

EE60032: Analog Signal Processing



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Module-5: Noise

Noise

Any unwanted disturbance that interferes/obscures with a signal of interest is referred as noise.

Examples : • input offset voltage, input offset current of op-amp \rightarrow dc noise.

- AC noise \rightarrow significantly degrades the performance
 - a) External / interference noise
 - b) Internal / inherent noise

External / interference noise :-

- Caused by unwanted interaction between the system and the outside or environment. It could be within different parts ~~within~~ of the system also.
 - i) Electric \rightarrow through parasitic capacitance. e.g. coupling, VDD/GND bounce.
 - ii) Magnetic \rightarrow through mutual inductance between ckts.
 - iii) Electromagnetic \rightarrow through each wire/traces as potential antenna.
 - iv) Electromechanical \rightarrow through transducers (microphone, piezoelectric) which converts non-electrical noise to electrical noise.
- It can be periodic, intermittent or completely random.
It can be minimised by filtering, decoupling, guarding, electrostatic or electromagnetic shielding, physical separation, low-noise power supplies etc.

Internal / Inherent Noise :-

This is generated inside ckt. and this noise is purely random.

Example : Thermal agitation of electrons in resistor.

Random generation of and recombination of electron-hole pairs in semiconductor.

Importance of signal-to-noise ratio :-

The noise degrades the quality of a signal.

$$SNR = 10 \log_{10} \frac{x_s^2}{x_n^2}$$

x_s = RMS value of signal

x_n = RMS value of noise.

Poorer the value of SNR, more difficult to rescue the signal from noise.

- Noise will be a concern based on performance requirement :-

$$\text{In 12 bit A/D converter, } \frac{1}{2} LSB = \frac{10V}{2^{12}} = 1.22 \text{ mV where } 10V = \text{full scale}$$

Lets assume, transducer is producing 10 mV signal.

To use full scale range of A/D, you have to amplify the signal by 1000 times.

Now $\frac{1}{2}$ LSB corresponds to a signal level of 1.22 mV.

If your amplifier has an input referred noise of 1 mV, then it will be invalidated.

Noise Properties

Noise is a random process, the instantaneous value of noise is unpredictable.
We have to deal with noise on a statistical basis.

① RMS value of noise:-

$$\text{RMS value of noise } X_n = \sqrt{\frac{1}{T} \int_0^T x_n^2(t) dt}$$

T = suitable averaging time interval.

X_n = RMS value of noise voltage/current.

Physically, X_n^2 represents the average power dissipated by $x_n(t)$ in a 1Ω resistor.

If voltage noise source, Power = $\frac{X_n^2}{R}$, if current noise source, power = $X_n^2 R$.

② Crest factor:-

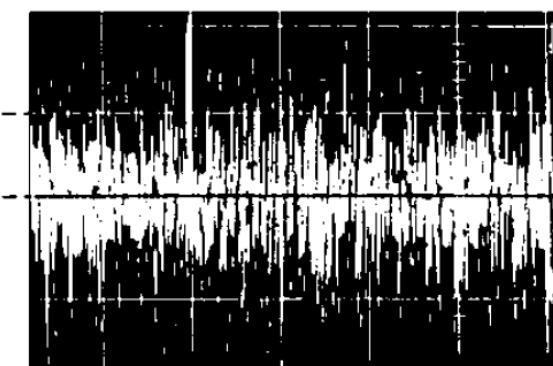
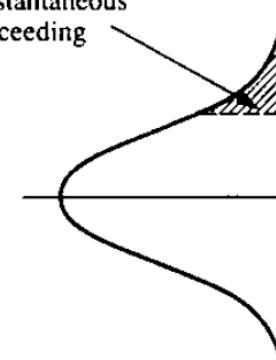
In many applications, such as A/D converter/comparator, the resolution or accuracy etc. are affected by instantaneous value rather

than RMS value of noise. Peak noise is more a concern.

Most noise has a Gaussian distribution, instantaneous values can be predicted in terms of probability.

$$\text{Crest factor} = \frac{\text{Peak value of the noise}}{\text{RMS value of the noise}}$$

Probability of instantaneous value of $x_n(t)$ exceeding value X



Voltage noise (right), and Gaussian distribution of amplitude.

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Noise Spectrum :-

X_n^2 represents average power dissipated by $x_n(t)$ in a $1\text{-}\Omega$ resistor.

For an AC signal, power is concentrated at one frequency.

However, for noise, power is spreaded over all frequencies due to random nature.

For noise, we must specify average noise power over a frequency band.

The rate of change of noise power with frequency is called noise power spectral density.

$$S_n(f) = \frac{dX_n^2}{df}$$

Unit of $S_n^2(f)$ $\rightarrow V^2/\text{Hz}$

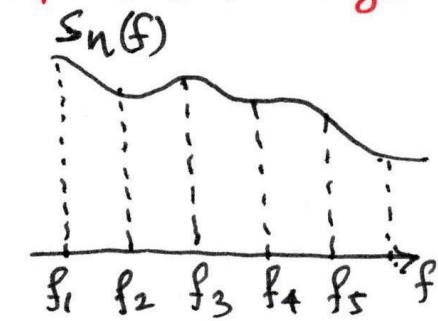
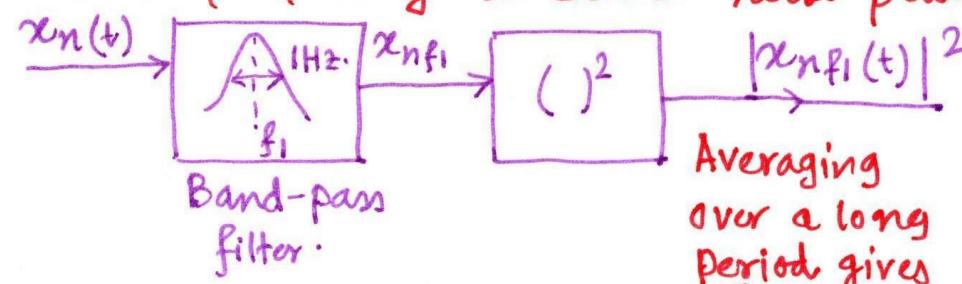
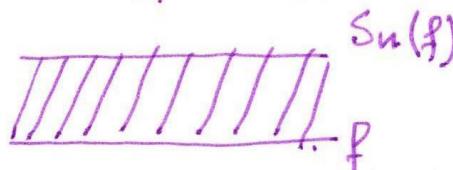
A^2/Hz

Unit of $S_n(f)$ $\rightarrow V/\sqrt{\text{Hz}}$

$A/\sqrt{\text{Hz}}$.

Example :

White spectrum



Power spectral density of a random process may be random. However, most of noise sources exhibit a predictable spectrum.

Total power carried out by white noise is "infinite", which is impractical. In practice, any noise spectrum that is flat in the band of interest is called white.

