Digital Signal Processing

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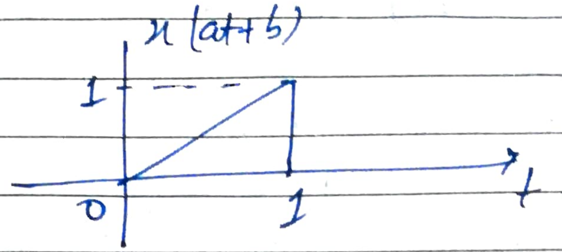
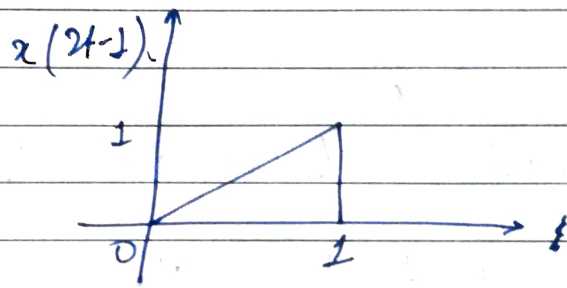
Q1. (a)

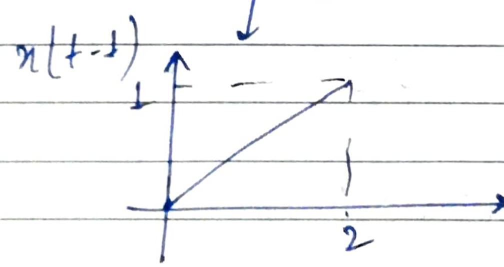


When $a = 2$, $b = -1$

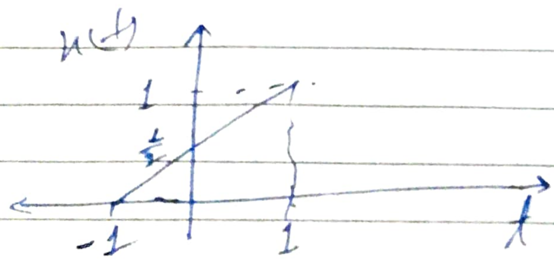
$y = x(2t-1)$



$\downarrow x(t/2)$



$\downarrow x(t+1)$

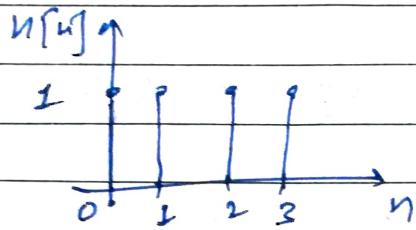


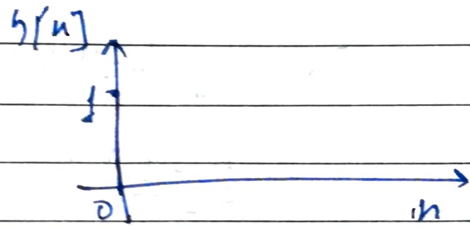
This is the final plot of $x(2t-1)$.

2. (b)

$$x[n] = u[n] - u[n-4]$$

$$h[n] = \delta[n] - \delta[n-2]$$

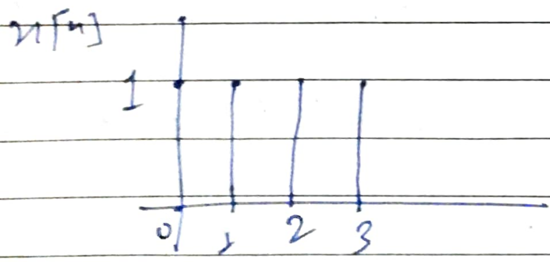


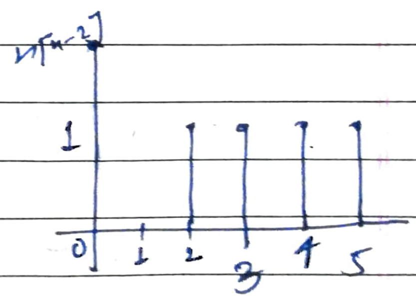


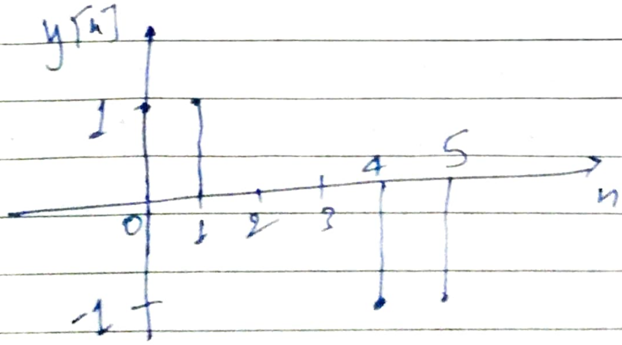
$$y[n] = x[n] * h[n]$$

$$= x[n] * [\delta[n] - \delta[n-2]]$$

$$= x[n] - x[n-2]$$







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Q2. (a) Accumulation z-transform.

$$\sum_{k=-\infty}^{n} g[k] \xleftrightarrow{z} \frac{z}{z-1} G(z)$$

where $g[k] \xleftrightarrow{z} G(z)$

Backward difference in z-transform.

$$g[n] - g[n-1] \xleftrightarrow{z} (1-z^{-1}) G(z)$$

$$= \left(\frac{z-1}{z}\right) G(z)$$

where. $g[n] \xleftrightarrow{z} G(z)$

Here, we can see that if we apply Backward difference and accumulation system, back to back on a sequence $g[n]$:

$$Y[n] = \text{Backward difference}\left(\text{Accumulate}(g[n])\right)$$

$$Y[z] = \left(\frac{z-1}{z}\right)\left(\frac{z}{z-1}\right) G(z)$$

$$= G(z)$$

We can see that Backward difference & accumulator systems are inverse of each other.

