Assignment-1

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y. Sample space look like :-
HH, TT, HTT, THH, HTHH, THTT, ...

X: 2 3 4

as, experiment only ends if last two outcomes are HH or TT and previous outcomes should be alternating.

Let random variable X denote number of tosses at which experiment ends.

$$\therefore P(X=n) = \begin{cases} \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}} & \text{if } n \geq 2 \\ 0 & n \leq 1 \end{cases}$$

a) ~~P(A)~~ for event A to occur, X is either 2, 3, 4 or 5.

$$\therefore P(A) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= \sum_{K=1}^{4} \frac{1}{2^K}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$$

b) for event B to occur, X is even.

$$\therefore P(B) = P(X=2) + P(X=4) + \ldots$$

$$= 2 \times \sum_{K=1}^{\infty} \frac{1}{2^{2K}} = 2 \cdot \sum_{K=1}^{\infty} \frac{1}{4^K}$$

$$= 2 \times \frac{1/4}{1-1/4} = \frac{2}{3}$$

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c) • $P(A\cap B) = P(X=2) + P(X=4)$

$= \frac{1}{2} + \frac{1}{2^3} = \frac{5}{8}$

• $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{15}{16} + \frac{2}{3} - \frac{5}{8}$

$= \frac{45+32-30}{48} = \frac{47}{48}$

• $P(A\cap B') = P(A) - P(A\cap B)$

$= \frac{15}{16} - \frac{5}{8}$

$= \frac{5}{16}$

• $P(A'\cap B') = 1 - P(A\cup B)$

$= 1 - \frac{47}{48} = \frac{1}{48}$

• $P(A'\cap B) = P(B) - P(A\cap B)$

$= \frac{2}{3} - \frac{5}{8}$

$= \frac{16-15}{24} = \frac{1}{24}$

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2) Number of ways to choose three numbers = $^{100}C_3$

i) Let us arrange these 3 numbers in ascending order, then if they are in A.P, they are of form

$$\underline{a,\ a+k,\ a+2k}\ . \qquad ;a,k \to \text{integers}$$

also, $1 \le a \le 100$

$$a+2k \le 100 \qquad (\text{max. term}) \le 100$$

$$1 \le k \le \left\lfloor \frac{100-a}{2} \right\rfloor \quad \to \text{floor of } \left(\frac{100-a}{2}\right)$$

for, all arranged triplets, if their first number in sequence, is a, then number series possible is

$$\left\lfloor \frac{100-a}{2} \right\rfloor$$

$\therefore$ total number of A.P. $= \displaystyle\sum_{a=1}^{100} \left\lfloor \frac{100-a}{2} \right\rfloor$

$$= 2\sum_{p=1}^{49} p = \underline{\underline{49 \times 50}}$$

(by observation)

$\therefore$ Probability it is A.P $= \dfrac{49\times50}{^{100}C_3} = \dfrac{\cancel{49\times50}}{\cancel{100\times99\times98}}$

$$= \frac{49\times50}{100\times99\times98}\times 6 = \frac{1}{66}$$

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ii) let us arrange, all triplets in ascending order,

$$a,\ ak,\ ak^2 \qquad , a, k \text{ integers.}$$

with, $2 \le k \le 10$, max is 10 for $a=1$

and, $1 \le a \le 100$

and, $ak^2 \le 100$

$$\Rightarrow a \le \left\lfloor \frac{100}{k^2} \right\rfloor$$

∴ each unique triplet has unique pair (a,k);

∴ total no. of G.P $= \sum_{k=2}^{10} \left\lfloor \frac{100}{k^2} \right\rfloor$

$$= 25 + \left\lfloor \frac{100}{9} \right\rfloor + \left\lfloor \frac{100}{16} \right\rfloor + \left\lfloor \frac{100}{25} \right\rfloor + \left\lfloor \frac{100}{36} \right\rfloor + \left\lfloor \frac{100}{49} \right\rfloor + \left\lfloor \frac{100}{64} \right\rfloor + \left\lfloor \frac{100}{81} \right\rfloor + \left\lfloor \frac{100}{100} \right\rfloor$$

$$= 25 + 11 + 6 + 4 + 2 + 2 + 1 + 1 + 1$$
$$= 53$$

Probability of G.P $= \dfrac{53}{\binom{100}{3}}$

Q3) Given, three games have already happened,
with 2 wins for A, 1 win for B, 0 win for C.

Let after n matches C wins series.

Also, before C wins for last time winning the series, it wins 3 more times, A wins 0/1 time, B wins 0/1/2 times.

∴ Let us denote the players winning in games as
AABC ; if A wins then A wins, then B wins, then C wins.

Case I:- n = 4 matches;
No. of favourable outcome = CCCC

Case II: n = 5 matches;

No. of Favourable outcomes = 4+4 = 8

$\boxed{BCCC}\ C \rightarrow 4!/3! = 4$ (permutation of bracket)

$\boxed{ACCC}\ C \rightarrow 4!/3! = 4$

Case III:- n=6 matches.
No. of Favourable outcomes = 20+10 = 30

$\boxed{ABCCC}\ C \rightarrow \frac{5!}{3!} = 20$ (permutation of bracket)

$\boxed{BBCCC}\ C \rightarrow \frac{5!}{3!2!} = 10$

Case IV: n=7 match

$\boxed{ABBCCC}\ C \rightarrow \frac{6!}{2!3!} = 60$

∴ $P(C\text{ wins}) = \frac{1}{3^7}(60 + 3\times 30 + 9\times 8 + 27) = \frac{249}{3^7} = \left(\frac{83}{729}\right)$

Q4) Let, $U_1 \rightarrow$ ~~u~~ event that urn 1 is chosen

$U_2 \rightarrow$ " " urn 2 " "

$R \rightarrow$ event that red ball is drawn

Given,

$$P(U_1) = 1/2 = P(U_2)$$

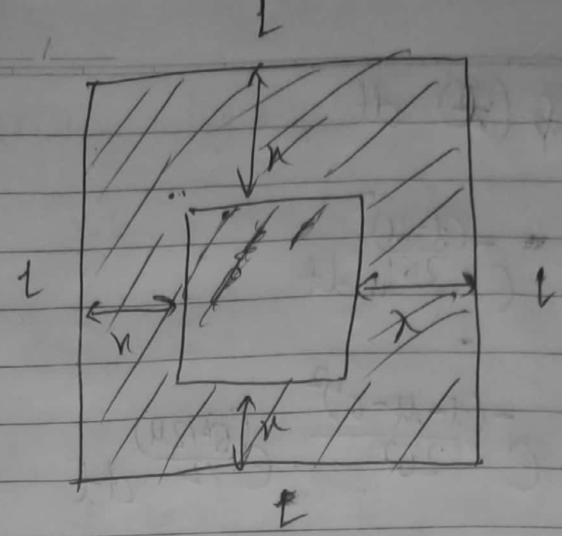
$$P(R|U_1) = \frac{5}{8} \qquad P(R|U_2) = \frac{3}{7}$$

$$\therefore P(U_1|R) = \frac{P(R|U_1)\cancel{P(U_1)}}{P(R|U_1)\cancel{P(U_1)} + P(R|U_2)\cancel{P(U_2)}}$$

$$= \frac{\frac{5}{8}}{\frac{5}{8}+\frac{3}{7}} = \frac{1}{1+\frac{24}{35}}$$

$$= \frac{35}{59}$$

(85)



Case I:- $\left(x \le \frac{1}{2}\right)$

It is clear, that if point P is chosen in shaded area then, distance of P from nearest side $\le x$ cm

also, area of square = 1 cm^2
and, it is equally likely to choose P in all of 1 cm^2.
Favourable area of P = $1-(1-2x)^2$

$\therefore$ Probability $= 1-1-4x^2+4x$
$= 4(x-x^2)$

Case II:- $x > \frac{1}{2}$

Clearly, any point in square, is having minimum distance from nearest side $\le \frac{1}{2}$ cm by symmetry

as, if $y > x$,
and $x > \frac{1}{2}$
$1-x < \frac{1}{2}$ nearest distance.

$\therefore$ probability = 1.

$\therefore$ Probability = $\begin{cases} 4(x-x^2), & x \le \frac{1}{2} \\ 1, & x > \frac{1}{2} \end{cases}$

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Q6) Let r.v X denote number of faces that are equal. then

$X = 0, \cancel{1}, 2, 3, 4, 5, 6$.

and, $A \rightarrow$ event that atleast two faces are equal

$\therefore P(A) = P(X \geq 2)$

$= 1 - P(X < 2)$

$= 1 - P(X = 0)$ ~~$P(X=1)$~~

also, as rolls are independent,

$P(X=0) = \frac{6!}{6^6}$

$\therefore P(A) = \left(1 - \frac{6!}{6^6}\right)$

$= 1 - \frac{\cancel{6} \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{6} \times 6 \times \cancel{6} \times \cancel{6} \times 6 \times 6}$

$= 1 - \frac{5}{324} = \frac{319}{324}$

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Q7) $K \rightarrow$ event that examinee knows answer to a given question
$C \rightarrow$ event that " answers question correctly.

