SSP End Sem

Pratyush Jaiswal
18EE35014

Q.2. (a) The second order AR process $u(n)$ is described by the difference equation:

$$u(n) = u(n-1) - 0.5\,u(n-2) + v(n)$$

The time-domain description of the autoregressive process is governed by the second-order difference eqn.

$$u(n) = -a_1 u(n-1) - a_2 u(n-2) + v(n)$$

Comparing with above,
The AR parameters equal

$$a_1 = -1 \quad \& \quad a_2 = 0.5$$

Thus $\quad w_1 = 1 \quad \& \quad w_2 = -0.5 \quad$ as $\{w_k = -a_k\}$

Accordingly we write the Yule Equations :-

$$\begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \end{bmatrix}$$

(b) Writing the Yule-Walker equations in expanded form:

$$r(0) - 0.5\,r(1) = r(1)$$

$$r(1) - 0.5\,r(0) = r(2)$$

Solving for $r(1)$ & $r(2)$,

$$r(1) = \frac{2}{3} r(0) \quad -① \qquad \& \qquad r(2) = \frac{1}{6}(r(0)) \quad -②$$

Also we know that

$$\sigma_v^2 = \sum_{k=0}^{2} a_k r(k)$$

$$= \cancel{r(0)} \cancel{a_1 r(2)}\ a_0 + a_1 r(1) + a_2 r(2) \quad -③$$

Substituting (1) & ② into ③ & solving for $r(0)$, we get

$$0.5 = r(0) + (-1)\frac{2}{3}r(0) + 0.5 \times \frac{1}{6} r(0)$$

$$\Rightarrow r(0) = \frac{0.5}{1 - \frac{2}{3} + \frac{1}{12}} = 1.2$$

then, $r(1) = \cancel{\frac{1}{8}}\ \frac{2}{3} r(0) = \frac{2}{3} \times 1.2 = 0.8$

$r(2) = \frac{1}{6} r(0) = \frac{1}{6} \times 0.2 = 0.2$

(C) Since the noise $v(n)$ has zero mean, so will the AR process $u(n)$. Hence,

$$\text{Var}[u(n)] = E[(u^2 n)]$$

$$= r(0)$$

$$= 1.2$$

Q.3: (a) The principal of orthogaility states that for the cost function

$$J = +[e(n)\, e^{*}(n)]$$

where $e(n)$ is the error,

to attain minimum value the corresponding estimation error ($e_o(n)$) needs to be the orthogonal to all the input samples that enters the filter at time n.

Mathematically,

$$E[u(n-k)\, e_o^{*}(n)] = 0$$

$k = 0, 1, 2 \ldots$

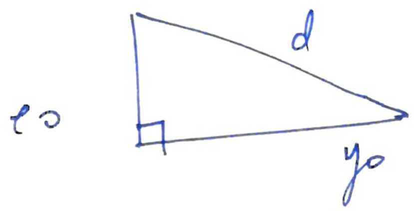
I/P → $u(n)$

$e_o(n)$ → optimal estimation error.

d → desired response.

y_o → optimum o/p.

e_o → estimation error



Statistics. Pythagora's Theorem

Q.3. (b)(i) For ensuring the convergence of the method of steepest descent,

$0 < \mu < \frac{2}{\lambda_{max}}$. where λ_{max} is the largest eigenvalue of the correlation matrix R.

Also, we are given

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Calculating eigen values,

$$|R - \lambda I| = 0$$

$$(1-\lambda)(1-\lambda) - 0.5^2 = 0$$

$$(1-\lambda-0.5)(1-\lambda+0.5) = 0$$

$$\lambda = 0.5 \text{ or } 1.5.$$

$$\lambda_1 = 0.5 \quad \& \quad \lambda_2 = 1.5$$

Hence, $\lambda_{max} = 1.5$.

