Analog Signal Processing

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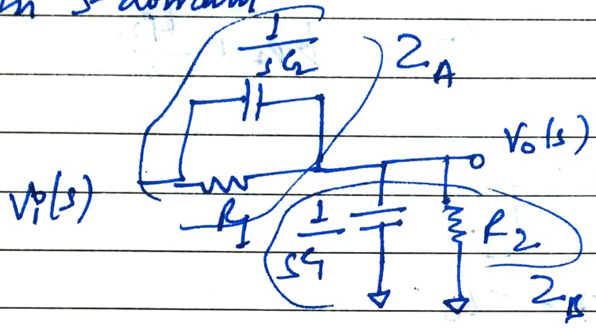
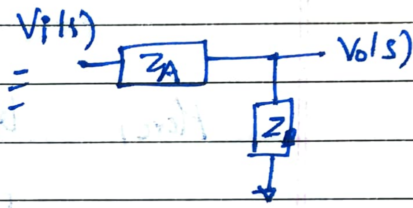
Lecture-11

Q1. In s-domain



$V_o(s) =$



Here, $Z_A = \frac{1}{sC_2} \parallel R_1$

$= \frac{R_1}{1+sC_2R}$

$Z_B = \frac{1}{sC_1} \parallel R_2 = \frac{R_2}{1+sC_1R_2}$

$$V_o(s) = \frac{Z_B}{Z_A+Z_B} V_i(s)$$

$$= \frac{\frac{R_2}{1+sC_1R_2}}{\frac{R_2}{1+sR_2C_1} + \frac{R_1}{1+sR_1C_2}} V_i(s)$$

$$= \frac{R_2(1+sR_1C_2)}{R_2+sR_1R_2C_2+R_1+sR_2R_1C_1} V_i(s)$$

$$V_0(s) = \frac{R_2 + s R_2 R_1 C_1}{(R_1+R_2) + s(C_1+C_2)(R_1R_2)} V_i(s)$$

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{R_2(1 + sR_1C_1)}{(R_1+R_2) + sR_1R_2(C_1+C_2)}$$

$$\text{DC gain} = \left| \lim_{s\to 0} H(s) \right|$$

$$= \frac{R_2}{R_1+R_2}$$

$$\text{High frequency Gain} = \left| \lim_{s\to\infty} H(s) \right|$$

$$= \frac{R_2R_1C_1}{R_1R_2(C_1+C_2)} = \frac{R_2C_1}{R_1(C_1+C_2)}$$

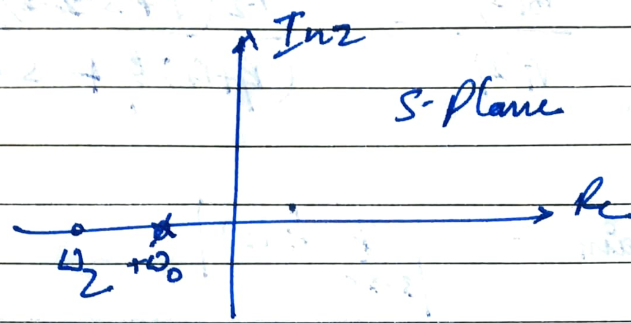
$$= \cancel{\frac{C_1}{C_1+C_2}} \quad \frac{C_2}{C_1+C_2}$$

$$\text{Pole } \omega_0 = -\frac{1}{(C_1+C_2)}\left(\frac{R_1+R_2}{R_1R_2}\right) = -\frac{1}{C_1+C_2}\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

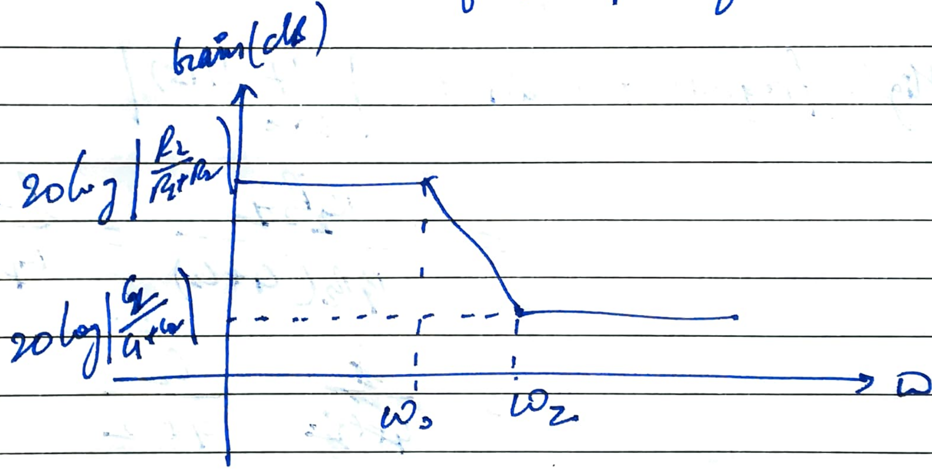
$$\text{zero } \omega_z = -\frac{1}{R_1C_2}$$

