Industrial Instrumentation Tutorial

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1) Maximum change in sensitivity

$\rightarrow k_M I_{Mmax} = 0.05k$

$I_M \sim N(\mu, \sigma)$ $\quad \mu = (I_{Mmax})/2$

$\sigma = 0.01 I_{Mmax}$

for confidence > 99%

I_M must lie in the range $[\mu - 3\sigma, \mu + 3\sigma]$

$I_M \in [\mu - 3\sigma, \mu + 3\sigma]$

$I_M \in [(0.5 - 3\times0.01) I_{Mmax}, (0.5 + 3\times0.01) I_{Mmax}]$

$I_M \in [0.47 I_{Mmax}, 0.53 I_{Mmax}]$

Input value of half of FS $= \frac{I_{max}}{2}$

$o/p = (k + k_M I_M)\frac{I_{max}}{2}$

for min o/p, $I_M \rightarrow I_M = 0.47 I_{Mmax}$

Min o/p $= (k + 0.47 k_M I_{Mmax})\frac{I_{max}}{2}$

$= (k + 0.47 \times 0.05k)\frac{I_{max}}{2} = 0.51175 k I_{max}$

for Max o/p $\rightarrow I_M = 0.53 I_{Mmax}$

Max o/p $= k(1 + 0.53\times0.05)\frac{I_{max}}{2} = 0.51325 k I_{max}$

Limits of Instruments at half of FS

$\Rightarrow$ (51.175% - 51.325%) max Output

2) Given,

Accelerometer is the input.

$V_s = 3V$, Temp $= 5-25°C$

Assuming zero output when sensor is aligned nominally to z-axis.

<u>Sources of errors:</u>

Non-linearity $= 0.3\%$ of F.S Span [part of α]

Input-range $= (-3.6g) - (3.6g)$

Sensitivity (Nominal) $= k = 300 mV/g$

Output range $= (-1.08V) - (1.08V)$

$N(I) = 0.3\%$ of FS $= \frac{0.3}{100} \times 1.08V = 3.24 mV$

Temperature is a modifying input (sensitivity changes w.r.t temperature) as well as an integrating factor input (causes an offset).

$k_M = 0.01\%$ of $k = 0.03 mV/g/°C$

Offset $= 1.5V = a$

Output range considering offset $\Rightarrow 0.42V \leq \text{Output}_{\text{nominal}} \leq 2.58V$

Also,

$k_I = 1mg/°C \times 300 mV/g = 300 \mu V/°C$

Temp range $= 5°-45°C$, $T_A = 25°C$, $\Delta T_{max} = 20°C$

Maximum error due to $k_I I_I = 300 \mu V/°C \times 20°C = 6 mV$

Alignment error $= 1 - \cos 1 = 1.5 \times 10^{-4} = 0.015\%$ of O/P

Since acceleration is single axis, no cross-axis alignment is considered.

For modifying input error, $\Delta T = 20°C$ $\{as\ T_A = 25°C\}$

$$\% \text{ error} = \frac{k_M I_{M\,max}}{KI} \times 100$$

$$= \frac{0.6}{300} \times 100 = 0.2\%$$ (Part of y as part of o/p)

Interferring input error = 6 mV, Span = 2.16 V

$$= \frac{0.006}{2.16} \times 100 = 0.278\%$$ (Part of x)

$$x\% = (0.1 + 0.278 + 0.3)\% = 0.678\%$$

$$y\% = (0.2 + 0.015)\% = 0.215\%$$

↳ (Interferring + Alignment Errors)

Resolution = 3600 μg (max)

3) Zero effect = 4 mA = a , Span = 16 mA = (20 − 4) mA

Thermal Sensitivity Shift = 0.018% of span/°C

$$= \pm \frac{0.018}{100} \times \frac{16}{15} \text{ mA}/°C$$

$$k_M = 1.92 \times 10^{-4} \text{ mA}/°C$$

Thermal Zero Shift = ±0.018% of span/°C

$$= \frac{0.018}{100} \times 16 \times T$$

$$\text{also } k_I I_I = \frac{0.018}{100} \times 16T =$$

$$k_I = \frac{0.018}{100} \times 16 = 2.88 \times 10^{-3} \text{ mA}/°C$$

$$o/p = KI + k_M I_M I + k_I I_I + a$$

$$K = \frac{\text{Output FS}}{I_{FS}} = \frac{16}{15} = 1.067 \text{ mA/PSI}$$

$$o/p = 1.067 \times I + 1.92 \times 10^{-4} \times I \times T + 2.88 \times 10^{-3} \times T + 4 \text{ mA}$$

