Assignment-4

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(1) The given tf is

$$\frac{Y(s)}{R(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

(a) for Controller canonical form

$$\frac{Y(s)}{R(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \cdot \frac{X(s)}{X(s)}$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{\left(b_2 + \frac{b_1}{s} + \frac{b_0}{s^2}\right) X(s)}{\left(1 + \frac{a_1}{s} + \frac{a_0}{s^2}\right) X(s)}$$

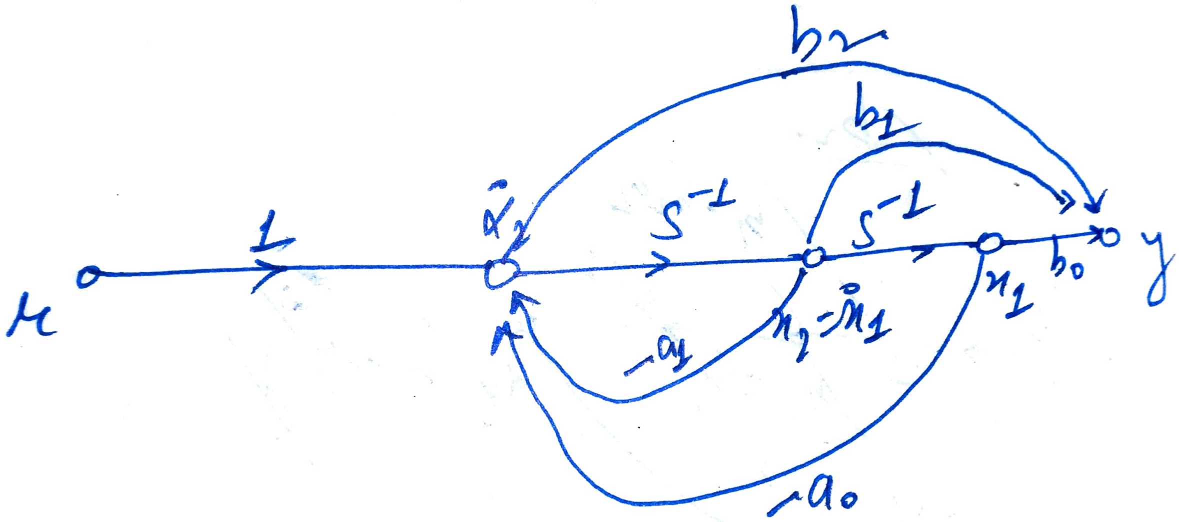
$$R(s) = \left(1 + a_1 s^{-1} + a_0 s^{-2}\right) X(s)$$

$$\boxed{X(s) = R(s) - \left(a_1 s^{-1} + a_0 s^{-2}\right) X(s)}$$

$$Y(s) = \left(b_2 + b_1 s^{-1} + b_0 s^{-2}\right) X(s)$$

$$Y(s) = b_2 \left[R(s) - \left(a_1 s^{-1} + a_0 s^{-2}\right) X(s)\right] + \left(b_1 s^{-1} + b_0 s^{-2}\right) X(s)$$

$$Y(s) = b_2 R(s) + \left[\left(b_1 \cancel{s^{-1} + b_0} - b_2 a_1\right) s^{-1} + \left(b_0 - b_2 a_0\right) s^{-2}\right] X(s)$$



$$\boxed{\dot{x}_2 \rightarrow X(s)}$$

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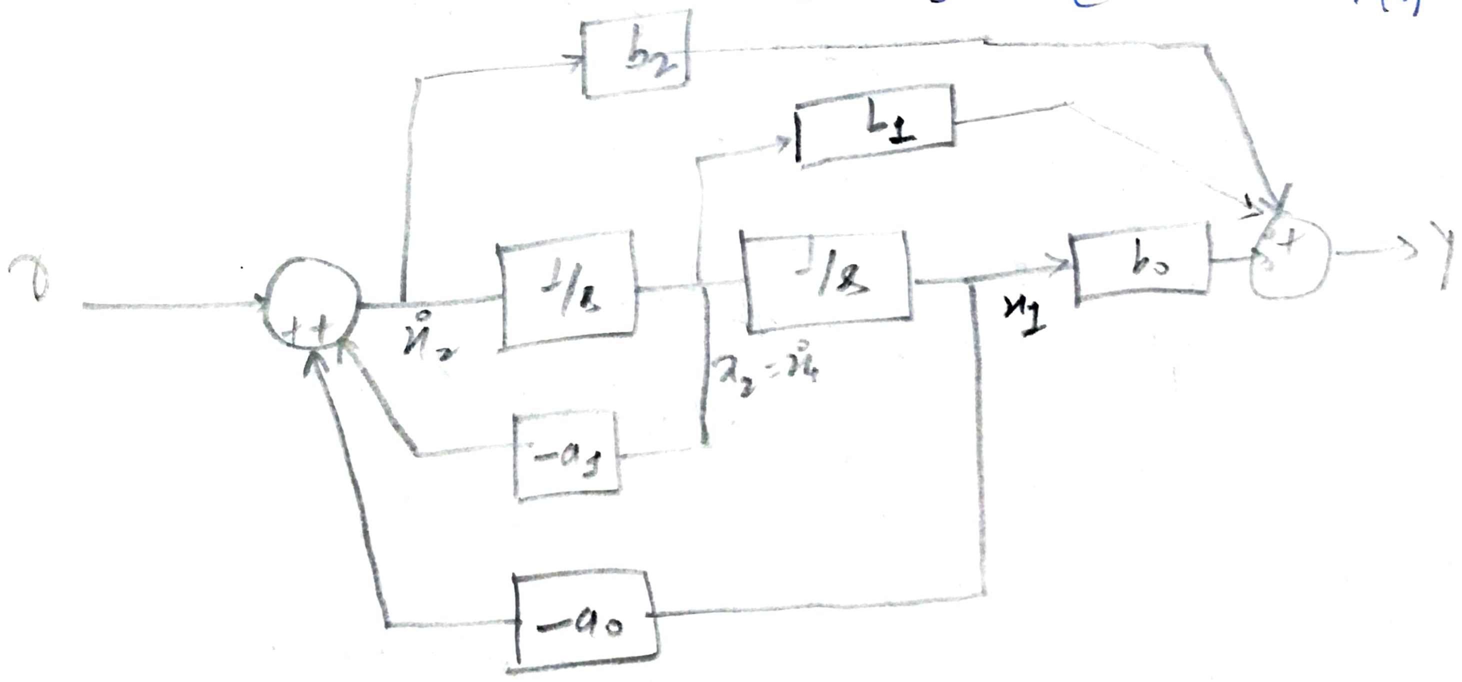
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -a_0 & a_1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} b_0 - b_2 a_0 & b_1 - b_2 a_1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} b_2 \end{bmatrix}}_{D} u$$

Controllability Matrix : $S = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -a_1 \end{bmatrix}$

$\Rightarrow$ Rank $= 2$

Observability Matrix : $V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} b_0 - b_2 a_0 & b_1 - b_2 a_1 \\ -a_0(b_1 - b_2 a_1) & (b_0 - b_2 a_0) + (b_1 - b_2 a_0) \end{bmatrix}$



