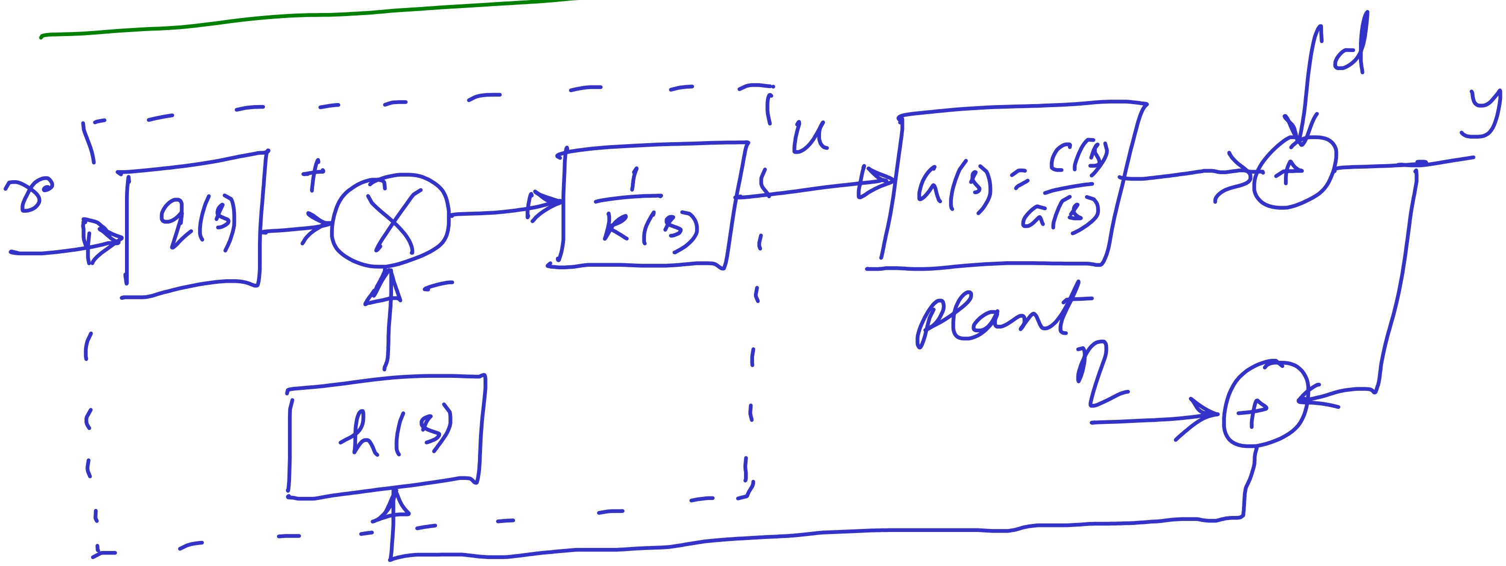
Lecture - 9

2-DOF Controller



2-dof controller

- In 2-dof control, r & y are independently treated since $h(s) \neq q(s)$. In case of 1-dof control $h(s) = q(s)$ and therefore the input to the controller is $r - y = e$ = tracking error.