Assuming adjustment is not suitable,

$a = 4 \pm 0.16$ mA , span $= 16 \pm 0.16$ mA

$$K_M = \frac{0.018}{100} \times \frac{16}{15} \pm \frac{0.018}{100} \times \frac{0.16}{15} = 1.92 \times 10^{-4} \pm 1.92 \times 10^{-6} \text{ mA/}^{\circ}\text{C/PSI}$$

Similarly,

$$K_I = \frac{0.018}{100} \times 16 \pm \frac{0.018}{100} \times 0.16 = 2.88 \times 10^{-3} \pm 2.88 \times 10^{-5} \text{ mA/}^{\circ}\text{C}$$

$$K = \frac{16}{15} \pm \frac{0.16}{15} = 1.067 \pm 0.01067 \text{ mA/PSI}$$

Worst Case Errors

Accuracy $= \pm 0.5\%$ of span $= \pm \frac{0.5}{100} \times 16$ ~~±0.5%~~ of span

Error due to $K_M I_M I$ $= 1.92 \times 10^{-4} \times (-34.25) \times 25$

$= -1.062\%$ of span

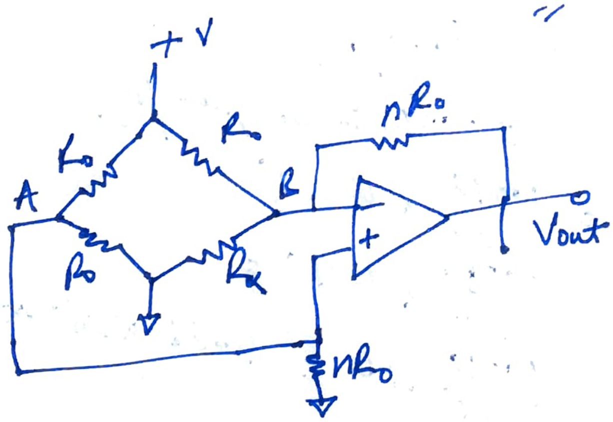
Error due to $K_I I_E$ $= -0.16922 = -1.062\%$ of span

Total worst case error $= -0.5 - 1.0692 - 1.062$

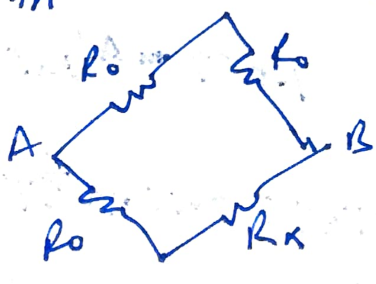
$= -2.624\%$ of span

$= -0.42$ mA

4)



R_{Th} across AB =

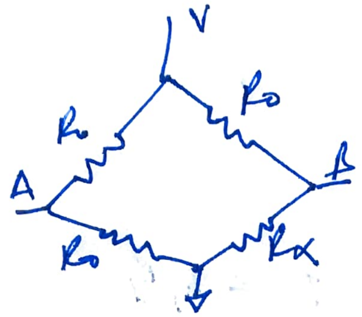


$$R_{AB} = (2R_0) \,\|\, (R_0 + R_x) = \frac{(2R_0)(R_0 + R_x)}{3R_0 + R_x}$$

$R_x = R_0(1+\alpha)$

$$R_{AB} = \frac{2(2+x)R_0}{(x+4)} = R_{Th}$$

V_{AB} (Thevenin Voltage across AB)



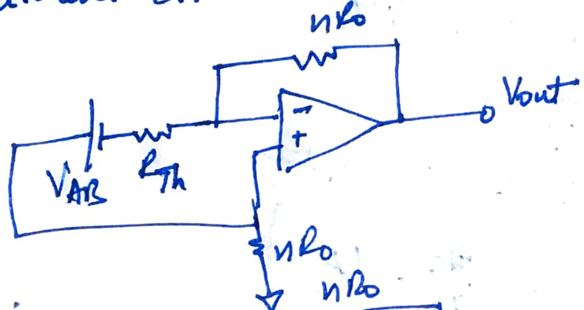
$$V_A = \frac{V}{2}, \quad V_B = \frac{R_x}{R_x+R_0}V$$

