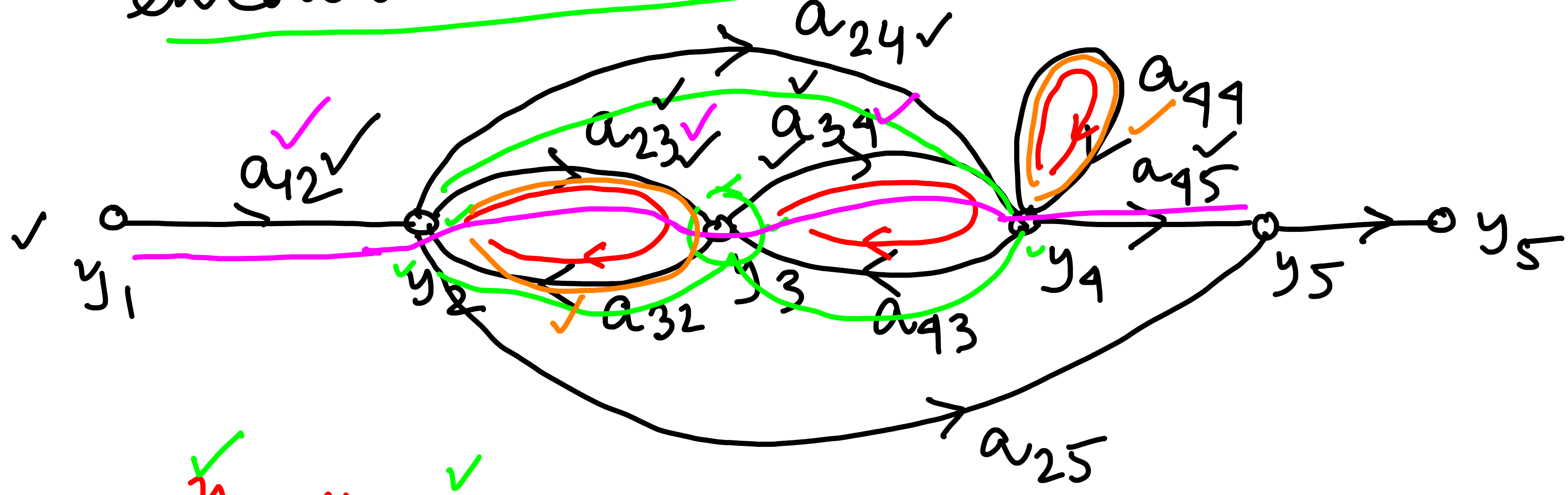
Signal Flow Graph

Loop: A loop is a path that originates and terminates on the same node and along which no other node is encountered more than once.



$y_2 - y_3 - y_2$, $y_3 - y_4 - y_3$, $y_4 - y_4$, $y_2 - y_4 - y_3 - y_2$

Path gain: The product of the branch gains encountered in traversing a path is called the path gain.

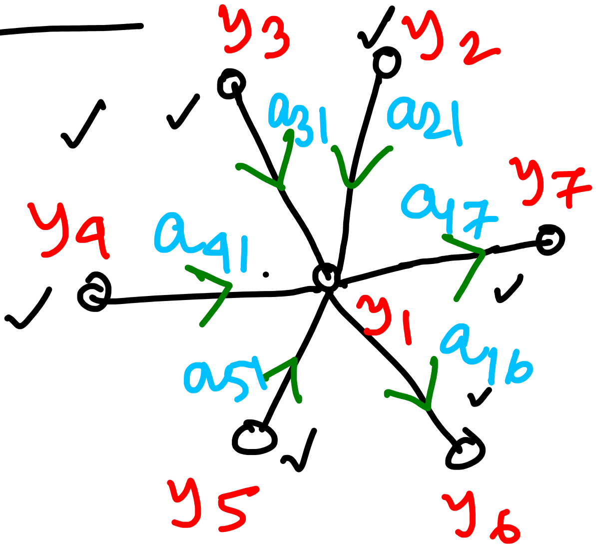
$y_1 - y_2 - y_3 - y_4 - y_5$, Path gain $= a_{12} a_{23} a_{34} a_{45}$

Nontouching loops: Two loops of a SFG are nontouching if they do not share a common node.

