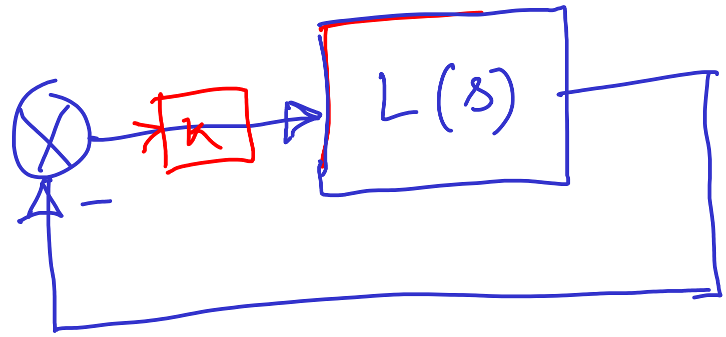
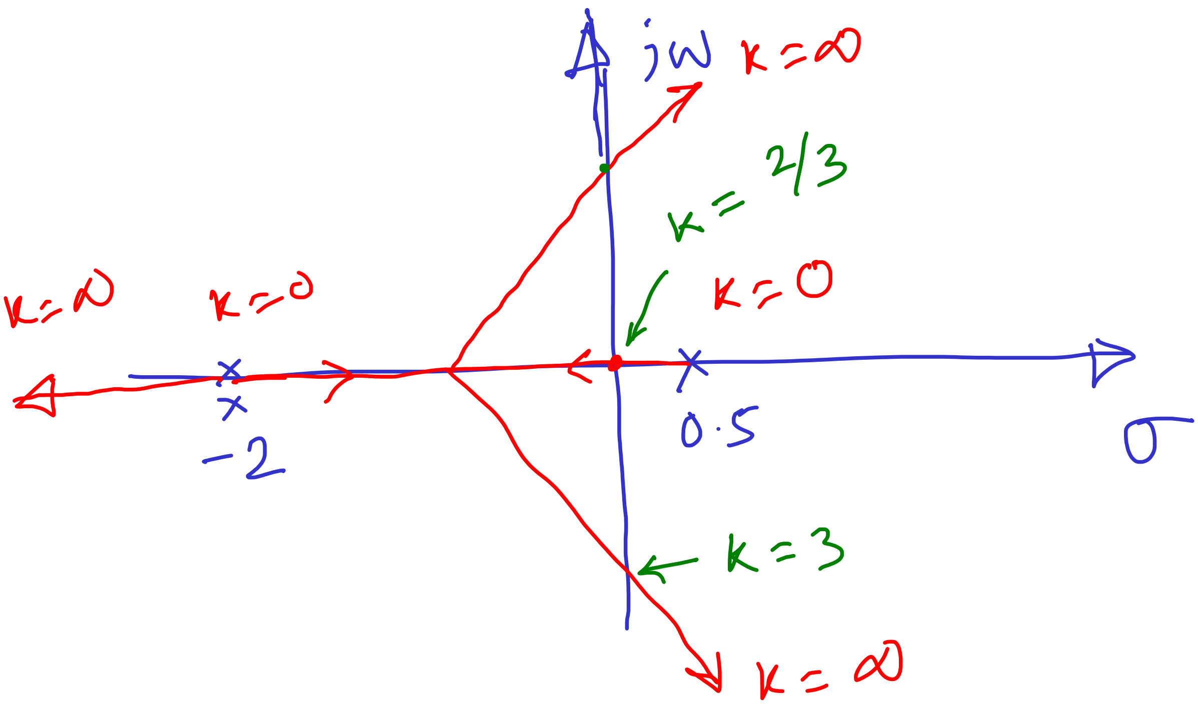
Ex 3

Lecture 2 (29-10-20)



$k = 1$ (Nominal gain)

