Assignment -1

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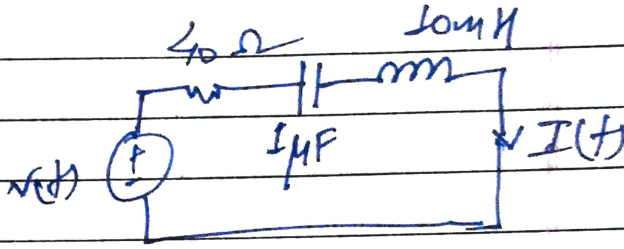
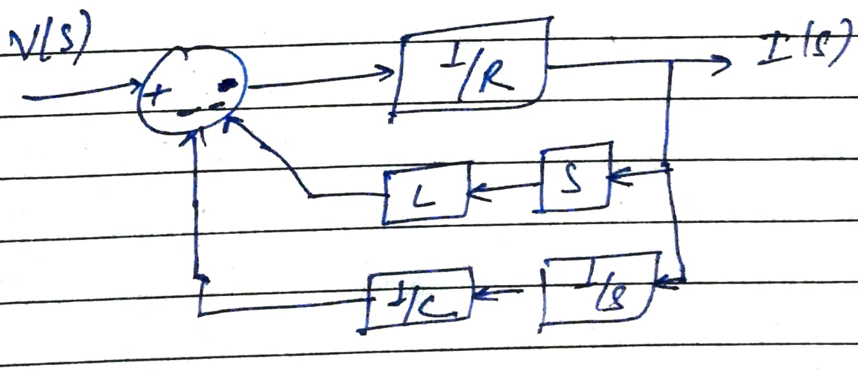
(1) (a) (i)



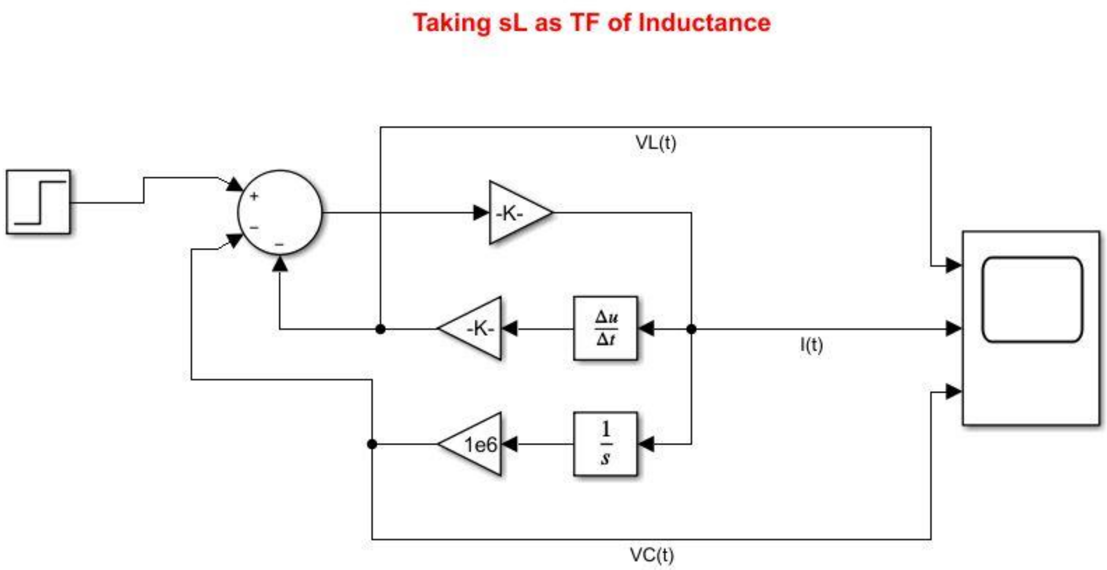
$$V(s) = R\,I(s) + sL\,I(s) + \frac{1}{sC}\,I(s)$$

$$\frac{I(s)}{V(s)} = \frac{1}{R\left(1+\frac{1}{R}\left(sL+\frac{1}{sC}\right)\right)}$$