Noise summation:-

Two noise sources: $x_{n1}(t)$ and $x_{n2}(t)$ and their corresponding rms values are known as X_{n1} and X_{n2} respectively.

$$x_{no}(t) = x_{n1}(t) + x_{n2}(t)$$

$$\text{Then, } X_{no}^2 = \frac{1}{T} \int_0^T x_{no}(t)^2 dt = \frac{1}{T} \int_0^T [x_{n1}(t) + x_{n2}(t)]^2 dt.$$

$$= \frac{1}{T} \int_0^T x_{n1}^2(t) dt + \frac{1}{T} \int_0^T x_{n2}^2(t) dt + \frac{1}{T} \int_0^T 2x_{n1}(t)x_{n2}(t) dt.$$

$$= X_{n1}^2 + X_{n2}^2 + \frac{2}{T} \int_0^T x_{n1}(t)x_{n2}(t) dt.$$

$$\text{If correlation coefficient } C = \frac{\frac{1}{T} \int_0^T x_{n1}(t)x_{n2}(t) dt}{X_{n1} X_{n2}}$$

$$\text{where } -1 \leq C \leq 1$$

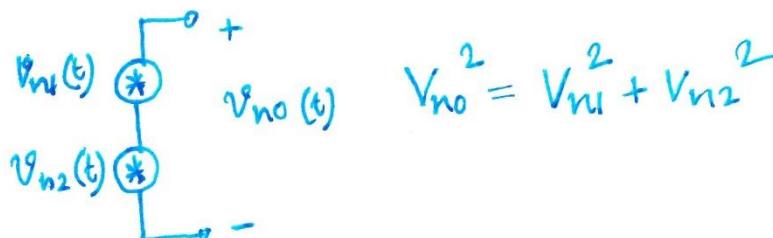
If $C = \pm 1$, then two signals are fully correlated.

If $C = 0$, then they are un-correlated.

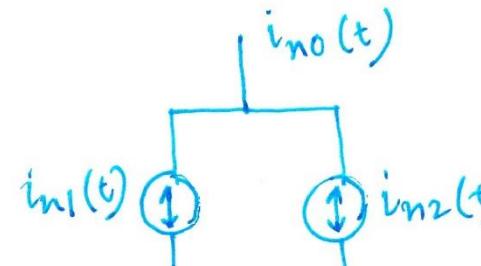
$$\text{Then } X_{no}^2 = X_{n1}^2 + X_{n2}^2 + 2C X_{n1} X_{n2}$$

Usually noise signals are uncorrelated,

$$\text{Then, } X_{no}^2 = X_{n1}^2 + X_{n2}^2$$



$$V_{no}^2 = V_{n1}^2 + V_{n2}^2$$

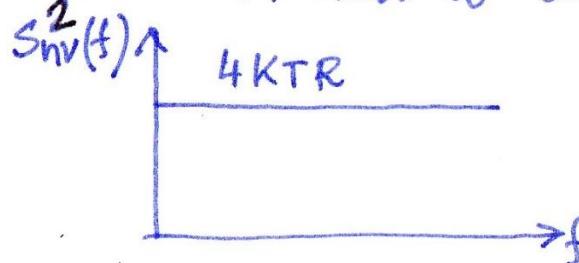
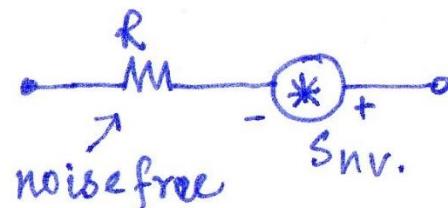


$$I_{no}^2 = I_{n1}^2 + I_{n2}^2$$

① Types of noise :- Different circuit components introduces noise.

1) Thermal noise :- (Also known as Johnson noise)

The random motions of electron introduces fluctuations in voltage measured across the conductor, even if average current is zero. This is thermal noise



$S_{nv}(f)^2 \rightarrow$ Power spectral density.

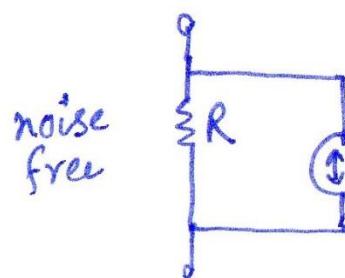
$S_{nv}(f) \Rightarrow$ Voltage spectral density. $= \sqrt{4KTR}$.

$$S_{nv}(f)^2 = 4KTR, f \geq 0.$$

K = Boltzmann Constant $= 1.38 \times 10^{-23} \text{ J/K}$.

T = Absolute temp.

R = Resistance value.



$S_{ni}(f) \rightarrow$ Current spectral density $= \sqrt{\frac{4KTR}{R}} = \sqrt{\frac{4KT}{R}}$

$S_{ni}(f)^2 \rightarrow$ Current power spectral density $= \frac{4KT}{R}$

2) Shot noise :-

This type of noise arises whenever charges crosses a potential barrier, such as in diodes or transistors. Barrier crossing is purely ~~non~~ random and produces random current noise.

Shot noise has a uniform power density. $S_{ni}^2(f) = 2qI$.

$$q = \text{charge of electron} = 1.602 \times 10^{-19} \text{ C.}$$

I = dc current through the barrier.

3) Flicker noise :- ($1/f$ noise or contact noise)

It is present in all active device and in some passive device.

In active device : it is due to traps. When current flows, these traps capture and release carriers randomly, causing random fluctuations of current.

E.g. in BJT - contamination and crystal defects at BE junction.

$$S_{ni}^2(f) = K \cdot \frac{I^a}{f}$$

K = device constant.

I = device current.

a = another device constant [range $\frac{1}{2}$ to 2]

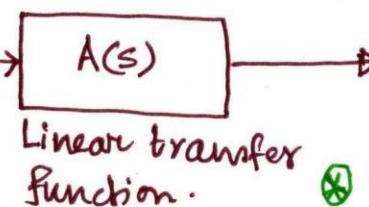
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① Noise filtering :-

I/P Noise power spect. $S_{nvip}(f)$
I/P Noise voltage spect. $S_{nvif}(f)$



$$S_{nvop}^2(f) = |A(j2\pi f)|^2 S_{nvip}(f) \quad \text{Q/P Noise power spec}$$

$$S_{nvop}(f) = |A(j2\pi f)| S_{nvip}(f) \quad \text{o/p noise volt. spect}$$

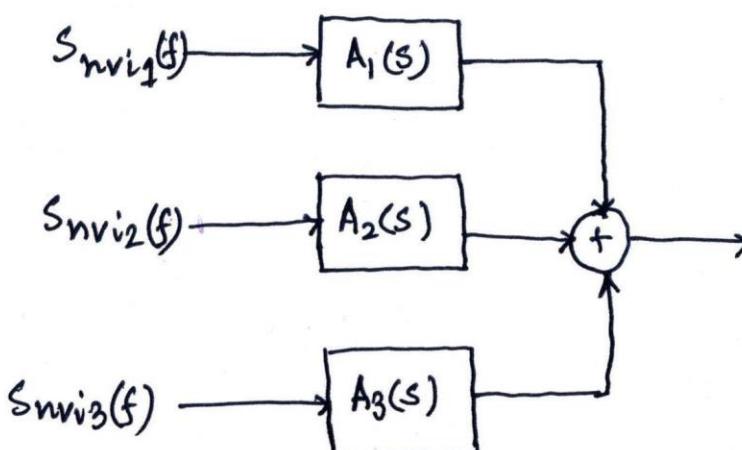
* Here $s = j\omega = j2\pi f$.