$y_2 - y_3 - y_2$, $y_4 - y_4$

SFG algebra

i)

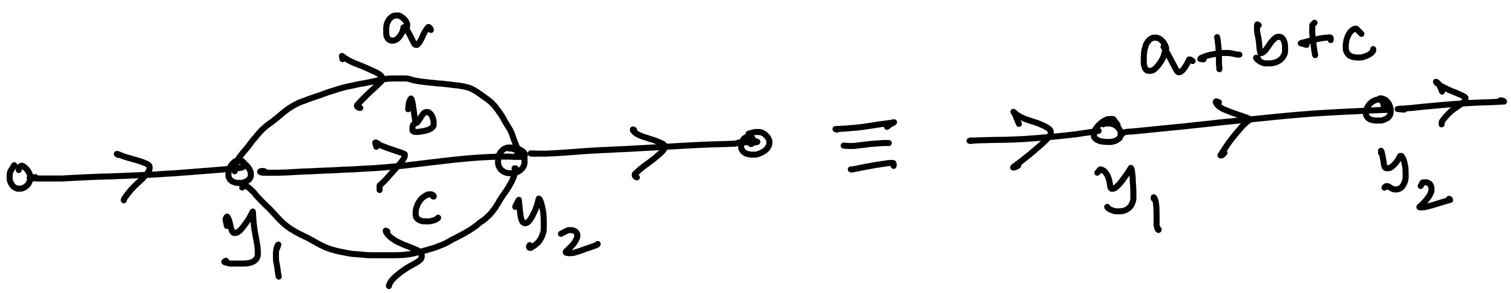


$$y_1 = a_{21}y_2 + a_{31}y_3 + a_{41}y_4 + a_{51}y_5$$

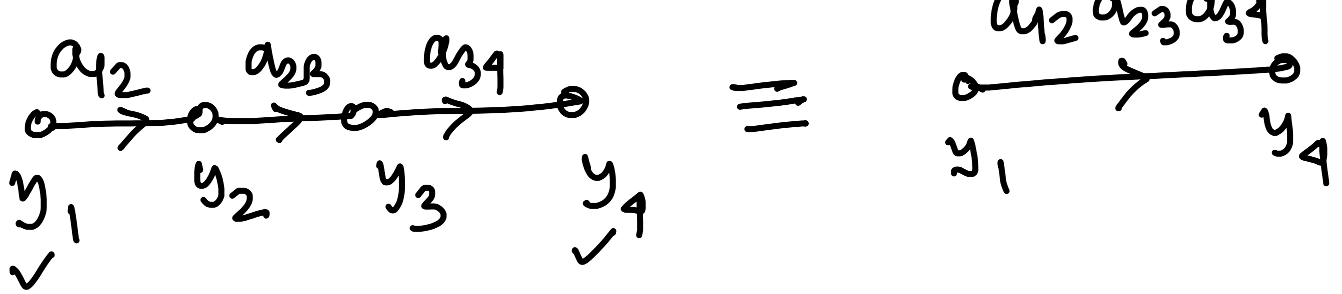
$$y_6 = a_{16}y_1$$

$$y_7 = a_{17}y_1$$

ii)



iii)



Mason's gain formula

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

N = Total number of forward paths between y_{in} and y_{out}.

