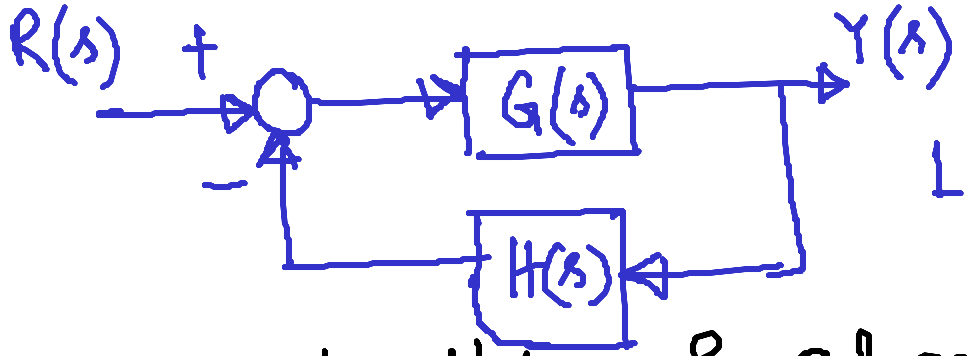
The Nyquist Stability Criterion



Loop TF = $G(s)H(s)$

- It relates the stability of closed-loop system to the loop TF frequency response and loop TF pole location.

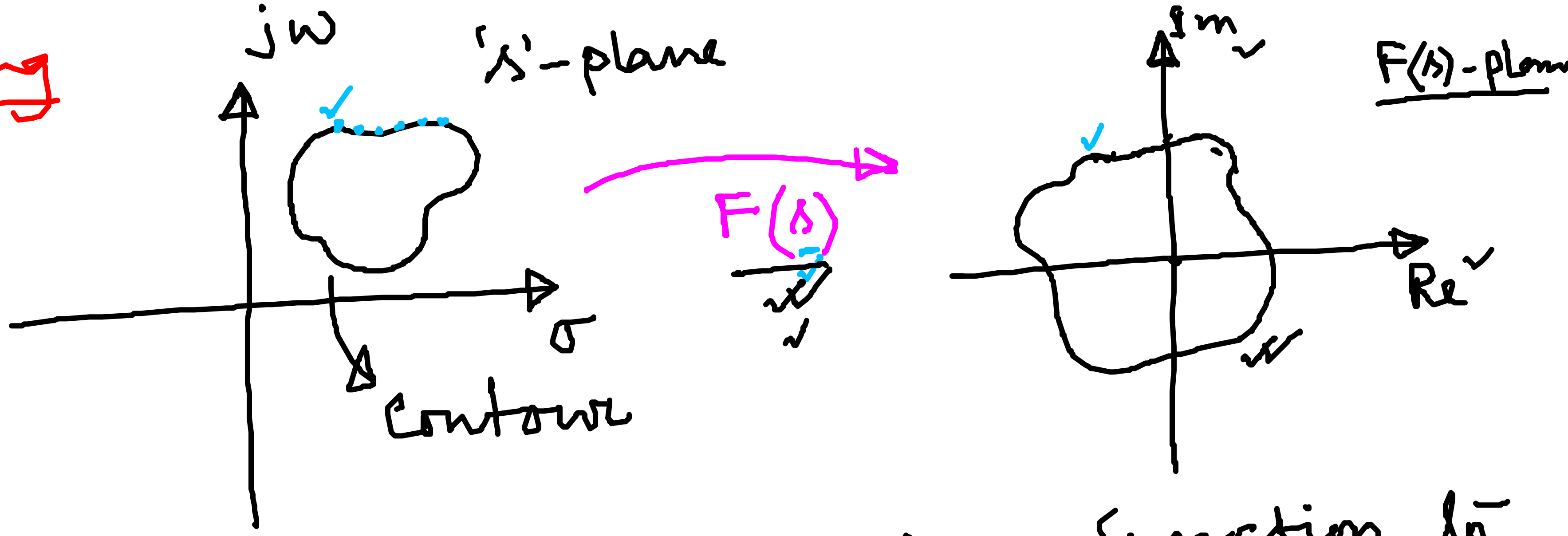
Let $G(s) = \dfrac{N_G}{D_G}$ and $H(s) = \dfrac{N_H}{D_H}$.

Then $G(s)H(s) = \dfrac{N_G N_H}{D_G D_H}$ and $1 + G(s)H(s) = \dfrac{D_G D_H + N_G N_H}{D_G D_H}$

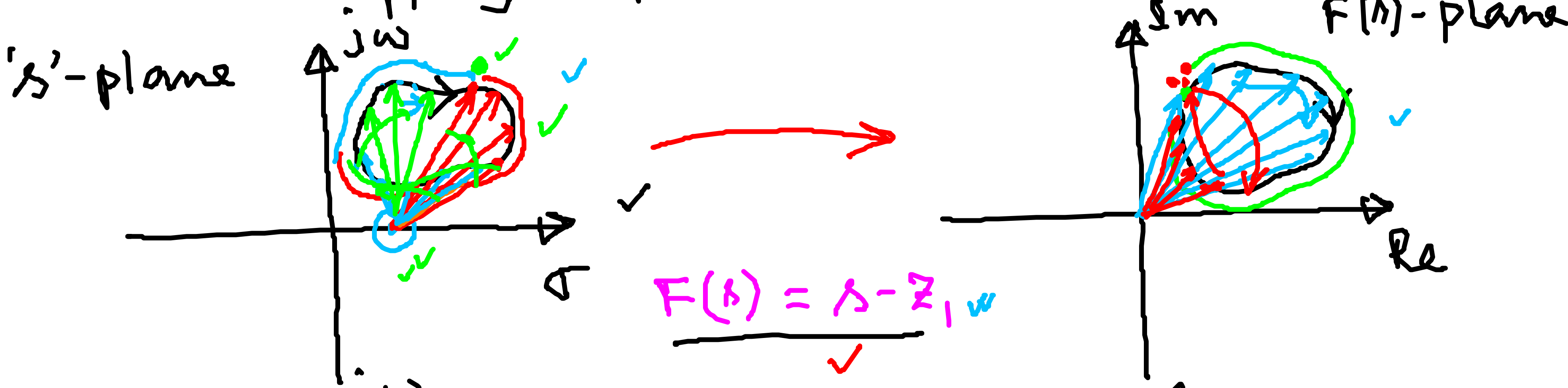
The closed-loop TF $= \dfrac{G(s)}{1 + G(s)H(s)} = \dfrac{N_G D_H}{D_G D_H + N_G N_H}$

