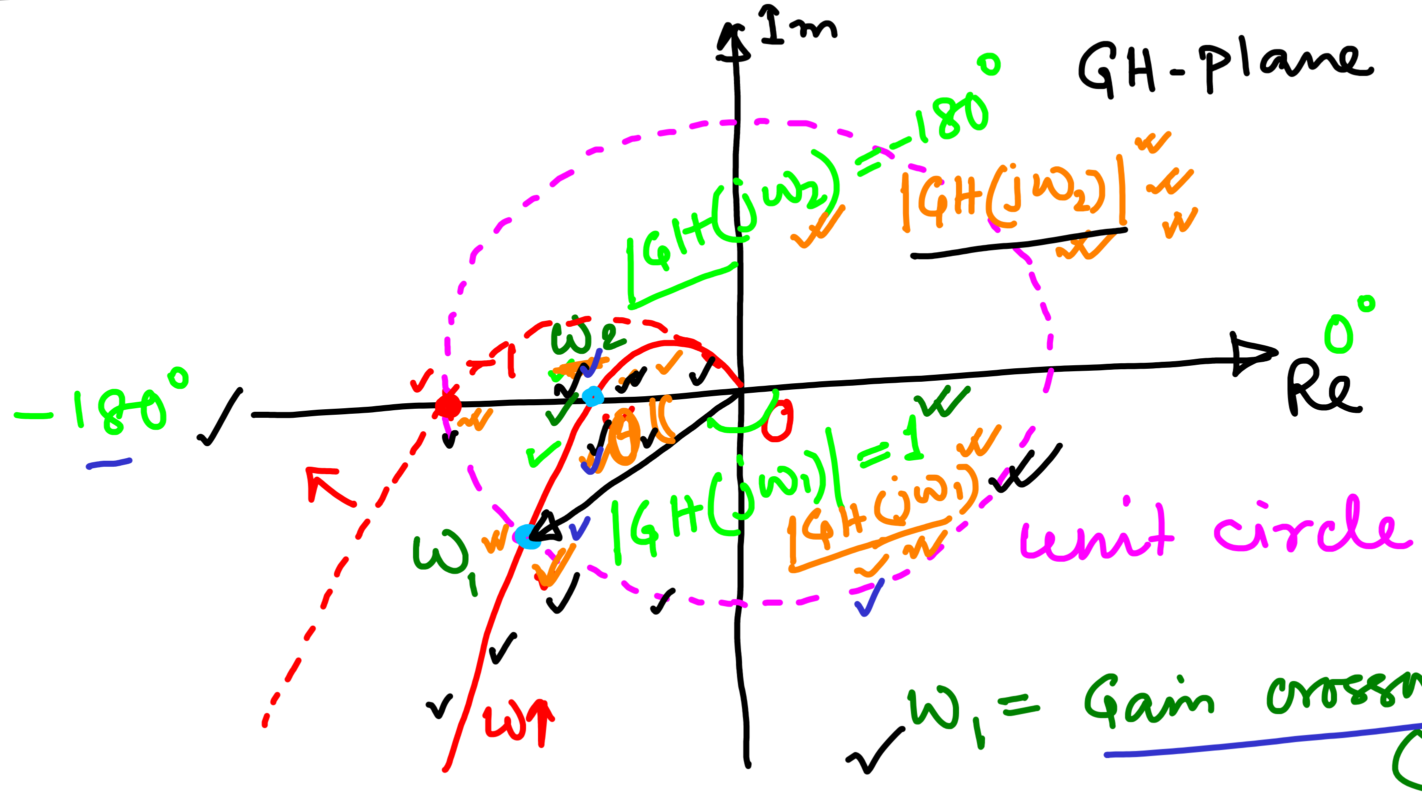
Gain Margin and Phase margin



ω_1 = Gain crossover freq. (GCF)

ω_2 = Phase crossover freq. (PCF)

Gain crossover frequency (GCF): The frequency at which $|GH(j\omega)| = 1$.

Phase crossover frequency (PCF): The frequency at which $\angle GH(j\omega) = -180°$.

