# ASSIGNMENT - 3

17EE35004
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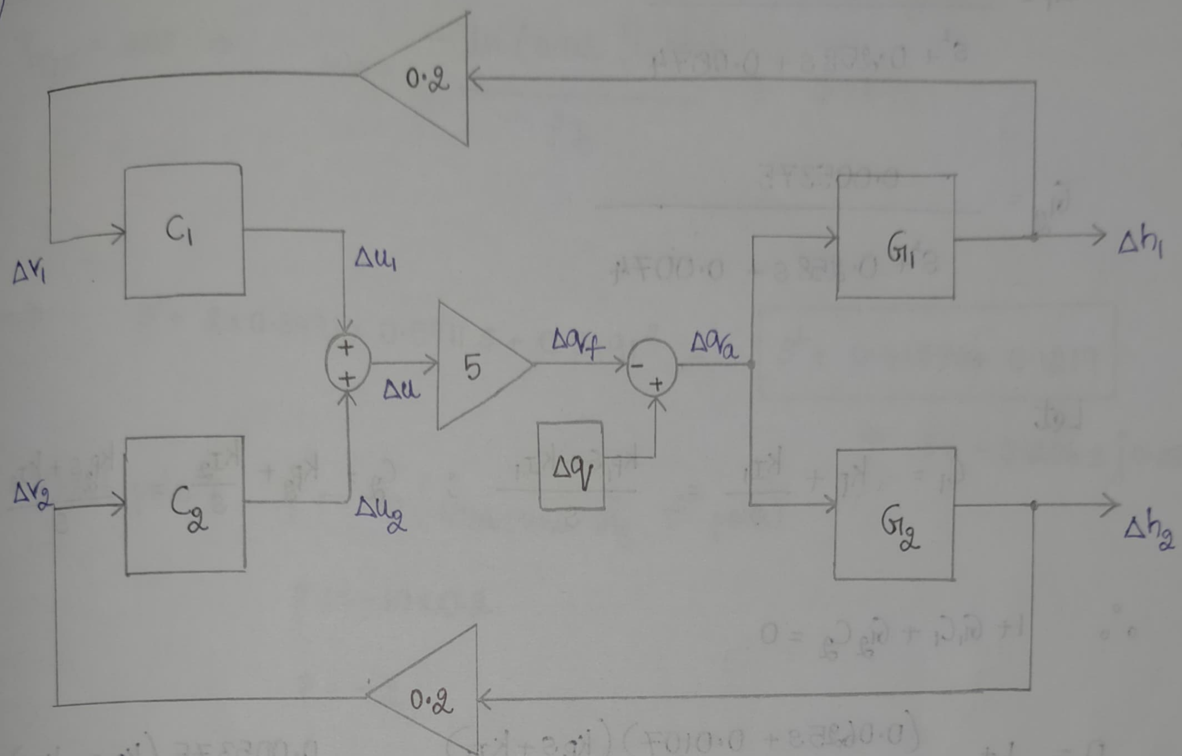
(1)




we have. $\Delta q_a = \Delta q - \Delta q_f$

$$\Delta q_f = 5(\Delta u) = 5(C_1 \Delta r_1 + C_2 \Delta r_2)$$

$$= 5(0.2 C_1 \Delta h_1 + 0.2 C_2 \Delta h_2)$$

$$= C_1 \Delta h_1 + C_2 \Delta h_2$$

$$= (G_1 C_1 + G_2 C_2) \Delta q_a$$

$$\Rightarrow \boxed{\Delta q_a = \frac{\Delta q}{1 + G_1 C_1 + G_2 C_2}}$$

$\therefore$ Characteristic equation $= 1 + G_1 C_1 + G_2 C_2 = 0$

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We have.

$$G_1 = \frac{0.0625 s + 0.0107}{s^2 + 0.258 s + 0.0074}$$

$$G_2 = \frac{0.005375}{s^2 + 0.258 s + 0.0074}$$

Let

$$C_1 = k_{P_1} + \frac{k_{I_1}}{s} = \frac{k_{P_1} s + k_{I_1}}{s} \; ; \quad C_2 = k_{P_2} + \frac{k_{I_2}}{s} = \frac{k_{P_2} s + k_{I_2}}{s}$$

$$\therefore \quad 1 + G_1 C_1 + G_2 C_2 = 0.$$

$$0 = 1 + \frac{(0.0625 s + 0.0107)(k_{P_1} s + k_{I_1})}{(s^2 + 0.258 s + 0.0074) s} + \frac{0.005375 (k_{P_2} s + k_{I_2})}{(s^2 + 0.258 s + 0.0074) s}$$

$$s(s^2 + 0.258 s + 0.0074) + (0.0625 s + 0.0107)(k_{P_1} s + k_{I_1}) + (0.005375)(k_{P_2} s + k_{I_2}) = 0$$

$$\boxed{s^3 + (0.258 + 0.0625 k_{P_1}) s^2 + s(0.0074 + 0.0107 k_{P_1} + 0.0625 k_{I_1} + 0.005375 k_{P_2}) + (0.0107 k_{I_1} + 0.005375 k_{I_2}) = 0.}$$

Since PI controllers are used, Steady state error will be zero.

Desired characteristic equation properties :

$P.O \leq 10\%$ ; $T_{s(2\%)} \leq 20s$.

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$$P.O = 10\% \Rightarrow e^{\frac{-\rho\pi}{\sqrt{1-\rho^2}}} = 0.1 \Rightarrow \rho = 0.5911$$

$$T_{s(2\%)} = 20s \Rightarrow \omega_n = \frac{-\ln\left(0.02\sqrt{1-\rho^2}\right)}{\rho T_s} = 0.3491$$

$$\Rightarrow s^2 + 2 \times 0.3491 \times 0.5911\, s + 0.3491^2 = \boxed{s^2 + 0.4127s + 0.1219}$$

$$\Rightarrow s = -0.206 \pm j0.2816$$

Third pole : $P \approx 10$ (Real part of $1^{st}$ pole)

$P \approx -10 \times 0.2$

$P = -2$

$\therefore$ $\boxed{(s+2)(s^2 + 0.4127s + 0.1219) = 0}$ is the desired char. eq.

