Statistical Signal Processing (EE60102)

Test 1, Spring 2021-2022

Time: 1 hour **Total marks: 20**

Q1. Let X be a continuous random variable with the cumulative distribution function (cdf) $F_X(x)$. Let $Y = F_X(X)$. Show that Y is a uniform random variable over $(0,1)$.

(3)

Q2. Suppose that X and Y are independent standard normal ($\mathcal{N}(0,1)$) random variables. Find the pdf of $Z = X + Y$.

(3)

Q3. A voltage V is a function of time t and is given by

$$V(t) = X\cos\omega t + Y\sin\omega t$$

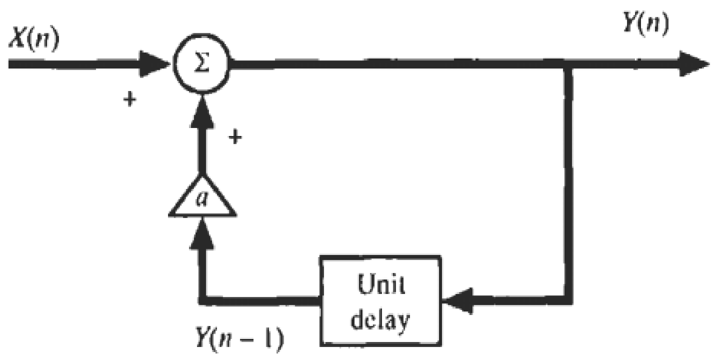
in which ω is a constant angular frequency and $X = Y = N(0;\sigma^2)$ and they are independent.

(a) Show that $V(t)$ may be written as $V(t) = R\cos(\omega t - \Theta)$

(b) Find the pdf's of random variables R and Θ and show that they are independent.

(4)

Q4. The discrete-time system shown in Fig. below consists of one unit delay element and one scalar multiplier (a $<$ 1). The input $X(n)$ is discrete-time white noise with average power σ^2. Find the spectral density and average power of the output $Y(n)$.



(4)

Q5. A zero mean Normal random vector $\boldsymbol{X} = (X_1, X_2)^T$ has the covariance matrix given by

$$K = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

Find a transformation $\boldsymbol{Y} = D\boldsymbol{X}$ such that $\boldsymbol{Y} = (Y_1, Y_2)^T$ is a Normal random vector with decorrelated components of unity variance.

(3)

Q6. Prove that the correlation matrix of a discrete-time stochastic process is always non-negative definite.

(3)