Industrial Instrumentation
Assignment-2

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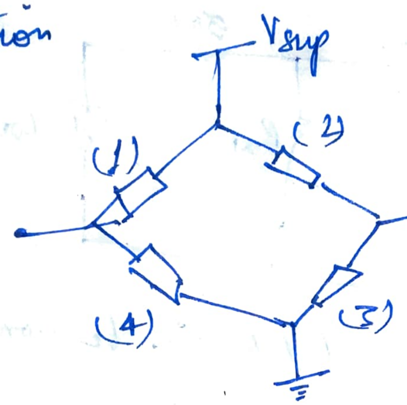
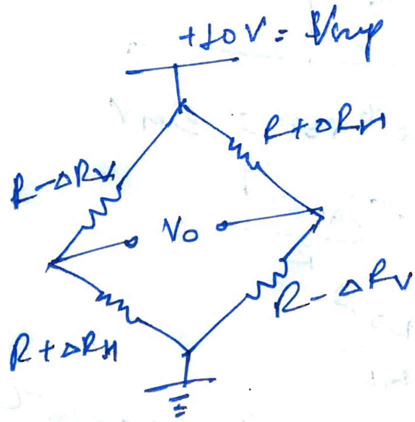
Q1. (a) Connection



(b) Let applied force be F, area of cross-section

$$A = \frac{\pi d^2}{4} \quad [d = 0.3m]$$



$$V_o = \left[\frac{R+\Delta R_H}{2R+\Delta R_H-\Delta R_V} - \frac{R-\Delta R_V}{2R+\Delta R_H-\Delta R_V}\right] V_{sup}$$

$$V_o = 10 \times \frac{\Delta R_H+\Delta R_V}{2R+\Delta R_H-\Delta R_V} \simeq 10\left[\frac{\Delta R_V+\Delta R_H}{2R}\right]$$

$$|\varepsilon_{vertical}| = \frac{F}{AY} \Rightarrow \Delta R_V = \frac{\alpha F R}{AY}$$

$\alpha \Rightarrow$ Gauge's ~~constant~~ factor

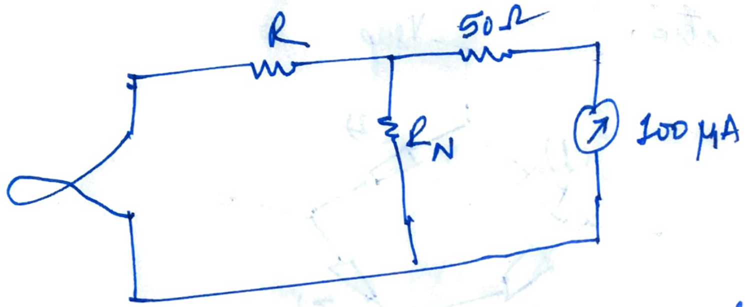
$Y \Rightarrow$ Young's Modulus

$$|\varepsilon_{horizontal}| = \frac{F\mu}{AY} \Rightarrow \Delta R_H = \frac{\alpha F R \mu}{AY}, \quad \mu \Rightarrow \text{Poisson Ratio}$$

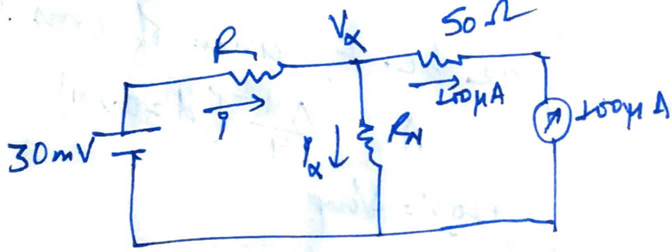
$$V_o = \frac{10 \times F R \times}{A Y \times 2R}\left[1+\mu\right]$$

$$\frac{V_o}{F} = \frac{5 \times}{A Y}(1+\mu) = \frac{5\times}{\frac{\pi d^2}{4} Y}(1+\mu) = 9.1956 \times 10^{-4}\ V/N$$

Q. 3)



At $T_A = 20^\circ C$ (let $20^\circ C$ be normal temperature)



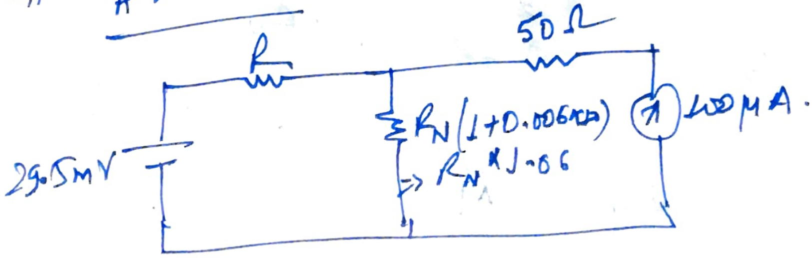
$$V_x = 100 \times 10^{-6} \times 50 = 5\,mV$$

$$i = \frac{(30-5)\times 10^{-3}}{R} =$$

$$i_x = \frac{5 \times 10^{-3}}{R_N}$$

$$\frac{25 \times 10^{-3}}{R} = \frac{5 \times 10^{-3}}{R_N} + 100 \times 10^{-6}$$

