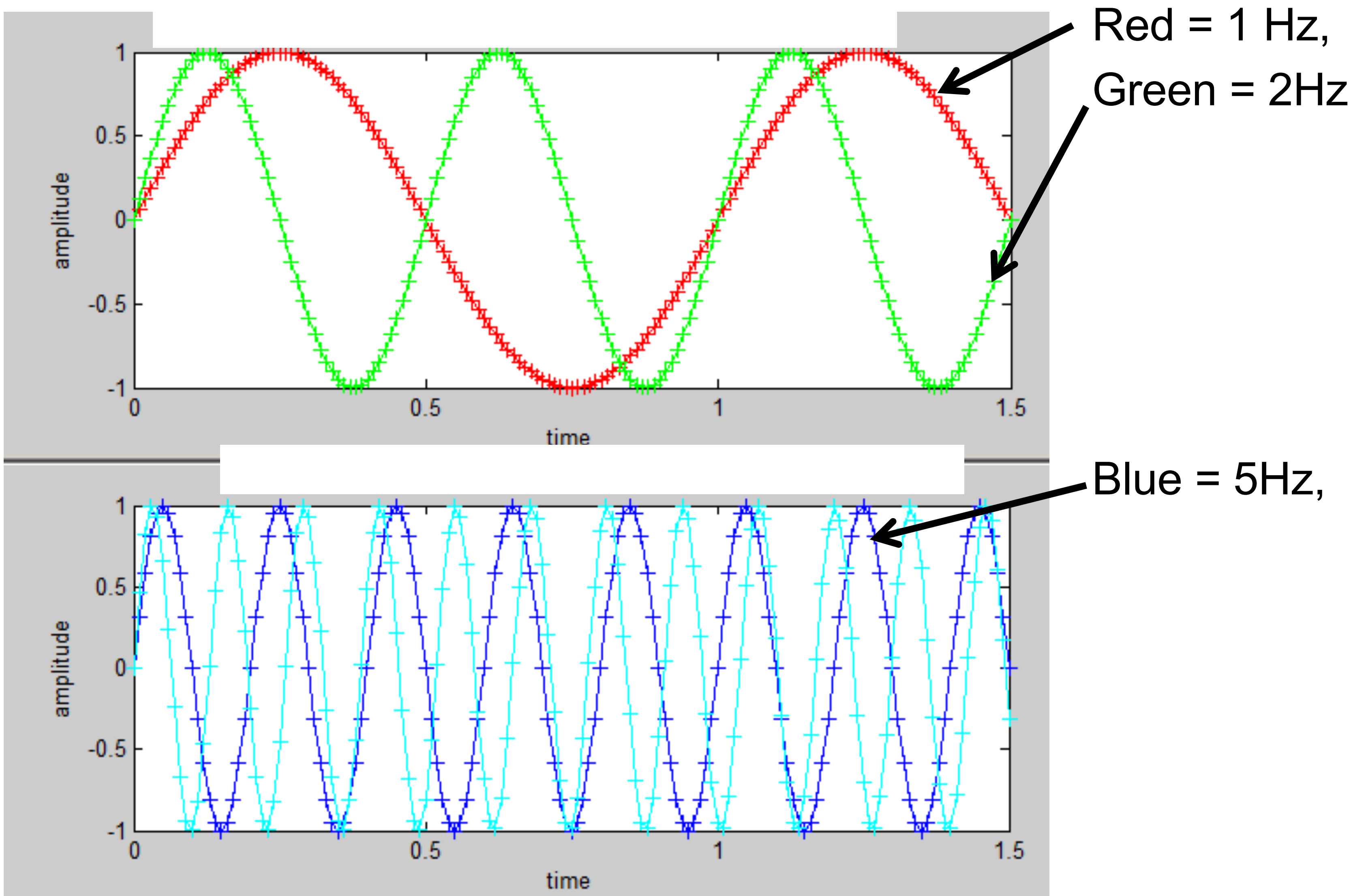


In this example, the impulse response h[k] is finite length (FIR type) with 53 taps, i.e., h[0],b[1],…,h[52]

