

Time Complexity AssignmentQues 1)Given code \rightarrow

```

for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= i; j++) {
        sum++;
    }
}

```

So the time complexity will be $\rightarrow O(n^2)$.Ques 2)Find $T(2)$, given $T(n) = 3T(n-1) + 12n$

$$T(0) = 5$$

~~$$60 - 5 = 55$$~~

$$\begin{aligned}
 T(1) &= 3T(0) + 12 \\
 &= 15 + 12 \\
 &= 27
 \end{aligned}$$

$$\begin{array}{r}
 2 \\
 27 \\
 \times 3 \\
 \hline
 81
 \end{array}$$

$$\begin{aligned}
 T(2) &= 3T(1) + 24 \\
 &= (27 \times 3) + 24 \\
 &\Rightarrow 81 + 24 = \underline{105} \quad (\text{Ans})
 \end{aligned}$$

Ques 3)Solve $T(n) = T(n-1) + c$ using substitution

$$\begin{aligned}
 T(n) &= T(n-1) + c \\
 &= (T(n-2) + c) + c \\
 &= T(n-2) + 2c
 \end{aligned}$$

$$\text{Similarly } = T(n-3) + 3c$$

Repeating this n times

$$\text{So for } T(0) = T(0) + nc$$

=

Given $T(0)$, you get a final solution. The time complexity is $O(n)$.

Ques 4)

Ans) Time Complexity using masters theorem
Given $T(n) = 16T(n/4) + n^2 \log n$

Comparing with standard form $\rightarrow T(n) = aT(n/b) + f(n)$

$$a = 16$$

$$b = 4$$

$$f(n) = n^2 \log n$$

$$\log_a b = \log_4 16 = 2$$

Since $f(n) = n^2 \log n$, grows faster than n^2 , the time complexity is dominated by $f(n)$. So by Master's theorem the complexity is

$$T(n) = O(n^2 \log n)$$

Ques 5) Recursive tree for $T(n) = 2T(n/2) + n$

- At the root, work done is n
- At the next level, the work is $2(n/2) = n$
- At the next level, the work is $4(n/4) = n$

Since each level does work of n , and the height of the tree is $(\log n)$ the total work is $(n \times \log n)$

$$\Rightarrow O(n \log n)$$

Ques 6) As in question 5, the tree structure is same, but now the additional work at each level is a constant k . The total work is \rightarrow
 $k \cdot \log n$

So the time complexity will be \rightarrow

$$O(k \log n)$$