At $T_A = 30^\circ C$



$$i = \frac{24.5 \times 10^{-3}}{R} \; ; \; i_x = \frac{5 \times 10^{-3}}{1.06 R_N}$$

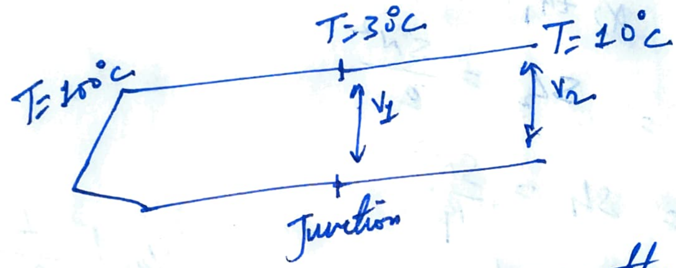
$$\frac{24.5\times10^{-3}}{R} - \frac{5\times10^{-3}}{1.06R_N} = 100\times10^{-6}$$

Solving the above two equations,

$$\frac{1}{R} = 6.185567\times10^{-3} , \quad \frac{1}{R_N} = 0.0109278$$

$$\Rightarrow R = 161.67\,\Omega , \quad R_N = 91.5094\,\Omega$$

Q4.



At the Junction, voltage across the terminal is

Type E (T= 100°C) − Type E (T= 30°C)

$$V_1 = 6.317\,mV - 1.801\,mV = 4.516\,mV$$

Voltage due to copper − constantan blade

Type T (T= 30°C) − Type T (T= 0°C)

$$V_2 - V_1 = \text{Type T (T=30°C)} - \text{Type T (T=0°C)}$$

$$= 1.916\,mV$$

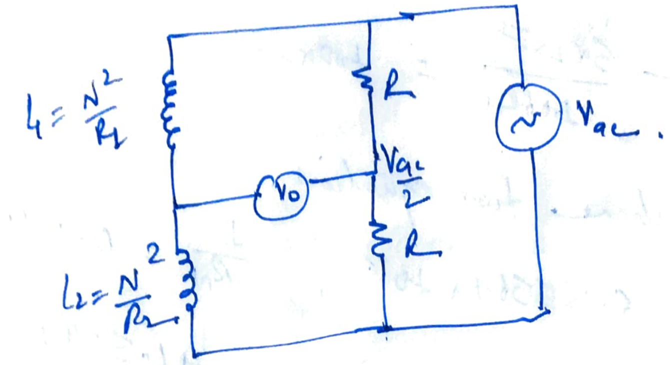
$$\therefore V_2 = 5.712V$$

Measured Temperature (assuming chromel constantan thermocouple)

$$T = \frac{10\times(5.712-5.646)}{(6.312-5.646)} + 90$$

$$90.99°C$$

Q6.



$$V_0 = \frac{V_{ac} X_2}{X_2 + X_1} - \frac{V_{ac}}{2}$$

$$X_2 = sL_2 = \frac{sN^2}{R_2}$$

$$X_1 = sL_1 = \frac{sN^2}{R_1}$$

$$V_0 = \frac{V_{ac} \times \frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} - \frac{V_{ac}}{2} = V_{ac}\left[\frac{1}{\frac{R_2}{R_1} + 1} - \frac{1}{2}\right]$$

$$\frac{R_2}{R_1} = \frac{R_0 + K(\theta + d)}{R_0 + K(\theta - d)}$$

$$V_0 = V_{ac}\left[\frac{R_0 + K(\theta - d)}{R_0 + K(\theta + d) + R_0 + K(\theta - d)} - \frac{1}{2}\right]$$

$$V_0 = V_{ac}\left[\frac{-kd}{2(R_0 + k\theta)}\right]$$

$$\boxed{V_0 \propto d}$$

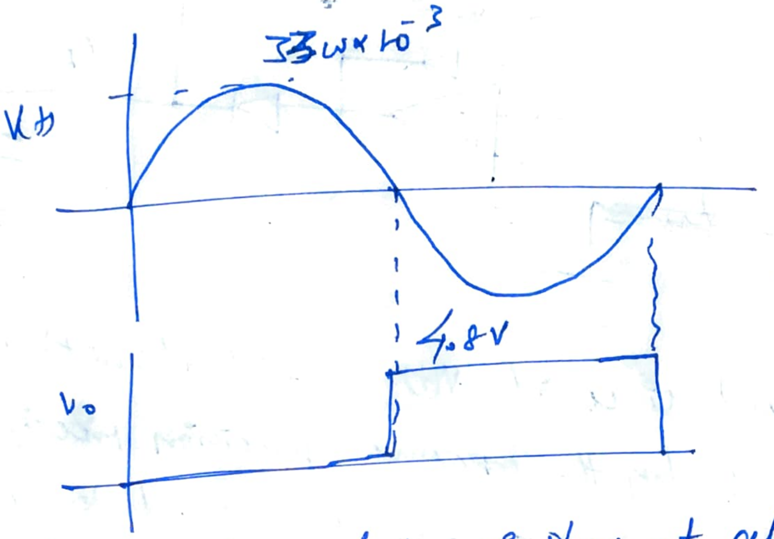
Q7.