A) 3 input signals

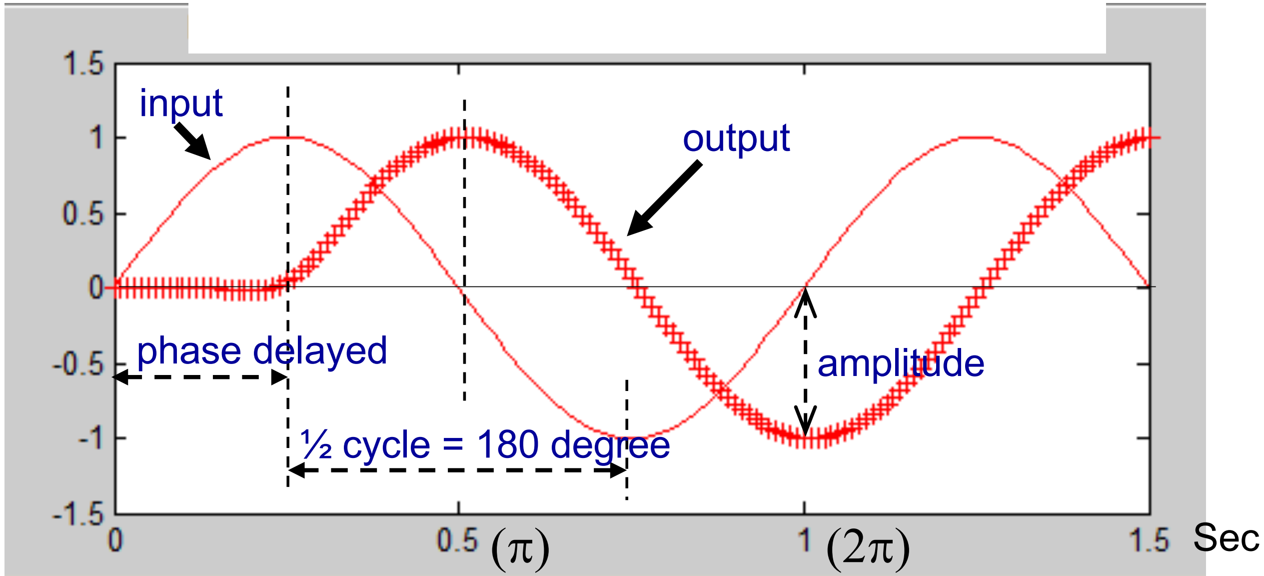
3 input signals with amplitude = 1, initial phase = 0 radian, and frequencies 1Hz, 2 Hz, 5 Hz, and sampling frequency = 100Hz.



A) Input and Output signals: showing magnitude and phase change

2 output signals plotted with the input signals

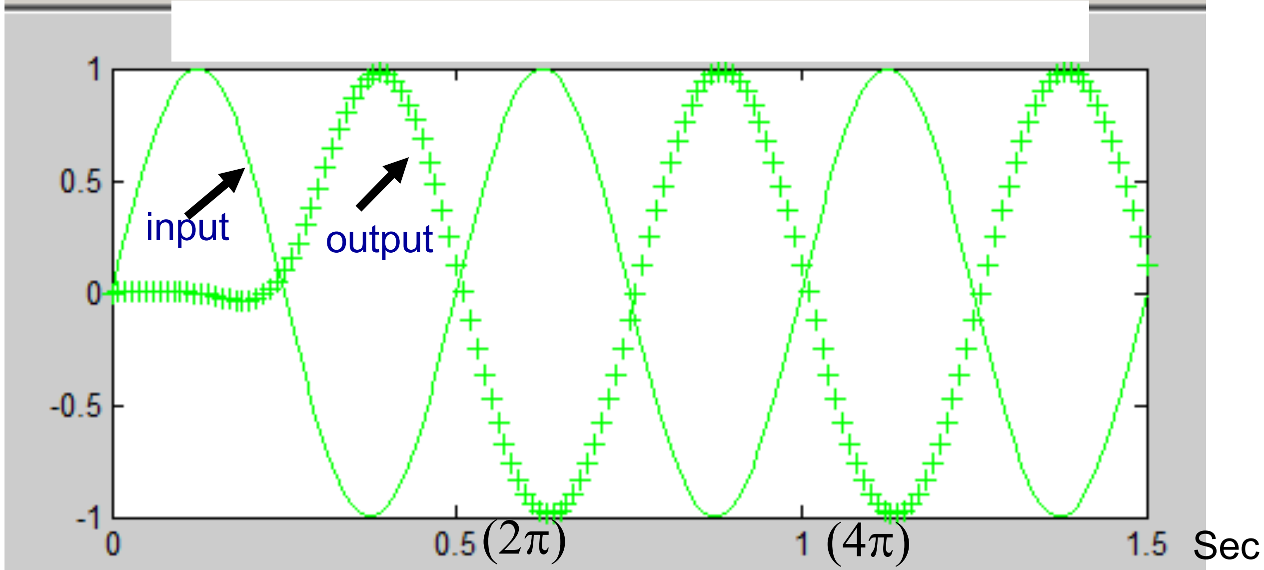
The lines with ‘-’ are the inputs and with ‘+’ are the outputs