To attain low pass filter,

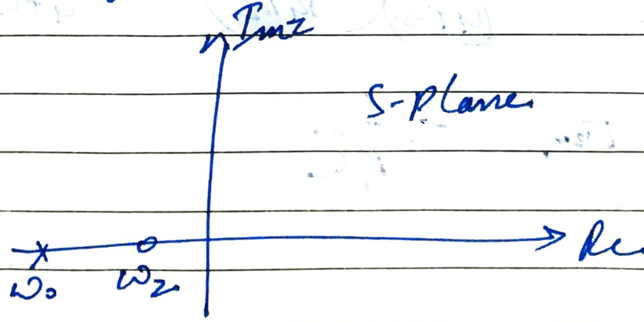
pole ω_0 < zero ω_2



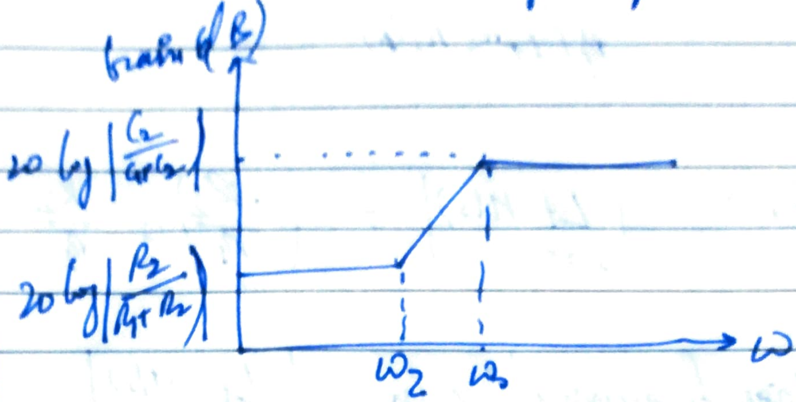
Gain Characteristics of low pass filter



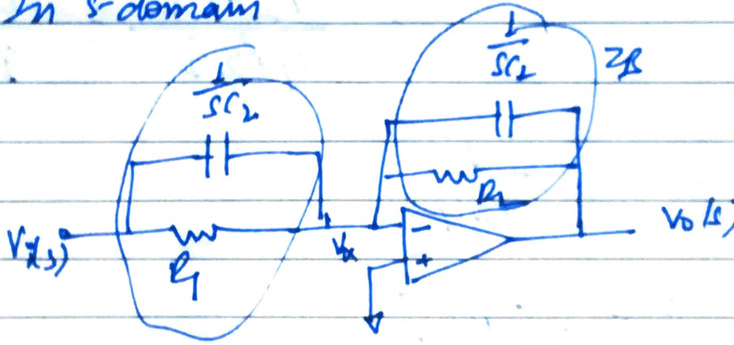
To attain High Pass filter, zero ω_2 < pole ω_0



Gain Characteristics of High Pass filter



Q2. In s-domain



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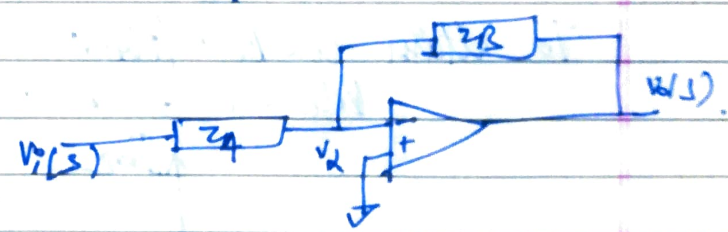


Here $v_x = 0$ (using virtual grounding concept $v^+ = v^-$)

$$Z_A = \frac{R_1}{1+sR_1C_2}, \quad Z_B = \frac{R_2}{1+sR_2C_1}$$

& $$\frac{V_o(s)}{V_i(s)} = \frac{-Z_A}{Z_A} = \frac{-R_2(1+sR_1C_2)}{R_1(1+sR_2C_1)}$$

$$H(s) = \frac{-R_2(1+sR_1C_2)}{R_1(1+sR_2C_1)}$$

$$\text{DC Gain} = \left|\underset{s\to 0}{Lt}\, H(s)\right| = \left|\frac{-R_2}{R_1}\right| = \frac{R_2}{R_1}$$

High ~~voltage~~ frequency Gain $= \left|\underset{s\to\infty}{Lt}\, H(s)\right|$

