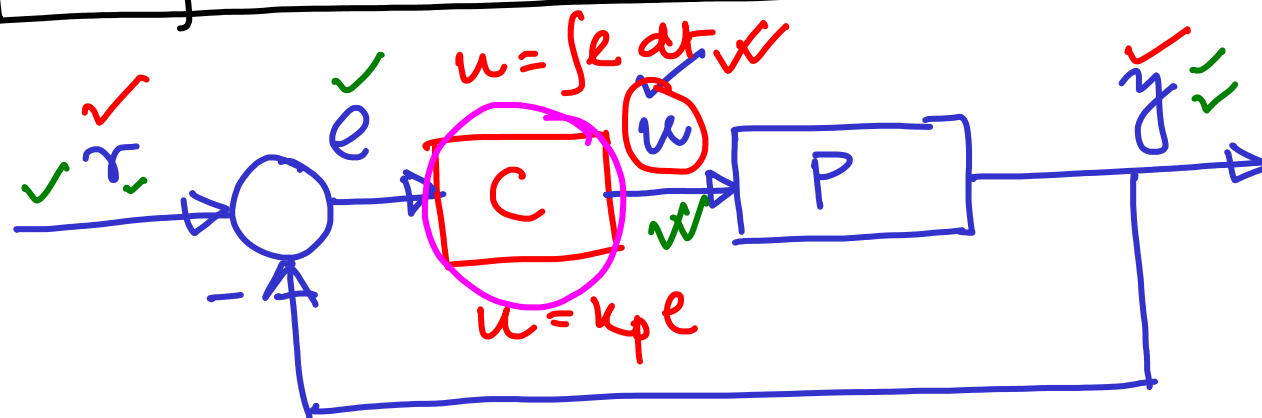


Simple forms of feedback



Error signal
 $e = r - y$

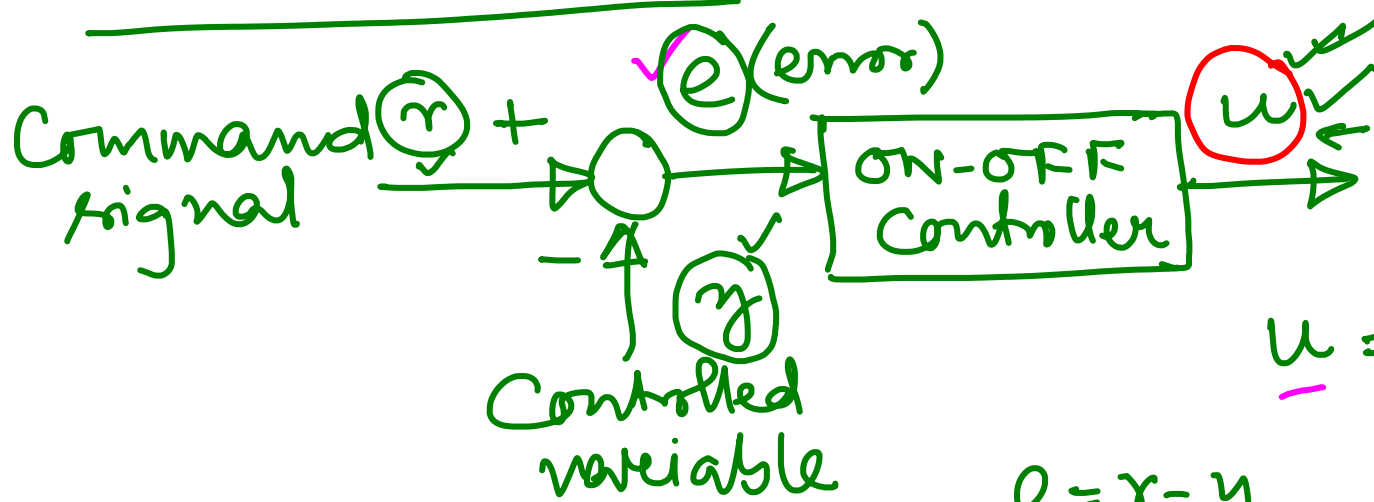
The idea of feedback to make corrective actions based on the difference between the desired and the actual values of a quantity can be obtained in different ways.