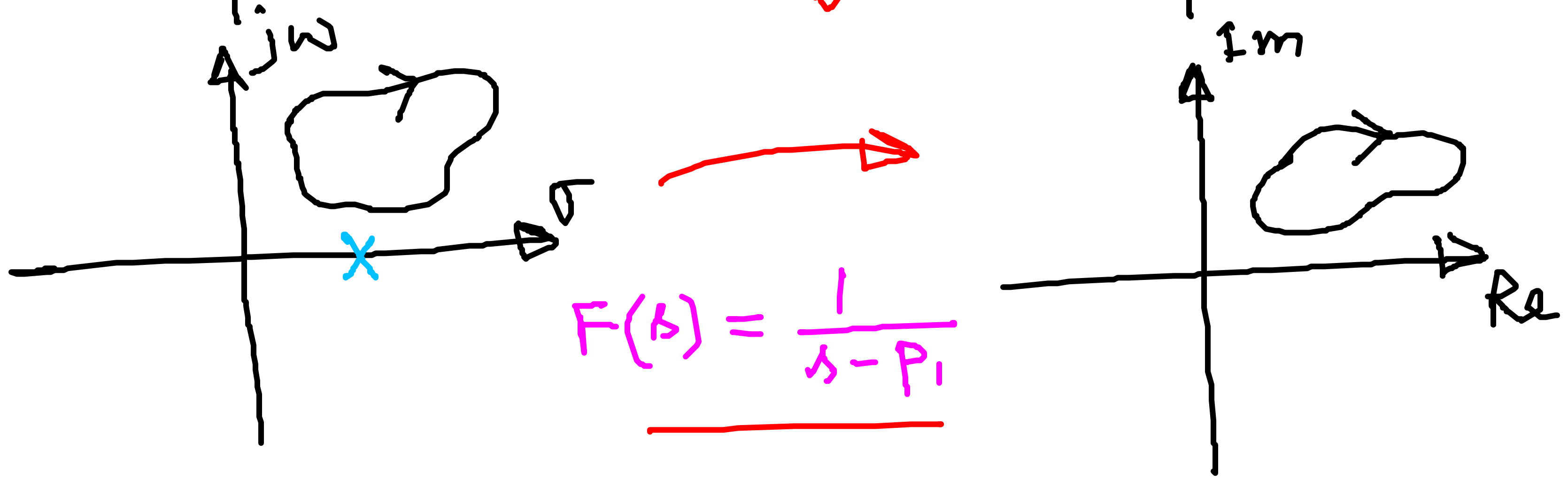
- Poles of $G(s)H(s)$ are same as the poles of $1 + G(s)H(s)$.
- Zeros of $1 + G(s)H(s)$ are same as the poles of closed-loop system.

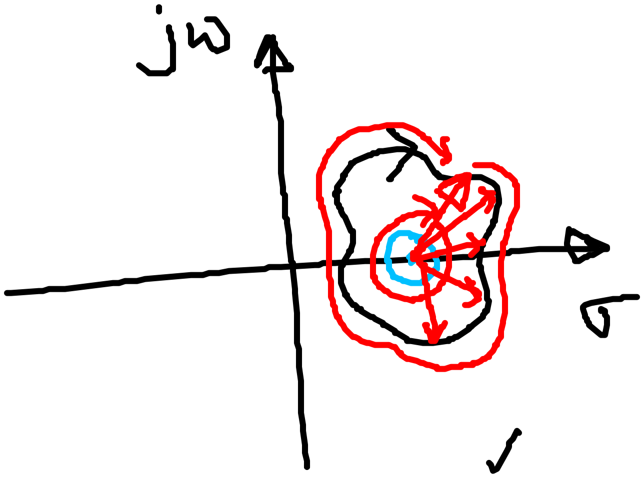
Mapping



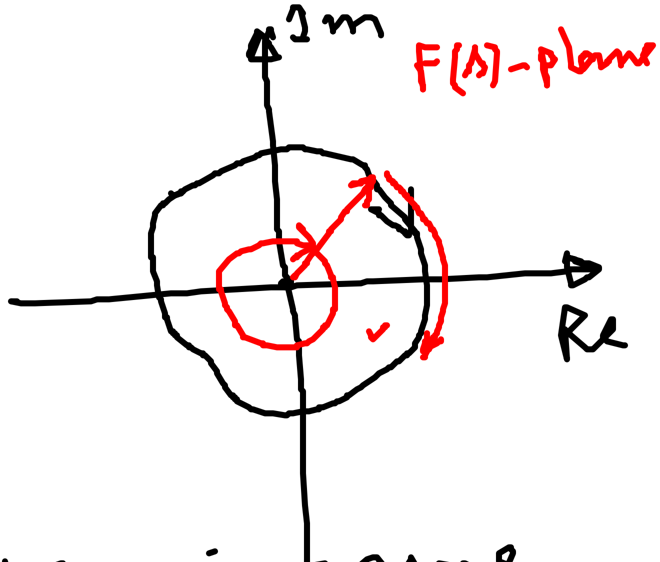
- A Contour is mapped through a function to another Contour in F(s)-plane.
- The mapping depends on F(s).