$$\Rightarrow \boxed{s^3 + 2.4127s^2 + 0.9473s + 0.2438 = 0}$$

Let $K_{I_1} = 5$ ;

$$K_{P_1} = \frac{2.4127 - 0.258}{0.0625} = 34.4752$$

$$K_{I_2} = \frac{0.2438 - 0.0107 \times 5}{0.005375} = 35.3914$$

$$K_{P_2} = \frac{0.9473 - 0.0074 - 0.0107 \times 34.4752 - 0.0625 \times 5}{0.005375}$$

$$K_{P_2} = 48.0877$$

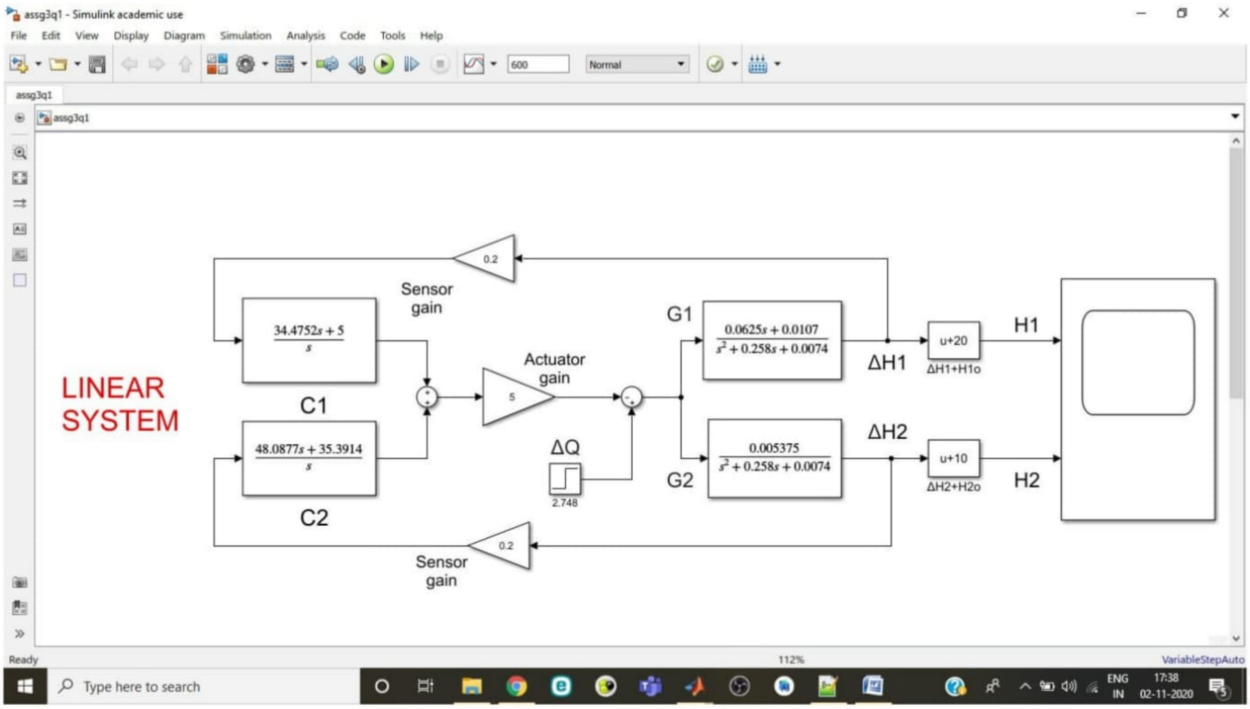
$\therefore$ $\boxed{K_{P_1} = 34.4752,\ K_{I_1} = 5,\ K_{P_2} = 48.0877,\ K_{I_2} = 35.3914}$

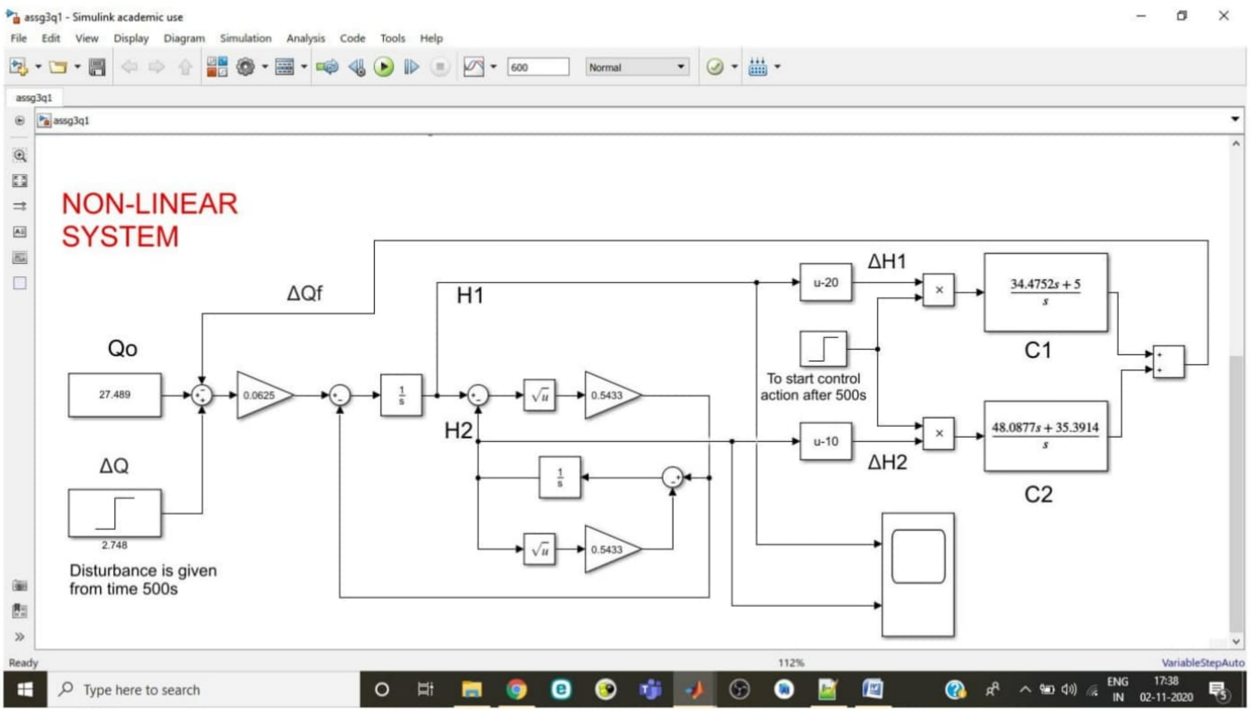
# Problem 1 : Coupled tanks controller design

