ASSIGNMENT - 1

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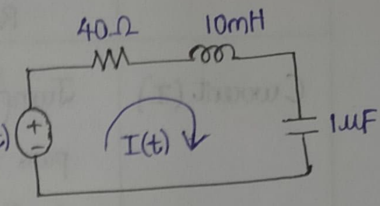
① (i) $V(s) = R\,I(s) + sL\,I(s) + \frac{1}{sC} I(s)$

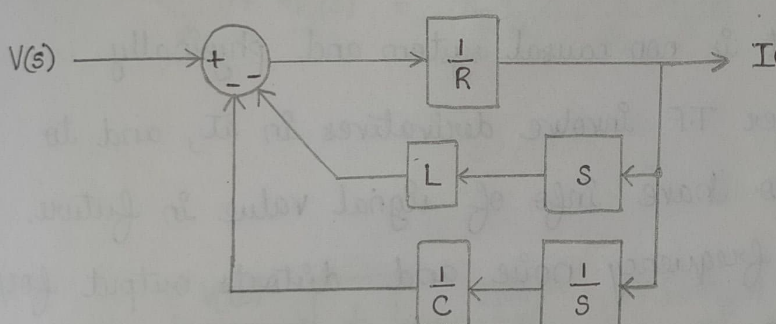


$$\frac{I(s)}{V(s)} = \frac{1}{R + sL + \frac{1}{sC}} = \frac{sC}{s^2LC + sRC + 1}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.2 \,,\quad \omega_0 = \frac{1}{\sqrt{LC}} = 10^4 \text{ rad/sec} \Rightarrow \text{Settling time} \approx \frac{4}{\xi\omega_0} \approx \boxed{2\text{ms}}$$

using 'sL' as TF :

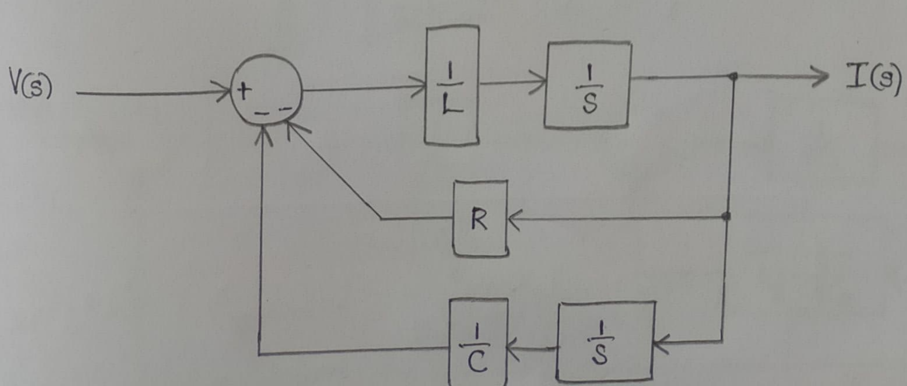
$$\frac{I(s)}{V(s)} = \frac{1}{R\left(1 + \frac{1}{R}\left(sL + \frac{1}{sC}\right)\right)} = \frac{\frac{1}{R}}{1 + \frac{1}{R}\left(sL + \frac{1}{sC}\right)}$$



(ii)

using "$\frac{1}{sL}$" as TF :

$$\frac{I(s)}{V(s)} = \frac{1}{sL\left(1 + \frac{1}{sL}\left(R + \frac{1}{sC}\right)\right)} = \frac{\frac{1}{sL}}{1 + \frac{1}{sL}\left(R + \frac{1}{sC}\right)}$$



Assignment 1 :