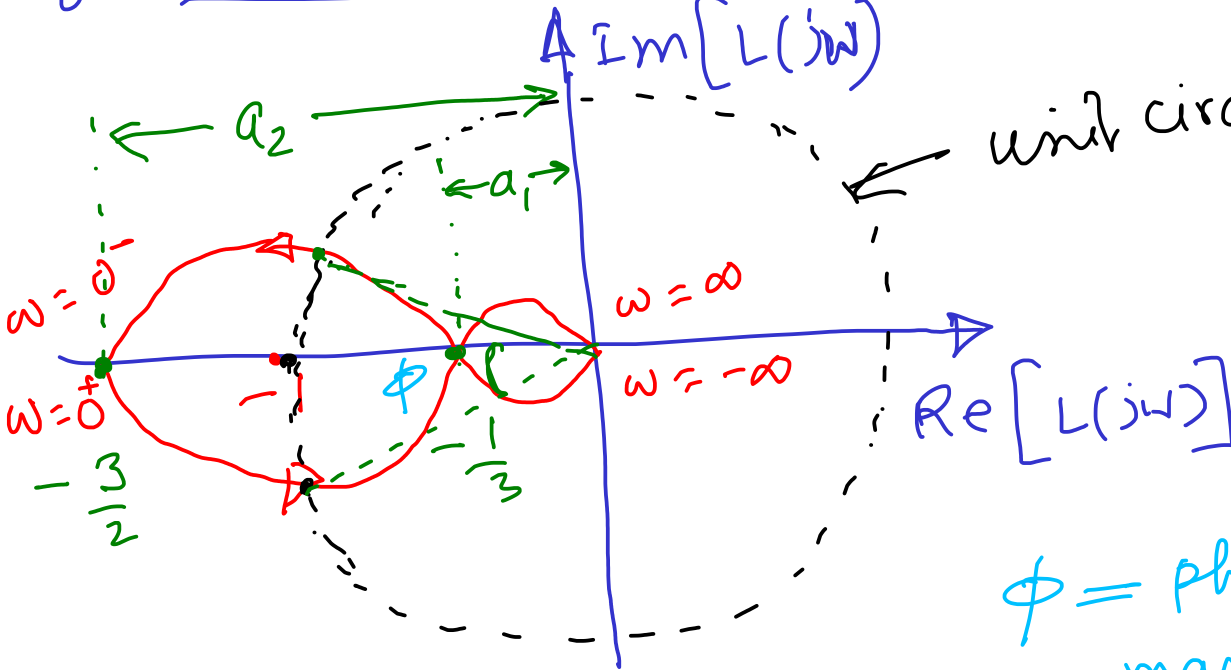
$$L(s) = \frac{3}{(s-0.5)(s+2)^2}$$



upper side GM = 3

Lower side GM = 2/3

using Nyquist plot



ϕ = phase margin

Upper side GM = $\frac{1}{a_1} = 3$

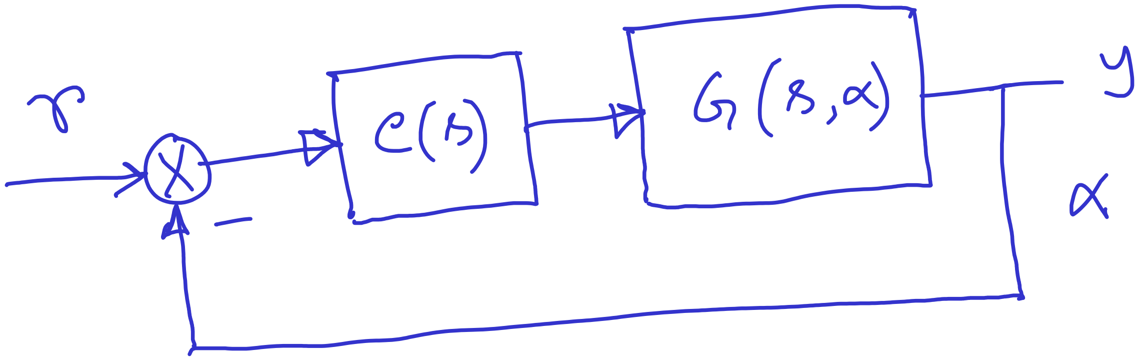
Lower side GM = $\frac{1}{a_2} = 2/3$

Note: Nyquist plot is symmetric w.r.t. x axis and therefore there is no term like +ve PM, −ve PM.