- For an m-th order 2-dof controller

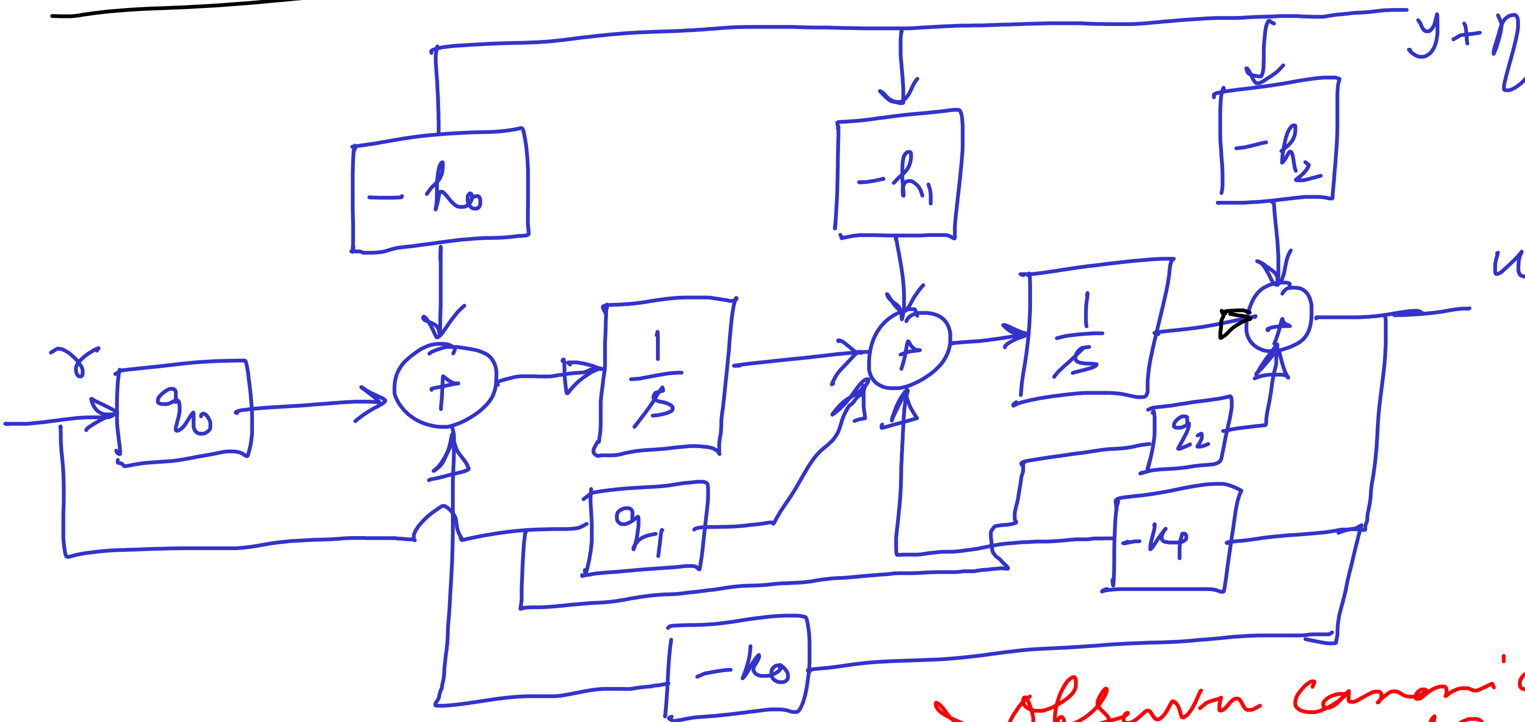
$$K(s) = s^m + k_{m-1} s^{m-1} + \cdots + k_1 s + k_0$$

$$h(s) = h_m s^m + h_{m-1} s^{m-1} + \cdots + h_1 s + h_0$$

$$q(s) = q_m s^m + q_{m-1} s^{m-1} + \cdots + q_1 s + q_0$$

[A minimum $m = n-1$ where n = plant order]

Realization of a 2nd order 2-dof Control



→ observer canonical form realization

$$u = -\frac{(h_2 s^2 + h_1 s + h_0)}{s^2 + k_1 s + k_0}(y+\eta) + \frac{(q_2 s^2 + q_1 s + q_0)}{s^2 + k_1 s + k_0}\, r$$

[verify this]

- Some important TFs with 2-dof control

1) r-to-y TF : $\frac{y}{r} = \frac{cq}{\delta}$ $\left(\text{with 1-dof } \frac{cn}{\delta}\right)$ ✓

2) d-to-y TF : $\frac{y}{d} = \frac{ak}{\delta}$ (same as 1-dof)

3) η-to y TF : $\frac{y}{\eta} = \frac{ch}{\delta}$ (same as 1-dof)

4) r-to u TF: $\frac{u}{r} = \frac{aq}{\delta}$ (using 1-dof $\frac{ah}{\delta}$)

Where $\delta = ak + ch$

5) Loop TF $L(s) = \frac{ch}{ak}$ (same as 1-dof)

Advantages of 2-DOF Configuration

- The additional polyn $q(s)$ helps achieve superior shaping of tracking characteristics & control effort characteristics with the same δ.

- The presence of $q(s)$ helps achieve some independency in shaping r-to-y & r-to-u behaviours while keeping the other loop behaviour same as 1-dof control

Design procedure of 2-dof control

Given ref. to output model

$$\boxed{T_{yr} = \frac{\alpha\, C(s)}{\hat{\delta}(s)}}$$

[where α is chosen such that $T(0) = 1$]

$\hat{\delta}(s)$ order = order of $a(s)$

[• Relative order of T_{yr} should be $\geq$ relative order of $G(s)$]

Here we choose =

[• The zeros of the plant are retained in T_{yr}. RHP zeros should always be retained. Otherwise the 2-DOF control can not be designed.]