$$N(\phi) = 4.0 + 1.5\cos(22\theta) \text{ mWb.}$$

(a) Number of teeth $= 22$ (Coeff. of θ)

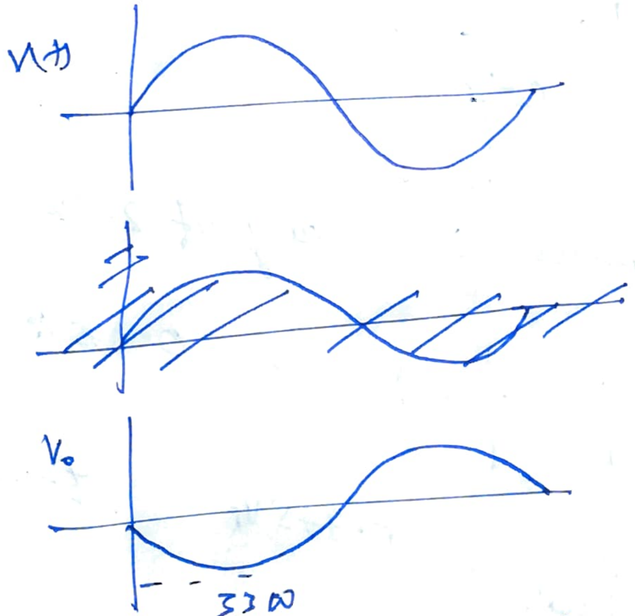
(b) $V(\theta) = -\frac{dN(\theta)}{dt} = 33\omega \sin(22\theta) \times 10^{-3}$ V $\quad \omega = \frac{d\theta}{dt}$

when $V(t)$ is negative, $V_o = 4.8\,V$
when $V(t)$ is positive, $V_o = 0\,V$



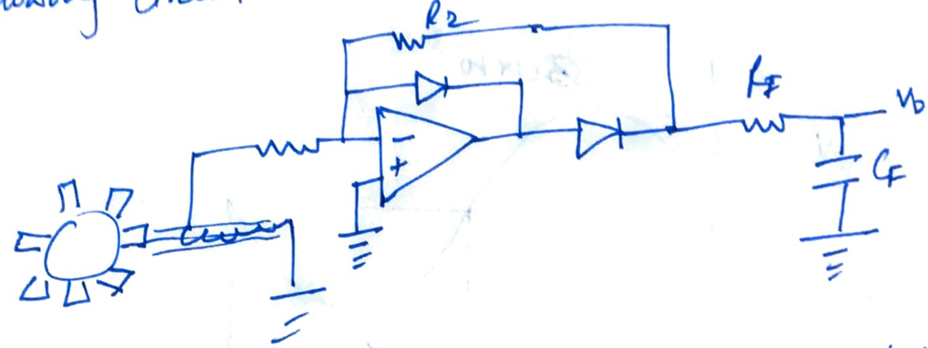
This waveform looks similar at all frequencies.

(c) When $R_F = 1\,k\Omega$, $V_o = -V(t)$



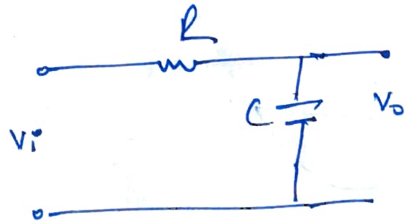
(d) The zener diode output is a square wave which makes measurement of frequency easy (using a digital counter). Moreover, the output amplitude is constant.

(e) To obtain a DC voltage proportional to ω, the following circuit can be used.



The input sinusoid $V(t) = 330 \sin(2200t)$ is full wave rectified by the opamp-precision rectifier. The low pass filter circuit is used to remove the ripple to obtain a DC voltage proportional to ω.