Limitations of GM, PM: GM, PM can guarantee robustness against pure gain variations & phase variation

Gain phase factor = $K e^{-j\phi}$ ← phase
↑ gain

Robustness against parameter variations



α = uncertain parameter

S_α^G = Sensitivity of G w.r.t. α

$$= \lim_{\Delta\alpha \to 0} \frac{\Delta G / G}{\Delta \alpha / \alpha}$$

$$\boxed{T = \frac{GC}{1+GC}}$$

$$= \frac{\partial G}{G} \Big/ \frac{\partial \alpha}{\alpha} = \text{open-loop Sensitivity}$$

S_α^T = Sensitivity of T w.r.t. α

$$= \frac{\partial T}{T} \Big/ \frac{\partial \alpha}{\alpha} = \text{closed-loop sensitivity}$$

Now one can show that

$$S_\alpha^T = \frac{1}{1+GC} \cdot S_\alpha^G$$

[Try] Simple!

$$\Rightarrow S_\alpha^T = S \, S_\alpha^G$$

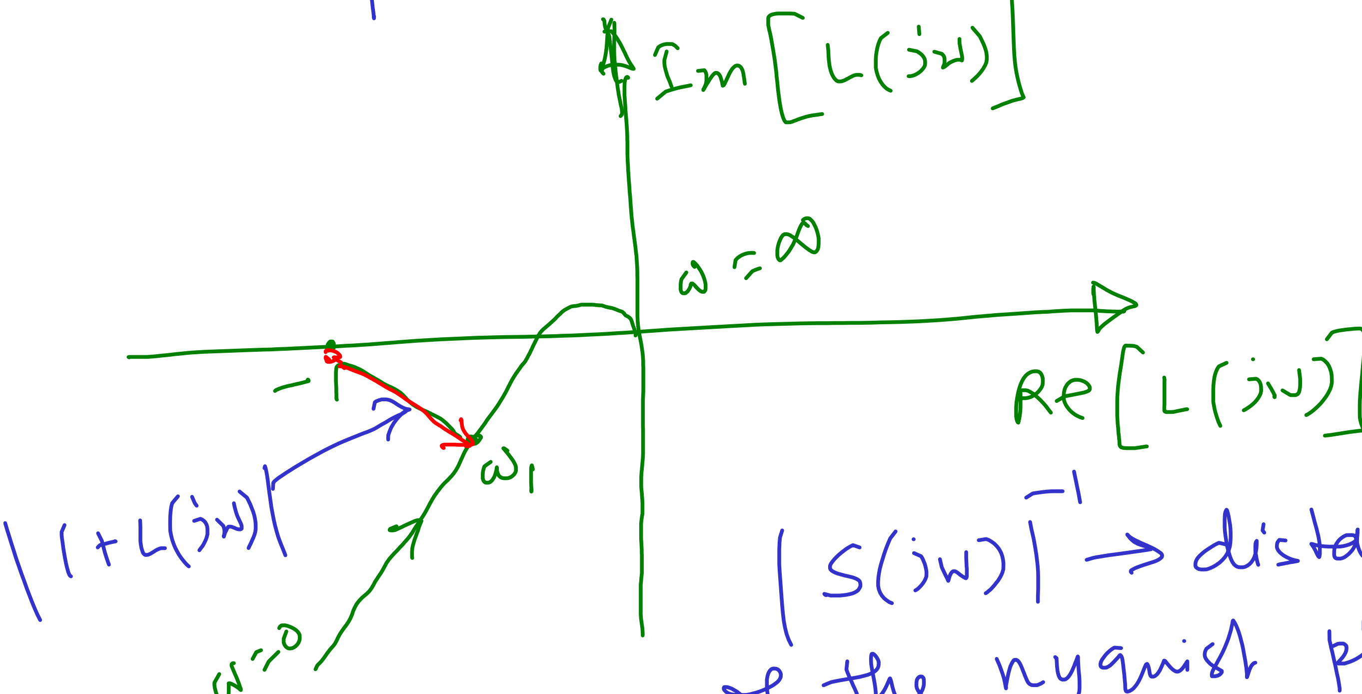
$$\boxed{S = \frac{1}{1+GC}} = \text{sensitivity function}$$

$$S = \frac{1}{1+L}\ , \quad L = \text{loop TF} = GC$$

Note: To reduce S, L should be as much high as possible.

Now $$S(j\omega) = \frac{1}{1+L(j\omega)}$$

$$\Rightarrow |S(j\omega)|^{-1} = |1+L(j\omega)|$$



$|S(j\omega)|^{-1} \rightarrow$ distance of the nyquist plot from $-1+j0$ pt at freq ω.