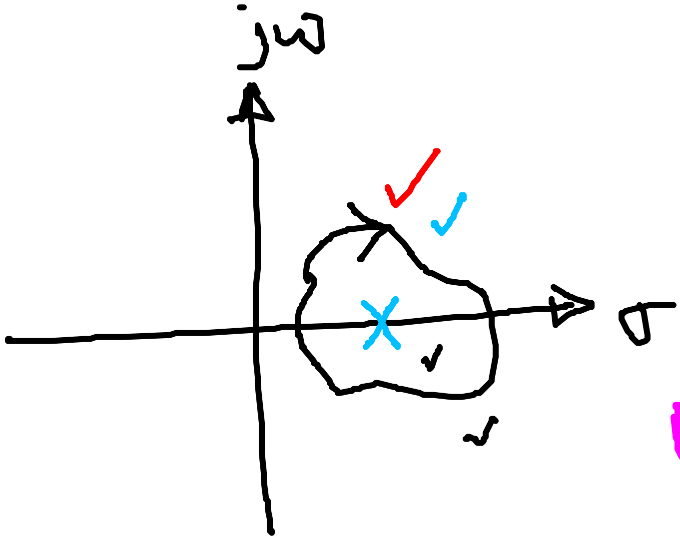




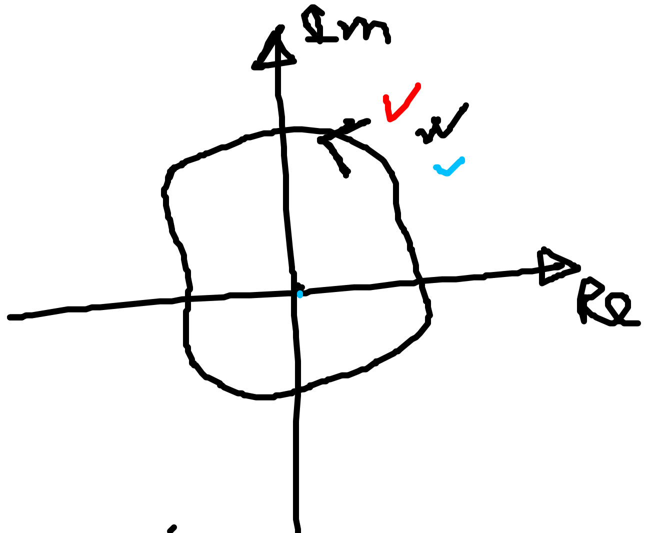
$$F(s) = s - Z_1$$



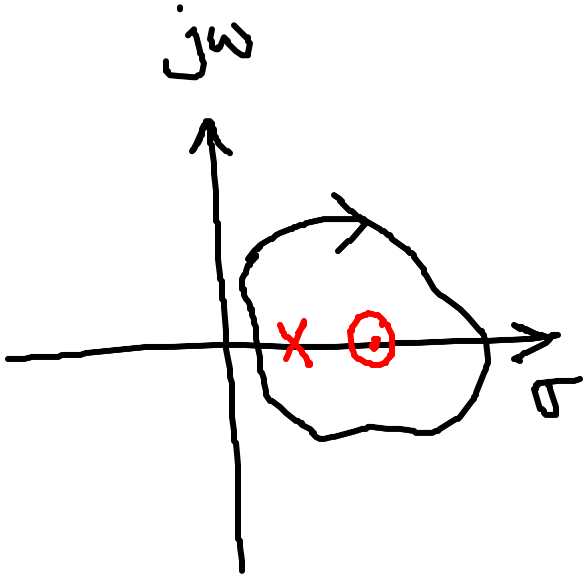
Rotation is same and encircles origin.



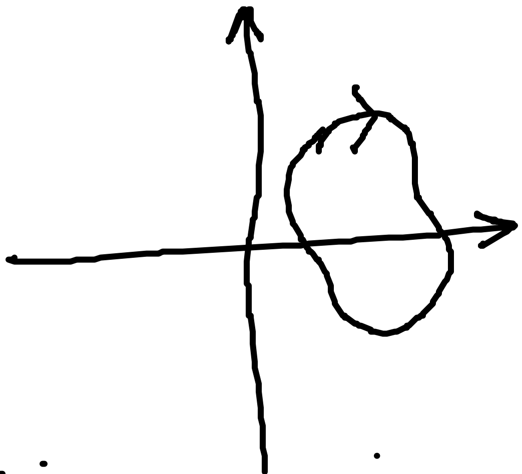
$$F(s) = \frac{1}{s - P_1}$$



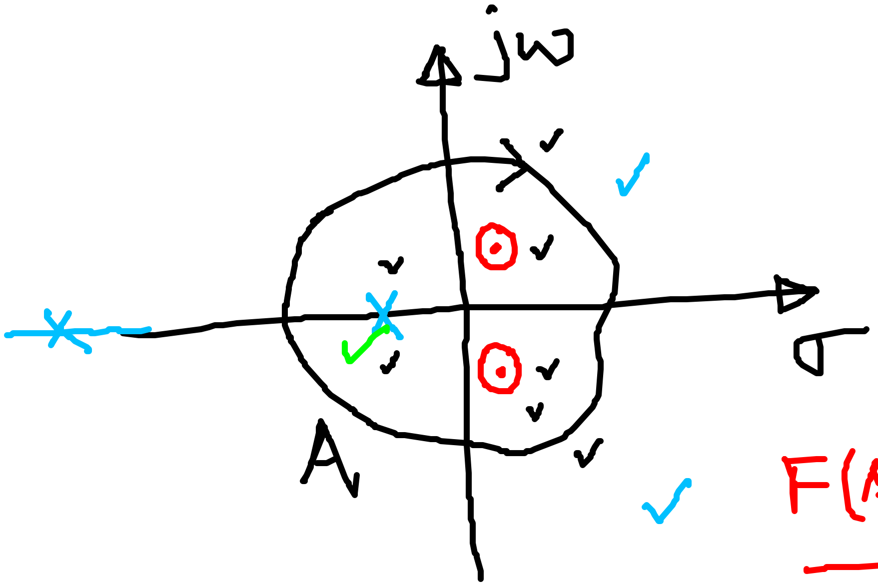
Rotation is reverse and encircles origin