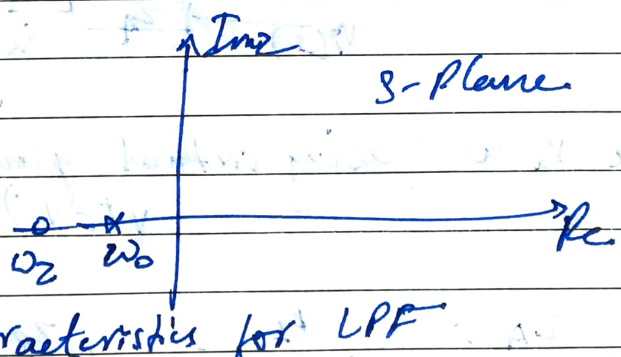
$$= \frac{R_2 R_1 C_2}{R_1 R_2 C_1} = \frac{C_2}{C_1}$$

$$\text{Pole } \omega_0 = -\frac{1}{R_2C_1}$$

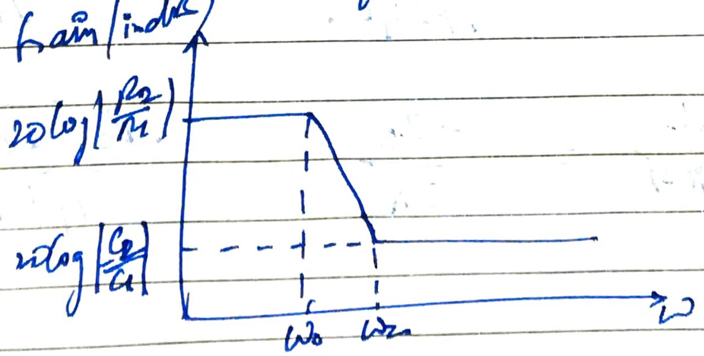
$$\text{Zero } \omega_z = -\frac{1}{R_1C_2}$$

To attain Low Pass filter

Pole ω_0 < zero ω_z

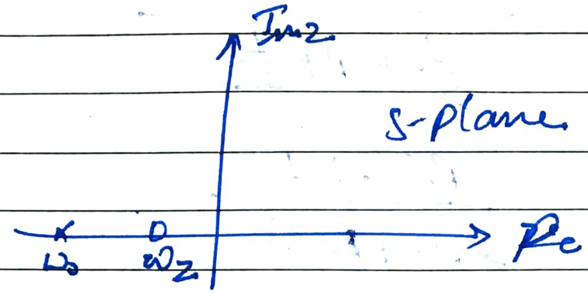


Gain Characteristics for LPF

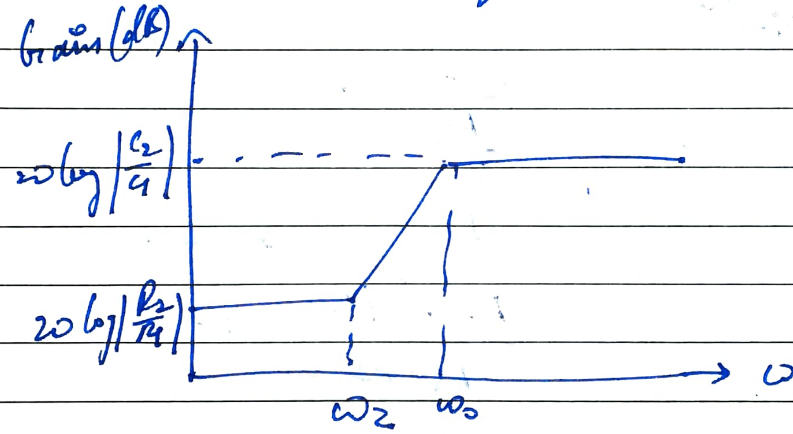


To attain High Pass filter,

pole ω_o > zero ω_z.

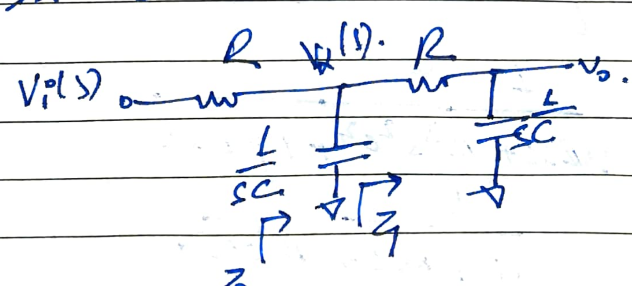


Gain Characteristics for HPF



Lecture 15

Q1. In s-domain



$\boxed{Z_1 = R + \frac{1}{sC}}$; $Z_2 = \frac{1}{sC} \parallel Z_1$

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$$Z_2 = \frac{1}{sC} \parallel Z_1$$