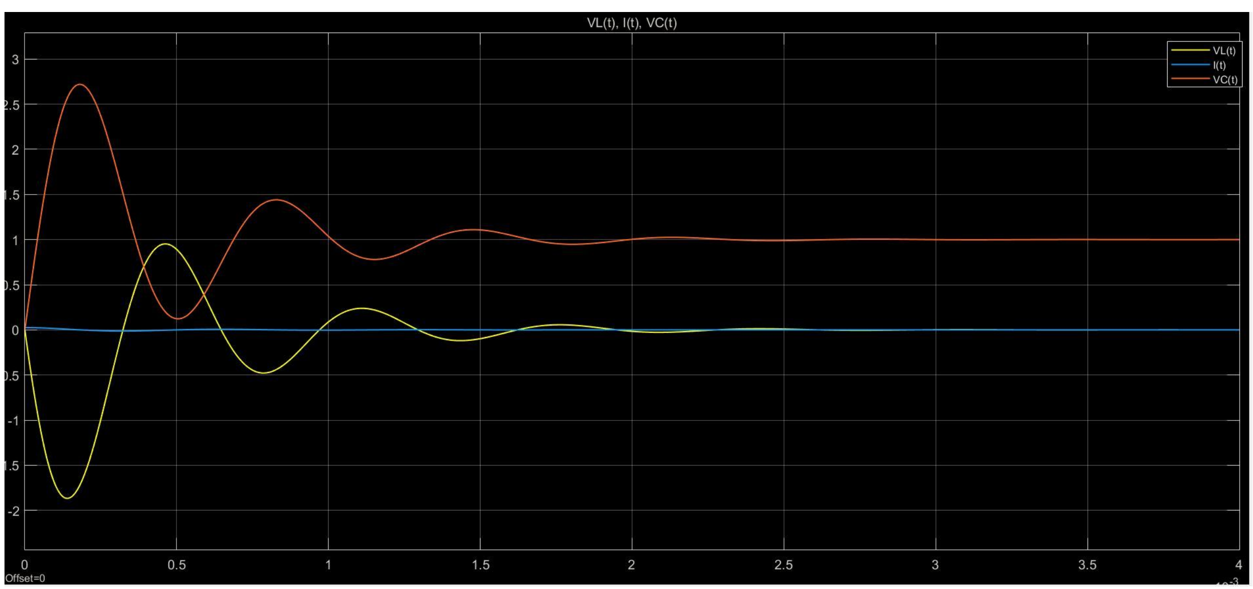
$$= \frac{\frac{1}{R}}{1+\frac{1}{R}\left(sL+\frac{1}{sC}\right)}$$



Block Diagram Representation:



Using sL as the transfer function. VL(t) is the voltage across the inductor, VC(t) is the voltage across the capacitor and I(t) is the current in the circuit