Gain margin (GM): The gain margin is the reciprocal of the magnitude $|GH(j\omega)|$ at PCF.

$$K_g = \frac{1}{|GH(j\omega_{PCF})|}$$

in dB $20 \log k_g = -20 \log |GH(j\omega_{PCF})|$

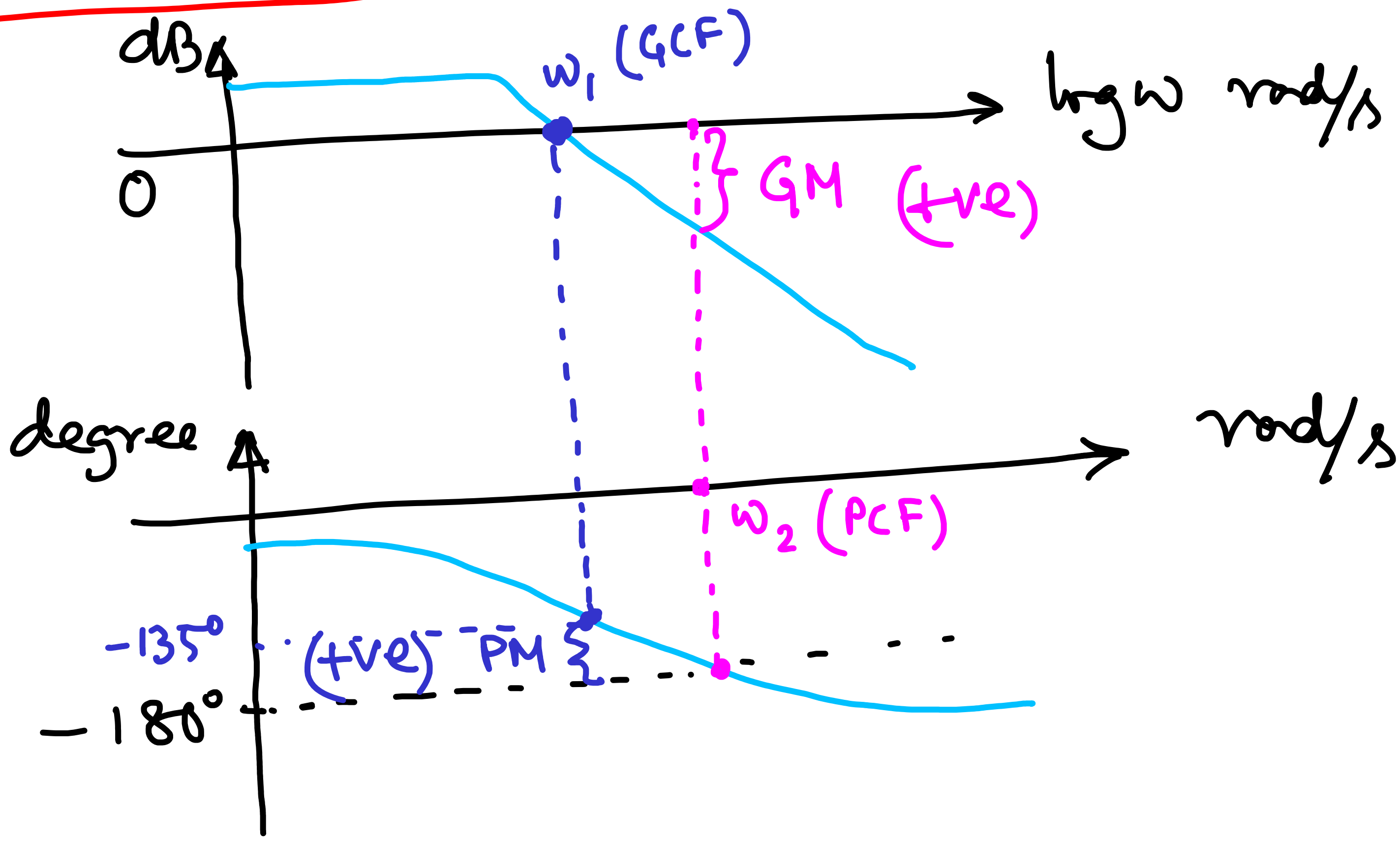
Phase margin (PM): The amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

