~~Given~~

Statistical Signal Processing
Assignment-1

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Given the random process

$$y(n) = A\cos(\omega_0 n + \phi)$$

where $A = 0.5$, $f_0 = 0.05$

The phase is coupled by AWGN

$$V(n) \sim N(0, \sigma_v^2)$$

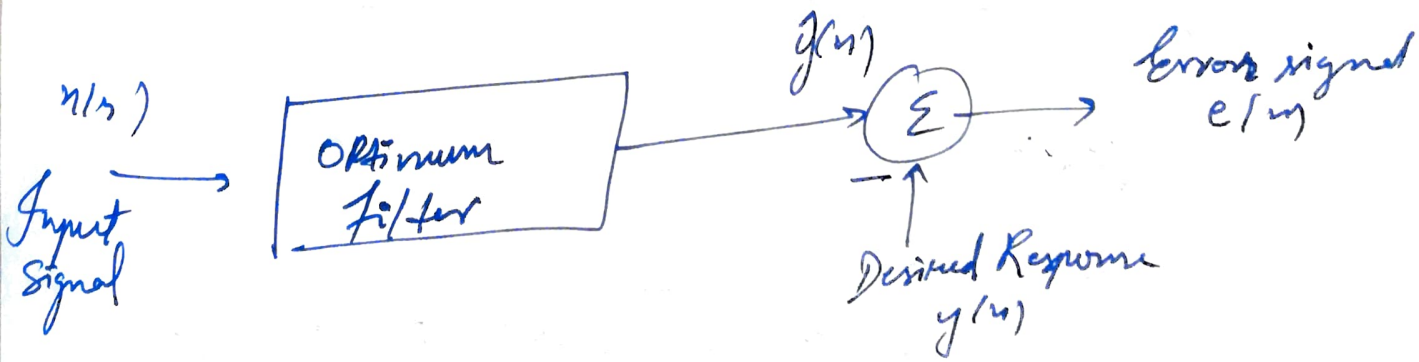
$$\sigma_v^2 = 0.5$$

$$x(n) = y(n) + V(n)$$

$V(n) \rightarrow$ AWGN $\sim N(0, \sigma_v^2)$

$y(n) \rightarrow$ Random Process

$x(n) \rightarrow$ Coupled Random Process



$y(n)$ & $v(n)$ are uncorrelated, autocorrelation of the input signal,

$$r_x(l) = r_y(l) + r_v(l)$$

$$r_{yu}(l) = E\left[y(n)\left[y(n-l) + v(n-l) \right] \right]$$

The auto-correlation matrix

$$R = E\left[x(n)\, x^H(n) \right]$$

$$x(n) = \left[x(n),\ x(n-1) \ \cdots \ x(n-M+1) \right]^T$$

$$R = \begin{bmatrix} r(0) & r(1) & \cdots & r(M-1) \\ r(1) & r(0) & \cdots & r(M-2) \\ \vdots & \vdots & & \vdots \\ r(M-1) & \cdots & & r(0) \end{bmatrix}$$

Cross-Correlation Vector

$$P = E\left[x(n) y^*(n) \right]$$

$$P = \left[p(0),\ p(-1),\ p(-2) \ \cdots \ p(1-M) \right]^T$$

$$R w_o = P$$

$$w_o = R^{-1} P$$

$$\boxed{w_o = R^{-1} r_{yu}(l)} \rightarrow \text{Optimum weight vector}$$

$$\hat{y}(n) = \sum_{k=0}^{M-1} w_{ok}\, x(n-k)$$

↳ Output of filter

w_{ok} → are the optimum weights.