## MATLAB code :

```
%%natural frequency and damping factor
zeta = 0.5911;  %P.O = 10 percentage
Ts = 20;
wn = (log(50/((1-zeta^2)^0.5))/(Ts*zeta));

%%Dominant pole
p = 2;
z1 = 2*wn*zeta;
z2 = wn*wn;

%% Desired characteristic equation coefficients
K2 = p+z1;      %coefficient of s^2
K1 = p*z1+z2;   %coefficient of s^1
K0 = p*z2;      %coefficient of s^0

%% Defining matrices
P = [K2; K1; K0];
C = [0; 0; 0.005375];
%Matrices if Ki2 is fixed
A = [0.0625 0 0; 0.0107 0.005375 0.0625; 0 0 0.0107];
B = [0.258; 0.0074; 0];
%Matrices if Ki1 is fixed
D = [0; 0.0625; 0.0107];
E = [0.0625 0 0; 0.0107 0.005375 0; 0 0 0.005375];

Ki11 = 5;                %Desired Ki1(to be fixed)
Ki2 = 30;                %Desired Ki2(to be fixed)

%% Finding the other coefficients
X = A\(P-B-Ki2*C);       %X = [Kp1; Kp2; Ki1]
% X is matrix with controller coefficients for a fixed Ki2

Y = E\(P-B-Ki11*D);      %Y = [Kp11; Kp12; Ki12]
% Y is matrix with controller coefficients for a fixed Ki1

Kp1 = X(1);
Kp2 = X(2);     %Controller Coefficients for a fixed Ki2
Ki1 = X(3);

Kp11 = Y(1);
Kp12 = Y(2);    %Controller Coefficients for a fixed Ki1
Ki12 = Y(3);
```

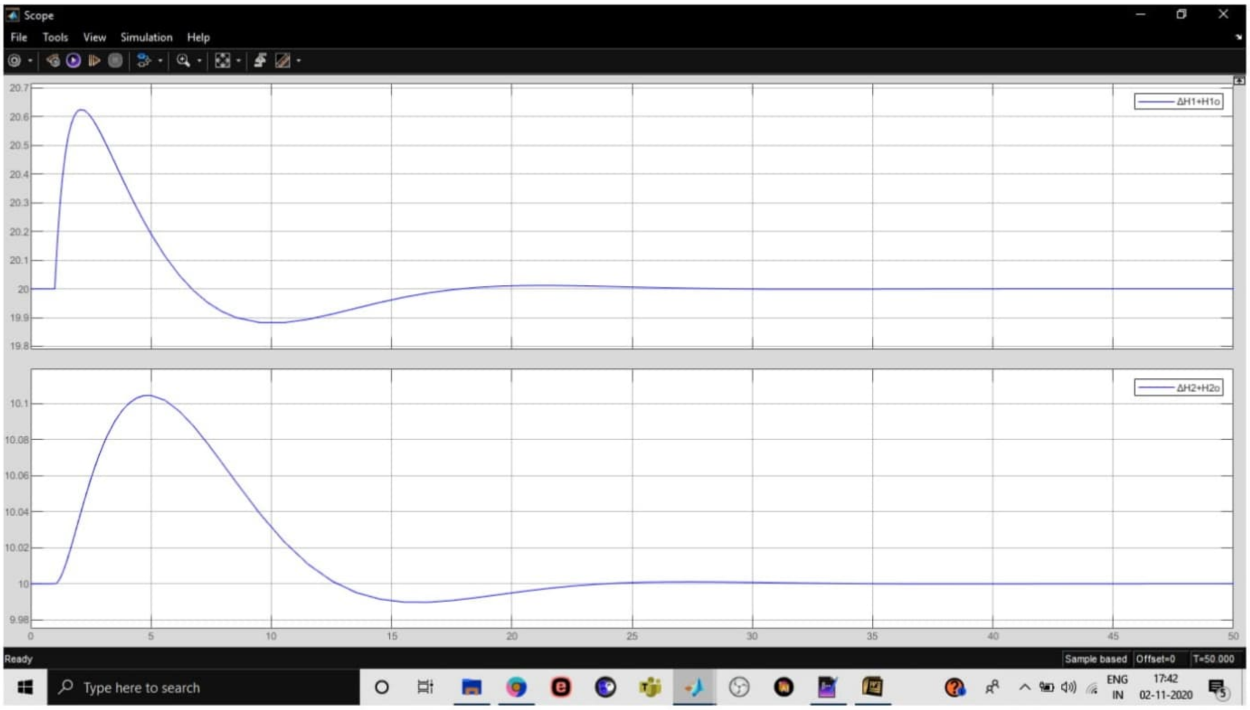
# SIMULINK Diagram :