$P(K) = p$
$P(C|K) = 1$
$P(C|K') = \frac{1}{m}$

$$\therefore P(K|C) = \frac{P(C|K)P(K)}{P(C|K)P(K) + P(C|K')P(K')}$$

$$= \frac{1 \times p}{1 \times p + \frac{1}{m} \times (1-p)}$$

$$= \left(\frac{mp}{mp + 1 - p}\right)$$

Q8) let A, B are independent, $P(A) > 0$, $P(B) > 0$,
then $P(A \cap B) = P(A) P(B)$

also, A and B are mutually exclusive $\Rightarrow$ $P(A \cap B) = 0$

but, $P(A \cap B) = P(A) P(B) = 0$
but $P(A), P(B) \neq 0$

$\therefore$ A and B can't be mutually exclusive if they are independent by theory of contradiction.

Similarly, if A, B are mutually exclusive then, A, B can't be independent.

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Q9) Let r.v. K denote number of events out of A, B, C that occur. then.

* $\Sigma P(A) = P(A) + P(B) + P(C)$
* $\Sigma P(A)P(B) = P(A)P(B) + P(B)P(C) + P(C)P(A)$
* $\Pi P(A) = P(A)P(B)P(C)$

i) Case I:- $K=0$

$P(K=0) = (1-P(A))(1-P(B))(1-P(C))$
$= 1 - \Sigma P(A) + \Sigma P(A)P(B) - \Pi P(A)$

Case II:- $K=1$

$P(K=1) = P(A)(1-P(B))(1-P(C)) + P(B)(1-P(A))(1-P(C))$
$+ P(C)(1-P(B))(1-P(A))$
$= \Sigma P(A) - 2\Sigma P(A)P(B) + 3\Pi P(A)$

Case: $K=2$

$P(K=2) = P(A)P(B)(1-P(C)) + P(B)P(C)(1-P(A))$
$+ P(C)P(A)(1-P(B))$
$= \Sigma P(A)P(B) - 3\Pi P(A)$

Case: $K=3$

$P(K=3) = P(A)P(B)P(C)$
$= \Pi P(A)$

* P(exach. K event happen) $= P(K=3)$

ii) P(~~exactly~~ atleast K happen)

$= \sum_{j=K}^{3} P(K=j)$

~~Case I:- atleast 0 happen~~

* P(atleast 0 event happen) $= 1$
* P(" 1 event happens) $= 1 - P(K=0)$
$= \Sigma P(A) - \Sigma P(A)P(B) + \Pi P(A)$

* P(" 2 event happen) $= 1 - P(K=0) - P(K=1)$
$= \Sigma P(A)P(B) - 2\Pi P(A)$

* P(atleast 3 event happens) $= P(K=3) = \Pi P(A)$

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iii) $P(\text{atmost } K \text{ happen}) = \sum_{j=0}^{K} P(K=j)$

- $P(\text{atmost } 0 \text{ happen}) = P(K=0) = 1 - \sum P(A) + \sum P(A)P(B) - \prod P(A)$
- $P(\text{atmost } 1 \text{ event happen}) = P(K=0) + P(K=1)$
  $= 1 - \sum P(A)P(B) + 2\prod P(A)$
- $P(\text{ " } 2 \text{ " } \text{ " }) = 1 - P(K=3)$
  $= 1 - \prod P(A)$
- $P(\text{ " } 3 \text{ " } \text{ " }) = 1$

Q10) Given A, B are independent,
$P(A \cap B) = \frac{1}{6} = P(A)P(B)$

i) and, $P(A' \cap B') = 1 - P(A \cup B) = \frac{1}{3}$

$\Rightarrow P(A \cup B) = \frac{2}{3} = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = \left(\frac{2}{3} + \frac{1}{6}\right) = \frac{5}{6}$

$\therefore$ Let us denote, $P(A)$ by a, $P(B) = b$
i.e. $P(A) = a$
$P(B) = b$

$\therefore a + b = \frac{5}{6}$

$ab = \frac{1}{6}$

$\therefore$ clearly $(a, b) = \left(\frac{1}{3}, \frac{1}{2}\right)$ or, $(a, b) = \left(\frac{1}{2}, \frac{1}{3}\right)$

ii) $P(A\cap B') = \frac{1}{3}$

$= P(A) - P(A\cap B)$

$= \frac{1}{3}$

$\Rightarrow P(A) = \frac{1}{3} + P(A\cap B) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$\therefore P(B) = \frac{1}{3}$

iii) $\therefore$ for part (ii) $P(A)$ and $P(B)$ are uniquely determined but not for part (i)

Q11) Given, A, B are independent

$\Rightarrow P(A\cap B) = P(A)P(B)$

$\therefore P(A'\cap B') = 1 - P(A\cup B)$ (definition)

$= 1 - P(A) - P(B) - P(A\cap B)$

$= 1 - P(A) - P(B) - P(A)P(B)$

$= (1-P(A))(1-P(B))$

$= P(A')\,P(B')$

Hence, A', B' are independent by definition

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Q17) $P(A) = \frac{3}{6} = \frac{1}{2}$ (2, 4, 6 → even faces, 1, 3, 5 → odd faces)

$P(B) = \frac{1}{2}$

* also, for any number on top face of dice I, exactly 3 numbers on that of dice II, lead to odd sum.

$\therefore$ favourable outcomes for odd sum = 6×3

Total outcomes = 6×6

$\therefore P(C) = \frac{6 \times 3}{6 \times 6} = \frac{1}{2}$

* $P(A \cap B) = \frac{3 \times 3}{36} = \frac{1}{4}$ (as, 3 faces of dice I and dice II are independently favourable)

$= P(A)P(B)$

$\therefore$ A, B are independent.

* $P(B \cap C) = \frac{3 \times 3}{36} = \frac{1}{4} = P(B)P(C)$

$\therefore$ B, C are independent.

* $P(A \cap C) = \frac{3 \times 3}{36} = \frac{1}{4} = P(A)P(C)$

A, C are independent

* $P(A \cap B \cap C) = \frac{3 \times 3}{36} \neq P(A)P(B)P(C)$ [as, if A and B happen then, C automatically happen]

$\therefore$ A, B, C are pairwise independent

but A, B and C are not mutually independent.

Q13) (a) $f(x)$ is a valid p.m.f., if $\sum f(x) = 1$

a) $\sum_{x=1}^{4} f(x) = \frac{1}{2}\left(\sum_{x=1}^{4} x\right) - 4$

$= \frac{1}{2} \times \frac{4 \times 5}{2} - 4 = 1$

$\therefore f$ is valid

b) $\sum_{x=1}^{\infty} \frac{e^{\lambda}\lambda^{x}}{x} = -e^{\lambda} \sum_{x=1}^{\infty} -\frac{\lambda^{x}}{x}$

we know $\ln(1-x) = \sum_{n=1}^{\infty} -\frac{x^n}{n}$ if $|x| < 1$

Case I :- $|\lambda| < 1$

$\therefore \sum_{x=1}^{\infty} \frac{e^{\lambda}\lambda^{x}}{x} = -e^{\lambda} \ln(1-\lambda) \neq 1$

$\therefore f$ is not valid.

Case II: $|\lambda| \geq 1$

$\sum f \to \infty$ $\therefore$ not valid

$\therefore$ It can't serve as pmf

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Q14) A → event that clock made by A works after 360 days
B → " " " " " B " " "
C → " " " " " C " " "

Given
$P(A) = 0.95$
$P(B) = 0.9$
$P(C) = 0.8$

Case I :- $X = 0$
$P(X=0)$ = No clock works correctly
$= P(A')\,P(B')\,P(C')$
$= 0.05 \times 0.1 \times 0.2$
$= 0.001$

Case II :- $X=1$
$P(X=1)$ = One of the clocks work
$= 0.95 \times 0.1 \times 0.2 + 0.05 \times 0.9 \times 0.2 + 0.05 \times 0.1 \times 0.8$
$= 0.032$

Case III :- $X=2$

$P(X=2)$ = only one of the clocks doesn't work
$= 0.95 \times 0.9 \times 0.2 + 0.95 \times 0.1 \times 0.8 + 0.05 \times 0.9 \times 0.8$
$= 0.283$

Case IV :- $x=3$
$P(X=3)$ = all three work
$= 0.95 \times 0.9 \times 0.8$
$= 0.684$

$$\therefore P_X(x) = \begin{cases} 0.001 & , x = 0 \\ 0.032 & , x = 1 \\ 0.283 & , x = 2 \\ 0.684 & , x = 3 \\ 0 & , \text{o.w} \end{cases}$$

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Q15) clearly, only 2 of six faces i.e. (1, 4) are perfect square

$\therefore P(X_i = 1) = \frac{1}{3}$

$$\therefore p_{X_i}(x) = \begin{cases} 1/3, & x = 1 \\ 2/3, & x = 0 \\ 0, & \text{o.w} \end{cases}$$

$Y = X_1 + X_2 + X_3$

$$p_Y(y) = \begin{cases} (2/3)^3, & y = 0 \\ {}^3C_1 \times (2/3)^2 \times (1/3), & y = 1 \\ {}^3C_2 \times 2/3 \times (1/3)^2, & y = 2 \\ (1/3)^3, & y = 3 \\ 0, & \text{o.w} \end{cases}$$

