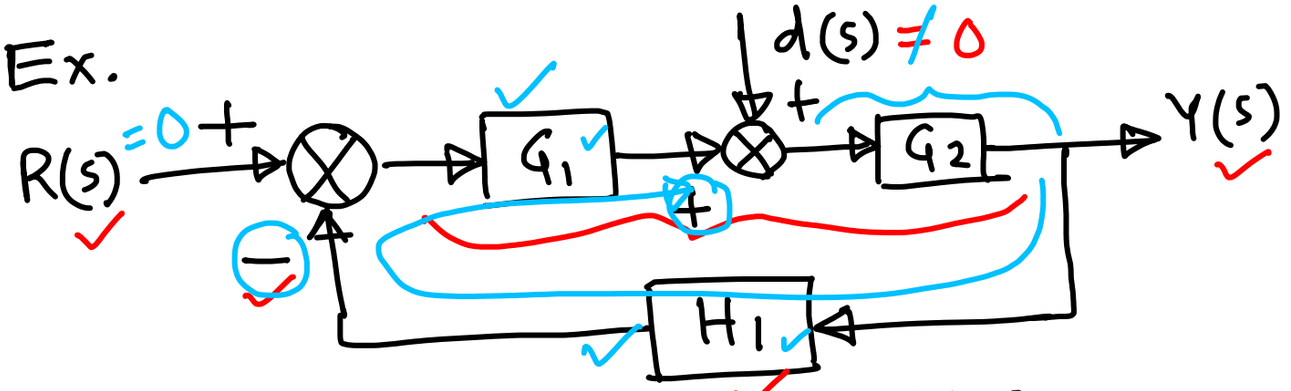
Block-diagram representation

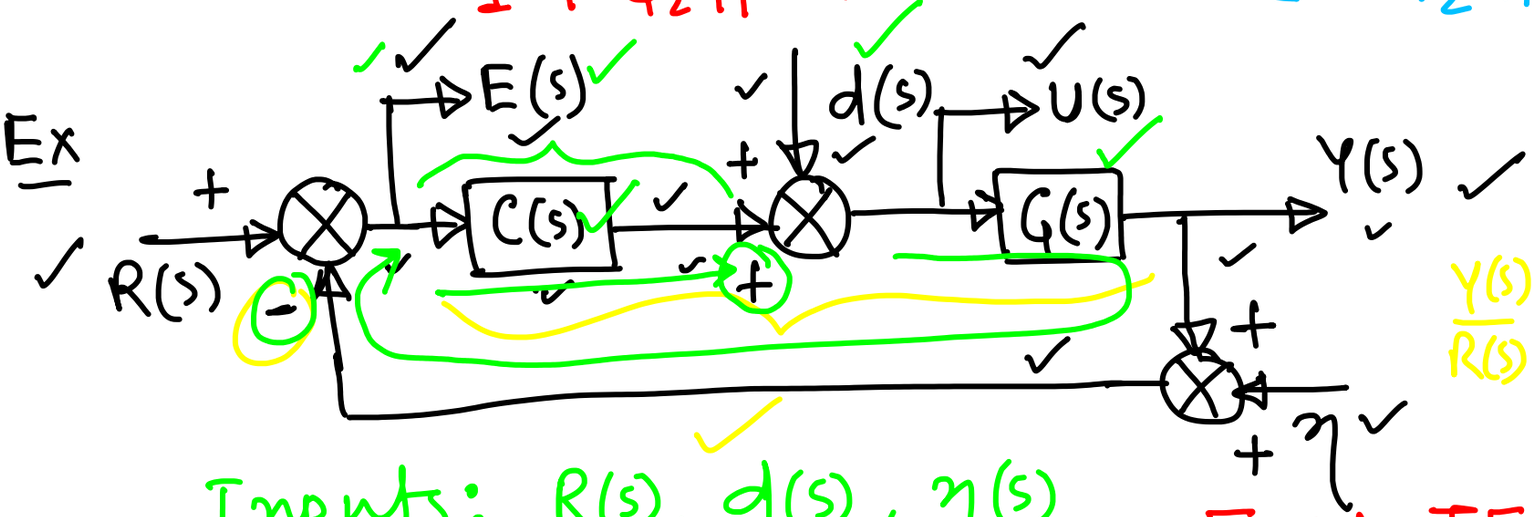
Ex.



