Indian Institute of Technology Kharagpur
Department of Electrical Engineering

Subject Name: Real-Time Signal Processing Laboratory
Subject Number: EE69014
Programme: M. Tech Students of Instrumentation and Signal Processing

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1. Consider the following 8×8 block of a gray scale digital image,

$$\mathbf{G} = \begin{bmatrix} 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 200 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 255 & 255 & 50 & 255 & 255 & 255 \\ 50 & 50 & 50 & 50 & 255 & 255 & 255 & 255 \end{bmatrix}$$

   use Singular Value Decomposition (SVD) for factorization of G, and reconstruct the approximate image, $\hat{G}$, using singular vectors corresponding to $1-5$ largest singular values. Use the available SVD routine.
   Next, instead of using the available SVD routine, write your own code of the SVD.

   Help: Read [1],[2].

2. Assume a uniform linear array (ULA) with $M = 10$ omnidirectional sensors placed along a line on x-axis. Assume that there are two uncorrelated narrow-band, coherent point sources ($F_1 = F_2 = 2$ GHz) lying on the X-Y plane, $\theta_1 = -10°$ and $\theta_2 = 20°$ (from the vertical Y-axis) in the far-field of the array. Let the inter-sensor spacing be $d = \frac{\lambda}{2}$, where λ is the wavelength of the sources. Let the SNR of the two sources be 5 dB each. Assume that the sources are emitting normally distributed random signals. The sensor noise is assumed to be both temporally and spatially white Gaussian, and is uncorrelated with the signal. Take the total number of data snapshots as $L = 4500$. Using the MUltiple SIgnal Classification abbreviated as MUSIC algorithm, find the estimates of bearing $\hat{\theta}_1$ and $\hat{\theta}_2$.

   Help: Read [3, 4, 5]

3. A beamformer is a processor used in conjunction with an array of sensors to provide a versatile form of spatial filtering. The sensor array collects spatial samples of propagating wave fields, which are processed by the beamformer. The objective is to estimate the signal arriving from a desired direction in the presence of noise and interfering signals. A beamformer performs spatial filtering to separate signals that have overlapping frequency content but originate from different spatial locations.
   Use the Minimum Variance Distortion-less Response (MVDR) beamformer to find estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ where there are two narrowband, coherent point sources ($F_1 = F_2 = 2$ GHz) lying on the

x-y plane, located at an angle $\theta = -10°$ and $\theta_2 = 20°$ (from the vertical Y-axis) in the far-field of the ULA array with $N = 10$ sensors placed along a line on x-axis. Note that this MVDR, known as Capon beamformer is an adaptive (data-driven) beamformer.

Help: Read [6, 7, 3]

4. Consider an $N-$length real-valued zero-mean white noise process with σ^2. Generate a third-order (stable) Auto-regressive (AR) process $x[n]$ with the suitable choice of (known) AR parameters a_1, a_2, and a_3 with record-length $N = 128$ and $\sigma^2 = 1.0$. Use Levinson-Durbin Algorithm to estimate a_1, a_2, a_3 and σ^2. Vary the record-length $N = 128, 256, 512, 1024, 2048$ and check the parameter (a_i, σ^2) estimation quality. For a given record-length $N = 128$, calculate the equivalent representation $r[0]$, k_1, k_2 and k_3, where k_i's are reflection coefficients.

   Help: Read [8, 9]

5. Construct a slowly time-varying signal of length 50, using a 1-D Gaussian random walk, $x_i^0 = x_{i-1}^0 + cq_i$, where x_i^0 is the ground truth value of the signal at time i, $q_i \sim \mathcal{N}(0, 1)$, and $c < 1$ is a non-negative constant.

   A. Corrupt the ground truth with an AWGN of SNR between $[-5, 15]$ dB, such that the noisy signal becomes,
   $$\mathbf{y} = \mathbf{x}^0 + \mathbf{n},$$
   where $\mathbf{y} \in \mathbb{R}^{50}$ is the noisy measurement signal, and $\mathbf{n} \in \mathbb{R}^{50}$ is the noise.

   B. Create a burst of sparse outliers drawn from the Poisson distribution (parameters are free to be chosen by the student.)
   $$\mathbf{y} = \mathbf{x}^0 + \mathbf{n} + \mathbf{e},$$

   The goal is to estimate the true signal $\mathbf{x}^0$. Solve using the least squares (LS) method using the alternating direction method of multipliers (ADMM) to recover the true signal. Given below is the model of the problem:
   $$J(\mathbf{x}) = \frac{1}{2}\|\mathbf{y} - \mathbf{x} - \mathbf{e}\|_2^2 + \gamma\|\mathbf{e}\|_1$$
   where γ is the regularization parameter.

