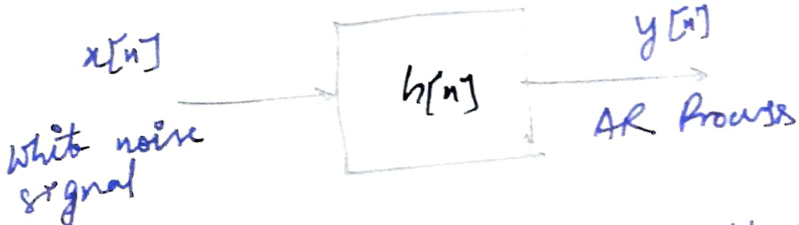
Assignment-4

Submitted By:
18EE35014

Levinson Durbin Algorithm to calculate LPC:

LD algorithm is used most commonly to estimate the all-pole AR model parameters, because the design equations used to obtain the best-fit AR model are simpler than those used for MA or ARMA modelling.



Consider a general all-pole filter (order P) : $H(z) = \frac{Y(z)}{X(z)}$

$$\frac{Y(z)}{X(z)} = \frac{G}{1 - \sum_{k=1}^{P} \alpha_k^{(P)} z^{-k}}$$

applying inverse z transform, we get

$$y[n] = Gx[n] + \sum_{k=1}^{P} \alpha_k^{(P)} y[n-k]$$

Now we want to obtain a filter T.F to an arbitrary desired filter T.F $H_d(z)$. This is done by minimizing the average square error between magnitudes of frequency response of desired filter $H_d(e^{j\omega})$ and all pole filter $H(e^{j\omega})$

$$e^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left| H(e^{j\omega}) - H_d(e^{j\omega}) \right|^2 d\omega$$

applying parseval's theorem.

$$e^2 = \sum_{n=0}^{N-1} \left(h[n] - h_d[n] \right)^2$$

Since $h[n]$ is impulse response : system response to $\delta[n]$.

$$h[n] = G \cdot \delta[n] + \sum_{k=1}^{P} \alpha_k^{(P)} h[n-k]$$

$$\Rightarrow e^2 = \sum_{n=0}^{N-1} \left(G\delta[n] + \sum_{k=1}^{P} \alpha_k^{(P)} h[n-k] - h_d[n] \right)^2$$

for error to be minimum,

$$\frac{de^2}{da_k} = 0$$

$$\sum_{n=0}^{N-1} \left(G\delta[n] \cdot h[n-k] + \sum_{l=1}^{P} \alpha_l^{P} h[n-l]\, h[n-k] \right) = \sum_{n=0}^{N-1} h_d[n]\, h[n-k]$$

Since system is casual, G won't enter the solution.

$$\sum_{l=1}^{P} \alpha_l^{(P)} \cdot \frac{\left(\sum_{n=0}^{N-1} h[n-l]\, h[n-k] \right)}{N} = \frac{\sum_{n=0}^{N-1} h_d[n] \cdot h[n-k]}{N}$$

let $\phi_{yy}[m] = \frac{1}{N} \sum_{n=0}^{N-1} h[n] \cdot h[n-m] = \phi[m]$

$$\boxed{\sum_{l=1}^{p} \alpha_l^{(p)} \phi[k-l] = \phi[k]}$$

for $k = 1, 2, \ldots p$

p set of linear equations

$$\boxed{R\alpha = P}$$

where $R = \begin{bmatrix} \phi[0] & \phi[1] & \ldots & \phi[p-1] \\ \phi[1] & \phi[0] & \ldots & \phi[p-2] \\ \vdots & \vdots & & \vdots \\ \phi[p-1] & \phi[p-2] & \ldots & \phi[0] \end{bmatrix}$

$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_p \end{bmatrix}$, $P = \begin{bmatrix} \phi[1] \\ \phi[2] \\ \vdots \\ \phi[p] \end{bmatrix}$

* a direct solution is given by [complexity $O(N^3)$]

$\alpha = R^{-1}P$ but it is tough to compute inverse for the large number. So instead we find solution is a recursive method to reduce the complexity.