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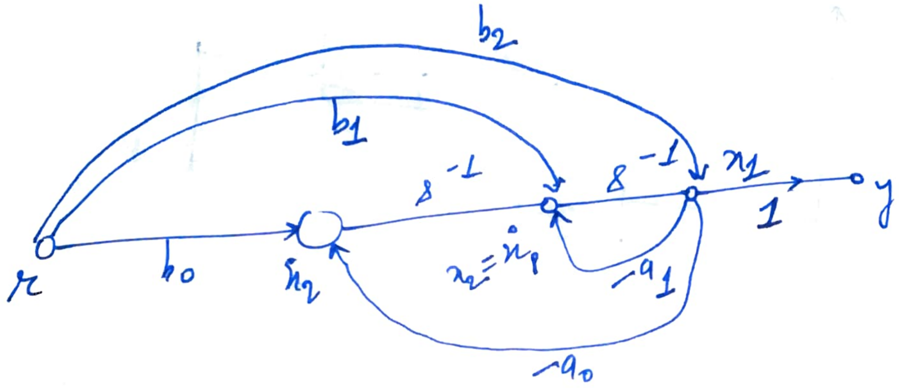
(b) Observer Canonical form

$$\frac{Y(s)}{R(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{b_2 + b_1 s^{-1} + b_0 s^{-2}}{1 + a_1 s^{-1} + a_0 s^{-2}}$$

$$Y(s) = (b_2 + b_1 s^{-1} + b_0 s^{-2}) R(s) - (a_1 s^{-1} + a_0 s^{-2}) Y(s)$$



$$\dot{x}_2 = r b_0 \cancel{+} - a_0 x_1 .$$

$$\dot{x}_1 = r b_1 - a_1 x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -a_1 & 1 \\ -a_0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1 \\ b_0 \end{bmatrix}}_{B} r$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D.} r .$$

Combatibility Matrix : $S = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} b_1 & b_0 - a_1 b_1 \\ b_0 & -a_0 b_1 \end{bmatrix}$

$\Rightarrow$ Rank = 2

Observability Matrix : $V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -a_1 & 1 \end{bmatrix}$

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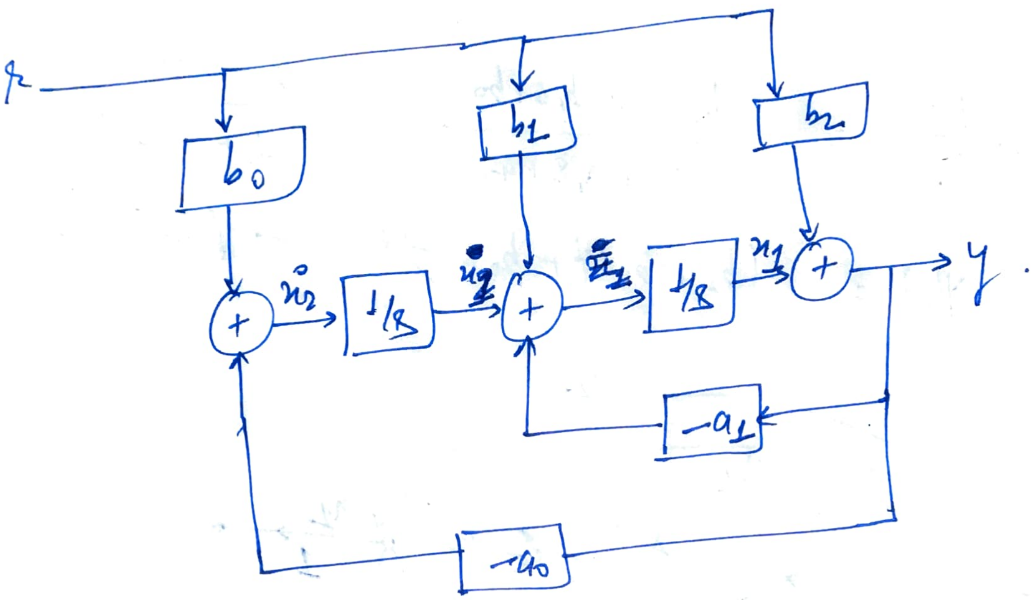

(2) (a)

(i)
$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{s+1} \\ \dfrac{s+3}{(s+1)(s+2)} \end{bmatrix} U(s)$$

$$= \begin{bmatrix} \dfrac{s+2}{(s+1)(s+2)} \\ \dfrac{s+3}{(s+1)(s+2)} \end{bmatrix} U(s)$$

$$\frac{Y_1(s)}{U(s)} = \frac{s+2}{s^2+3s+2} = \frac{(s^{-1}+2s^{-2})\cancel{U(s)}}{(1+3s^{-1}+2s^{-2})\cancel{U(s)}}$$

$$\frac{Y_2(s)}{U(s)} = \frac{s+3}{(s+1)(s+2)} = \frac{(s^{-1}+3s^{-2})\cancel{U(s)}}{(1+3s^{-1}+2s^{-2})\cancel{U(s)}}$$

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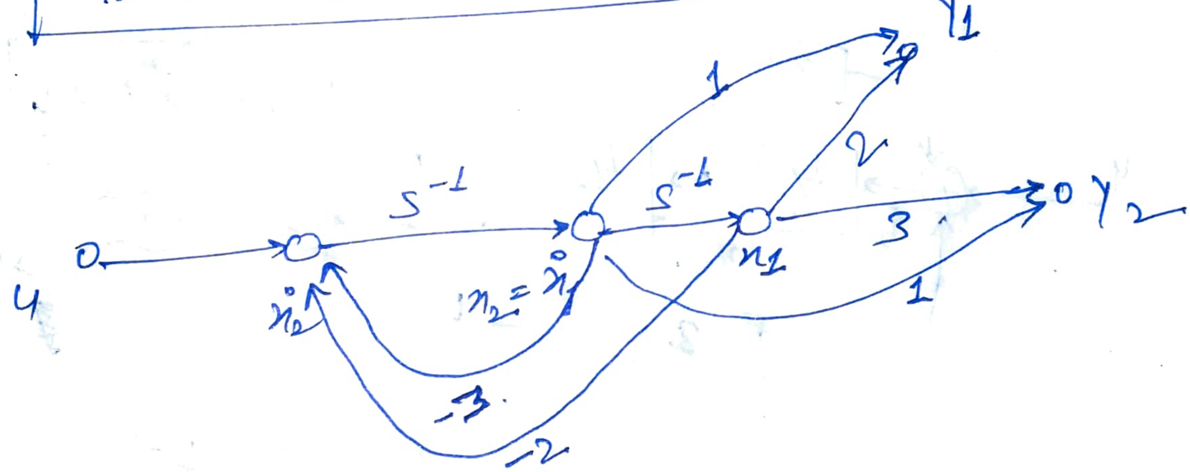
$$U(s) = (1 + 2s^{-1} + 2s^{-2}) X(s)$$

$$\boxed{X(s) = \cancel{}\, U(s) - (3s^{-1} + 2s^{-2}) X(s)}$$

$$\boxed{Y_1(s) = (s^{-1} + 2s^{-2}) X(s)}$$

$$\boxed{Y_2(s) = (s^{-1} + 3s^{-2}) X(s)}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

