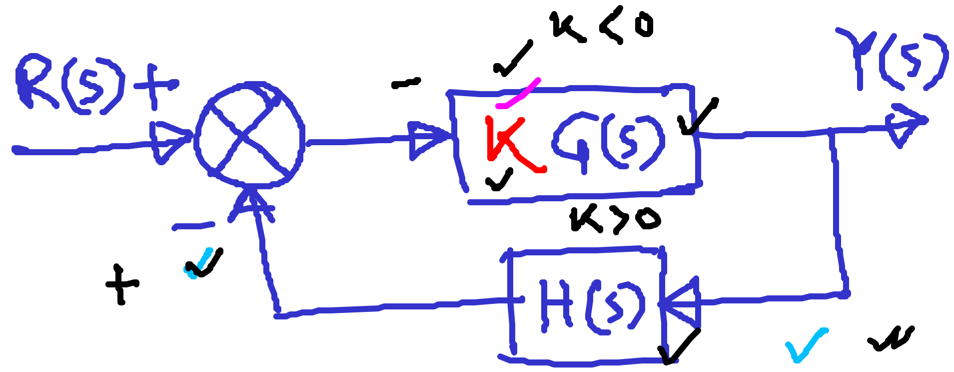
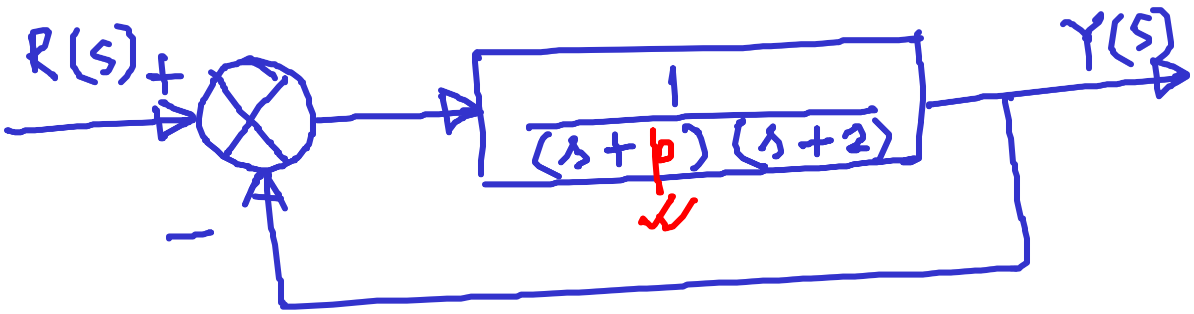
Root Locus Technique: A graphical tool for control system analysis and design



Closed-loop TF = $\dfrac{K G(s)}{1 + K G(s) H(s)}$

- A graphical representation of the closed-loop poles as a system parameter (gain, a pole, a zero) is varied.
- When K varies, the locations of roots of $1 + K G(s) H(s) = 0$ also vary.
- The plot of varying roots is root locus.
- When $K > 0$, it is with −ve feedback structure; $K < 0$ indicates positive feedback structure. In +ve feedback the ch. eq^n is $1 - K G(s) H(s) = 0$.

A pole is varying



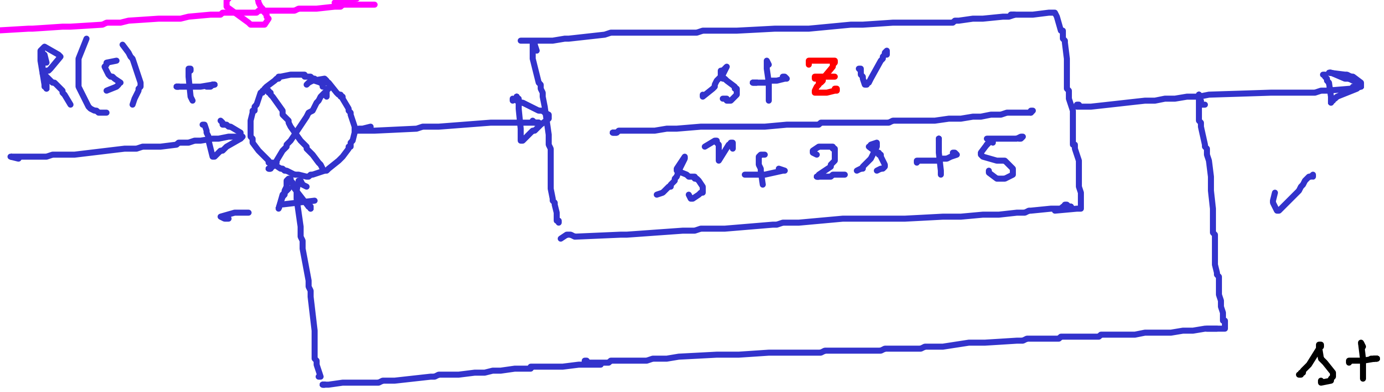
$$\text{Closed-loop TF} = \frac{1}{s^2+2s+ps+2p+1}$$