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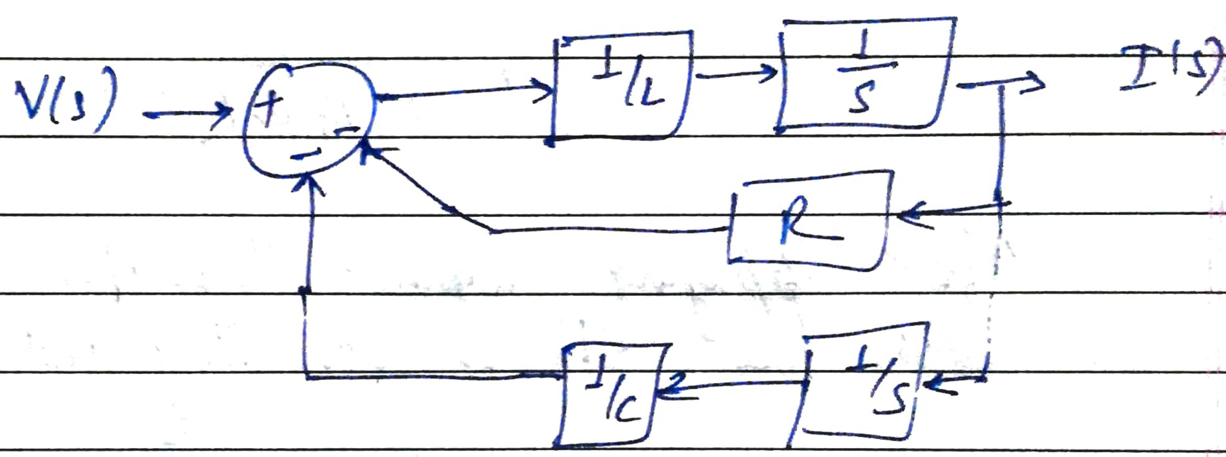
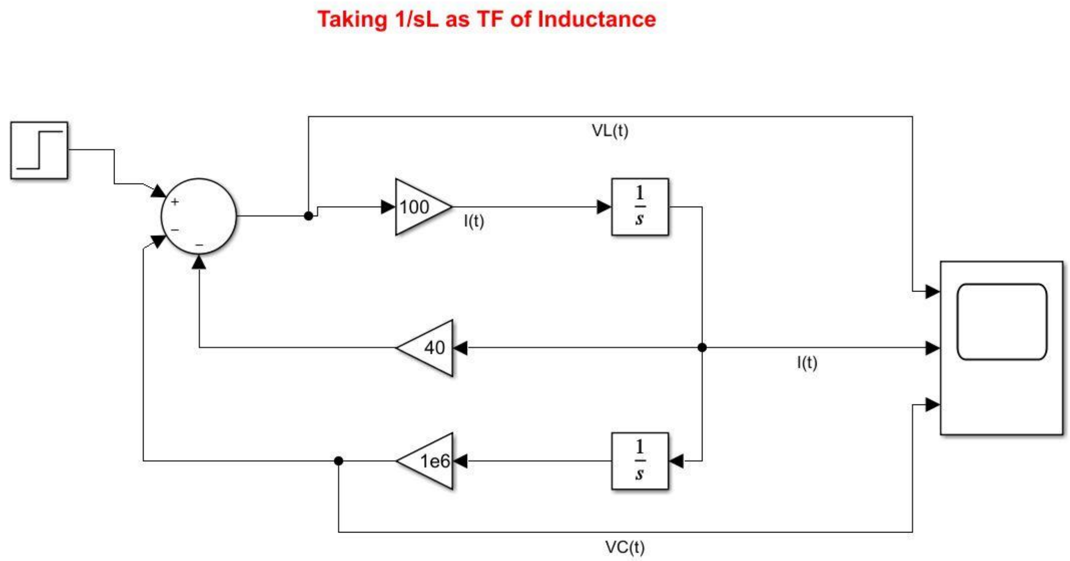
(ii) Using $\frac{1}{sL}$ as tf for inductor.

$$\frac{I(s)}{V(s)} = \frac{1}{sL\left(1+\frac{1}{sL}\left(R+\frac{1}{sC}\right)\right)}$$