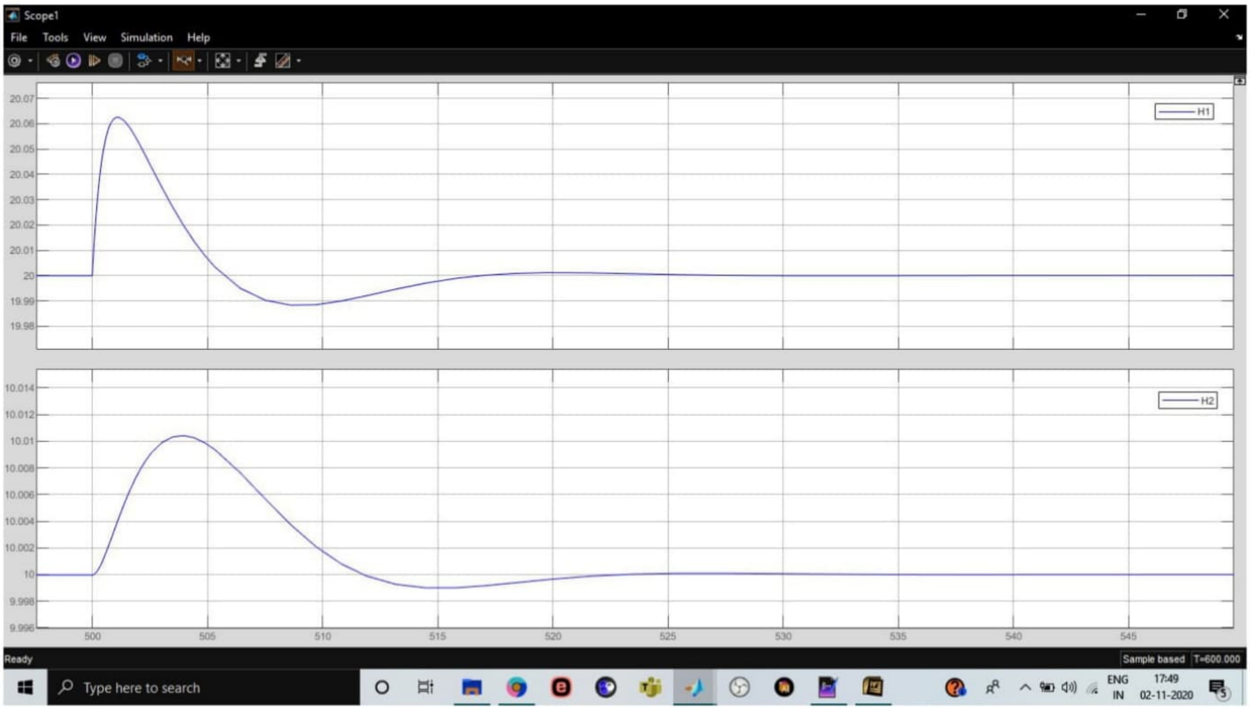



# Waveforms :

## (a) Linear model : H1o+ ΔH1 and H2o+ ΔH2




## (b) Non-Linear model : H1 and H2



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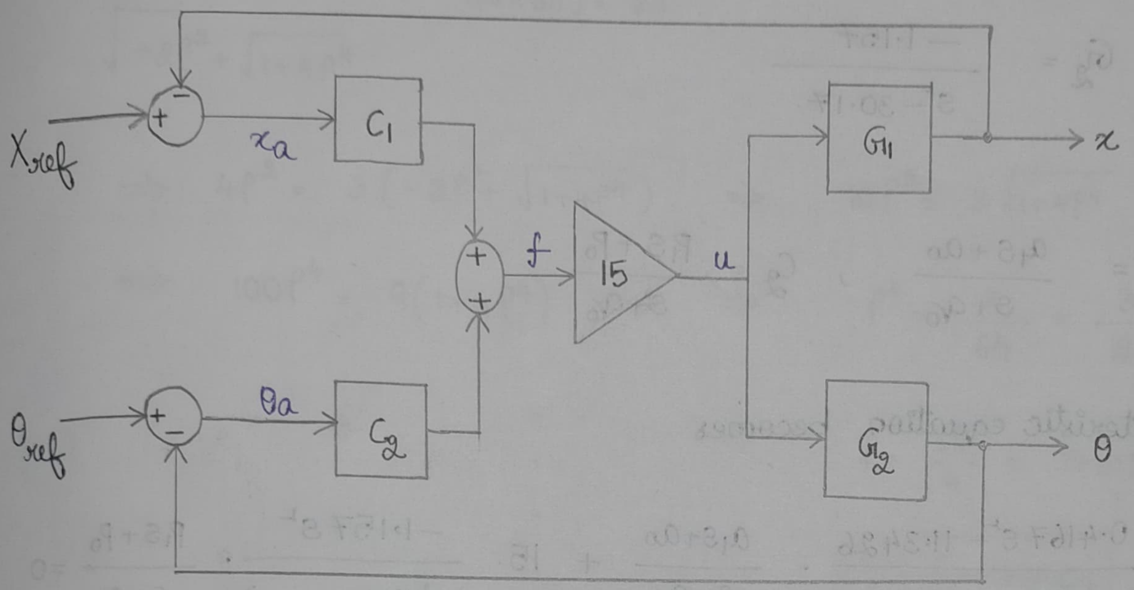


We have $x_a = X_{ref} - x$, $\theta_a = \theta_{ref} - \theta$.

$$u = 15f = 15(C_1 x_a + C_2\theta_a)$$

$$u = 15(C_1 X_{ref} + C_2\theta_{ref}) - 15(C_1 x + C_2\theta)$$

$$u = 15(C_1 X_{ref} + C_2\theta_{ref}) - 15(G_1C_1u + G_2C_2u)$$

$$u = \frac{15(C_1 X_{ref} + C_2\theta_{ref})}{1 + 15(G_1C_1 + G_2C_2)}$$

Characteristic equation is $\boxed{1 + 15(G_1C_1 + G_2C_2) = 0}$

<u>NOTE</u>: Here $\overset{=0}{X_{ref}}$ & $\theta_{ref} = 0$, so steady state will be (0,0). So we will just observe transient response due to initial conditions.

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We have : $G_1 = \dfrac{0.4167 s^2 - 11.3426}{s^2(s^2 - 30.17)}$

$$G_2 = \frac{-1.157}{s^2 - 30.17}$$

Let $C_1 = \dfrac{a_1 s + a_0}{s + q_0}$ , $C_2 = \dfrac{P_1 s + P_0}{s + q_0}$

$\therefore$ Characteristic equation becomes

$$1 + 15 \cdot \frac{0.4167 s^2 - 11.3426}{s^2(s^2 - 30.17)} \cdot \frac{a_1 s + a_0}{s + q_0} + 15 \cdot \frac{-1.157 s^2}{s^2(s^2 - 30.17)} \cdot \frac{P_1 s + P_0}{s + q_0} = 0$$

$\Rightarrow$

$$s^2(s^2 - 30.17)(s + q_0) + 15(0.4167 s^2 - 11.3426)(a_1 s + a_0) - 15(1.157 s^2)(P_1 s + P_0) = 0$$

$\Rightarrow$

$$\boxed{s^5 + (q_0) s^4 + (-30.17 + 6.25 a_1 - 17.355 P_1) s^3 + (-30.17 q_0 + 6.25 a_0 - 17.355 P_0) s^2 + (-170.139 a_1) s + (-170.139 a_0) = 0}$$

Desired ~~Steady State~~ Characteristic equation properties :

