

Q1. Step 1: Initial Computations

Convert all the loads in per-unit values

$$P_{L2} = \frac{150}{100} = 1.5 \text{ pu}$$

$$Q_{L2} = \frac{50}{100} = 0.5 \text{ pu}$$

$$P_{L3} = \frac{50}{100} = 0.5 \text{ pu}$$

$$Q_{L3} = \frac{20}{100} = 0.2 \text{ pu}$$

Convert all the generation in per-unit values,

$$P_{g2} = \frac{70}{100} = 0.7 \text{ pu}$$

$$Q_{g2} = \frac{30}{100} = 0.3 \text{ pu}$$

$$P_{g3} = 0$$

$$Q_{g3} = 0$$

Compute net-injected power at bus 2 and 3.

$$P_2 = P_{g2} - P_{L2} = (0.7 - 1.5) = -0.8 \text{ pu}$$

$$Q_2 = Q_{g2} - Q_{L2} = (0.3 - 0.5) \text{ pu} = -0.2 \text{ pu}$$

$$P_3 = P_{g3} - P_{L3} = (0 - 0.5) = -0.5 \text{ pu}$$

$$Q_3 = P_{33} Q_{g2} - Q_{L3} = (0 - 0.2) = -0.2 \text{ pu.}$$

Step 2: Formation of Ybus matrix:

$$Y_{BUS} = \begin{bmatrix} 53.85 \angle -61.2^\circ & 22.36 \angle 116.4^\circ & 31.62 \angle 108.4^\circ \\ 22.36 \angle 116.4^\circ & 58.13 \angle -63.9^\circ & 35.77 \angle 116.6^\circ \\ 31.62 \angle 108.4^\circ & 35.77 \angle 116.6^\circ & 67.23 \angle -64.2^\circ \end{bmatrix}$$

Step 3: Iterative Computation

$$V_2^{(p+1)} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^{(p)})^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} \right]$$

~~Slack bus voltage~~

$$V_3^{(p+1)} = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^{(p)})^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)} \right]$$

Slack bus Voltage

$$V_1 = (1 + j0.00)$$

Starting Voltage

$$V_2^{(0)} = (1 + j0)$$

$$V_3^{(0)} = (1 + j0)$$

Now,

$$\frac{R_2 - j0.2}{Y_{22}} = \frac{(-0.8 + j0.2)}{58.13 \angle -62.4^\circ} = 0.01418 \angle -120.64^\circ$$

$$\frac{Y_{21}}{Y_{22}} = \frac{22.36 \angle 116.6^\circ}{58.13 \angle -62.4^\circ} = 0.3846 \angle 179^\circ$$

$$= -0.3846$$

$$\frac{Y_{23}}{Y_{22}} = \frac{55.77 \angle 116.6^\circ}{58.13 \angle -62.4^\circ} = -0.6153$$

$$V_2^{(P+1)} = \left[\frac{0.0142 \angle -120.64^\circ + 0.3846 V_1^{(P)}}{(V_P^{(P)})^*} + 0.6153 V_2^{(P)} \right]$$

Now,

$$\frac{R_3 - j0.3}{Y_{33}} = \frac{-0.5 + j0.2}{67.23 \angle -67.2^\circ} =$$

$$8.01 \times 10^{-3} \angle -134.60^\circ$$

$$\frac{Y_{31}}{Y_{33}} = \frac{31.62 \angle 108.4^\circ}{67.23 \angle -67.2^\circ} = 0.47 \angle 175.6^\circ$$

$$\frac{Y_{32}}{Y_{33}} = \frac{55.77 \angle 116.6^\circ}{67.23 \angle -67.2^\circ} = 0.532 \angle 183.8^\circ$$

$$V_3^{(p+1)} = \left[\frac{0.008 \angle -134.60^\circ}{(V_3^{(p)})^*} + 0.47 \angle 175.6^\circ V_1 - 0.532 \angle 183.8^\circ V_2^{(p+1)} \right]$$

Now, $p=0$.

$$V_2^{(1)} = \frac{0.0142 \angle -130.64^\circ + 0.3846 \times 1}{(1+j0)^*} + 0.6153 \times (1+j0)$$

$$= 0.9907 \angle -0.623^\circ$$

$$V_3^{(1)} = \frac{0.008 \angle -134.60^\circ}{(1+j0)^*} - 0.47 \angle 175.6^\circ \times 1 - 0.532 \angle 183.8^\circ \times 0.9907 \angle -0.623^\circ$$

$$= 0.9893 \angle -0.726^\circ$$

After 1st Iter.

$$V_2^{(1)} = 0.9907 \angle -0.623^\circ$$

$$V_3^{(1)} = 0.9893 \angle -0.726^\circ$$

No. 1, $p = 1$.