   Help: Read [10]

6. Consider a cell with one Base Station (BS) and one User Equipment (UE) shown in Fig. 1. The BS is equipped with N_t antenna elements in the form of Uniform Linear Array (ULA). The inter element spacing between the elements of ULA is d with λ being the carrier wavelength. The UE has an ULA with N_r antennas.

   The mmWave downlink channel between the BS and the UE is given by:
   $$\mathbf{H}_{N_r \times N_t} = \sum_{p=1}^{L} \alpha_{\mathbf{p}}\, \mathbf{a_r}(\theta_{\mathbf{p}}^{\mathbf{r}})\mathbf{a_t}(\theta_{\mathbf{p}}^{\mathbf{t}})^*$$

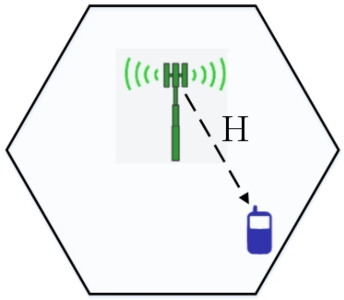

Figure 1: A Base Station and a User Equipment.

where $\alpha_p \sim \mathbb{CN}(0,1)$ and $\mathbf{a}(\theta_p) = [1, e^{j\frac{2\pi d}{\lambda}\sin(\theta_p)}, \cdots, e^{j\frac{2\pi(N-1)d}{\lambda}\sin(\theta_p)}]^T$, N being the number of antenna elements in the array and θ_p being the Angle of Arrival (AoA)/ Angle of Departure (AOD) if UE/BS is concerned. The BS applies a precoding matrix $\mathbf{F} \in \mathbb{C}^{N_t \times K}$ to the transmit symbol $\mathbf{s} \in .\mathbb{C}^{K\times 1}$. The received signal at the UE is given by:

$$\mathbf{y} = \sqrt{\rho}\mathbf{HFs} + \mathbf{n}$$

where ρ is the signal to noise ratio (SNR), $\mathbb{E}[ss^H] = \frac{1}{K}\mathbf{I}_K$ is the transmit signal power constraint and $\mathbf{n} \sim \mathbb{CN}(0,1)$ is the normalized receive noise at the UE. Assume complete Channel Side Information(CSI) at the transmitter and the receiver. The receiver applies a decoder $\mathbf{W} \in \mathbb{C}^{N_r \times K}$ to the received signal $\mathbf{y}$ to obtain $\mathbf{r}$ as

$$\mathbf{r} = \sqrt{\rho}\mathbf{W}^*\mathbf{HFs} + \mathbf{W}^*\mathbf{n}$$

where * is conjugate transpose operator.
The Spectral Efficiency (SE) of the system is given as:

$$\mathbf{SE} = \log_2\left|1 + \frac{\rho}{K}(\mathbf{W}^*\mathbf{W})^{-\mathbf{1}}\mathbf{W}^*\mathbf{HFF}^*\mathbf{H}^*\mathbf{W}^*\right|$$

Perform the Singular Value Decomposition (SVD) of $\mathbf{H} = \mathbf{U\Lambda V}^*$. For fully digital beamforming $\mathbf{F}$ is the K columns of $\mathbf{V}$ corresponding to the K largest singular values of $\mathbf{H}$ and $\mathbf{W}$ is the K columns of $\mathbf{U}$ corresponding to the K largest singular values. Find out the SE for different SNR and plot the same. Take $N_t = 64, N_r = 4, \lambda = 0.01m, d = \frac{\lambda}{2}m, L = 10, \theta_p \sim \mathbb{U}(0, 2\pi), K = 2, SNR \in [-20, 20]$ dB.

Help: Read [11]

7. Consider a sinusoidal signal $s(t)$, with amplitude A, angular frequency Ω and phase ϕ; i.e. $s(t) = A\sin(\Omega t - \phi)$. Let $A = 1.2$v, $\phi \in \{0,\ \pi\}$ and $\Omega = 1000\pi$ rad/s.

   A. Construct a Neural Network (NN) with one hidden layer to classify two signals with zero initial phase and 180° phase shift respectively. The training data consists of two samples as shown below:

      Do not use any in-built machine learning library. Plot the training loss versus epoch curve and save the weights of the NN after training.

   B. Using this NN model, perform bit detection as zero or one over 10^6 bits. The noisy samples, $x(t)$ will be generated by adding white noise with power σ^2, $w(t)$ to the signal, $s(t)$. The SNR values are -5 dB, -10 dB, 0 dB, 5 dB, and 10 dB. Plot the Bit Error Rate (BER) with SNR to observe the detection (classification) performance of the NN model.

Amplitude (volt)	Phase (rad/s)	Output Label
1.2	0	1
1.2	π	0

References

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