Control and Electronic Systems Design

<u>Assignment 2</u>

1. (i) In a coupled-tank system, water is poured at the rate q_0 in Tank 1, which discharges into Tank 2, which in turn discharges in the open. Each of them has a cross-sectional area A and assume that both the tanks are placed on same base. Suppose the flow out of each tank is proportional to the square root of the hydrostatic pressure causing it, the proportionality constant for Tank 1 being ρ_1 and that for Tank 2 being ρ_2.
   a) Obtain first the equilibrium condition for the system, and then the *incremental* transfer function between the height of the water level in Tank 2 and q_0 about the equilibrium point.
   b) Also obtain a state-space representation of the system.
   c) Present an electrical equivalent of the system.

   (ii) In above, assume the height of each tank is 30 cm and A=16 cm^2. The orifices through which, Tank 1 discharges to Tank 2, which in turn discharges in the open, have diameter 0.5 cm each. The desired operating water level of Tank 2 is 10 cm. The pump used to pour water in Tank 1 has flow constant 5 cm^3/s/V. The maximum voltage that can be applied to the pump is 25 V. The sensitivity of the level sensors used is 5 cm/V. With all these parameters, simulate the open-loop system to check whether the behavior of the linearized system is similar to the nonlinear system around the operating point.

2. A Pendulum consisting of a mass-less rod of length $l = 0.36m$ and a bob of mass $m = 0.26kg$ is attached to a cart of mass $M = 2.4kg$ moving on a rail. Let the position of the cart on the rail be denoted by x and the angle of the rod with respect to the upward vertical by θ. The cart is propelled by a horizontal force of u along the rail. The pendulum is to be balanced in the inverted position with the cart being within the rail by applying a suitable u.
   a) Obtain the transfer functions $\theta(s)/U(s)$ and $X(s)/U(s)$ for θ being close to zero.
   b) Explain if the pendulum can be balanced using only θ or only x feedback.
   c) Bring out the robustness issue, if any, associated with this control problem.