The feedback system will be internally u/s.

1) *choose $\delta(s) = ak + ch = \hat{\delta}(s)\, \hat{q}(s)$ where $\hat{q}(s)$ is selected so as to meet loop-performances

2) Set $q(s) = \alpha\, \hat{q}(s)$

* once $\delta(s)$ is obtained the $x(s)$ & $h(s)$ polyⁿ can be found using sylvester

- with the above design

$$T_{yr} = \frac{cq}{\delta} = \frac{\alpha c \cancel{\hat{q}}}{\hat{\delta} \cdot \cancel{\hat{q}}} = \frac{\alpha c}{\hat{\delta}}$$

make appr ord

ie we get the desired ref. input to output model.

- Also $T_{ur} = \dfrac{aq}{\delta} = \dfrac{a \alpha \hat{q}}{\hat{\delta} \cdot \hat{q}}$

$$\Rightarrow \boxed{T_{ur} = \frac{\alpha a}{\hat{\delta}}} \Rightarrow \text{ref. i/p}$$

to control effort chan. is independent of choice of $\hat{q}$ ie q

Example: Suppose $G(s) = \dfrac{3}{s^2 - s}$

Let a desired $T_{yr}(s) = \dfrac{2.44}{s^2 + 2.4s + 2.44}$

$\alpha c(s) \rightarrow 2.44$

$\hat{\delta}(s) \rightarrow s^2 + 2.4s + 2.44$

[Note T_{yr} yields $\xi = 0.768$, $\omega_n = 1.562$

$$t_s = \frac{4}{\xi \omega_n} = 3.33\,\text{sec}$$

Chice of $\hat{q}(s)$.

Case 1: The roots of $\hat{q}(s)$ closer to $j\omega$ axis (ie low value)

Let $\hat{q}(s) = s + 0.2$

$$\Rightarrow \delta(s) = \hat{\delta}(s) \cdot \hat{q}(s)$$

$$\Rightarrow \delta(s) = (s^2 + 2.4s + 2.44)(s + 0.2)$$

$$\Rightarrow \delta(s) = s^3 + 2.6s^2 + 2.92s + 0.688 \quad -① $$

On the other hand,

$$\delta(s) = ak + ch$$

$$\Rightarrow \delta(s) = (s^2 - s)(s + k_0) + 3(h_1 s + h_0)$$

$$\Rightarrow \delta(s) = s^3 + (k_0 - 1)s^2 + (3h_1 - k_0)s + 3h_0 \quad -②$$

Comparing ① & ②, we get

$h_0 = 0.163$, $k_0 = 3.6$, $h_1 = 2.17$

$$\Rightarrow \boxed{h(s) = 2.17s + 0.163}, \quad \boxed{k(s) = s + 3.6}$$

We get $\alpha\, c(s) = 2.44$

$$\Rightarrow \alpha \times 3 = 2.44$$

$$\Rightarrow \alpha = 0.813$$

$$\therefore\ q(s) = \alpha\, \hat{q}(s)$$

$$\Rightarrow \boxed{q(s) = 0.813(s+0.2)}$$

Case 2 : $\hat{q} = s+2$ (medium value)