$$PM = 180^\circ + \angle GH(j\omega_{gcF})$$ ✓

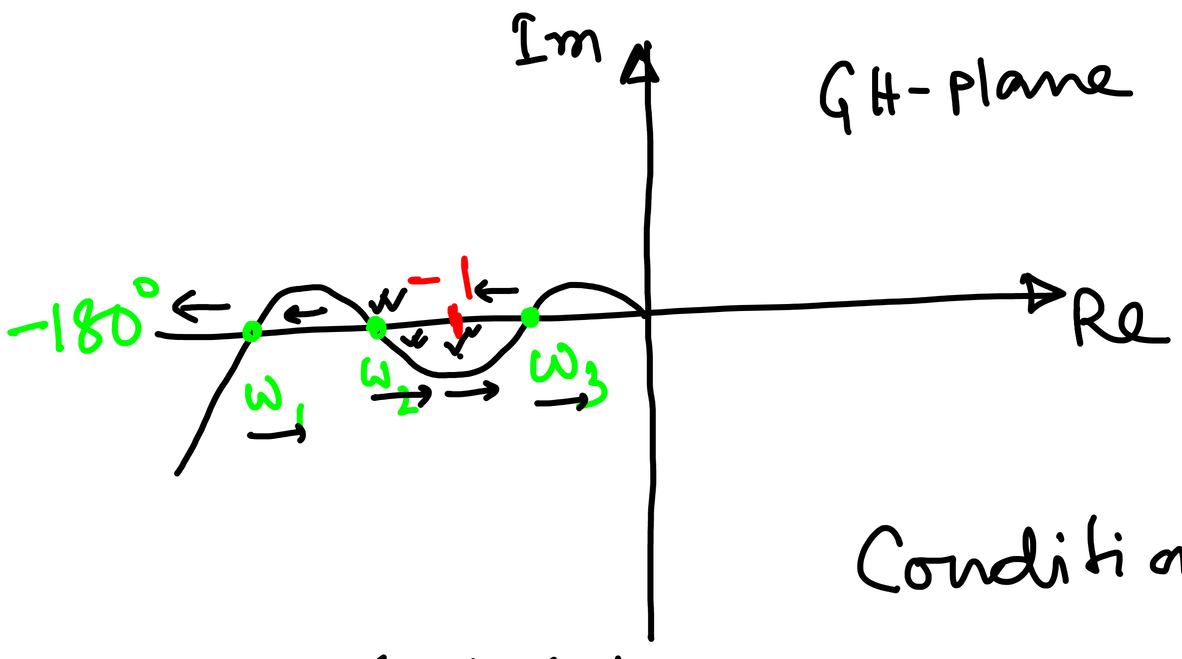
-135° $= 45^\circ$ ✓✓

$\boxed{\text{Expected } PM = 30^\circ \text{ to } 60^\circ \text{ and } GM \geq 6\,dB}$