Q8



for a maximum inaccuracy of 2.5% at 100 rad/s

$$\left| \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \right| \times V_i = V_o = 0.975 V_i \quad \text{at } \omega = 100$$

$$\frac{V_i}{1 + j\omega CR} = 0.975 V_i \quad , \tau = RC$$

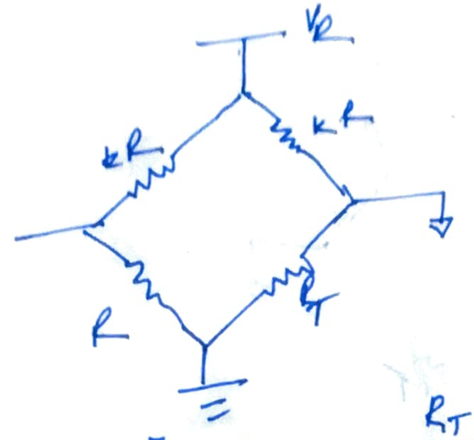
$$\frac{1}{|1 + j\omega \tau|} = 0.975$$

$$\sqrt{1 + \omega^2 \tau^2} = \frac{1}{0.975}$$

$$1 + 100^2 \tau^2 = 1.0519395 13$$

$$\boxed{\tau = 2.279 \text{ ms}}$$

Q3. (a)



$$V_o = V_R\left[\frac{1}{k+1} - \frac{R_T}{R_T + kR}\right]$$

Let the three points be:

V_o	0	2.5	5.0
ΔT	0	25	50

(i) $\Delta T = 0 \quad (T = 273\,K)$

$$R_T = 1.68\, e^{3050\left(\frac{1}{273} - \frac{1}{298}\right)} = 4.289\,k\Omega$$

$$\therefore V_R\left[\frac{1}{k+1} - \frac{4.289}{4.289 + kR}\right] = 0\,V$$

(ii) $\Delta T = 25 \quad (T = 298)$

$$R_T = 1.68\, e^{3050\left(\frac{1}{298} - \frac{1}{298}\right)} = 1.68\,k\Omega$$

$$\therefore V_R\left[\frac{1}{k+1} - \frac{1.68}{1.68 + kR}\right] = 2.5\,V$$

(iii) $\Delta T = 50 \quad (T = 323\,K)$

$$R_T = 1.68\, e^{3050\left(\frac{1}{323} - \frac{1}{298}\right)} = 0.7608\,k\Omega$$

$$\therefore V_R\left[\frac{1}{k+1} - \frac{0.7608}{0.7608 + kR}\right] = 5.0\,V$$

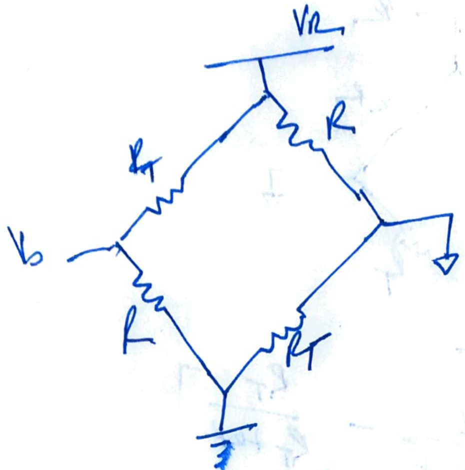
Solving these equations yield,

$V_R = 12.7898\,V$

$R = 4.289\,k\Omega$

$k = 0.2701$

(b)



$$V_o = V_R\left[\frac{R}{R_T+R} - \frac{R_T}{R+R_T}\right]$$

$$V_o = V_R\left[\frac{R-R_T}{R+R_T}\right]$$

Since there are only 2 degrees of freedom, i.e, R, V_R. This configuration cannot be fit to 3-points.