The basic idea of the recursion is to find the solution α_{p+1} for the $(p+1)^{th}$ order case from the solution α_p for the p^{th} order case.

$$R_{p+1} = \left[\begin{array}{c|c} R_p & \begin{matrix}\phi[p] \\ \phi[p-1] \\ \vdots \\ \phi[1]\end{matrix} \\ \hline \phi[p]\ \phi[p-1] \dots \phi[1] & \phi[0] \end{array}\right]$$

let $p = q$

$$q_{p+1} = \left[\begin{array}{c} q_p \\ \vdots \\ \hline \phi[p+1] \end{array}\right] \quad \text{and} \quad \rho_p = \begin{bmatrix} \phi[p] \\ \phi[p-1] \\ \vdots \\ \phi[1] \end{bmatrix}$$

$$R_{p+1} = \left[\begin{array}{c|c} R_p & \rho_p \\ \hline \rho_p^T & \phi[0] \end{array}\right]$$

$$\alpha_{p+1} = \left[\begin{array}{c} \gamma_p \\ \hline \alpha_{p+1} \end{array}\right] = \left[\begin{array}{c} \alpha_p \\ \hline 0 \end{array}\right] + \left[\begin{array}{c} \varepsilon_p \\ \hline k_{p+1} \end{array}\right]$$

where ε_p is correction term

k_{p+1} is new α_{p+1} → reflection coefficients

$\therefore \quad R_{p+1}\alpha_{p+1} = q_{p+1}$

$$\begin{bmatrix} R_p & \rho_p \\ \rho_p^T & \phi[0] \end{bmatrix} \left(\begin{bmatrix} \alpha_p \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_p \\ k_{p+1} \end{bmatrix} \right) = \begin{bmatrix} q_p \\ \phi[p+1] \end{bmatrix}$$

$$R_p\alpha_p + R_p\varepsilon_p + \rho_p k_{p+1} = q_p$$

and $\quad \rho_p^T\alpha_p + \rho_p^T\varepsilon_p + \phi[0]\, k_{p+1} = \phi[p+1]$

On simplifying and approximately,

$$k_{p+1} \doteq \frac{-\phi[p+1] \cancel{+} - \cancel{*}\rho_p^T\alpha_p}{E_p}$$

where $E_p = (1-k_p)^2 E_{p-1}$

$E_0 = \phi[0]$

$\therefore$ Recursion algorithm is as follows:

$$E_0 = \phi[0]$$

$$k_i = \frac{-\phi[i] - \sum_{j=1}^{i-1} \alpha_j^{(i-1)}\phi[i-j]}{E_{i-1}} \qquad \text{for } 1 \le i \le p$$

$$\alpha_i^{(i)} = k_i$$

$$\alpha_j^{(i)} = \alpha_j^{(i-1)} + k_i\alpha_{i-j}^{(i-1)} \qquad \text{for } j = 1, 2, \ldots, i-1$$

$$E_i = (1-k_i^2)\, E_{i-1}$$

after P steps, we arrive at pth order estimate

here

$$e_p = \left[\prod_{j=1}^{p} (1-k_j^2)\right] E_0 = \left[\prod_{j=1}^{p} (1-k_j^2)\right] \phi[0]$$

gives the estimate of variance of $x[n]$

Levinson

Reflection coefficients $\phi[0], k_1, k_2, \ldots k_p$ $\xleftrightarrow{\text{interconversion}}$ AR Parameters $a[1], a[2], \ldots a[p], \sigma^2$

Step down

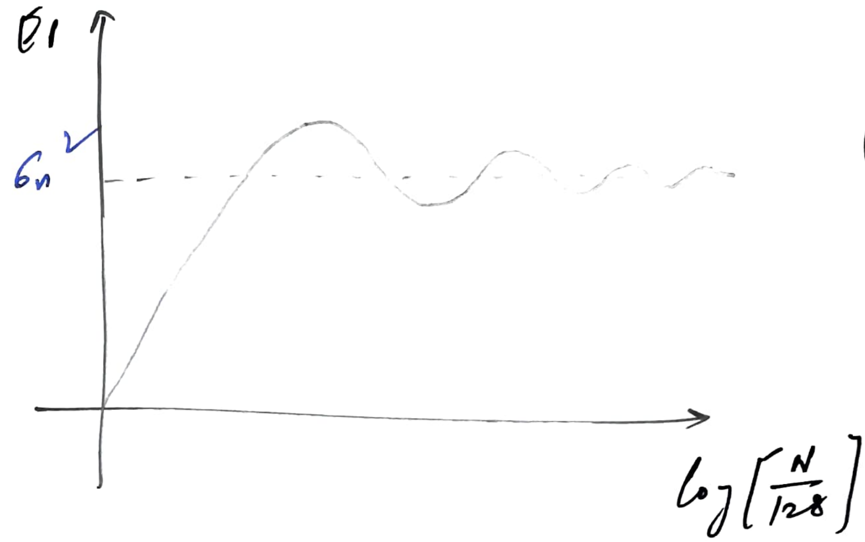
<u>Note:</u>

→ for all pole AR process generator to be stable, the poles must all lie inside unit circle in the z-plane.