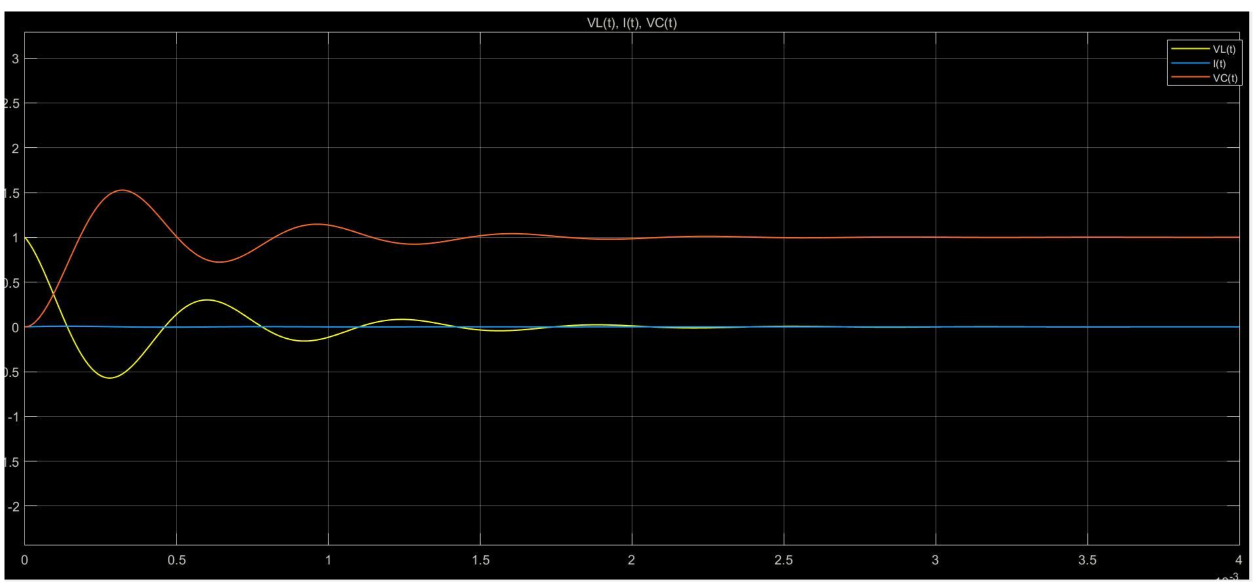
$$= \frac{\left(\frac{1}{sL}\right)}{1+\left(\frac{1}{sL}\right)\left(R+\frac{1}{sC}\right)}$$



Block Diagram Representation:



Using 1/sL as the transfer function. VL(t) is the voltage across the inductor, VC(t) is the voltage across the capacitor and I(t) is the current in the circuit



(iii) The responses obtained in (i) & (ii) are not identical.

	(i)	(ii)
V_c	Overshoot is more than 100%., $V_{c\,peak)} > 2V_\infty$	Overshoot is around 50%. $V_{c\,peak} \approx 1.5V_c(\infty)$
V_L	At $t=0$, its value is 0.	At $t=0$, its value is equal to 1V
I	Jump of 0.025A at $t=0$, later damped (exp) oscillation,	Continuous at $t=0$ & later damped osci.

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Reason is that "s" is an Improper Transfer function. Also we know that improper TF is physically unrealizable.

Proof:-
Suppose we have a state space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $A \in R^{n \times n}$

After applying laplace Transform on both sides, the TF is coming out to be

$$G(s) = C(sI_n - A)^{-1} B + D$$

$$(sI_n - A)^{-1} = \frac{adj(sI_n - A)}{det(sI_n - A)}$$

Also we know that
→ each entry of the adjugate is s is of degree at most equal to n.
→ the determinant of $sI_n - A$ is of degree n.

So, every n^2 TF in $G(s)$ has the property that the degree of numerator is less than or equal to the degree of denominator.

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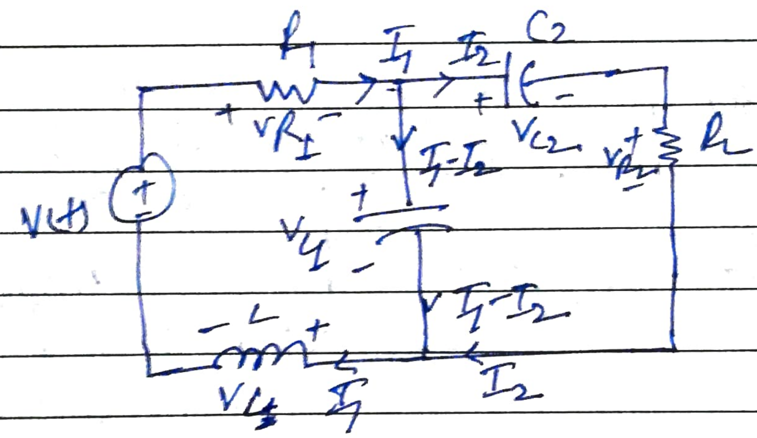
So, the improper transfer function cannot have a state-space realization & hence they are physically unrealizable.

