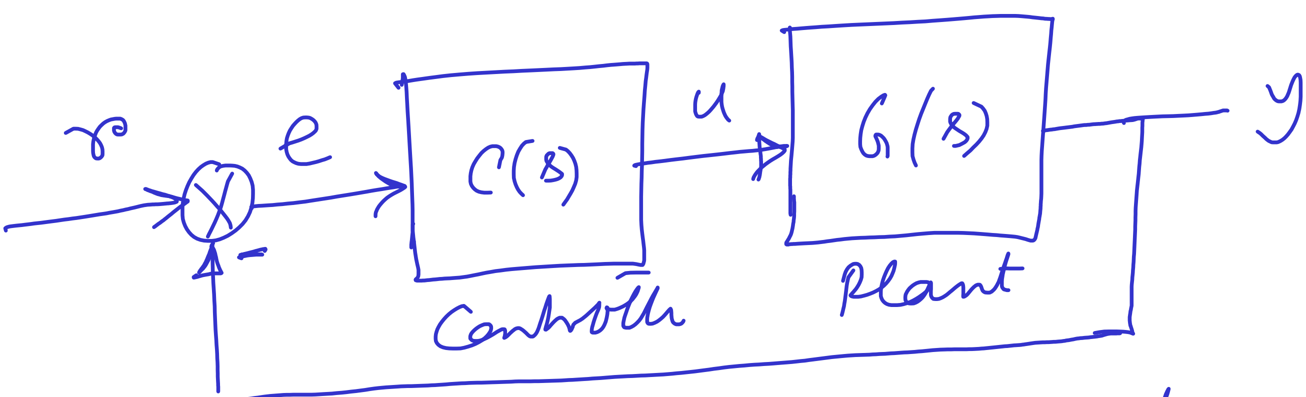
Lecture - 7

Designing lag, lead & lag-lead controller

specifications

- Steady-state error constraints (e.g. proportional error constraint (K_p) velocity error constraint (K_v), etc)
- phase margin (PM)
- gain cross-over freq (gcf)



$C(s) = K H(s)$ — cascaded controller

K — static gain used to meet s.s error constraints

$H(s)$ — Normalized lag, lead or lag-lead controller
(used to meet PM & g.c.f specifications.

To meet S.S specifications:

$$e_{ss} = \frac{1}{1+K_p} \rightarrow \text{for step i/p}$$

$$= \frac{1}{K_v} \rightarrow \text{for ramp input}$$

Where

$$K_p = \lim_{s\to 0} L(s) = \lim_{s\to 0} G(s) \cdot K H(s)$$

$$= \lim_{s\to 0} K G(s) \quad [\because H(0) = 1]$$

$$K_v = \lim_{s\to 0} s L(s) = \lim_{s\to 0} K s G(s)$$

To meet PM & g.c.f

- From given PM specification, determine the phase of the compensated system at gcf as $-180^\circ + PM = \phi_{com}$

- Determine the phase of the uncompensated system at the given gcf, ϕ_{uc}, say
- Determine the gain (in dB), M_{uc} of the uncompensated system at the given gcf
- If $\phi_{com} > \phi_{uc}$ & M_{uc} is $-$ve in dB, then use lead controller with $\phi_c = \phi_{com} - \phi_{uc}$ & $M_c = -M_{uc}$
- If $\phi_{com} < \phi_{uc}$ & M_{uc} is $+$ve in dB, then use lag controller where $\phi_c = \phi_{com} - \phi_{uc}$, $M_c = -M_{uc}$
- If $\phi_{com} > \phi_{uc}$ but M_{uc} is $+$ve then use lag-lead controller with $M_c = -M_{uc}$, $\phi_c = \phi_{com} - \phi_{uc}$

Note: If ω_c is not mentioned, then the following are important:

- $\omega_c > \omega_{uc}$, use lead controller such that M_c is $+$ve & ϕ_c is positive
- $\omega_c < \omega_{uc}$ - - lag controller such that M_c & ϕ_c are negative

- use lag-lead for a freq if M_c is -ve but ϕ_c is +ve

Example: $G(s) = \dfrac{1}{s(s+5)(s+10)}$

Design a suitable controller to meet the following specification

a) $K_v = 10$, b) $PM = 45^0$

(choose suitable g.c.f ω_c)

Soln To determine servo gain K

$$K_v = \lim_{s \to 0} s K G(s)$$

$$\Rightarrow K_v = \frac{K}{50}$$

$$\Rightarrow K = 50 K_v$$

$$\Rightarrow \boxed{K = 500}$$

choice of suitable gcf ω_c

Freq response data (for $KG(s) \to$ uncompensated loop TF)