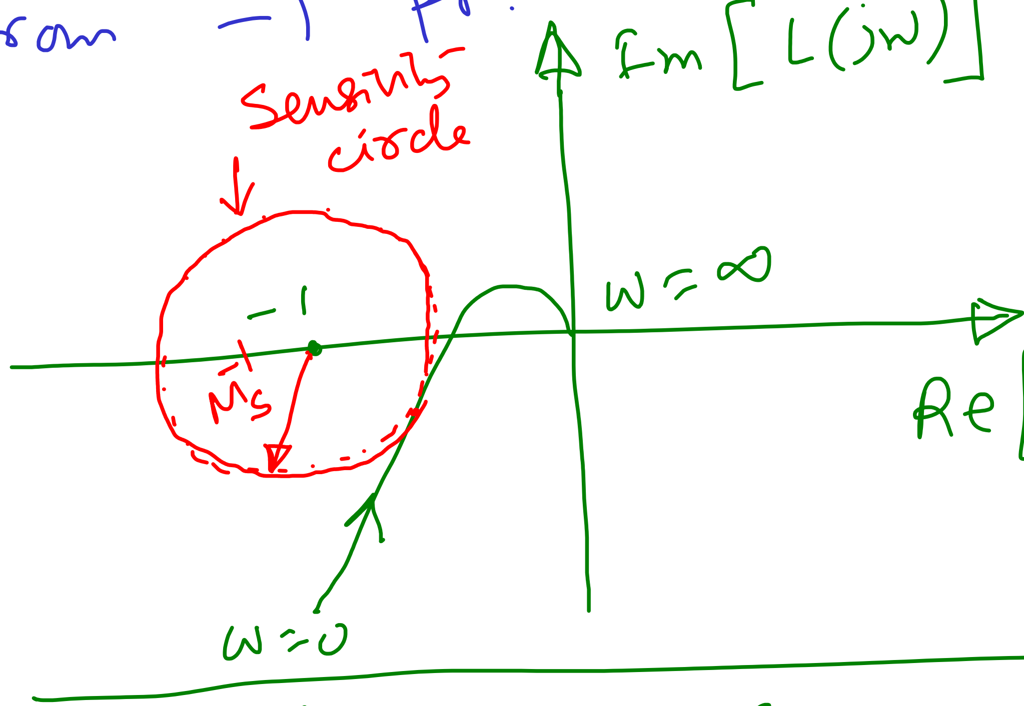
• For RS, the nyquist plot should be as much far away as possible from $-1+j0$ point.

Let us define $M_s = \max_{\omega} |S(j\omega)|$

$$\Rightarrow M_s^{-1} = \frac{1}{\max_{\omega} |S(j\omega)|}$$

$$\Rightarrow M_s^{-1} = \min_{\omega} |1+L(j\omega)|$$

$\Rightarrow$ M_s^{-1} is the minimum distance of the nyquist plot of $L(j\omega)$ from -1 pt.



$\Rightarrow$ $\boxed{M_s \text{ should as small as possible for RS}}$

$$S = \frac{1}{1+L}$$

As $L(s)$ has low pass behaviour

$\Rightarrow$ $|L(j\omega)| << 1$ at high freq

$\Rightarrow$ $|S(j\omega)| \approx 1$ at high freq

So for R.S, at least $|S(j\omega)|$ should be made small at low freq.

- For u/s & NMP plants there is always peak in $|S(j\omega)|$ plot

For example,

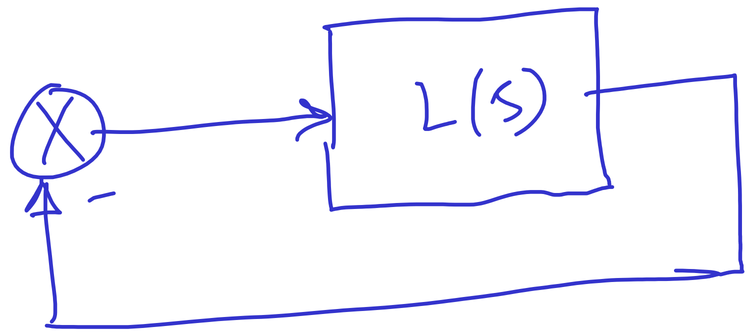
$$M_s \geq \frac{|z+p|}{|z-p|} \geq 1$$

where z, p are u/s zero & pole location of a plant.