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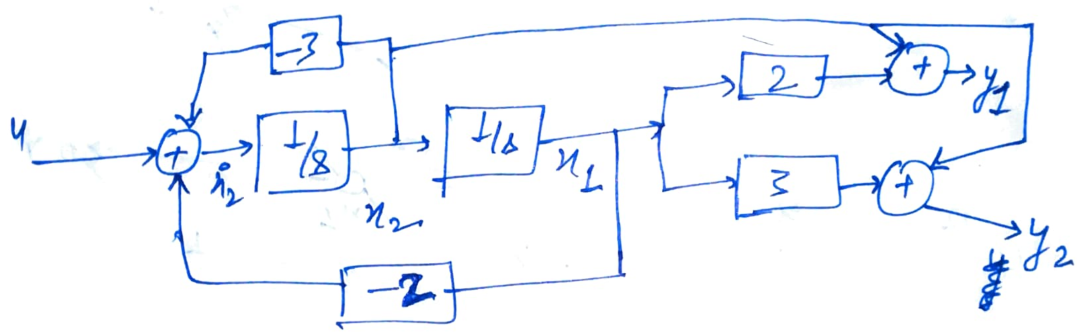
Controllability Matrix $= S = \begin{bmatrix} B & AB \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \Rightarrow \text{Rank} = 2$$

Observability Matrix $= V = \begin{bmatrix} C \\ CA \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ -2 & -1 \\ -2 & 0 \end{bmatrix} \quad \text{Rank} = 2$$



(ii)

$$Y(s) = \begin{bmatrix} \dfrac{s+2}{(s+1)(s+2)} & \dfrac{s+3}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$Y(s) = \frac{s+2}{s^2+3s+2} U_1(s) + \left(\frac{s+3}{s^2+3s+2}\right) U_2(s)$$

$$\boxed{(1+3s^{-1}+2s^{-2})Y(s) = (s^{-1}+2s^{-2})U_1(s) + (s^{-1}+3s^{-2})U_2(s)}$$

$$\boxed{\cancel{Y(s) = s^{-1}\left(U_1(s)+U_2(s) - 3Y(s)\right) + \left(2U_1(s)+3U_2(s) - 2Y(s)\right)s^{-2}}}$$

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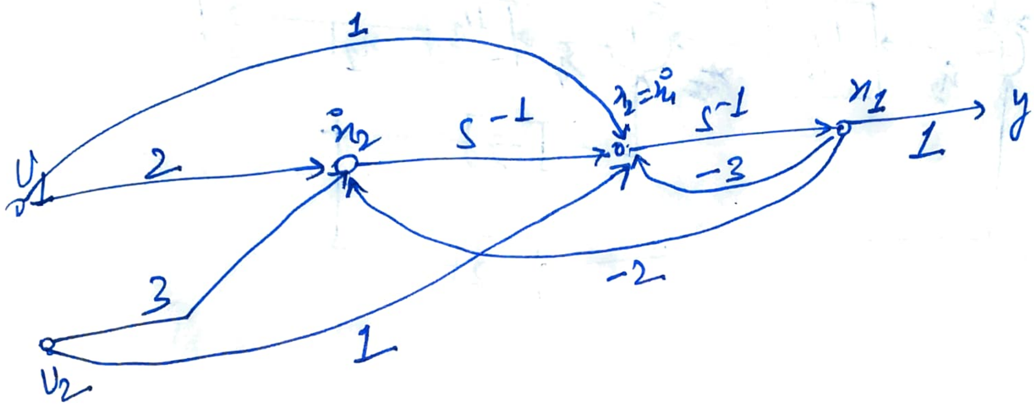
$$Y(s) = \left[(s^{-1} + 2s^{-2})U_1(s) + (s^{-1} + 3s^{-2})U_2(s)\right]$$
$$- (3s^{-1} + 2s^{-2})Y(s)$$

$$Y(s) = \left[s^{-1}(U_1(s) + U_2(s)) + (2U_1(s) + 3U_2(s))s^{-2}\right]$$
$$- (3s^{-1} + 2s^{-2})Y(s)$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[0]}_{D} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Controllability Matrix: $S = [B \quad AB] = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & 3 & -2 & -2 \end{bmatrix}$

Rank = 2

Observability Matrix: $V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

Rank = 2

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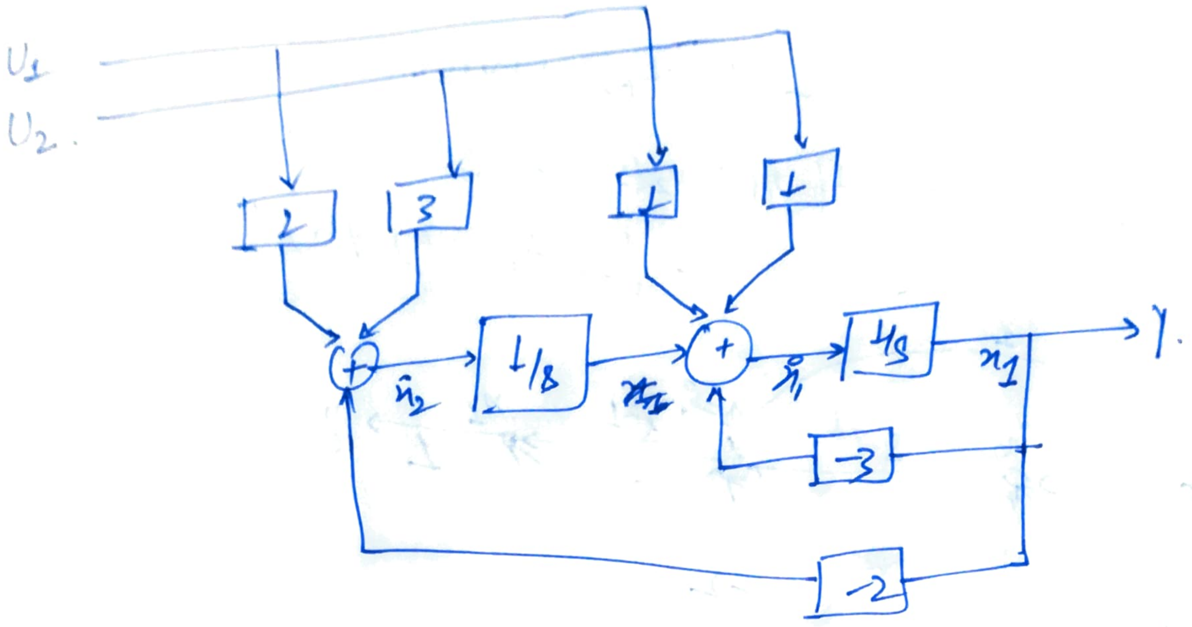
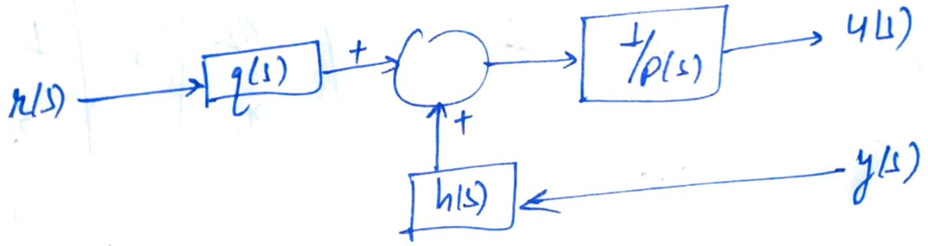