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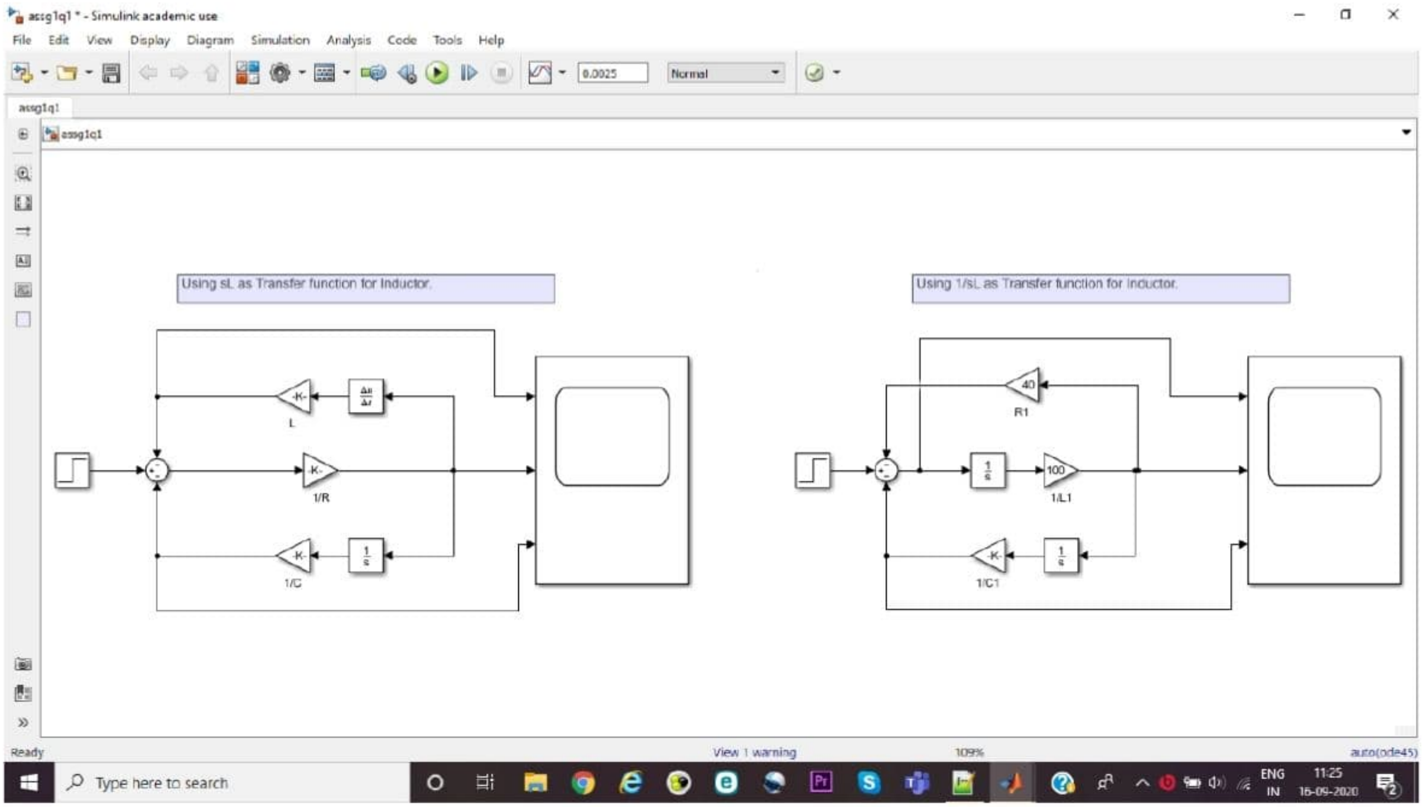
Block Diagram representation:



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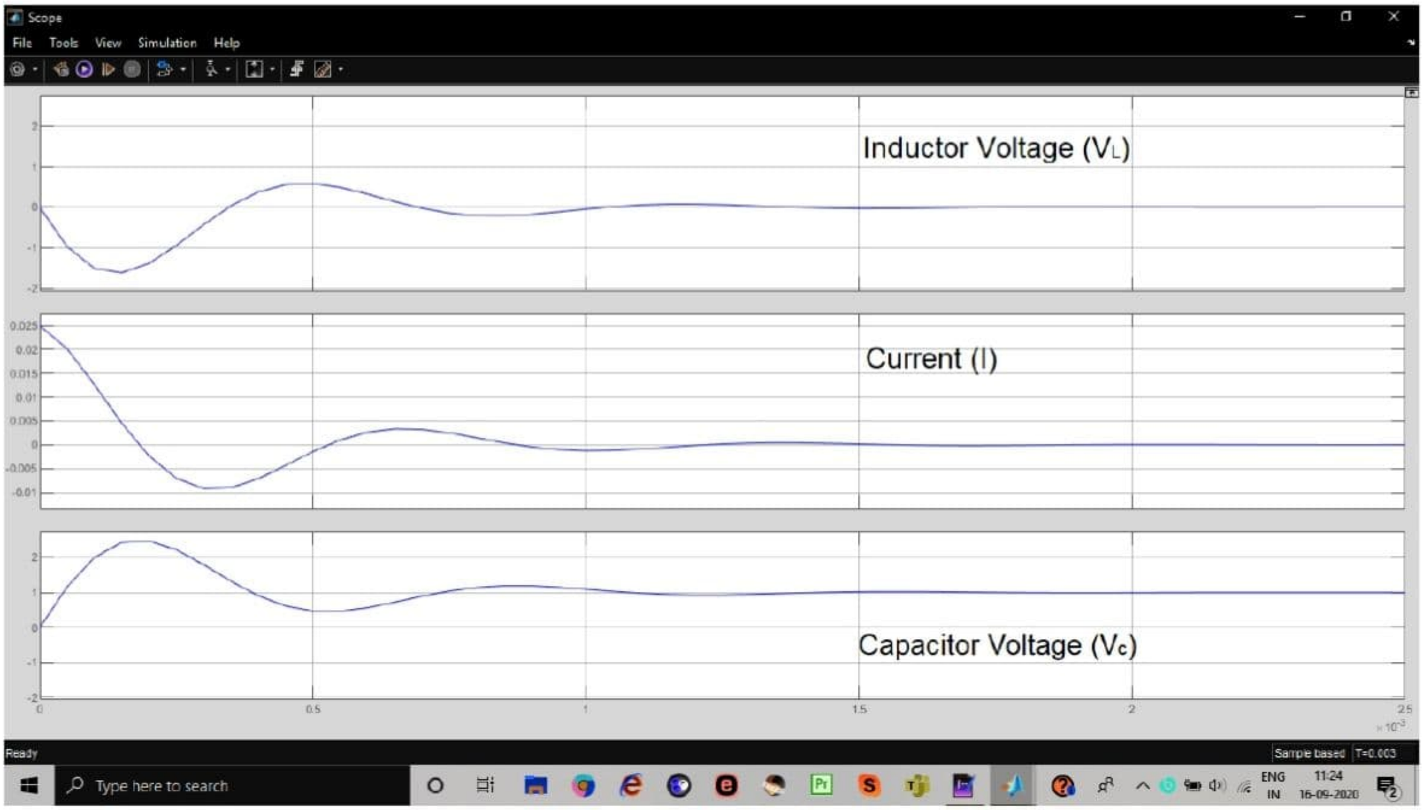