ω (rad/s)	ϕ_{uc} (degree)	M_{uc} (dB)	ω rad/s	ϕ_{uc} (deg)	M_{uc} (dB)
1	-107.02°	19.79	4	-150.46°	5.166
1.5	-115.21°	16.01	6	-171.16°	0.772
2	-123.11°	13.164	8	-186.65°	-5.725
2.5	-130.6	10.81	9	-192.93°	-7.935
3.0	-136.66°	8.75	10	-198.43°	-10

For lead Controller:

Compensated system phase needed at desired ω_c is $\phi_{com} = -180^\circ + PM = -180^\circ + 45^\circ$

$\Rightarrow \phi_{com} = -135^\circ$

From the table $\phi_{com} > \phi_{uc}$ & $M_{uc} = -ve$ for the range $8 \leq \omega \leq 10 \Rightarrow$ ω_c should be chosen from 8 to 10 rad/s for lead controller design

~~Let~~ $\boxed{\omega_c = 8}$ rad/s

Then $\phi_c = -135° - (-186.65°)$

$\Rightarrow \boxed{\phi_c = 51.65°}$

& $\boxed{M_c = 5.725 \text{ dB}}$

The lead controller becomes

$$\boxed{H(s) = \frac{12.725\,(s+4.78)}{s+60.82}}$$

(check it)

For lag controller design:

From freq response data,

$\phi_{uc} > \phi_{com}$ & $M_{uc} = +ve$

for $1 \le \omega \le 2.5$. Thus

the ω_c should lie in between

$\boxed{1 \text{ to } 2.5}$ rad/s.

Let $\boxed{\omega_c = 2 \text{ rad/sec.}}$

Then $\phi_c = \phi_{cm} - \phi_{uc}$

$$\Rightarrow \boxed{\phi_c = -135 - (-123.11) = 11.89^\circ \text{ (Phase lag)}}$$

$$\boxed{M_c = -M_{uc} = -13.164 \text{ dB}}$$

Then the lag controller becomes

$$\boxed{H(s) = \frac{0.2124(s+0.543)}{s+0.1153}}$$

(check it)

For lag-lead controller:

Here $3 \le \omega_c \le 6$ rad/s

[since in this range M_{uc} is +ve but $\phi_{uc} < \phi_{cm}$

Let $\boxed{\omega_c = 4 \text{ rad/s}}$

Then $\phi_c = -135^\circ - (-150.46^\circ)$

$$\Rightarrow \boxed{\phi_c = 15.46^\circ}$$

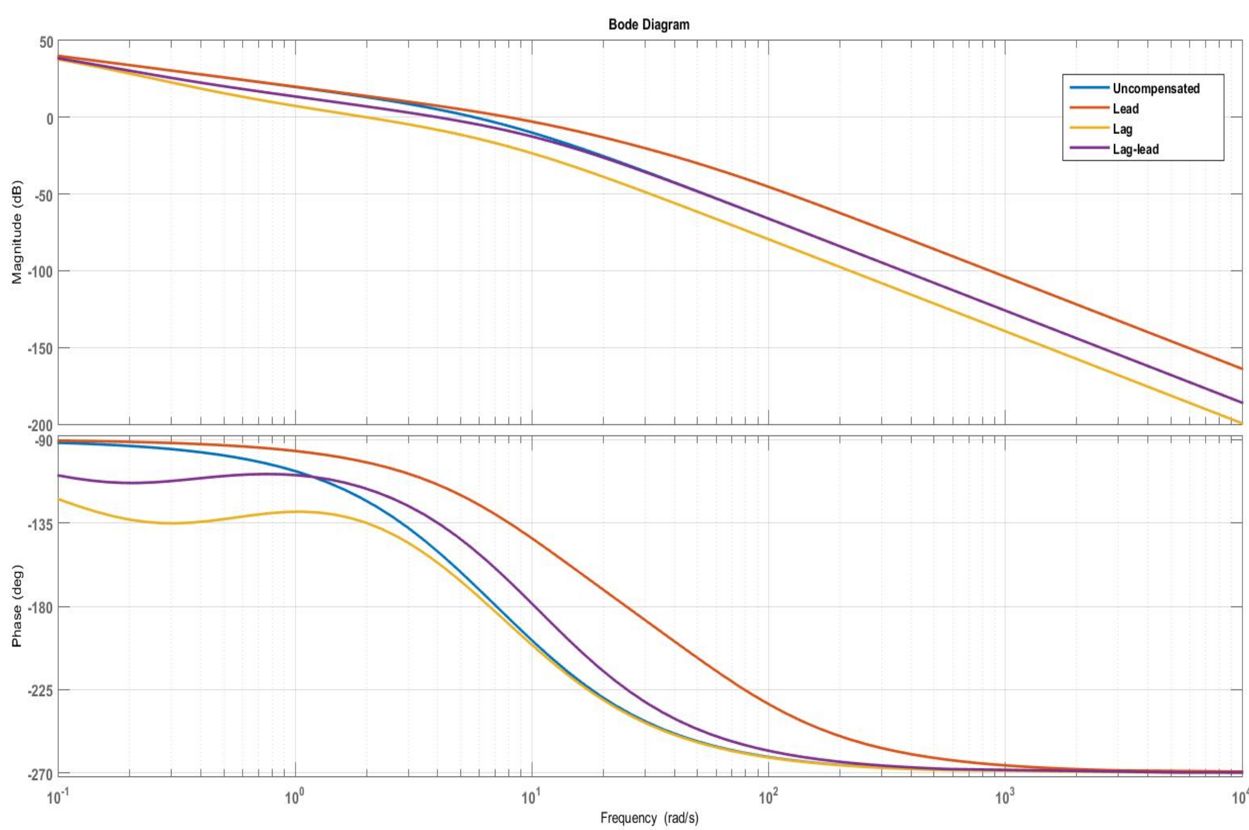
$$\boxed{M_c = -M_{uc} = -5.166 \text{ dB}}$$