$$= \frac{1}{s^2+2s+p(s+2)+1}$$

$$= \frac{1/s^2+2s+1}{1+p\,\frac{s+2}{s^2+2s+1}}$$

$$\text{Ch. eq}^n: \quad 1+p\,\underbrace{\frac{s+2}{s^2+2s+1}} = 0$$

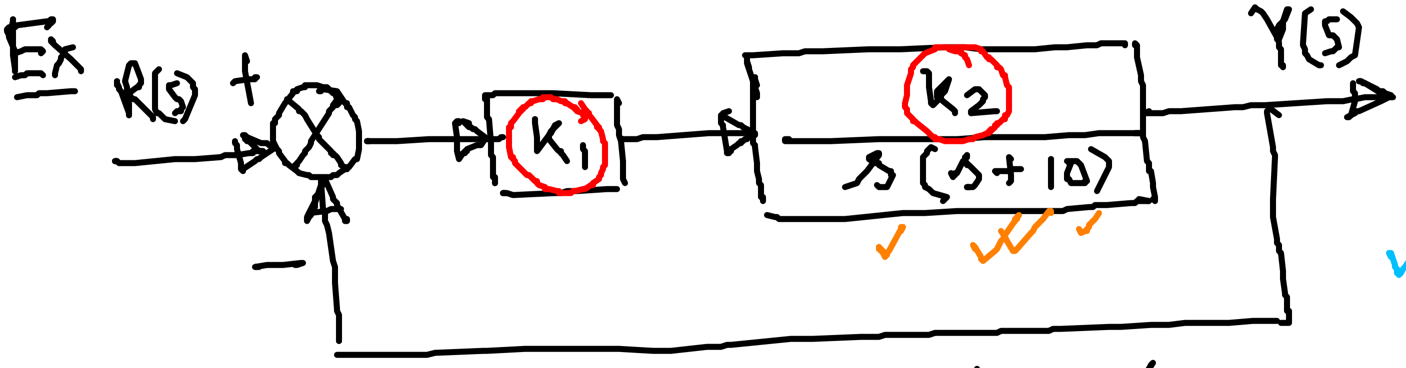
A zero is varying



$$\text{Closed-loop TF} = \frac{s+z}{s^2+2s+5+s+z} = \frac{s+z/s^2+3s+5}{\underbrace{1+\frac{z}{s^2+3s+5}}}$$

Ch. eq^n: $1 + z\,\frac{1}{s^2+3s+5} = 0$

Ex



Closed-loop TF: $$\frac{K_1K_2/s(s+10)}{1+\frac{K_1K_2}{s(s+10)}}$$

Ch. eq^n: $1 + K_1K_2\,\frac{1}{s(s+10)} = 0$

K

$\boxed{K = K_1K_2}$

Let Pole 1 = −10 and Pole 2 = 0.

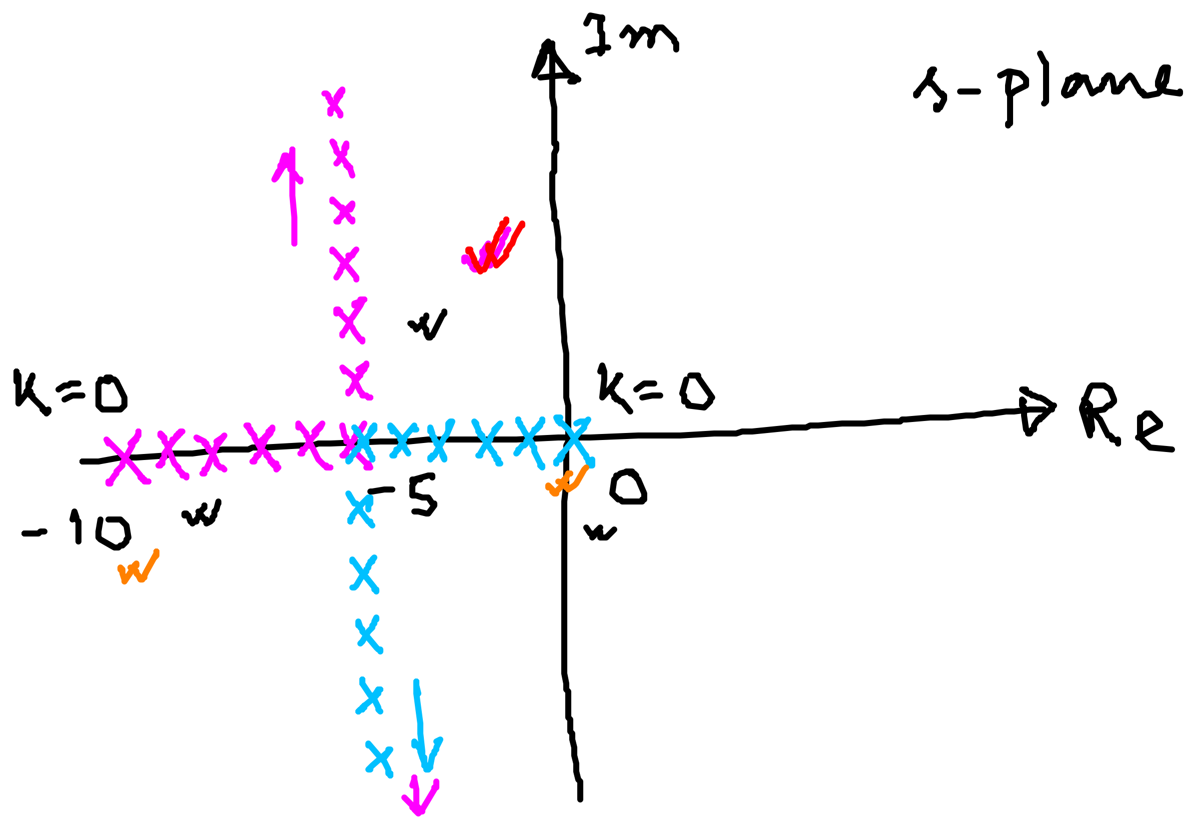
$$1 + K\,\frac{1}{s(s+10)} = 0 \Rightarrow s(s+10) + K = 0$$