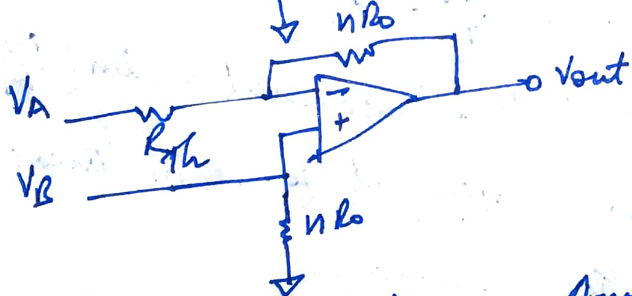
$$V_{AB} = \frac{(R_0 - R_x)}{2(R_0+R_x)}V$$

$$= \frac{-R_0\alpha}{2(2R_0+R_0\alpha)}V = \frac{-\alpha}{2(2+\alpha)}V$$

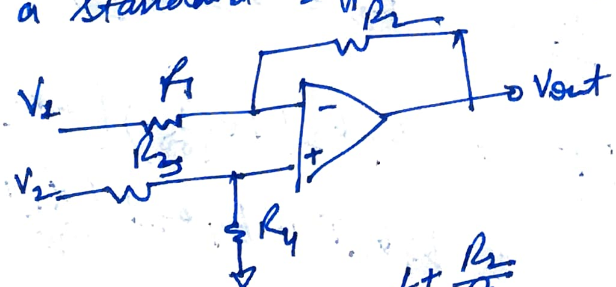
~~Eq~~

Equivalent Circuit





For a standard Difference Amplifier.



$R_2 = nR_0, \quad R_1 = R_{Th}$

$R_3 = 0, \quad R_4 = nR_0$

$V_1 = V_A, \quad V_2 = V_B$

$$A_{CM}\text{(common mode)} = \frac{1+\frac{R_2}{R_1}}{1+\frac{R_3}{R_4}} - \frac{R_2}{R_1} = 1$$

$$A_d\text{(differential mode)} = \frac{1}{2}\left(\frac{R_2}{R_1} + \frac{1+R_2/R_1}{1+\frac{R_3}{R_4}}\right) = \frac{1}{2}\left(1+\frac{2R_2}{R_1}\right)$$

$$= \frac{1}{2} + \frac{R_2}{R_1} = \frac{1}{2} + \frac{nR_0}{\frac{2(2+x)}{(x+4)}R_0} = \frac{n(x+4)}{2(x+2)}$$

$$V_{out} = A_d V_{AB} = \left[\frac{n(n+4)}{2(n+2)} + \frac{1}{2}\right]\left[\frac{-\alpha}{2(\alpha+2)}\right] V$$

$V = 10V$, $n = 100$

i) $x = 0.1$, $V_{out} = -23.36\,V$

ii) $x = 0.01$, $V_{out} = -2.5\,V$

5) Two instruments in Series

$$I_1 \rightarrow \boxed{k_1} \xrightarrow{O_1,\ I_2} \boxed{k_2} \rightarrow O_2$$

$I_1 \le R_1$, $I_2 = O_1 = k_1 I_1 \Rightarrow I_2 \le k_1 R_1$

also, $I_2 \le R_2$.

$\therefore I_2 = \min(R_2, k_1 R_1)$

Now, $I_2 = k_1 I_1$

$\le \min(R_2, k_1 R_1)$

$k_1 I_1 \le \min(R_2, k_1 R_1)$

$$\boxed{I_1 \le \min\left(\frac{R_2}{k_1}, R_1\right)}$$

$\Rightarrow$ FS input of the instrument $= \min\left(\frac{R_2}{k_1}, R_1\right)$

Assuming the entire instrument to be in linear region

Accuracy of $S_1 \to \alpha_1$, $S_2 \Rightarrow \alpha_2$.

