Second-order system

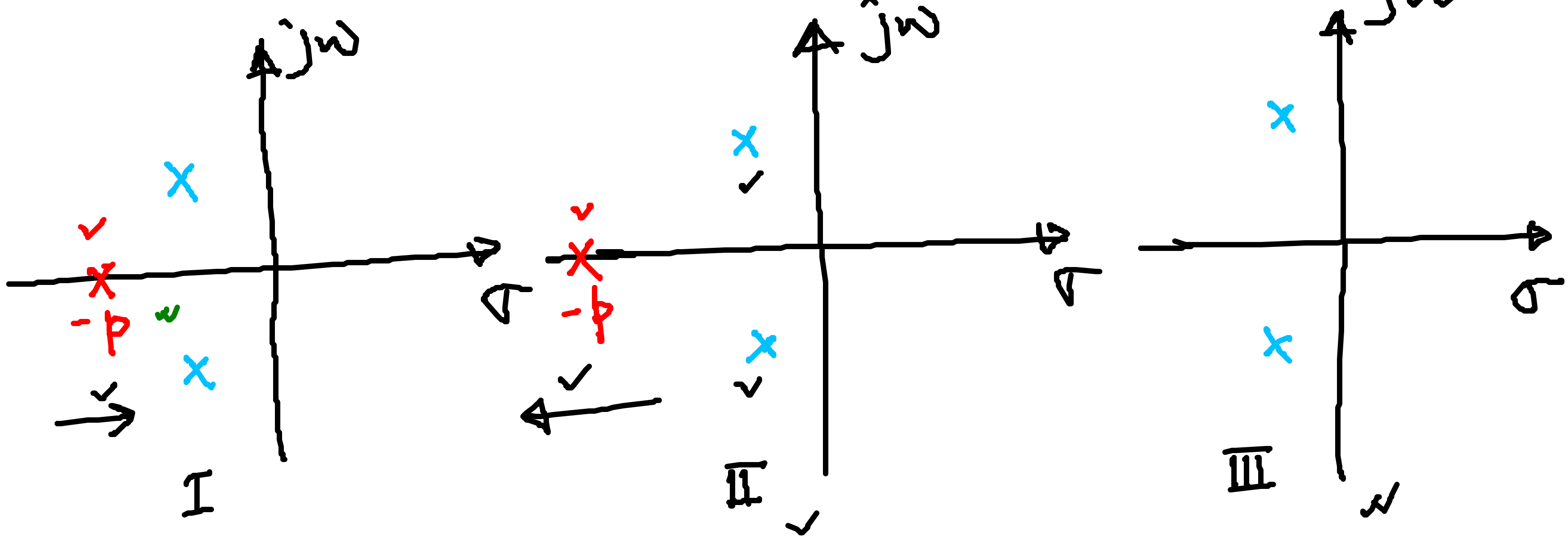
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Adding a pole:

$$G(s) = \frac{p \times \omega_n^2}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$y(t) = A\,u(t) + e^{-\zeta\omega_n t}\left(B\cos\omega_d t + C\sin\omega_d t\right) + D\,e^{-pt}$$

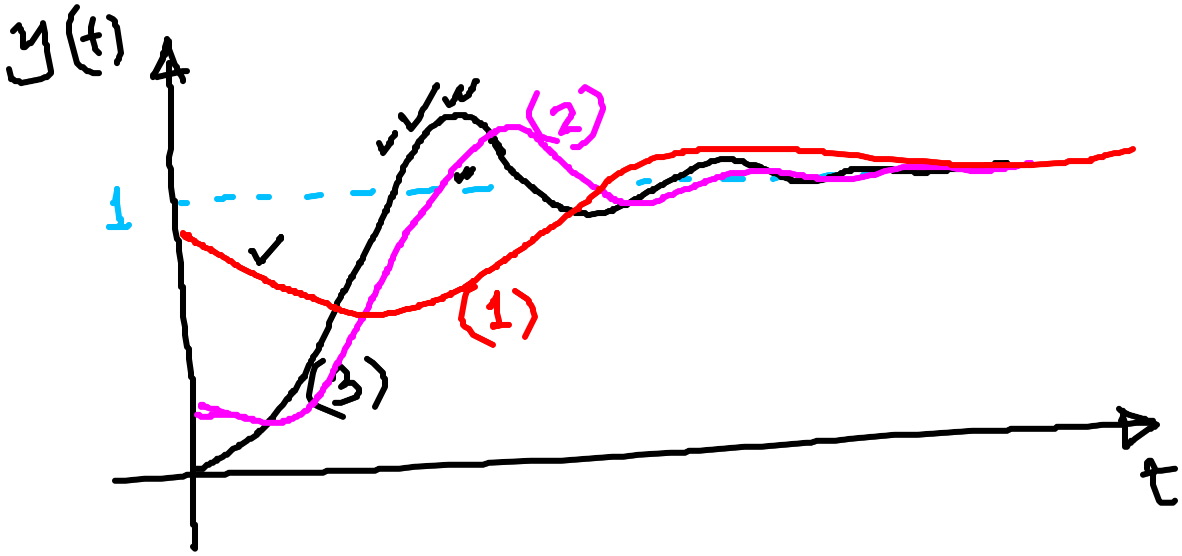
If p is sufficiently large, the response can be characterized by ζ and ω_n.