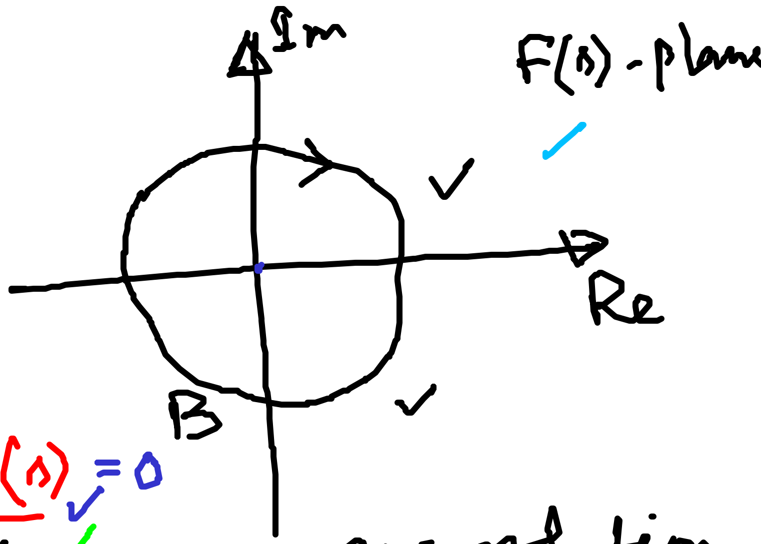
$$F(s) = \frac{s - Z_1}{s - P_1}$$



Rotation remains same and doesn't encircle origin



$F(s) = 1 + G(s)H(s) = 0$



CW rotation

CW rotation

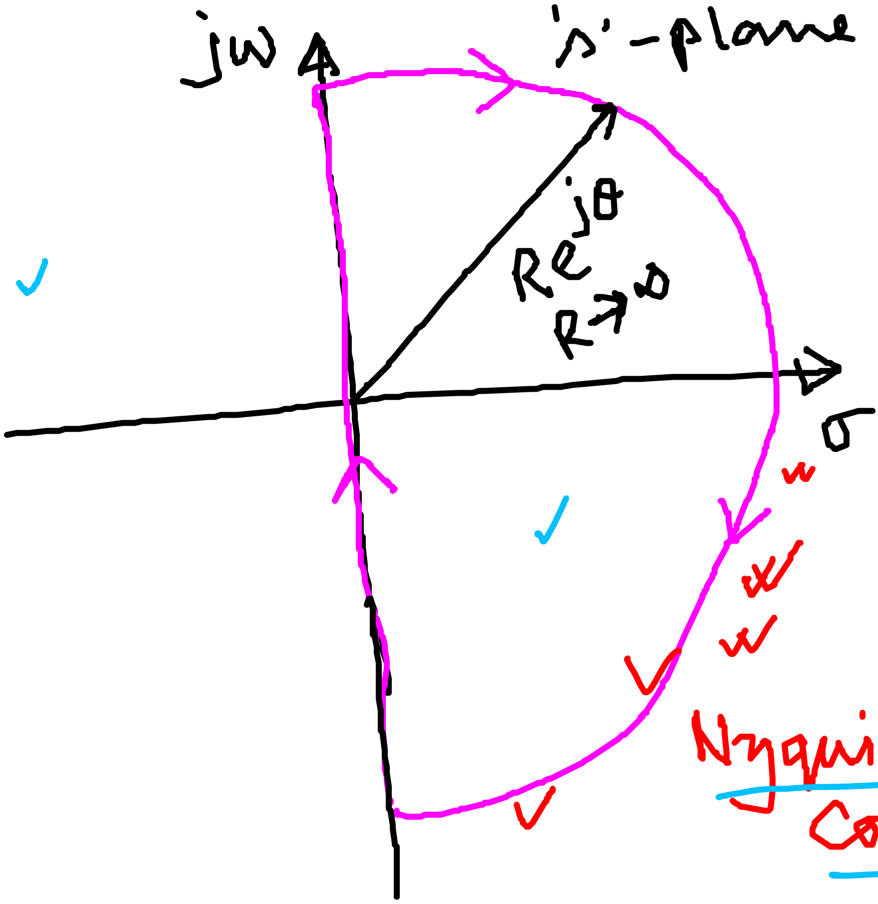
$$\boxed{N = \overset{1}{P} - \overset{2}{Z}} = -1$$

N = The number of CCW rotation of Contour B about the origin.

P = The number of poles of 1 + G(s) H(s) inside the contour A.

Z = The number of zeros of 1 + G(s) H(s) inside the contour A.

This is known as 'Principle of argument'.

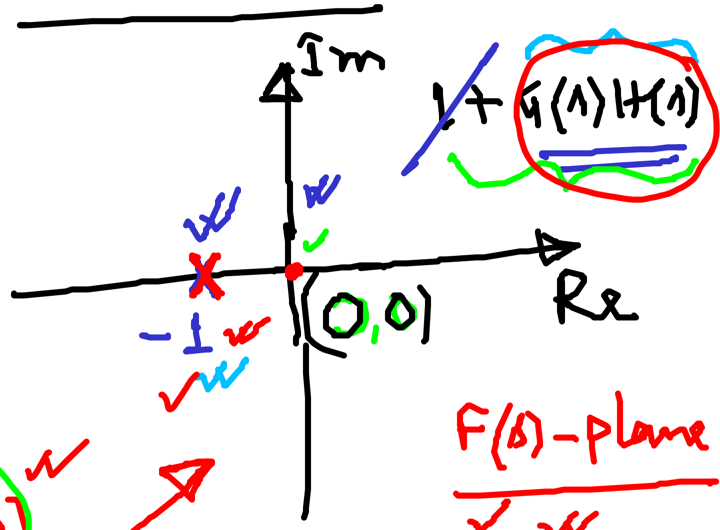


$\boxed{N = P - Z}$ from principle of argument.