* Output noise power spectral density is a function ~~of~~ ONLY of the magnitude of transfer function, NOT its phase.

* The total output mean squared value is given by :-

$$V_{nop(\text{rms})}^2 = \int_0^\infty S_{nvop}(f) df = \int_0^\infty |A(j2\pi f)|^2 S_{nvip}(f) df.$$

* In case multiple uncorrelated noise are contributing :-

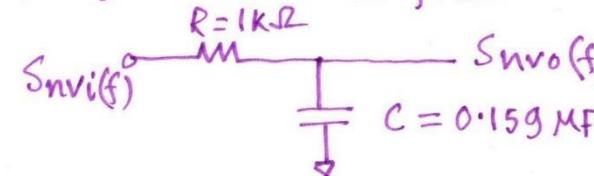
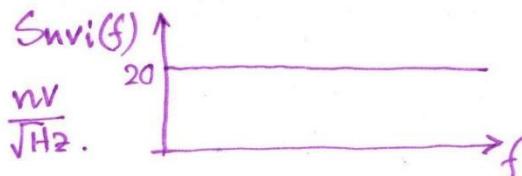


$$S_{nvop}^2(f) = \sum_{1 \text{ to } 3} |A(j2\pi f)|^2 S_{nvif_{1,2,3}}^2(f)$$

$$S_{nvop}(f) = \left[\sum_{1 \text{ to } 3} |A(j2\pi f)|^2 S_{nvif_{1,2,3}}^2(f) \right]^{1/2}$$

Example:- Consider a noise signal, $S_{ni}(f)$ that has a voltage spectral density of $20 \text{ nV}/\sqrt{\text{Hz}}$ as shown in Fig. Find the total noise rms value between dc and 100 kHz.

What is the total noise rms value if it is filtered by RC ckt as shown in Fig. Where it is assumed the RC filter is noise free.



$$A(j2\pi f) = \frac{1}{1 + j\frac{f}{f_0}}$$

$$f_0 = \frac{1}{2\pi RC} = 1 \text{ kHz}$$

$$\begin{aligned} V_{ni(\text{rms})}^2 &= \int_0^{100 \text{ kHz}} (20)^2 df \\ &= 4 \times 10^7 (\text{nV})^2 \end{aligned}$$

Ans:- The noise mean square value from 0 to 100 kHz $\Rightarrow V_{ni(\text{rms})}^2 = \int_0^{100 \text{ kHz}} (20)^2 df$
So, the root mean square value $V_{ni(\text{rms})} = \underline{6.324 \text{ nV}}$.

$$S_{no}(f) = S_{ni}(f) |A(j2\pi f)| = \frac{20 \times 10^{-9}}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}}$$

Observation:- $V_{no(\text{rms})} = \frac{1}{\sqrt{2}} V_{ni(\text{rms})}$
Since noise is also filtered above 1 kHz
• We should not use larger BW than required otherwise noise performance suffer.

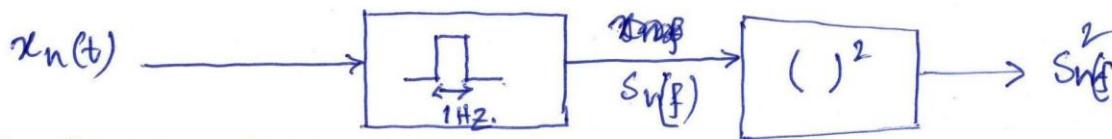
Total mean squared value of output noise $V_{no(\text{rms})}^2$ between dc to 100 kHz -

$$\begin{aligned} V_{no(\text{rms})}^2 &= \int_0^{100 \text{ kHz}} S_{no}(f)^2 df = \int_0^{100 \text{ kHz}} \frac{(20 \times 10^{-9})^2}{1 + \left(\frac{f}{f_0}\right)^2} df \\ &= (20)^2 f_0 \left[\arctan\left(\frac{f}{f_0}\right) \right]_0^{100 \text{ kHz}} (\text{nV})^2 = 400 \times 1 \text{ k} \left[\arctan(100) - \arctan(0) \right] \\ &= 400 \times 1 \text{ k} \times 1.56 (\text{nV})^2 = 6.24 \times 10^{-5} (\text{nV})^2 \end{aligned}$$

$$\text{So, } V_{no(\text{rms})} = \underline{0.79 \text{ mV(rms)}}$$

- Noise bandwidth :-

Definition of power spectral density = noise power within 1 Hz bandwidth.

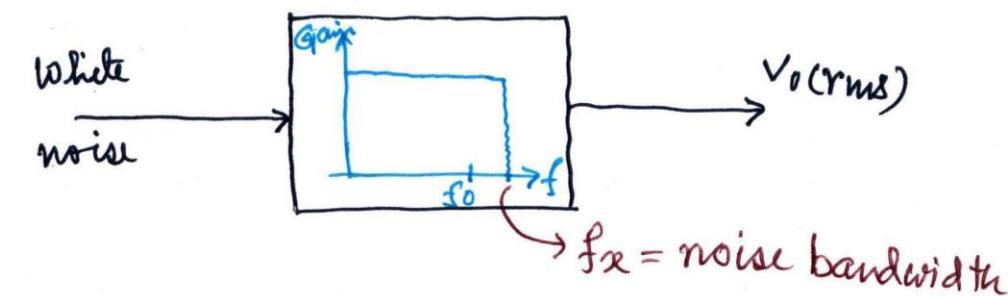
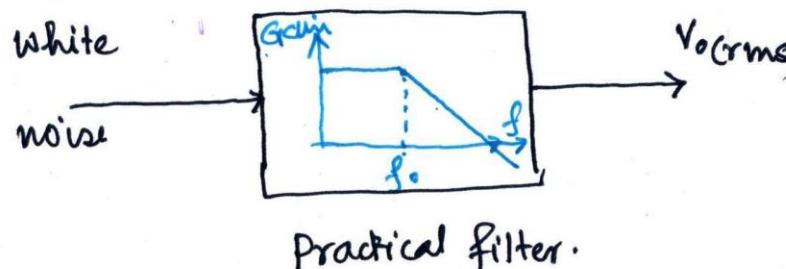


Requirement :- Brick wall band pass filter with 1 Hz passband.

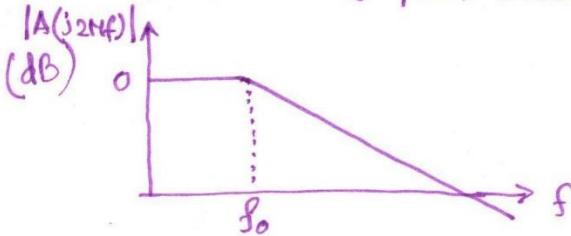
In practice: We usually use lower order filter to avoid complexity. Then more noise is passed to the output.