PM $\geq 30^\circ$

$T_s$ (2%) $\leq \dfrac{20s}{4} = 5s$

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Let $PM = 60^\circ$,

$$\frac{2\rho}{\sqrt{-2\rho^2 + \sqrt{1+4\rho^4}}} = \tan(60^\circ) = \sqrt{3}$$

$$\Rightarrow 4\rho^2 = 3\left(-2\rho^2 + \sqrt{1+4\rho^4}\right) \Rightarrow 10\rho^2 = 3\sqrt{1+4\rho^4}$$

$$\Rightarrow 100\rho^4 = 9(1+4\rho^4) \Rightarrow \rho^4 = \frac{9}{64} = \frac{36}{256}$$

$$\Rightarrow \rho = \frac{\sqrt{6}}{4} = 0.6124$$

$$\therefore \boxed{\rho = 0.6124}$$

$T_s = 5s$
(2%)

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\rho^2}\right)}{\rho T_s} = 1.3544 \quad \therefore \boxed{\omega_n = 1.3544}$$

$$\Rightarrow s^2 + 2 \times 0.6124 \times 1.3544\, s + 1.3544^2 = 0$$

$$\boxed{s^2 + 1.6588s + 1.8344 = 0} \Rightarrow s = -0.8294 \pm j\,1.0707.$$

Dominant poles : $P_1, P_2, P_3 \geq 10(+0.8294)$

$(s^2 + 1.6588s + 1.8344)(s+P_1)(s+P_2)(s+P_3) = 0$ is the desired char. eq.

$$\Rightarrow \boxed{\begin{aligned} &s^5 + \left(1.6588 + (P_1+P_2+P_3)\right)s^4 + \left(1.8344 + 1.6588(P_1+P_2+P_3) + (P_1P_2+P_2P_3+P_3P_1)\right)s^3 \\ &+ \left(1.8344(P_1+P_2+P_3) + 1.6588(P_1P_2+P_2P_3+P_3P_1) + P_1P_2P_3\right)s^2 \\ &+ \left(1.8344(P_1P_2+P_2P_3+P_3P_1) + 1.6588(P_1P_2P_3)\right)s + 1.8344(P_1P_2P_3) = 0 \end{aligned}}$$

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$\Rightarrow s^5 + K_4 s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_0 = 0$ is the desired char eq.

$\Rightarrow$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 6.25 & 0 & -17.0355 & 0 & 0 \\ 0 & 6.25 & 0 & -17.0355 & -30.17 \\ -170.139 & 0 & 0 & 0 & 0 \\ 0 & -170.139 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \\ p_1 \\ p_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} K_4 \\ K_3 \\ K_2 \\ K_1 \\ K_0 \end{bmatrix} - \begin{bmatrix} 0 \\ -30.17 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, we get.

| | | |
|---|---|---|
| $a_1 = -24.0673$ | $P_1 = -42.9143$ | $q_0 = 41.6588$ |
| $a_0 = -20.7014$ | $P_2 = -242.1421$ | |

$$\therefore \quad C_1 = \frac{-24.0673s + (-20.7014)}{s + 41.6588}$$

$$C_2 = \frac{-42.9143s + (-242.1421)}{s + 41.6588}.$$

Non-linear model: $I \approx 0$.

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$$u = (M+m)\ddot{x} + ml \frac{d^2}{dt^2}(\sin\theta)$$

$$u = (M+m)\ddot{x} + ml \frac{d}{dt}[\cos\theta \cdot \dot{\theta}]$$

$$u = (M+m)\ddot{x} + ml[\cos\theta \cdot \ddot{\theta} - \sin\theta \cdot (\dot{\theta})^2]$$

$$\boxed{u = (M+m)\ddot{x} + (ml\cos\theta)\ddot{\theta} - (ml\sin\theta)(\dot{\theta})^2} \longrightarrow 1$$

$V = mg + ml \frac{d^2}{dt^2}(\cos\theta)$ ; $H = m\ddot{x} + ml \frac{d}{dt^2}(\sin\theta)$ ;

$V\sin\theta = H\cos\theta.$

$$\Rightarrow \sin\theta\left[\cancel{m}g - \cancel{m}l\frac{d}{dt}[\sin\theta \cdot \dot{\theta}]\right] = \cos\theta\left[\cancel{m}\ddot{x} + \cancel{m}l\frac{d}{dt}[\cos\theta \cdot \dot{\theta}]\right]$$

$$\sin\theta\left[g - l\sin\theta \cdot \ddot{\theta} - l\cancel{\cos\theta \cdot (\dot{\theta})^2}\right] = \cos\theta\left[\ddot{x} + l\cos\theta \cdot \ddot{\theta} - l\cancel{\sin\theta \cdot (\dot{\theta})^2}\right]$$

$$g\sin\theta = \ddot{x}\cos\theta + l\ddot{\theta}[\cos^2\theta + \sin^2\theta]$$