$k_1' = k_1\left(1 + \frac{\alpha_1}{100}\right)$, $k_2' = k_2\left(1 + \frac{\alpha_2}{100}\right)$

Max. Error

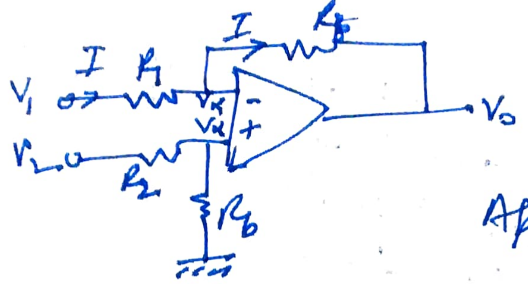
$$O = k_1' k_2' I = k_1 k_2 \left(1 + \frac{\alpha_1}{100}\right)\left(1 + \frac{\alpha_2}{100}\right) I$$

$$= k_1 k_2 I + k_1 k_2 I \left(\frac{\alpha_1}{100} + \frac{\alpha_2}{100} + \frac{\alpha_1 \alpha_2}{10^4}\right)$$

$$\% \text{ error} = \frac{O - \bar{O}}{\bar{O}} \times 100\% = \frac{k_1 k_2 I \left(1 + \frac{\alpha_1}{100} + \frac{\alpha_2}{100} + \frac{\alpha_1 \alpha_2}{10^4}\right) - k_1 k_2 I}{k_1 k_2 I} \times 100\%$$

$$= \left(\alpha_1 + \alpha_2 + \frac{\alpha_1 \alpha_2}{100}\right)\%$$

6)



$$V_x = \frac{R_b V_2}{R_2 + R_b} \quad \text{— (i)}$$

Applying KCL,

$$\frac{V_1 - V_x}{R_1} = \frac{V_x - V_o}{R_f}$$

$$\Rightarrow V_x\left(\frac{1}{R_1} + \frac{1}{R_f}\right) = \frac{V_1}{R_1} + \frac{V_o}{R_f}$$

$$\Rightarrow V_x = \frac{V_1 R_f + V_o R_1}{R_1 + R_f} \quad \text{— (ii)}$$

Putting value of V_x from (i),

$$\frac{R_b V_2}{R_b + R_2} = \frac{V_1 R_f + V_o R_1}{R_1 + R_f}$$

$$\Rightarrow \boxed{V_o = \frac{V_2\left(1 + \frac{R_f}{R_1}\right)}{\left(1 + \frac{R_2}{R_b}\right)} - \frac{V_1 R_f}{R_1}} \quad \text{— (iii)}$$

$V_{differential} = V_d = V_2 - V_1$

$V_{CM} = V_{common\ mode} = \frac{V_1 + V_2}{2}$

$$\left.\begin{aligned} V_1 &= V_{cm} - \frac{V_d}{2} \\ V_2 &= V_{cm} + \frac{V_d}{2} \end{aligned}\right\}$$

Substituting values of V_1 & V_2 in (iii), we get,

$$V_o = \underbrace{\left[\frac{1 + \frac{R_f}{R_1}}{1 + \frac{R_2}{R_b}} - \frac{R_f}{R_1}\right]}_{A_{cm}} V_{cm} + \underbrace{\frac{1}{2}\left[\frac{R_f}{R_1} + \frac{1 + \frac{R_f}{R_1}}{1 + \frac{R_2}{R_b}}\right]}_{A_d} V_d$$

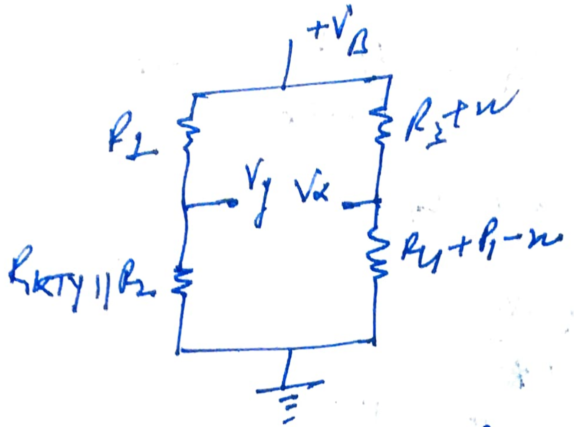
$R_f = 100\,k\Omega$, $R_1 = 10\,k\Omega$, $R_2 = 1\,k\Omega$, $R_b = 100\,k\Omega$

$$A_{cm} = \frac{-10}{111} \ \& \ A_d = \frac{1105}{111}$$

$$\therefore CMRR = \left|\frac{A_d}{A_{cm}}\right|$$

$$\text{in dB} = 20 \log\left|\frac{A_d}{A_{cm}}\right| = 40.867\ dB$$

7)