(b)



$$q = q_2 s^2 + q_1 s + q_0$$

$$p = p_2 s^2 + p_1 s + p_0$$

$$h = h_2 s^2 + h_1 s + h_0$$

$$\frac{1}{p} = \frac{1}{p_2 s^2 + p_1 s + p_0}$$

$$u = (hy + qr)\frac{1}{p}.$$

$$u = \begin{bmatrix} \frac{h}{p} & \frac{q}{p} \end{bmatrix} \begin{bmatrix} y \\ r \end{bmatrix}$$

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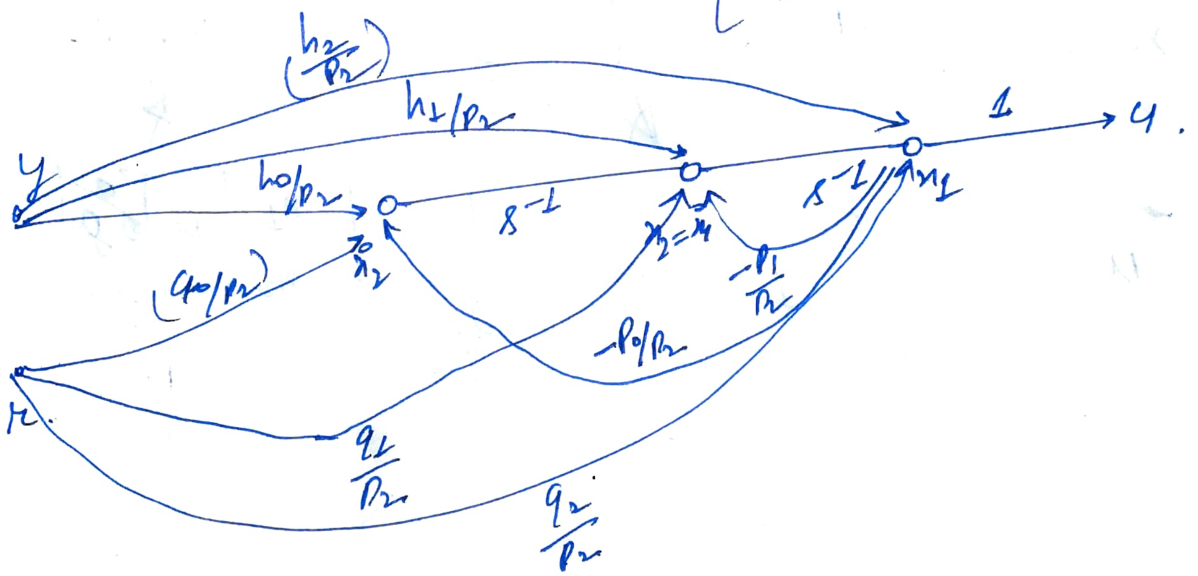
Realising q & h.

$$u = \frac{h_2 s^2 + h_1 s + h_0}{p_2 s^2 + p_1 s + p_0} y + \frac{q_2 s^2 + q_1 s + q_0}{p_2 s^2 + p_1 s + p_0} r$$

$$\left(1 + \frac{p_1}{p_2}\frac{1}{s} + \frac{p_0}{p_2} s^{-2}\right) u = \left(\frac{h_2}{p_2} + \frac{h_1}{p_2} s^{-1} + \frac{h_0}{p_2} s^{-2}\right) y + \left(\frac{q_2}{p_2} + \frac{q_1}{p_2} s^{-1} + \frac{q_0}{p_2} s^{-2}\right) r$$

$$u = \left[\left(\frac{h_2}{p_2} y + \frac{q_2}{p_2} r\right) + \left(\frac{h_1}{p_2} y + \frac{q_1}{p_2} r\right) s^{-1} + \left(\frac{h_0}{p_2} + \frac{q_0}{p_2}\right) s^{-2}\right] - \left[\frac{p_1}{p_2} s^{-1} + \frac{p_0}{p_2} s^{-2}\right] u$$



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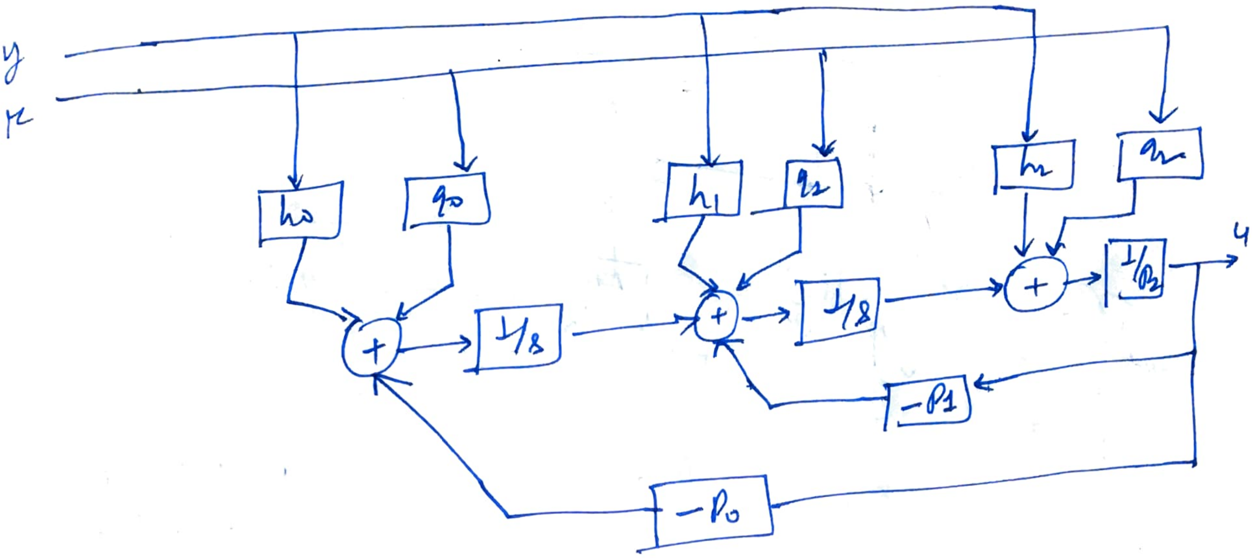

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -p_1/p_2 & 1 \\ -p_0/p_2 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} h_1/p_2 & q_1/p_2 \\ h_0/p_2 & q_0/p_2 \end{bmatrix}}_{B} \begin{bmatrix} y \\ r \end{bmatrix}$$

$$u = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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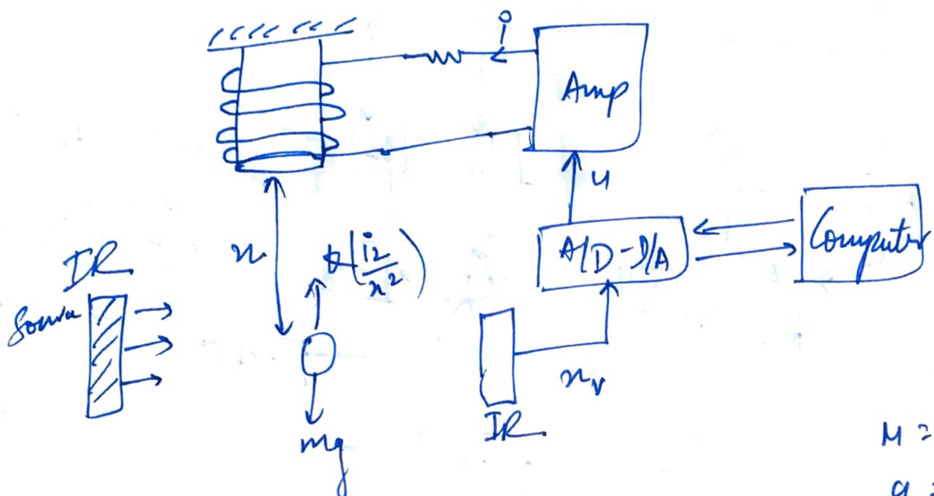
(3)