Then

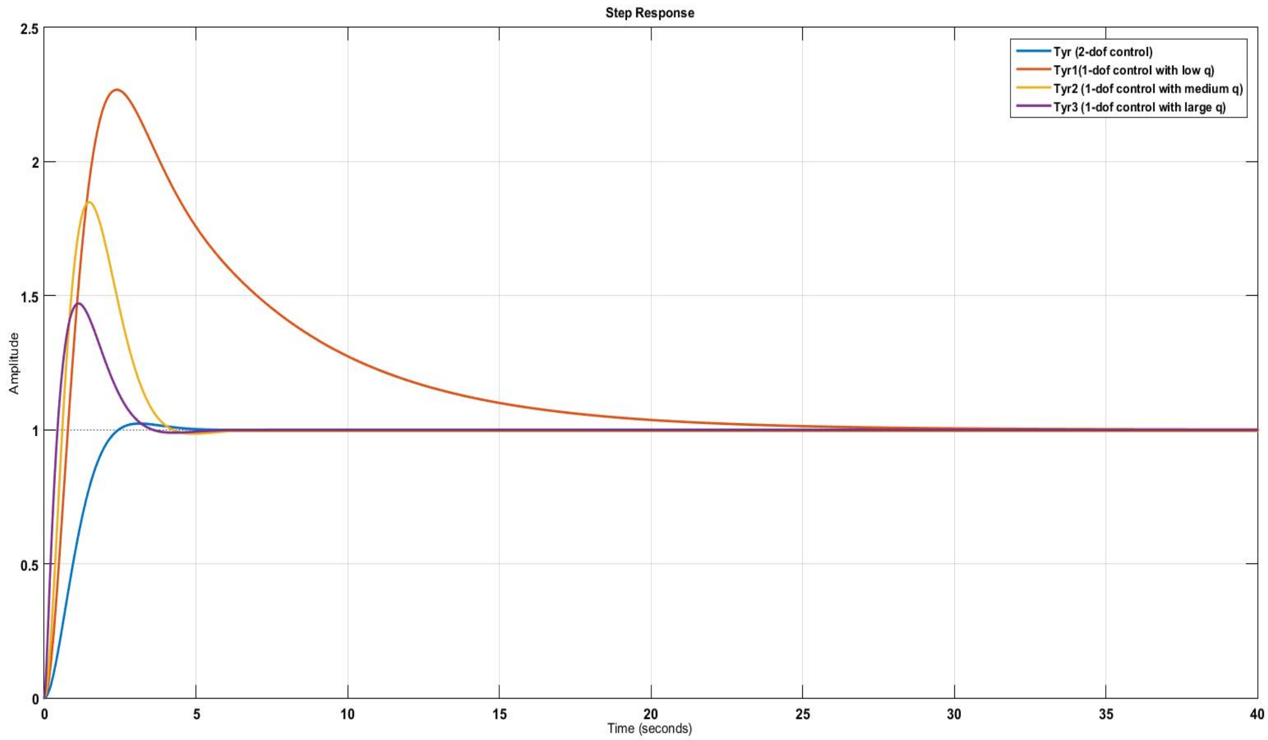
$$\boxed{\begin{aligned} h(s) &= 4.21s + 1.63 \\ k(s) &= s + 5.4 \\ q(s) &= 0.813(s+2) \end{aligned}}$$

Case 3 : $\hat{q}(s) = s+20$ (ie large value)

Then

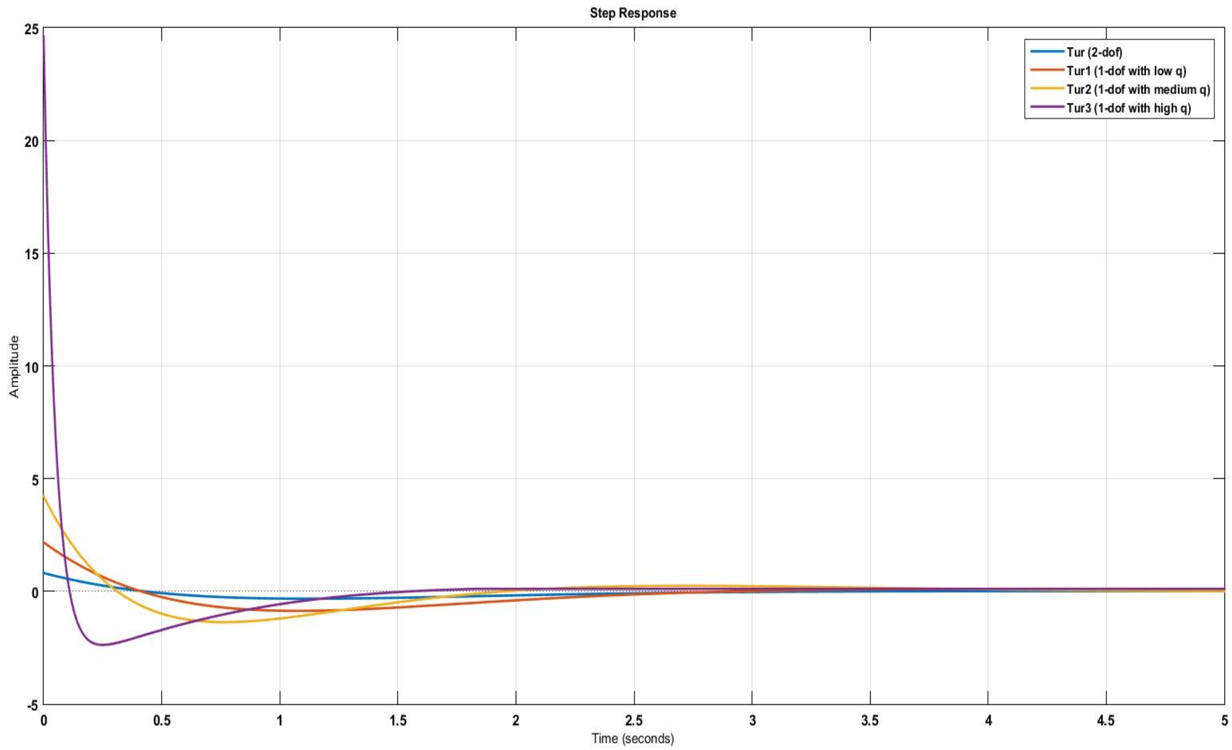
$$\boxed{\begin{aligned} h(s) &= 24.61s + 16.26 \\ k(s) &= s + 23.4 \\ q(s) &= 0.813(s+20) \end{aligned}}$$