$$G_1(s) = \frac{73.626}{(s+3)(s^2 + 4s + 24.542)}$$

$$G_2 = \frac{245.42}{(s+10)(s^2 + 4s + 24.542)}$$

$$G_3 = \frac{24.542}{s^2 + 4s + 24.542}$$



Adding a zero:

$$G(s) = \frac{1\,(s+1)}{s^2 + 0.5s + 1}$$

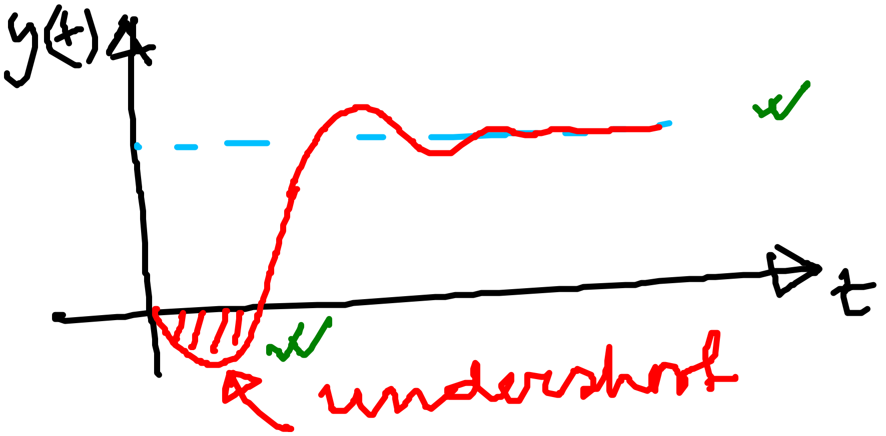
$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 0.5s + 1} + s\,\frac{1}{s^2 + 0.5s + 1}$$

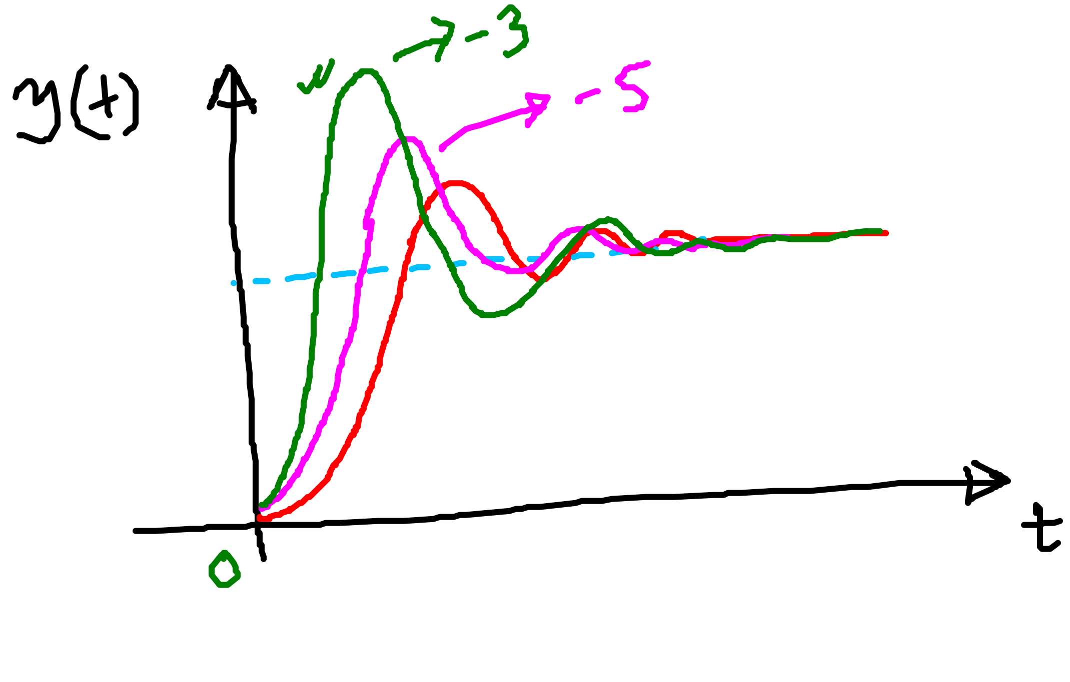
$$Y(s) = \frac{1}{s^2 + 0.5s + 1} \times U(s) + s\,\frac{1}{s^2 + 0.5s + 1}\,U(s)$$