Red = 1 Hz,

The output amplitude = 1,

Phase delayed by -93.6 degree



Green = 2Hz

The output amplitude = 0.99,

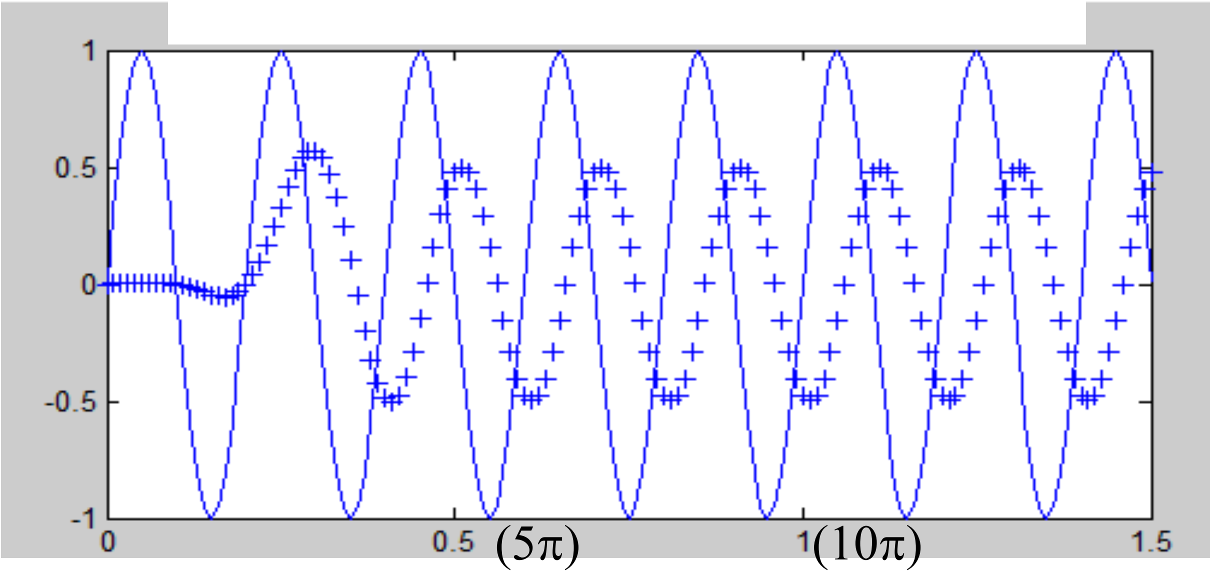
Phase delayed by

- 187.2 degrees

A) Input and Output signals : showing magnitude and phase change.

The other output signal plotted with the input signals

The lines with '-' are the inputs and with '+' are the outputs



Blue = 5 Hz,

The output amplitude = 0.4995,

Phase delayed by

-468 degree

A) The effect of the filter

- Example: 3 sine waves, with amplitude 1, and frequencies 1Hz, 2Hz and 5 Hz, phase = 0 rad, passing through a filter, and the output are:

for 1 Hz, amplitude = 1, phase = -93.6 degrees

for 2 Hz, amplitude = 0.99, phase = -187.2 degrees

for 5 Hz, amplitude = 0.5, phase = -468 degrees

- How do we describe the system?

Its amplitude and phase response at all the frequencies.

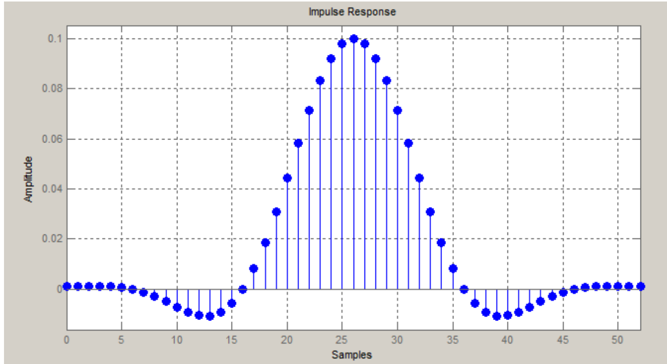
Let us sketch $H(\omega)$; what about $H(\Omega)$?

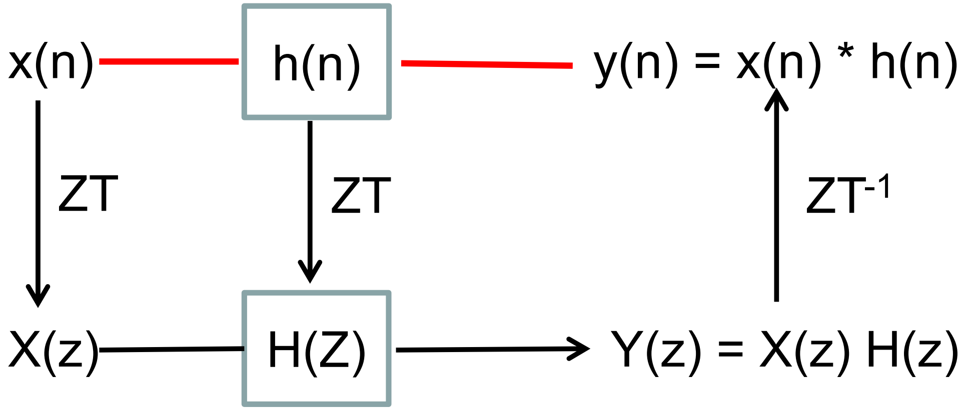
A) How do we calculate H(ω) and Phase response