Mapping via $G(s)H(s)$

Nyquist Contour.

$F(s) = G(s)H(s)$



$N = P - Z$

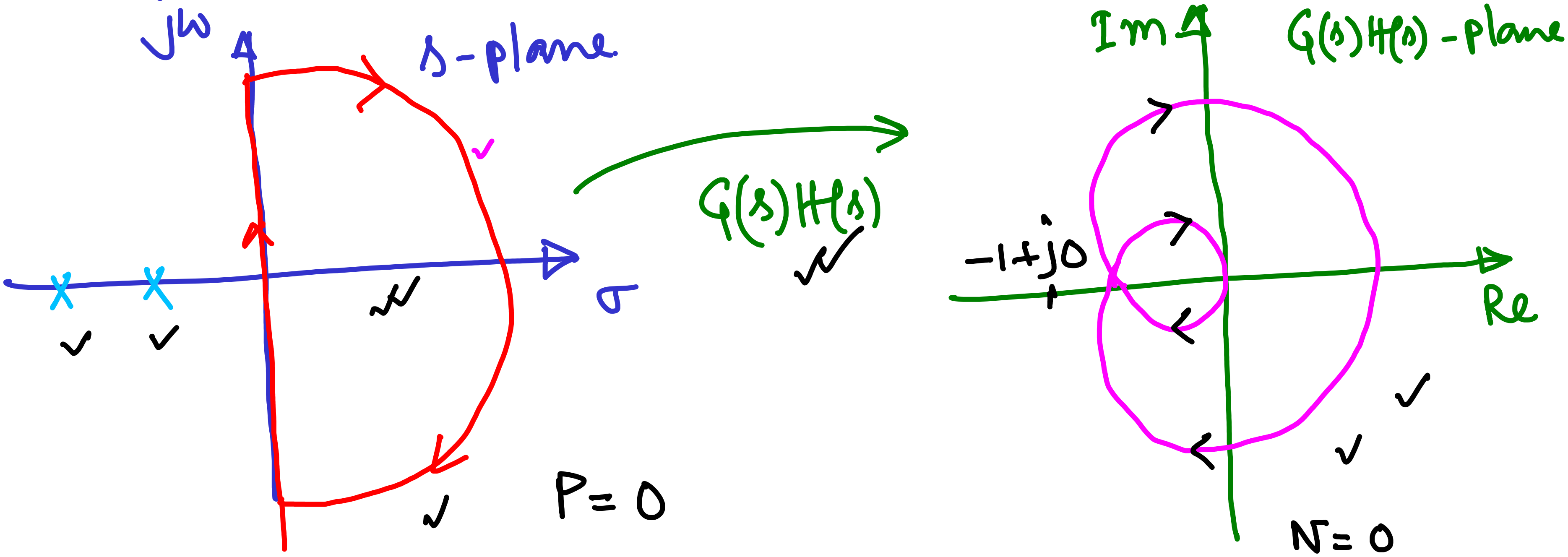
$\Rightarrow Z = P - N$ ← CCW rotation enclosing $-1+j0$.

P: No of poles of loop TF $(G(s)H(s))$ enclosed by the Nyquist Contour

Z: No of closed-loop poles enclosed by the Nyquist Contour.

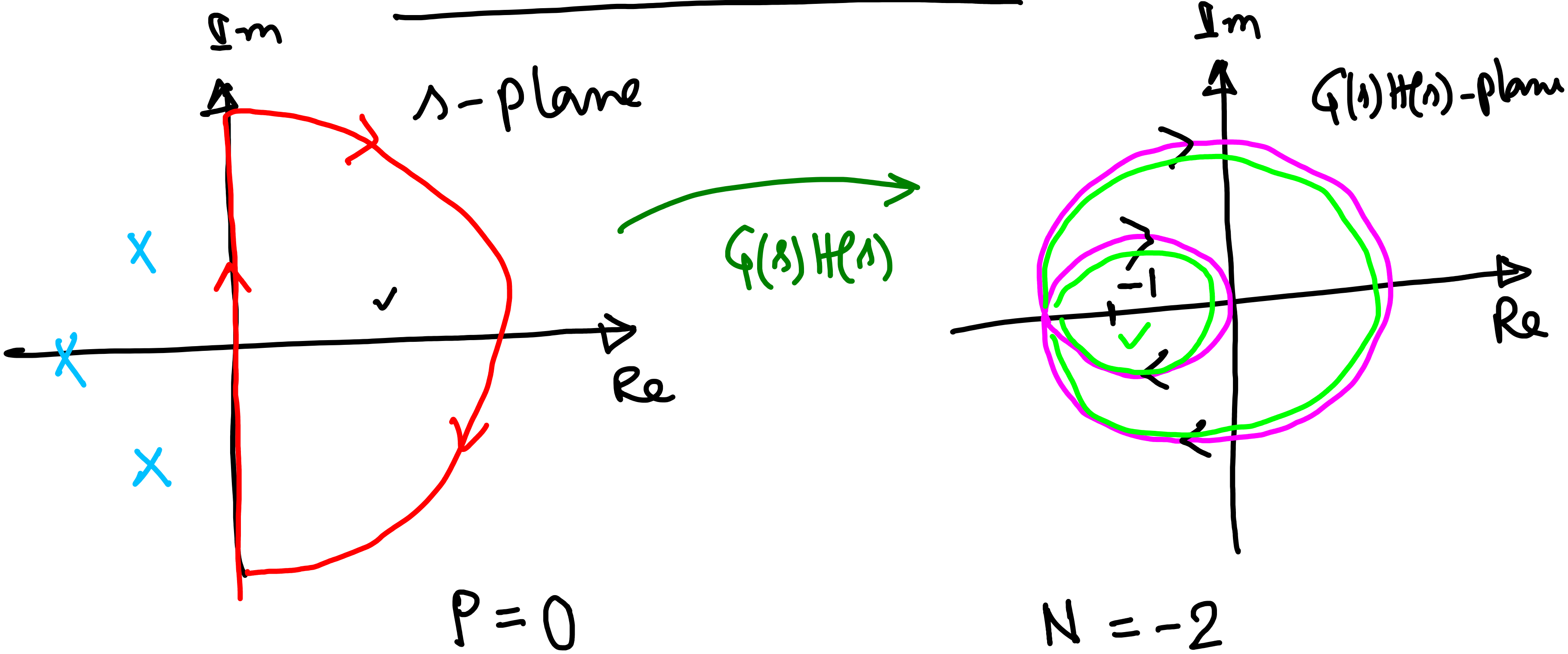
The closed-loop system is stable if the number of encirclement about $-1+j0$ in CCW direction is equal to the number of loop TF poles in RHP.