✓ ON-OFF control ✓

✓ PID control ✓

(Proportional-integral-derivative)

ON-OFF control

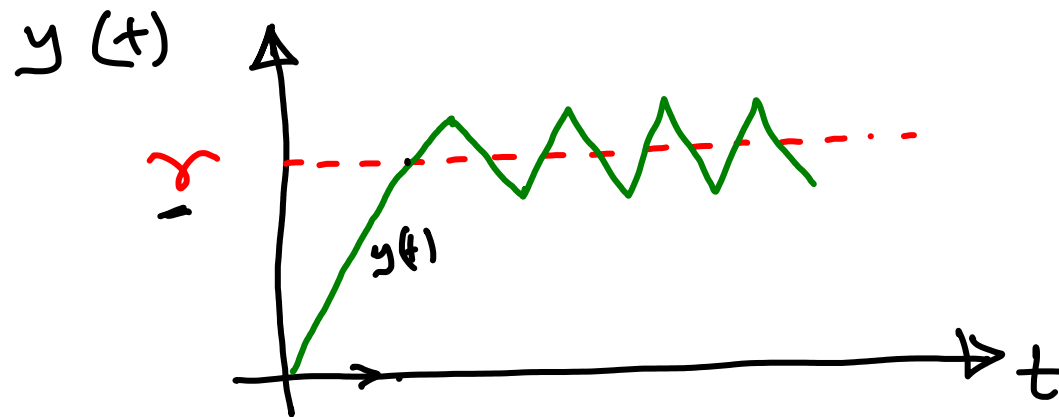


$$e = r - y$$

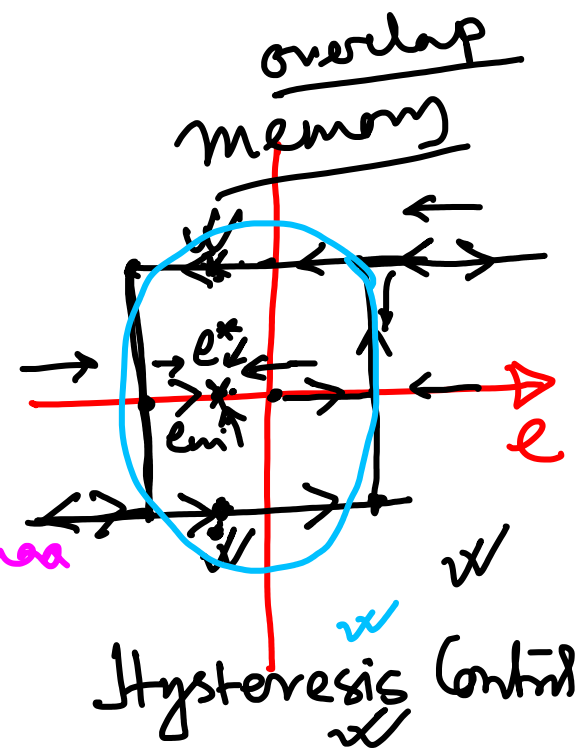
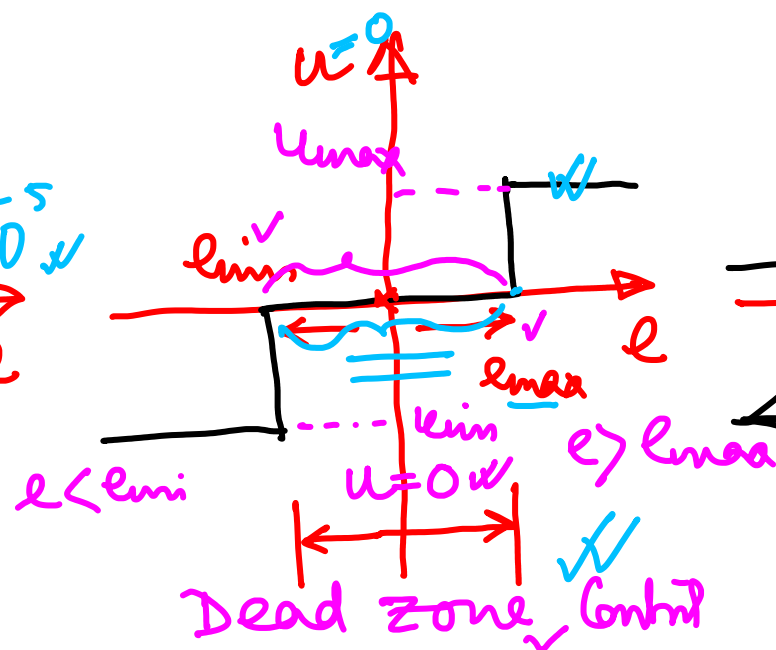
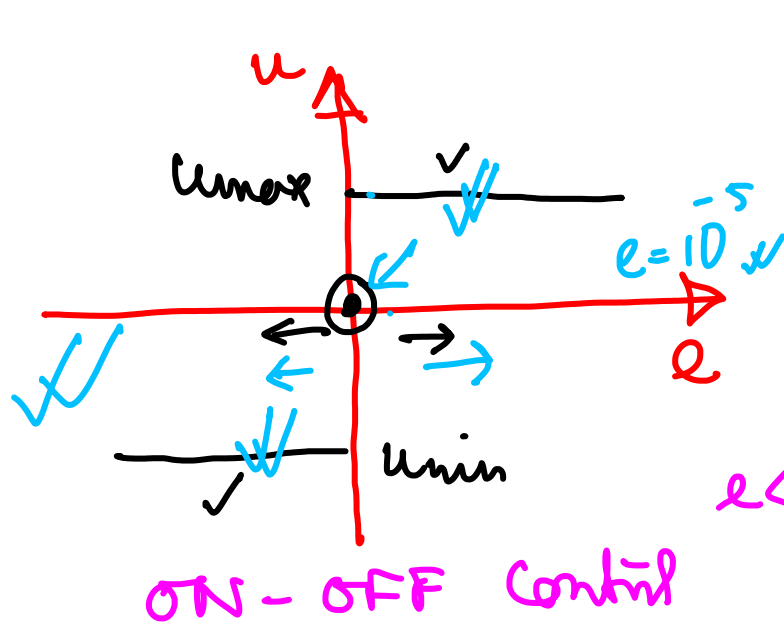
$$u = \begin{cases} u_{\max} & \text{if } e > 0 \\ u_{\min} & \text{if } e < 0 \\ = 0 & \text{if } e = 0 \end{cases}$$