To account the gradual stopband of practical filter, the term "noise bandwidth" is defined.

Noise Bandwidth :- Noise bandwidth of a practical filter is equal to the frequency span of a brick wall filter that has the same rms output noise as the practical filter when white noise is applied to both filter, and they have same dc gain



Example :- Using first order RC filter



$$A(s) = \frac{1}{1 + \frac{s}{2\pi f_0}} \text{ where } f_0 = \frac{1}{2\pi RC}.$$

$$|A(j2ref)| = \left[\frac{1}{1 + \left(\frac{f}{f_0}\right)^2} \right]^{1/2}$$

Input signal, $S_{ni}(f) = S_{nw}$ white noise

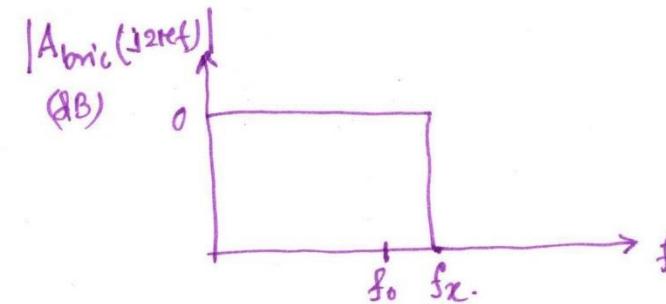
→ When passed through the RC filter

Output noise power spectrum

$$S_{nvo}(f) = |A(j2ref)|^2 S_{ni}(f)$$

Output rms noise ~~V_{no}~~ $V_{no(rms)}$

$$\begin{aligned} V_{o(rms)}^2 &= \int_0^\infty \frac{S_{nw}}{1 + \left(\frac{f}{f_0}\right)^2} df \\ &= S_{nw} \left[f_0 \arctan\left(\frac{f}{f_0}\right) \right]_0^\infty \\ &= \frac{S_{nw} \pi f_0}{2} \end{aligned}$$



Once the same input $S_{ni}(f) = S_{nw}$ is applied to a brick wall filter :-

$$\begin{aligned} V_{o,brick(rms)}^2 &= \int_0^\infty S_{nw}^2 df \\ &= S_{nw}^2 f_x \end{aligned}$$

Equating the two O/P noises -

$$V_{o(rms)} = V_{o,brick(rms)}$$

$$f_x = \frac{\pi f_0}{2}$$

Thus, the noise bandwidth of a first order low pass filter having 3dB BW equal to f_0 , equals to $\pi(f_0/2)$

$$f_0 = \frac{1}{2\pi RC}, \quad f_x = \frac{f_0}{4RC}$$

$$\text{So, } V_{o(rms)}^2 = S_{nw}^2 \frac{f_0}{4\pi RC}$$

For higher order filter, similar analysis can be done.

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Piecewise Integration of Noise :-

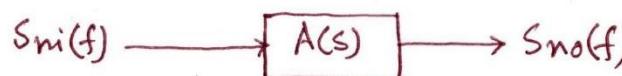
Useful when two or more sources are present in the system.

Provides an approximate value of noise at an early stage of design.

Computer simulation gives a more accurate figure at final stage.

Useful tool : Integrating piecewise-linear fn. over frequency similar to Bode diagram.

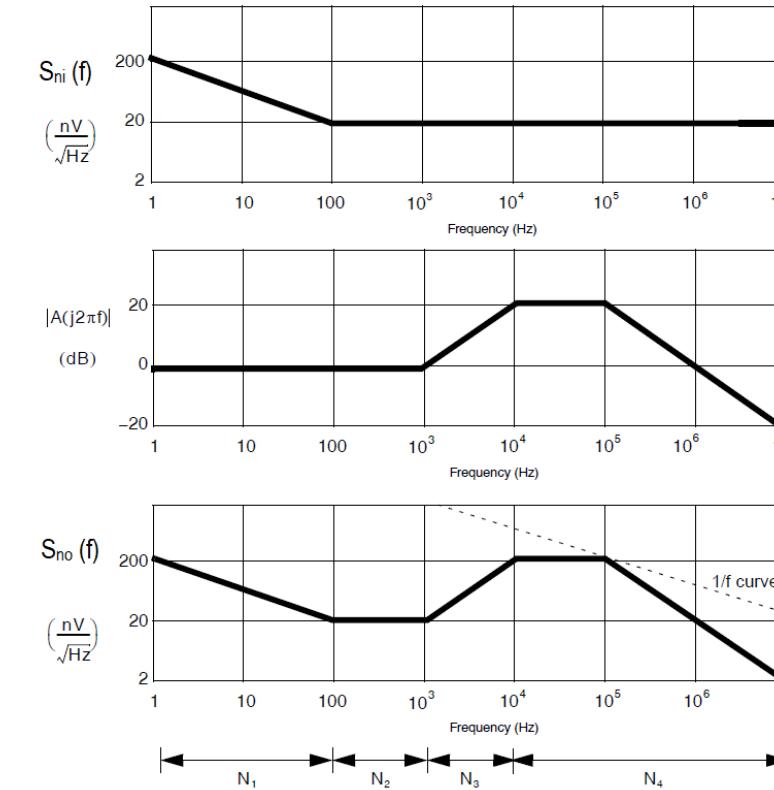
- Example: Consider an input noise signal $S_{ni}(f)$ being applied to the amplifier $A(s)$ as shown in Fig. Find the output rms noise of $V_{no(rms)}$ considering only frequencies above 1 Hz.



$$\text{As } S_{no}(f) = |A(j2\pi f)| S_{ni}(f)$$

In dB scale (Bode diagram) $S_{ni}(f)$ and $|A(j2\pi f)|$ are additive.

$$\text{For } N_1 \text{ region: } V_{N1}^2(\text{rms}) = \int_1^{100} \left(\frac{200}{\sqrt{f}} \right)^2 df = (200)^2 \left[\ln(f) \right]_1^{100} = 1.84 \times 10^5 (\text{mV})^2$$



$$\begin{aligned} \text{For } N_2 \text{ region: } V_{N2}^2(\text{rms}) &= \int_{100}^{1K} (20)^2 df = (20)^2 \left[f \right]_{100}^{1K} \\ &= 3.6 \times 10^5 (\text{mV})^2 \\ \text{For } N_3 \text{ region: } V_{N3}^2(\text{rms}) &= \int_{1K}^{10K} \left(\frac{20f}{1K} \right)^2 df = \left(\frac{20}{1K} \right)^2 \left[\frac{1}{3} f^3 \right]_{1K}^{10K} \\ &= 1.33 \times 10^8 (\text{mV})^2 \\ \text{For } N_4 \text{ region: } V_{N4}^2(\text{rms}) &= \int_{10K}^{\infty} \frac{(200)^2}{1 + \left(\frac{f}{f_0} \right)^2} df = \int_0^{\infty} \frac{(200)^2}{1 + \left(\frac{f}{10^5} \right)^2} df - \int_0^{10K} \frac{(200)^2}{1 + \left(\frac{f}{10^5} \right)^2} df \\ &= (200)^2 \frac{\pi}{2} 10^5 - (200)^2 10^4 = 5.88 \times 10^9 (\text{mV})^2 \\ V_{total}^2(\text{rms}) &= [V_{N1}^2(\text{rms}) + \dots + V_{N4}^2(\text{rms})]^{1/2} = 77.5 \text{ mV} \end{aligned}$$

$1/f$ noise tangent principle:

In the previous example: RMS noise in N_4 region $V_{NA}(\text{rms}) = \sqrt{5.88 \times 10^9} \text{ mV}$
 $= 7.67 \mu\text{V} (\text{rms})$

* $1/f$ noise tangent principle explain this.