$$V_2^{(2)} = \frac{0.0142 \angle -130.64^\circ}{(0.9907 \angle -0.623^\circ)^*}$$

$$+ 0.3846 \angle 11^\circ + 0.6153 \angle 0.9893 \angle -0.726^\circ$$

$$= 0.9839 \angle -1.076^\circ$$

$$V_3^{(2)} = \frac{0.008 \angle -134.6^\circ - 0.47 \angle 175.6^\circ}{(0.9893 \angle -0.726^\circ)^*}$$

$$- 0.532 \angle 183.8^\circ \times 0.9839 \angle -1.076^\circ$$

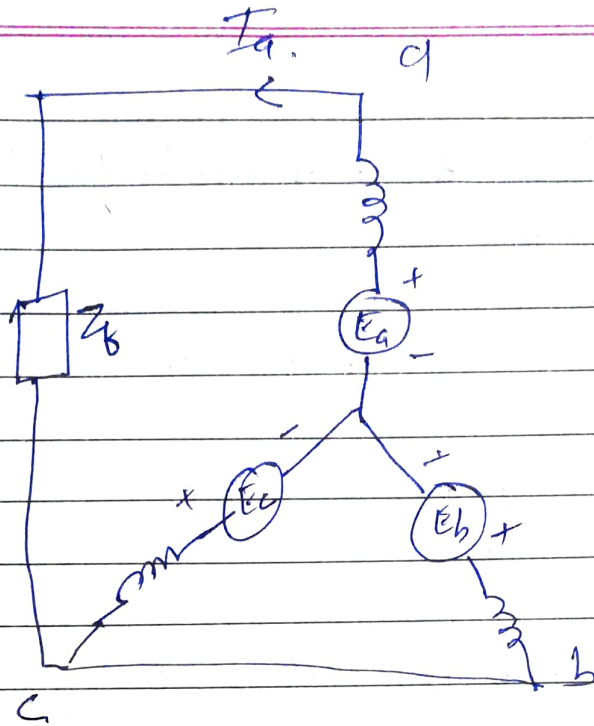
$$= 0.9858 \angle -0.98^\circ$$

After 2nd Iter,

$$V_2^{(2)} = 0.9839 \angle -1.076^\circ$$

$$V_3^{(2)} = 0.9858 \angle -0.98^\circ$$

Q2:



$$I_a + I_b + I_c = 0.$$

$$V_b = V_c.$$

$$V_a = V_b + Z_f I_a.$$

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_b + Z_f I_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a1} = \frac{1}{3} [V_b + Z_f I_a + \beta V_b + \beta^2 V_b]$$

$$= \frac{1}{3} [V_b + Z_f I_a - V_b]$$

$$= \frac{Z_f I_a}{3}$$

$$V_{a2} = \frac{1}{3} [V_b + Z_f I_a + \beta^2 V_b + \beta V_b]$$

$$= \frac{Z_f I_a}{3}$$

$$V_{a0} = \frac{1}{3} [3V_b + Z_f I_a]$$

$$= V_b + \frac{Z_f I_a}{3}$$

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a1} = \frac{1}{3} [I_a + \beta I_b + \beta^2 I_c]$$

$$I_{a2} = \frac{1}{3} [I_a + \beta^2 I_b + \beta I_c]$$

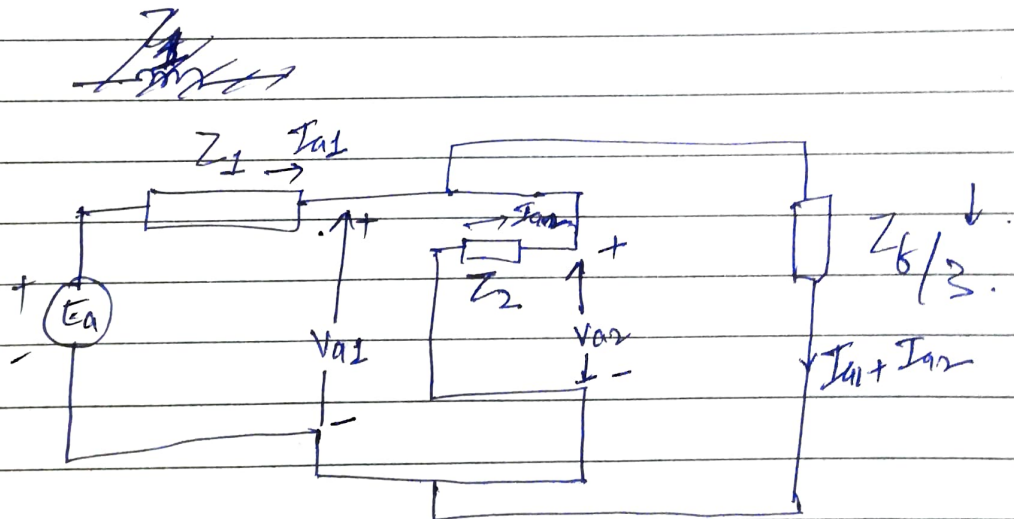
$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c] = \frac{0}{3} = 0$$

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$= I_{a1} + I_{a2}$$

$$V_{a1} = V_{a2} = (I_{a1} + I_{a2}) \frac{Z_f}{3}$$

Then, the sequence network :-



Sequence network.