Find TF $\frac{Y(s)}{R(s)}$ and $\frac{Y(s)}{d(s)}$.

$$\frac{Y(s)}{R(s)} = \frac{G_2 G_1}{1 + G_2 G_1 H_1} \qquad \frac{Y(s)}{d(s)} = \frac{G_2}{1 + G_2 G_1 H_1}$$

Ex



$$\frac{Y(s)}{R(s)} = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)}$$

Inputs: R(s), d(s), η(s)

Outputs: Y(s), E(s), U(s).

Find TFM

$$\begin{bmatrix} R(s) \\ d(s) \\ \eta(s) \end{bmatrix} \text{ to } \begin{bmatrix} Y(s) \\ E(s) \\ U(s) \end{bmatrix}.$$

$$Y(s) = G(s)\big(d(s) + C(s)\,E(s)\big) = G(s)\big(d(s) + C(s)\big(R(s) - \eta(s) - Y(s)\big)\big)$$

$$= G(s)\,d(s) + G(s)\,C(s)\,R(s) - G(s)\,C(s)\,\eta(s) - G(s)\,C(s)\,Y(s)$$

$$Y(s) = \frac{G(s)}{1+G(s)\,C(s)}\,d(s) + \frac{G(s)\,C(s)}{1+G(s)\,C(s)}\,R(s) - \frac{G(s)\,C(s)}{1+G(s)\,C(s)}\,\eta(s)$$

$$U(s) = d(s) + C(s)\,R(s) - C(s)\,\eta(s) - C(s)\,Y(s)$$

$$= d(s) + C(s)\,R(s) - C(s)\,\eta(s) - \frac{G(s)\,C(s)}{1+G(s)\,C(s)}\,d(s) - \frac{G(s)\,C(s)\,C(s)}{1+G(s)\,C(s)}\,R(s) + \frac{G(s)\,C(s)\,C(s)}{1+G(s)\,C(s)}\,\eta(s)$$

$$= \frac{1}{1+G(s)\,C(s)}\,d(s) + \frac{C(s)}{1+G(s)\,C(s)}\,R(s) - \frac{C(s)}{1+G(s)\,C(s)}\,\eta(s)$$

$$E(s) = R(s) - \eta(s) - Y(s) = \frac{1}{1+G(s)\,C(s)}\,R(s) - \frac{1}{1+G(s)\,C(s)}\,\eta(s) - \frac{G(s)}{1+G(s)\,C(s)}\,d(s)$$

($R(s)$: $=1$; $\eta(s)$: $=0$; $d(s)$: $=1$)