$$e = 0$$

- simple, no parameter to choose
- typically results in oscillation of controlled variable



- Control variable is not defined when the error is zero.



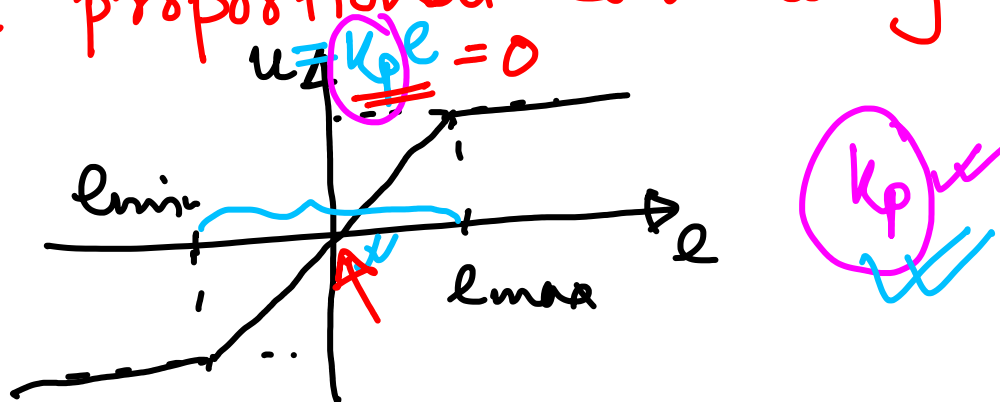
- ON-OFF Control overreacts
- A small change in the error makes the activated variable change over the full range.