$R_1 = 3.3k\Omega$ $\quad V_B = 5V$

$R_2 = 22k\Omega$

$R_3 = 4k\Omega$

$R_4 = 1k\Omega$

$$V_x - V_y = \left[\frac{R_4 + P_1 - x}{R_3 + R_4 + P_1} - \frac{R_{KTY}||R_2}{R_1 + R_{KTY}||R_2} \right] V_B$$

$P_1 = 220\Omega$, $\quad x \in [0, P_1]\,\Omega$, $\quad R_{KTY} \in [886\,\Omega, 1696\,\Omega]$

Case I $\quad x = 0$, $R_{KTY} = 886\,\Omega$

$V_x - V_y = 160\,mV$

Case II $\quad x = 0$, $R_{KTY} = 1696\,\Omega$

$V_x - V_y = -441.532\,mV$

Case III $\quad x = 220\,\Omega$, $R_{KTY} = 886\,\Omega$

$V_x - V_y = -50.08\,mV$

Case IV $\quad x = 220\,\Omega$, $R_{KTY} = 1.696\,V$

$V_x - V_y = -657.26\,mV$

$V_x - V_y \in [-657.26\,mV,\ 160\,mV]$

Gain of op-amp $= \dfrac{-R_f}{R_i}$, $\quad R_f \in [68k\Omega, 72.7k\Omega]$

$R_i = 33k\Omega$

$2.061 \le |\text{Gain}| \le 2.203$ $\quad \left[-\text{ve as } -\dfrac{R_f}{R_i}\right]$

$V_o = 5 + \text{Gain}\,(V_x - V_y)$

$V_{o\,min} = 5V - 2.203 \times 160\,mV$
$= +4.64752\,V$

$V_{o\,max} = 5V + 2.203 \times 657.26\,mV$
$= 6.447945\,V > 5V$

So clipped at $5V$

$\therefore V_o \in [4.647V, 5V]$

As temperature changes, the non linearity would also get affected.

$$871.70 \leq R_{BTY} \| R_2 \leq 1574.61$$

In absence of R_2

$$886 \leq R_{BTY} \leq 1196\ \Omega$$

By the virtue of R_2 in parallel ~~resulting~~, results in the decreased range of resistance which results in lesser variation of output voltage.

Assuming a nominal voltage temperature of 50°C

$$\Rightarrow R_{BTY} = 1209\ \Omega$$

$$V_x - V_y = \left(\frac{1220 - x}{5220} - \frac{1446.02}{4446.02}\right) 5$$

When $x = 0$,

$$V_x - V_y \Rightarrow -0.1214\ V$$

When $x = 220\ \Omega$

$$V_x - V_y = -0.3321\ V$$

Hence at 50°C, $|V_x - V_y|$ lies between ⇒

$$|V_x - V_y| \rightarrow 121.4\ mV - 332.1\ mV$$

8) at 0°C, Assuming unit surface area, $A = 1m^2$

$$Y = 2 \times 10^{11}\ Nm^{-2}, \quad R_L = 0.5 \times 5 = 2.5\ \Omega$$

$$V_s = (2R_g + 2R_L) i = 5.125\ V$$

$$\text{Sensitivity} = \frac{V_s}{4} \times \frac{GF}{AY} = 10^{-5}$$

$$GF = \frac{10^{-5} \times AY \times 4}{V_s} = 1.56 \times 10^{6}$$

at 50°C,

$$GF = 1.56 \times 10^{6} \times \left(\frac{50}{18} \times \frac{0.75}{100} + 1\right) = 1.88 \times 10^{6}$$