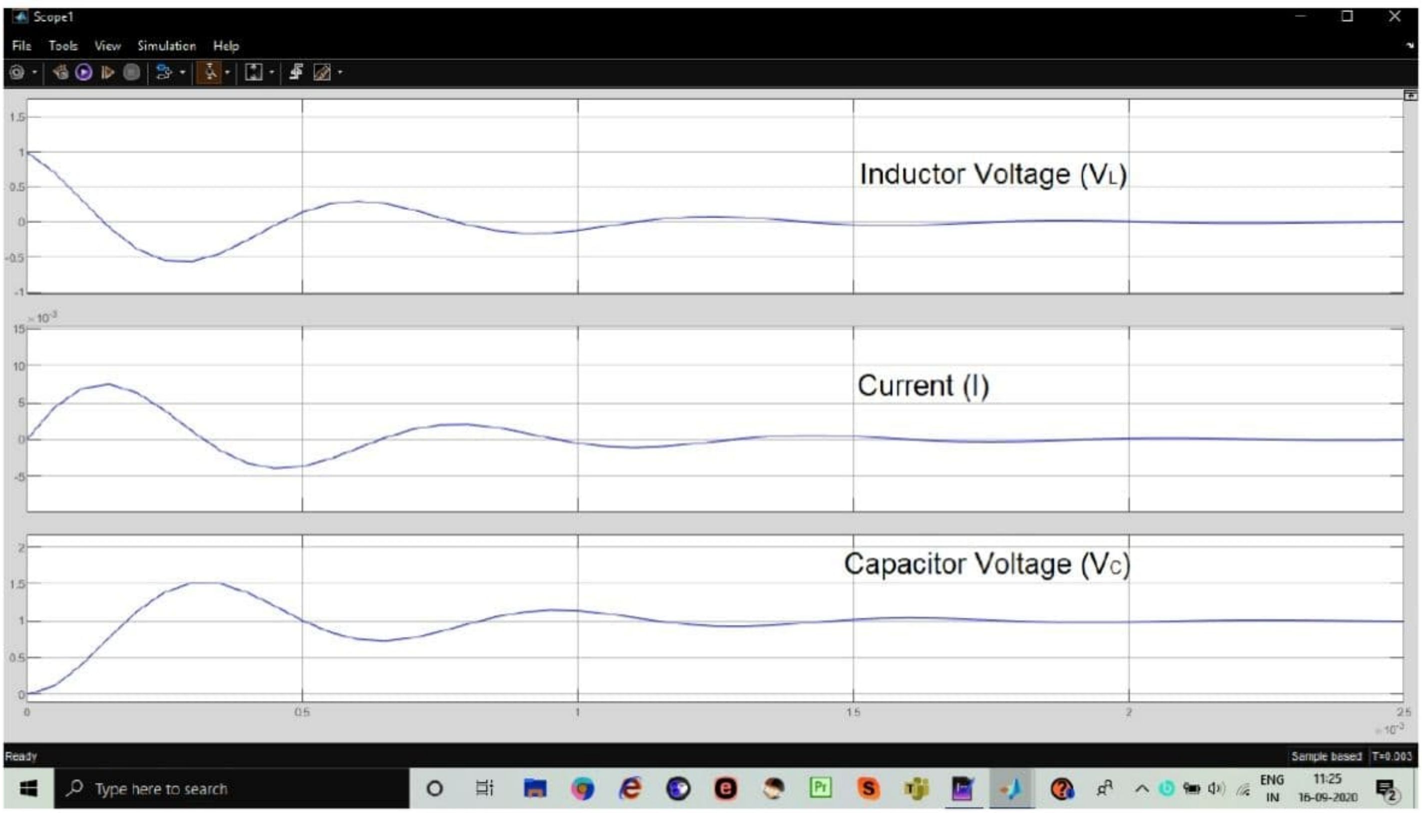
(i) Response using “sL” as TF of Inductance