$$y'(t) = y(t) + \frac{dy(t)}{dt}$$

$$y'(t) = \mathcal{L}^{-1}\underbrace{\left[(s+a)\;Y(s)\right]} = a\,y(t) + \frac{dy(t)}{dt}$$

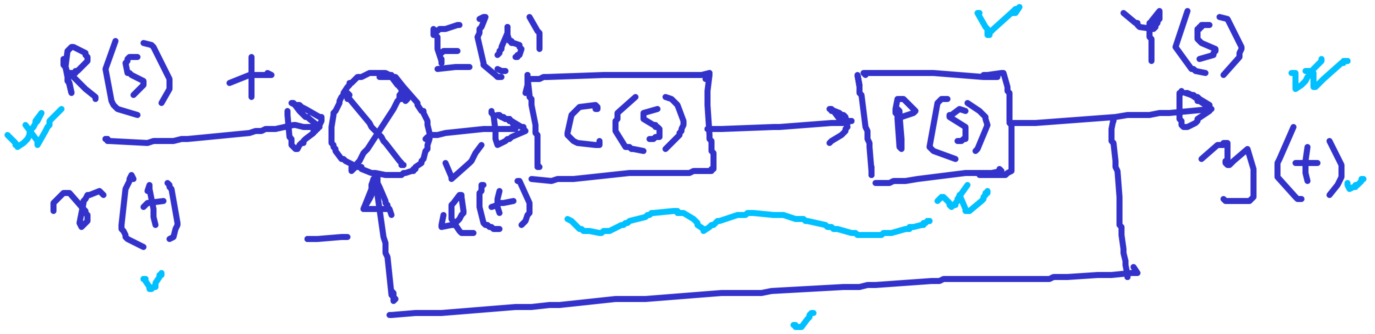
- If a is very large, we can neglect $\frac{dy}{dt}$ effect and it is a scaled version of the original response.
- If a is not very large, the response has an additional component. Typically, $\frac{dy}{dt}$ is positive at the start and hence it increases the overshoot. When a is -ve (nonminimum phase system), a negative kick is observed.





Steady-state error

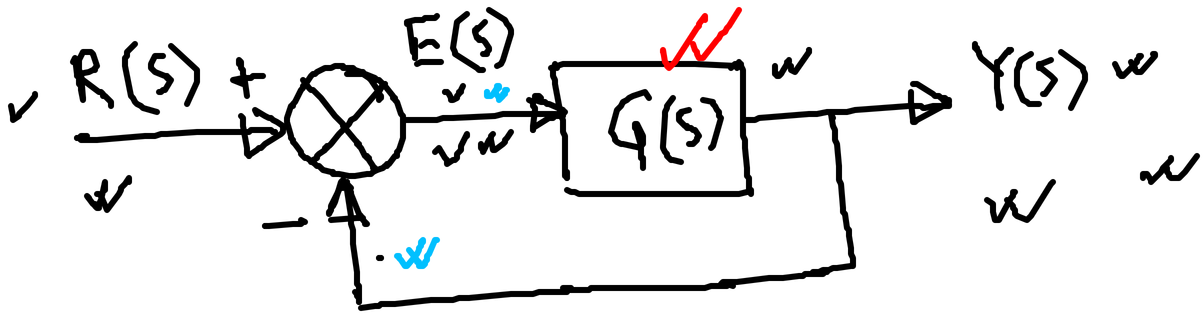
- SS error is the difference between the input and the output for a prescribed test input as $t \to \infty$.