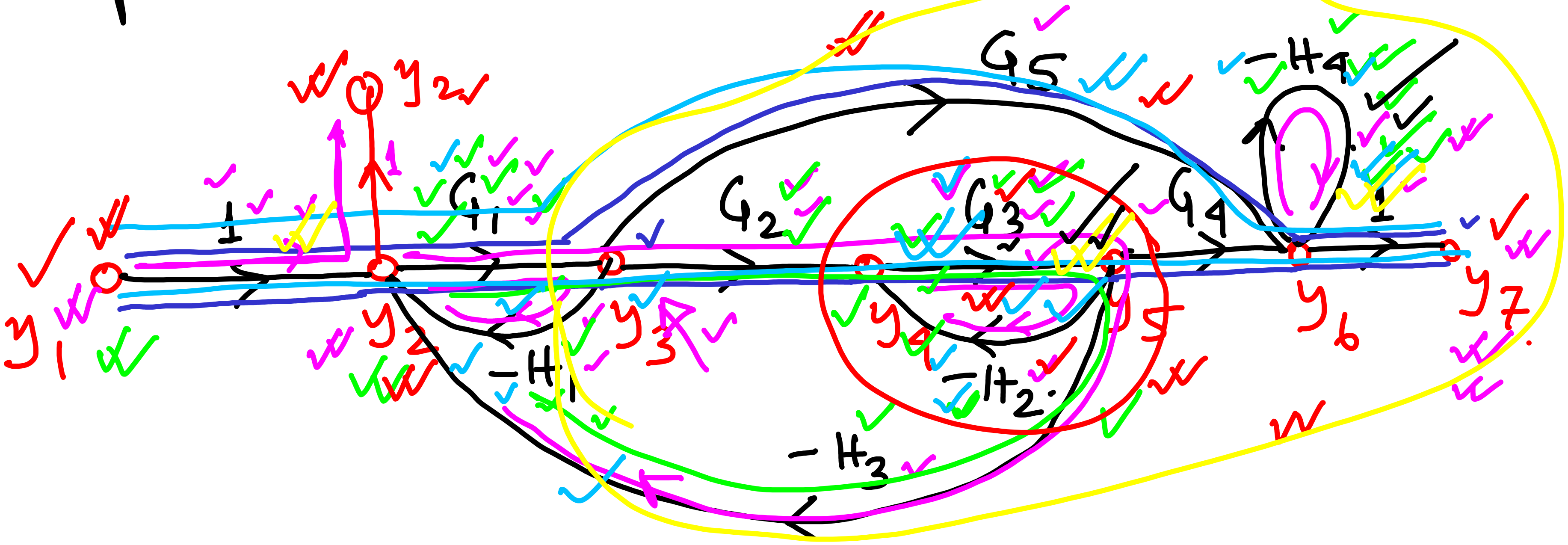
M_k – gain of the k-th forward path

$\Delta = 1 -$ (sum of the gains of all individual loops) $+$ (sum of the products of gains of all possible combinations of two nontouching loops) $-$ (- - - three nontouching loops) $+$ (- - - -

Δ_k = the Δ for that part of the SFG that is nontouching with the k-th forward path.

k=2

Ex



Forward paths: $y_1 - y_2 - y_3 - y_4 - y_5 - y_6 - y_7$, gain $= G_1G_2G_3G_4$

$y_1 - y_2 - y_3 - y_6 - y_7$ gain $= G_1G_5$

$$M = \sum_{k=1}^{2} \frac{M_k \Delta_k}{\Delta} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 (\Delta_1) + G_1 G_5 (\Delta_2)}{\Delta}$$

$$\Delta = 1 - (-G_1 H_1 - G_3 H_2 - G_1 G_2 G_3 H_3 - H_4) + (G_1 H_1 G_3 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4) - (-G_1 H_1 G_3 H_2 H_4)$$

$$\Delta_1 = 1, \quad \Delta_2 = 1 - (-G_3 H_2) = 1 + G_3 H_2$$

$$\frac{y_7}{y_1} = M = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 (1 + G_3 H_2)}{1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + H_4 + G_1 H_1 G_3 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4 + G_1 H_1 G_3 H_2 H_4 = \Delta}$$