(ii) Response using “1/sL” as TF of Inductance



(iii) The responses obtained in (i) and (ii) are not identical.

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	Response in (i)	Response in (ii)
Current (I)	Jump of 0.025A at t=0, pass t=0 a damped(exponential) oscillation.	Continuous at t=0, and later damped oscillation
V_L	At t=0, it's value is zero and decreasing just after t=0.	Jump of 1V at t=0, and later a damped oscillation
V_C	% overshoot is more than 100% i.e., $V_{C,peak} > 2V_C(\infty)$	% Overshoot is around 50% i.e; $V_{C,peak} \approx 1.5V_C(\infty)$

Reason behind it is that "SL" is an improper Tranfer Fuction. The problem with improper T.F is it is non-causal system and physically unrealizable. Because improper T.F involve derivatives in it, and to define a derivative we need to have info of signal value in future. Also derivative amplifies high frequency noise and distorts output for/at input discontinuities. But this effects are removed in case of "$\frac{1}{SL}$" as it is a proper Transfer Function.

Therefore using "$\frac{1}{SL}$" T.F to represent Inductor is proper and correct.

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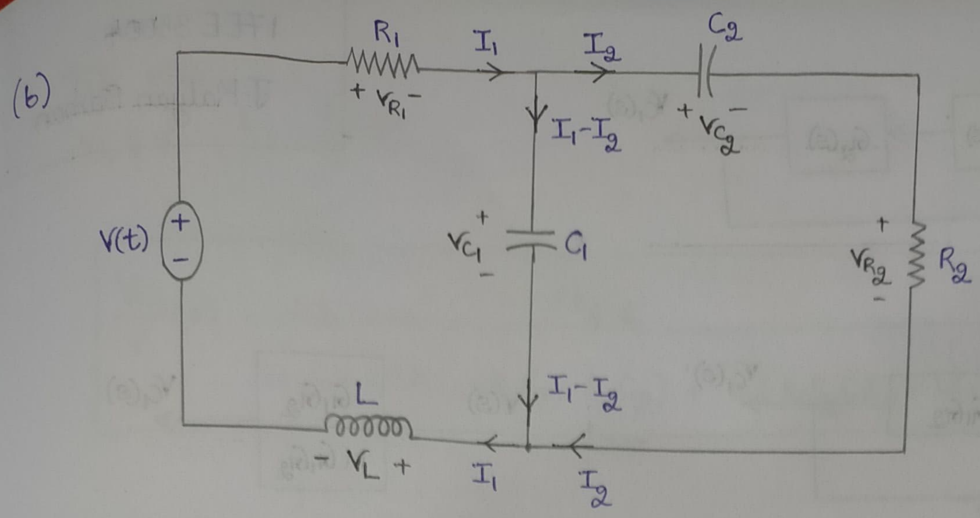
(b)

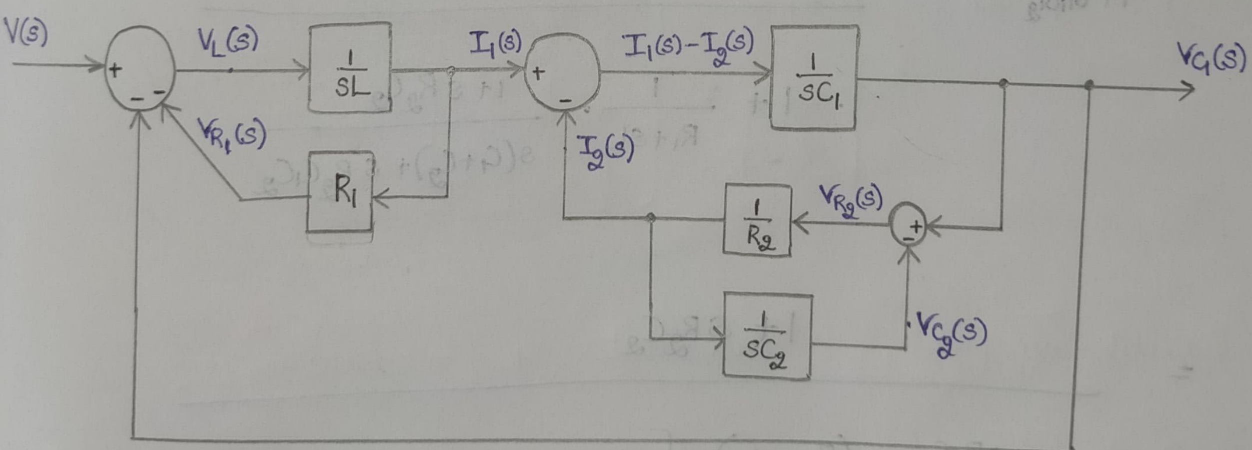


$$V(s) = V_{R_1}(s) + V_L(s) + V_{C_1}(s) \Rightarrow I_1(s) = \frac{V(s) - V_{C_1}(s) - V_L(s)}{R_1} \quad (\text{OR}) \quad \frac{V_L(s)}{sL}$$

