Digital Signal Processing (EE41013/EE60033)

Mid-semester Examination (Autumn 2021-22)

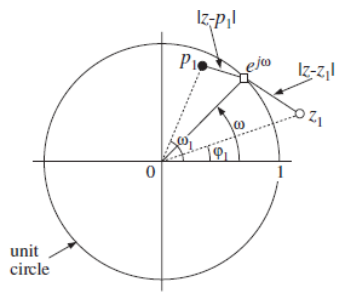
Total Marks: 30 **Time: 2 hours**

Q1. A stable filter has a system function specified as

$$H(z) = \frac{z - z_1}{z - p_1}$$

where z_1 and p_1 denote the zero and pole, respectively.

Given the pole-zero locations shown in the Figure below, discuss the behaviour of variation of $|H(e^{j\omega})|$ as the vector $z = e^{j\omega}$ moves around the unit circle. Plot $|H(e^{j\omega})|$.



(4)

Q2. Determine the transfer function and causal impulse response of a filter described by the difference equation

$$y(n) = 0.25y(n-2) + x(n)$$

Plot the location of poles and zeroes and sketch the variation of $|H(e^{j\omega})|$ over $0 < \omega < \pi$.

Discuss the behaviour of $|H(e^{j\omega})|$ and why it is so. **(5)**

Q3. Consider an input signal $x[n] = \{\cos(0.1n) + \cos(0.4n)\}u(n)$ containing two cosine sequences of angular frequencies 0.1 rad/samples and 0.4 rad/samples. Design a high-pass filter that will pass the high frequency component and block the low frequency part. Assume the filter to be an FIR filter of length 3 with an impulse response $h[0] = h[2] = \alpha_0,\ h[1] = \alpha_1$.

Specifically, obtain the suitable values of α_0 and α_1 such that the output of the high-pass filter is the cosine sequence with higher frequency. **(4)**

Q4. Use the linearity and time shifting properties of DTFT to obtain $Y(e^{j\omega})$ when

$$y[n] = \begin{cases} \alpha^n & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases} \qquad |\alpha| < 1$$

(3)

Q5. Determine the inverse Z-transform of

$$G(z) = \frac{z^3}{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)^2} \quad , \quad |z| > \frac{1}{2}$$

(4)

Q6. Determine all possible signals $x(n)$ associated with the z-transform

$$X(z) = \frac{5z^{-1}}{(1 - 2z^{-1})(3 - z^{-1})}$$

(4)

Q7. (a) Compute the step response of the system

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

(b) Show that any discrete-time signal $x(n)$ can be expressed as

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]u(n-k)$$

(3 + 3)