$$R_L = 2.5(1 + \alpha_{cu} \Delta T) = 3.04\ \Omega$$

$$R_g = 100(1 + \alpha_{Brass} \Delta T) = 110\ \Omega$$

Let us assume we are measuring a force of 1kN

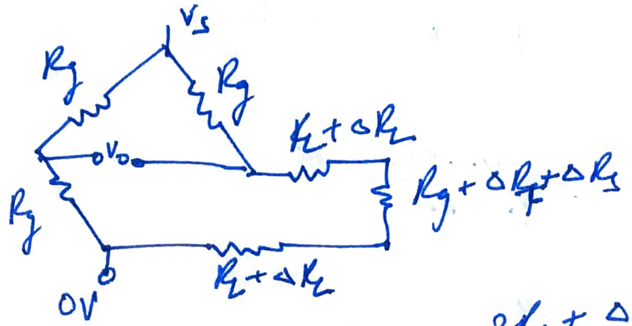
$\Rightarrow \quad Y\varepsilon = \frac{F}{A} = 10^3$

$$\varepsilon = \frac{10^3}{2\times10^{11}} = 5\times10^{-9}$$

$$\frac{\Delta R}{R} = \varepsilon\, G.F = 9.4\times10^{-3}$$

$$\Delta R_{gauge} = \Delta R = 9.4\times10^{-3}\times100 = 1.034\,\Omega$$

The bridge for Strain Management,

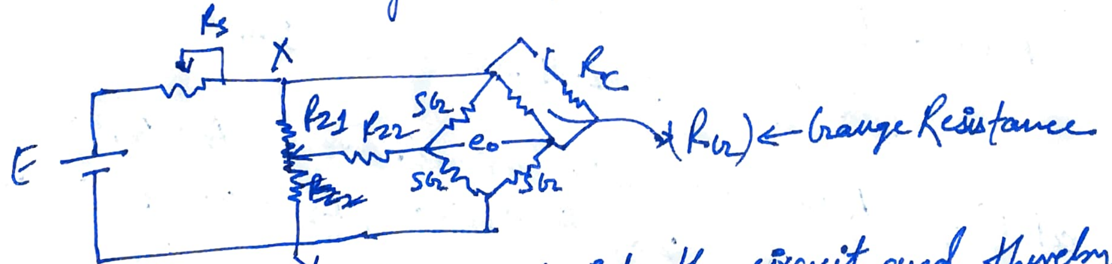


where, ΔR_T & ΔR_L are due to temperature change & ΔR_S due to strain.

$$\text{Now,} \quad V_o = \frac{V_s}{4}\times\frac{2R_L+\Delta R_T+\Delta R_S}{R_g+R_L+\Delta R_L+\left(\frac{\Delta R_L+\Delta R_T}{2}\right)}$$

$$\& \quad V_s = i\left(R_g+2(R_L+\Delta R_L)+R_g+\Delta R_T+\Delta R_S\right)$$

9)



R_S controls the flow of current into the circuit and thereby adjusts the sensitivity of the bridge.
R_{Z1}, R_{Z2} are the potentiometer resistances and they are used in getting very smooth variations as they allow increment or decrement in small steps and hence change the zero output point. R_C helps in calibration.

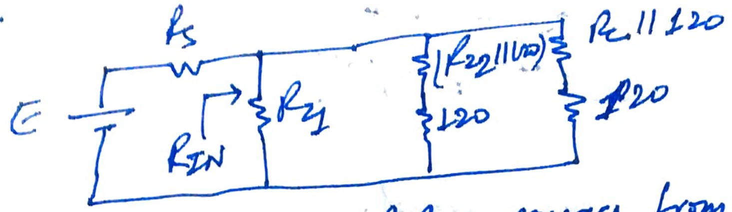
There are 4 structures possible.

1) R_C is closed & R_{Z2} is connected to X.
2) R_C is closed & R_{Z2} is connected to Y.

3) R_C is opened & R_{Z2} is connected to X.

4) R_C is opened & R_{Z2} is connected to Y.

Case I



Here, R_{Z1} & R_{Z2} ranges from 10–15 kΩ,

Then, $R_{Z2} \| 120 \simeq 120$

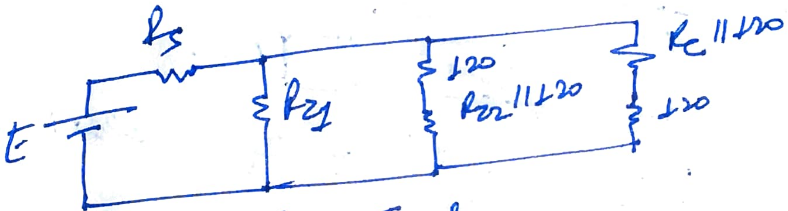
Here, to limit the current flow, R_S must be greater than R_{IN}.