The step-size parameter μ must therefore satisfy the condition

$$0 < \mu < \frac{2}{1.5} = 1.334$$

We may thus choose $\mu = 1.00$

(b) (ii) As we know,

$$w(n+1) = w(n) + \mu[P - Rw(n)]$$

with $\mu = 1$ and

$$P = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

Putting in above equation,

$$w(n+1) = w(n) + \left\{ \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} w(n) \right\}.$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \right) w(n) + \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix} \{ w(n) + \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

That is,

$$\begin{bmatrix} w_1(n+1) \\ w_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} w_1(n) \\ w_2(n) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

$$w_1(n+1) = -0.5\, w_2(n) + 0.5$$

$$w_2(n+1) = -0.5\, w_1(n) + 0.25$$

(c)

Q.3. (c) As we know that

$$\hat{\omega}(n+1) = \hat{\omega}(n) + \frac{\beta u(n)}{\|u(n)\|^2} e^*(n)$$

→ When $u(n)$ is large, LMS suffers from gradient noise amplification. With the normalization of the step size by $\|u(n)\|$ in the LMS algorithm, gradient noise amplification is reduced.

→ $\|u(n)\|^2$ alters the direction of estimated gradient vector.

→ The rate of convergence of the normalized LMS algorithm is potentially faster than traditional LMS algorithm.

Q. 4. (a) Here the observations are

$x[n]$, $x[n-1]$ & $x(n-2)$,

So, we will be using optimal filter of order $M=2$.

i.e, $s[n]$ is estimated using present & last two observations.

$$\therefore \hat{s}(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2)$$

$$\begin{bmatrix} R_x[0] & R_x[-1] & R_x[-2] \\ R_x[1] & R_x[0] & R_x[-1] \\ R_x[2] & R_x[1] & R_x[0] \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \end{bmatrix} = \begin{bmatrix} R_{sx}[0] \\ R_{sx}[1] \\ R_{sx}[2] \end{bmatrix}$$

Minimum Mean Squared Error (MSE) =

$$R_s[0] - \sum_{i=0}^{2} h[i]\, R_x[i].$$

$$R_s[l] = R_x[l] = 2(0.8)^{|l|}$$

$$R_{sx}[l] = E\{s(n)\, x(n-l)\}$$

$$= E\{s(n)\,(s(n-l) + w(n-l))\}$$

$$= R_s(l) = 2(0.8)^{|l|}$$

$$R_x[l] = E\{(s(n) + w(n))\,(s(n-l) + w(n-l))\}$$

$$= R_s(l) + R_w(l) = 2(0.8)^{|l|} + 2\delta(l)$$

The required values are:

$R_s[0] = 2$

$R_x[0] = R_s(0) + R_w(0) = 4$

$R_x[2] = R_x[-2] = \cancel{1.28}\ 2\times 0.8^2 = 1.28$

$R_{sx}[0] = 2.00.$

$2\times 0.8^1 + 2\times\delta(0)) = 1.6$

$R_{sx}[1] = \cancel{1.60}\ 2\times 0.8^1$

$R_{sx}[2] = 1.28.$

Hence, equation for filter parameter becomes,

$$\begin{bmatrix} 4 & 1.6 & 1.28 \\ 1.6 & 4 & 1.6 \\ 1.28 & 1.6 & 4 \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \end{bmatrix} = \begin{bmatrix} 2 \\ 1.6 \\ 1.28 \end{bmatrix}$$

After ~~form~~ solving the above eq^n,

$h(0) = 0.3824, \quad h(1) = 0.2000$

$h(2) = 0.1176.$

$$MMSE = R_s[0] - \sum_{l=0}^{2} h[l]\, R_x[l]$$

$$= 2 - 0.3824\times 2.000 - 0.200\times(1.60) - 0.1176\times 1.28$$

$$= 0.7647$$

$$\boxed{MMSE = 0.7647}$$

Q.5.
(a) from Levinson-Durbin algorithm, we need to find b_1, b_2 & b_3.