$\therefore$ pdf $$p_Y(y) = \begin{cases} 8/27, & y = 0 \\ 12/27, & y = 1 \\ 6/27, & y = 2 \\ 1/27, & y = 3 \end{cases}$$

cdf $$F_Y(y) = \begin{cases} 0, & y < 0 \\ 8/27, & 0 \le y < 1 \\ 20/27, & 1 \le y < 2 \\ 26/27, & 2 \le y < 3 \\ 1, & y \ge 3 \end{cases}$$

Q16) $f_X(x) = (1-K)K^x \quad , x = 0,1,2,\ldots$

f_X is valid iff
$\sum_{x=0}^{\infty} f_X(x) = 1$

which is possible only for $|K|<1$

$\therefore (1-K)\sum_{x=0}^{\infty} K^x = (1-K) \times \frac{1}{(1-K)} = 1$

$$M_X(t) = E[e^{tX}]$$

$$= \sum_{x=0}^{\infty} e^{tx}\cdot K^x(1-K)$$

$$= (1-K)\cdot\sum_{x=0}^{\infty}(Ke^t)^x$$

$$= \frac{(1-K)}{(1-Ke^t)}$$

$$\therefore E[X] = \left.\frac{d}{dt}M_X(t)\right|_{t=0}$$

$$= \left.\frac{(1-K)}{(1-Ke^t)^2}\times Ke^t\right|_{t=0} = \left(\frac{K}{1-K}\right)$$

$$E[X^2] = \left.\frac{d^2}{dt^2}M_X(t)\right|_{t=0} = \left.\frac{d}{dt}\left(\frac{(1-K)Ke^t}{(1-Ke^t)^2}\right)\right|_{t=0}$$

$$= \left.\frac{K(1-K)\left[(1-Ke^t)^2e^t + 2Ke^{2t}(1-Ke^t)\right]}{(1-Ke^t)^4}\right|_{t=0}$$

$$= \left.\frac{K(1-K)(1+Ke^t)e^t}{(1-Ke^t)^3}\right|_{t=0} = \frac{K(1+K)}{(1-K)^2}$$

$\therefore Var[X] = E[X^2] - E^2[X]$

$$= \frac{K(1+K)}{(1-K)^2} - \frac{K^2}{(1-K)^2}$$

$$= \frac{K}{(1-K)^2}$$

Q17) i)

- $F(0) = 0$
- $F(\infty) \to 1$
- $F(x)$ is strictly increasing

$\therefore F(x)$ is a cdf.

~~i) $P(X \le 6) = F(6^-) = 1 - \frac{2}{3}e^{-\frac{6}{3}} - \frac{1}{3}e^{-[2^-]}$~~

~~$= P(X \le 6)$~~

~~$=$~~

~~$= 1 - \frac{2}{3}e^{-2} - \frac{1}{3}e^{-1}$~~

i) $P(X>6) = 1 - P(X \le 6)$

$= 1 - F(6)$

$= 1 - \left(1 - \frac{2}{3}e^{\frac{-6}{3}} - \frac{1}{3}e^{-\lfloor\frac{6}{3}\rfloor}\right) = e^{-2}$

ii) $P(X=5) = F(5) - F(5^-)$

$$= \left(\cancel{1} - \cancel{\frac{2}{3}e^{\frac{5}{3}}} - \frac{1}{3}e^{-\lfloor\frac{5}{3}\rfloor}\right) - \left(\cancel{1} - \cancel{\frac{2}{3}e^{\frac{5}{3}}} - \frac{1}{3}e^{-\lfloor\frac{5^-}{3}\rfloor}\right)$$

$= 0$

iii) $P(5 < X \le 8) = F(8) - F(5^-) = \frac{2}{3}\left(e^{-\frac{5}{3}} - e^{-\frac{8}{3}}\right) + \frac{1}{2}\left(e^{-1} - e^{-2}\right)$

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Q18) $f_X(x) = \frac{x}{9} e^{-\frac{x}{3}}, \; x > 0$

i) $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx$

$= \int_{0}^{x} \underbrace{\frac{x}{9}}_{u} \underbrace{e^{-\frac{x}{3}}}_{v} dx$

$= \left(\frac{x}{9} \int e^{-\frac{x}{3}} dx\right)\Big|_0^x - \int_0^x \frac{1}{9} \times -3 e^{-\frac{x}{3}} dx$

$= \left(-\frac{x}{3} e^{-\frac{x}{3}} - e^{-\frac{x}{3}}\right)\Big|_0^x$

$= -\frac{x}{3} e^{-\frac{x}{3}} - e^{-\frac{x}{3}} + 1$

$= 1 - e^{-\frac{x}{3}}\left(\frac{x}{3} + 1\right)$

$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$

$= \int_{0}^{\infty} \underbrace{\frac{x^2}{9}}_{u} \underbrace{e^{-\frac{x}{3}}}_{v} dx.$

$= \frac{x^2}{9} \times -3 e^{-\frac{x}{3}}\Big|_0^\infty + \frac{2}{3} \int_0^\infty \underbrace{x}_{u} \underbrace{e^{-\frac{x}{3}}}_{v} dx$

$= \left(\frac{-x^2}{3} e^{-\frac{x}{3}} + \frac{2}{3} x \times -3 e^{-\frac{x}{3}}\Big|_0^\infty - \frac{2}{3} \int_0^\infty -3 e^{-\frac{x}{3}} dx\right)$

$= \left(\frac{-x^2}{3} e^{-\frac{x}{3}} - 2x e^{-\frac{x}{3}} - 6 e^{-\frac{x}{3}}\right)\Big|_0^\infty = 6$

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$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_0^{\infty} \frac{x^3}{9} e^{-\frac{x}{3}}\,dx$$

$$= \cancel{\frac{x^3}{9} \times -3 e^{-\frac{x}{3}} \Big|_0^{\infty}} + 9 \int_0^{\infty} \frac{x^2}{9} e^{-\frac{x}{3}}\,dx$$

$$= 9E[X] = 9 \times 6 = 54$$

$$\therefore Var[X] = E[X^2] - E^2[X]$$
$$= 54 - 36$$
$$= 18$$

ii) $P(X \le 6) = F_X(6) = 1 - e^{-2}(2+1)$

$$= 1 - 3e^{-2}$$

Q19) $f(x) = 12x^2(1-x)$, $0 < x \le 1$, 0.00

$$\therefore f'(x) = 24x - 36x^2 = 12x(2-3x)$$

$$f''(x) = 24 - 72x = 24(1-3x)$$

$\therefore f'(x) = 0$
at, $x = \left(\frac{2}{3}\right)$

$$f''\left(\frac{2}{3}\right) = 24 \times \left(1 - 3 \times \frac{2}{3}\right) = -24 < 0$$

$\therefore x = 2/3$ is a maxima ~~and~~ mode of distribution

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(Q20) Given, A_1, A_2, A_3, A_4, A_5 are independent.

* Let $B_1 = A_1 \cup A_3$
  $B_2 = A_2 \cap A_4$
  $B_3 = A_5^c$

* $P(B_1 \cap B_2) = P((A_1 \cup A_3) \cap (A_2 \cap A_4))$
  $= ~~P((A_1 \cap A_2) \cup (A_3~~$ ~~(distribution)~~
  $= P((A_1 \cap A_2 \cap A_4) \cup (A_3 \cap A_2 \cap A_4))$ → distribution law
  $= P(A_1 \cap A_2 \cap A_4) + P(A_3 \cap A_2 \cap A_4)$
  $- P(A_1 \cap A_2 \cap A_3 \cap A_4)$ → independent

  $= P(A_1)P(A_2)P(A_4) + P(A_3)P(A_2)P(A_4)$
  $- P(A_1)P(A_2)P(A_3)P(A_4)$
  $= (P(A_1) + P(A_3) - P(A_1)P(A_3)) P(A_2)P(A_4)$
  $= P(A_1 \cup A_3) P(A_2 \cap A_4)$
  $= P(B_1)P(B_2)$

  $\therefore B_1, B_2$ are independent.