$M = 0.02\,kg$
$g = 9.81\,m/s^2$
$k_1 = 1.05\,A/V$
$k_2 = 143.48\,V/m$
$k_3 = -2.8\,V$

$$M\frac{d^2x}{dt^2} = Mg - k\left(\frac{i^2}{x^2}\right)$$

$$i = k_1 u$$

$$x_v = k_2 x + k_3$$

State Space Equation

Assume $x_1 = x$, $x_2 = \dot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{k}{M}\cdot\frac{i^2}{x^2} \end{bmatrix} \begin{array}{l} \rightarrow f_1 \\ \rightarrow f_2 \end{array}$$

for Equivalent Condition :

$$i_0 = 0.8A, \quad x_0 = 0.009\,m$$

$$\Rightarrow x_{10} = 0.009\,m \quad \& \quad \dot{x}_2 = \dot{x}_1 = 0$$
$$\hookrightarrow x_{20} = 0$$

If $\dot{x}_2 = 0$

$$Mg - k\frac{i_0^2}{x_0^2} = 0$$

$$\Rightarrow k = \frac{Mg\,x_0^2}{i_0^2} = \frac{0.02 \times 9.81 \times 0.009^2}{0.8^2}\ Nm^2/A^2$$

$$= 24.83\ \mu Nm^2/A^2$$

Also,

$$i_0 = b_1 u_0 \Rightarrow u_0 = \frac{i_0}{b_1} = \frac{0.8}{1.05} V$$

$$= 0.762 V$$

State Space:

$$f_1 = x_2 \quad \& \quad f_2 = g - \frac{k}{M}\frac{i^2}{x^2} = g - \frac{k}{M} b_1^2 \frac{u^2}{x_1^2}$$

$$\frac{\partial f_1}{\partial x_1} = 0, \quad \frac{\partial f_1}{\partial x_2} = 1, \quad \frac{\partial f_1}{\partial u} = 0, \quad \frac{\partial f_2}{\partial x_2} = 0$$

$$\frac{\partial f_2}{\partial x_1} = \frac{2k}{M} b_1^2 \frac{u^2}{x_1^3}\Big|_{u_0, x_{10}} = \frac{2k}{M} b_1^2 \times \frac{u_0^2}{x_{10}^3}$$

$$= \frac{2 \times g}{x_{10}} \times \frac{k i_0^2}{M g \times x_{10}^2}$$

$$= \frac{2g}{x_{10}} = 2180$$

$$\frac{\partial f_2}{\partial u} = \frac{-2k b_1^2 u}{M x_1^2}\Big|_{u_0, x_{10}} = \frac{-2k b_1^2 u_0}{M x_{10}^2} = \frac{-2b_1 g}{i_0}$$

$$= -25.75$$

$$\frac{\partial x_v}{\partial x_1} = b_2, \quad \frac{\partial x_v}{\partial x_2} = 0, \quad \frac{\partial x_v}{\partial u} = 0$$

$\therefore$ Incremental T.F

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 2180 & 0 \end{bmatrix}}_{A} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -25.75 \end{bmatrix}}_{B} \Delta u$$

$$\Delta x_v = \underbrace{\begin{bmatrix} b_2 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

So, transfer function of $\frac{\Delta x_v}{\Delta u}$ is given by

$C(SI-A)^{-1}B$

$$= \begin{bmatrix} 143.48 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ -2180 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -25.75 \end{bmatrix}$$

$$= \begin{bmatrix} 143.48 & 0 \end{bmatrix} \left(\frac{1}{s^2-2180}\right) \begin{bmatrix} s & 1 \\ 2180 & s \end{bmatrix} \begin{bmatrix} 0 \\ -25.75 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 143.48 & 0 \end{bmatrix} \begin{bmatrix} -25.75 \\ -25.75s \end{bmatrix}}{(s^2-2180)}$$

$$= \frac{143.48 \times (-25.75)}{s^2-2180} = \frac{-3694.61}{s^2-2180}$$

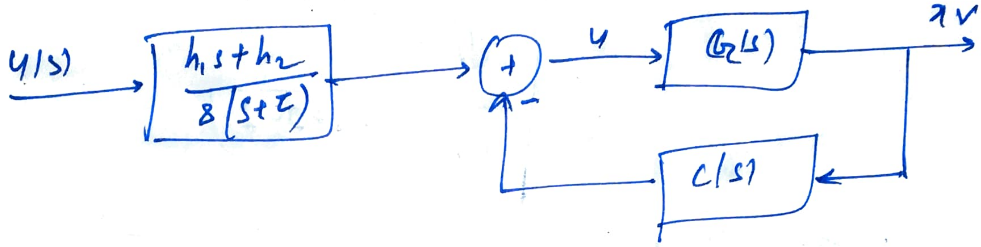
$$\boxed{\frac{\Delta x_v(s)}{\Delta u(s)} = \frac{-3694.61}{s^2-2180}}$$

$$G(s) = \frac{-3694.61}{s^2-2180}$$

The order of the system is 2.

So, let us consider II order Controller

$$C(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(s+z)}$$



Characteristics Eqn is

$$1 + b_2 C = 0$$

$$1 - \frac{(3694.61)}{(s^2 - 2180)}\left[\frac{q_2 s^2 + q_1 s + q_0}{s(s+z)}\right] = 0$$

$$s(s^3 + zs^2 - 2180 s - 2180 z) - 3694.61\, q_2 s^2 - 3694.61\, q_1 s - 3694.61\, q_0 = 0$$

$$\boxed{s^4 + zs^3 - s^2(3694.61 q_2 + 2180) - s(2180z + 3694.61 q_1) \cancel{-369509} - 3694.61\, q_0 = 0} \quad (1)$$

Desired Characteristics Equation Properties.

$|GM| \geq 6\,dB$ $|PM| \geq 60°$

$t_s \leq 20\,sec.$ $PM = 80°$

$\Rightarrow \dfrac{4}{\xi\omega_n} = 2$ $\boxed{\xi_d = 0.8}$

$\Rightarrow \boxed{\omega_n = 2.5}$

Desired CE.

$(s+a)^2(s^2 + 2\omega_n\xi s + \omega_n^2) = 0.$

$(s^2 + 2as + a^2)(s^2 + 2\omega_n\xi s + \omega_n^2) = 0$

$\Rightarrow s^4 + (2\omega_n\xi + 2a)s^3 + (\omega_n^2 + a^2 + 4a\omega_n\xi)s^2$

$\Rightarrow (2a\omega_n^2 + 2a^2\omega_n\xi)s + a^2\omega_n^2 = 0$

$$\Rightarrow \boxed{s^4 + (2\omega_n\xi + 2a)s^3 + (\omega_n^2 + a^2 + 4a\omega_n\xi)s^2 + (2a\omega_n^2 + 2a^2\omega_n\xi)s + a^2\omega_n^2 = 0} \quad -(2)$$

from pole placement technique, we can take.