Given h(n) the impulse response, to calculate H(ω), we use:

1) Z-transform of h(n) to get H(z)
2) Evaluate H(z) with $z=e^{j\omega}$ for ω = 0.. π

h(n)



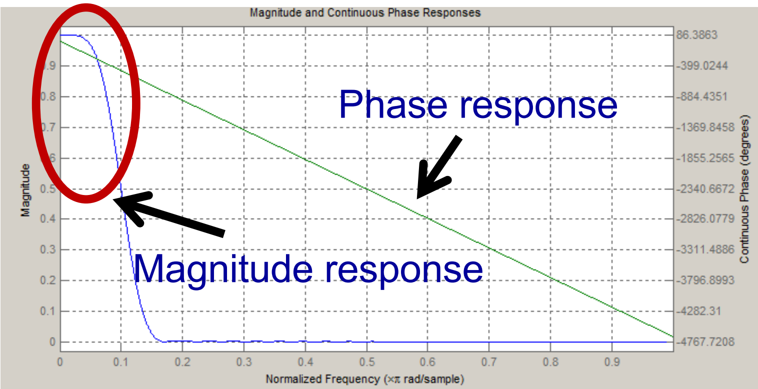


A block diagram of LTI systems in both time-domain and z-domain

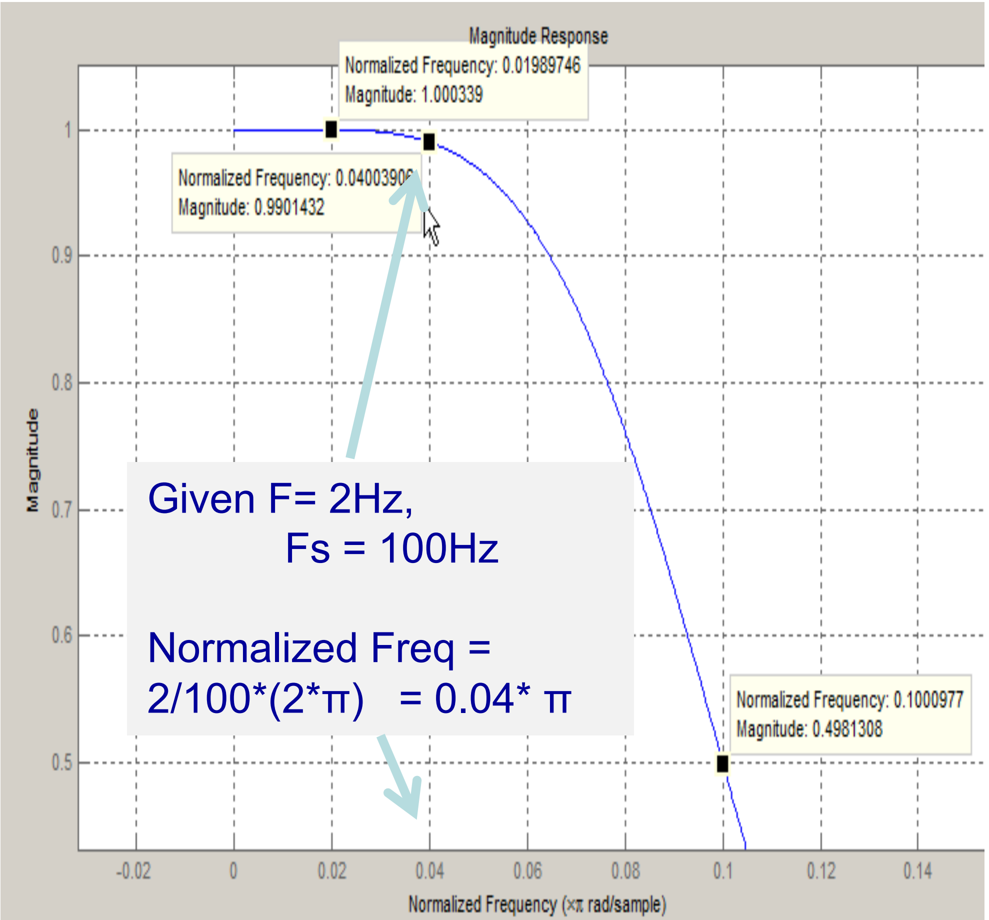
$$H(z)\big|_{z=e^{j\omega}} = \sum_{-\infty}^{\infty} h(n) z^{-n}\big|_{z=e^{j\omega}}$$