Tyr characteristics



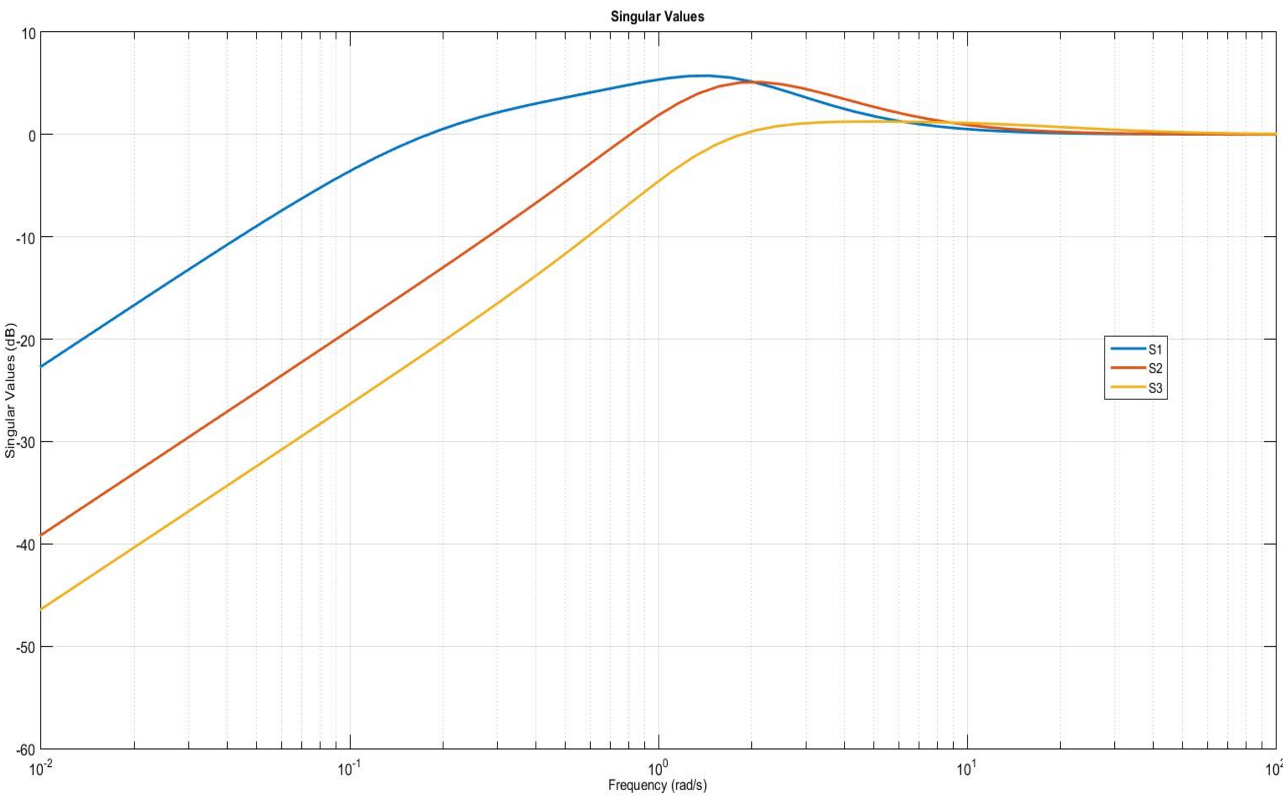
- 2-DOF response is independent on choice of $\hat{q}(s)$
- 1-DOF response becomes peaky because of the root of $h(s)$ close to the $j\omega$ axis.