* $P(B_2 \cap B_3) = P((A_2 \cap A_4) \cap A_5')$
  $= P(A_2 \cap A_4) - P(A_2 \cap A_4 \cap A_5)$ → law
  $= P(A_2)P(A_4) - P(A_2)P(A_4)P(A_5)$ → independent
  $= P(A_2)P(A_4)P(A_5')$
  $= P(B_2)P(B_3)$

  $\therefore B_2, B_3$ are independent

* $P(B_1 \cap B_3) = P((A_1 \cup A_3) \cap A_5')$
  $= P(A_1 \cup A_3) - P((A_1 \cup A_3) \cap A_5)$
  $= P(A_1) + P(A_3) - P(A_1 \cap A_3) - P((A_1 \cap A_5) \cup (A_3 \cap A_5))$
  $= P(A_1) + P(A_3) - P(A_1 \cap A_3) - P(A_1 \cap A_5) - P(A_3 \cap A_5)$
  $+ P(A_1 \cap A_3 \cap A_5)$
  $= P(A_1 \cup A_3)(1 - P(A_5)) = P(B_1)P(B_3) \therefore B_1, B_3$ → independent

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* $P(B_1 \cap B_2 \cap B_3) = P\left((A_1 \cup A_3) \cap (A_2 \cap A_4) \cap A_5'\right)$

$= P(A_1 \cap A_2 \cap A_4 \cap A_5') + P(A_3 \cap A_2 \cap A_4 \cap A_5') - P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5')$

$= P(A_1 \cap A_2 \cap A_4) - P(A_1 \cap A_2 \cap A_4 \cap A_5) + P(A_3 \cap A_2 \cap A_4) - P(A_3 \cap A_2 \cap A_4 \cap A_5) - P(A_1 \cap A_2 \cap A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)$

$= P(A_1)P(A_2)P(A_4)P(A_5') + P(A_3)P(A_2)P(A_4)P(A_5') - P(A_1)P(A_2)P(A_3)P(A_4)P(A_5')$

$= P(A_2)P(A_4)P(A_5')\left(P(A_1) + P(A_3) - P(A_1)P(A_3)\right)$

$= P(B_2)P(B_3)P(B_1)$

$\therefore B_1, B_2, B_3$ are independent

proved

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Q21) We know, that number of ways n objects corresponding to distinct n places, then number of ways in which all objects are at wrong place is denoted by

$$D(n) = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots \frac{1}{n!}\right)$$

$= \cancel{n!}($

$$\therefore D(4) = 4!\left(\cancel{1} - \cancel{\frac{1}{1}} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$$

$$= 4!\left(\frac{12-4+1}{24}\right)$$

$$= \cancel{4!} \times \frac{9}{\cancel{24}} = 9$$

then, P(all letters are wrongly placed)

$$= \frac{D(4)}{4!} = \frac{\cancel{9}^{3}}{\cancel{24}_{8}} = \frac{3}{8}$$

Q22) U1 → event that ball is drawn from urn 1

U2 → " " " " " urn 2

W → event that drawn ball is white

Given, $P(U1) = P(U2) = 1/2$

$$P(W|U1) = \frac{4}{7}, \quad P(W|U2) = \frac{3}{10}$$

$$\therefore P(U1|W) = \frac{P(W|U1)\,P(U1)}{P(W|U1)\,P(U1) + P(W|U2)\,P(U2)}$$

$$= \frac{4/7}{\frac{4}{7} + \frac{3}{10}} = \frac{1}{1 + \frac{21}{40}} = \frac{40}{61}$$

* P(W) =

$$= \left(\frac{4}{7} + \frac{3}{10}\right) \times \frac{1}{2} = \frac{40+21}{70 \times 2} = \frac{61}{140}$$

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Q23) Let r.v. X denote no. of heads in 6 tosses.

$x = 0, 1, 2, \ldots, 6$

$\therefore E \rightarrow$ event that atleast 5 heads appear.

$\therefore P(E) = P(X=5) + P(X=6)$

$P(X=6) = \frac{1}{2^6}$

$P(X=5) = {}^6C_1 \times \frac{1}{2^{\cancel{6}}} \times 1$ (1 out of 6 in Tell)

$= \frac{6}{2^6}$

$\therefore P(E) = \left(\frac{7}{64}\right)$

Q24) ~~Let r.v. X denote number of times~~

P(none of missiles hit) $= (0.9)^5$

P(one of missiles hit) $= {}^5C_1 \times (0.9)^4 \times 0.1$

$\therefore$ P(ship is still afloat) $=$ P(no hits) $+$ P(1 hit)

$= (0.9)^4 (0.9 + 0.1 \times 5)$

$= 1.4 \times 0.9^4$

$= 0.91854$

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Q25) Let $P(A)$ denote prob. A is selected.

$\therefore P(A) = P(B) = \cdots = P(F) = \frac{1}{6}$

we have to find :-

$P((A \cap B) \mid (C' \cap D'))$?

$= \cancel{\frac{P(A \cap B \cap C' \cap D')}{P(C' \cap D')}}$

method I :-

${}^6C_3 = \underline{\underline{20}} \Rightarrow$ number of ways 3 people are selected

Which are :-

A, B, C	A, B, D	A, B, E	A, D, F
B, C, D	B, C, E	B, C, F	B, E, F
C, D, E	C, D, F	A, C, D	A, C, E
D, E, F	A, C, F	A, D, E	A, D, F
A, E, F	B, D, E	B, D, F	C, E, F

$\therefore P((A \cap B) \mid C' \cap D') = \frac{N(A \cap B \cap C' \cap D')}{N(C' \cap D')}$

clearly, $N(C' \cap D') = 4$

$N(A \cap B \cap C' \cap D') = 2$

$\therefore P(A \cap B \mid C' \cap D') = \frac{2}{4} = \frac{1}{2}$

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method II:- $P((A\cap B)|(C'\cap D'))$

$$= \frac{P(A\cap B\cap C'\cap D')}{P(C'\cap D')}$$

$$= \frac{P(A\cap B\cap C') - P(A\cap B\cap C'\cap D)}{1-P(C\cup D)}$$

$$= \frac{P(A\cap B) - P(A\cap B\cap C) - P(A\cap B\cap D) + P(A\cap B\cap C\cap D)}{1-P(C)-P(D)+P(C\cap D)}$$

$\times\, P(A\cap B\cap C\cap D) = 0$ as committee has only 3 people.

also, $P(A\cap B\cap C) = \frac{1}{20}$ as, out of ${}^{20}C_3$ possible selection this is one of them

$= P(A\cap B\cap D) \rightarrow$ symmetry

also $P(A\cap B) = P(C\cap D) \ldots \rightarrow$ symmetry.

$= \frac{4}{20}$ (as, 3rd member is one of 4 remaining ones)

$P(A) = P(B) = \ldots = P(F)$

$= \frac{{}^5C_2}{20} = \frac{10}{20}$ (as other members, 2 selected out of $\underline{5}$)

$$\therefore P((A\cap B)|(C'\cap D')) = \frac{\frac{4}{20} - 2\times\frac{1}{20} + 0}{\cancel{1} - \frac{10}{20}\times 2 + \frac{4}{20}} = \frac{\frac{2}{20}}{\frac{4}{20}} = \frac{1}{2}$$

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Q26) Total no. of ways to choose 2 socks = $^{12}C_2$

ways to select only 2 black socks = $^{2}C_2$

" " " " 2 ~~black~~ blue " = $^{4}C_2$

" " " " brown " = $^{6}C_2$

$$\therefore P(\text{same color}) = \frac{^{2}C_2 + {^{4}C_2} + {^{6}C_2}}{^{12}C_2}$$

$$= \frac{1 + \frac{4\times3}{2} + \frac{6\times5}{2}}{\frac{12\times11}{2}}$$

$$= \frac{1+6+15}{66} = \frac{22}{66} = \frac{1}{3}$$

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Q27) let, X_i is the the error committed in ith cheque.

then total error, $S = \sum_{i=1}^{100} X_i$

~~from CLT we know,~~

also, X_i are i.i.d, with $\mu_i = 0$, $\sigma_i^2 = \frac{1}{12}$

as, X_i is uniform r.v b/w $[-0.5, 0.5]$

from CLT we know,

$$\frac{S - n\mu}{\sqrt{n\sigma^2}} \sim N(0,1) \quad \text{as } n \to \text{large}$$

as $n = 100$, which is large, we can apply CLT;

$$\frac{S}{10 \times \frac{1}{\sqrt{12}}} \sim N(0,1)$$

also, Chebyshev inequality says :-

$$P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2} \quad \text{if } E[X] = \mu, \ var(X) = \sigma^2$$

putting $\mu = 0$, $\sigma = 1$, $X = \frac{S}{\left(10 \times \frac{1}{\sqrt{12}}\right)} \sim N(0,1)$

$$P\left(\left|\frac{S}{\frac{100}{\sqrt{12}}}\right| \geq k\right) \leq \frac{1}{k^2}$$

we want to find; $P(|S| \geq 5)_{max}$?

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$$\therefore P\left(|S| \geq \frac{10}{\sqrt{12}}k\right) \leq \frac{1}{k^2}$$

$$\therefore \frac{10}{3\sqrt{12}}k = 5$$

$$\Rightarrow k = \sqrt{3}$$

$$\therefore P(|S| \geq 5) \leq \frac{1}{3}$$

Q28) F → event that a girl is in first year

S → " " " " in second year

T → " " " " in third "

C → event that a girl is enrolled in computer course

∴ girl is randomly chosen.

Given:-

$P(F) = 0.4$

$P(S) = P(T) = 0.3$

$P(C|F) = 0.3 = P(C|S)$

$P(C|T) = 0.5$

$$\therefore P(C) = \sum P(C|F)P(F)$$

$$= 0.3 \times 0.4 + 0.3 \times 0.3 + 0.5 \times 0.3$$

$$= 0.12 + 0.09 + 0.15$$

$$= 0.36$$

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Q29) $K \rightarrow$ event that student knows answer a question
$C \rightarrow$ event that student answer a question correctly.