If we lower a $1/f$ noise line until it touches the spectral density curve - the total noise can be approximated by the noise in the vicinity of $1/f$ line.

Few properties of $1/f$ curve: $S_{Nv}(f) = K\nu^2/f$ → power spectral density.

$S_{Nv}(f) = K\nu/\sqrt{f}$ → spectral density.

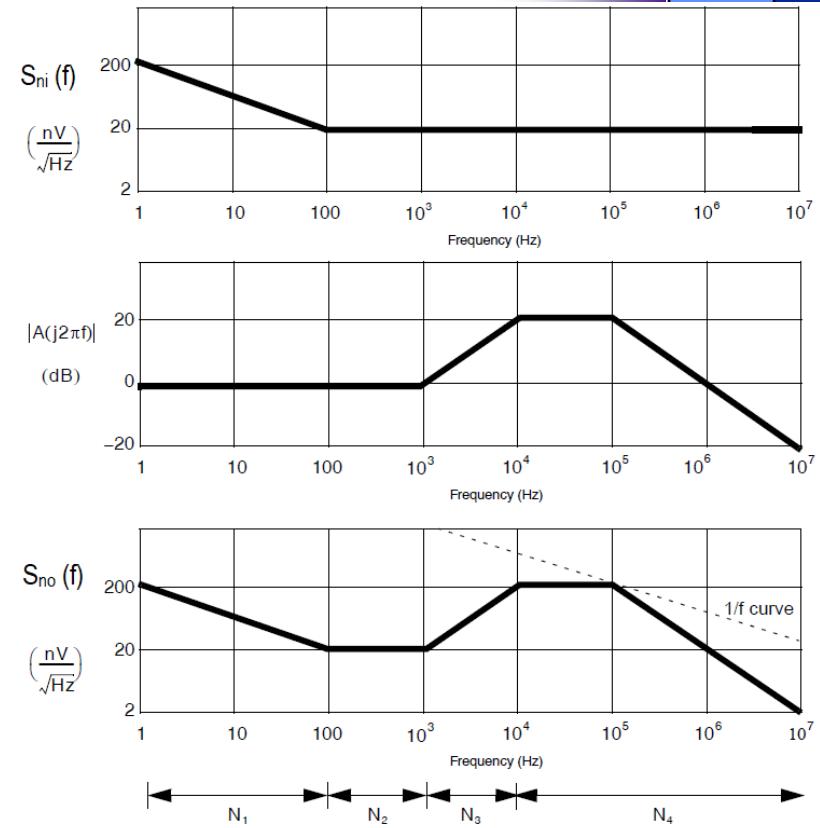
Power spectral density has a slope -1 dec/dec .

Spectral density has a slope -0.5 dec/dec .

Mean square noise power. $= K\nu^2 \int_{f_L}^{f_H} \frac{1}{f} df = K\nu^2 \ln \left[\frac{f_H}{f_L} \right]$

$1/f$ noise has same power content in each frequency decade. \Rightarrow constant power/freq

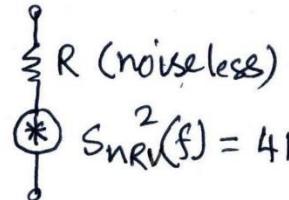
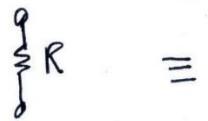
Therefore, lowering this constant power/frequency curve, the largest power contribution will touch first.



Noise models for circuit elements :-

1. Resistors :- Major source \rightarrow thermal noise \rightarrow power spectral density $S_{nRv}^2(f) = 4kTR$
where k = Boltzmann constant $= 1.38 \times 10^{-23}$ J/K, T = absolute temp in K, R = Resistance

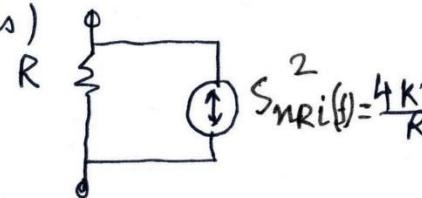
Noise models :-



(noisless)

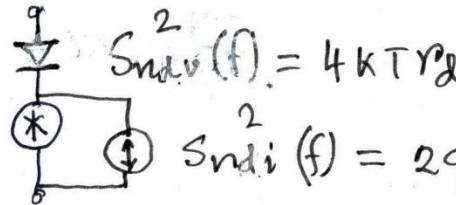
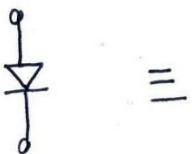
$$S_{nRv}^2(f) = 4kTR$$

=



$$S_{nRi}^2(f) = \frac{4kT}{R}$$

2. Diodes :- Major noise source \rightarrow Shot noise



$$S_{nqv}^2(f) = 4kT\gamma_d$$

$$S_{ndi}^2(f) = 2qI_D$$

$$\text{Thermal power spectral density } S_{nqv}^2(f) = 4kT\gamma_d$$

γ_d = diode on resistance

Shot noise current spectral density $S_{ndi}^2(f)$

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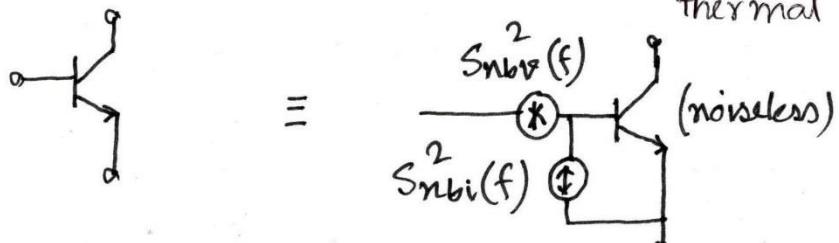


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3. BJT :- Main noise sources \rightarrow shot noise of both the collector and base currents.

flicker noise of the base current

thermal noise of the base resistance.



$$S_{nbb}(f) = 4kT \left(r_b + \frac{1}{2g_m} \right)$$

where r_b = base resistance

$$g_m = \frac{qI_c}{kT}$$

$$S_{nbi}(f) = \underbrace{2qI_B}_{\text{Shot noise from base current}} + \underbrace{\frac{2qkI_B}{f}}_{\text{Flicker noise from base current}} + \underbrace{\frac{2qI_c}{|I_B(f)|^2}}_{\text{Input referred collector current shot noise (very small)}}$$

4. MOSFET :-

Main noise sources \rightarrow flicker and thermal.

$$\text{Flicker noise } S_{nmv}(f) = \frac{K}{WL C_{ox} f}, \text{ where } K = \text{device parameter}$$

* y_f noise is inversely proportional to the transistor area.
Larger the device, less y_f noise.

