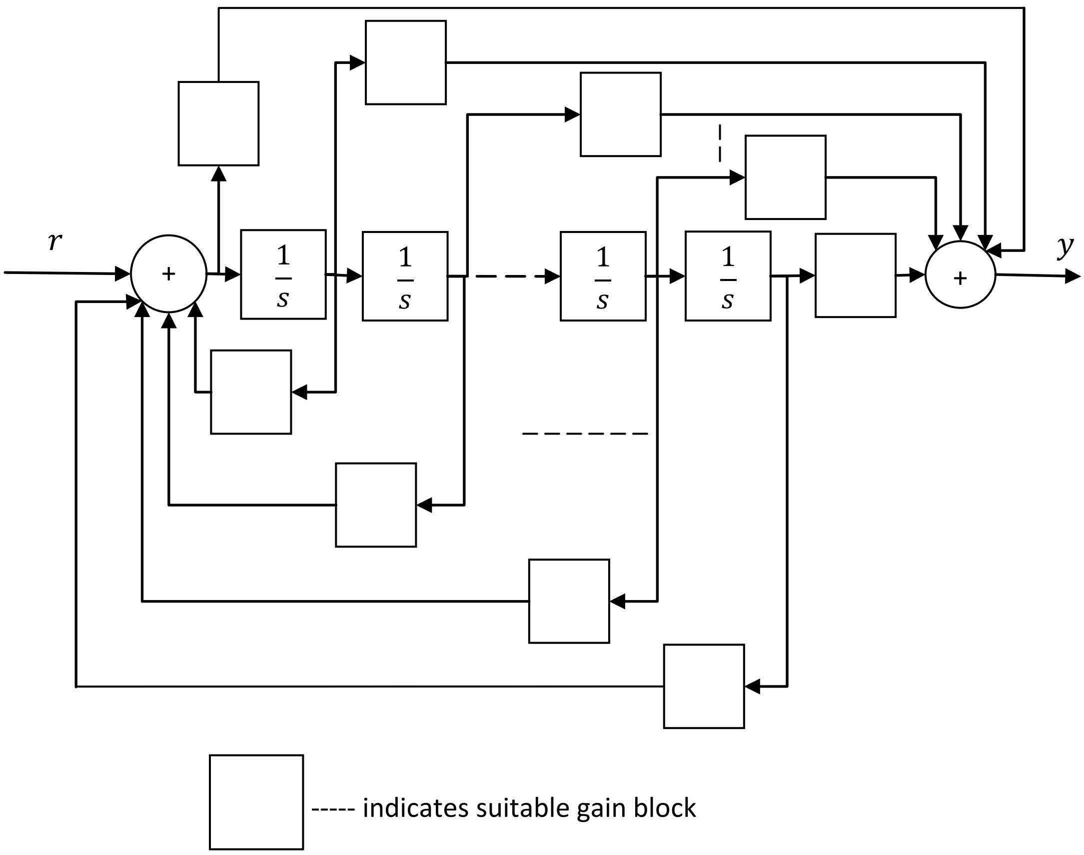
Assignment 4

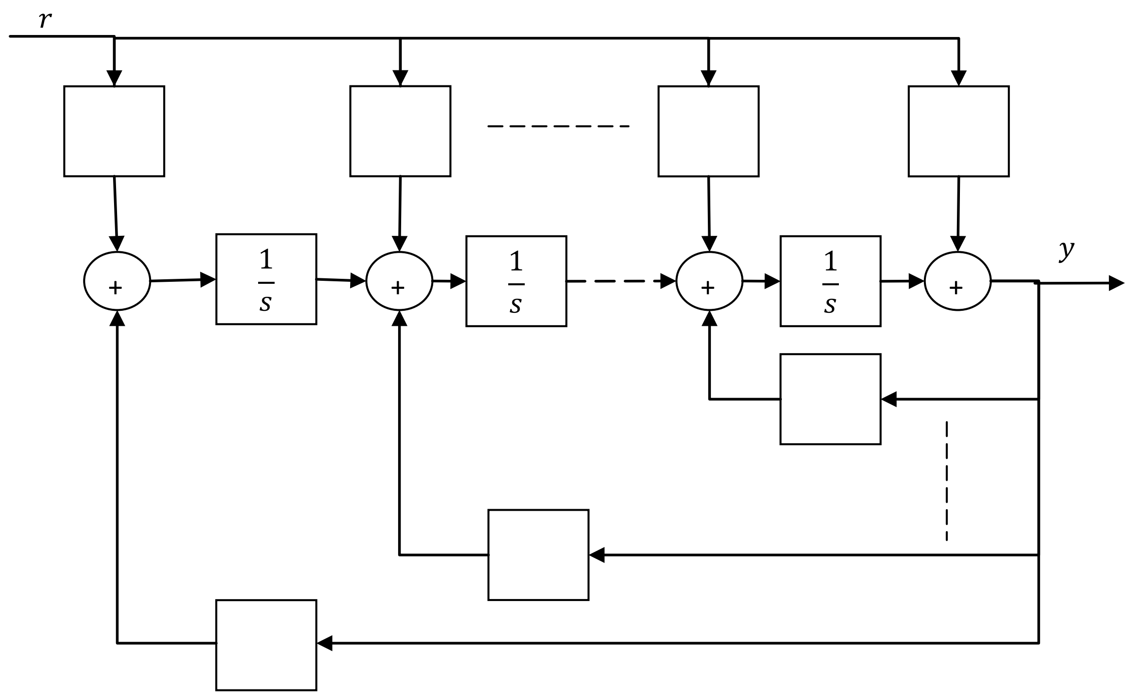
1. Implement the transfer function $(b_2 s^2 + b_1 s + b_0)/(s^2 + a_1 s + a_0)$ in the controller and the observer canonical forms.