Then with $\phi_2 = 2^\circ$, one obtains the lag-lead controller

$$\boxed{H(s) = \frac{(s+5.36236)(s+0.26331)}{(s+11.44735)(s+0.1233)}}$$

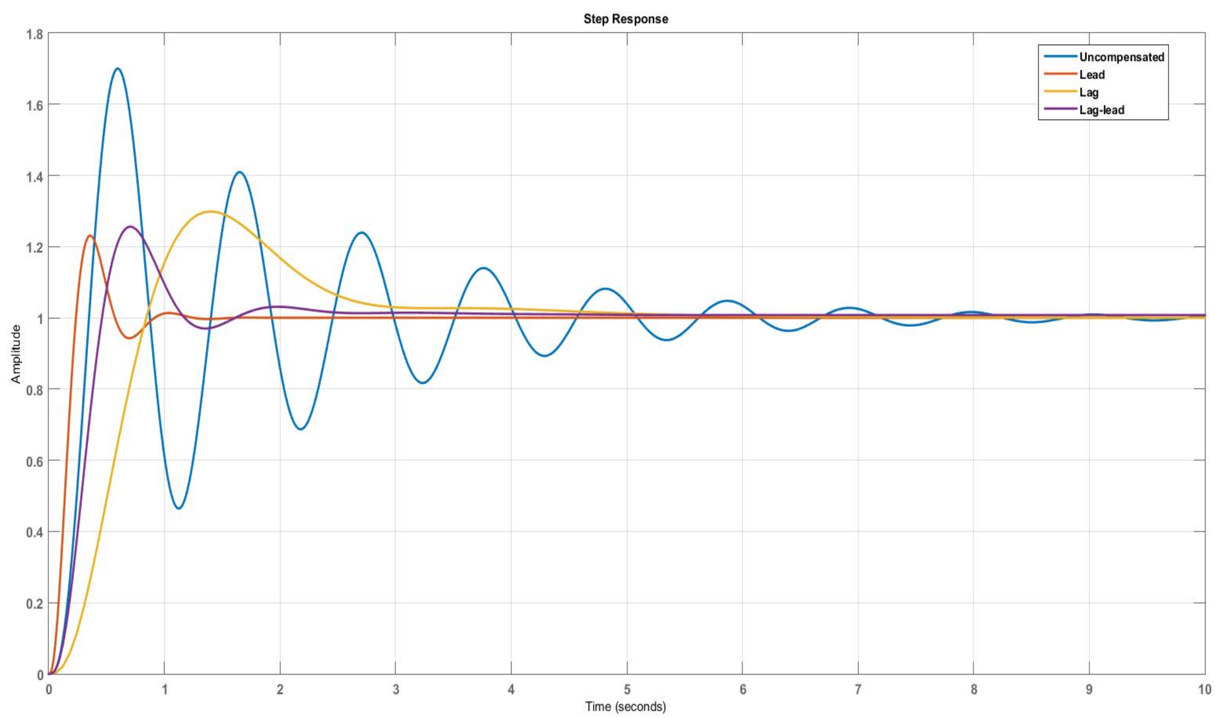
(check it)

Bode-plot



- with lead contr. dB and Phase curves gets shifted up
- - - - lag - - - dB & Phase - - - - - down
- lag-lead contr. a compensator is made.

Time response



- uncompensated response is oscillati-
- with lead contr. system is fast
- - - - lag - - - - - becomes slow
- - - lag-lead - - - response is traded-off.