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Q2(b): Let us consider an accumulator as

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Now, by variance, let us delay the output by n_0.

$$y[n-n_0] = \sum_{k=-\infty}^{n-n_0} x[k]. \quad \text{—(1)}$$

Now, we find,

$$y_1[n] = \sum_{k=-\infty}^{n} x_1[k]. \qquad \left\{ \text{Assuming, } x_1[n] = x[n-n_0] \right\}.$$

$$= \sum_{k=-\infty}^{n} x[k-n_0] \quad \text{—(2)}$$

Substitute the shift in variables as $k_1 = k - n_0$ into (2)

$$y_1[n] = \sum_{k_1=-\infty}^{n-n_0} x[k_1].$$

Since the index k in eq (1) & index k_1 in eq (2) are dummy indices of summation and can be replaced by any other labels, eq (1) & eq (2) are equal. & therefore $y_1[n] = y[n-n_0]$. So the accumulator is a time invariant system.

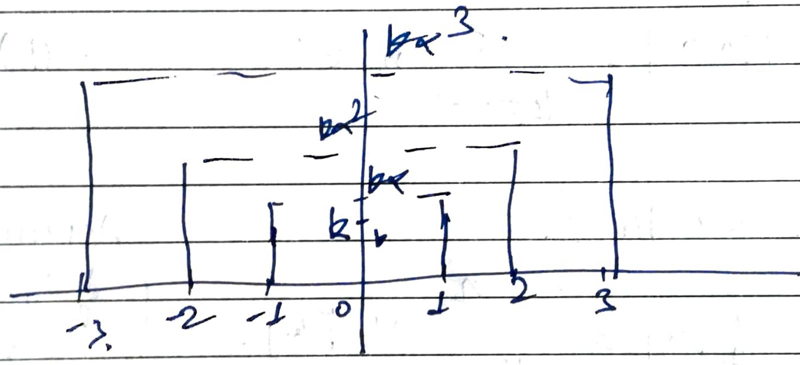
Q 3)

$x(n) = k\alpha^{|n|}$

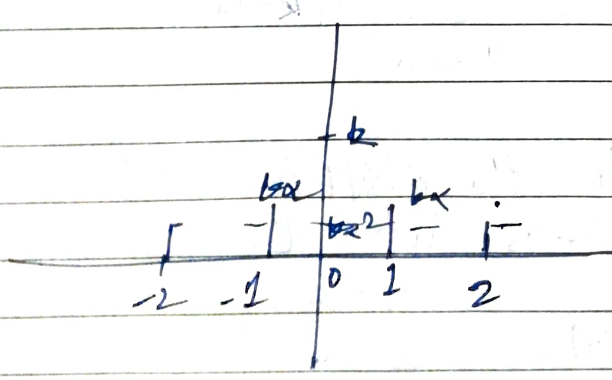
Case I : if $\alpha > 1$

$x(n) = k\alpha^{|n|}$

$\begin{cases} k\alpha^{n} & n \geq 0 \\ k\alpha^{-n} & \text{if } n < 0. \end{cases}$



Case II : $\alpha < 1$.



for $\alpha > 1$.

The graph exponentially increases to both sides. therefore there will not be a common ROC to left sided & right sided plots.

Common ROC doesn't exist

$\therefore$ Z-transform for $x[n] = k\alpha^{|n|}$, $\alpha > 1$ doesn't exist.

for $\alpha < 1$,

The plot converges, so both z-transform & ROC exists.

$$X(z) = k\left(\frac{-z}{z-\alpha^{-1}} + \frac{z}{z-\alpha}\right)$$

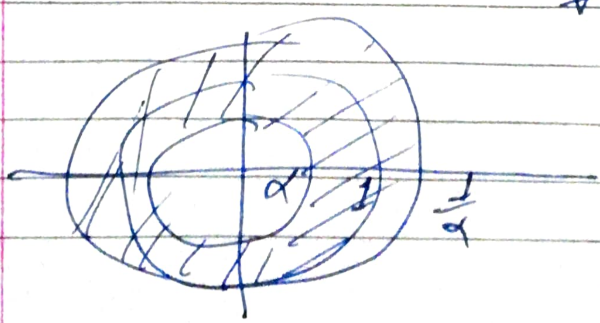
$$= \frac{kz\left(\alpha-\alpha^{-1}\right)}{\left(z-\alpha^{-1}\right)\left(z-\alpha\right)}$$

$R_1 = |z| < |\alpha^{-1}|$, $R_2 = |z| > |\alpha|$

ROC: $R_1 \cap R_2$

$: |\alpha| \le |z| \le |\alpha^{-1}|$

$\forall\ |\alpha| < 1$.



$|\alpha| \le |z| \le |\alpha^{-1}|$.