Finding b_1:

$$\Delta_0 = \gamma(1) = 0.8 \;,\; P_0 = \gamma(0) = 1$$

$$\Rightarrow b_m = \frac{-\Delta_{m-1}}{P_{m-1}}, \text{ Putting } m=1$$

$$b_1 = \frac{-\Delta_0}{P_0} = -0.8$$

Finding b_2

$$b_2 = \frac{-\Delta_1}{P_1}$$

We have, $P_m = P_{m-1}(1-|b_m|^2)$

let $m=1$

$$P_1 = P_0(1-|b_1|^2)$$

$$= 1(1-(-0.8)^2)$$

$$= 1(1-0.64) = 0.36$$

$$=$$

Also, we have $\Delta_{m-1} = \sum_{k=0}^{m-1} a_{m-1,k}\, \gamma(m-k)$

let $m=2$, $\Delta_{2-1} = \sum_{k=0}^{2-1} a_{2-1,k}\, \gamma(2-k)$

$$\Delta_1 = \sum_{k=0}^{1} a_{1,k}\, \gamma(2-k)$$

$$\Delta_1 = a_{1,0}\,\gamma(2) + a_{1,1}\,\gamma(1)$$

$$\Delta_1 = 1\,\gamma(2) + b_1\,\gamma(1)$$ $\left[\because a_{m,m} = b_m \atop a_{m,0} = 1 \right]$

$$\Delta_1 = 1 \times 0.6 + (-0.8) \times 0.8$$

$$\Delta_1 = -0.04$$

$$k_2 = \frac{-\Delta_1}{P_1} = \frac{-(-0.04)}{0.36} = 0.1111$$

<u>finding b_3:</u>

$$k_3 = \frac{-\Delta_2}{P_2}$$

$$P_m = P_{m-1}\left(1 - |b_m|^2\right)$$

Putting, $m = 2$,

$$P_2 = P_1\left(1 - (k_2)^2\right)$$

$$= 0.36\left(1 - (0.1111)^2\right)$$

$$= 0.3556$$

$$\Delta_{m-1} = \sum_{k=0}^{m-1} a_{m-1,k}\,\gamma(m-k)$$

$m = 3$

$$\Delta_2 = \sum_{k=0}^{2} a_{2,k}\,\gamma(3-k)$$

$$\Delta_2 = a_{2,0}\gamma(3) + a_{2,1}\gamma(2) + a_{2,2}\gamma(1)$$

$a_{2,0} = 1$

$a_{2,2} = b_2 = 0.1111$

$a_{2,1} = b_1 + b_2 b_1 = -0.8 + 0.1111(-0.8)$

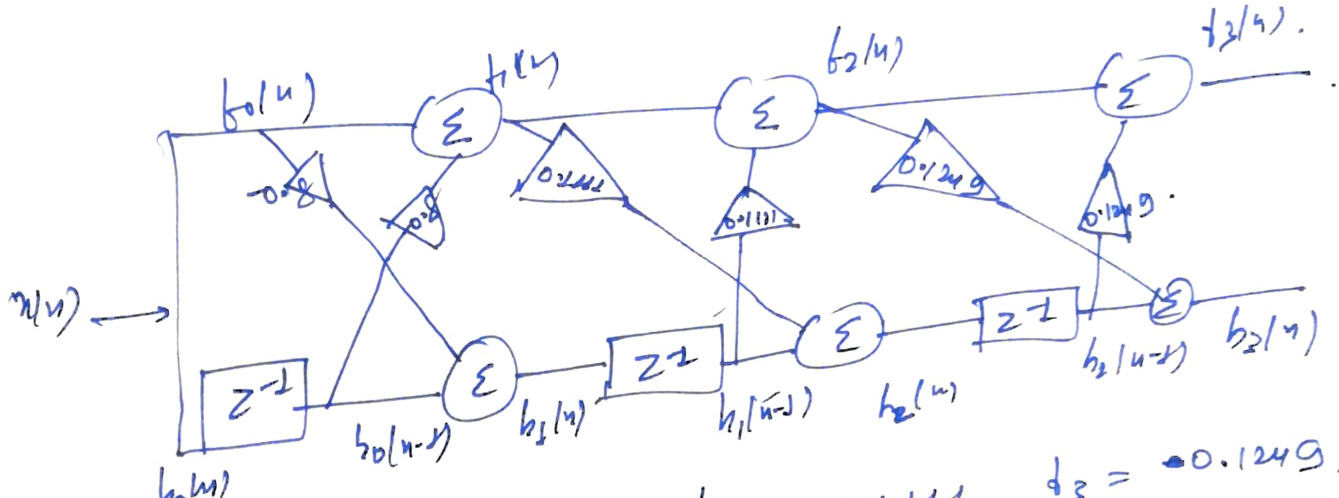
$= -0.8889$

$$\Delta_2 = 1(0.4) + (-0.8889)(0.6) + (0.1111)(0.8)$$

$$\Delta_2 = -0.0444$$

$$b_3 = -\frac{\Delta_2}{P_2} = \frac{-(-0.4444)}{0.3556} = 0.1249$$

(ii) Setup of three stage lattice predictor is.