K = 0

When $K=0$, $s=0$ and $s=-10$ are two roots.

K	Pole 1	Pole 2
0	−10	0
5	−9.47	−0.53

10	-8.87	-1.13
15	-8.16	-1.84
20	-7.24	-2.76
25	-5	-5
30	$-5+j2.25$	$-5-j2.24$
35	$-5+j3.16$	$-5-j3.16$
40	$-5+j3.87$	$-5-j3.87$
45	$-5+j4.47$	$-5-j4.47$
50	$-5+j5$	$-5-j5$



Properties of root locus (Negative feedback case, $K>0$)

Ch. eqⁿ: $1 + K\,G(s)\,H(s) = 0$

$$K\,G(s)\,H(s) = -1 = 1\angle(2k+1)180^\circ$$

$k = 0, \pm1, \pm2, \dots$

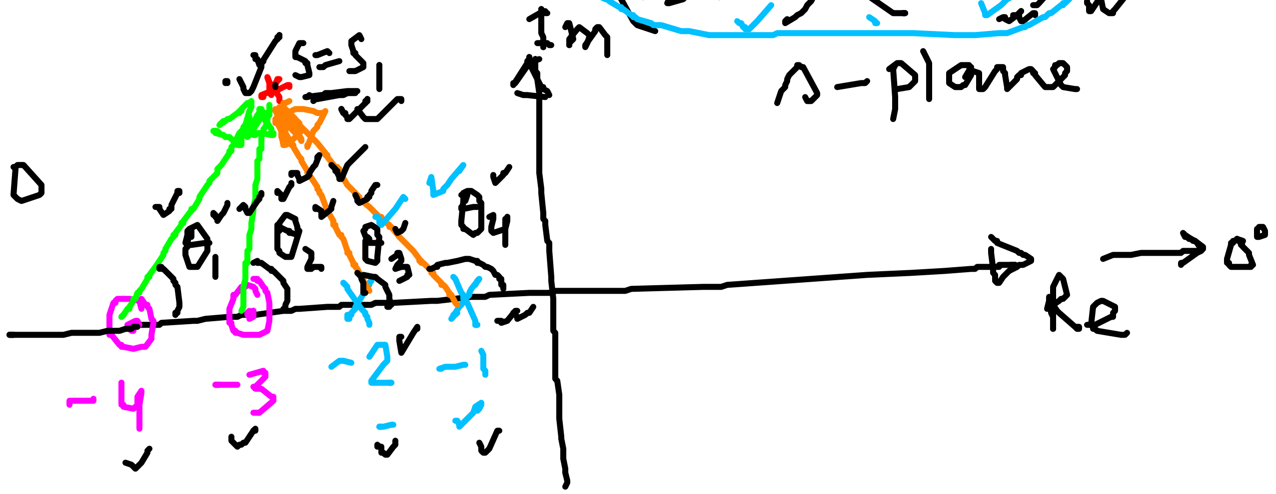
$$\boxed{\begin{aligned} |K\,G(s)\,H(s)| &= 1 \\ \angle K\,G(s)\,H(s) &= \angle(2k+1)180^\circ \end{aligned}}$$

If at $s = s_1$ the angle criterion is satisfied, s_1 is a closed-loop pole when $K = \dfrac{1}{|G(s)\,H(s)|}$ is set.

Ex.

Let $K\,G(s)\,H(s) = \dfrac{K(s+3)(s+4)}{(s+1)(s+2)}$

$1 + K\,G(s)\,H(s) = 0$



If $-\theta_3 - \theta_4 + \theta_1 + \theta_2 = (2k+1)\,180^\circ$, ✓✓

then $s = s_1$ is a closed-loop pole.

The corresponding value of

$$K = \frac{\prod \text{pole lengths}}{\prod \text{zero lengths}}.$$