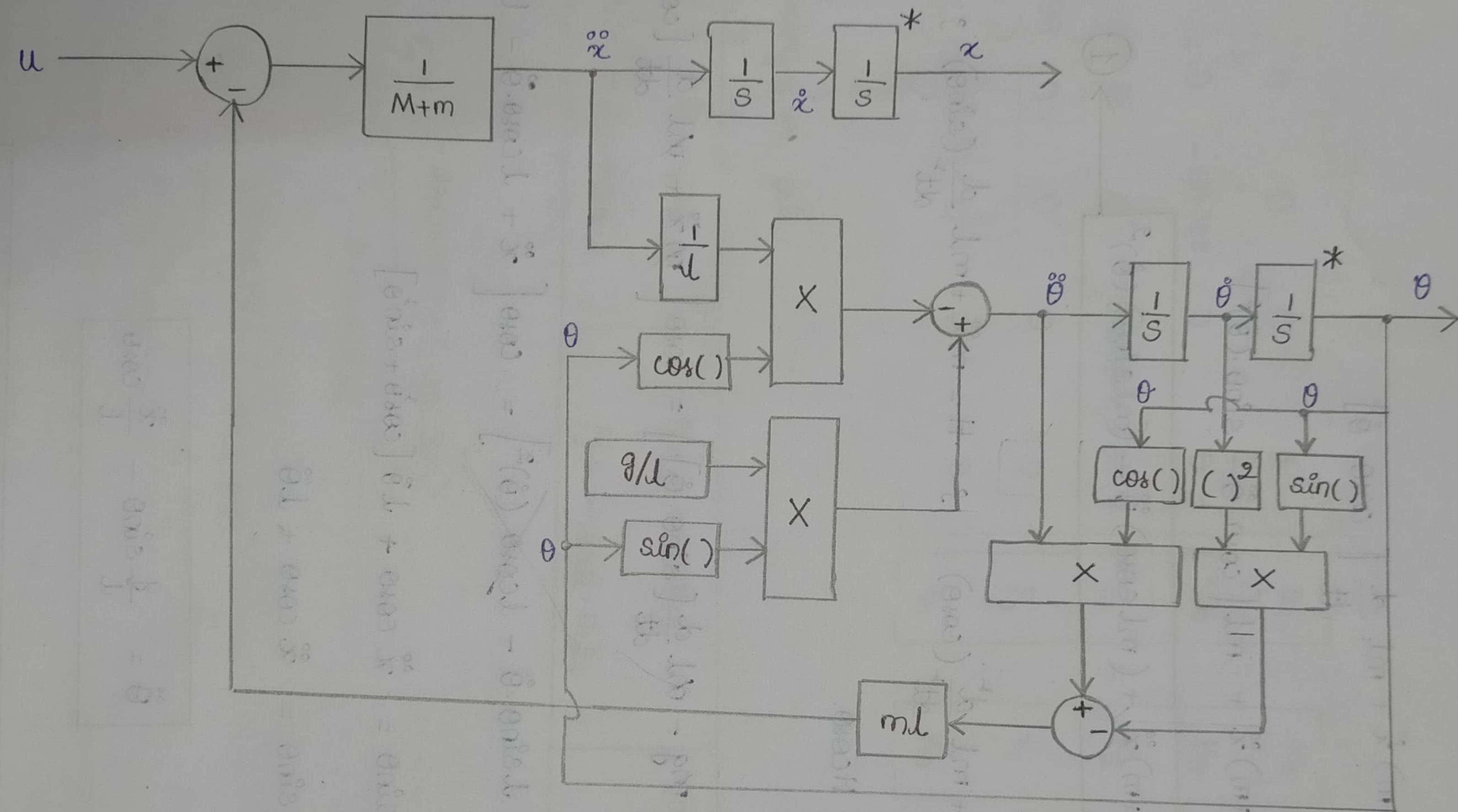
$$g\sin\theta = \ddot{x}\cos\theta + l\ddot{\theta}$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{g}{l}\sin\theta - \frac{\ddot{x}}{l}\cos\theta}$$

1:

$$\boxed{\ddot{x} = \frac{1}{M+m}\left[u + ml\left(-\cos\theta \cdot \ddot{\theta} + \sin\theta \cdot (\dot{\theta})^2\right)\right]}$$

NON-LINEAR BLOCK DIAGRAM :




"*" marked integrators must be updated with initial conditions of $x$ and $\theta$.

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# Problem 2 : Inverted Pendulum controller design

# MATLAB code :

```
%%natural frequency and damping factor
zeta = 6^0.5/4;
Ts = 5;
wn = (log(50/((1-zeta^2)^0.5))/(Ts*zeta));
%% Three dominant poles
p1= 8;
p2 = 12;
p3 = 20;

z1 = 2*zeta*wn;
z2 = wn*wn;

%% Desired characteristic equation coefficients
K4 = z1 + (p1+p2+p3);                                            %coeff of s^4
K3 = z2 + z1*(p1+p2+p3) + (p1*p2+p2*p3+p3*p1);                   %coeff of s^3
K2 = z2*(p1+p2+p3) + z1*(p1*p2+p2*p3+p3*p1) + (p1*p2*p3);        %coeff of s^2
K1 = z2*(p1*p2+p2*p3+p3*p1) + z1*(p1*p2*p3);                     %coeff of s^1
K0 = z2*(p1*p2*p3);                                              %coeff of s^0

%% Defining matrices
P = [K4;K3;K2;K1;K0];
A = [0 0 0 0 1; 6.25 0 -17.355 0 0; 0 6.25 0 -17.355 -30.17; -170.139 0 0 0 0; 0 -
170.139 0 0 0];
B = [0; -30.17; 0; 0; 0];

X = A\(P-B);              % X = [a1; a0; p1; p0; q0]

%% Bode plot
a1 = X(1);
a0 = X(2);
b1 = X(3);
b0 = X(4);
c0 = X(5);

G1 = tf([0.4167 0 -11.3426],[1 0 -30.17 0 0]);
G2 = tf(-1.157,[1 0 -30.17]);
C1 = tf([a1 a0],[1 c0]);
C2 = tf([b1 b0],[1 c0]);

CL_TF = G1*C1+G2*C2;
figure;
bode(CL_TF);
title('Bode plot');
```