Given;

$P(K) = 3/5$

$P(C|K) = 1$
$P(C|K') = 1/2 \rightarrow$ fair toss

$$\Rightarrow P(K|C) = \frac{P(C|K)\,P(K)}{P(C|K)\,P(K) + P(C|K')\,P(K')}$$

$$= \frac{1 \times 3/5}{3/5 + 2/5 \times \frac{1}{2}}$$

$$= \frac{3}{4}$$

Q30) ~~P(E)~~ Sample space of two tosses $= \{HH, TT, HT, TH\}$
each outcome equally likely.

$\therefore P$ (not more than one head) $= P(E) = P(TT, HT, TH)$
$= \frac{3}{4}$

$P(F) = P(TH) + P(HT) = \frac{2}{4}$

$P(E \cap F) = P(TH) + P(HT) = \left(\frac{2}{4}\right) \neq P(E)\,P(F)$

clearly ~~P(E)~~ $\therefore E, F$ are not independent

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Q31) E, F are independent $\Leftrightarrow P(E\cap F) = P(E)P(F)$

i) $P(E\cap F') = P(E) - P(E\cap F)$ → set probability
$= P(E) - P(E)P(F)$
$= P(E)P(F')$
∴ E, F' are independent

ii) $P(E'\cap F) = P(F) - P(E\cap F)$
$= P(F) - P(E)P(F)$
$= P(F)(1-P(E))$
$= P(F)\,P(E')$
∴ E', F are independent

iii) $P(E'\cap F') = 1 - P(E\cup F)$
$= 1 - P(E) - P(F) + P(E\cap F)$
$= 1 - P(E) - P(F) + P(E)P(F)$
$= (1-P(E))(1-P(F))$
$= P(E')\,P(F')$
E', F' are independent

Q32) $P(E_1) = 0.5$, $P(E_2) = 0.6$, $P(E_3) = 0.8$

P(only one gun hits) $= P(E_1)P(E_2')P(E_3')$ → only 1 hits
$+ P(E_2)P(E_3')P(E_1')$ → only 2 hits
$+ P(E_3)P(E_1')P(E_2')$ → only 3 hits
disjoint events

$P(E_1\cap E_2'\cap E_3') = P(E_1)P(E_2')P(E_3')$
as E_1, E_2', E_3' are independent

∴ P(only one gun hit) $= 0.5\times0.4\times0.2 + 0.6\times0.5\times0.2$
$+ 0.8\times0.4\times0.5$
$= 0.5(0.08 + 0.12 + 0.32)$
$= 0.5\times0.52 = 0.26$

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Q33) $W_1 \to$ event that engine works in first hour

$W_2 \to$ " " " " in second hour

$P(W_1) = P(W_2) = 0.98$.

$\therefore P(W_1 \cap W_2) = P(W_1)P(W_2)$ as,

as, W_1, W_2 are independent

as, ~~the~~ time slots do not overlap

and independent on interval

like poisson.

$\therefore P(W_1 \cap W_2) = (0.98)^2 = 0.9604$

Engine works for 2 hours

Q34) $A \to$ ~~...~~ event that house prefers brand A

$\therefore P(A) = 0.6$

i) P(exactly 5 house have to interview, to get first house, which prefer A)

$\Rightarrow$ P(first 4 houses don't prefer A, and but 5th does)

$= (0.4)^4 \times 0.6 = 0.01536$

$0.6 \times 0.4^4 + 0.6 \times 0.4^5 + \cdots$

ii) P(atleast 5) $= 0.\cancel{6} \cdots$ ~~$(0.4)^4 \times 0.6$~~

$= \cancel{0.6 \sum (0.4)^k}$ ~~$0.6 \sum$~~ $0.6 \times 0.4^4 \sum_{k=0} 0.4^k$

$= \cancel{0.6 \times \frac{(1-0.4)}{(1-0.4)}}$ $0.6 \times 0.4^4 \times \frac{1}{1-0.4}$

$= \cancel{(1-0.4^5)} = 0.4^4 = 0.0256$

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Q35) $\lambda = 3$

$t = 2$

$N(t)$ = number of particles between in t period

$P(N(t) \geq 10) = 1 - P(N(t) \leq 9)$

$= 1 - \sum_{n=0}^{9} \frac{e^{-\lambda t}(\lambda t)^n}{n!}$

$= \left(1 - e^{-6} \sum_{n=0}^{9} \frac{6^n}{n!}\right)$

$= 1 - e^{-6}(1 + 6 + 18 + 36 + 54 + 64.8 + 64.8 + 55.54 + 41.66 + 27.77)$

$= 1 - \frac{369.57}{403.43}$

$= 0.0839$

Q36) $\lambda = 16$

$P(N(1) > 24)$ = P(switch board will saturate)

$\Rightarrow P(N(1) \geq 24) = 1 - P(N(1) \leq 24)$

$= 1 - P(N(1) < 25)$

$\approx 1 - P\left(Z \leq \frac{24.5 - 16}{\sqrt{16}}\right)$ (CLT result)

$= 1 - P(Z \leq 2.125)$

$= \Phi(-2.125)$

$= 0.0168$

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Q37) Three cases are possible:-

Case I:- Girls outnumber Boys → G event
Case II:- Boys outnumber Girls → B event
Case III:- Girls = Boys ⇒ not possible as 5 children.

also, as $P(Boy) = P(Girl) = 1/2$

by symmetry:- case I and case II have equal probability.
also, G, B events are mutually exclusive and exhaustive.

$\therefore P(G) = P(B)$
$P(G) + P(B) = 1$
$\Rightarrow P(G) = \frac{1}{2}$ → Girls outnumber boys

Q38) Let r.v. X be the number of question answered correctly

then, clearly, $X \sim Bin(20, 1/2)$

$$\therefore P(X \geq 12) = 1 - P(X \leq 11)$$
$$= 1 - \sum_{x=0}^{11} \binom{n}{x} p^x q^{n-x} \quad (p = q = 1/2)$$
$$= 1 - \sum_{x=0}^{4} \binom{20}{x} \left(\frac{1}{2}\right)^n$$
$$= 1 - \frac{1}{2^n} \sum_{x=0}^{4} \binom{20}{x}$$

we know $\sum_{x=0}^{20} {}^{20}C_x = 2^{20} = \sum_{x=0}^{9} {}^{20}C_x + {}^{20}C_{10} + \sum_{x=11}^{20} {}^{20}C_x$
$$= 2\sum_{x=0}^{9} C_x + {}^{20}C_{10}$$

$[{}^nC_x = {}^nC_{n-x}]$

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$$\therefore P(X \geq 12) = 1 - \frac{1}{2^{20}}\left(\left(\frac{2^{20} - {}^{20}C_{10}}{2}\right) + {}^{20}C_{10} + {}^{20}C_{11}\right)$$

$$= \frac{1}{2} - \frac{1}{2^{20}}\left(\frac{{}^{20}C_{10}}{2} + {}^{20}C_{11}\right)$$

$$= \frac{1}{2} - \frac{1}{2^{20}}\left(\frac{20!}{(10!)^2 \times 2} + \frac{20!}{11!\,9!}\right)$$

$$= \frac{1}{2} - \frac{1}{2^{20}}\left(\frac{20!}{(10!)^2 \times 2} + \frac{20! \times 10}{(10!)^2 \times 11}\right)$$

$$= \frac{1}{2} - \frac{1}{2^{20}} \times \frac{20!}{(10!)^2} \times \left(\frac{31}{22}\right)$$

$$= \frac{1}{2} - 0.248 = \underline{\underline{0.252}}$$

Q39) ~~$P = {}^5C_2 \times {}^{10}C_1 \times \frac{1}{2^{10}}$~~

$$P(\text{nine heads in 10 tosses}) = {}^{10}C_1 \times \frac{1}{2^{10}}$$

each 10 tosses are independent.

$$\therefore P(\text{exactly nine heads in 2 out of 5 (10 tosses)}) = {}^5C_2 \times \left({}^{10}C_1 \times \frac{1}{2^{10}}\right)^2 \cancel{\left(1 - \frac{{}^{10}C_1}{2^{10}}\right)^3} \to 1$$

$$= 60\left(\frac{10}{2^{10}}\right)^2 = \underline{\underline{0.0057}}$$

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Q40) let r.v. X denote number of error pulses.

then

$X \sim Bin(10, 0.001)$

$\therefore$ P(atleast one error / second)

$= P(X \geq 1)$

$= 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - (0.999)^{10}$

$= 1 - 0.99 = 0.01$

Q41) let r.v X denote number of trials to get a productive well

$X \sim Geo(0.2)$

i) P(3rd hold is productive)

$= P(X=3)$

$= (0.8)^2 \times 0.2 = 0.128$

ii) P(all 10 wells fail)

$= (0.8)^{10} = 0.107$

Q42) let N(t) denote number of accidents in a given period t.

~~bet~~ t_0 is time during weekend.