$$\text{SS error} = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} r(t) - y(t)$$

Test signals: Step input (1) ✓ Ramp input (t) Parabolic $\left(\frac{1}{2}t^2\right)$

Steady-state error for unity feedback system



SS error: $E(s) = R(s) - Y(s) = R(s) - G(s)E(s)$

$$E(s) = \frac{R(s)}{1 + G(s)}$$ ✓

Applying the final value theorem

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{s\to 0}{Lt}\ sE(s) = \underset{s\to 0}{Lt}\ \frac{sR(s)}{1+G(s)} \quad \frac{1}{s},\ \frac{1}{s^2}$$

Step input:

$$e_{ss} = \frac{1}{1+\underset{s\to 0}{Lt}\ G(s)} = \frac{1}{1+K_p}$$

$$K_p = \underset{s\to 0}{Lt}\ G(s) \quad [\text{Position Constant}]$$

Ramp input:

$$e_{ss} = \underset{s\to 0}{Lt}\ \frac{sR(s)}{1+G(s)} = \underset{s\to 0}{Lt}\ \frac{\cancel{s}\cdot\frac{1}{s^{\cancel{2}}}}{1+G(s)}$$

$$= \underset{s\to 0}{Lt}\ \frac{1}{s+sG(s)} = \frac{1}{\underset{s\to 0}{Lt}\ sG(s)} = \frac{1}{K_v}$$

$$K_v = \underset{s\to 0}{Lt}\ sG(s) \quad [\text{Velocity Constant}]$$

Parabolic input:

$$e_{ss} = \underset{s\to 0}{Lt}\ \frac{sR(s)}{1+G(s)} \qquad R(s) = \frac{1}{s^3}$$

$$= \frac{1}{\underset{s\to 0}{Lt}\ s^2G(s)} = \frac{1}{K_a}$$

$$K_a = \underset{s\to 0}{Lt}\ s^2G(s) \quad [\text{Acceleration Const.}]$$

- We call these constants as static error constants.

- The number of pure integrators in the TF is known as the type of system.

No integrator – $G(s) = \frac{1}{s+1}$ type-0

1 integrator $G(s) = \frac{1}{s(s+1)}$ type-1

2 integrators $G(s) = \frac{1}{s^2(s+1)(s+2)}$ type-2

and so on.

$G(s) = \frac{1}{s+2}$

$\lim_{s\to 0} G(s) = \frac{1}{2} = K_p$

$$\boxed{K_p = \lim_{s\to 0} G(s)} \qquad \boxed{K_v = \lim_{s\to 0} s\,G(s)} \qquad \boxed{K_a = \lim_{s\to 0} s^2 G(s)}$$

$K_v = \lim_{s\to 0} \frac{s}{s(s+1)}$ $= 0$

$e_{ss} = \frac{1}{1+K_p}$ $\qquad e_{ss} = \frac{1}{K_v}$ $\qquad e_{ss} = \frac{1}{K_a}$

Input	SS error	Type 0: static constant	Type 0: SS error	Type-1: static const.	Type-1: SS error	Type-2: static constant	Type-2: SS error
Step	$\frac{1}{1+K_p}$	$K_p = \text{Const.}$	$\frac{1}{1+K_p}$	∞	0	∞	0
Ramp	$1/K_v$	$K_v = 0$	∞	$K_v = \text{Const.}$	$\frac{1}{K_v}$	$K_v = \infty$	0

Parabolic $\quad 1/K_a \quad \checkmark K_a = 0 \quad \infty \quad K_a = 0 \quad \infty \quad K_a = \text{Cont.} \quad \frac{1}{K_a}$

$$G(s) = \frac{1}{s(s+1)} \quad \text{[Type-1]}$$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{1}{s(s+1)} = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}(s+1)} = 1 \text{ (finite)}$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^{\cancel{2}} \frac{1}{\cancel{s}(s+1)} = 0$$

