Statistical Signal Processing (EE60102)

Mid-semester examination, Spring 2021-22

Time: 2 hours **Total Marks: 30**

Q1. A time series $u(n)$ obtained from a wide-sense stationary stochastic process of zero mean and correlation matrix R is applied to an FIR filter with impulse response defined by the coefficient vector $\mathbf{w}$.

(a) Show that the average power of the filter output is equal to $\mathbf{w}^H R\mathbf{w}$.

(b) How is the result in part **(a)** modified if the stochastic process at the filter input is a white noise with variance σ^2?

(2+2=4)

Q2. Use the Yule-Walker equations to determine the auto-correlation coefficients of the following AR models assuming that $w(n) \sim WN(0,1)$:

(i) $x(n) = 0.5x(n-1) + w(n)$

(ii) $x(n) = 1.5x(n-1) - 0.6x(n-2) + w(n)$

(2+3=5)

Q3. Consider a Wiener filtering problem characterized as follows. The correlation matrix R of the tap-input vector $\mathbf{u}(n)$ is

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

The cross-correlation vector between the tap-input vector $\mathbf{u}(n)$ and the desired response $d(n)$ is $\mathbf{p} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$.

(a) Evaluate the optimum tap-weights of the Wiener filter.

(3)

(b) What is the minimum mean-square error produced by this Wiener filter?

(3)

(c) Express the optimum tap weights and minimum mean square error, J_{min}, of the Wiener filter in terms of the eigenvalues of the matrix R and associated eigenvectors and verify the solution of **(a)**, **(b)**.

(5)

Q4. A process $y(n)$ with the autocorrelation $r_y(l) = a^{|l|}$, $-1 < a < 1$, is corrupted by additive, uncorrelated white noise $\nu(n)$ with variance σ_ν^2. To reduce the noise in the observed process $x(n) = y(n) + \nu(n)$, we use a first-order Wiener filter.

Express the optimal tap weights and the MMSE J_{min} in terms of the parameters a and σ_ν^2.

(5)

Q5. **(a)** Draw the signal flow graph for the LMS algorithm.

(b) What is the difference between steepest descent algorithm and LMS ?

(3+2=5)