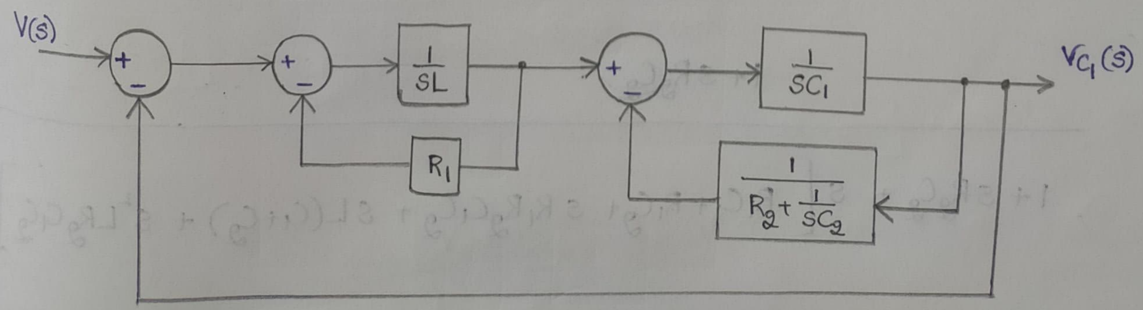
$$\Rightarrow I_1(s) = \frac{1}{sL}\left(V(s) - V_{C_1}(s) - V_R(s)\right) \quad \text{and} \quad V_R(s) = R\, I_1(s)$$

$$V_{C_1}(s) = \frac{1}{sC_1}\left(I_1(s) - I_2(s)\right)$$

$$V_{C_1}(s) = V_{C_2}(s) + V_{R_2}(s) = R_2 I_2(s) + \frac{1}{sC_2} I_2(s) \Rightarrow I_2(s) = \frac{V_{C_1}(s)}{R_2 + \frac{1}{sC_2}} = \frac{\frac{1}{R_2} V_{C_1}(s)}{1 + \frac{1}{R_2}\cdot\frac{1}{sC_2}}$$



$\Downarrow$



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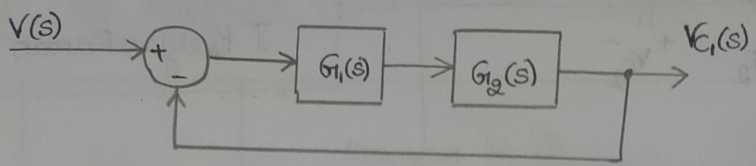

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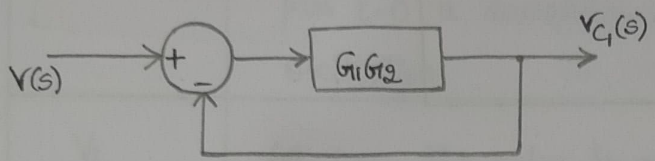
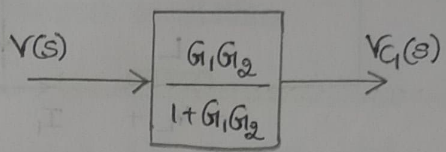


$\Longrightarrow$



$$G_1 = \frac{\frac{1}{sL}}{1+R_1\frac{1}{sL}} = \boxed{\frac{1}{R_1+sL}}$$

$$G_2 = \frac{\frac{1}{sC_1}}{1+\frac{1}{sC_1}\cdot\frac{1}{R_2+\frac{1}{sC_2}}} = \frac{R_2+\frac{1}{sC_2}}{1+sC_1\left(R_2+\frac{1}{sC_2}\right)}$$

$$= \boxed{\frac{1+sR_2C_2}{sC_2+sC_1+s^2R_2C_1C_2}}$$

$$\frac{V_{C_1}}{V}(s) = \frac{G_1G_2}{1+G_1G_2} = \frac{\frac{1}{R_1+sL}\cdot\frac{1+sR_2C_2}{s(C_1+C_2)+s^2R_2C_1C_2}}{1+\frac{1}{R_1+sL}\cdot\frac{1+sR_2C_2}{s(C_1+C_2)+s^2R_2C_1C_2}}$$

$$= \frac{1+sR_2C_2}{1+sR_2C_2+(R_1+sL)\,s\left[C_1+C_2+sR_2C_1C_2\right]}$$

$$= \frac{1+sR_2C_2}{1+sR_2C_2+s\left[R_1C_1+R_1C_2+sR_1R_2C_1C_2+sL(C_1+C_2)+s^2LR_2C_1C_2\right]}$$