$\boxed{a = 750\,\xi\omega_n = 1500}$

Comparing the coefficients from eq(1) & eq(2), we get.

$s^4, \quad 1 = 1$

$s^3, \quad Z = 2\omega_n\xi + 2a$

$s^2, \quad -(3694.619_2 + 2180\cancel{k}) = \omega_n^2 + a^2 + 4a\omega_n\xi$

$s, \quad -(\cancel{2180}+ 3694.619_1) = 2a\omega_n^2 + 2a^2\omega_n\xi$

$s^0, \quad -3694.619_0 = a^2\omega_n^2$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ -3694.61 & 0 & 0 & 0 \\ 0 & -3694.61 & 0 & -2180 \\ 0 & 0 & -3694.61 & 0 \end{bmatrix} \begin{bmatrix} q_2 \\ q_1 \\ q_0 \\ z \end{bmatrix} = \begin{bmatrix} 2\omega_n \zeta + 2a \\ \omega_n^2 + a^2 + 4a\zeta\omega_n + 2180 \\ 2a\omega_n^2 + 2a^2\omega_n\zeta \\ a^2\omega_n^2 \end{bmatrix}$$

from Matlab,

$q_2 = -612.8350$

$q_1 = -4.2085 \times 10^3 = -4208.5$

$q_0 = -3806.2$

$z = 3004$

$$C = \frac{-612.83(s+5.796)(s+1.072)}{s(s+3004)}$$

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So, the Characteristics Equation is:-

$$s^4 + 3004s^3 + 226201.25s^2 + 9018750s + 11.25 \times 10^6 = 0$$

```
wn = 2.5;
zeta = 0.8;
a = 750*wn*zeta;
```

```
X = [0 0 0 1
    -3694.61 0 0 0
    0 -3694.61 0 -2180
    0 0 -3694.61 0];
Y = [2*wn*zeta+2*a
    wn^2+a^2+4*a*wn*zeta+2180
    2*wn^2+2*a^2*wn*zeta
    a^2*wn^2];
Q = inv(X)*Y
```

```
Q = 4×1
10^3 ×
   -0.6128
   -4.2085
   -3.8062
    3.0040
```

```
q2 = Q(1)
```

```
q2 = -612.8350
```

```
q1 = Q(2)
```

```
q1 = -4.2085e+03
```

```
q0 = Q(3)
```

```
q0 = -3.8062e+03
```

```
tau = Q(4)
```

```
tau = 3004
```

```
s = zpk("s");
G = (-3694.61)/(s^2-2180)
```

```
G =

        -3694.6
  -------------------
  (s+46.69) (s-46.69)

Continuous-time zero/pole/gain model.
```

```
C = (q2*s^2+q1*s+q0)/(s*(s+tau))
```

```
C =

  -612.83 (s+5.796) (s+1.072)
  ---------------------------
          s (s+3004)
```

```
Continuous-time zero/pole/gain model.
```

```
allmargin(G*C)
```

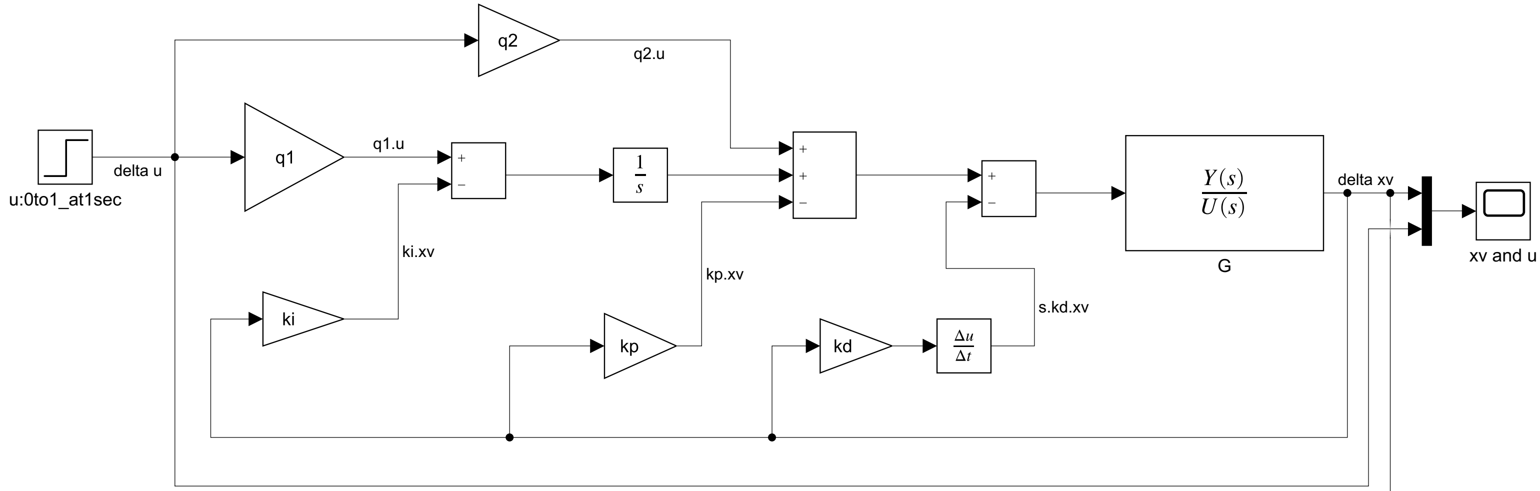
```
ans = struct with fields:
      GainMargin: [0.4224 Inf]
     GMFrequency: [2.4950 Inf]
     PhaseMargin: 75.8117
     PMFrequency: 729.4575
     DelayMargin: 0.0018
     DMFrequency: 729.4575
          Stable: 1
```

```
syms s
expand(s^4+(2*wn*zeta+2*a)*s^3+(wn^2+a^2+4*a*wn*zeta)*s^2+(2*a*wn^2+2*a^2*wn*zeta)*s+a^2*wn*2)
```

```
ans =
```