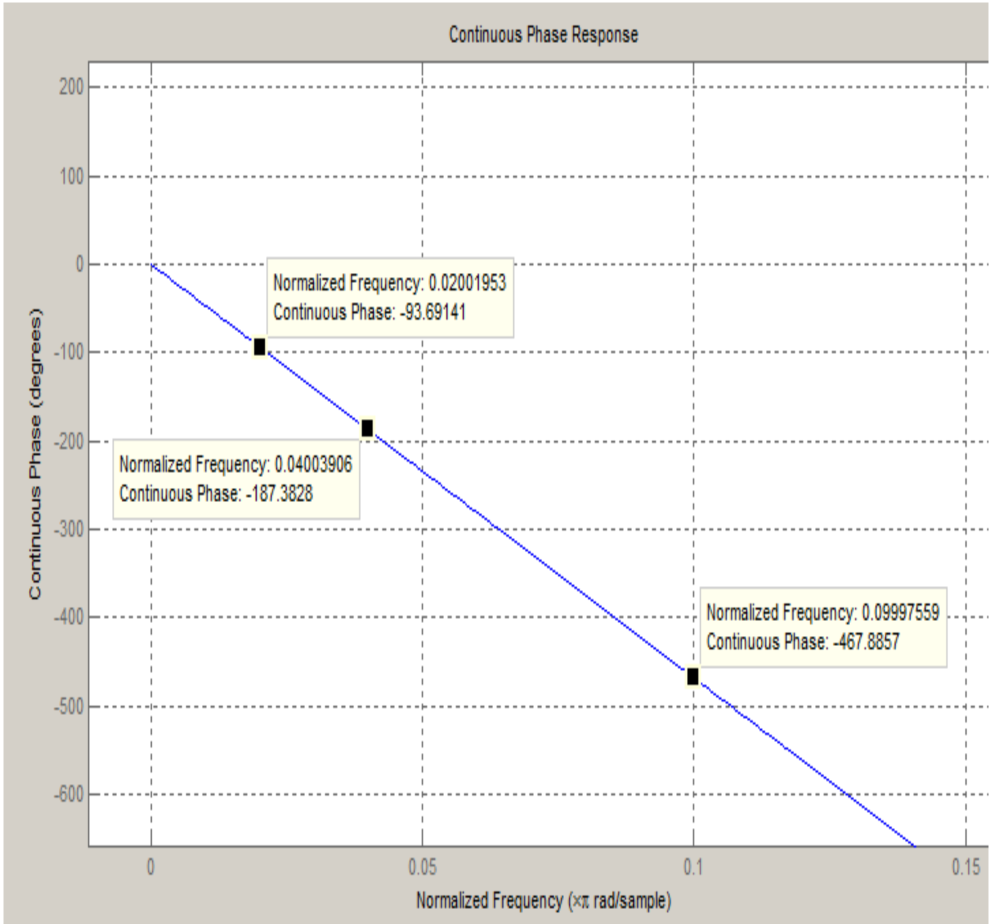
$$\Longrightarrow \sum_{-\infty}^{\infty} h(n) e^{-j\omega n} = H(\omega)$$

A) Input and Output signal (the same case above): showing magnitude and phase change.



Zoomed-in image: showing magnitude and phase response of the three frequencies.





B) Introduction to Digital Filters

- We only consider stable LTI filters. We also focus primarily on causal filters. How do we classify filters?
  a. Impulse response h(t) length: IIR (infinite impulse response) or FIR (finite impulse response).
  b. Types of frequency (magnitude) response: low pass, high bass, band pass, band-stop.
  c. Types of phase response: linear/non-linear

See the slides next.

B) Comparing FIR and IIR filters

FIR	IIR
Has finite impulse response and hence is stable	Has infinite impulse response due to feedback, and hence may be unstable.
Can be easily designed to be linear phase	Difficult to achieve Linear phase
Has desirable numerical property (no feedback)	Has poorer numerical property (sensitive to quantization errors)
Usually requires more computation to achieve same filter specification (magnitude) than IIR	Requires less computation than FIR to achieve same filter (magnitude) specification and can achieve better filter performance in terms of magnitude response