Tur characteristics

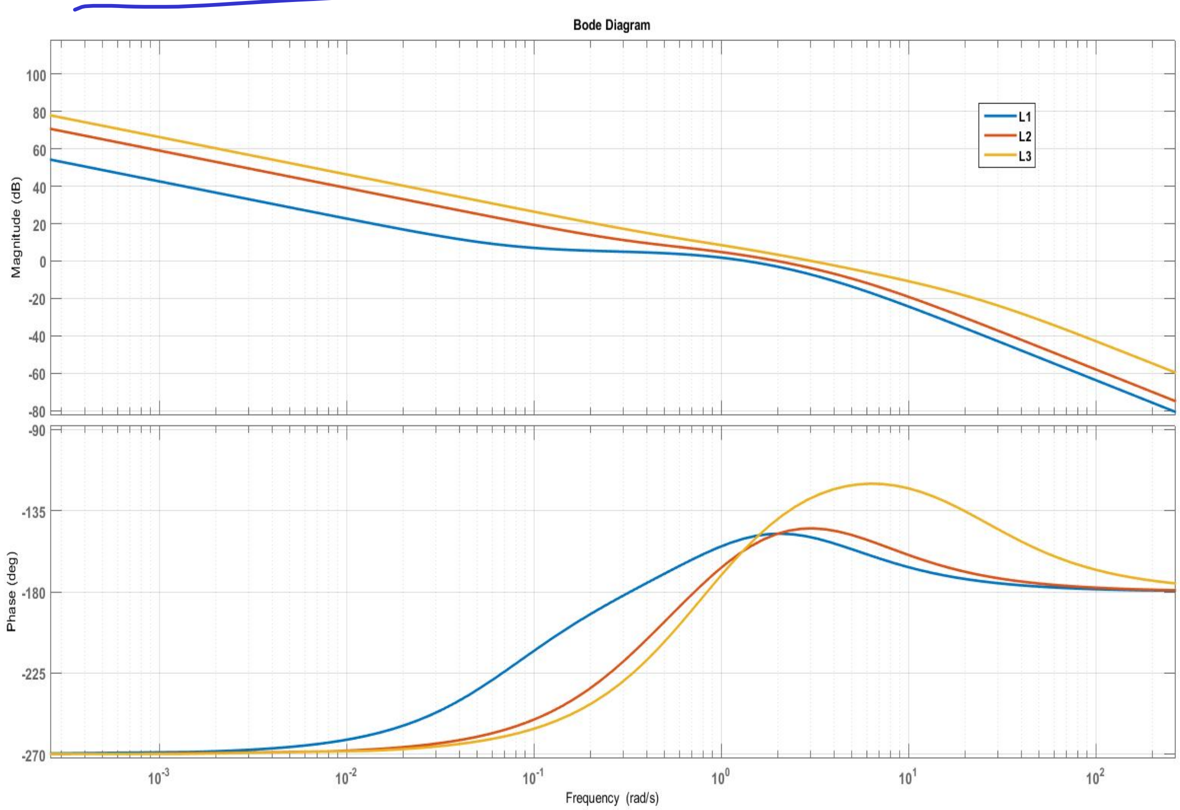


- 2-DOF Tur characteristic is independent of $\hat{q}$
- 1-DOF Tur - - - - - - - has higher mag. compared to the 2-DOF control

S - plot



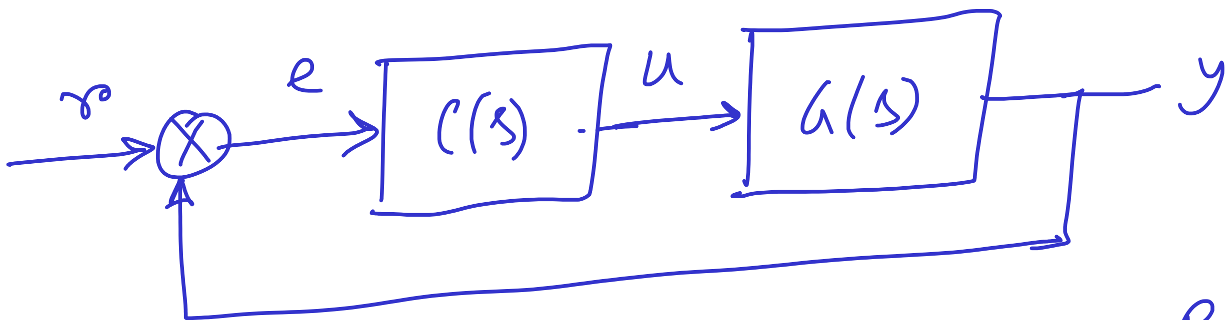
L(jw) plot



- If $\hat{q}$ is small (ie the roots of $\hat{q}$ are close to $j\omega$ axis) then
  - M_S is more (?)
  - Disturbance attenuation is poor
  - noise attenuation is better
- If $\hat{q}$ is high then
  - M_S is less
  - Disturbance attenuation is good
  - noise attenuation is poor
- If $\hat{q}$ is chosen medium then the above performances are traded-off well.
- Loop behaviours for both 1-DOF & 2-DOF controls are same.