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T.F btw V_{C_1} & V : $\frac{V_{C_1}}{V}(s)$

$$\boxed{\frac{V_{C_1}}{V}(s) = \frac{1+sR_2C_2}{1+s(R_1C_1+R_1C_2+R_2C_2)+s^2(R_1R_2C_1C_2+LC_1+LC_2)+s^3LR_2C_1C_2}}$$

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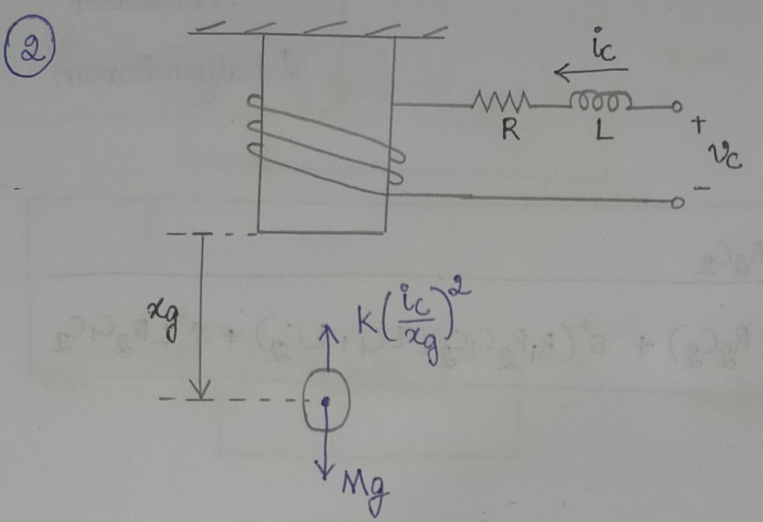
(2)



$R = 20\Omega$

$M = 2kg$

$L = 0.5H$

$x_0 = 5\times10^{-3}m$

$k = 3\times10^{-4}\ Nm^2A^{-2}$

$$M\ddot{x}_g = Mg - k\left(\frac{i_c}{x_g}\right)^2$$

$$v_c = Ri_c + L\frac{di_c}{dt}.$$

INPUT : v_c

OUTPUT : x_g

(a) $\ddot{x}_g$, x_g, i_c and $\frac{di_c}{dt}$ are involved, and we need to use minimum states.

$\therefore$ $x_1 = x_g$, $x_2 = \dot{x}_g$, $x_3 = i_c$ $\Rightarrow$ $\dot{x}_1 = \dot{x}_g$, $\dot{x}_2 = \ddot{x}_g$, $\dot{x}_3 = \frac{di_c}{dt}$ [All variables are covered]

State Space equation :

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix} x_2 \\ g-\frac{k}{M}\frac{x_3^2}{x_1^2} \\ -\frac{R}{L}x_3+\frac{1}{L}v_c\end{bmatrix}\begin{matrix}\to f_1\\ \to f_2\\ \to f_3\end{matrix}$$

(b) Equilibrium point : given that $x_{10} = \boxed{x_0 = 5\times10^{-3}m} \to x_0$

$\dot{x}_1 = 0 \Rightarrow \boxed{x_{20} = 0} \to \dot{x}_0$

$\dot{x}_2 = 0 \Rightarrow g - \frac{k}{M}\frac{x_{30}^2}{x_{10}^2} \Rightarrow x_{30} = x_{10}\sqrt{\frac{Mg}{k}}$

$$x_{30} = 5\times10^{-3}\times\sqrt{\frac{2\times9.8}{3\times10^{-4}}} \approx \boxed{1.278A} \to i_0$$