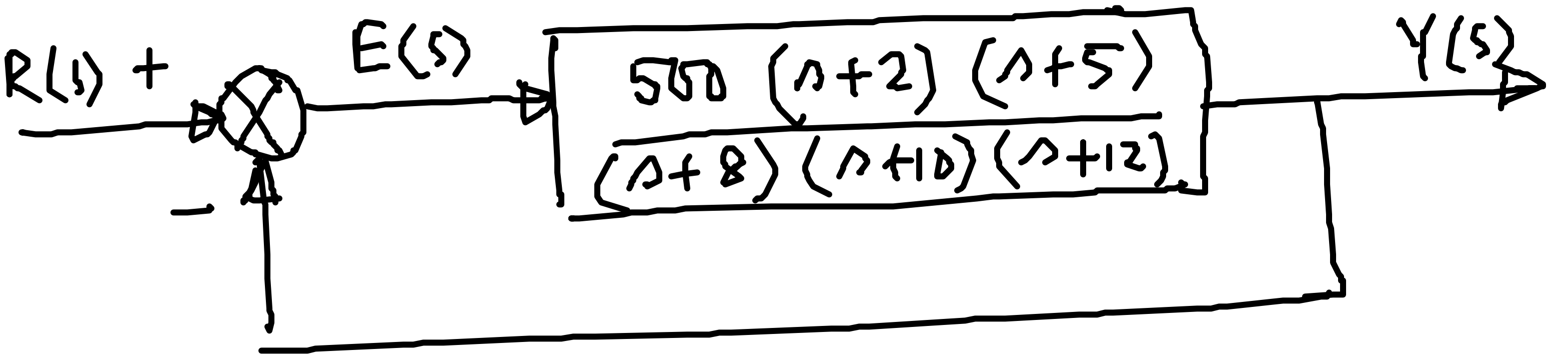
$$G(s) = \frac{1}{s^2(s+1)} \quad \text{[Type-2]}$$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{1}{s^2(s+1)} = \infty \checkmark$$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{\cancel{s}}{s^{\cancel{2}}(s+1)} = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{\cancel{s^2}}{\cancel{s^2}(s+1)} = 1 \text{ (finite)}$$

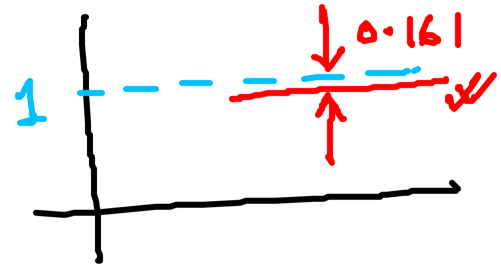
Example:



Type 0 system.

$$k_p = \lim_{s\to 0} G(s) = \frac{125}{24} = 5.208$$

$$\lim_{t\to\infty} e(t) = r(t) - y(t)$$



$$k_v = 0 \quad , \quad k_a = 0$$

$$e_{ss_{step}} = \frac{1}{1+5.208} = 0.161$$

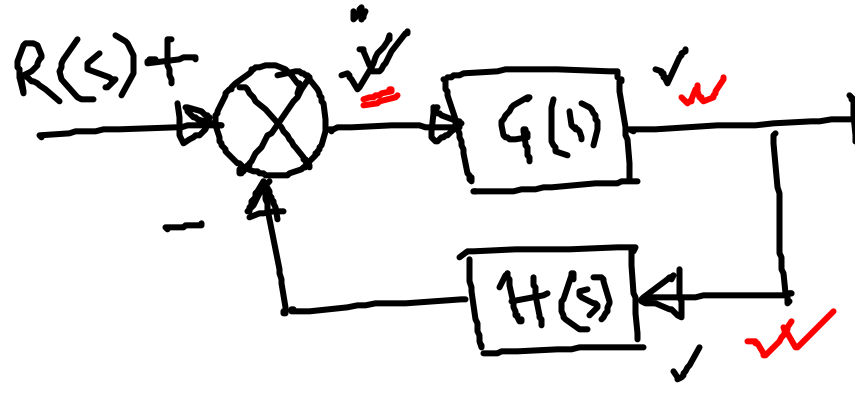
Example:

What information is contained in the specification $k_p = 1000$?

1) System stable (2) Type 0 (3) Step input.