* Extremely important as it dominates at low freq.

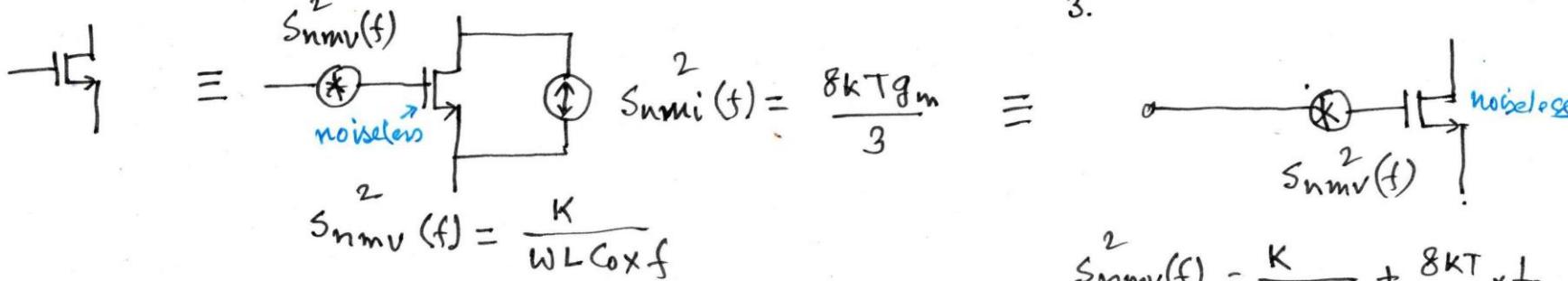
* K is smaller for PMOS, preferred for reducing y_f noise.

$$\text{Thermal noise : In triode region } \rightarrow S_{nmo}(f) = 4kT r_{ds}, S_{nmf}(f) = 4kT/r_{ds}$$

$$\text{In saturation region } \rightarrow S_{nmo}(f) = 4kT \gamma \frac{1}{g_m}$$

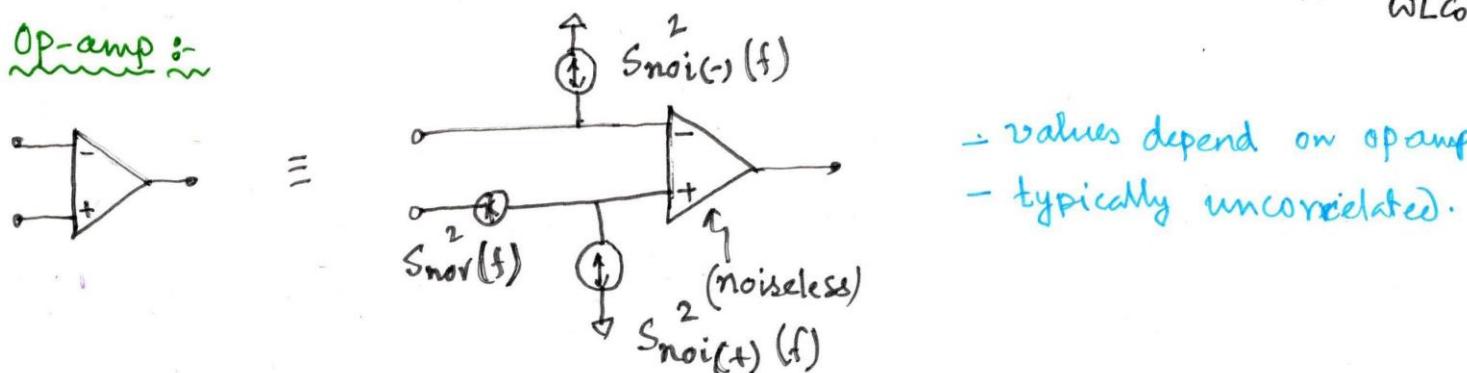
In saturation region $\rightarrow S_{nmv}^2(f) = 4kT \cdot \frac{1}{g_m} = \frac{8kT}{3} \cdot \frac{1}{g_m}$.
 $\Rightarrow 2/3$ as channel is not homogeneous.

Current spectral density $S_{nmi}^2(f) = \frac{8kT g_m}{3}$



$$S_{nmv}^2(f) = \frac{K}{WLCo \cdot f} + \frac{8kT}{3} \cdot \frac{1}{g_m}$$

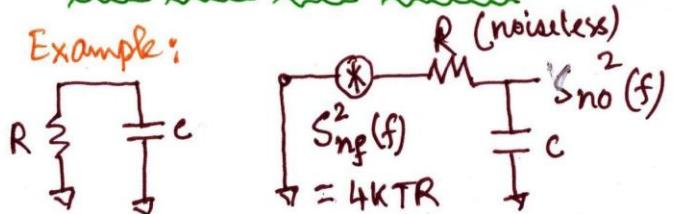
5. Op-amp :-



- values depend on opamp
- typically uncorrelated.

6. Capacitors and inductors:- Do NOT generate a noise, however, they do accumulate noise

Example:



$$V_{no(rms)}^2 = S_{nf}^2(f) \left(\frac{R}{2}\right) f_0$$

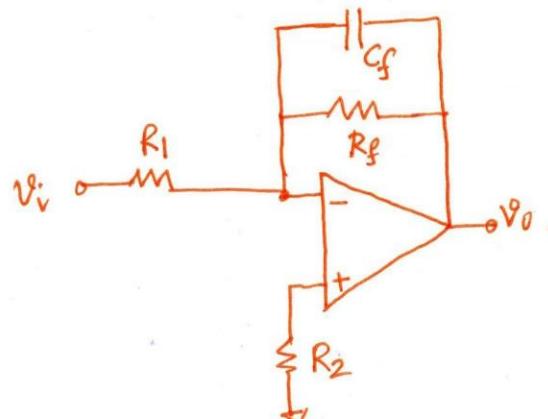
$$= 4kTR \left(\frac{1}{2}\right) \frac{1}{2\pi RC}$$

$$S_{no}^2(f) = S_{nf}^2(f) |H(j\omega_f)|^2$$

$$V_{no(rms)}^2 = \int_0^\infty S_{no}^2(f) df$$

$$= \int_0^\infty S_{nf}^2(f) \left[\frac{1}{1 + (\omega/\omega_p)^2} \right] df$$

Op-amp noise analysis



A low pass filter

Assumption: all noise sources are uncorrelated.