$$R_{IN} = R_{Z1} \,\|\, 240 \,\|\, (R_C \| 120 + 120)$$

$$\simeq 240 \,\|\, (R_C \| 120 + 120) \quad [\text{approximate}]$$

So, $\boxed{R_S > 240 \,\|\, (R_C \| 120 + 120)}$

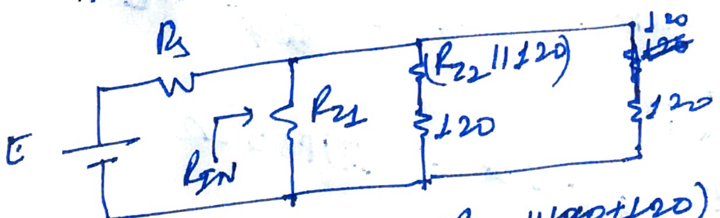
Case 2



Same as Case I, So,

$$\boxed{R_S > 240 \,\|\, (R_C \| 120 + 120)}$$

Case 3 & Case 4 will be having the structure as:



$$R_{IN} = R_{Z1} \,\|\, (R_{Z2} \| 120 + 120) \,\|\, 240$$

$$\simeq (R_{Z2} \| 120 + 120) \,\|\, 240 \quad [\because R_{Z1} \sim 10\text{–}15 \text{ k}\Omega]$$

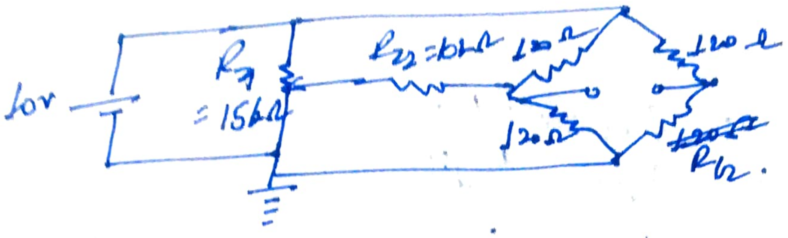
$$\simeq (120 + 120) \,\|\, 240$$

$$\simeq (240 \| 240) \simeq 120\,\Omega$$

$\boxed{\therefore R_S > 120\,\Omega}$

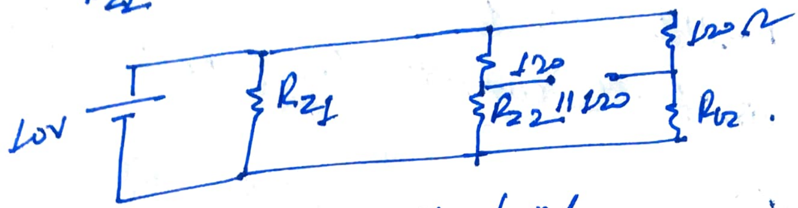
$\therefore \quad R_s > \max\left(240 \,||\, (120 + R_c || 120),\ 120\,\Omega\right)$

10)



<u>Case I.</u>

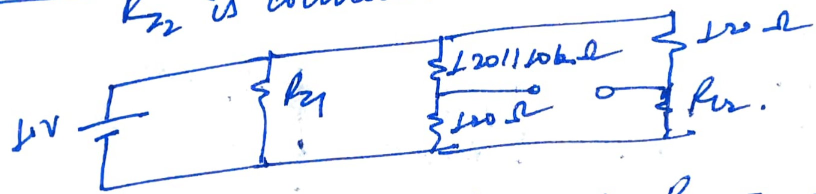
R_{22} is connected to ground,



for balancing the bridge

$R_{b2} = R_{22} || 120 = 120 || 10k\Omega = 118.57\,\Omega$

<u>Case II</u> R_{22} is connected to <u>10V</u>.



$$\frac{120 || 10k}{120} = \frac{120}{R_{b2}} \Rightarrow R_{b2} = 121.44\,\Omega$$

$\therefore$ Range of $R_{b2} \rightarrow [118.57\,\Omega - 121.44\,\Omega]$

11)

$$\varepsilon = \frac{R_o}{GF(R_o + R_c)}$$

$$R_c = \frac{R_o}{\varepsilon \times GF} - R_o$$

a) $\varepsilon = 800 \times 10^{-6}$

$$R_c = \frac{120}{800 \times 10^{-6} \times 2} - 120 = 199880\,\Omega$$

b) $$\varepsilon = \frac{120}{1000 \times 10^{-6} \times 2} - 120 = 59880\,\Omega$$