Therefore using "$\frac{1}{SL}$" T.F to represent Inductor is proper & correct.

(b)



$$V(s) = V_{R_1}(s) + V_{C_1}(s) + V_{L_1}(s)$$

$$\Rightarrow I_1(s) = \frac{V(s) - V_{C_1}(s) - V_L(s)}{R} = \frac{V_L(s)}{SL}$$

$$\Rightarrow I_1(s) = \frac{1}{SL}\left(V(s) - V_1(s) - V_R(s)\right)$$

$$V_{R_1}(s) = R_1 I_1(s)$$

$$V_{C_1}(s) = \frac{1}{SC_1}\left(I_1(s) - I_2(s)\right)$$

$$V_{C_1}(S) = V_{C_2}(s) + V_{R_2}(s)$$

$$= R_2 I_2(s) + \frac{1}{SC_2} I_2(s)$$

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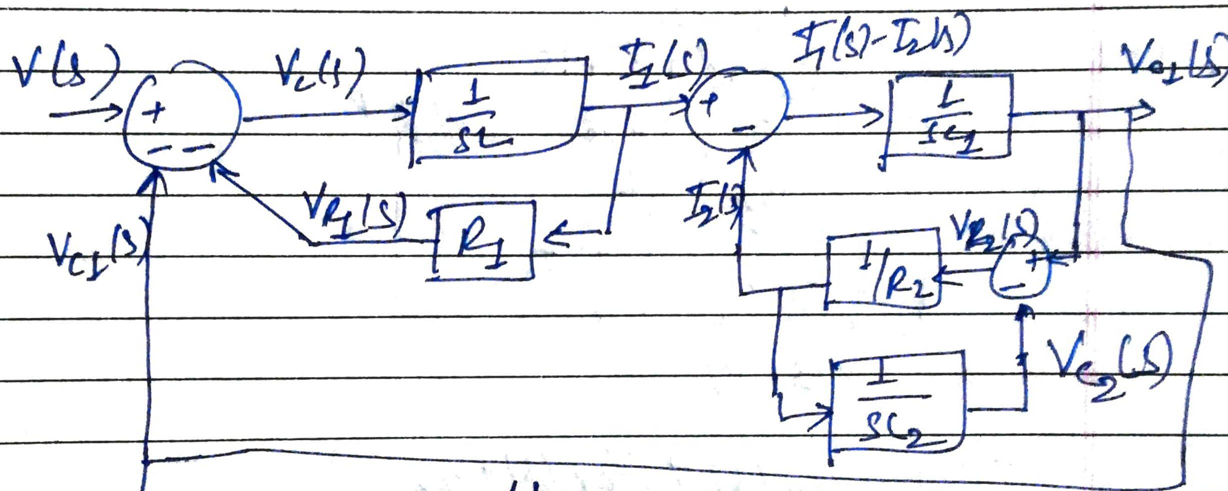
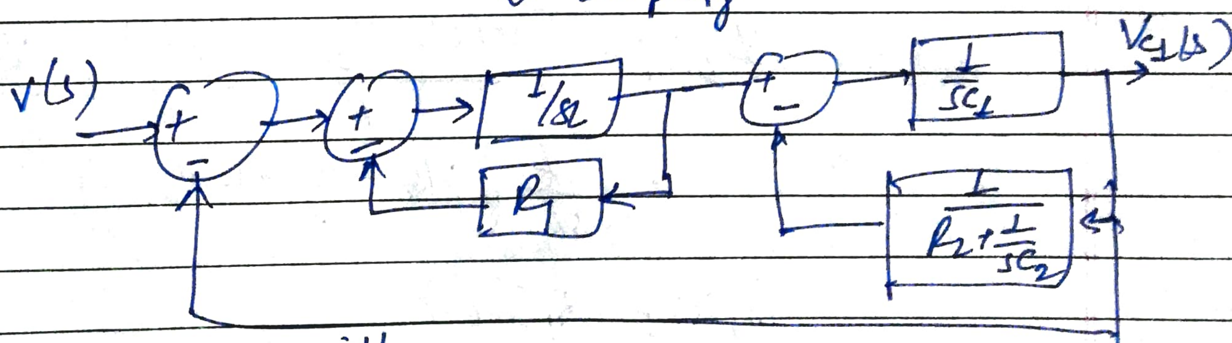
$$\Rightarrow \quad I_2(s) = \frac{V_{C_1}(s)}{R_2 + \frac{1}{sC_2}}$$