PID Controller

- widely used in process control industry — because of simplicity in structure, easy to implement in both analog & digital fashions.



$$C(s) = K_p + \frac{K_i}{s} + K_d s$$ — PID Controller TF

$$= K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$ — P-I-D Controller in time-const form

K_p = proportional gain $\Rightarrow$ If $K_p \uparrow$ then it reduces S.S. error but makes the system more oscillating.

K_I = integral gain — $K_i \neq 0$ makes zero s.s. error w.r.t. step ref. i/p. but makes the system

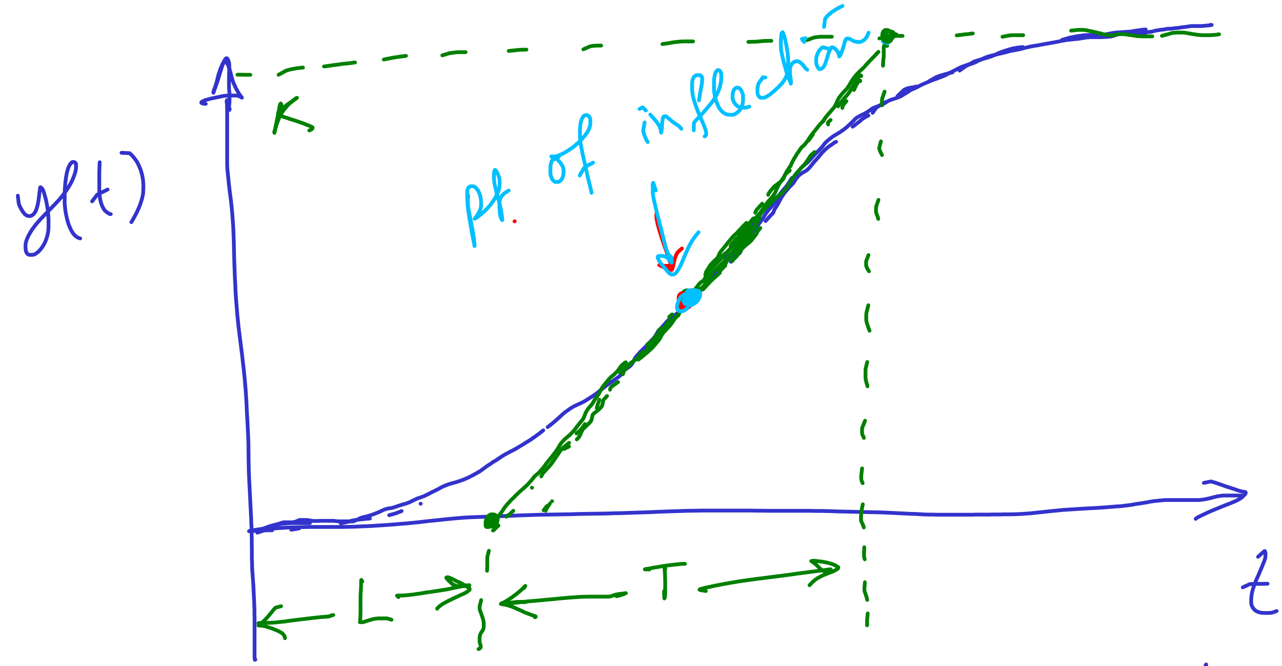
Sluggish because of addition of pole at $s = 0$. [As k_i increases the system becomes more slower]

k_d = derivative gain → increases speed of the response.

- One important advantage of PID control is that it can be designed even if the model TF is not known.
- Some standard tuning rules are there for PID which makes attractive to the process industry

Ziegler - Nichol's tuning rules

Case I: For overdamped system that have S-shaped open-loop response.



The plant is modelled as

$$G(s) = \frac{K e^{-Ls}}{Ts+1}, \quad \begin{array}{l} T = \text{time constant} \\ L = \text{delay} \\ K = \text{S.S. Value} \end{array}$$

First order time-delayed model

Tuning rule

Controller type	K_p	T_I	T_D
P	T/L	∞	0
PI	$0.9\,T/L$	$\frac{L}{0.3}$	0
PID	$1.2\,T/L$	$2L$	$0.5L$

Case 2 : The systems that have overshoot in the open-loop response. .