First step:- what is output noise $S_{\text{out}1}(f)$ due to noise sources at inverting

Three noise sources are added together = $S_{\text{nr}i}(f)^2 + S_{\text{no}i}(f)^2 + S_{\text{nf}i}(f)^2$ path²

$$\text{If passes through } \left(R_f \parallel \frac{1}{sC_f} \right) = \frac{R_f}{1 + sR_f C_f}$$

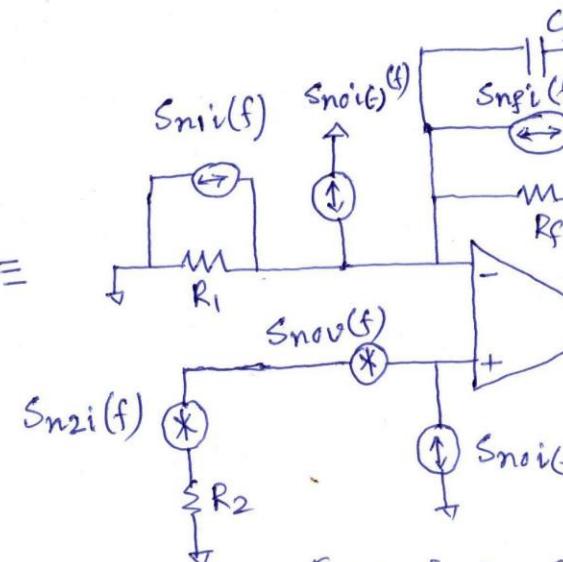
$$\therefore S_{\text{out}1}(f)^2 = \left[S_{\text{nr}i}(f)^2 + S_{\text{no}i}(f)^2 + S_{\text{nf}i}(f)^2 \right] \left| \frac{R_f}{1 + j2\pi f R_f C_f} \right|^2 \quad \begin{matrix} \text{3dB freq} \\ f_0 = \frac{1}{2\pi R_f C_f} \end{matrix}$$

Second step:- What is the output noise $S_{\text{out}2}(f)$ due to the noise sources at non-inverting path?

$$S_{\text{out}2}(f)^2 = \left[S_{\text{no}i(+)}(f)^2 R_2 + S_{\text{nr}i}(f)^2 + S_{\text{no}i}(f)^2 \right] \left| 1 + \frac{R_f/R_1}{j2\pi f R_f C_f} \right|^2$$

Case 1:- If $R_f \ll R_1$, then gain is close to 1.

Case 2:- If $R_f \gg R_1$, then low freq gain $\approx R_f/R_1$, 3-dB freq. $f_0 = \frac{1}{2\pi R_f C_f}$
Gain decreases above f_0 and reaches to unity at $f_1 = (R_f/R_1) f_0$



- Opamp inherent noises:

$S_{\text{no}v}(f), S_{\text{no}i(-)}(f), S_{\text{no}i(+)}(f)$

- Component noises:

Thermal noise of resistor.

$S_{\text{noise}}(f)$

- for R_1 and R_f , we use current noise source

- for R_2 , we use voltage noise source.

Equivalent noise model.

One should include op-amp's positive input noise integrated over the region between f_1 and unity gain freq of the opamp.

So, total output noise mean squared value -

$$S_{\text{out}}^2(f) = S_{\text{out}_1}^2(f) + S_{\text{out}_2}^2(f)$$

Example: Estimate the total noise rms value for a 10 kHz low pass filter as previous figure. When $C_f = 160 \text{ pF}$, $R_f = 10^8 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 9.1 \text{ k}\Omega$. Assume that the noise voltage of the op-amp is given as $S_{\text{no}v}(f) = 20 \text{ nV}/\sqrt{\text{Hz}}$, ~~$S_{\text{no}i(-)}(f) = S_{\text{no}i(+)}(f) = 0.6 \text{ pA}/\sqrt{\text{Hz}}$~~ and the unity gain frequency equals to 5 MHz.

Assume that the system is in room temperature :-

$$S_{\text{no}i}(f) = \sqrt{\frac{4kT}{R_2}} = \sqrt{\frac{4 \times 1.38 \times 10^{-23} \times 300}{9.1 \text{ k}}} = 12.2 \text{ nV}/\sqrt{\text{Hz}}$$

$$S_{\text{ni}}(f) = \sqrt{\frac{4kT}{R_1}} = \sqrt{\frac{4 \times 1.38 \times 10^{-23} \times 300}{10 \text{ k}}} = 1.28 \text{ pA}/\sqrt{\text{Hz}}$$

$$S_{\text{nf}i}(f) = \sqrt{\frac{4kT}{R_f}} = \sqrt{\frac{4 \times 1.38 \times 10^{-23} \times 300}{10^8 \text{ k}}} = 0.406 \text{ pA}/\sqrt{\text{Hz}}$$

The low frequency value of $S_{\text{no}u1}(f)$ is found by $f=0$

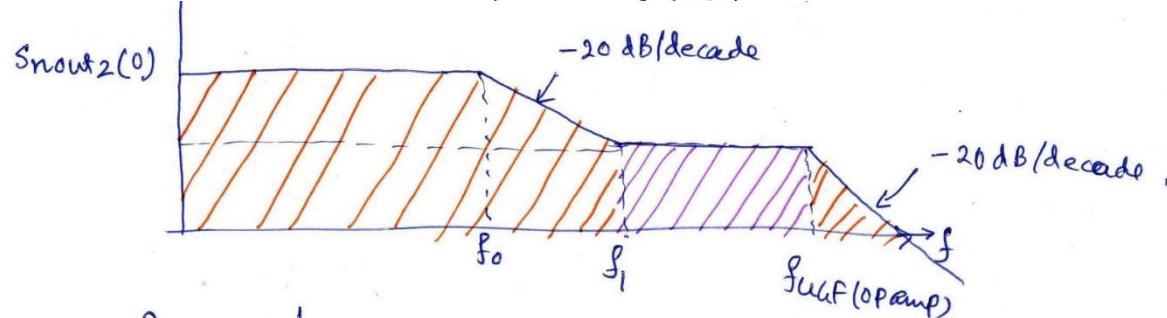
$$\begin{aligned} S_{\text{no}u1}^2(0) &= [S_{\text{ni}}^2(0) + S_{\text{no}i}^2(0) + S_{\text{nf}i}^2(0)] R_f^2 \\ &= [(0.406)^2 + (1.28)^2 + (0.6)^2] (1 \times 10^{-12})^2 (10^8 \text{ k})^2 \\ &= (147 \text{ nV}/\sqrt{\text{Hz}})^2 \end{aligned}$$

$$\text{Noise bandwidth } f_x = \frac{f_0}{2} = \frac{10 \text{ kHz}}{2} = \frac{1}{2 \pi R_f C_f} = \frac{1}{4 R_f C_f}$$

$$\text{So, } V_{\text{no}u1(\text{rms})}^2 = \int_0^{f_x} (147 \text{ nV}/\sqrt{\text{Hz}})^2 \cdot df = (147 \text{ nV}/\sqrt{\text{Hz}})^2 \cdot \frac{1}{4 R_f C_f} = (18.4 \mu\text{V})^2$$