12) $$P = \frac{V_{ref}^2}{R_1 + R(1+x)} + \frac{V_{ref}^2}{R_1 + R}$$

Given: $P \leq 0.2 \times 10^{-3}\, W$; assuming $x << 1$

$$\Rightarrow P = \frac{2V_{ref}^2}{R_1 + R} \leq 0.2 \times 10^{-3}\, W.$$

$$\Rightarrow R_1 \geq \frac{2V_{ref}^2}{0.2 \times 10^{-3}} - R$$

$$\Rightarrow R_1 \geq 2249900\,\Omega \simeq \cancel{22.499 \times 1\Omega}\ 2.25\,M\Omega$$

Output of the setup $= A \cdot V_{Th}$

$$= -A\left[\frac{R(1+x)}{R(1+x)+R_1} - \frac{R}{R+R_1}\right]V_{ref}$$

$$= -RAV_{ref}\left[\frac{(1+x)(R+R_1) - R(1+x) - R_1}{(R+R_1)(R(1+x)+R_1)}\right]$$

$$= \frac{-RR_1AV_{ref}x}{(R+R_1)(R(1+x)+R_1)} \simeq \frac{-RR_1AV_{ref}x}{(R+R_1)^2}$$

$\{x << 1\}$

$x = \alpha\Delta T$

let reference temperature $= 0\,°C$

$x = \alpha T$

$$O/P = \frac{-RR_1AV_{ref}\alpha T}{(R+R_1)^2}$$

Here, $R_1 \geq 2.25\,M\Omega$, let $R_1 = 2.25\,M\Omega$

Sensitivity $= 0.1\,V/°C$

$$\therefore \frac{O/P}{T} = 0.1 = \frac{-ARR_1V_{ref}\alpha}{(R+R_1)^2}$$

$$\Rightarrow |A| = \frac{0.1(R+R_1)^2}{RR_1V_{ref}\alpha}$$

$= 38268.707$

$R = 100$

$\alpha = 0.003$

$V_{ref} = 15$

at 100°C

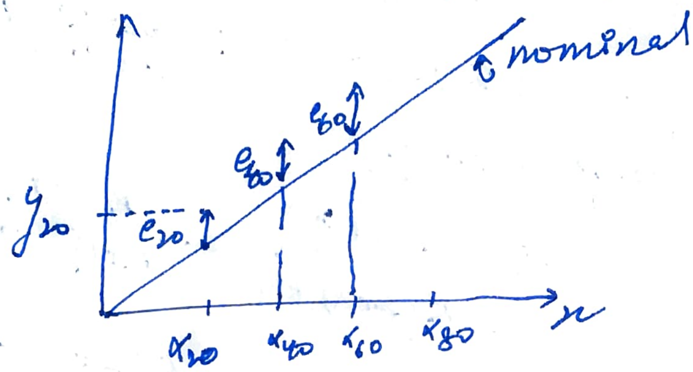
$$|V_0| = \frac{A R R_1 V_{ref} T}{(R_1 + R(1+\alpha \Delta T))(R+R_1)} = 9.999825773$$

V_0 assumed = $0.1 \times 100 V = 10V$

error due to non-linearity = 0.000174227

$\approx 1.74 \times 10^{-4}$ V

13)



Let's consider x_{20}, e_{20}, y_{20}, say the error production, x_{20}% of FS output. Find the maximum errors for all the other points x_{40}, x_{60} & x_{80} in ~~the~~ terms of y.
Now, for other inputs, iterate doing the same as above & store x & y values.
Now finding which pair to choose :-
Let's say we have 2 pairs : (x_1, y_1) & (x_2, y_2)
& $(x_1 < x_2)$ & $y_1 < y_2$). Then, (x_1, y_1) gives the lesser margin in limit of error, so it is a better choice.
Let say y is the measured output & e_1 & e_2 are the limits of error given by a (x_1, y_1) & x be the actual value & $y - x$ be the error e. So here the the objective would be to minimize the range of e_1, e_2 considering they should also include e in the range.