Assignment 1

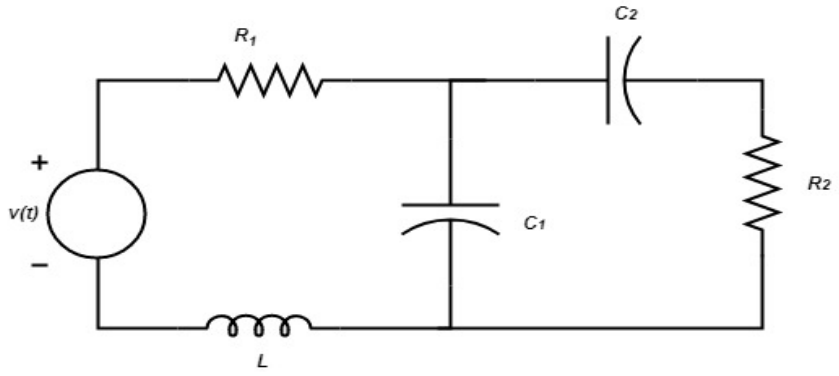
1. (a) The transfer function (TF) of a resistance R may be represented either as $\frac{V(s)}{I(s)} = R$ or as $\frac{I(s)}{V(s)} = \frac{1}{R}$. Similarly, it appears at first sight that the TF of a (relaxed) inductance L may be either sL or $\frac{1}{sL}$, and that of a (relaxed) capacitor C may be either $\frac{1}{Cs}$ or Cs.

   (i) Representing an inductance as a block with TF of sL and a capacitance as a block with TF of $\frac{1}{Cs}$, obtain a block diagram representation of a series RLC circuit excited by a voltage source $v(t)$. Using any standard simulation software (e.g. MATLAB-SIMULINK), obtain the voltage responses $v_L(t), v_c(t)$ and the current $i(t)$ for $v(t) = 1(t)$, $R = 40\ \Omega$, $L = 10$ mH, $C = 1$ μF.

   (ii) Repeat the above considering the TF of inductance as $\frac{1}{sL}$.

   (iii) Are the responses obtained in (a) and (b) identical? If not, explain the reason behind and decide which of the above representations is *proper* and hence correct.

   (b) For the RLC circuit shown below, obtain a proper block diagram representation. Also obtain the TF between $V_{C_1}(s)$ and $V(s)$ using block diagram reduction method.



2. An electromagnet having a $(0.5H, 20\Omega)$ coil is to be used to suspend a $2kg$ steel ball at a nominal gap of $x_0 = 5 \times 10^{-3}m$. The lifting force exerted on the mass is given by $3 \times 10^{-4}(i_c/x_g)^2 N$, where i_c is the current through the coil and $x_g = x_0 + x$ is the gap between the magnet and the ball. Let the voltage applied to the coil be $v_c = v_0 + v$, v_0 being the nominal value.

a) Make a suitable choice of the states and obtain the corresponding state-space equation in the form $\dot{X} = f(X, u)$ where X is the state vector, u the input, and f is a (possibly nonlinear) vector function. (Justify that one may choose $x_g, \dot{x}_g, i_c$ as the states.)

b) Obtain the equilibrium point characterized by $x_g = x_0, \dot{x}_g = \dot{x}_0, i_c = i_0$, and $v_g = v_0$.

c) Find the incremental state-space representation in terms of $x, \dot{x}, i,$ and v where $i = i_c - i_0$.

d) Hence find the transfer function from v to x. Comment on the *order*, *type* and *stability* of the system?