$\dot{x}_3 = 0 \Rightarrow -\frac{R}{L}x_{30} + \frac{1}{L}v_0 = 0$

$\Rightarrow v_0 = Rx_{30} = 20\times1.278 \approx \boxed{25.56V} \to v_0$

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$\therefore \quad x_0 = 5\times10^{-3}\,m \;;\; \dot{x}_0 = 0 \;;\; i_0 = 1.278\,A \;;\; v_0 = 25.56\,V$

(c) Incremental State Space Representation

$$\boxed{\frac{\partial f_1}{\partial x_1} = 0} \quad \boxed{\frac{\partial f_1}{\partial x_2} = 1} \quad \boxed{\frac{\partial f_1}{\partial x_3} = 0} \quad \boxed{\frac{\partial f_1}{\partial v_g} = 0}$$

$$\frac{\partial f_2}{\partial x_1} = \frac{2k}{M}\cdot\frac{x_3^2}{x_1^3}\Bigg|_{x_{10},x_{30}} = \frac{2k}{M}\cdot\frac{x_{30}^2}{x_{10}^3}$$

$$\boxed{\frac{\partial f_2}{\partial x_1} = \frac{2g}{x_{10}}}$$

$$\boxed{\frac{\partial f_2}{\partial x_2} = 0} \quad \boxed{\frac{\partial f_2}{\partial v_g} = 0}$$

$$\frac{\partial f_2}{\partial x_3} = -\frac{2k}{M}\cdot\frac{x_3}{x_1^2}\Bigg|_{x_{10},x_{30}} = -\frac{2k}{M}\cdot\frac{x_{30}}{x_{10}^2}$$

$$\boxed{\frac{\partial f_2}{\partial x_3} = -\frac{2}{x_{10}}\sqrt{\frac{kg}{M}}}$$

$$\boxed{\frac{\partial f_3}{\partial x_1} = 0} \quad \boxed{\frac{\partial f_3}{\partial x_2} = 0} \quad \boxed{\frac{\partial f_3}{\partial x_3} = -\frac{R}{L}} \quad \boxed{\frac{\partial f_4}{\partial v_g} = \frac{1}{L}}$$

$$\frac{2g}{x_{10}} = \frac{2\times9.8}{5\times10^{-3}} = 3920 \;;\; -\frac{2}{x_{10}}\sqrt{\frac{kg}{M}} = -\frac{2}{5\times10^{-3}}\times\sqrt{\frac{3\times10^{-4}\times9.8}{2}} \approx -15.34$$

$$-\frac{R}{L} = -\frac{20}{0.5} = -40 \;;\; \frac{1}{L} = \frac{1}{0.5} = 2$$

$\therefore$ Incremental T.F

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 3920 & 0 & -15.34 \\ 0 & 0 & -40 \end{bmatrix}}_{\text{“A”}}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{\text{“B”}} v \quad \text{and} \quad y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\text{“C”}}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}$$

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(d) T.F from v to x is given by $G(s) = C(sI-A)^{-1}B$

$$(sI-A) = \begin{bmatrix} s & -1 & 0 \\ -3920 & s & 15.34 \\ 0 & 0 & s+40 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{(s+40)(s^2-3920)} \begin{bmatrix} +s(s+40) & -(-3920)(s+40) & +0 \\ -(-1)(s+40) & +s(s+40) & -0 \\ +(-1)(15.34) & -s(15.34) & s^2-3920 \end{bmatrix}^T$$

$$= \frac{1}{(s+40)(s^2-3920)} \begin{bmatrix} s(s+40) & s+40 & -15.34 \\ 3920(s+40) & s(s+40) & -15.34s \\ 0 & 0 & s^2-3920 \end{bmatrix}$$

$$C(sI-A)^{-1}B = \frac{2\times1\times(-15.34)}{(s+40)(s^2-3920)} \Rightarrow \boxed{G(s) = \frac{-30.68}{(s+40)(s^2-3920)}}$$

→ order of $G(s) = 3$. (degree of denominator)

→ Type 0 system (no integrators)

→ There is Right Hand Plane pole at $s = +\sqrt{3920}$, therefore the system in unstable