Find $\frac{y_7}{y_2}$

$$\frac{y_7}{y_2} = \frac{y_7 / y_1}{y_2 / y_1}$$

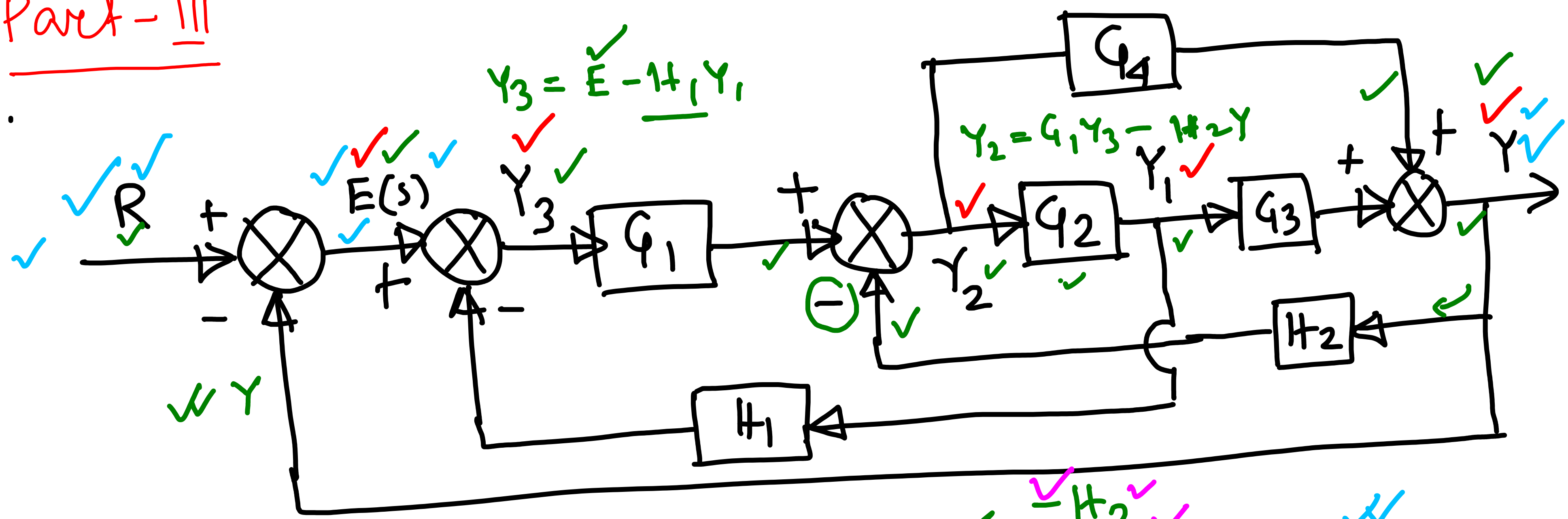
$$\frac{y_2}{y_1} = \frac{1 \left(1 - (-G_3 H_2 - H_4) + (G_3 H_2 H_4)\right)}{\Delta}$$

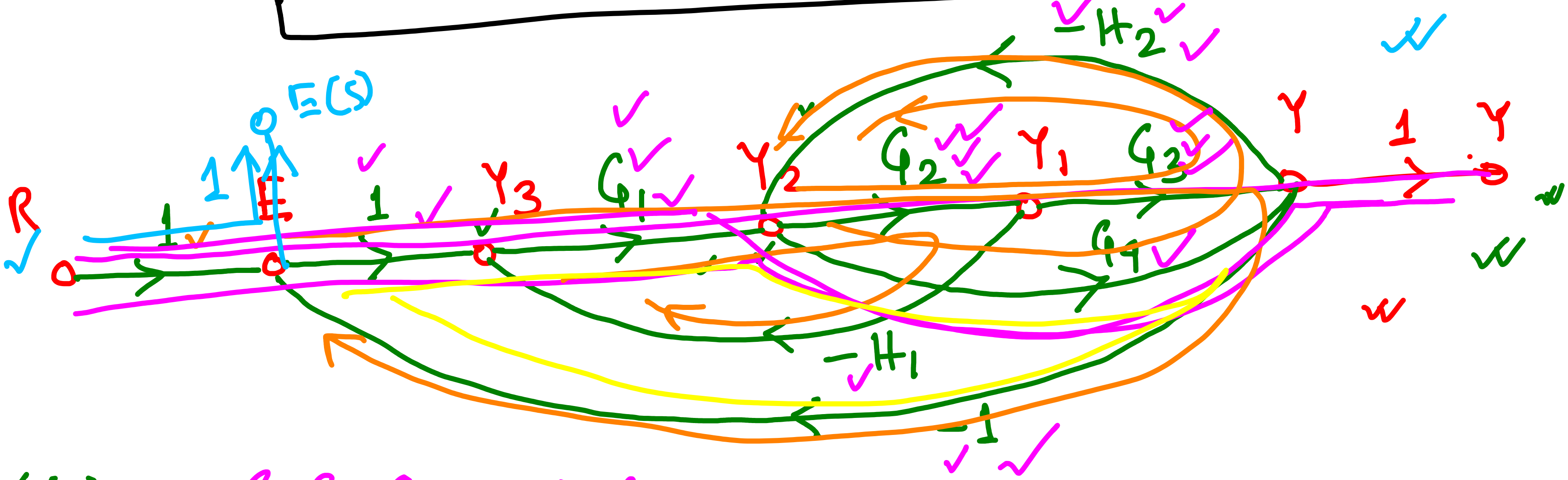
$$= \frac{1 + H_4 + G_3 H_2 + G_3 H_2 H_4}{\Delta}$$