Then Set $K_I = 0$, $K_d = 0$ & close the loop and increase the gain K_p from 0 to K_{cr} at which the closed-loop response just starts oscillating. Let the period of the sustained oscillations be P_{cr}. Then

Controller type	K_p	T_I	T_d
P	$0.5 K_{cr}$	∞	0
PI	$0.45 K_{cr}$	$\frac{P_{cr}}{1.2}$	0
PID	$0.6 K_{cr}$	$0.5 P_{cr}$	$0.125 P_{cr}$

Practical implementation of PID Controllers

The problems with PID Controllers are

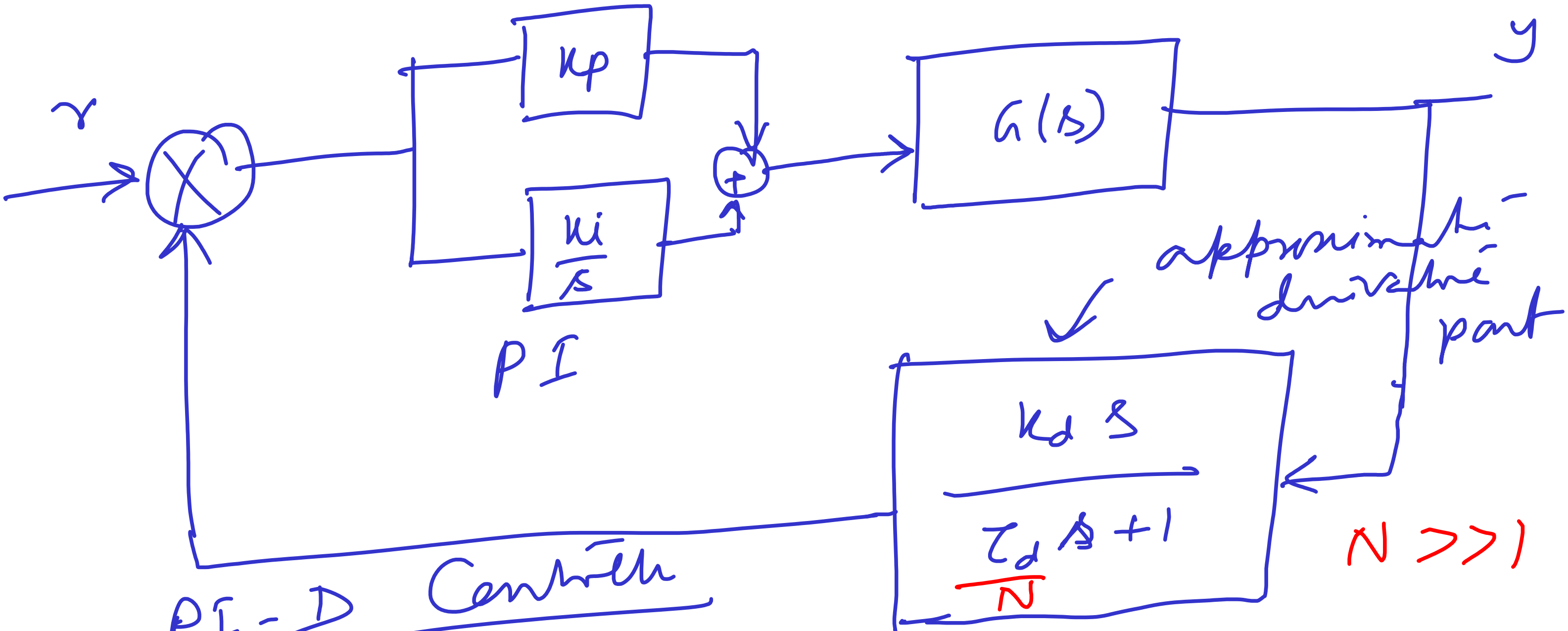
1) The derivative term, being improper is difficult to realize in practice

2) The pure derivative term amplifies the noise which cause actuator saturation. It causes derivative kick in presence step reference. which increases the control i/p suddenly.

To solve the above —

a) $s \cong \dfrac{s}{\tau_d s + 1}$ with $\tau_d << T_d$

b) A minor loop implementation



PI-D Controller

Takes care of both noise amplification problem as well as derivative implementation problem

* Apply pole-placement method to design PID controller for a 2nd order system [Do yourself]

$$G(s) = \frac{b_2 s + b_1}{s^2 + a_2 s + a_1}$$

Note: complete pole-placement can be achieved here using PID control