$$= \frac{\left(\frac{1}{sC}\right) \times \left(R + \frac{1}{sC}\right)}{\frac{1}{sC} + R + \frac{1}{sC}}$$

$$= \frac{RsC + 1}{sC(2 + RsC)}$$

$$\frac{V_x(s)}{V_i(s)} \; \cancel{\frac{V_o(s)}{V_i(s)}} = \frac{Z_2}{R + Z_2}$$

$$= \frac{1}{\frac{R}{Z_2} + 1}$$

$$= \frac{1}{\frac{RsC(2 + RsC)}{RsC + 1} + 1} \quad \text{—(i)}$$

$$\frac{V_o(s)}{V_x(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{(1 + RsC)} \quad \text{—(ii)}$$

Multiplying eq (i) & (ii).

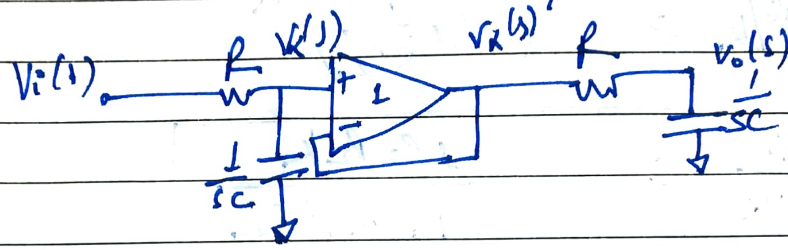
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{2RsC + R^2 s^2 C^2 + 1 + RsC}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + 3RsC + R^2 s^2 C^2}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{C^2R^2\left(s^2 + \frac{3s}{RC} + \frac{1}{R^2C^2}\right)}}$$

Q2. In s-domain

from virtual grounding concept.



$$\frac{V_x(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+RSC} \quad \text{—(i)}$$

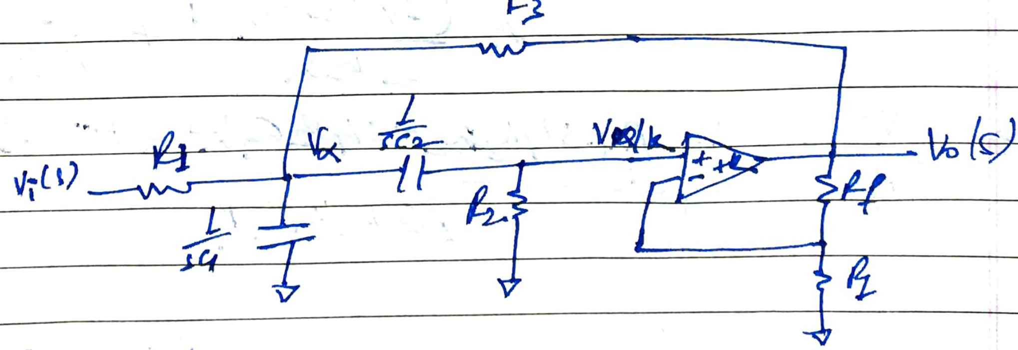
$$\frac{V_o(s)}{V_x(s)} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+RSC} \quad \text{—(ii)}$$

Multiplying Eq (i) & (ii)

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{(1+RSC)^2} = \frac{1}{R^2C^2\left(s^2+\frac{2s}{RC}+\frac{1}{R^2C^2}\right)}$$

Lecture -16

Q1. In s-domain



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Here,

$$V_0/k = \left(\frac{R_2}{R_2 + \frac{1}{sC_2}}\right) V_\alpha$$

$$\boxed{\frac{V_0}{V_\alpha} = \frac{sC_2R_2k}{1+R_2sC_2}}$$

Here,

$$\frac{V_i - V_\alpha}{R_1} = \frac{V_\alpha - V_0}{R_3} + \frac{V_\alpha}{\frac{1}{sC_1}} + \frac{V_\alpha}{\frac{1}{sC_2} + R_2}$$

$$\Rightarrow \frac{V_i}{R_1} = V_\alpha\left(\frac{1}{R_1} + \frac{1}{R_3} + \cancel{\frac{1}{s}}\, sC_1 + \frac{sC_2}{1+R_2sC_2}\right) - \frac{V_0}{R_3}$$

$$\Rightarrow \frac{V_i}{R_1} = \frac{V_0(1+R_2sC_2)}{sC_2R_2k}\left(\frac{1}{R_1} + \frac{1}{R_3} + sC_1 + \frac{sC_2}{1+R_2sC_2}\right) - \frac{V_0}{R_3}.$$