y(n)= $\sum_{l=0}^{L-1} b_l x(n-l)$

$$\mathrm{H(z)} = \sum_{l=0}^{L-1} b_l \, z^{-l}$$

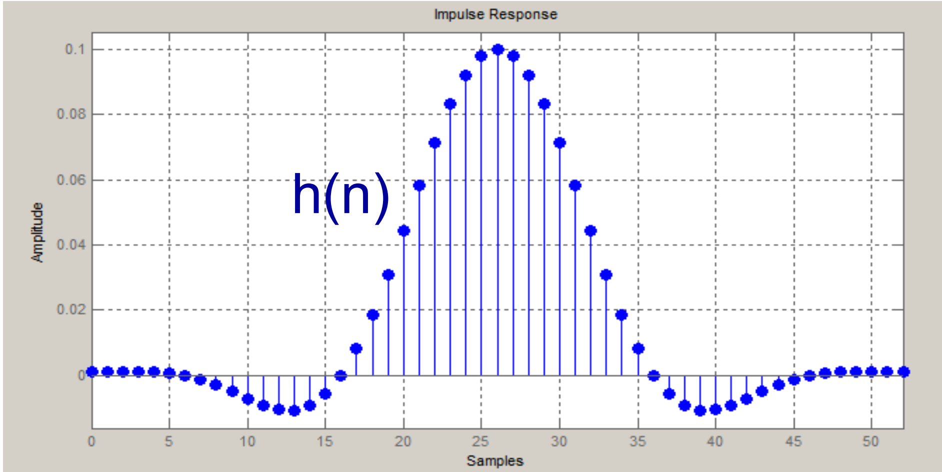
y(n)= $\sum_{l=0}^{L-1} b_l x(n-l)$ + $\sum_{m=1}^{M} a_m y(n-m)$

$$H(z) = \frac{\sum_{l=0}^{L-1} b_l \, z^{-l}}{1 + \sum_{\mathrm{m}=1}^{M} a_m \, z^{-m}}$$

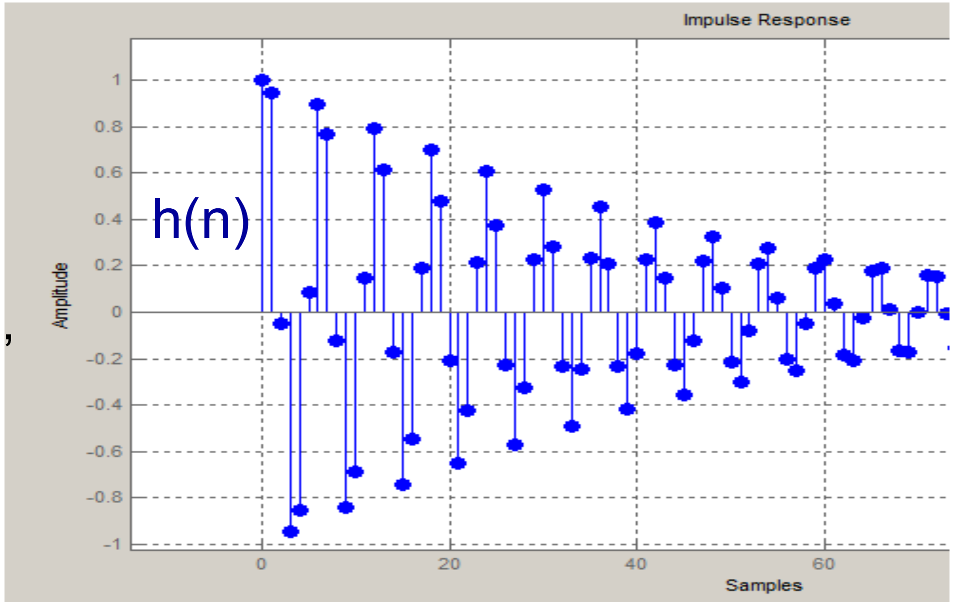
B) Impulse response length

Causal Filters, response comes after appearance of input impulse to filter.