$\therefore E[N(t_0)] = \lambda t_0 = 0.7$

• P(atleast 3 accident) = $P(N(t_0) \geq 3)$

$= 1 - P(N(t_0)=0) - P(N(t_0)=1) - P(N(t_0)=2)$

$= 1 - e^{-0.7} \sum_{n=0}^{2} \frac{(0.7)^n}{n!} = 1 - e^{-0.7}\left(1 + 0.7 + \frac{0.49}{2}\right)$

$= 0.034$

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Q4)

* Clearly, $F(x)$ is non-decreasing. as. $f'(x) \geq 0$ always

* and $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to \infty} F(x) = 1$

* checking right continuity at $x = -1$;

$F(-1) = F(-1^+) = 1/4$

and at $x = 1$

* $F(1) = F(1^+) = 1$

$\therefore F(x)$ is a valid cdf

Q44)

* continuity check of $F(x)$:-

i) at $x = 0$; $F(0) = F(0^-) = F(0^+) = 0$

ii) at $x = 1$; $F(1) = F(1^-) = F(1^+) = \frac{2}{5}$

iii) at $x = 2$, $F(2) = F(2^-) = F(2^+) = 1$

$\therefore X$ is continuous

$$p_X(x) = \begin{cases} 2/5 & 0 < x \leq 1 \\ 3/5 & 1 < x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

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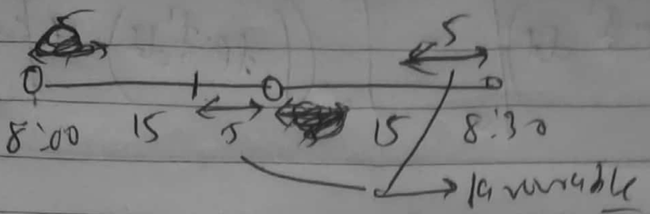
Q45)



Total length of possible arrival of bus = 30

Favourable length of arrival of bus = 10

as, each differential length is equally probable to arrived by passengers

$$P(\text{he/she waits} < 5 \text{ min}) = \frac{10}{30} = \frac{1}{3}$$

Q46) for, f to be valid,

$$\int_0^\infty f(x)\,dx = 1$$

$$\Rightarrow c \int_0^\infty \frac{x^2}{v} \frac{e^{-gx^2}}{v}\,dx = 1$$

$$\Rightarrow \cancel{\frac{c}{2}\int_0^\infty x e^{-gt} dx = 1}$$ $\cancel{(x^2 = t)}$

put, $gx^2 = \frac{t^2}{2}$ $\Rightarrow$, $\sqrt{2g}\,x = t$

$\Rightarrow \sqrt{2g}\,dx = dt$

$$\Rightarrow \frac{c}{\sqrt{2g}} \int_0^\infty \frac{t^2}{2g} e^{-\frac{t^2}{2}}\,dt = 1$$

$$\Rightarrow \frac{(2g)^{3/2}}{c} = \int_0^\infty \frac{t \cdot t\, e^{-\frac{t^2}{2}}}{\sqrt{\;}\;\sqrt{\;}}\,dt$$

$$= \cancel{\int 2dt\, t^2 x = \int_0^\infty e^{-\frac{t^2}{2}} dt} -$$

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$$\Rightarrow \left(t \cdot \int t e^{-\frac{t^2}{2}} dt \right)\Big|_0^\infty - \int_0^\infty \left(\int t e^{-\frac{t^2}{2}} dt \right) dt = \frac{(2g)^{3/2}}{c}$$

$$\boxed{\int t e^{-\frac{t^2}{2}} dt = \int e^{-p}\, dp \quad \left(\frac{t^2}{2} = p\right) \\ = \left(-e^{-\frac{t^2}{2}}\right)}$$

$$\Rightarrow \left(\cancel{-t e^{-\frac{t^2}{2}}\Big|_0^\infty} + \int_0^\infty e^{-\frac{t^2}{2}} dt \right) = \frac{(2g)^{3/2}}{c}$$

$$\Rightarrow \sqrt{2\pi} \times \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{t^2}{2}} dt = \frac{(2g)^{3/2}}{c}$$

$$\Rightarrow \sqrt{2\pi} \times \Phi(0) = \frac{(2g)^{3/2}}{c}$$

$$\Rightarrow c = \frac{(2g)^{3/2}}{\sqrt{\pi}} \times \sqrt{2} = 4g\sqrt{\frac{g}{\pi}}$$

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47) Given, $f_X(x) = \begin{cases} 4x^3 &, 0<x<1 \\ 0 &, o.w \end{cases}$

$\therefore F_X(x) = \int_0^x 4x^3\,dx$

$= x^4$

$\therefore F_X(x) = \begin{cases} x^4 &, 0<x<1 \\ 0 &, x \le 0 \\ 1 &, x \ge 1 \end{cases}$

25^{th} percentile, $x_{0.25}$ is given by:-

$F_X(x_{0.25}) = 0.25$

$\Rightarrow x_{0.25}^4 = 0.25$

$\Rightarrow x_{0.25} = \left(\frac{1}{4}\right)^{1/4} = \frac{1}{\sqrt{2}}$

48) $E[Y] = \int_{-\infty}^{\infty} y f(y)\,dy$

$= \int_{-1}^{1} y(1-|y|)\,dy$

$= \int_0^1 y(1-y)\,dy + \int_{-1}^{0} y(1+y)\,dy$

$= \left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1 + \left(\frac{y^2}{2} + \frac{y^3}{3}\right)\Big|_{-1}^{0}$

$= \frac{1}{2} - \frac{1}{3} - \frac{1}{2} + \frac{1}{3} = 0$

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$$E[Y^2] = \int_{-\infty}^{\infty} y^2 f(y)\,dy$$

$$= \int_0^1 y^2(1-y)\,dy + \int_{-1}^{0} y^2(1+y)\,dy$$

$$= \left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Bigg|_0^1 + \left(\frac{y^3}{3} + \frac{y^4}{4}\right)\Bigg|_{-1}^{0}$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{3} - \frac{1}{4}$$

$$= 2\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{6}$$

$$\therefore Var[Y] = E(Y^2) - E^2(Y)$$

$$= \frac{1}{6}$$

49) $F(U) = \begin{cases} 1-\cos U, & 0 \le U \le \pi/2 \\ 0, & U < 0 \\ 1, & U > \pi/2 \end{cases}$

$\therefore f(U) = \frac{d}{dU}(F(U)) = \sin U \quad , 0 \le U \le \pi/2$

$f(U) = \begin{cases} \sin U, & 0 < U \le \pi/2 \\ 0, & \text{O.W} \end{cases}$

$$\therefore E[U] = \int_{-\infty}^{\infty} U f(U)\,dU = \int_0^{\pi/2} U \sin U\,dU$$

$$= \left(U \int_0^{\pi/2} \sin U\,dU\right)\Bigg|_0^{\pi/2} + \int_0^{\pi/2} \cos U\,dU$$

$$= U\{-\cos U\}\Big|_0^{\pi/2} + \sin U\Big|_0^{\pi/2} = 1$$

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Q 50) $E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_0^1 \frac{3}{4} x\,dx + \int_2^3 \frac{1}{4} x\,dx$$

$$= \frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} \times (3^2 - 2^2)$$

$$= \frac{3}{8} + \frac{5}{8} = \underline{\underline{1}}$$

$E[X^2] = \int_0^1 \frac{3}{4} x^2\,dx + \int_2^3 \frac{1}{4} x^2\,dx$

$$= \frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}}\Big|_0^1 + \frac{1}{4} \times \frac{x^3}{3}\Big|_2^3$$

$$= \frac{1}{4} + \frac{\cancel{3^3}^{\,9}}{\cancel{3}\times 4} - \frac{\cancel{2^3}^{\,2}}{\cancel{4}\times 3}$$

$$= \frac{1}{4} + \frac{9}{4} - \frac{2}{3} = \frac{5}{2} - \frac{2}{3} = \frac{15-4}{6} = \frac{11}{6}$$

$\therefore Var[X] = E[X^2] - E[X]^2$

$$= \frac{11}{6} - 1$$

$$= \frac{5}{6}$$

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Sl)

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{1}^{\infty} \frac{2}{x^2}\, dx$$

$$= \left(\frac{-2}{x}\right)\Big|_{1}^{\infty}$$

$$= 2$$

$$Var[X] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{1}^{\infty} \frac{2}{x}\, dx$$

$$= 2\ln x \Big|_{1}^{\infty} \to \infty$$

So. $E[X] = 2 \Rightarrow$ finite

$Var[X] = \infty \Rightarrow$ infinite

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Q52) $f(x) = \begin{cases} \frac{1}{2}x^2e^{-x} & , 0 < x \le \infty \\ 0 & , o\omega. \end{cases}$

$\therefore f'(x) = \frac{1}{2}\times 2xe^{-x} - \frac{x^2e^{-x}}{2}$

$= \frac{e^{-x}}{2}(2x - x^2)$

equating $f'(x) = 0$:-

$x^2 = 2x$

$\Rightarrow x = 2$

also, $f''(x) = e^{-x} - xe^{-x} - xe^{-x} + \frac{x^2}{2}e^{-x}$

$= e^{-x}(1 - 2x + x^2/2)$

$f''(2) = e^{-2}(1 - 4 + 2)$

$= -e^{-2} < 0$

$\therefore$ maxima at $x = 2$

$\therefore x_0 = 2$ is a mode of density f.