$$\Rightarrow \frac{V_i}{R_1} = V_0\left(\left(\frac{1+R_2sC_2}{sC_2R_2k} - 1\right)\frac{1}{R_3} + \frac{1+R_2sC_2}{(sC_2R_2k)R_1} + \frac{(1+R_2sC_2)C_1}{C_2R_2k} + \frac{1}{R_2k}\right)$$

$$\Rightarrow \frac{V_i}{R_1} = V_0\left(\frac{R_1 + sR_1R_2C_2 - sR_1R_2C_2k + R_2 + sC_2R_2R_3 + sR_3R_1(1+R_2sC_2)C_1 + sC_2R_3R_1}{sC_2R_2kR_3R_1}\right)$$

$$\Rightarrow \frac{V_i}{R_1} = \frac{v_0\left(s^2 R_1R_2R_3C_2C_1 + s\left(R_1R_2C_2 - R_1R_2C_2k + R_2R_3C_2 + R_1R_3C_1 + R_1R_3C_2\right) + R_1 + R_3\right)}{sC_2R_2kR_1R_3}$$

$$\Rightarrow \frac{V_o}{V_i}(s) = \frac{s\,C_2R_2kR_3}{s^2R_1R_2R_3C_1C_2 + s\left(R_1R_2C_2 - R_1R_2C_2k + R_2R_3C_2 + R_1R_3C_1 + R_1R_3C_2\right) + (R_1+R_3)}$$

$$\Rightarrow \frac{V_o(s)}{V_i(1)} = \frac{\frac{sk}{R_1C_1}}{s^2 + s\frac{\left((1-k)R_1R_2C_2 + (R_1+R_2)R_3C_2 + R_1R_3C_1\right)}{R_1R_2R_3C_1C_2} + \frac{(R_1+R_3)}{R_1R_2R_3C_1C_2}}$$

Comparing denominator with the standard form,

$$s^2 + \frac{s\omega_0}{Q} + \omega_0^2 = 0;$$

We got,

$$\boxed{\omega_0 = \sqrt{\frac{R_1+R_3}{R_1R_2R_3C_1C_2}} = \sqrt{\frac{1+\frac{R_1}{R_3}}{R_1R_2C_1C_2}}}$$

$Q =$

$$\frac{\omega_0}{Q} = \frac{(1+k)}{R_3C_1} + \frac{1}{R_2C_1} + \frac{1}{R_1C_1} + \frac{1}{R_2C_2}$$

$$\cancel{\omega_0}\ \frac{1}{Q} = \left(\frac{\sqrt{1+\frac{R_1}{R_3}}}{\sqrt{R_1R_2C_1C_2}}\right)^{-1}\left(\frac{1+k}{R_3C_1} + \frac{1}{R_2C_1} + \frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right)$$

$$\Rightarrow \quad Q = \frac{\sqrt{1+\frac{R_1}{R_3}}}{\sqrt{R_1R_2C_1C_2}\left(\frac{1-k}{R_3C_1} + \frac{1}{R_2C_1} + \frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right)}$$

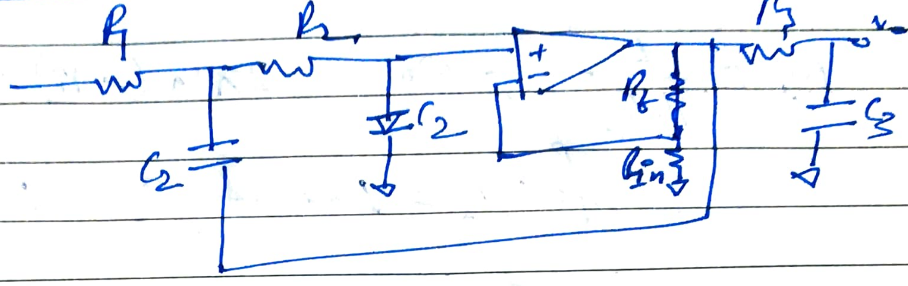
$$\sqrt{1+\frac{R_1}{R_3}}$$

$$= \frac{(1-k)}{R_3}\sqrt{\frac{R_1R_2C_2}{C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}}$$

$$\boxed{Q = \frac{\sqrt{1+\frac{R_1}{R_3}}}{\left(1+\frac{(1-k)R_1}{R_3}\right)\sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}}}}$$

lecture-20

Q1.



