Routh-Hurwitz Criterion

Ex. Find the number of poles in the LHP, in the RHP and on the imaginary axis.

$$s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 128$$

s^8	1	10	48	128	128
s^7	$\cancel{3}\ 1$	$\cancel{24}\ 8$	$\cancel{96}\ 32$	$\cancel{192}\ 64$	
s^6	$\cancel{2}\ 1$	$\cancel{16}\ 8$	$\cancel{64}\ 32$	$\cancel{128}\ 64$	
s^5	$\cancel{0}\ \cancel{6}\ 3$	$\cancel{0}\ \cancel{32}\ 16$	$\cancel{0}\ \cancel{64}\ 32$		
s^4	$\cancel{8/3}\ 1$	$\cancel{64/3}\ 8$	$\cancel{64}\ 24$		
s^3	$\cancel{-8}\ -1$	$\cancel{-40}\ -5$			
s^2	$\cancel{3}\ 1$	$\cancel{24}\ 8$			
s	3				
s^0	8				

change

change

$$A(s) = s^6 + 8s^4 + 32s^2 + 64$$

$$\frac{dA(s)}{ds} = 6s^5 + 32s^3 + 64s$$

✓ Two roots of the even polynomial in the RHP.

✓ Two roots of the even polynomial in the LHP. (due to symmetry w.r.t. origin)

Two roots in LHP +

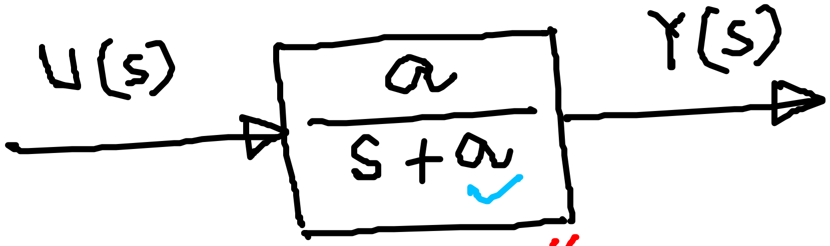
✓ Two roots on the imaginary axis.

