AST

# Assignment - 2

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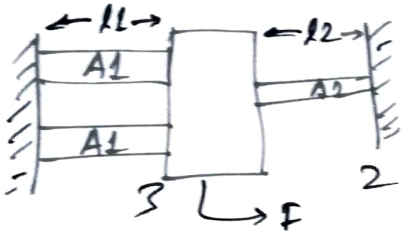
1.




$F = 1000\,N$, along node 3

$A1 = 16 \times 10^{-4}\ m^2$, $l1 = 1m$ $\quad E = 2 \times 10^{11}\ N/m^2$

$A2 = 9 \times 10^{-4}\ m^2$, $l2 = 0.5m$

$$\begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix} = E \begin{bmatrix} 2A1/L1 & -2A1/L1 & 0 \\ -2A1/L1 & \frac{2A1}{L1} + \frac{A2}{L2} & -\frac{A2}{L2} \\ 0 & -\frac{A2}{L2} & \frac{A2}{L2} \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ v3 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 64 & -64 & 0 \\ -64 & 100 & -36 \\ 0 & -36 & 36 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times 10^7$$

Stiffness Matrix $= 10^7 \begin{bmatrix} 64 & -64 & 0 \\ -64 & 100 & -36 \\ 0 & -36 & 36 \end{bmatrix}$ N/m

$v_1 = 0$, $v_2 = 0$

$F = (-64 v_1 + 100 v_3 - 36 v_2) \times 10^7$

$= 100 v_3 \times 10^7 \Rightarrow$ **$v_3 = 1\,\mu m$**

2.

$$k_1 = \frac{E \cdot A_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 32\times10^7\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = \frac{E \cdot A_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 36\times10^7\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_3 = \frac{E \cdot A_3}{l_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 32\times10^7\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(i) Combined Stiffness Matrix

$$K = 10^7 \times \begin{bmatrix} 32 & -32 & 0 & 0 \\ -32 & (32+36) & -36 & 0 \\ 0 & -36 & (36+32) & -32 \\ 0 & 0 & -32 & 32 \end{bmatrix}$$

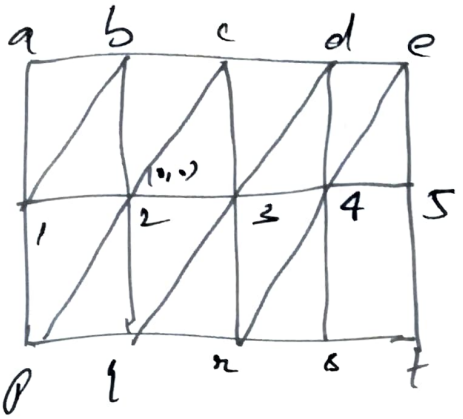
(ii) $F = 9.4 \Rightarrow u = \begin{bmatrix} k_0 = 0 \\ k_1 \\ k_2 \\ k_3 \end{bmatrix}$ $\quad F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 5\,kN \end{bmatrix}$

$F_2 = F_3 = 0$, $k_0 = 0$

$$\Rightarrow 10^7\begin{bmatrix} 68 & -36 & 0 \\ -36 & 68 & -32 \\ 0 & -32 & 32 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10^3 \end{bmatrix}$$

$k_1 = 3.125\,\mu m$, $k_2 = 5.903\,\mu m$

$k_3 = 9.028\,\mu m$




Let point (2) = (0,0) $\quad p = (-1,-1), \quad c = (+1,1)$

$r = (1,-1)$

$V_p = V_r = 0, \quad V_c = 100\ V$

$V_2 = a + bx + cy$

$V_p = 0 \Rightarrow \quad a - b - c = 0 \quad -(1)$

$V_c = 100 \Rightarrow \quad a + b + c = 100 \quad -(2)$

$V_r = 0 \Rightarrow \quad a + b - c = 0 \quad -(3)$

from (1), (2) & (3),

$a = 50, \quad b = 0, \quad c = 50$

$V_2 = a + b \times 0 + c \times 0 = 50\ V$

$V_3 = a + b(+1) + c \times 0 = 50\ V$

$V_1 = a + b(-1) + c \times 0 = 50\ V$

$V_4 = a + b \times 2 + c \times 0 = 50\ V$

$V_5 = a + b \times 3 + c \times 0 = 50\ V$

4. Total energy stored $= w$

$= \frac{1}{2} b x^2 + \frac{1}{2} c V^2$

$C = \frac{\epsilon_0 A}{d - x}$

$$F = -\frac{dW}{dx} = -kx + \frac{1}{2}\frac{\varepsilon_0 A}{(d-x)^2}V^2$$

At equilibrium,

$$kx = \frac{1}{2}\frac{\varepsilon_0 A}{(d-x)^2}AV^2$$

$$V = \sqrt{\frac{2kx(d-x)^2}{\varepsilon_0 A}} = 3.0059\ V$$

$k = 10\ N/m$, $d = 5\mu$, $x = 4\mu$, $A = 4\ mm^2$

$$\text{At } V_p = \frac{d^2W}{dx^2} = 0 = k - \frac{\varepsilon_0 A}{(d-x)^3}V_p^2$$

$$V_p = \sqrt{\frac{k(d-x)^3}{\varepsilon_0 A}}$$

$$F = \frac{dW}{dx} = 0$$

$$-kx + \frac{1}{2}\frac{\varepsilon_0 A}{(d-x)^2} \propto \frac{k(d-x)^3}{\varepsilon_0 A} = 0$$

$$\Rightarrow -x + \frac{d-x}{2} = 0$$

$$\Rightarrow x = \frac{d}{3}$$

$$V_p = \sqrt{\frac{k(d-\frac{d}{3})^3}{\varepsilon_0 A}} = 3.2338\ V$$

Pull-in $k = 10\ N/m$, $d = 5\mu$, $A = 4\ mm^2$

$\varepsilon_0 = 8.854 \times 10^{-12}$

5. (b) $V_{pi} = \sqrt{\frac{k(d-d_3)^3}{\epsilon_0 A}}$ (from Q4)

$= 1.1247\ V$

(ii) At equilibrium,

$k x = \frac{1}{2} \frac{\epsilon_0 A}{(d-x)^2} V^2$

Let $d = \frac{\epsilon_0 A V^2}{2k} = 0.1171164 \times 10^{-18}$

$x = \frac{d}{(d-x)^2}$

$\Rightarrow x(d-x)^2 - d = 0 = f(x)$

Using Newton Raphson Method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, $x_0 = 0$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{d}{d^2} = 0.1171164 \times 10^{-16}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.1622131 \times 10^{-6}$

$= 0.162\ \mu m$

6. $C = \frac{\epsilon A}{d}$, $C_{total} = \frac{4\epsilon_0 k_m}{d}$

$E = \frac{1}{2} \times \frac{4\epsilon k x^2}{d}$

energy $= \frac{1}{2} \times \frac{4\epsilon k (x-\Delta x) V^2}{d}$

$\frac{1}{2} \times \frac{4\epsilon k (x-\Delta x)}{d} x^2 = \frac{1}{2} k (\Delta x)^2$

$\Rightarrow \boxed{\Delta x = \frac{4\epsilon k x V^2}{k d}}$