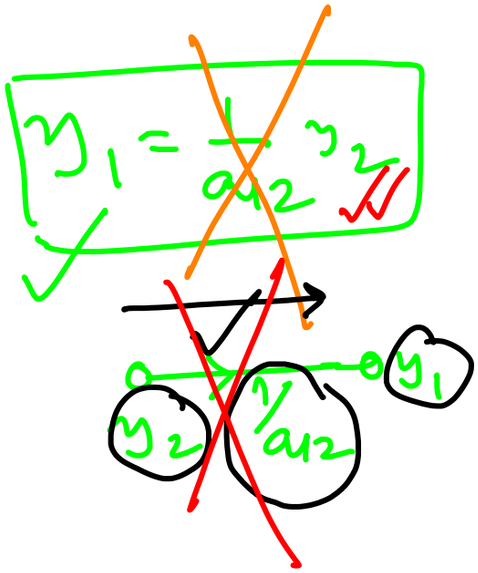
$1 - (-G(s))\,C(s)$

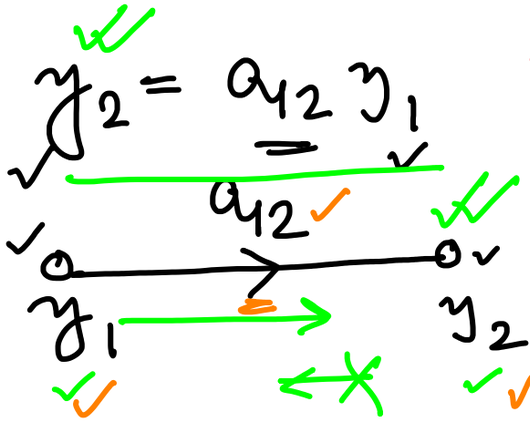
$\frac{E(s)}{d(s)} =$

$$\begin{bmatrix} Y(s) \\ E(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} & -\frac{GC}{1+GC} \\ \frac{1}{1+GC} & -\frac{G}{1+GC} & -\frac{1}{1+GC} \\ \frac{C}{1+GC} & \frac{1}{1+GC} & -\frac{C}{1+GC} \end{bmatrix} \begin{bmatrix} R(s) \\ d(s) \\ \eta(s) \end{bmatrix}$$

Signal Flow Graph (SFG)

- It is a simplified version of block-diagram
- It is a cause-effect representation of linear system modeled by algebraic equations.
- It was introduced by S. J. Mason.





This diagram does not imply the relationship $y_1 = \frac{1}{a_{12}} y_2$.

y_1 – input node
y_2 – output node
– variables are represented by nodes.

a_{12} – gain
– branch from y_1 to y_2
– Signal flows along the direction of arrow only.