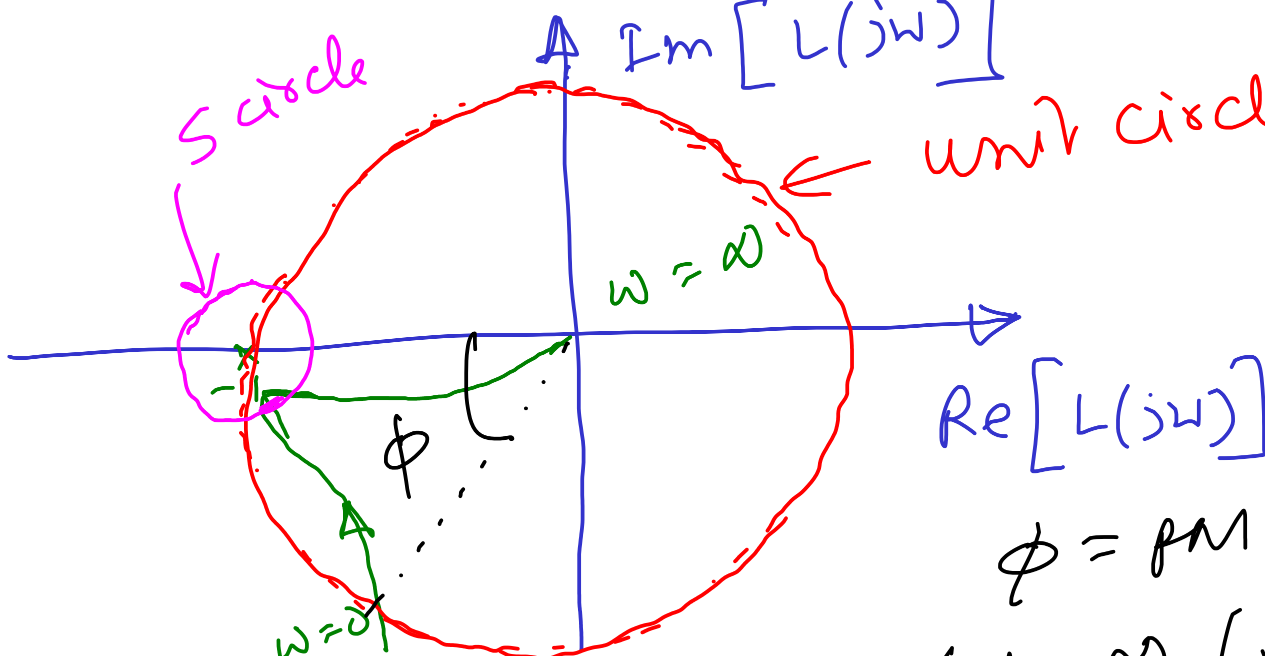
$\Rightarrow$ u/s, NMP plants suffer from poor robustness $\longrightarrow$ difficult to control.

An example where GM, PM are good but M_s is high $\Rightarrow$ poor Robustness

Example of Flexible structure



$$L(s) = \frac{0.38\left(s^2 + 0.1s + 0.55\right)}{s(s+1)\left(s^2 + 0.06s + 0.5\right)}$$



$\phi = PM = 70^\circ$

$GM = \infty$ (upper side)

$M_s = 3.67$

$\Rightarrow M_s^{-1} = 0.2725$

Relation among GM, PM, M_s

$$GM_{max} \geq \frac{M_s}{M_s - 1}$$

$$PM \geq 2\sin^{-1}\frac{1}{2M_s}$$

GM_{max} = upperside GM

- Suppose we want $GM_{max} \geq 2$ & $PM \geq 30^\circ$. What should be M_s?