PID Control

- The above disadvantage can be avoided in proportional control.

$$u = \begin{cases} u_{\max} & \text{if } e \geq e_{\max} \\ K_p e & \text{if } -e_{\min} < e < e_{\max} \\ u_{\min} & \text{if } e \leq -e_{\min} \end{cases}$$

- K_p is the proportional controller gain.



- the interval (e_{\min}, e_{\max}) is called the proportional band.

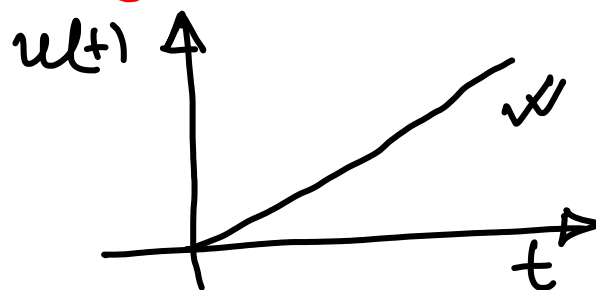
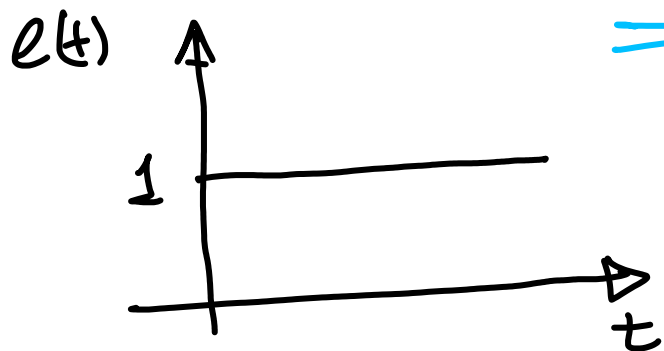
- Controlled variable deviates from the reference. In order to maintain it, e should not be equal to zero since some control input is required.

✓
(draw back
of propor-
tional
controller)

- This can be avoided by making the control action proportional to the integral of the error.