The equal component design,

$$R_1 = R_2 = R \ , \ C_1 = C_2 = C$$

from generalised expression of Q,

$$= \frac{1}{(1-k)\sqrt{\frac{C_1R_1}{C_2R_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{C_2R_1}{C_1R_2}}}$$

$$= \frac{1}{1+1+(1-k)} = \frac{1}{3-k}.$$

If $Q = 1$,

$$\Rightarrow k = 2 \Rightarrow 1 + \frac{R_F}{R_{IN}} = 2$$

$$\boxed{R_F = R_{IN}} = 1\,k\Omega$$

Here, $\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = \frac{1}{RC} = 2\pi \times 1.45\ MHz$.

let us assume $R = 1k\Omega$

$$C = \frac{1}{2\pi \times 1.45}\ nF \simeq 109\ pF$$

$R_1 = R_2 = 1k\Omega$, $C_1 = C_2 = 109\,pF$

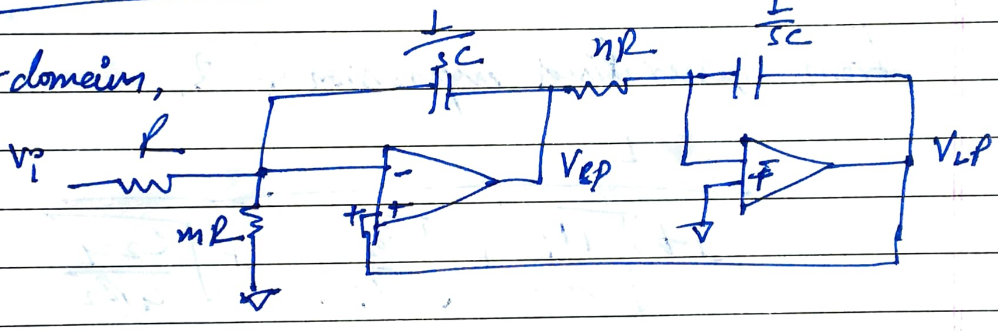
$\omega_3 = \frac{1}{R_3 C_3} = \omega_0 = 2\pi \times (1.45)\,MHz$

for $R_3 = 1k\Omega$ & $C_3 = 109\,pF$

So, $R_3 = 1k\Omega$
& $C_3 = 109\,pF$

Try Yourself:

Q2. In s-domain,



$$\frac{V_{BP}}{nR} = \frac{-V_{LP}}{1/sC}$$

$$\Rightarrow \boxed{\frac{V_{BP}}{V_{LP}} = -sCnR}$$

$$\frac{V_i - V_{LP}}{R} = \frac{V_{LP}}{mR} + \frac{V_{LP} - V_{BP}}{\frac{1}{sC}}$$

$\Rightarrow$

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$$\Rightarrow \frac{V_i}{R} = V_{LP}\left(\frac{1}{R} + \frac{1}{mR} + sC + s^2C^2nR\right)$$

$$\Rightarrow \frac{V_i}{R} = C^2nR\,V_{LP}\left(\frac{(1+m)}{C^2R^2nm} + \frac{1}{nRC}s + s^2\right)$$

$$\boxed{\frac{V_{LP}}{V_i} = \left(\frac{1}{R^2nC^2}\right)\cdot\frac{1}{\left(s^2 + \frac{s}{nRC} + \frac{1+m}{C^2R^2nm}\right)}}$$

$$V_{BP} = -sCnR\,V_{LP}$$

$$\boxed{\frac{V_{BP}}{V_i} = \frac{-s}{RC\left(s^2 + \frac{s}{nRC} + \frac{1+m}{C^2R^2nm}\right)}}$$

Comparing denominator with

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2$$

$$\omega_0^2 = \frac{1+m}{nmR^2C^2} \Rightarrow \omega_0 = \sqrt{\frac{1+m}{mnR^2C^2}}$$

$$= \frac{1}{RC}\sqrt{\frac{1}{n} + \frac{1}{mn}}$$

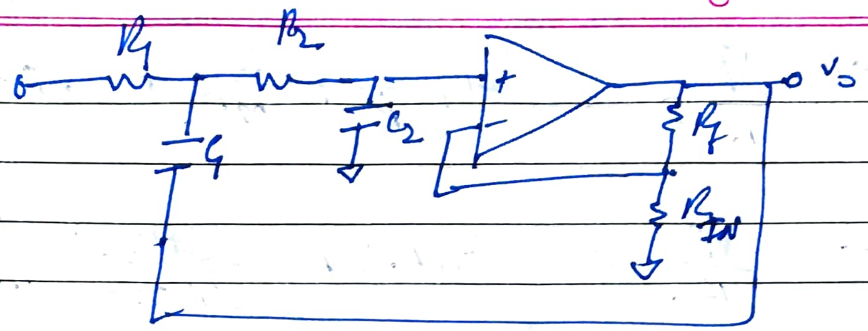
$$\frac{\omega_0}{Q} = \frac{1}{nRC} \Rightarrow \boxed{\omega_0 = \frac{Q}{nRC}}$$

$$Q = \omega_0 nRC$$

$$= \frac{nRC}{RC}\sqrt{\frac{1}{n} + \frac{1}{mn}} = \sqrt{n\left(1 + \frac{1}{m}\right)}$$

$$\boxed{Q = \sqrt{n\left(1 + \frac{1}{m}\right)}} \qquad \boxed{\omega_0 = \frac{1}{RC}\sqrt{\frac{1}{n}\left(1 + \frac{1}{m}\right)}}$$

Q3.