Output noise due to the sources at the positive terminal :-

$$\begin{aligned} S_{\text{nout2}}(0) &= [S_{\text{noi}(+)}^2 R_2 + S_{\text{n2i}}^2(f) + S_{\text{nov}}^2(f)] \left[1 + \frac{R_f}{R_i}\right]^2 \\ &= (24.1 \text{ nV}/\sqrt{\text{Hz}})^2 \times 11^2 \quad \text{as } \frac{R_f}{R_i} = 10 \\ &= (265 \text{ nV}/\sqrt{\text{Hz}})^2 \end{aligned}$$



$$f_0 = \frac{1}{2\pi R_f C_f} = \frac{1}{2\pi \times 100K \times 160 pF} = 9.95 \text{ kHz.}$$

$$f_x = \frac{1}{4 R_f C_f} = \frac{1}{4 \times 100K \times 160 pF} = 15.625 \text{ kHz.}$$

$$f_1 = \left(\frac{R_f}{R_i}\right) f_0 = \frac{100K}{10K} \times f_0 = 10 f_0 = 99.5 \text{ kHz.}$$

$$f_{\text{uaf}} = 5 \text{ MHz}$$

$$\begin{aligned} \text{Output noise } V_{\text{nout2}}^2(\text{rms}) &= \int_0^{f_x} (265 \text{ nV}/\sqrt{\text{Hz}})^2 df + (24.1 \text{ nV}/\sqrt{\text{Hz}})^2 \int_{f_1}^{f_{\text{uaf}}} df. \\ &= (265 \text{ nV}/\sqrt{\text{Hz}})^2 \times \frac{1}{4 R_f C_f} + (24.1 \text{ nV}/\sqrt{\text{Hz}})^2 (f_{\text{uaf}} - f_1) \\ &= (83 \mu\text{V})^2 + (53.3 \mu\text{V})^2 \\ &= (86 \mu\text{V})^2 \end{aligned}$$

$$\text{Total output noise } V_{\text{no}}(\text{rms}) = \sqrt{(18.4 \mu\text{V})^2 + (86 \mu\text{V})^2} = 87.9 \mu\text{V. (rms)}$$

EE60032: Analog Signal Processing



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Summary of the course

Module-1: Signal processing using operational amplifier

Class-2: Operational amplifier and various DC non-idealities [1] (input bias current, input offset current, input offset voltage, total output offset voltage, thermal drift, PSRR)

Class-3: AC performance of the op-amp, concept of stability, dominant pole compensation [1].

Class-4: Pole-zero compensation, internal compensation, small signal response [1].

Class-5: Slew rate limitation of the op-amp with tutorials problems [1].

Class-6: Various applications of op-amp in signal processing: Summing amplifier, difference amplifier, instrumentation amplifier, adder/subtractor, integrator [1].

Class-7: Differentiator, phase-lag circuit, phase-lead circuit, logarithmic amplifier [1].

Class-8: Antilogarithmic amplifier, applications of log and anti-log amplifiers, half wave and full wave precision rectifier, peak detector, voltage clamper circuit [1].

Class-9: Positive clipper and negative clipper. Tutorial problems. [1].

Summary of the course

Module-2: Analog and switched capacitor Filters

Class-10: Introduction of Filters, classifications, generalized transfer function of filter [5].

Class-11: First order filters with passive and active realization [5, 6].

Class-12: Second order LCR filter implementation: low pass filter, high pass filter and band pass filter. [5].

Class-13: Second order LCR filter implementation: band reject filter, all pass filter. [5].

Class-14: General impedance converter (GIC), second order active filter realization [5]

Class-15: KRC/Sallen-Key filter and implementation of low pass filter [5]

Class-16: Implementation of KRC/Sallen-Key high pass and band pass filters, sensitivity analysis of KRC/Sallen-Key filter [5]

Class-17: KHN/State variable filter and its sensitivity analysis [5]

Class-18: Butterworth approximation functions and tutorial problems [6]

Class-19: Chebyshev approximation functions and tutorial problems [6]

Class-20: Tutorial problems

Class-21: Switched capacitor filter: basic operation and its different elements [2]

Class-22: Introduction of z-transform, switched capacitor integrator and its parasitic effects, development of parasitic insensitive delayed integrators [2]

Class-23: Delay-free integrators, signal-flow graph analysis [2]

Class-24: First order filter [2]

Summary of the course

Module-3: Data converters

Class-25: Introduction, Static Characteristics of DAC [7]

Class-26: Dynamic characteristics of the DAC, Different architectures of DAC, Current scaling parallel DAC [7]

Class-27: Voltage scaling DAC, Charge scaling DAC, Serial DAC [7]

Class-28: Expanding the resolution of parallel DAC [7]

Class-29: Aliasing error [7], Sample and Hold Circuits without feedback and with feedback [2]

Class-30: Static and dynamic Characteristics of ADC [7]

Class-31: Single slope ADC, Dual slope ADC, Successive Approximation ADCs [7]

Class-32: Flash ADCs, Over sampling method for A/D and D/A conversion, Delta-Sigma converter: First order [7]

Class-33: Delta-Sigma modulator: Second and higher orders [7]

Summary of the course

Module-4: Phase locked loop and Oscillators

- Class-34: Phase locked loop architecture, phase detector, Locking process and steady-state behavior, static phase error [8]
- Class-35: Dynamic model of type-I PLL and understanding various design trade-offs, Capture range and lock-in range of PLL [8]
- Class-36: Basic oscillator theory, Wein bridge oscillator, ring oscillator and voltage controlled oscillator

Module-5: Noise

- Class-37: External/interference noise, Internal/inherent noise, noise properties [3]
- Class-38: Noise spectrum, noise summation, different sources of noise such as thermal noise, shot noise, flicker noise [3]
- Class-39: Noise filtering, Noise bandwidth [2]
- Class-40: Peicewise integration of noise, 1/f noise tangent principle [2]
- Class-41: Noise models in circuit elements (resistor, diode, BJT, MOSFET, op-amp, capacitor and inductor), Noise analysis of an active low pass filter. [2]
- Class-42: Summary

Final Systematic Evaluation Criteria

1. Try yourself: 30%
 - ✓ Meet the strict deadlines: Every Saturday by 11:59 PM.
 - ✓ Those who have internet connectivity issue, plan to submit early.
2. Class Tests on Each Module (Class Test-1, 2, 3): 30%
 - ✓ After completion of each module, one class test will be conducted.
3. Class Test on Whole Syllabus (Class Test-4) : 40%
 - ✓ At the end, one class test on whole syllabus will be conducted.

- Please provide your feedback on the course on ERP!
- Do NOT share the lecture slides or videos over internet!
- Thank you for attending the course!