→ Time complexity of LD algorithm is $O(N^2)$
  Space " " " " is $O(N)$.

Results

→ for lower length, the estimation is too weak; but as length increases the estimation is becoming more perfect.



Comparision of LD with other algorithms:

→ LD v/s Cholesky decomposition:

* The cholesky decomposition is a method used to find inverse of matrix which has Hermitian Symmetry.

	Computational Complexity	Space
LD	$O(N^2)$	$O(N)$
Cholesky	$O(N^3)$	$O(N^2)$

* LD, exploits the fact that LPC analysis has Toeplitz Symmetry.

→ LD v/s LMS algorithm

* LMS algo is an adaptive filter techniques, But it does not gurantee minimum phase systems & stability while LD does. Although LMS is more elegant and accurate method of prediction, it is considerably slower than LD algorithm.

```
% Levinson Durbin Recursive algorithm
% LD algorithm is used for linear prediction filter coefficients
clc;
clear;
%Initialization of global parameters
M=3;    %Order of filter used
maxLen = 20; %Maximum length of white noise signal
variance =1 ; % variance of white noise signal

%Poles of the filter
%All poles are chosen to be within unit circle for STABILITY
p1=0.4;
p2=0.5;
p3=0.6;

%Finding filter coefficients as per given poles
a1 = -(p1+p2+p3);
a2 = (p1*p2+p2*p3+p3*p1);
a3 = -p1*p2*p3;

% AR parameters found b designed LD algorithm
alpha = zeros(maxLen,M+1);
% AR parameters found by Matlab inbuilt functions
alpha1 = zeros(maxLen,M+1);
% Reflection coefficients in designed LD algorithm
k = zeros(maxLen,M);
% Reflection coefficients in Matlab inbuilt functions
k1 = zeros(maxLen,M);
% Variance estimate in designed LD algorithm
err = zeros(maxLen,1);
% Variance estimate in Matlab inbuilt function
err1 = zeros(maxLen,1);
% Loop running across all possible lengths
for n=1:1:maxLen
    N = 128*2^(n-1); %length of the signal
    v = wgn(N,1,10*log10(variance)); %Noise signal with given specs
    %var(v)
    u1 = zeros(1,N+3)';
    for i=1:1:N
        %Code for AR process generatio
        u1(i+3) = -a1*u1(i+2)-a2*u1(i+1)-a3*u1(i)+v(i);
    end
    u=u1(4:N+3);
    Rx = zeros(M+1,1); %Autocorrelation Sequence
    for i=1:M+1
        for j=i+1:N+1
            Rx(i) = Rx(i) + u(j-1)*u(j-i);
        end
        Rx(i) = Rx(i)/(N-i+1);
    end
    temp = zeros(M,M); %Temporary filter coefficients variable
    E = zeros(M+1,1); %Temporary variance collecting variable
    total=0;
```

```
    %i=0 : zero level iteration
    E(1) = Rx(1); %Initializing estimates
    k(n,1) = -Rx(2)/Rx(1); %First reflection coefficient
    temp(1,1) = k(n,1);
    E(2) = (1-(k(n,1))^2)*E(1); %First level variance estimate

    %Iteration or LD Recursion
    for i=2:1:M
        total=0;
        for j=1:1:i-1
            %dot product of autocorrelation sequence and filter
            %coefficients
            total = total + temp(i-1,j)*Rx(i+1-j);
        end
        k(n,i) = -(Rx(i+1)+total)/E(i); %Finding new reflection coefficients
        temp(i,i) = k(n,i); %Finding new filter coefficients
        for j=1:1:i-1
            %Updating lower order filter coefficients
            temp(i,j) = (temp(i-1,j)+k(n,i)*temp(i-1,i-j));
        end
        E(i+1) = (1-((k(n,i))^2))*E(i); %Updating variance estimates
    end

    alpha(n,:) = [1,temp(3,:)]; % final filter coefficients solution
    err(n) = E(M+1); %Final variance estimates
    %Using Matlab inbuilt Levinson Durbin Function
    [alpha1(n,:),err1(n),k1(n,:)] = levinson(Rx,3);
end
%Plotting the Results
figure
plot(1:1:maxLen, err); xlabel('20*log2(N/64)');
ylabel('variance estimate ');
```