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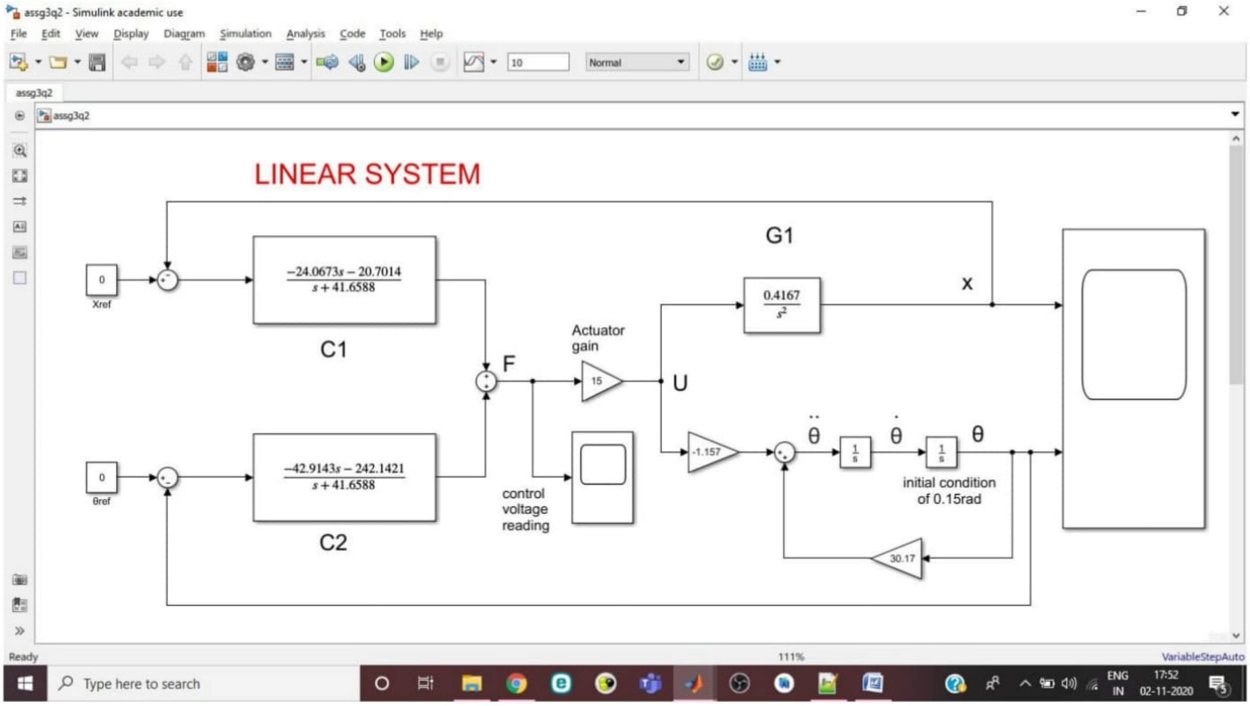
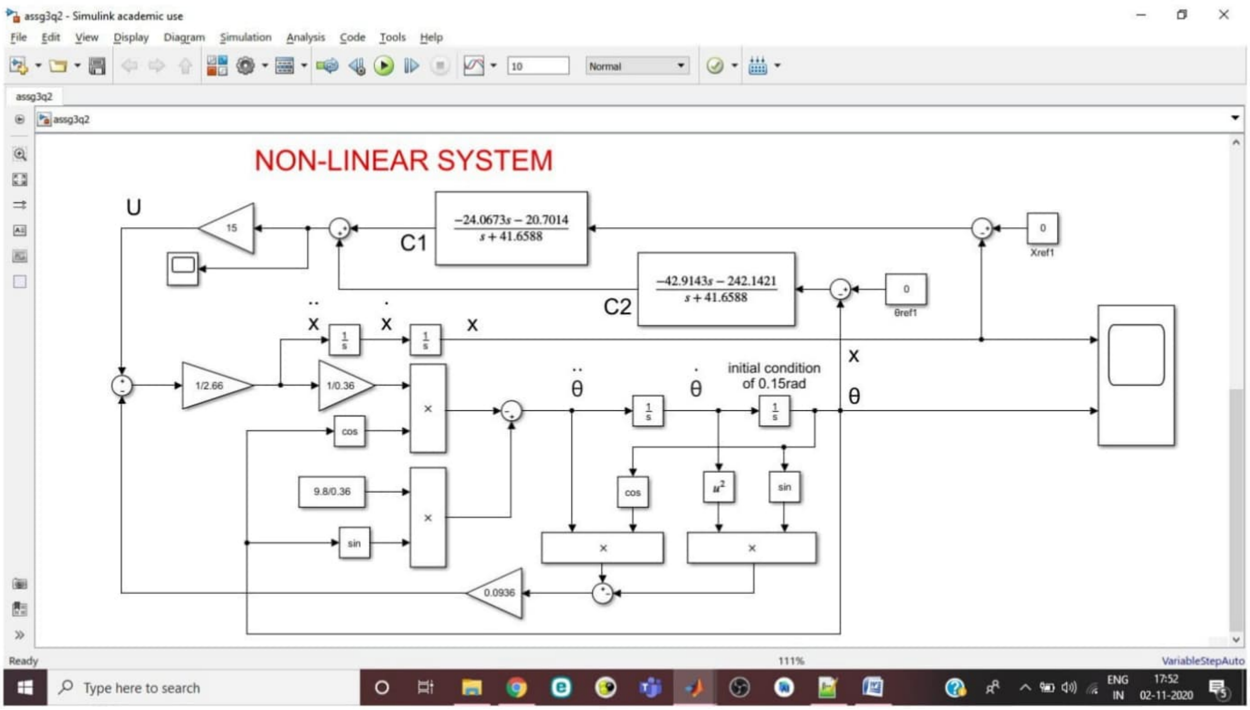
# SIMULINK Diagram :



