Q3: forward difference in 1D:

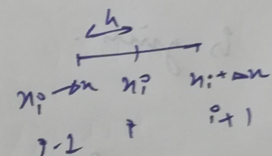
$$f'(x_i) = \frac{f(x_i+\Delta x) - f(x_i)}{\Delta x}$$

Backward difference in D.

$$f'(x_i) = \frac{f(x_i) - f(x_i-\Delta x)}{\Delta x}$$

$$f''(x_i) = \frac{f'(x_i+\Delta x) - f'(x_i)}{\Delta x}$$

$$= \frac{f(x_i+\Delta x) - 2f(x_i) + f(x_i-\Delta x)}{(\Delta x)^2}$$



$$= \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}$$

Now, using Laplace Eqn.

$$\nabla^2 f = 0$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{h^2}$$

Substituting in above Laplace Eqⁿ.

$$\frac{f_{i+1,j} + f_{i-1,j} - 2f_{i,j}}{h^2} + \frac{f_{i,j+1} + f_{i,j-1} - 2f_{i,j}}{h^2} = 0$$

$$f_{i,j} = \frac{f_{i+1,j} + f_{i-1,j} + f_{i,j-1} + f_{i,j+1}}{4}$$

$\Rightarrow$ Finite Difference ~~method~~ Equation.