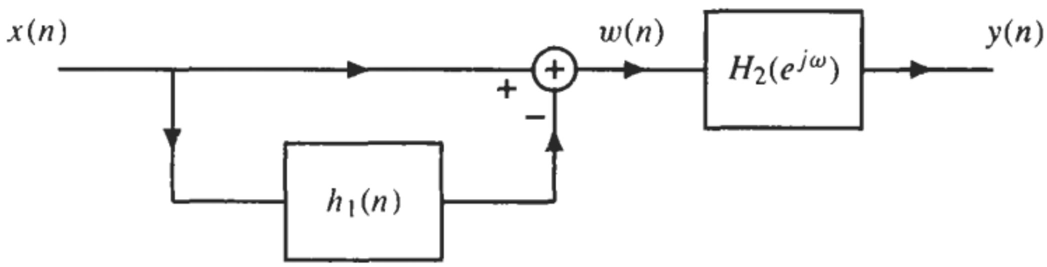
Digital Signal Processing (EE41013/EE60033)

End-semester Examination (Autumn 2021-22)

Total marks: 50 **Time: 3 hours**

Q1. **(a)** Consider the following inter-connection of linear time-invariant systems



where $h_1(n) = \delta(n-1)$ and $H_2(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\omega| \leq \pi \end{cases}$

Find the frequency response and the unit sample response of the system.

(4)

(b) Find the inverse DTFT of

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j10\omega}}$$

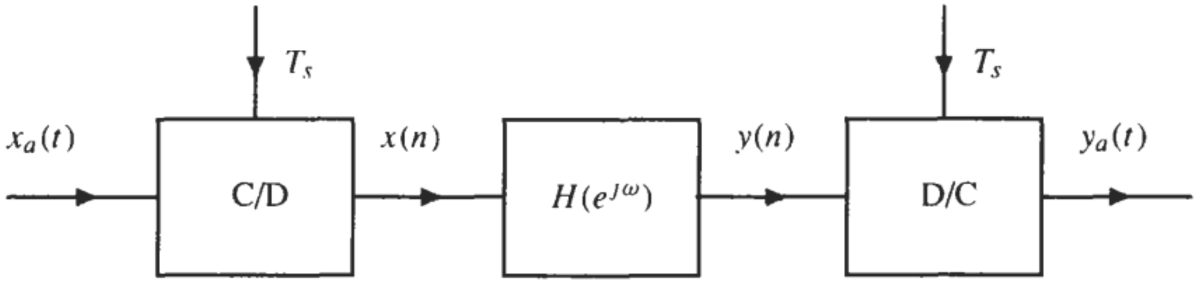
(3)

Q2. **(a)** A continuous-time signal $x_a(t)$ is to be filtered to remove frequency components in the range $5\,\text{kHz} \leq f \leq 10\,\text{kHz}$. The maximum frequency present in $x_a(t)$ is 20 kHz. The filtering is to be done by sampling $x_a(t)$, filtering the sampled signal and reconstructing back the analog signal using an ideal D/C converter.

Find the minimum sampling frequency to avoid aliasing and for this sampling rate find the frequency response of an ideal digital filter $H(e^{j\omega})$ that will remove the desired frequencies from $x_a(t)$.

(3)

(b) Consider the system shown below for processing a continuous-time signal with a discrete-time system.



The frequency response of the discrete-time filter is

$$H(e^{j\omega}) = \frac{2(\frac{1}{3} - e^{-j\omega})}{1 - \frac{1}{3}e^{-j\omega}}$$

If $f_s = 2\,\text{kHz}$ and $x_a(t) = \sin(1000\pi t)$, find the output $y_a(t)$.

(4)

Q3. (a) Suppose that we are given the following information about an LTI system:

1. If the input to the system is $x_1[n] = (1/6)^n u[n]$, then the output is

$$y_1[n] = \left[a\left(\frac{1}{2}\right)^n + 10\left(\frac{1}{3}\right)^n\right] u(n)$$

where a is a real number.

2. If $x_2[n] = (-1)^n$, then the output is $y_2[n] = \frac{7}{4}(-1)^n$.

Derive the value of the constant a and the system function $H(z)$ for the above system and also determine the ROC.

What can you conclude about the causality and stability of this system?

(5)

(b) The following is known about a discrete-time LTI system with input $x[n]$ and output $y[n]$:

1. If $x[n] = (-2)^n$ for all n, then $y[n] = 0$ for all n.
2. If $x[n] = (1/2)^n u[n]$ for all n, then $y[n]$ for all n is of the form

$$y[n] = \delta[n] + a\left(\frac{1}{4}\right)^n u[n]$$

where a is a constant.

(i) Determine the value of the constant a.

(ii) Determine the response $y[n]$ if the input $x[n]$ is

$$x[n] = 1, \text{ for all } n$$

(4)

Q4. (a) (i) Derive the group delay of a first order all-pass filter with a real pole $|\alpha| < 1$.

(ii) Prove that the group delay is non-negative for all ω.

(3+2)

(b) A non-minimum phase causal sequence $x[n]$ has a Z-transform

$$X(z) = \frac{(1 - \frac{3}{2}z^{-1})(1 + \frac{1}{3}z^{-1})(1 + \frac{5}{3}z^{-1})}{(1 - z^{-1})^2(1 - \frac{1}{4}z^{-1})}$$

For what values of the constant α will the sequence $y[n] = \alpha^n x[n]$ be minimum phase?

(3)

Q5. A notch filter has a transfer function with zeros at $z = e^{\pm j\omega_0}$. For the filter with system function $H(z) = 1 - z^{-1} + z^{-2}$

(i) Determine the notch frequency ω_0 and the form of the corresponding sinusoidal sequence to be suppressed

(ii) Verify by computing the output $y[n]$ that in the steady state, $y[n] = 0$ when the sinusoidal sequence is applied at the input of the filter.

(2+2)

Q6. (a) Consider two 4-point sequences $g[n] = \{1, 2, 0, 1\}$, $h[n] = \{2, 2, 1, 1\}$.
Obtain the output $y_C[n]$ of the circular convolution between $g[n]$ and $h[n]$.

(4)

(b) Verify your answer in (a) by using 4×4 DFT matrix W_4 to compute the forward and inverse DFT.

(4)

Q7. (a) Why is the DFT matrix, W_N said to be a unitary matrix ?

(b) What is the advantage that you can derive from this property of W_N ?

(c) Derive the Parseval's relation for $N-$ dimensional discrete-time signals using the above property.

(2+2+3)