Assignment - 1

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17EE35004

Singular Value Decomposition : SVD

SVD of an $m \times n$ matrix M is a factorization of the form $U \cdot S \cdot V^H$, where U is an $m \times m$ unitary matrix, S is an $m \times n$ rectangular diagonal matrix with non-negative real numbers on the diagonal, and V is an $n \times n$ unitary matrices. If M is real matrix, then U and V are orthogonal matrices.

$$G = \begin{bmatrix} 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 200 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 255 & 255 & 50 & 255 & 255 & 255 \\ 50 & 50 & 50 & 50 & 255 & 255 & 255 & 255 \end{bmatrix}$$

Factorizing : $U \cdot S \cdot V^H = G$.

or

$U \cdot S \cdot V^T = G$ here

Once we complete factorization; we must reconstruct the image.

(a) one largest eigenvalue.

$$Im1 = U(:,1)\cdot S(1,1)\cdot V(:,1)'$$

first column of U

first eigenvalue largest

first column of V

(b) two largest eigenvalues

$$Im2 = U(:,1:2)\cdot S(1:2,1:2)\cdot V(:,1:2)'$$

(c) three largest eigenvalue

$$Im3 = U(:,1:3)\cdot S(1:3,1:3)\cdot V(:,1:3)'$$

(d) four largest eigenvalue

$$Im4 = U(:,1:4)\cdot S(1:4,1:4)\cdot V(:,1:4)'$$

(e) five largest eigenvalue

$$Im5 = U(:,1:5)\cdot S(1:5,1:5)\cdot V(:,1:5)'$$

To check which reconstructed image is more close to original one, we need to find distance between two images, i.e; norm of difference of images. The one which is closer is a more approximate image.

$\text{dist}(G, Im1) = \| G - Im1 \|_2 = 334.0981$

$\text{dist}(G, Im2) = 267.8401$

$\text{dist}(G, Im3) = 186.5231$

$\text{dist}(G, Im4) = 109.0080$

$\text{dist}(G, Im5) = 0$ ✓ exact reconstruction.

Why SVD is used ?

SVD is robust and reliable orthogonal matrix decomposition method. It decomposes a matrix into orthogonal components with which optimal subrank approximations may be obtained. The largest object component in an image found using the SVD generally correspond to eigenimages associated with the largest singular values, while image noise corresponds to eigenimages associated with the smallest singular values. SVD is used to approximate the matrix decomposing the data into an optimal estimate of the signal and the noise components. This property is one of the most important properties of SVD in noise filtering, compression and forensic which could also treated as adding noise in a proper detectible way.

$$X = \sum_{i=1}^{k} B_i A_i C_i^T \approx B_1 A_1 C_1^T + B_2 A_2 C_2^T + \cdots + B_K A_K C_K^T$$

SVD applications :

→ SVD is used in noise filtering, image denoising, and watermarking.

→ SVD has maximum energy packing among the other transforms. With multiresolution SVD, following characteristics of an image may be measured, at each level of resolution: isotropy, spercity of principal components & self similarity under scaling.

→ Truncated SVD tranformations offer significant saving in storage without great loss of information. This property is used in Compression of image.

→ SVD has the ability to adapt to the variations in local statistics of an image. The stability of singular values (specifies energy of image layer) can be utilized by SVD based watermarking technique. Thus it is used in Image Forensic.

MATLAB CODE :

RTSP_Assg_1_17EE35004.m :

```matlab
%% Initializing all Matrices

clc;
clear;

G = [255 255 255 255 255 255 255 255;
     255 255 255 100 100 100 255 255;
     255 255 100 150 150 150 100 255;
     255 255 100 150 200 150 100 255;
     255 255 100 150 150 150 100 255;
     255 255 255 100 100 100 255 255;
     255 255 255 255 50  255 255 255;
     50  50  50  50  255 255 255 255];

[U,S,V] = svd(G);            % Matlab's SVD algo

%% Reconstruction of images using largest eigenvalues

im1 = U(:,1)*S(1,1)*V(:,1)';
im2 = U(:,1:2)*S(1:2,1:2)*V(:,1:2)';
im3 = U(:,1:3)*S(1:3,1:3)*V(:,1:3)';
im4 = U(:,1:4)*S(1:4,1:4)*V(:,1:4)';
im5 = U(:,1:5)*S(1:5,1:5)*V(:,1:5)';

%% Finding the distance between original and reconstructed images

dist1 = zeros(5,1);
dist1(1) = norm(G-im1);
dist1(2) = norm(G-im2);
dist1(3) = norm(G-im3);
dist1(4) = norm(G-im4);
dist1(5) = norm(G-im5);

%% ploting the reconstructed images

figure;
subplot(3,2,1);imagesc(G);
title("original image");
subplot(3,2,2);imagesc(im1);
title("one largest eigenvalue");
subplot(3,2,3);imagesc(im2);
title("two largest eigenvalues");
subplot(3,2,4);imagesc(im3);
title("three largest eigenvalues");
subplot(3,2,5);imagesc(im4);
title("four largest eigenvalues");
subplot(3,2,6);imagesc(im5);
title("five largest eigenvalues");
```

Reconstructed images using SVD algorithm :