$\therefore$ $\Delta T = 0$ $(T = 273K)$

$$V_R\left[\frac{R-4.289}{R+4.289}\right] = 0$$

$$R = 4.289\,k\Omega$$

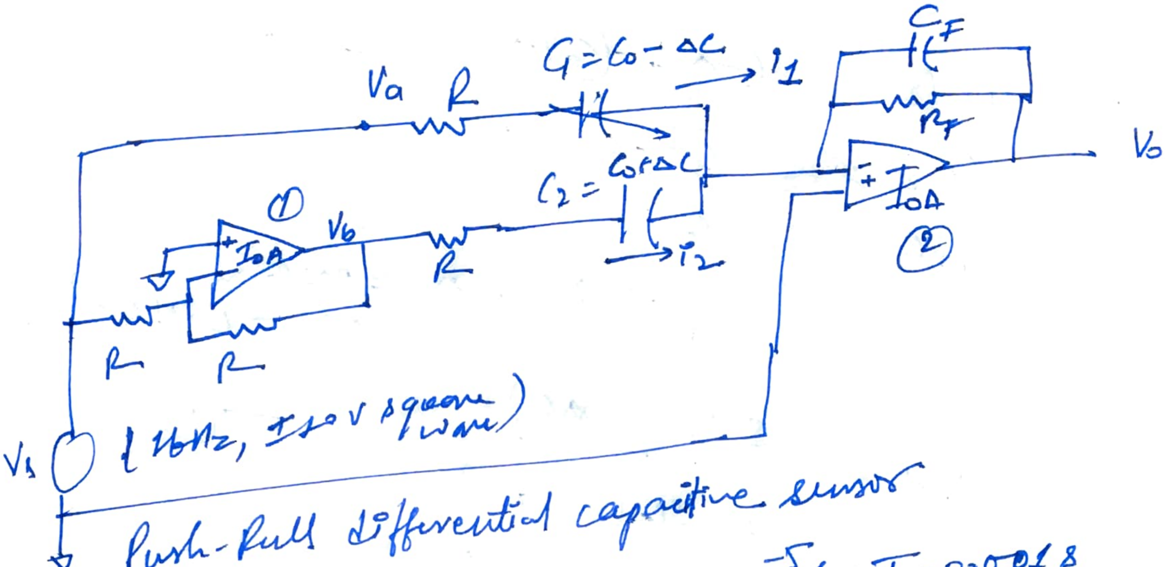
$\Delta T = 50$ $(T = 323K)$

$$V_R\left[\frac{R-0.7608}{R+0.7608}\right] = 5$$

$$V_R = 7.1563V$$

In this configuration, non-linearity is decreased whereas sensitivity remains ~~appropriomatly~~ approximately constant.

11)



Push-Pull differential capacitive sensor

$RC_0 << T << R_F C_F$; $RC_0 = 10^{-5}$ s ; $T = 0.001$ s

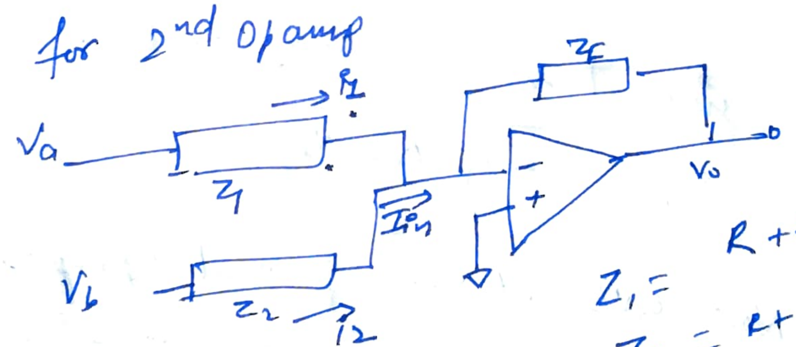
$R_F C_F = 0.1$ s.

Assumption = Ideal Opamps.

Here, $V_a = V_s$; KCL at (-) of ① :

$$\frac{V_s - 0}{R} = \frac{0 - V_b}{R} \Rightarrow \boxed{V_b = -V_s}$$

(a) for 2nd opamp



$Z_1 = R + \frac{1}{sC_1}$

$Z_2 = R + \frac{1}{sC_2}$

$\Rightarrow Z_F = \left(R_F \parallel \frac{1}{sC_F}\right)$

$$\boxed{Z_F = \frac{R_F}{1 + sC_F R_F}}$$

$$i_{in} = i_1 + i_2$$

$$= \frac{V_a}{Z_1} + \frac{V_b}{Z_2}$$

$$i_{in} = V_s\left(\frac{1}{Z_1} - \frac{1}{Z_2}\right)$$

$$i_{in} = V_s\left(\frac{sC_1}{(RsC_1+1)} - \frac{sC_2}{(RsC_2+1)}\right)$$

$$i_{IN} = \frac{sV_s(C_1 - C_2)}{1 + sR(C_1 + C_2) + s^2R^2C_1C_2}$$

$$C_1 - C_2 = C_0 - \Delta C - (C_0 + \Delta C) = -2\Delta C$$

$$C_1 C_2 = C_0^2 - \Delta C^2 \simeq C_0^2$$

$$C_1 + C_2 = 2C_0$$

$\left\{ \because \text{ given } \Delta C \simeq 0.1 C_0 \Rightarrow \Delta C^2 = 0.01 C_0^2 \right\}$.

$$i_{IN} = \frac{sV_s(-2\Delta C)}{1 + sR(2C_0) + s^2R^2C_0^2} \quad — ①$$

$$i_{IN}(s) = V(s) \left\{ \frac{\left(\frac{-2\Delta C}{R^2C_0^2}\right)s}{s^2 + \frac{2s}{RC_0} + \frac{1}{(RC_0)^2}} \right\}$$