   Hint: The structure of implementation of an n-th order transfer function is given below:

   a) Controller canonical form

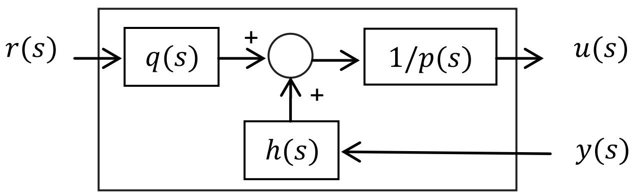


b) Observer canonical form



2. (a) Obtain minimal realizations of the MIMO transfer functions $[1/(s+1) \quad (s+3)/(s+1)(s+2)]^T$ and $[1/(s+1) \quad (s+3)/(s+1)(s+2)]$.

(b) Show that the 2-dergree-of-freedom controller given by



where p, h, q are polynomials can be implemented in a proper fashion.

3. (a) The schematic of a laboratory based magnetic ball suspension system is shown in Fig. 1. Here, the objective is to suspend the steel ball at equilibrium position x_0. Let the corresponding equilibrium current be i_0. The electromagnetic force exerted by the electromagnet is given by $k(i^2/x^2)$, where k is a constant depending on coil parameters. The system is equipped with an inner control loop providing a current proportional to the control voltage that is generated for control purpose, i.e., $i = k_1 u$. Assuming sensor's gain to be k_2, mass of the ball to be m and acceleration

due to gravity be g, obtain the expression for the small signal transfer function $\Delta x_v / \Delta u$ of the model. The parameters of the system are given in Table 1.

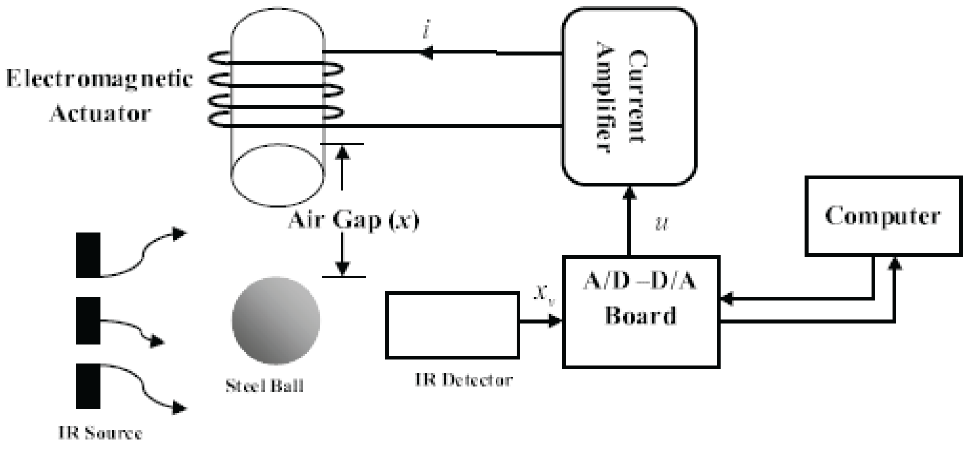


Fig. 1: Magnetic ball suspension system

Table 1: Parameters of the magnetic ball suspension system

Parameters	Value
m—Mass of the steel ball	0.02 kg
g—Acceleration due to gravity	9.81 m/s^2
i_0—Equilibrium value of current	0.8 A
x_0—Equilibrium value of position	0.009 m
k_1—Control voltage to coil current gain	1.05 A/V
k_2—Sensor gain, offset (η)	143.48 V/m, -2.8 V
Control input voltage level (u)	± 5 V
Sensor output voltage level (x_ν)	$+1.25$ V to -3.75 V

(b) Using pole-placement method, design a minimal order, proper, 2-DoF (degree-of-freedom) controller for the above magnetic ball suspension system to achieve settling time less than 2 s and peak overshoot less than 5%. In addition, choose the non-dominant closed-loop pole locations such that $|GM| \geq 6\ dB, |PM| \geq 60^0$. Check the tracking performance of the controller designed through simulations of both linear and nonlinear system. Also verify GM and PM obtained through simulations. Now if the above controller is made 1-DoF with all the closed-loop pole locations same as before then study its behaviour and verify the superiority of the 2-DoF controller in respect of tracking performance and control effort.