$\therefore f$ is unimodal

83) $f(x) = \frac{1}{\pi(1+x^2)}$, $x \in (-\infty, \infty)$

$F(x) = P(X \le x)$

$= \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)} dx$

$= \frac{1}{\pi}\left(\tan^{-1} x - \tan^{-1}(-\infty)\right)$

$= \left(\frac{\tan^{-1} x}{\pi} + \frac{1}{2}\right)$

for median, m, $F(x) = 0.5$

$\Rightarrow F(m) = 0.5$

$\Rightarrow \frac{\tan^{-1} m}{\pi} + \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \underline{\underline{m = 0}}$

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54) $M_X(t) = e^{t^2+3t}$

I) We know, m.g.f. of Normal Random variable is of form
$$e^{\mu t + \frac{\sigma^2}{2}t^2}$$

$\therefore X \sim N(3, 2)$

$\therefore$ Mean of $X = 3$

Var of $X = 2$

II) also, we know,
$$\frac{d}{dt} M_X(t) \Big|_{t=0} = E[X]$$

$$\Rightarrow E[X] = e^{(t^2+3t)} \times (2t+3) \Big|_{t=0}$$

$$= 3$$

$$\frac{d^2}{dt^2} M_X(t) \Big|_{t=0} = E[X^2]$$

$$\Rightarrow E[X^2] = \frac{d}{dt}\left(2te^{t^2+3t} + 3e^{t^2+3t}\right)\Big|_{t=0}$$

$$= 2e^{t^2+3t} + 2te^{t^2+3t}(2t+3) + 3e^{t^2+3t}(2t+3)\Big|_{t=0}$$

$$= 2 + 9 = 11$$

$\therefore Var[X] = E[X^2] - E^2[X] = 11 - 3^2 = 2$

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55) Given;

$$f_X(x) = \begin{cases} \frac{1}{2} & , \cancel{-1/2 \le x \le 1/2}\ -1 \le x \le 1 \\ 0 & , \text{o.w} \end{cases}$$

$$Y = X^2$$

~~for, $x \in [-1/2, 1/2]$~~

$\therefore$ ~~$F_X(x) =$~~

for, $x \in [-1, 1]$

$$F_X(x) = P(X \le x)$$

$$= \int_{-1}^{x} \frac{1}{2}\,dx = \frac{1}{2}(x+1)$$

$$\therefore F_X(x) = \begin{cases} \frac{1}{2}(x+1) & , x \in [-1, 1] \\ 0 & , x < -1 \\ 1 & , x > 1 \end{cases}$$

also, for, Y:-

Case I:- $y < 0$,

$F_Y(y) = P(Y \le y) = 0$, as, $Y \ge 0$ always

Case II:- $\cancel{0} \le y \le 1$

$$F_Y(y) = P(Y \le y)$$

$$= P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y})$$

$$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

$$= \frac{1}{2}(\sqrt{y} + \sqrt{y}) = \sqrt{y}$$

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Case III :- $y > 1$

$\therefore F_Y(y) = P(Y \le y)$

$= 1$ (as, $Y = X^2 \le 1$ always)

$\therefore f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{2\sqrt{y}}$

$$\therefore f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & y \in [0, 1] \\ 0, & y > 1 \end{cases}$$

Q56) Given, $P(X \le x) = F(x) \rightarrow$ cdf of X

$Y = F(X)$

Case I :- $0 \le y \le 1$

$\underset{\text{cdf of } Y}{G(y)} = P(Y \le y) = P(F(X) \le y)$

$= P(X \le F^{-1}(y)) \rightarrow$ as F is strictly increasing

$= F(F^{-1}(y)) = y$

$\therefore \underset{\text{pdf of } y}{g(y)} = \frac{d}{dy} G(y) = 1$

Case II :- $y < 0$:-

$G(y) = P(Y \le y) = P(F(X) \le y)$

$= 0$

as, range of F is $[0, 1]$

Case III :- $y > 1$:-

$G(y) = P(F(X) \le y)$

$= 1$ as, F is always less than 1

$\therefore$ Y has a uniform distribution on $[0, 1]$.

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Q57)

a) $P(0 \le X \le 0.87)$

$= \Phi(0.87) - \Phi(0)$

$= \Phi(0.87) - \frac{1}{2}$ $\quad\quad (\Phi(0) = \frac{1}{2}) \to$ symmetry

$= 0.80785 - 0.5 = 0.30785$

b) $P(-2.64 \le X \le 0)$

$= \Phi(0) - \Phi(-2.64)$

$= \frac{1}{2} + \Phi(2.64) - 1 = \Phi(2.64) - \frac{1}{2}$

$= 0.99585 - 0.5 = 0.49585$

c) $P(-2.13 \le X \le 0.56)$

$= \Phi(0.56) - \Phi(-2.13)$

$= \Phi(2.13) - \Phi(0.56)$

$= 0.98341 - 0.71226 = 0.27115$

d) $P(|X| \geq 1.39)$

$= 1 - P(|X| \le 1.39)$

$= 1 - P(-1.39 \le X \le 1.39)$

$= 1 - \Phi(1.39) + \Phi(-1.39)$

$= 1 - \Phi(1.39) + 1 - \Phi(1.39) = 2(1 - \Phi(1.39)) = 2(1 - 0.91774)$

$= 0.16452$

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Q58) $M_X(t) = e^{-6t+32t^2}$

we know, m.g.f of Normal variable $N(\mu, \sigma^2)$ is of form $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$\therefore$ clearly,

$X \sim N(-6, 8)$

$\therefore$ $P(-4 \le X \le 16)$

$= P\left(\frac{2}{\sqrt{8}} \le \frac{X+6}{\sqrt{8}} \le \frac{22}{\sqrt{8}}\right)$

$= P\left(\frac{1}{\sqrt{2}} \le Z \le \frac{11}{\sqrt{2}}\right)$ $\quad (Z \sim N(0,1))$

$= \Phi\left(\frac{11}{\sqrt{2}}\right) - \Phi\left(\frac{1}{\sqrt{2}}\right)$ $\quad \left(\Phi\left(\frac{11}{\sqrt{2}}\right) \approx 1\right)$

$= 1 - 0.76$

$= \underline{\underline{0.24}}$

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59) $X \sim \mathcal{N}(0, 6^2)$

$Y = -X$

$F_X(x) =$ cdf of X , $f_X(x) =$ pdf of $X = \frac{1}{\sqrt{2\pi}6} \exp\left(\frac{-x^2}{26^2}\right)$

$F_Y(y) =$ cdf of Y

Then;

$F_Y(y) = P(Y \le y) = P(-X \le y)$

$= P(X \ge -y)$

$= 1 - P(X < -y)$

$= 1 - F_X(-y)$

$\therefore f_Y(y) = \frac{d}{dy} F_Y(y)$

$= -\cancel{f_X(y)}\ - f_X(-y) \cdot \frac{d}{dy}(-y)$

$= f_X(-y)$

$= \frac{1}{\sqrt{2\pi 6}} \exp\left(\frac{-y^2}{26^2}\right)$

$\therefore Y \sim \mathcal{N}(0, 6^2)$

ie $-X \sim \mathcal{N}(0, 6^2)$ $\rightarrow$ proved

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60) $X \sim N(\mu, \sigma^2)$

$Y = |X - \mu|$

Let, pdf of X be f_X and cdf be F_X

Similar, f_Y, F_Y for Y.

Case I :- $y < 0$

$F_Y(y) = P(Y \leq y) = 0$ (as $Y \geq 0$ always clearly)

$\therefore f_Y(y) = 0$

Case II :- $y \geq 0$

$$F_Y(y) = P(Y \leq y)$$
$$= P(|X-\mu| \leq y)$$
$$= P(-y \leq X-\mu \leq y)$$
$$= P(-y+\mu \leq X \leq y+\mu)$$
$$\cancel{= P(X \leq y+\mu) - P(\ldots)}$$
$$= F_X(y+\mu) - F_X(-y+\mu)$$

$$\therefore f_Y(y) = \frac{d}{dy} F_Y(y) = f_X(y+\mu) + f_X(-y+\mu)$$
$$= \frac{2}{\sqrt{2\pi}\sigma}\left(\exp\left(-\frac{y^2}{2\sigma^2}\right)\right)$$
$$= \frac{2}{\sqrt{2\pi}\sigma}\exp\left(\frac{-y^2}{2\sigma^2}\right)$$

$$\therefore f_Y(y) = \begin{cases} \frac{2}{\sqrt{2\pi}\sigma}\exp\left(\frac{-y^2}{2\sigma^2}\right), & y \geq 0 \\ 0, & y < 0 \end{cases}$$

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$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y)\,dy$$

$$= \int_0^{\infty} \frac{2y}{\sqrt{2\pi}\sigma} \exp\left(\frac{-y^2}{2\sigma^2}\right) dy$$

put, $\frac{y^2}{2\sigma^2} = t$

$$\Rightarrow \frac{2y}{2\sigma^2}\,dy = dt$$

$$\Rightarrow E[Y] = \int_0^{\infty} \frac{2y}{\sqrt{2\pi}\sigma} \times \frac{\sigma^2}{y} \exp(-t)\,dt$$

$$= \sigma\sqrt{\frac{2}{\pi}} \int_0^{\infty} \exp(-t)\,dt$$

$$= \sigma\sqrt{\frac{2}{\pi}}\left(-\exp(-t)\right)\Big|_0^{\infty}$$

$$= \sigma\sqrt{\frac{2}{\pi}}$$ proved

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61) $X \sim N(\mu, \sigma^2)$ $, -\infty < x < \infty$