Putting $s = j\omega$ in ① $\left(T = \frac{2\pi}{\omega}\right)$

$$i_{IN} = V_s \left(\frac{-2j\omega\Delta C}{1 + 2j\omega RC_0 + (j\omega)^2R^2C_0^2} \right)$$

given $T \gg RC_0 \Rightarrow \omega RC_0 \ll 1$

$$\boxed{i_{IN} = -2j\omega\Delta C \cdot V_s}$$

(b) Now KVL at (−) of 2nd Opamp:-

$$\frac{0 - V_0}{Z_F} = i_{IN} \Rightarrow V_0 = -Z_F i_{IN}$$

Substituting i_{IN}, $V_0 = (2j\omega\Delta C V_s) Z_F$

$$\Rightarrow V_0 = \frac{2j\omega\Delta C V_s R_F}{1 + j\omega R_F C_F}$$

$T \ll R_F C_F$

$\omega R_F C_F \gg 1$

$$\simeq \frac{2\Delta C R_F V_s j\omega}{j\omega R_F C_F}$$

$$\Rightarrow \boxed{V_0 = \left(\frac{2\Delta C}{C_F}\right) V_s}$$

(c) If, $\Delta C = 0.1 C_0 \sin(2\pi f t)$

here, $f = \frac{10}{100} \times \frac{1}{T} = \frac{1}{10T}$

$$\Delta C = 0.1 C_0 \sin\left(2\pi \cdot \frac{1}{10T} \cdot t\right)$$

(9)(b) $\Rightarrow$ $V_o = \frac{2 \times 0.1 C_0}{C_f} \times \sin\left(\frac{2\pi t}{10T}\right) \times V_s$ — 11c

11) a) from 11a :

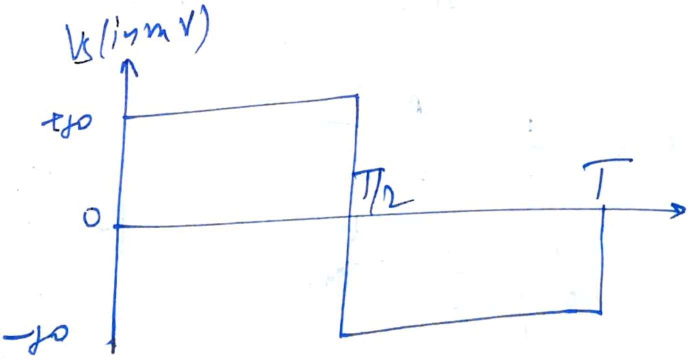
$$i_{IN} = -2j\omega \Delta C V_s \quad ; \quad \omega = \frac{2\pi}{T}$$

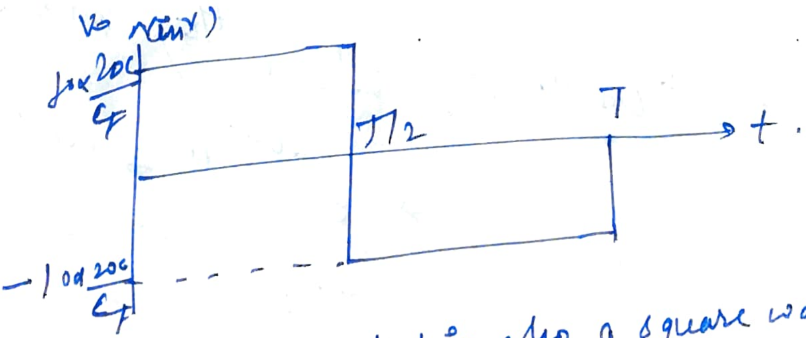
$T = 0.001$ s.

11b) from 11(b) : given constant ΔC,

$\Rightarrow$ V_o will also be square wave output.

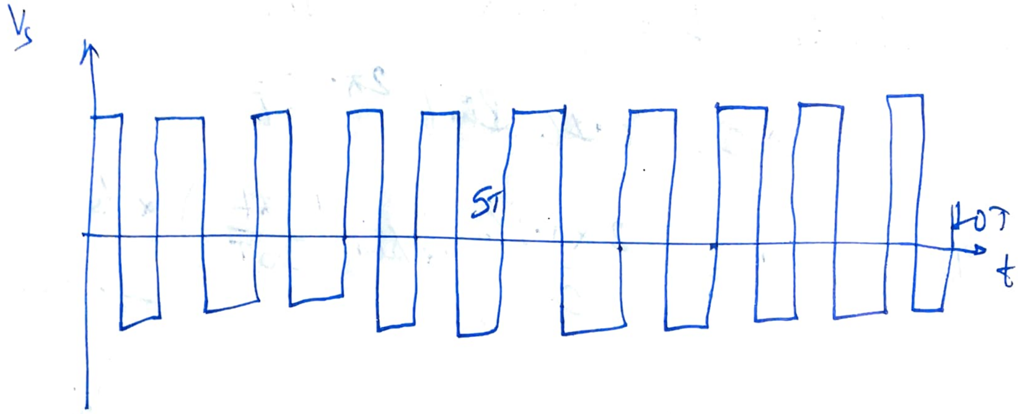
here $T = 0.001$ s.





$\Rightarrow$ Output is also a square wave, which is in phase with v_s, but of different amplitude.