GM and PM from Bode Plot

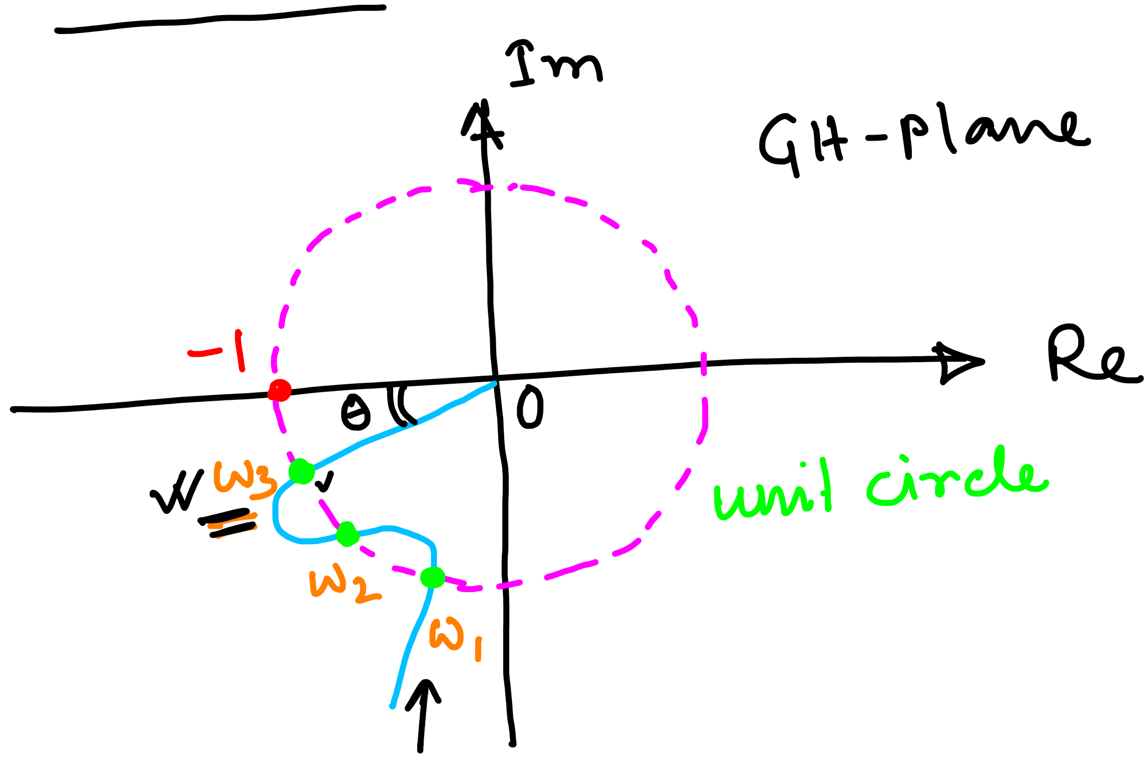


Multiple GCF and PCF



Conditionally stable ✓

$K_2 \leq K \leq K_4$



Delay margin

Characteristic eq^n ✓

$$1 + \underbrace{L(j\omega)}_{\text{loop TF}} e^{-\tau j\omega} = 0$$

$$L(j\omega)\, e^{-\tau j\omega} = -1$$

$$\angle L(j\omega_g) - \omega_g \tau \frac{180^\circ}{\pi} = 180^\circ \quad \text{and} \quad |L(j\omega_g)| = 1$$

(gain crossover freq.)

We know that

$$PM = 180^\circ + \angle L(j\omega_g)$$

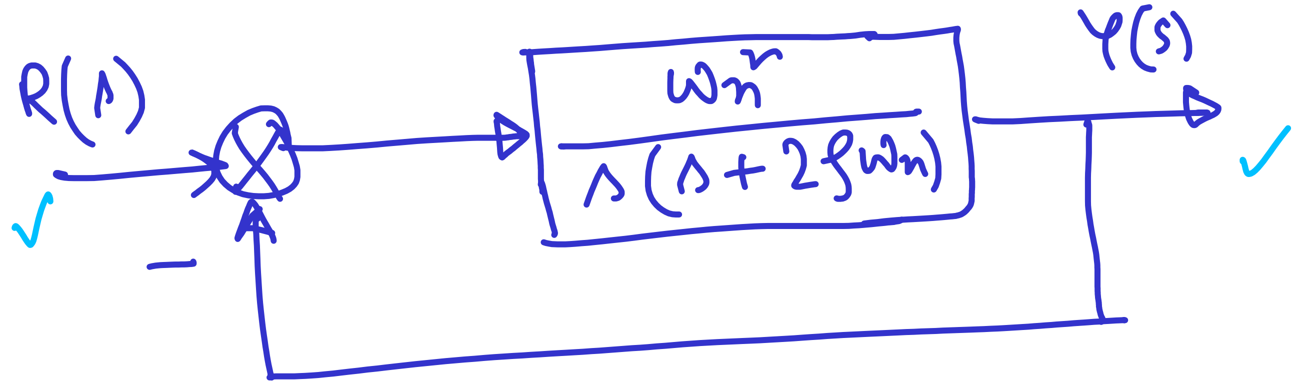
$$\angle L(j\omega_g) = PM - 180^\circ.$$