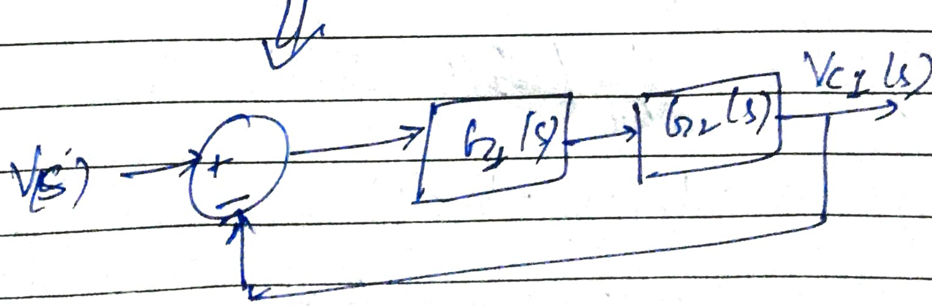
$$= \frac{\frac{1}{R_2}}{1 + \cancel{R}\frac{1}{sC_2R_2}} V_{C_1}(s)$$



⇓ Simplify



⇓



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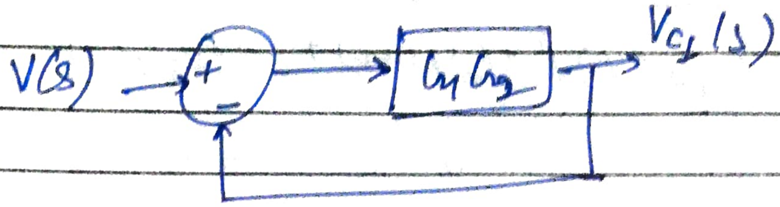

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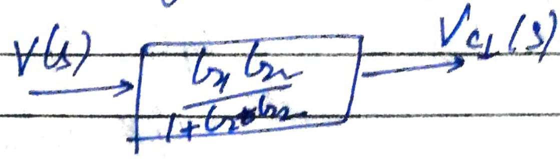

$$G_1(s) = \frac{\frac{1}{sL}}{1+\frac{R}{1}\times\frac{1}{sL}} = \frac{1}{R_1+sL}$$

$$G_2(s) = \frac{\frac{1}{sC_1}}{1+\frac{1}{sC_1}\times\frac{1}{R_2+\frac{1}{sC_2}}}$$

$$= \frac{1+sR_2C_2}{sC_2+sC_1+s^2R_2C_2C_1}$$

$$\frac{V_{C_1}(s)}{V(s)} = \frac{G_1G_2}{1+G_1G_2}$$

$$= \frac{\left(\frac{1}{R_1+sL}\right)\cdot\left(\frac{1+sR_2C_2}{sC_2+sC_1+s^2R_2C_2C_1}\right)}{1+\left(\frac{1}{R_1+sL}\right)\left(\frac{1+sR_2C_2}{sC_2+sC_1+s^2R_2C_2C_1}\right)}$$