Ex



$Z = P - N = 0 - 0 = 0$

Closed-loop system stable.

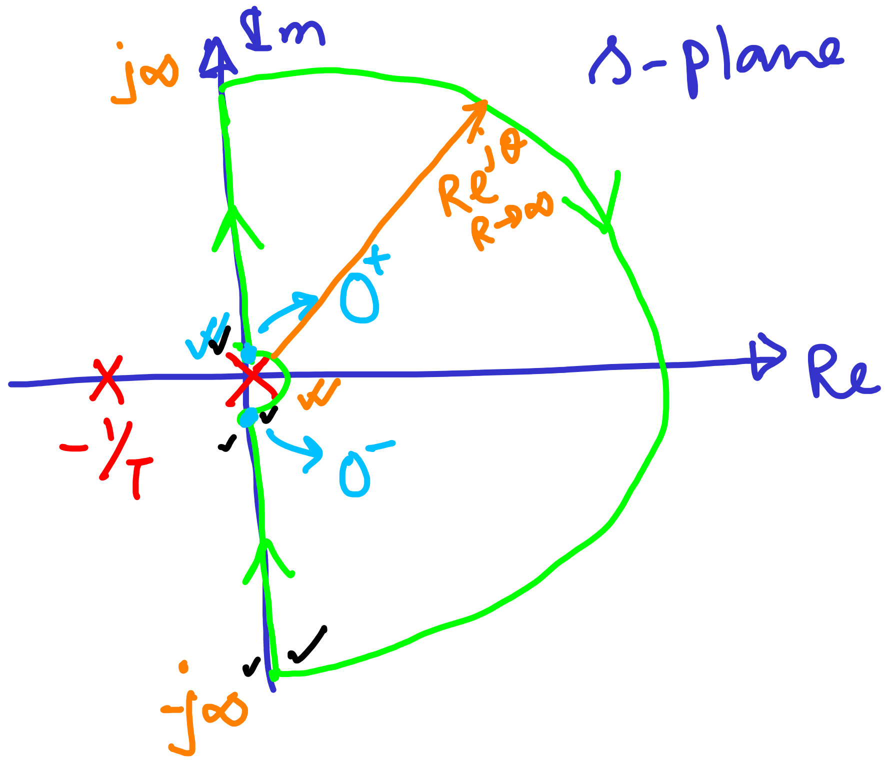


$Z = P - N$

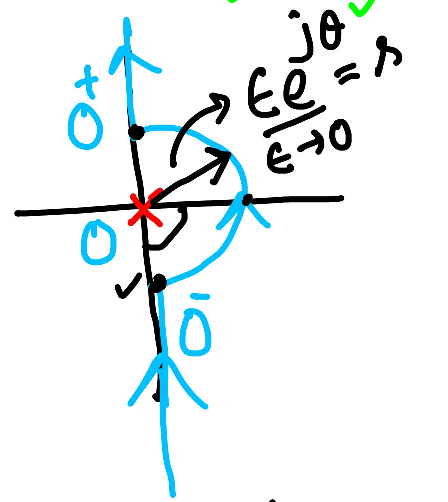
$= 0 - (-2) = 2$ (Two poles are in RHP)

The closed-loop system is unstable.