4 in LHP
2 in RHP
2 on imaginary axis

Time-domain Analysis

First order system

Three standard signals:
Impulse
Step
Ramp

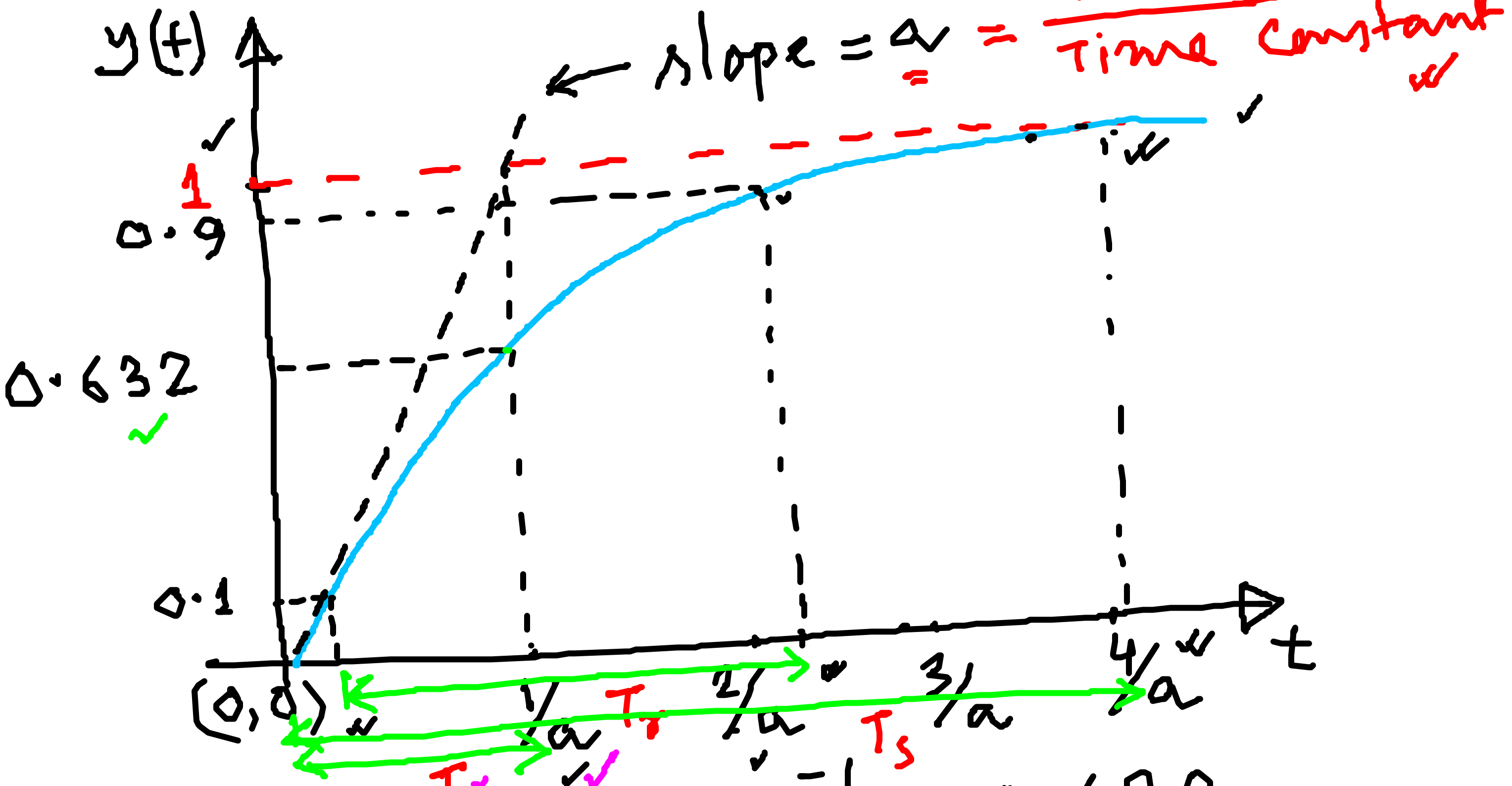


Input is unit step:

$$Y(s) = \frac{a}{s(s+a)}$$

$$\frac{a}{a\left(\frac{1}{a}s+1\right)} = \frac{1}{\frac{1}{a}s+1}$$

$$y(t) = \mathcal{L}^{-1}\, Y(s) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s+a}\right] = 1 - e^{-at}$$



When $t = \frac{1}{a}$, $y(t) = 1 - e^{-1} = 0.632$

$$\left.\frac{dy(t)}{dt}\right|_{t\to 0} = a e^{-at}\Big|_{t\to 0} = a$$

Time Constant: Time it takes for the step response to rise to 63.2% of its final value.
(T)

Time Constant $= \frac{1}{a}$

Rise time: The time required to go from 0.1 to 0.9 of its final value.
(T_r)

$$\left.\begin{aligned} y(t) = 1 - e^{-at_1} &= 0.1 \\ 1 - e^{-at_2} &= 0.9 \end{aligned}\right\} \quad T_r = t_2 - t_1 = \frac{2}{a}\ln 3 \approx \frac{2.2}{a} \approx 2.2T$$

Settling time: The time for the response to reach and stay within 2% of its final value.
(T_s)