4) SS error $= \frac{1}{1+k_p} = \frac{1}{1001}$

Steady-state error for non unity feedback



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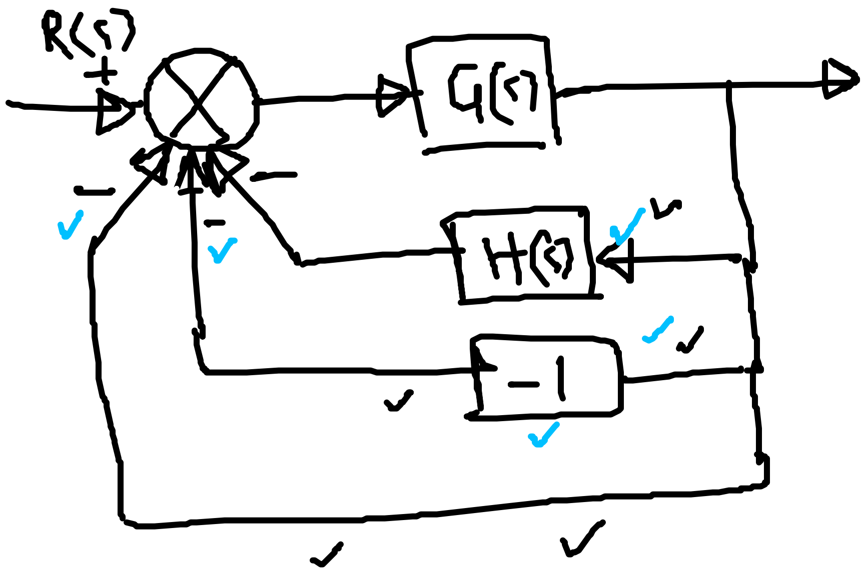

$$E(s) = R(s) - Y(s)$$

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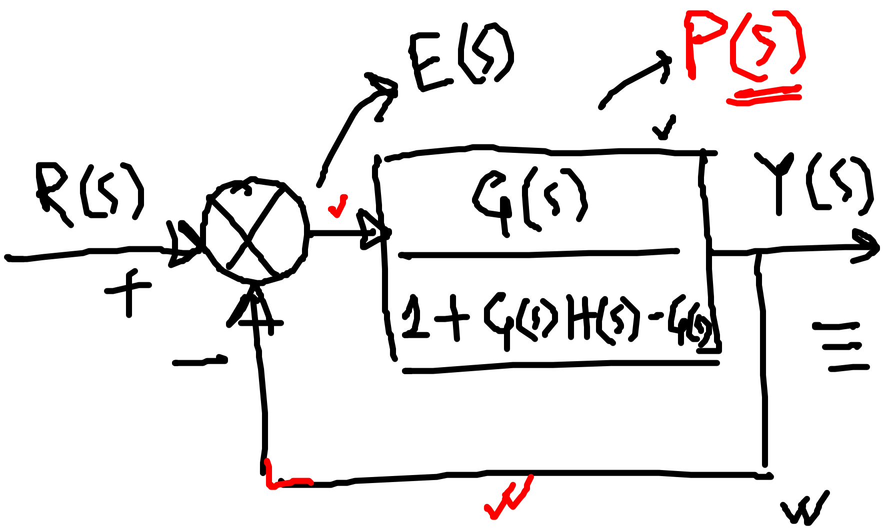

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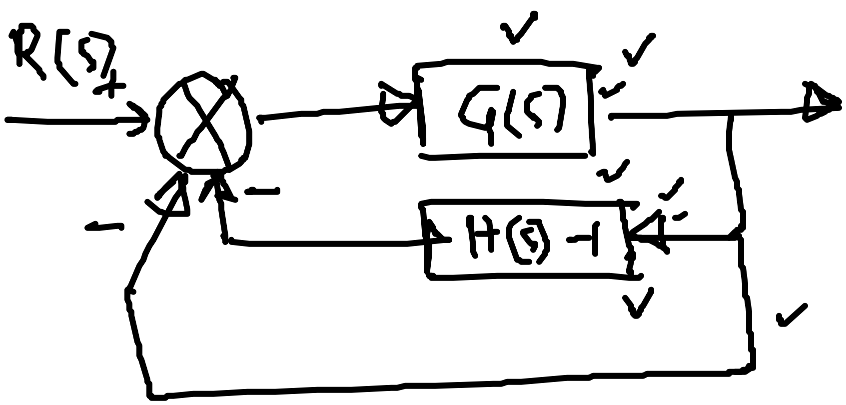

$$P(s) = \frac{G(s)}{1 + G(s)\,H(s) - G(s)}$$