$b_1 = -0.8$, $b_2 = 0.2222$, $b_3 = 0.1249$.

(iii) We need to find P_3 :-

$$P_3 = P_2(1 - |b_3|^2)$$

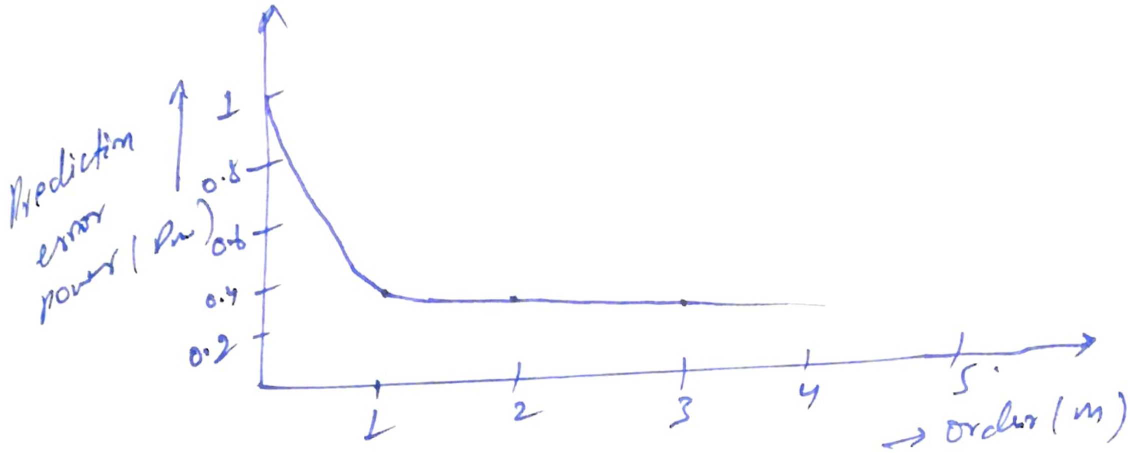
$$P_3 = 0.3556(1 - (0.1249)^2)$$

$$P_3 = 0.3501$$

Average power of prediction power is

$P_0 = 1$, $P_1 = 0.36$, $P_2 = 0.3556$, $P_3 = 0.3501$

Plot of prediction error power versus prediction order.



Q6 (b)

$$u^B(n) = [u(n-M+1), u(n-M+2) \dots u(n)]^T$$

Writing the Wiener Hopf Equation,

$$R g = r^{B*}$$

$$\begin{bmatrix} r(0) & r(1) & \dots & r(M-1) \\ r(-1) & r(0) & \dots & r(M-2) \\ \vdots & & & \\ r(-M+1) & r(-M+2) & \dots & r(0) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix} = \begin{pmatrix} r(M) \\ r(M-1) \\ \vdots \\ r(1) \end{pmatrix}$$

Equivalently we may be write,

$$\sum_{k=1}^{M} g_k r(k-i) = r(M+1-i) \quad i = 1, 2, \dots M.$$

let $k = M - l + 1$ or $l = M - k + 1$ then,

$$\sum_{l=1}^{M} g_{M-l+1} r(M-l+1-i) = r(M+1-i), \quad i = 1, 2 \dots M$$

Next, putting $M+1-i = j$ or $i = M+1-j$ then

$$\sum_{l=1}^{M} g_{M-l+1} r(j-l) = r(j) \quad j = 1, 2, \dots M.$$

Putting this relation into matrix,

$$\begin{bmatrix} r(0) & r(-1) & \dots & r(-M+1) \\ r(1) & r(0) & \dots & r(-M+2) \\ \vdots & & & \\ r(M-1) & r(M-2) & \dots & r(0) \end{bmatrix} \begin{bmatrix} g_M \\ g_{M-1} \\ \vdots \\ g(1) \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(M) \end{bmatrix}$$