When $G(s)H(s)$ has poles and/or zeros on the $j\omega$-axis

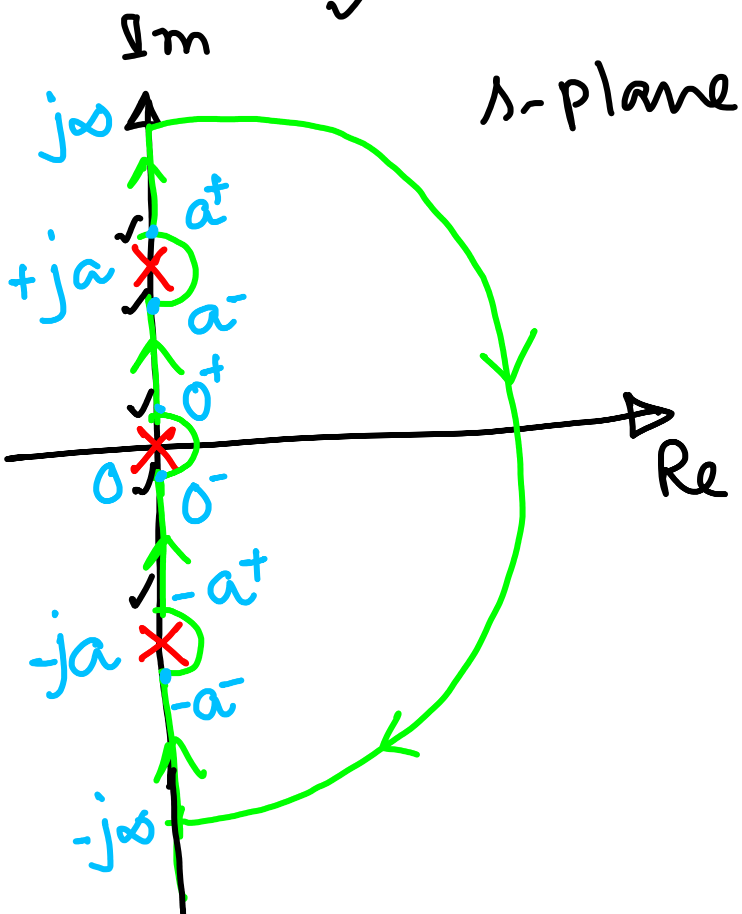


$$GH(s) = \frac{1}{s(Ts+1)}$$



$-\infty$ to 0^- to 0^+ to ∞ ✓

$\epsilon \to 0$ and
θ varies as
$-90°$ to 0 to $90°$

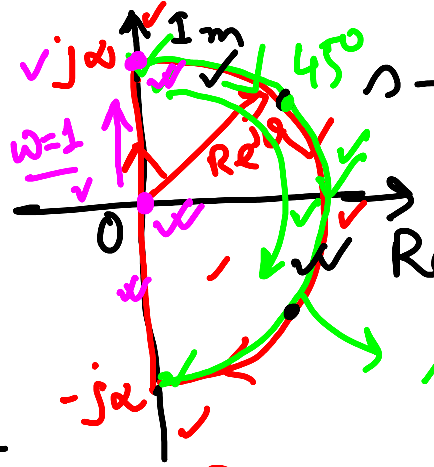


variation of ω

$-\infty$ to $-a^-$ to $-a^+$ to 0^-
to 0^+ to a^- to a^+ to ∞.

Ex

$$G(s)H(s) = \frac{1}{s^2 + 0.8s + 1}$$



$P = 0$ θ varies 90° to 0° to -90°

$$G(j\omega)H(j\omega) = \frac{1}{-\omega^2 + j0.8\omega + 1}$$

$$= \frac{(1-\omega^2) - j0.8\omega}{(1-\omega^2)^2 + 0.64\omega^2} = \frac{1-\omega^2}{(1-\omega^2)^2 + 0.64\omega^2} + j\,\frac{-0.8\omega}{(1-\omega^2)^2 + 0.64\omega^2}$$

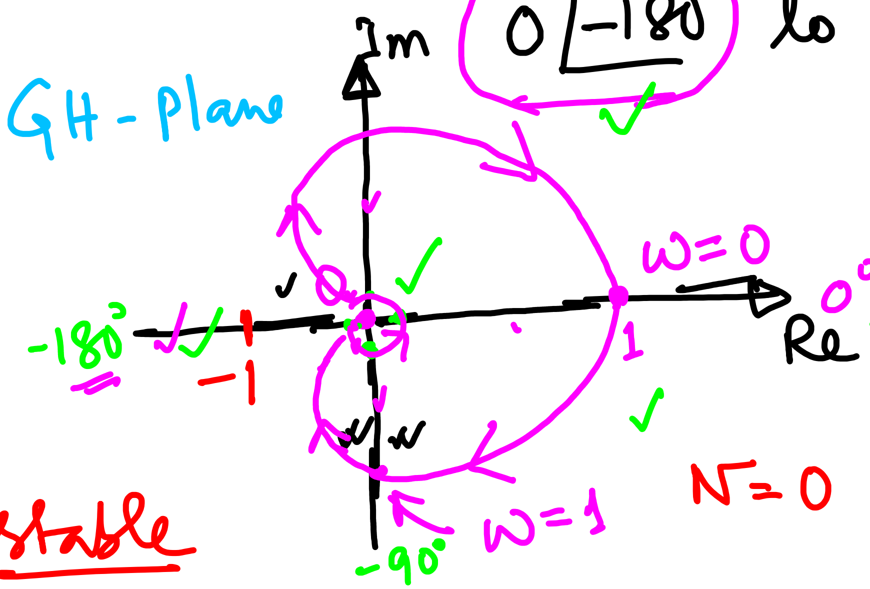
$$\lim_{\omega \to 0} GH(j\omega) = 1\angle 0^\circ$$

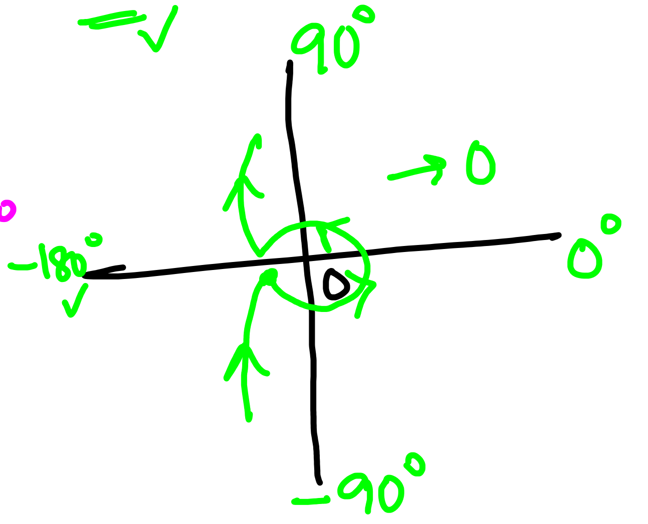
$$\lim_{\omega \to \infty} GH(j\omega) = 0$$

$$\lim_{R \to \infty} GH(Re^{j\theta}) = \lim_{R \to \infty} \frac{1}{R^2 e^{j2\theta} + 0.8Re^{j\theta} + 1} = 0\angle -2\theta$$

to $0\angle 90^\circ$

$0\angle -180^\circ$ to $0\angle 0^\circ$ to $0\angle 180^\circ$.