11) c) (11c) $\Rightarrow$ $V_o = \frac{0.2 C_o}{C_F} \sin\left(\frac{2\pi t}{10T}\right) \cdot V_s$



Q10. As it is of first order: $H(s) = \frac{k}{1+sT}$

Step Input $X(s) = \frac{1}{s}$ $\Rightarrow Y(s) = \frac{k}{s(sT+1)}$

$Y(s) = \frac{A}{s} + \frac{B}{sT+1}$ $\Rightarrow A = k$

$B = -AT \Rightarrow -Ak$

$Y(s) = \frac{K}{s} - \frac{AkT}{sT+1}$

$y(t) = L^{-1}\left\{ \frac{k}{s} - \frac{kT}{sT+1} \right\} = k\left(1 - e^{-t/\tau}\right)$

from 1st Condition: $k\left(1-e^{-t/\tau}\right) \geq 0.95k$

$\Rightarrow e^{-0.05/T} \leq 0.05$

$\Rightarrow T \leq \frac{0.05}{\ln 20} = 0.016698$

$= 16.69$ ms

Ramp Input of 100 PST/s

$X(s) = \frac{100}{s^2}$ $\Rightarrow Y(s) = \frac{100k}{s^2(sT+1)}$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{1+sT}$$

$$As(sT+1) + B(1+sT) + s^2 C = 100k.$$

$$\boxed{B = 100k}$$

$$AT + C = 0 \Rightarrow C = -AT$$

$$A + BT = 0$$

$$\Rightarrow A = -BT = -100kT$$

$$C = +100kT^2$$

$$Y(s) = \frac{-100kT}{s} + \frac{100k}{s^2} + \frac{100kT^2}{1+sT}$$

$$\boxed{y(t) = 100k\left(t - T + Te^{-t/T}\right)}$$

$t \to \infty$: $y(t) = 100kt - 100kT$; $x(t) = 100kt$

ss error : $\lim_{t\to\infty} x(t) - y(t) = 100kT \leq 2$

$$\boxed{kT \leq 0.02}$$

c) for a first order T.F & sine wave input,

$A \sin \omega t \Rightarrow H(s) = \frac{K}{1+sT}$

$$y(t) = Ak\left[\frac{\omega_0 T}{1+\omega_0^2 T^2} e^{-t/T} + \frac{1}{\sqrt{1+\omega_0^2 T^2}} \sin(\omega_0 t + \phi)\right].$$

$$\phi = \tan^{-1}(\omega_0 T)$$

as $t \to \infty \Rightarrow y(t) = \frac{Ak}{\sqrt{1+\omega_0^2 T^2}} \sin(\omega_0 t + \phi)$

given from 3rd Condition:

$$\frac{Ak}{\sqrt{1+(40\pi T)^2}} \geq 0.9\,Ak$$

$$1+40^2\pi^2T^2 \leq (0.9)^{-2}$$

$$\boxed{T \leq \frac{\sqrt{19}}{360\pi}\ s}$$

$$\boxed{T \leq 3.854\ ms}$$

from above,

$T \leq 3.854$ ms

$kT \leq 0.020$ PSI

$= 20$m PSI

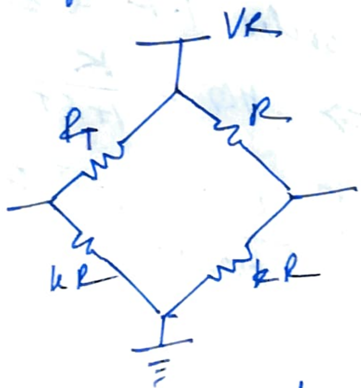
$k \rightarrow$ Steady State Gain

$T \rightarrow$ Time Constant

Q5. Extreme Scenario:-

$T_a = 0°C$

Bridge Voltage should be 0V.



$R_T = R_0(1+\alpha\Delta T)$, where $\Delta T = 0$

$\Rightarrow R_T = R_0 = 100\,\Omega$

for $\Delta V = 0 \Rightarrow R = R_0 = 100\,\Omega$

$T_a = 100°C$

Bridge Voltage should be $\Delta V = 5.268\,mV$

$$\Delta V = V_R\left[\frac{kR}{(k+1)R} - \frac{kR}{kR + R(1+\Delta T\alpha)}\right] = 5.268 \times 10^{-3}$$

$$\Rightarrow V_R\left[\frac{k}{k+1} - \frac{k}{k+1+0.00392 \times 100}\right] = 5.268 \times 10^{-3}$$

$$\Rightarrow \frac{k \times V_R \times \left[0.0392\right]}{(k+1)(k+1.392)} = 5.268 \times 10^{-3}$$