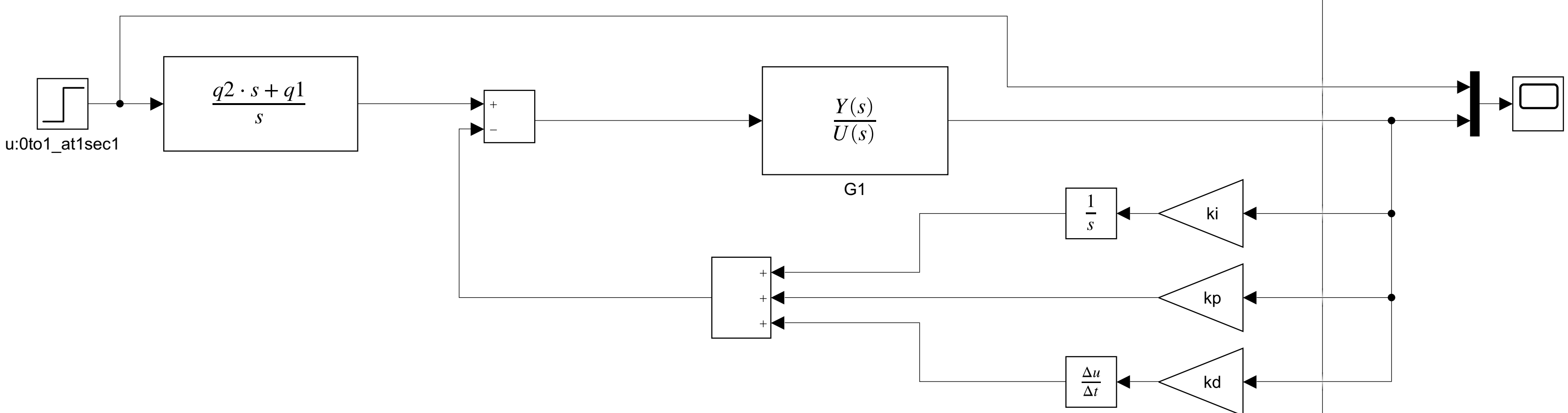
$$s^4 + 3004\,s^3 + \frac{9048025\,s^2}{4} + 9018750\,s + 11250000$$

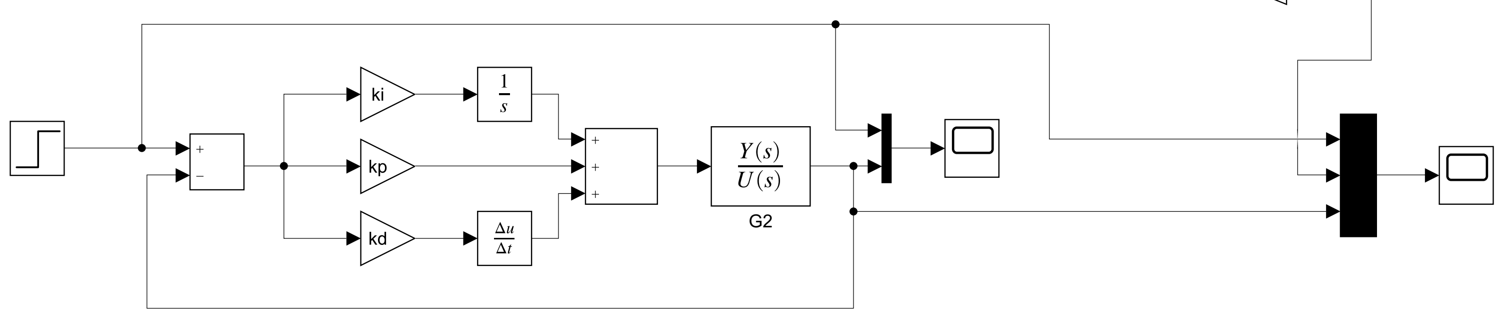
Question03: b)