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$$= \frac{1 + sR_2C_2}{(R_1 + sL)\left(s(C_1+C_2) + s^2R_2C_1C_2\right) + (1 + sR_2C_2)}$$

$$= \frac{1 + sR_2C_2}{1 + sR_2C_2 + s\left[R_1C_1 + R_1C_2 + sR_1R_2C_1C_2 + sL(C_1+C_2) + s^2LR_2C_1C_2\right]}$$

$$= \frac{1 + sR_2C_2}{1 + s(R_1C_1 + R_1C_2 + R_2C_2) + s^2(R_1R_2C_1C_2 + LC_1 + LC_2) + s^3LR_2C_1C_2}$$

So,

$$\boxed{\frac{V_{C_1}(s)}{V(s)} = \frac{1 + sR_2C_2}{1 + s(R_1C_1 + R_1C_2 + R_2C_2) + s^2(R_1R_2C_1C_2 + LC_1 + LC_2) + s^3LR_2C_1C_2}}$$

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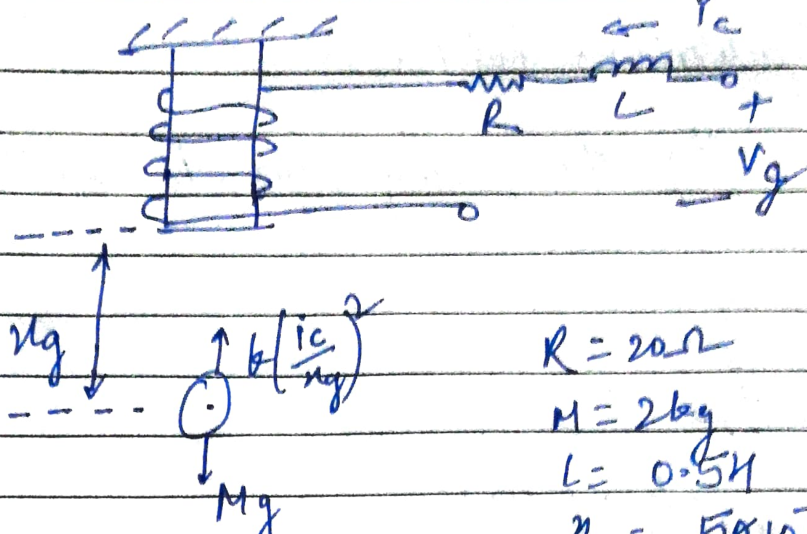
(2.)



$R = 20\Omega$
$M = 2kg$
$L = 0.5H$
$x_0 = 5\times10^{-3}m$
$k = 3\times10^{-4}\ Nm^2A^{-2}$

$$M\ddot{x}_g = Mg - k\left(\frac{i_c}{x_g}\right)^2$$

$$v_g = Ri_c + L\frac{di_c}{dt}$$

(a) $\ddot{x}_g$, i_c, x_g, $\frac{di_c}{dt}$ are involved, and number of states has to be minimized.

$\therefore x_1 = x_g,\ x_2 = \dot{x}_g,\ x_3 = i_c \Rightarrow \dot{x}_1 = \dot{x}_g,$

$\dot{x}_2 = \ddot{x}_g,\ \dot{x}_3 = \frac{di_c}{dt}$ [We covered all the variables]

State Space Equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ g-\left(\frac{k}{M}\right)\left(\frac{x_3}{x_1}\right)^2 \\ -\frac{R}{L}x_3 + \frac{V_g}{L} \end{bmatrix} \begin{array}{l} \rightarrow f_1 \\ \rightarrow f_2 \\ \rightarrow f_3 \end{array}$$

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(b) Equilibrium Point:

$x_g = x_0$, $\dot{x}_g = \dot{x}_0$, $i_c = i_0$ & $v_g = v_0$

$\dot{x}_g = 0$

$\dot{x}_{10} = 0$

$$g - \left(\frac{k}{M}\right)\left(\frac{x_{30}}{x_{10}}\right)^2 = 0$$

$$\Rightarrow \boxed{x_{30} = x_{10}\sqrt{\frac{Mg}{k}}}$$

$$= 5\times10^{-3} \times \sqrt{\frac{2\times9.8}{3\times10^{-4}}}\ A \simeq 1.28A$$

$$\boxed{i_{c0} = 1.28A}$$

$$\dot{x}_{30} = 0 \Rightarrow \frac{V_c}{L} = \frac{R}{L}x_3$$

$$V_{c0} = Rx_{30} = 25.56\ V$$

$$V_{c0} = \boxed{V_0 = 25.56\ V}$$

$\therefore$ $\boxed{x_0 = 0.005m,\ \dot{x}_0 = 0,\quad i_c = i_0 = 1.28A,\ V_g = V_0 = 25.56V}$

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(c) Incremental State Space Representation

$$\frac{\partial f_1}{\partial x_1} = 0, \quad \frac{\partial f_1}{\partial x_2} = 1, \quad \frac{\partial f_1}{\partial x_3} = 0, \quad \frac{\partial f_1}{\partial v_g} = 0$$

$$\frac{\partial f_2}{\partial x_1} = \frac{2k}{M} \cdot \frac{x_3^2}{x_1^3}\Bigg|_{x_{10}, x_{30}} = \frac{2k}{M} \frac{x_{30}^2}{x_{10}^3} = \frac{2g}{x_{10}}$$

$$\boxed{\frac{\partial f_2}{\partial x_1} = \frac{2g}{x_{10}}}$$

$$\boxed{\frac{\partial f_2}{\partial x_2} = 0 = \frac{\partial f_2}{\partial v_g} = 0}$$

$$\frac{\partial f_2}{\partial x_3} = \frac{-2k}{M} \cdot \frac{x_3}{x_1^2}\Bigg|_{x_{10}, x_{30}} = \frac{-2k}{M} \cdot \frac{x_{30}}{x_{10}^2}$$

$$\boxed{\frac{\partial f_2}{\partial x_3} = \frac{-2}{x_{10}}\sqrt{\frac{kg}{M}}}$$

$$\frac{\partial f_3}{\partial x_1} = 0, \quad \frac{\partial f_3}{\partial x_2} = 0, \quad \frac{\partial f_3}{\partial x_3} = \frac{-R}{L}, \quad \frac{\partial f_3}{\partial v_g} = \frac{1}{L}$$

$\frac{2g}{x_{10}} = \frac{2 \times 9.8}{0.005} = 3920$

$\frac{-2}{x_{10}}\sqrt{\frac{kg}{M}} = \frac{-2}{0.005}\sqrt{\frac{3 \times 10^{-4} \times 9.8}{2}} \simeq 15.34$

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$\frac{R}{L} = \frac{20}{0.5} = 40$

$\frac{1}{L} = \frac{1}{0.5} = 2$

$\therefore$ Incremental State Space Representation

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 3920 & 0 & -15.24 \\ 0 & 0 & -40 \end{bmatrix}}_{A}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B} v$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C}\begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}$$

(d) Transfer function from v to x

$\rightarrow G(s) = C(sI-A)^{-1}B.$

$$(sI-A) = \begin{bmatrix} s & -1 & 0 \\ -3920 & s & 15.24 \\ 0 & 0 & s+40 \end{bmatrix}$$

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$$(SI-A)^{-1} = \frac{1}{(S+40)(S^2-3920)} \begin{bmatrix} S(S+40) & S+40 & -15.34 \\ 3920(S+40) & S(S+40) & -15.34S \\ 0 & 0 & S^2-3920 \end{bmatrix}$$

$$C(SI-A)^{-1}B = \frac{2\times 1\times(-15.34)}{(S^2-3920)(S+40)} \Rightarrow$$

$$G(S) = \frac{-30.68}{(S+40)(S^2-3920)}$$

Order of $G(S) = 3$ (degree of denominator)

Type 0 system (no integrators)

There is a Right Hand plane pole at $s = +\sqrt{3920}$, making the system unstable.