$Y = \frac{(X-\mu)^2}{\sigma^2}$, ~~$0 \le y$~~

Let $X \sim$ pdf f , cdf F , ~~$f(x)$~~ $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$Y \sim$ pdf g , cdf G

$\therefore G(y) = P(Y \le y)$

$= P\left(\frac{(X-\mu)^2}{\sigma^2} \le y\right)$

For, $y < 0$

$G(y) = P(Y \le 0) = 0$ as $Y \ge 0$ w.p. 1

For $y \ge 0$:-

$G(y) = P(Y \le y) = P\left(\mu - \sqrt{y}\sigma \le X \le \mu + \sqrt{y}\sigma\right)$

$= \{ P(X \le \mu + \sqrt{y}\sigma) - P(X \le \mu - \sqrt{y}\sigma)$

$= F(\mu + \sqrt{y}\sigma) - F(\mu - \sqrt{y}\sigma) \}$

$\therefore g(y) = \frac{d}{dy} G(y) = f(\mu + \sqrt{y}\sigma) \times \sigma \times \frac{1}{2\sqrt{y}} + f(\mu - \sqrt{y}\sigma) \times \sigma \frac{1}{2\sqrt{y}}$

$= \frac{\cancel{\sigma}}{\cancel{2}\sqrt{y}} \{ \times \cancel{2} \times e^{-\frac{y}{2}} \times \frac{1}{\sqrt{2\pi}\cancel{\sigma}} = \left(\frac{e^{-\frac{y}{2}}}{\sqrt{2\pi y}}\right) = \frac{y^{-\frac{1}{2}} e^{-y/2}}{2^{1/2}\Gamma\frac{1}{2}}$ (y)

$\therefore Y \sim \chi_1^2$

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pdf

62) $X \sim N(5, 10) \Rightarrow \left(\frac{X-5}{\sqrt{10}}\right) \sim Z$

$$P\left(0.04 < (X-5)^2 < 38.4\right)$$

$$= P\left(\frac{0.04}{10} < \frac{(X-5)^2}{10} < \frac{38.4}{10}\right)$$

$$= P\left(0.004 < Z^2 < 3.84\right)$$

$$= P\left(\sqrt{0.004} < Z < \sqrt{3.84}\right) + P\left(-\sqrt{3.84} < Z < -\sqrt{0.004}\right)$$

$$= P(Z < \sqrt{3.84}) - P(Z < \sqrt{0.004}) + P(Z < \sqrt{0.004}) - P(Z < \sqrt{3.84})$$

$$= \Phi(\sqrt{3.84}) - \Phi(\sqrt{0.004}) + \Phi(\sqrt{0.004}) - \Phi(-\sqrt{3.84})$$

$$= 2\left(\Phi(1.96) - \Phi(0.063)\right) \qquad \left(\Phi(x) + \Phi(-x) = 1\right)$$

$$= 2(0.975 - 0.52392)$$

$$= \underline{0.9022}$$

63) Given, $\log X \sim \mathcal{N}(\mu, \sigma^2)$

Let, r.v. Y [redacted] $= \log X$

$\Rightarrow X = e^Y$

let, f_X be pdf of X, F_X be cdf of X
f_Y be " " Y, F_Y " " " Y

then,

Case I :- $x \leq 0$

$F_X(x) = P(X \leq x) = 0$ as, $x \geq 0$ always clearly.

$\therefore f_X(x) = 0$

Case II: $x > 0$;

$$F_X(x) = P(X \leq x)$$
$$= P(e^Y \leq x)$$
$$= P(Y \leq \ln x)$$
$$= F_Y(\ln x)$$

$$\therefore f_X(x) = \frac{d}{dx} F_X(x) = \underline{f_Y(\ln x) \cdot \frac{1}{x}} \quad -(I)$$

$$* \; E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^{\infty} x f_X(x)\,dx$$

$$= \int_0^{\infty} f_Y(\ln x)\,dx$$

putting, $\ln x = t \Rightarrow dx = x\,dt$

also, limit changes to $\infty \to -\infty$

as, $\lim_{x \to 0^+} \ln x \to -\infty$

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$$\therefore E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{t} \cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu-\sigma^2)^2}{2\sigma^2}} \cdot e^{\frac{(\sigma^2+2\mu)}{2}}\, dt$$

put, $\left(\frac{t-\mu-\sigma^2}{\sigma}\right) = p$

$\Rightarrow dt = \sigma\, dp$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\left(\frac{\sigma^2}{2}+\mu\right)} \cdot e^{-\frac{p^2}{2}}\, dp$$

$$= e^{\left(\frac{\sigma^2}{2}+\mu\right)} \times \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{p^2}{2}}\, dp$$

$$= e^{\left(\frac{\sigma^2}{2}+\mu\right)} \times \Phi(\infty)$$

$$= e^{\left(\frac{\sigma^2}{2}+\mu\right)}$$

$Z \sim N(0,1)$

$\Phi(\infty) = P(Z \le \infty)$

$= 1$

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$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_{0}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_{0}^{\infty} x f_X(\ln x)\, dx$$

similarly to E(X) calculation result

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu-2\sigma^2)^2}{2\sigma^2}} \cdot e^{2(\mu+\sigma^2)}\, dt$$

=, similarly to E[X] :-

$$= e^{2(\mu+\sigma^2)}$$

$$\therefore Var(X) = E[X^2] - E^2[X]$$

$$= e^{(2\mu+2\sigma^2)} - e^{(\sigma^2+2\mu)}$$

$$= (\exp(\sigma^2)-1)\exp(\sigma^2+2\mu)$$

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64) Given, $X \sim Gamma(\lambda, \alpha)$

we know;

$$E[X] = \frac{\alpha}{\lambda} = 2$$

$$var[X] = \frac{\alpha}{\lambda^2} = 7$$

$$\therefore \lambda = \frac{2}{7}$$

and, $\alpha = \frac{4}{7}$

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65) Given, $X_1, X_2, X_3, X_4 \sim N(100, 25)$ iid

we know, for, $X \sim N(\mu, \sigma^2)$

$$M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

Given and, X_i's are iids. then,

$$M_{\Sigma X_i}(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

Given $Y = X_1 - 2X_2 + X_3 - 3X_4$

$\therefore$ ~~$M_Y(t)$~~ $= e^{(\Sigma\mu_i)t + \cancel{(\Sigma\sigma_i)t}}$

Let us denote by, $\mu_i = E[X_i] = \mu = 100$

$\sigma_i^2 = Var[X_i] = \sigma^2 = 25$

and, $a_1 = 1$

$a_2 = -2$

$a_3 = 1$

$a_4 = -3$

$$\therefore M_Y(t) = e^{(\Sigma a_i\mu_i)t + \frac{(\Sigma a_i^2\sigma_i^2)}{2}t}$$

$$= e^{\mu t(\Sigma a_i) + (\Sigma a_i^2)\frac{\sigma^2}{2}t}$$

$$= e^{-3\mu t + \frac{(\sqrt{15}\sigma)^2}{2}t}$$

$\therefore Y \sim N(-3\mu, 15\sigma^2)$

$\therefore Y \sim N(-300, 375)$

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66) $f(x) = \begin{cases} kx(x-1) & \text{if } 0<x<1 \\ 0 & \text{o.w.} \end{cases}$

$\because$ for, $f(x)$ to be a valid pdf:-

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow k\int_0^1 (x - x^2)\,dx = 1$$

$$\Rightarrow k = \frac{1}{\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1} = 6$$

also, $P(X > \frac{1}{2}) = \int_{1/2}^{\infty} f(x)\,dx$

$$= \int_{1/2}^{1} (6x - 6x^2)\,dx$$

$$= 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{1/2}^{1} = 6\left(\frac{1}{2} - \frac{1}{3} - \frac{1}{8} + \frac{1}{24}\right)$$

$$= 6\left(\frac{12-8-3+1}{24}\right)$$

$$= \frac{\cancel{6}\times 2}{\cancel{24}} = \frac{1}{2}$$

$\therefore k = 6$, $P(X > \frac{1}{2}) = \frac{1}{2}$

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$$f(x) = \begin{cases} 0.5 & , \text{ if } x = -1 \\ e^{-x}/2 & , 0 < x < \infty \\ 0 & , \text{o.w} \end{cases}$$

$$M_X(t) = E[e^{tX}]$$

$$= \int_0^\infty e^{tx} \times \frac{e^{-x}}{2} dx + e^{-t} \times \frac{1}{2}$$

$$= \frac{e^{-t}}{2} + \frac{1}{2}\int_0^\infty e^{(t-1)x} dx$$

clearly, $M_X(t)$ exists only for, $t < 1$

$\therefore$ for $t < 1$:-

$$M_X(t) = \frac{e^{-t}}{2} - \frac{1}{2(t-1)}$$

$$= \frac{1}{2}\left(e^{-t} + \frac{1}{1-t}\right)$$

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