Finite impulse response



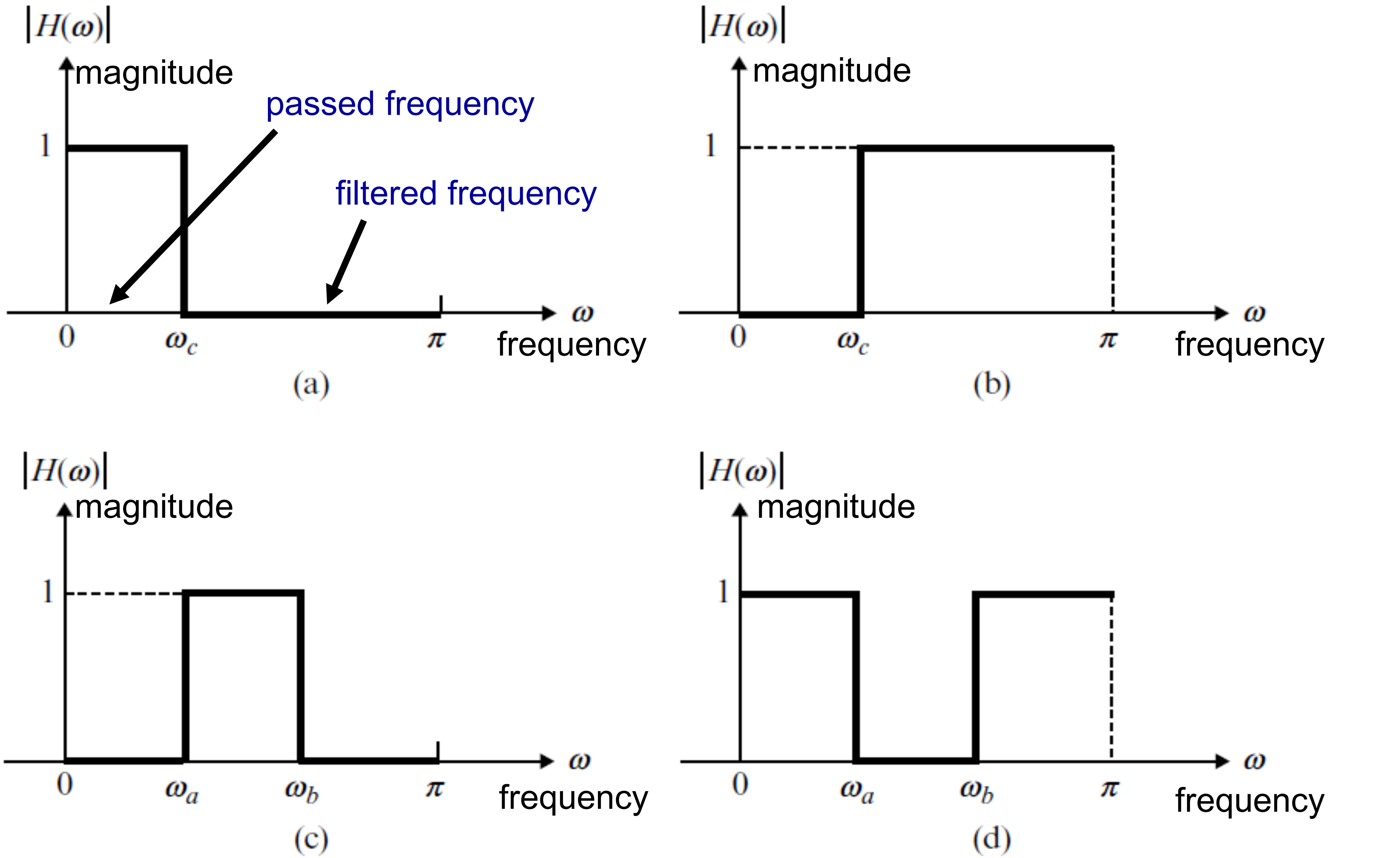
Infinite impulse response
Impulse response goes on infinitely.
Can be implemented using IIR filter.



IIR needs to satisfy the stability criterion,

$$\sum_{n=0}^{\infty} |h(n)| < \infty$$

B) Types of Digital filters



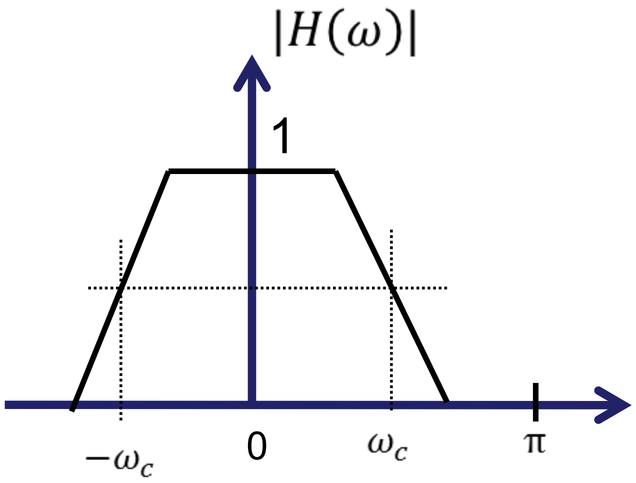
Magnitude response for ideal (a) low pass, (b) high-pass, (c) band pass, (d) band-stop filters

B) Magnitude frequency response: analog vs digital frequency in x-axis

Recall:

The conversion from digital freq to analog, and vice versa is by the following relationship: $\omega = \dfrac{\Omega}{F_s}$