In equal component design,

$R_1 = R_2$, $C_1 = C_2$, $Q = \frac{1}{3-k}$ & $\omega_0 = \frac{1}{R_1 C_1}$

known,

$Q = 5$, $f_0 = 10\,kHz$

$\omega_0 = 2\pi f_0 = 2\pi \times 10$ ~~kHz~~ $k\,rad/s$.

$\frac{1}{R_1 C_1} = 2\pi \times 10 \times 10^3\ rad/s$.

let us assume, $\boxed{R_1 = 1k\Omega}$

$C_1 = \frac{1}{2\pi \times 10^4 \times 10^3} F \simeq 15.915\,nF$

$\boxed{R_1 = R_2 = 1k\Omega}$, $\boxed{C_1 = C_2 = 15.915\,nF}$

$Q = 5$

$\frac{1}{3-k} = 5$ $\Rightarrow$ $1 + \frac{R_F}{R_{IN}} = 2.8$

$\Rightarrow \boxed{k = 2.8}$ $\Rightarrow \boxed{\frac{R_F}{R_{IN}} = 1.8}$

Taking $\boxed{R_{IN} = 1k\Omega}$

$\boxed{R_F = 1.78\,k\Omega}$

Gain in KRC low pass filter,

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{k}{s^2(C_1C_2R_1R_2) + s(C_2R_1 + C_2R_2 + R_1C_1 - kC_1R_1) + 1}$$

$$\text{DC gain} = \left| \lim_{s\to 0} H(s) \right|$$

$$= |k| = 2.8$$

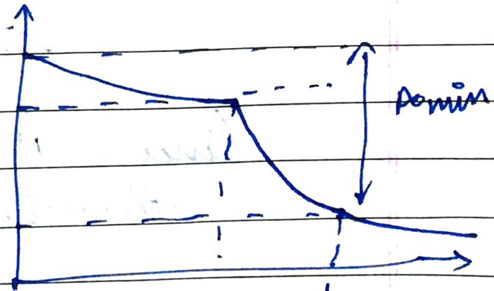
Lecture-24

Q1. Given,

$A_{max} = 0.5\,dB$

$f_p = 1\,MHz$

$f_s = 2\,MHz$



Order of filter $N \leq 5$

$f_p = 1MHz$ $f_s = 2MHz$

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N^{th}-Order Butterworth approximation function,

$$|H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon_1^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

Passband Attenuation $A_{max} = 10\log(1+\varepsilon_1^2)^{\frac{1}{2}}$ dB

$$10\log(1+\varepsilon_1^2)^{\frac{1}{2}} = 0.5$$

$$\Rightarrow \varepsilon_1^2 = 10^{0.05} - 1$$

$$\Rightarrow \boxed{\varepsilon_1 = \sqrt{10^{0.05}-1} \simeq 0.349}$$

Stopband attenuation A_{min} =

$$10\log\left[1+\varepsilon_1^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] dB =$$

$$= 10\log\left[1+0.349^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] dB$$

Since $N \leq 5$, the ~~ma~~ greatest stopband attenuation will occur for $N=5$.

Greatest Stopband Attenuation =

$$10\log\left[1+0.349^2 \times 2^{10}\right] dB = \cancel{20.99\,dB}$$

$$= 20.99\,dB$$

Q20

$A_{min} = 25 \, dB.$

$f_s = 5 \, MHz$

$f_p = 2 \, MHz$

Order of filter $N \leq 5$.

At stop-band, $\omega = \omega_s$.

$$10 \log \left[1 + \xi^2 \cosh^2 \left[N \cosh^{-1} \left(\frac{\omega_s}{\omega_p} \right) \right] \right] = 25 \, dB.$$

As N increases, the value of ξ decreases, we also know

$$A_{max} = 10 \log (1 + \xi^2)$$

To achieve maximum ripple, ξ must be minimum & that occurs when $N = 5$.