$$PM - 180^\circ - \omega_g \tau \frac{180^\circ}{\pi} = 180^\circ$$

$$PM = \omega_g \tau \frac{180^\circ}{\pi}$$

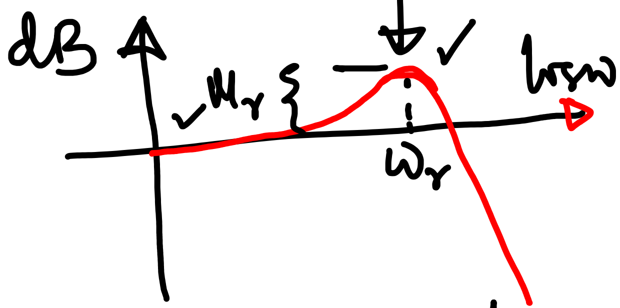
$$\boxed{\tau = \frac{PM}{\omega_g} \times \frac{\pi}{180^\circ}}$$

Relationship with time-domain analysis



$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonance peak



Maximum peak in Bode Plot occurs at $\omega_r = \omega_n\sqrt{1-2\zeta^2}$.

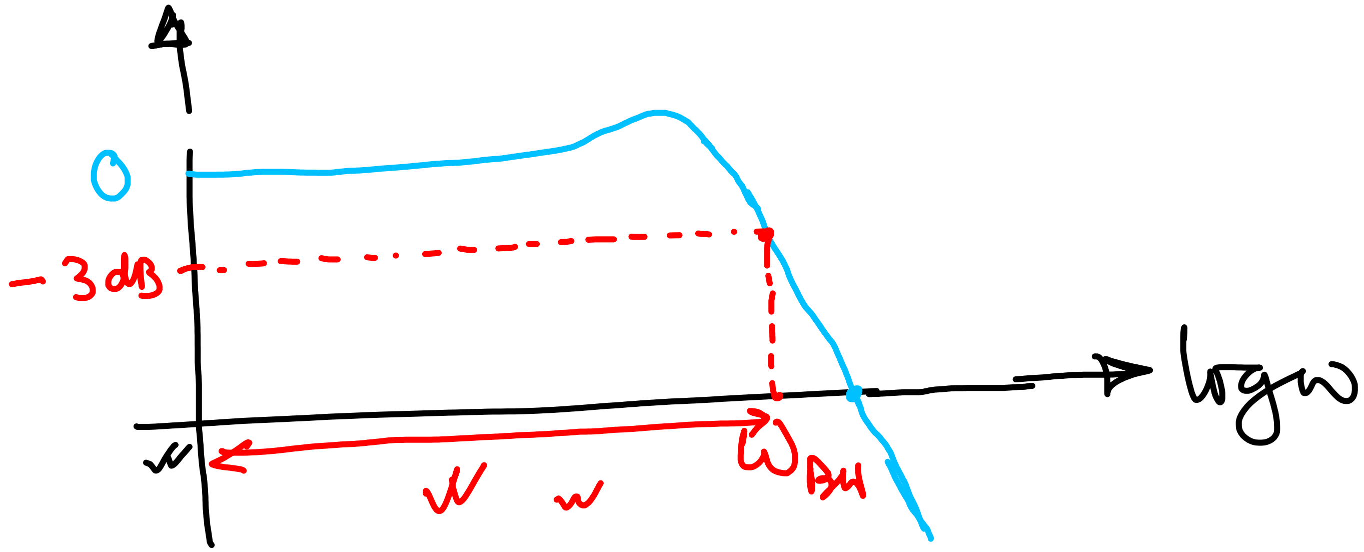
Resonance frequency

Resonant peak occurs when $0 \le \zeta \le 0.707$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$ (Resonant Peak)

- Large M_r indicates damping is poor.
  Expected M_r is $1 < M_r \le 1.4$
- For $0 \le \zeta \le 0.6$, $\zeta \approx \frac{PM}{100}$
- For small value of ζ, ω_d and ω_r are almost same.

Bandwidth



The frequency range $0 \leq \omega \leq \omega_{BW}$ in which the magnitude of the closed-loop does not drop -3 dB is called the bandwidth of the system.

$\downarrow$ Rise time $\propto \frac{1}{BW \uparrow}$