B) Magnitude frequency response: Y-axis



- The y-axis of magnitude frequency response can be in different units, e.g,

$|H(\omega)|$ = magnitude response

$|H(\omega)|^2$ = squared magnitude response

10*log10 $|H(\omega)|^2$ = squared magnitude response in dB (decibel)

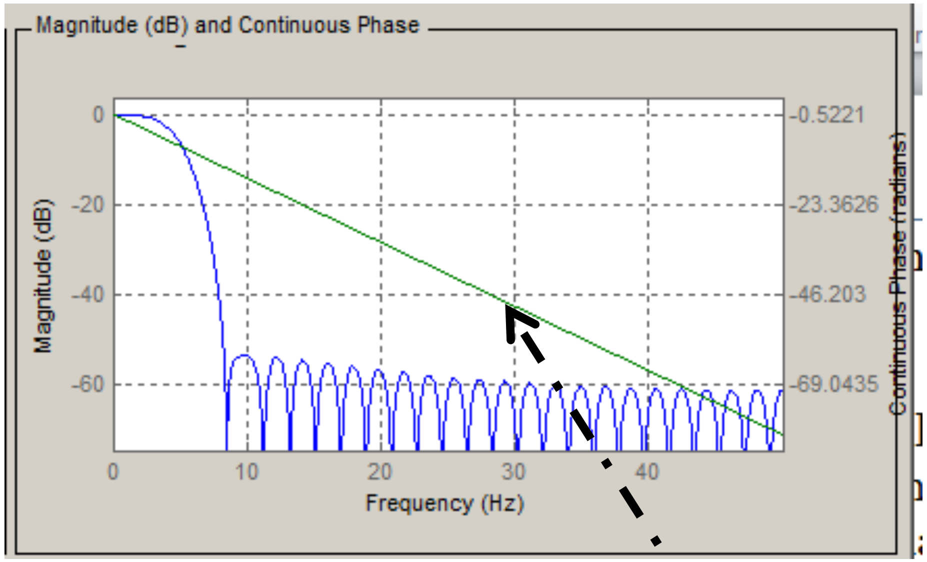
- The important numbers for low pass filter!

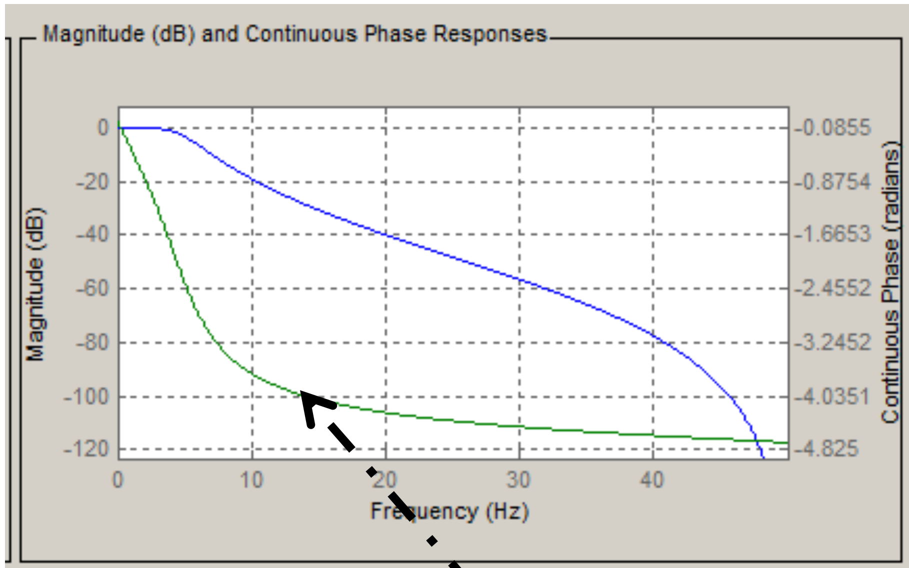
When $|H(\omega)|^2$ = 0.5, i.e, half power point, then the corresponding frequency ω_c is called the cutoff frequency.
At the cutoff frequency, $|H(\omega)|$ = sqrt(0.5) = 0.7071, and
Response in dB = 10*log10$|H(\omega)|^2$ = 10*log10(0.5) = -3.01 dB
(the 3 dB also called the half power point)
In designing filters, we usually deal with magnitude response in dB.

B) Linear and non-linear phase

- The phase response indicates the amount of phase the input signal (at a particular frequency) will be affected.
- If a system has linear phase, then signal for all frequencies will be delayed by the same amount of time (but different phase), and hence the signal will not be phase-distorted.—to be explored more in Section P.2.2.1.

Two filters designed with same requirement,
low pass filter with cutoff frequency 5Hz, sampling rate = 100Hz.





53 tap linear FIR filter
Linear Phase (green line)

3rd order IIR filter, nonlinear phase
Nonlinear phase (green line)