So, order of filter $= 5$

$$10 \log \left[1 + \xi^2 \cosh^2 \left[5 \cosh^{-1} (2.5) \right] \right] = 2.5$$

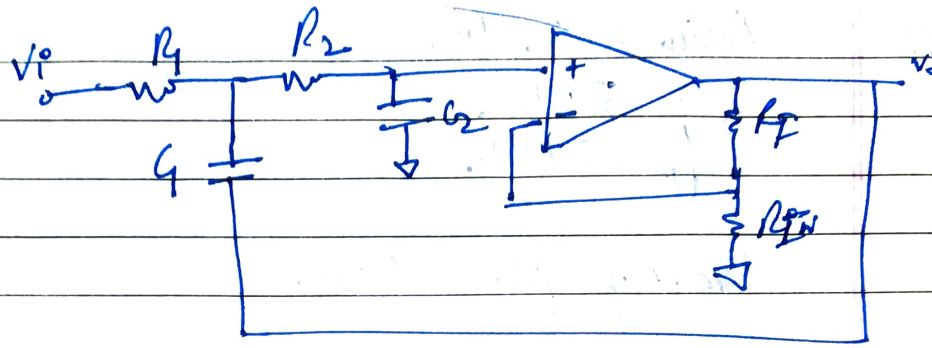
$$\Rightarrow \xi^2 \cosh^2 (7.8339) = 10^{2.5} - 1$$

$$\Rightarrow \boxed{\xi = 0.014}$$

Maximum ripple $= 10 \log (1 + \xi^2)$

$= 8.5113 \times 10^{-4} \, dB.$

Q3:



$$\frac{V_o(s)}{V_i(s)} = \frac{k}{s^2 R_1 R_2 C_1 C_2 + s\left[C_2 R_1 + C_2 R_2 + C_1 R_1 - kC_1 R_1\right] + 1}$$

Low frequency Gain = 2

$$\lim_{s \to 0} \frac{V_o(s)}{V_i(s)} = \boxed{k = 2} \quad \text{— (i)}$$

$$\Rightarrow 1 + \frac{R_F}{R_{IN}} = 2$$

$$\Rightarrow \boxed{R_F = R_{IN}}$$

Taking $\boxed{R_{IN} = 1k\Omega}$

$\boxed{R_F = 1k\Omega}$

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a) In equal component choices.

$$Q = \frac{1}{3-k} = 1.5$$

$$\Rightarrow \boxed{k = 7/3}$$

But from the relation (i), $\boxed{k=2}$,

So, we cannot consider this design.

b) Taking equal resistance design,

$R_1 = R_2$, $C_1 \neq C_2$.

$$Q = \frac{1}{\sqrt{\frac{C_2 R_1}{C_1 R_2}} + \sqrt{\frac{C_2 R_2}{C_1 R_1}} + \sqrt{\frac{C_1 R_1}{R_2 C_2}}\,(1-k)}$$

Given,

$Q = \frac{3}{2}$, $k = 2$, $R_1 = R_2$, $C_1 \neq C_2$.

$$\frac{3}{2} = \frac{1}{2\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}}}$$

$$2\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}} = \frac{2}{3}$$

$\Rightarrow$ Taking $\frac{C_2}{C_1} = x$

$$\Rightarrow \quad 2\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{2}{3}.$$

Squaring both sides,

$$4x + \frac{1}{x} - 4 = 4/9$$

$$\Rightarrow \quad 4x^2 + 1 - 4x = \frac{4x}{9}$$

$$\Rightarrow \quad 4x^2 - \frac{40x}{9} + 1 = 0.$$

$$x = 0.7977,\ 0.3134$$

When $x = 0.7977$

$$\frac{C_2}{C_1} = 0.7977$$

Taking $\boxed{C_1 = 50pF}$, $\cancel{C_2} =$

$C_2 = \cancel{39.90}$

$$\boxed{C_2 = 39.885\,pF}$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi \times 50\,MHz.$$

$$\frac{1}{\sqrt{R_1^2 \times C_1^2 \times 0.7977}} = 2\pi \times 50 \times 10^6$$

$$\Rightarrow \frac{1}{R_1C_1\sqrt{0.7977}} = 2\pi \times 50 \times 10^6.$$

$$\Rightarrow R_1 = \frac{1}{50\times10^{-12} \times \sqrt{0.7977} \times 100\pi \times 10^6}\ \Omega$$

$$= 71.2787\,\Omega$$

$$\boxed{R_1 = R_2 = 71.2787\,\Omega}$$

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When $n = 0.3134$

$$\frac{C_2}{C_1} = 0.3134$$

Taking $\boxed{C_1 = 50\ pF}$

$$\boxed{C_2 = 15.67\ pF}$$

$$\frac{1}{R_1 C_1 \sqrt{0.3134}} = 2\pi \times 50\ MHz$$

$$\Rightarrow R_1 = \frac{1}{50 \times 10^{-12} \times \sqrt{0.3134} \times 100\pi \times 10^{6}}\ \Omega$$

$$= 113.71837\ \Omega$$

$$\therefore \boxed{R_1 = R_2 = 113.7184\ \Omega}$$