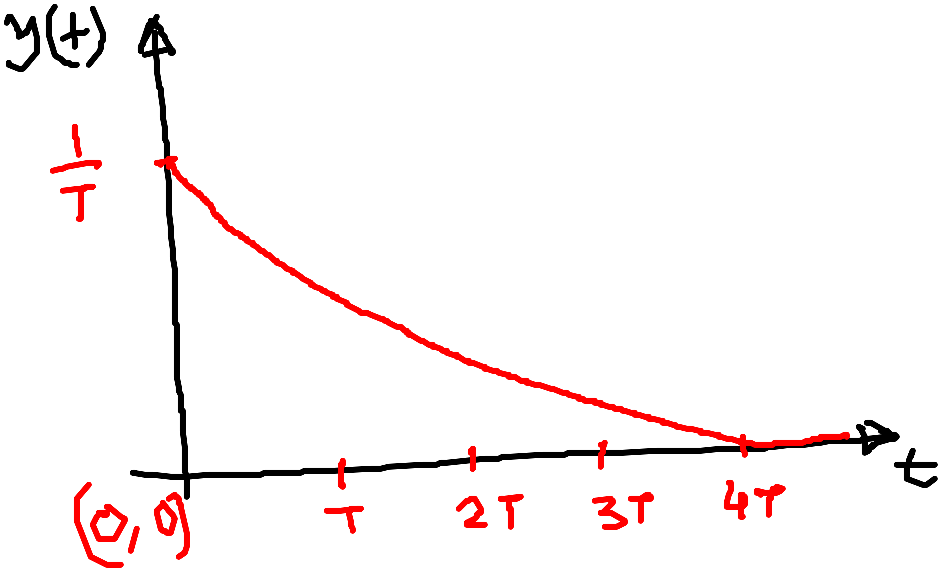
$$0.98 = 1 - e^{-at}$$

$$T_s = t = -\frac{1}{a}\ln(0.02) \approx \frac{4}{a} = 4T$$

Unit-impulse response:

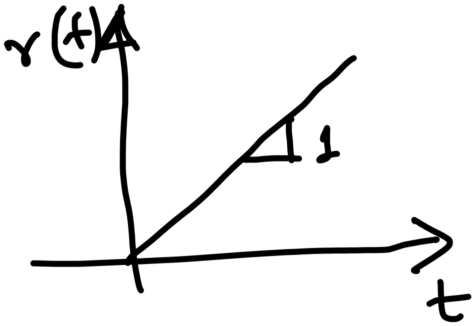
$$G(s) = \frac{1}{sT+1}$$

$$Y(s) = \frac{1}{sT+1} \Rightarrow y(t) = \frac{1}{T} e^{-t/T}, \quad t \geq 0.$$



Unit ramp response:

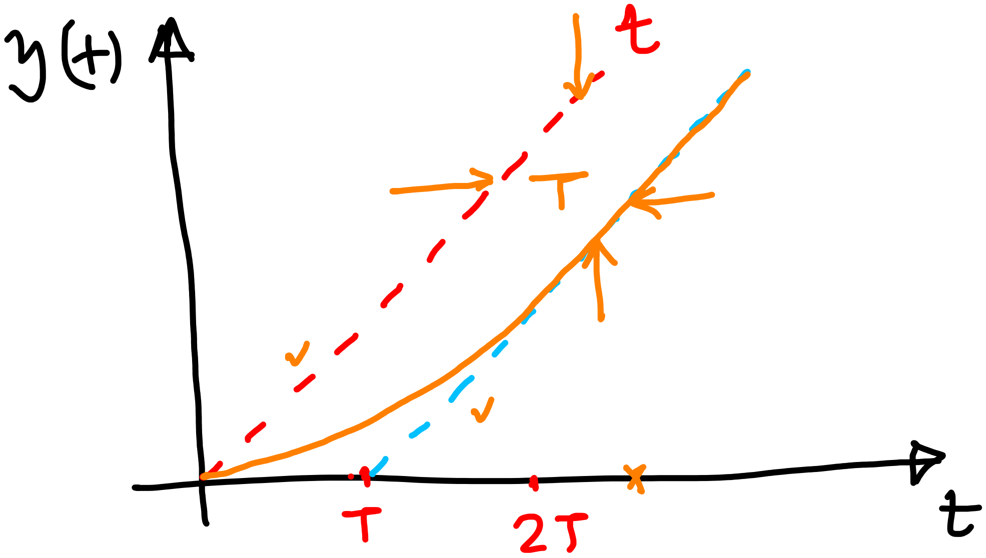
$$Y(s) = \frac{1}{sT+1} \times \frac{1}{s^2} = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{sT+1}$$



$$y(t) = \mathcal{L}^{-1} Y(s) = t - T + Te^{-t/T}, \quad t \geq 0$$

$$\underline{y(\infty) = t - T}$$

Error at $t \to \infty$, $= t - t + T = \underline{T}$



Unit - impulse: $y(t) = \frac{1}{T} e^{-t/T}$

Unit - step: $y(t) = 1 - e^{-t/T}$

Unit - ramp: $y(t) = t - T + Te^{-t/T}$

$\frac{d}{dt}$

$t \geq 0.$