$$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1 + H_4 + G_3H_2 + G_3H_2H_4}$$

Part - III

Ex.





$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 - (-G_1G_2H_1 - G_1G_2G_3 - G_2G_3H_2 - G_4H_2 - G_1G_4)}$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_1G_2G_3 + G_2G_3H_2 + G_4H_2 + G_1G_4}$$

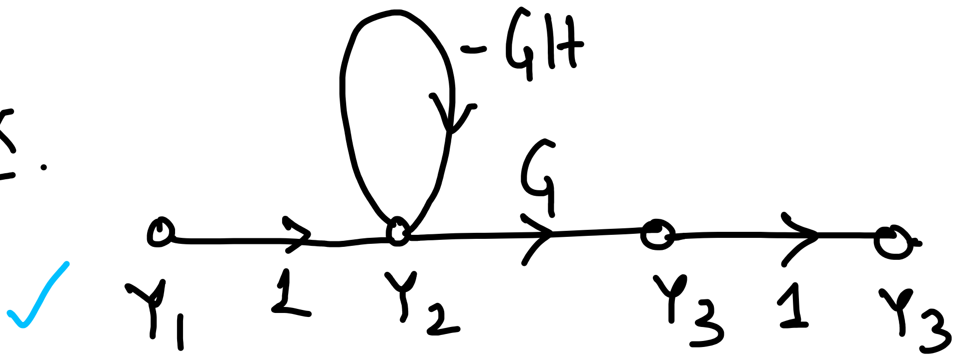
Δ

$$\frac{E(s)}{R(s)} = \frac{1-\left(1-\left(-G_1G_2H_1 - G_2G_3H_2 - G_4H_2\right)\right)}{\Delta}$$

$$= \frac{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2}{\Delta}$$

$$\frac{Y(s)}{E(s)} = \frac{Y(s)/R(s)}{E(s)/R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2}$$

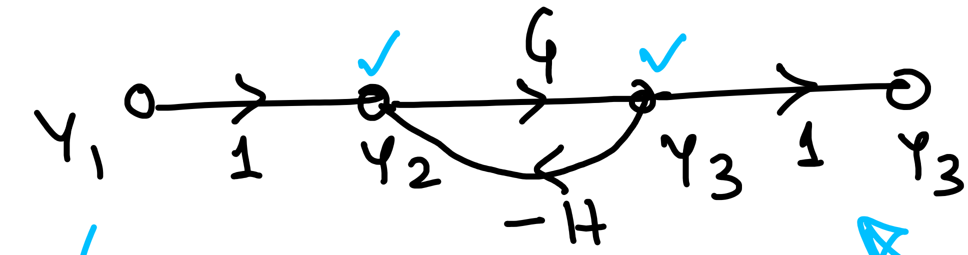
Ex.



Are they equivalent?

YES

$Y_2 = Y_1 - GH\,Y_2, \quad Y_3 = G\,Y_2$



$Y_2 = Y_1 - H\,Y_3, \quad Y_3 = G\,Y_2$

$= Y_1 - HG\,Y_2$