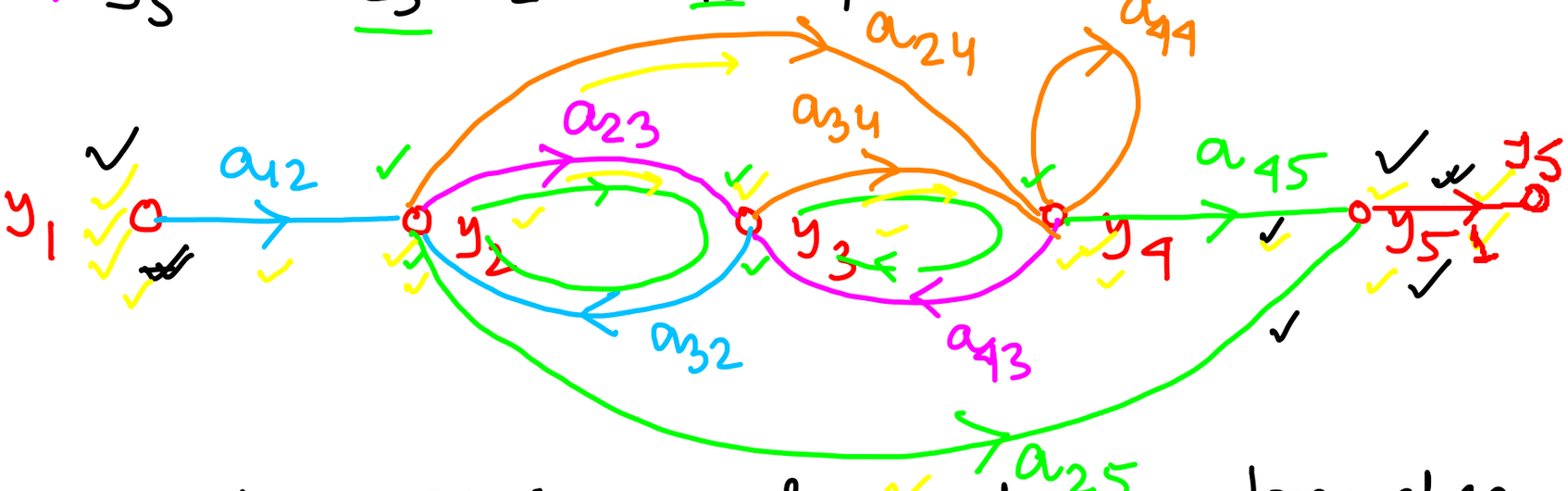
Ex

$$y_2 = a_{12} y_1 + a_{32} y_3$$

$$y_3 = a_{23} y_2 + a_{43} y_4$$

$$y_4 = a_{24} y_2 + a_{34} y_3 + a_{44} y_4$$

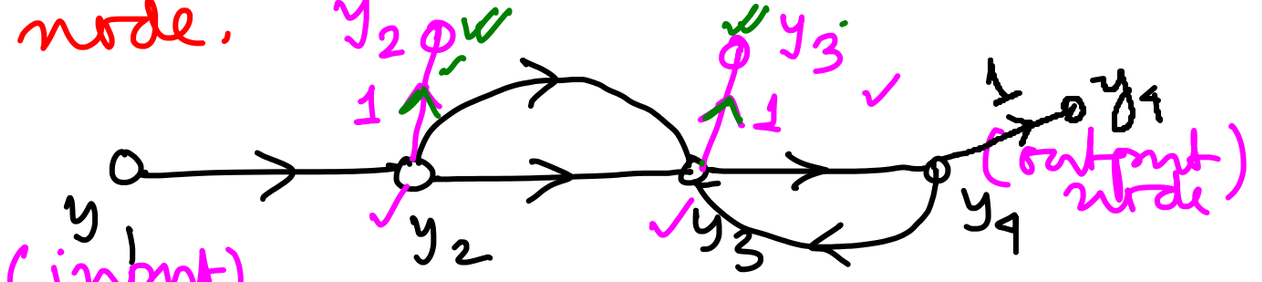
$$y_5 = a_{25} y_2 + a_{45} y_4$$



Input node: It has only outgoing branches (y_1).

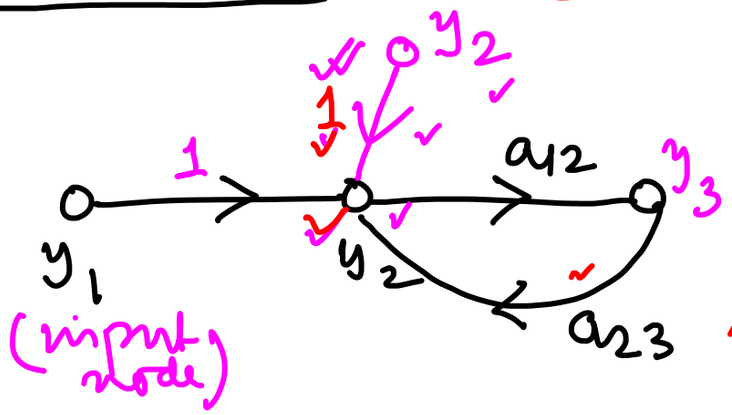
Output node: It has only incoming branches. (y_5)

We can make any non-input node as output node.



$$y_2 = y_2, \quad y_3 = y_3$$

We cannot make any node as input node.



$y_2 = y_1 + a_{23}y_3$

$y_2 = y_1 + a_{23}y_3 + y_2$

Path: A path is any collection of a continuous succession of branches traversed in the same direction.

$y_2 - y_3 - y_2$, $y_3 - y_4 - y_3$, $y_2 - y_3 - y_4 - y_3 - y_2$

It does not prevent any node from being traversed more than once.

Forward path: A forward path is a path that starts at an input node and ends at an output node, and along which no node is traversed more than once.

$y_1 - y_2 - y_5$, $y_1 - y_2 - y_4 - y_5$, $y_1 - y_2 - y_3 - y_4 - y_5$.