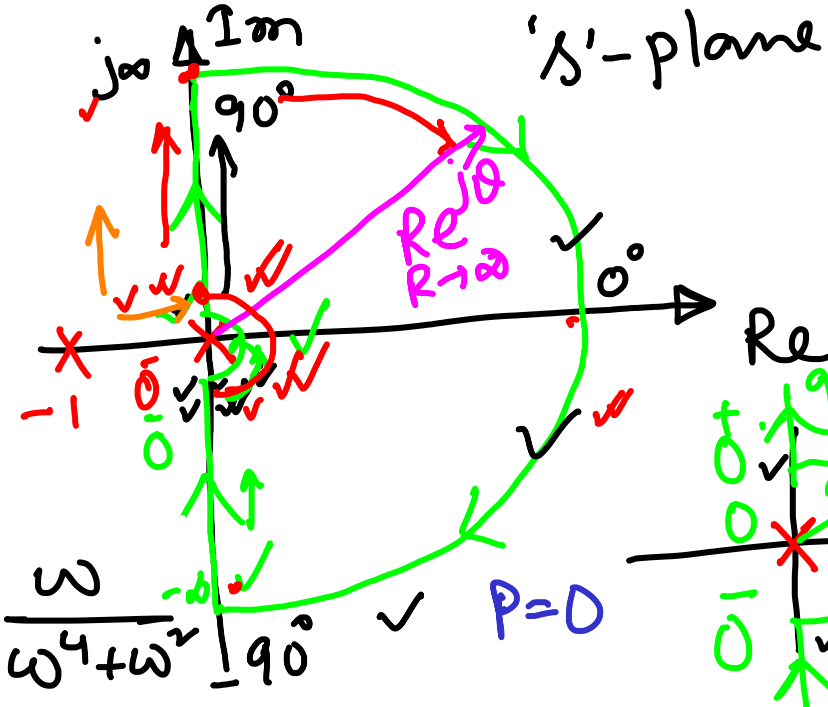
$Z = P - N$
$= 0 - 0 = 0$

stable

$N = 0$

Ex

$$GH(s) = \frac{1}{s(s+1)}$$

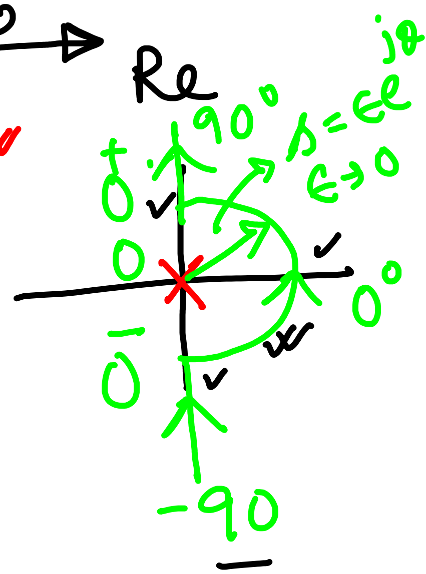


$$GH(j\omega) = \frac{1}{-\omega^2 + j\omega}$$

$$= \frac{-\omega^2 - j\omega}{\omega^4 + \omega^2} = -\frac{\omega^2}{\omega^4+\omega^2} - j\frac{\omega}{\omega^4+\omega^2}$$

$$= -\frac{1}{\omega^2+1} - j\frac{1}{\omega(\omega^2+1)}$$

$-1 - j\infty$



$$GH(\epsilon e^{j\theta}) = \lim_{\epsilon\to 0} \frac{1}{\epsilon e^{j\theta}(\epsilon e^{j\theta}+1)} = \lim_{\epsilon\to 0} GH(\epsilon e^{j\theta}) = \infty \angle -\theta$$

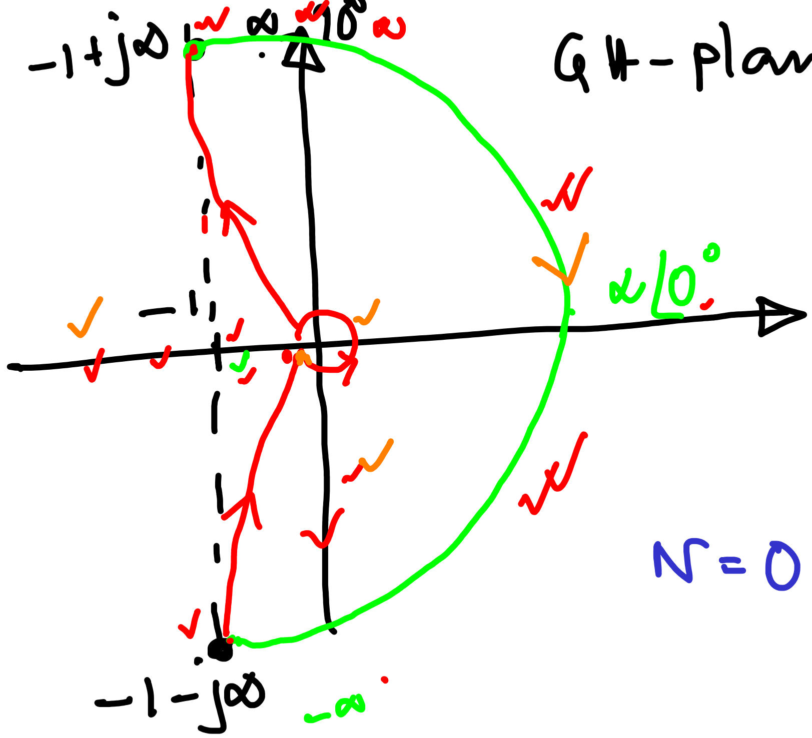
$\infty \angle 90°$ to $\infty \angle 0°$ to $\infty \angle -90°$

$$\lim_{\omega\to\infty} GH(j\omega) = 0$$

$$GH(Re^{j\theta}) = \frac{1}{Re^{j\theta}(Re^{j\theta}+1)}$$

$$\lim_{R\to\infty} GH(Re^{j\theta}) = 0 \angle -2\theta$$

$0 \angle -180°$ to $0 \angle 0°$ to $0 \angle 180°$



Z = 0
(stable)

N = 0