Programmable Embedded Systems

Class Work. (22/10/2021)

Submitted By:
Pratyush Jaiswal
18EE35014

Unsigned Division

Let $U(a_3, b_3) = \dfrac{U(a_1, b_1)}{U(a_2, b_2)}$

and consider the largest possible result:

$$\text{largest result} = \frac{\text{largest dividend}}{\text{smallest divisor}}$$

$$= \frac{2^{a_1} - 2^{-b_1}}{2^{-b_2}} = 2^{a_1+b_2} - 2^{b_2-b_1} \quad (1)$$

Thus we require

$$2^{a_3} - 2^{-b_3} \geq 2^{a_1+b_2} - 2^{b_2-b_1} \quad (2)$$

It is natural to let $a_3 = a_1 + b_2$, in which case the inequalities below result:

$$2^{a_3} - 2^{-b_3} \geq 2^{a_3} - 2^{b_2 - b_1}$$

$$-2^{-b_3} \geq \cancel{1} - 2^{b_2 \cancel{- a_3} - b_1}$$

$$2^{-b_3} \leq 2^{b_2 - b_1}$$

$$-b_3 \leq b_2 - b_1$$

$$\boxed{b_3 \geq b_1 - b_2} \quad — (3)$$

Thus we have constraint on b_3 due to b_1 & b_2.

Now, we consider the smallest possible result:

$$\text{Smallest result} = \frac{\text{Smallest dividend}}{\text{Largest divisor}}$$

$$= \frac{2^{-b_1}}{2^{a_2} - 2^{-b_2}} \quad — (4)$$

This then requires b_3 to obey the following constraint:

$$2^{-b_3} \leq \frac{2^{-b_1}}{2^{a_2} - 2^{-b_2}}$$

$$\boxed{b_3 \geq b_1 + \log_2\left(2^{a_2} - 2^{-b_2}\right)} \quad — (5)$$

Date: __/__/__
Page: ____

If we assume b_2 is positive,

the above constraint (5) is the more stringent of the above two constraints (3) & (5) on b_3.

We then express (5) in simpler form:

$$b_3 \geq \log_2\left(2^{a_2+b_1} - 2^{b_1-b_2}\right) \quad -(6)$$

Then final result is

$$U(a_1, b_1) / U(a_2, b_2) = U\left(a_1 + b_2, \left\lceil \log_2\left(2^{a_2+b_1} - 2^{b_1-b_2}\right)\right\rceil\right)$$

$-(7)$