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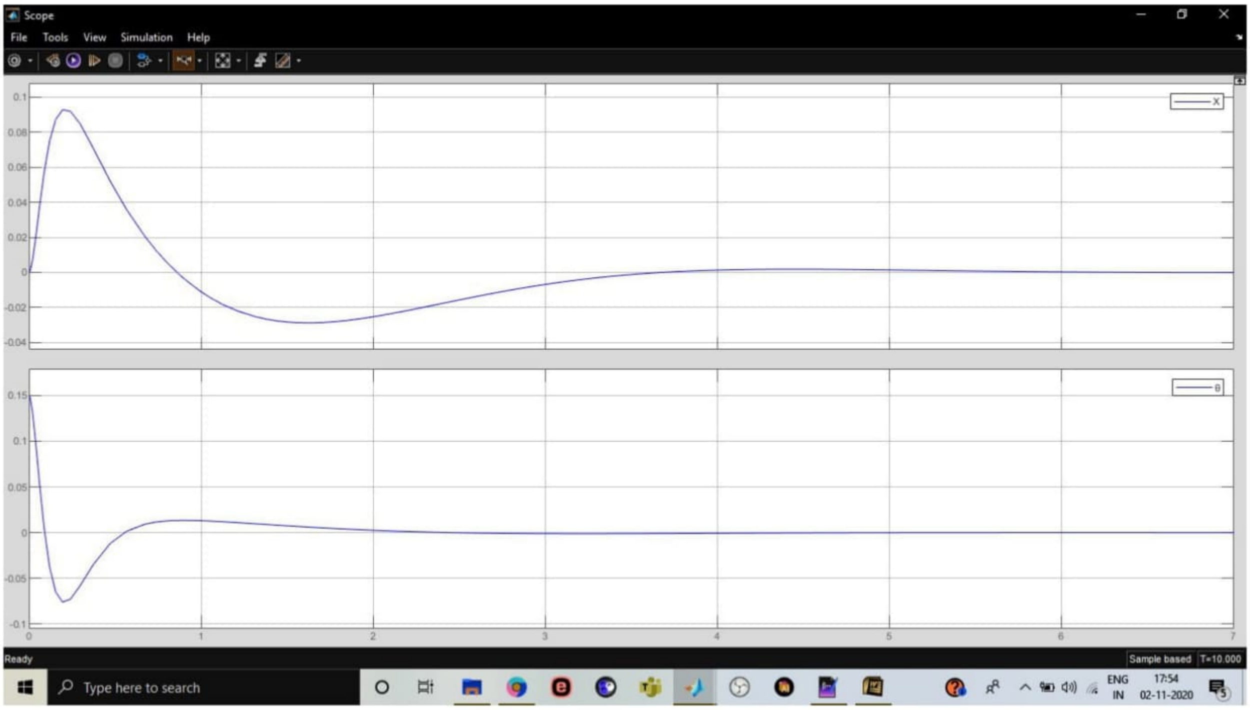
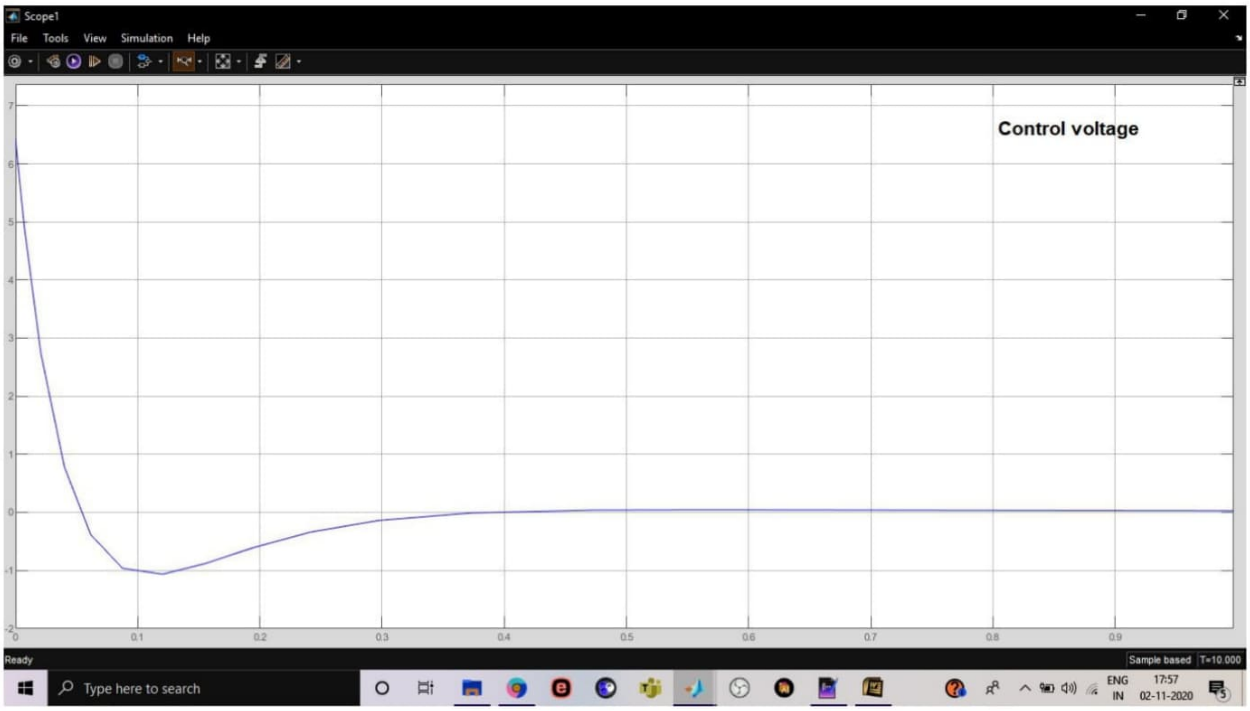
# Waveforms :

## (a) Linear model :

### X and θ

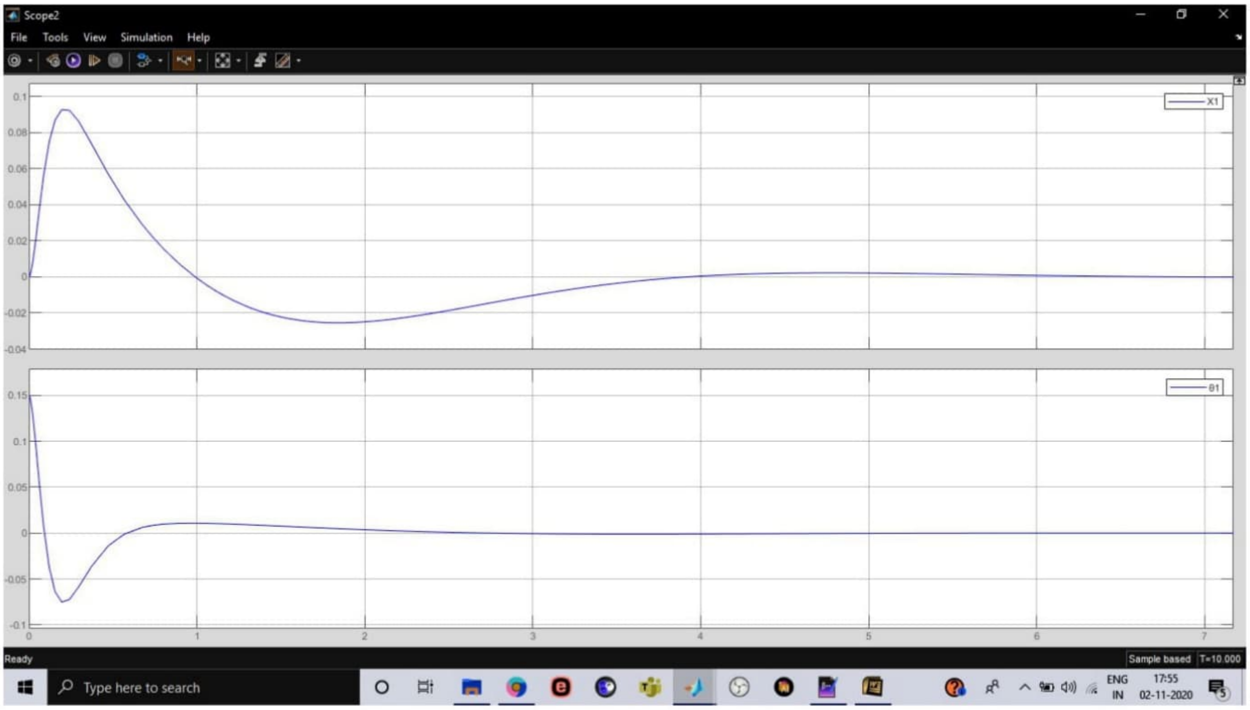



### Control voltage(u)



# (b) Non-Linear model :

## X and θ




## Control voltage(u)