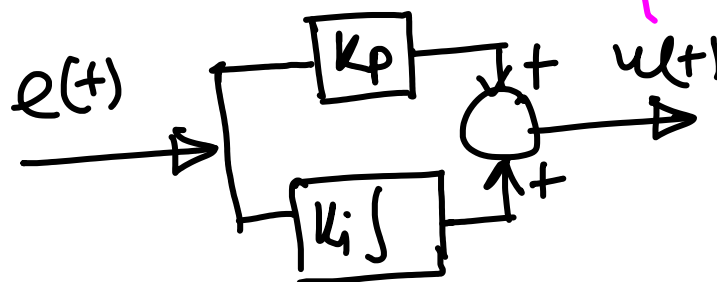
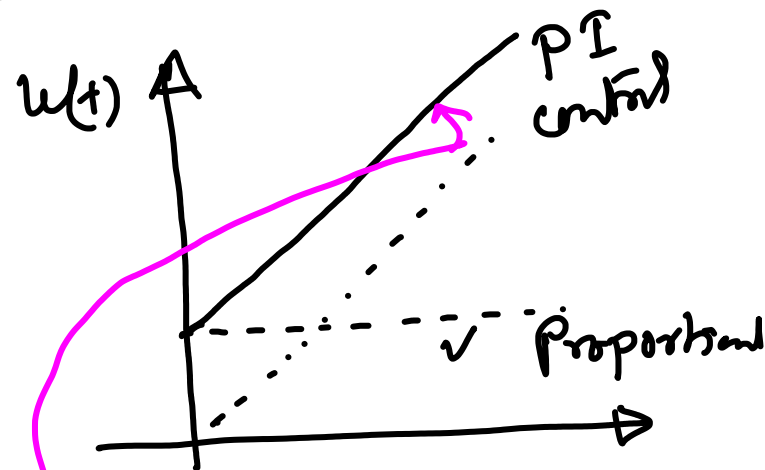
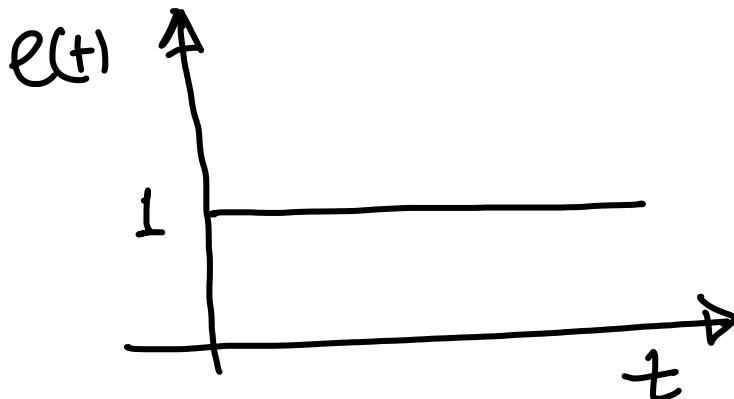
$$\underline{u(t)} = \underline{k_i} \int_0^t \underline{e(\tau)} d\tau$$

- k_i Integral Control gain.



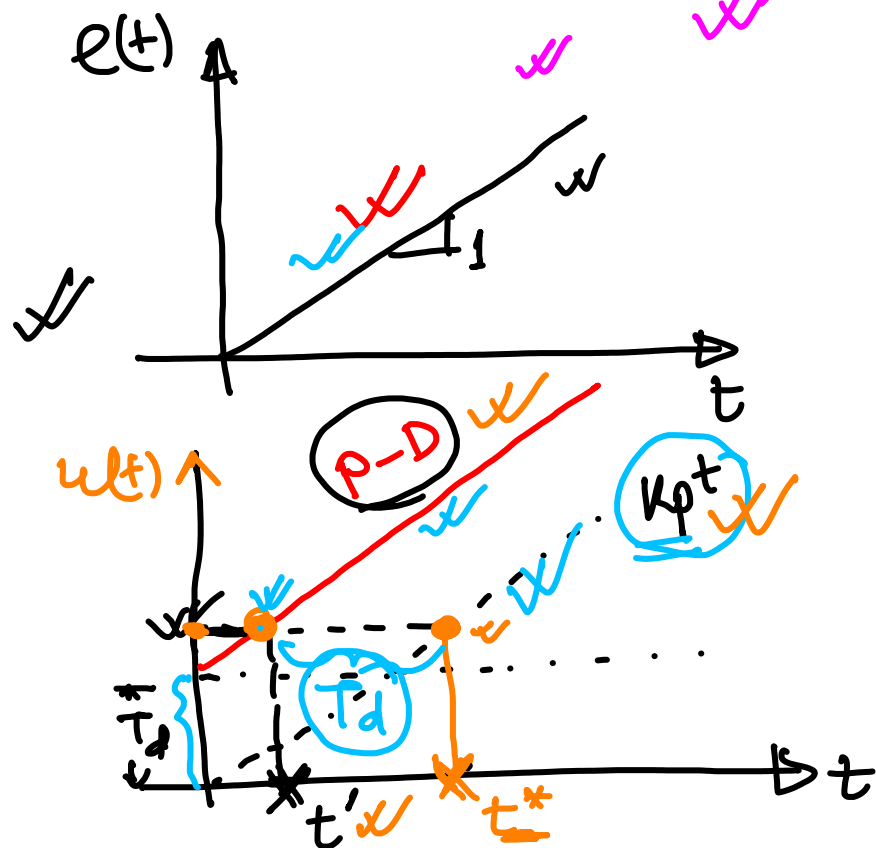
- The steady-state error becomes zero.