Soln: To guarantee $GM_{max} \geq 2$

$$\frac{M_s}{M_s - 1} \geq 2 \Rightarrow M_s \geq 2M_s - 2$$
$$\Rightarrow \boxed{M_s \leq 2}$$

To guarantee $PM \geq 30^\circ$

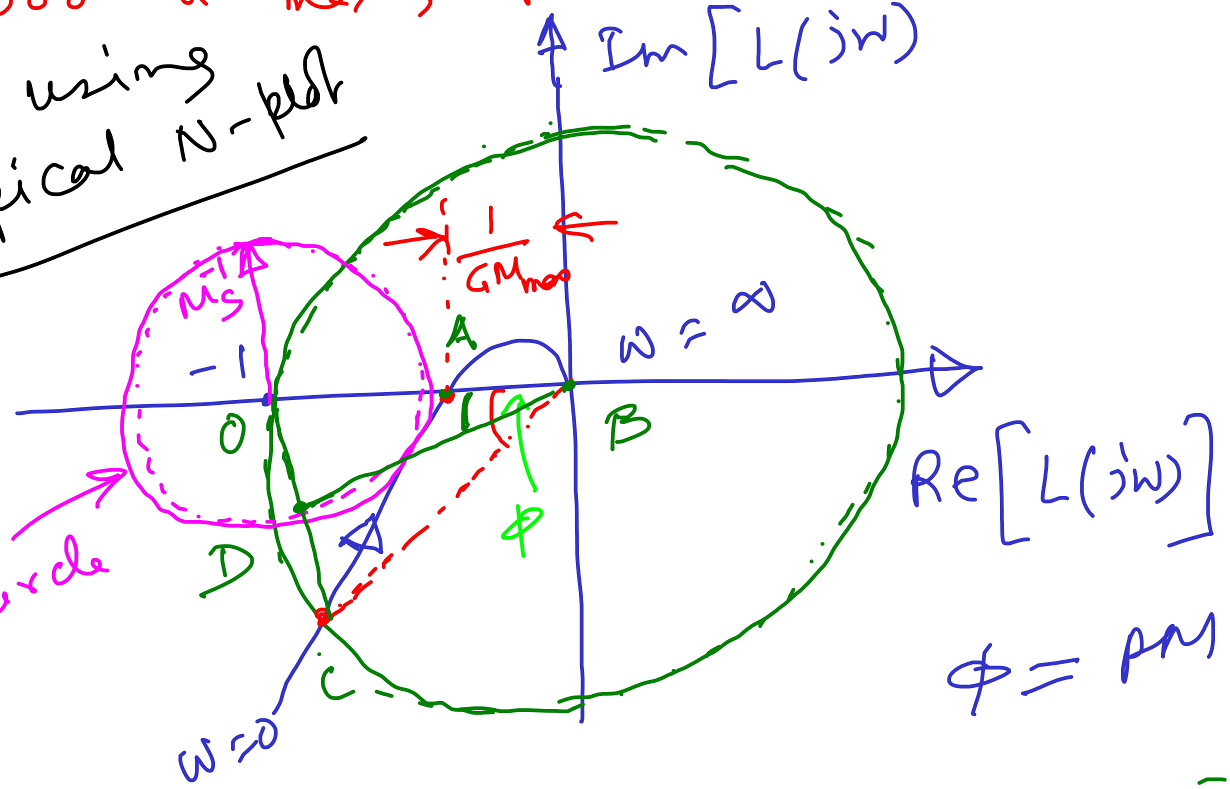
$$2\sin^{-1}\frac{1}{2M_s} \geq 30^\circ$$

$$\Rightarrow \boxed{M_s \leq 1.93}$$

So, to guarantee both $GM_{max} \geq 2$ & $PM \geq 30^\circ$, $\boxed{M_s \leq 1.93}$

· Smaller value (<2) of M_s guarantees good GM_{max}, PM

Proof using a typical N-plot



$\phi = PM$

$$OA > M_s^{-1} \Rightarrow 1 - \frac{1}{GM_{max}} > M_s^{-1}$$

$$\Rightarrow GM_{max} \geq \frac{M_s}{M_s - 1}$$

From ΔOBC,

$$OC \geq M_s^{-1}$$

$$\Rightarrow \frac{OC}{2} \geq \frac{1}{2M_s}$$

$$\Rightarrow OD \geq \frac{1}{2M_s}$$

$$\Rightarrow \sin\frac{\phi}{2} \geq \frac{1}{2M_s}$$

$$\Rightarrow PM \geq 2\sin^{-1}\frac{1}{2M_s}$$

[In ΔOBC, $OB = OC = 1$]