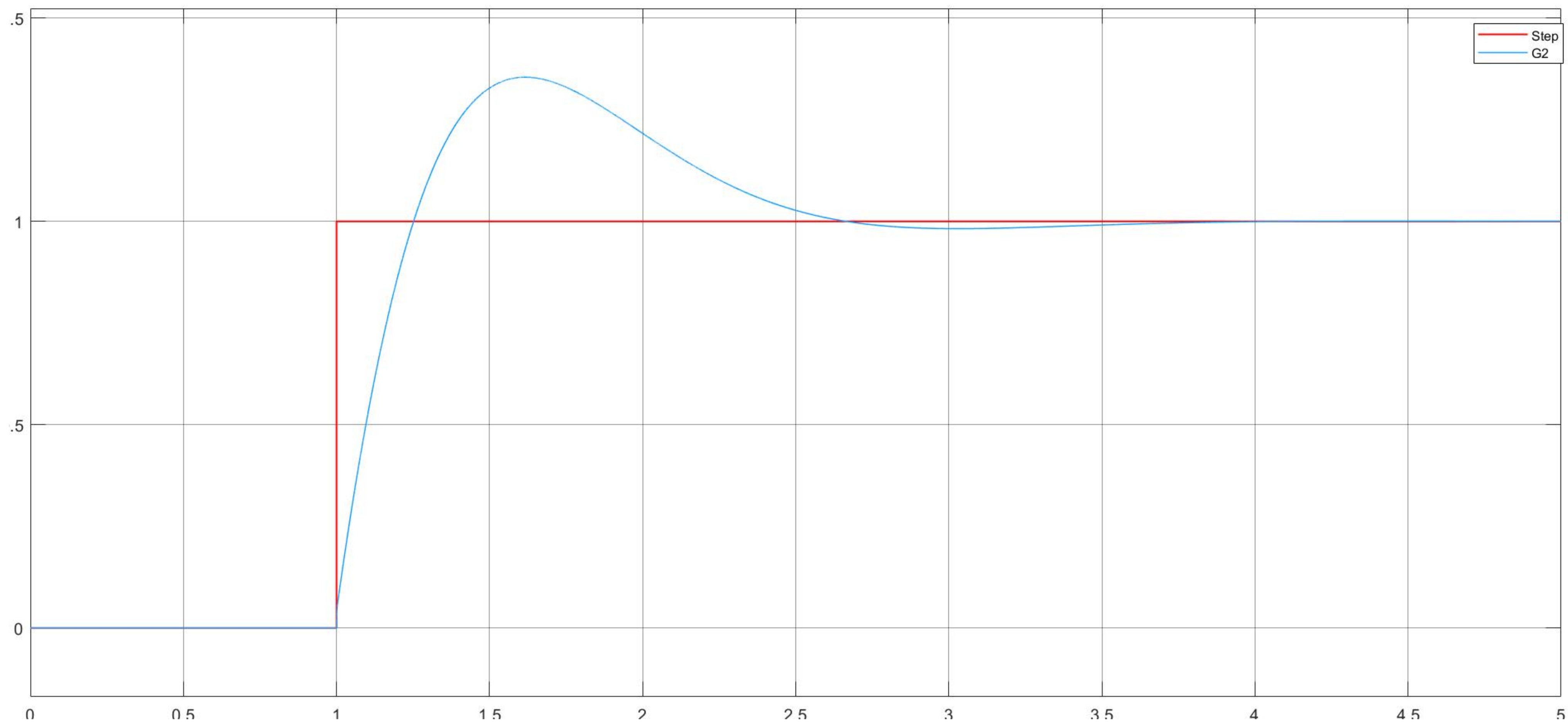
Linear Model for 2-DoF Controller

Another model for 2-DOF controller

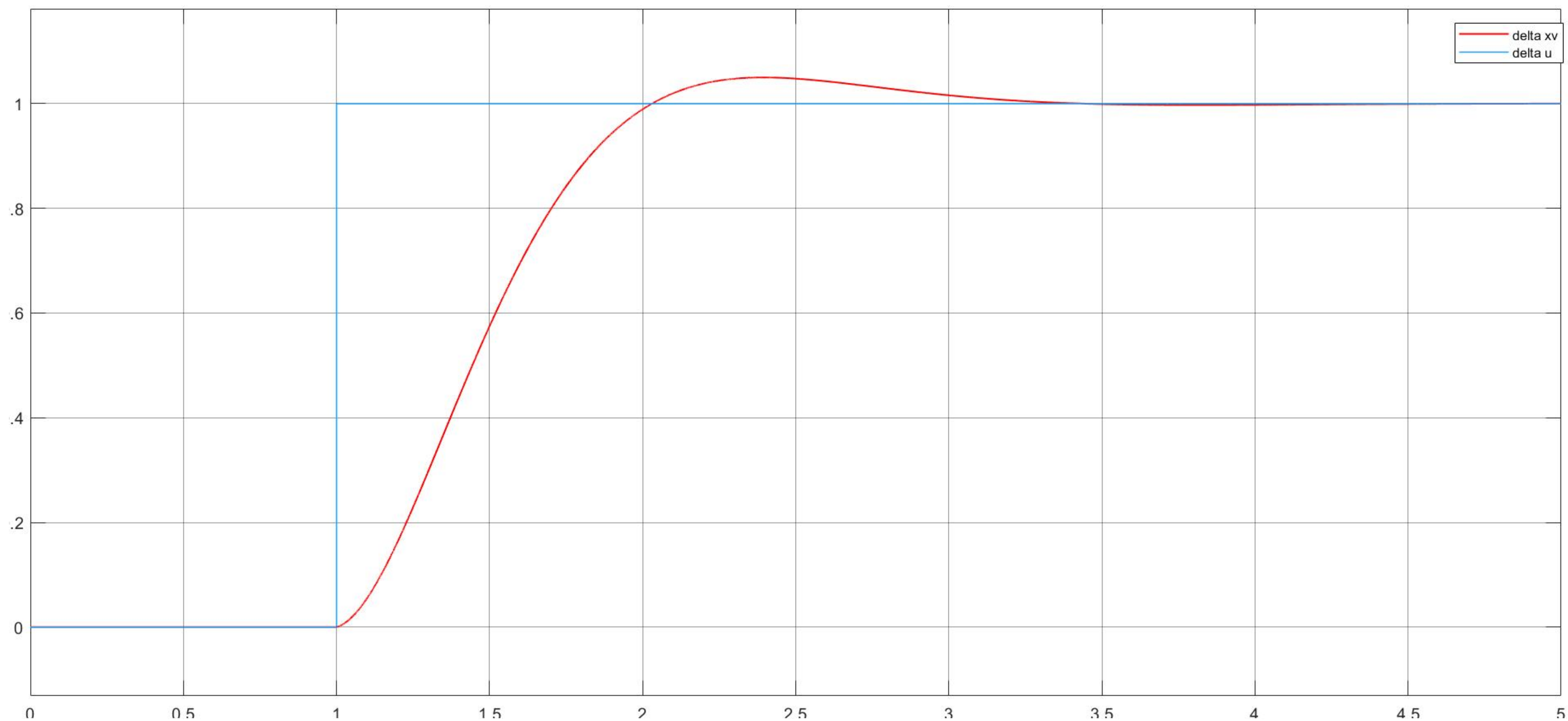




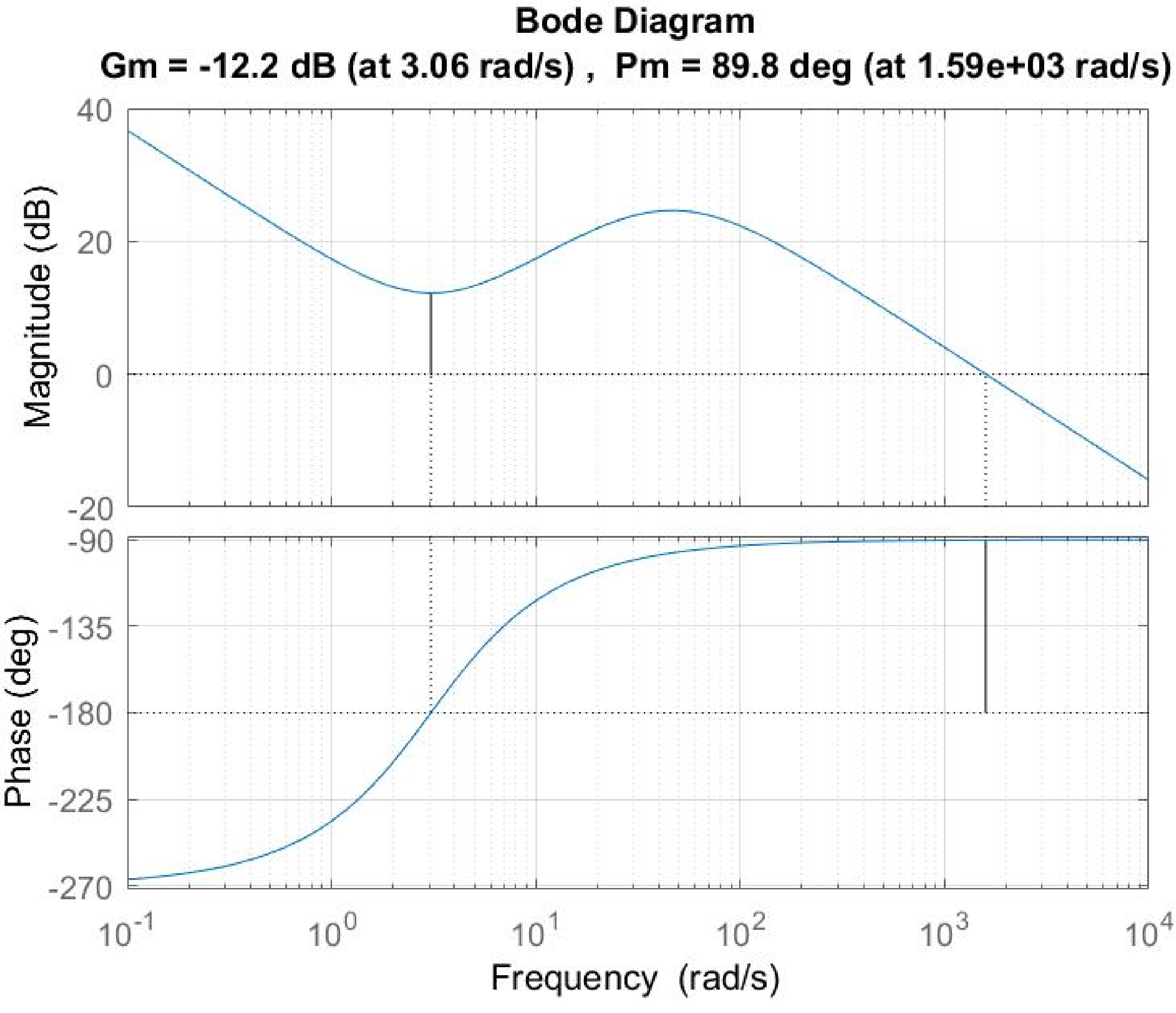
1 DOF model



Step
G2
.5
1
.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



delta xv
delta u
1
.8
.6
.4
.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Bode Diagram
Gm = -12.2 dB (at 3.06 rad/s) , Pm = 89.8 deg (at 1.59e+03 rad/s)
40
20
0
-20
Magnitude (dB)
-90
-135
-180
-225
-270
Phase (deg)
10^-1
10^0
10^1
10^2
10^3
10^4
Frequency (rad/s)