P-I (Proportional-Integral) Control $\check{P} + I$



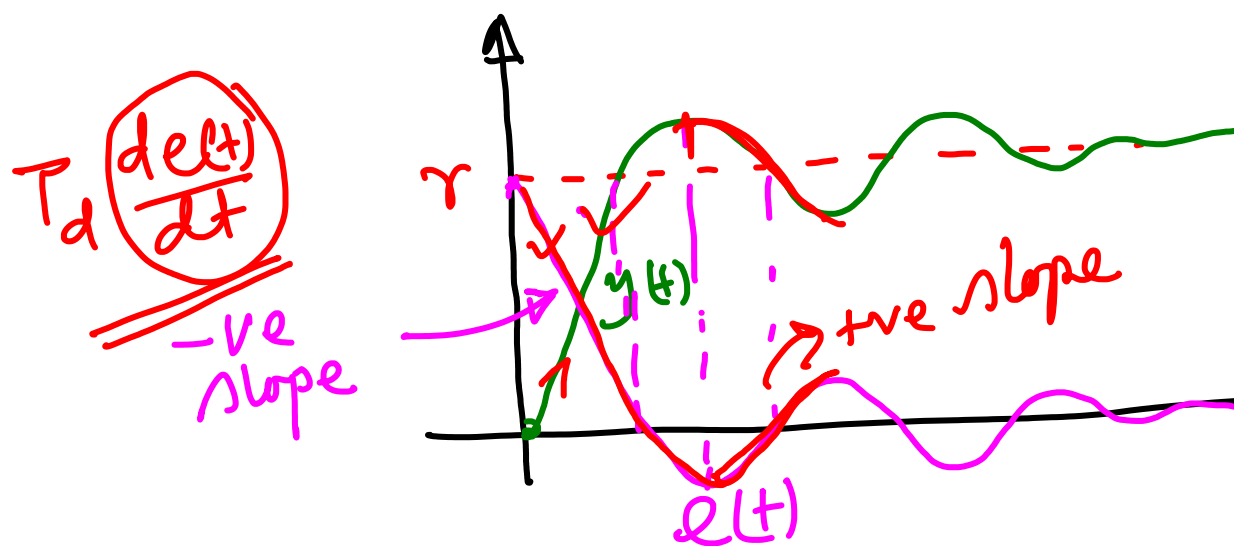
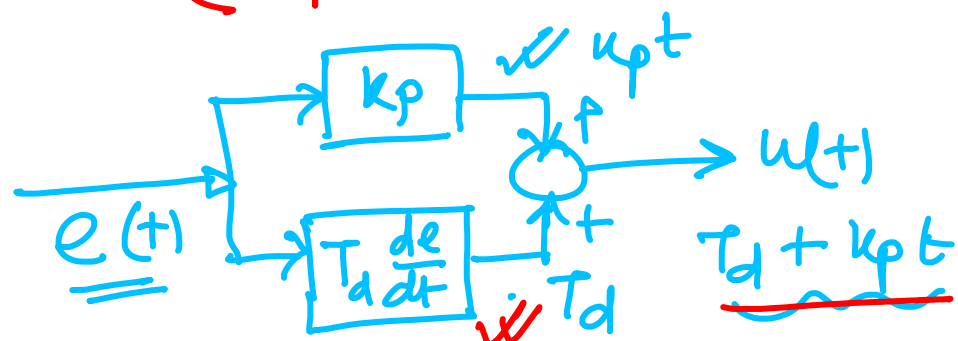
An additional refinement is to provide the controller with an anticipative ability by using a prediction of the error

$$e(t + T_d) \approx e(t) + T_d \frac{de(t)}{dt} \quad \text{at } \frac{de(t)}{dt} = 0$$



— It predicts the error T_d time units ahead.

— PD (Proportional + derivative)



$$e = r - y$$

PID