let $V_R = 10V \Rightarrow k = 0.0018767$

$\therefore kR = 0.18767\ \Omega$

The error due to non-linearity can be defined as:-

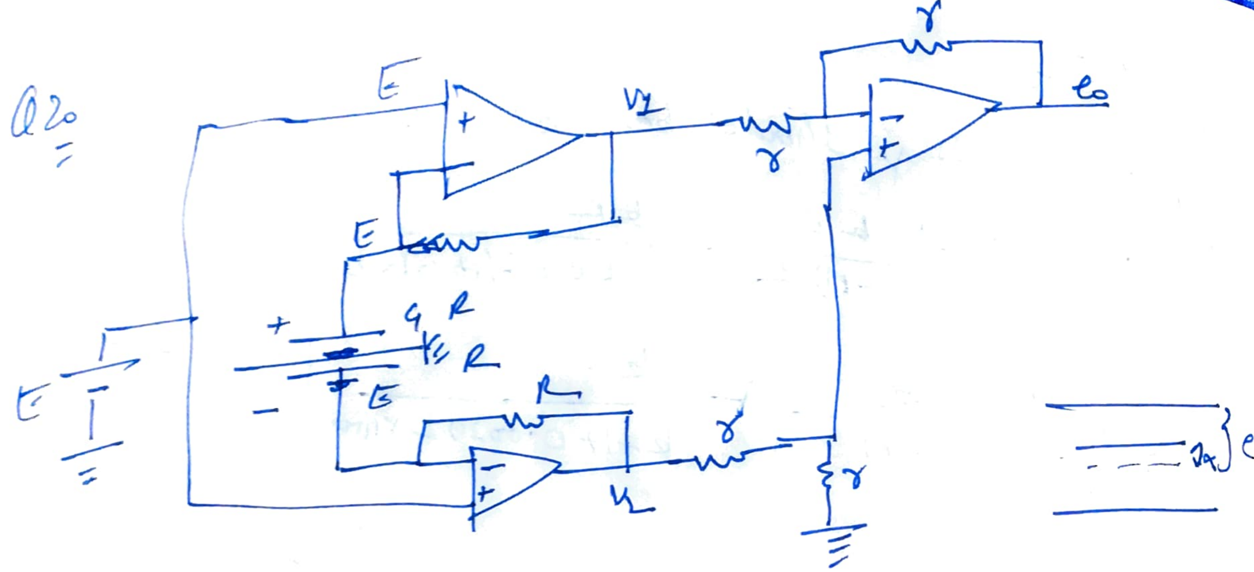
$$\text{error} = V_{compensated}\big|_{T_a=T} - 5.268 \times 10^{-3}$$

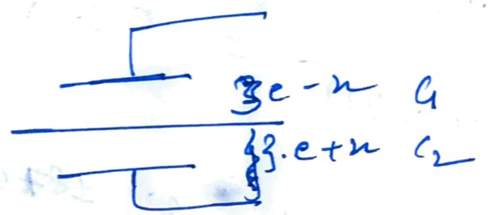
where $V_{compensated}\big|_{T_a=T} = -V_{thermocouple}\big|_{T_a=T} + 5.268 \times 10^{-3} + \Delta V$

Temperature (Ambient °C)	Error (mV)
0	0
10	0.1983
20	0.3404
30	0.4318
40	0.4769
50	0.4800
60	0.4464
70	0.3784
80	0.2795
90	0.1534
100	0

$$\therefore \text{Non-linearity} = \frac{0.4800}{5.268} \times 100\% = 9.11\%$$

Q20





$$C_1 = \frac{\epsilon A}{d_1} = \frac{\epsilon A}{e-x}$$

$$C_2 = \frac{\epsilon A}{e+x}$$

$$V_1 = E + ER\frac{dC_1}{dt}$$

$$V_1 = E + ER\frac{dC_2}{dt}$$

Then there is a difference amplifier stage

$$e_0 = V_2 - V_1$$

$$= ER\left\{\frac{dC_2}{dt} - \frac{dC_1}{dt}\right\}.$$

$$e_0 = ER\alpha\left\{\frac{dC_2}{dt} - \frac{dC_1}{dt}\right\}.$$

$$\frac{dC_2}{dt} = \frac{-\epsilon A}{(e+x)^2}\frac{dx}{dt}$$

$$\frac{dC_1}{dt} = + \frac{\epsilon A}{(e-x)^2}\frac{dx}{dt}$$

$$e_o = -ER\epsilon A \left\{ \frac{(e+x)^2+(e-x)^2}{(e+x)^2(e-x)^2} \right\} \frac{dx}{dt}$$

$$= -\epsilon A \epsilon R \left\{ \frac{2(e^2+x^2)}{(e^2-x^2)^2} \right\} \frac{dx}{dt}$$

$e^2 >> x^2$ {Assumption for linearity}.

$$e_o = -\epsilon A E R \frac{2e^2}{e^4} \times \frac{dx}{dt}$$

$$e_o = \frac{-2\epsilon AER}{e^2} \frac{dx}{dt}$$

Taking L.T

$$E_o(s) = \frac{-2\epsilon AER}{e^2} s\,x(s)$$

$$\boxed{\frac{E_o(s)}{x(s)} = \frac{-2\epsilon EAR}{e^2} s}$$