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Sol:

current $I = \dfrac{V_1 - V_2}{R + jX}$

$$= \frac{|V_1|\angle\delta_1 - |V_2|\angle\delta_2}{R + jX}$$

$$I^* = \frac{|V_1|\angle-\delta_1 - |V_2|\angle-\delta_2}{R - jX} \quad \text{— (i)}$$

Power at Area 1

$$S_1 = P_{tie\,1,2} + j\,Q_{tie\,1,2}$$

$$= V I^*$$

$$= |V_1|\angle\delta_1 \,(I^*)$$

$$= \frac{|V_1|^2 - |V_1||V_2|\angle\delta_1 - \delta_2}{R - jX}$$

$$= \frac{(|V_1|^2 - |V_1||V_2|\angle\delta_1 - \delta_2)(R + jX)}{R^2 + X^2}$$

$$P_{tie\,1,2} + jQ_{tie\,1,2} = \frac{\left(|V_1|^2 - |V_1||V_2|\cos(\delta_1 - \delta_2) - j|V_1||V_2|\sin(\delta_1 - \delta_2)\right)(R + jX)}{R^2 + X^2}$$

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$$= \frac{|V_1|^2 R - |V_1||V_2| R \cos(\delta_1-\delta_2) + |V_1||V_2| X \sin(\delta_1-\delta_2)}{R^2+X^2}$$

$$+ j\left[\frac{-|V_1||V_2| \sin(\delta_1-\delta_2) R + |V_1|^2 X - |V_1||V_2| X \cos(\delta_1-\delta_2)}{R^2+X^2}\right]$$

So,

$$P_{tie\,1,2} = \frac{|V_1|^2}{R^2+X^2} R + \frac{|V_1||V_2|}{R^2+X^2}\left[-R\cos(\delta_1-\delta_2) + X\sin(\delta_1-\delta_2)\right]$$

$$Q_{tie\,1,2} = \frac{|V_1|^2}{R^2+X^2} X - \frac{|V_1||V_2|}{R^2+X^2}\left[X\cos(\delta_1-\delta_2) + R\sin(\delta_1-\delta_2)\right]$$

Similarly,

$$S_2 = P_{tie\,2,1} + j\,Q_{tie\,2,1}$$

$$= V_2 I^*$$

$$= |V_2|\angle\delta_2\, I^*$$

$$= \frac{|V_2||V_1|\angle(\delta_2-\delta_1) - |V_2|^2}{R^2+X^2}(R+jX)$$

$$= \left[\frac{\left(|V_1||V_2|\cos(\delta_2-\delta_1) - |V_2|^2\right) + j|V_1||V_2|\sin(\delta_2-\delta_1)}{R^2+X^2}\right](R+jX)$$

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$$P_{tie\,2,1} + jQ_{tie\,2,1} =$$

$$\left[\frac{|V_1||V_2|R\cos(\delta_2-\delta_1) - |V_2|^2 R) - |V_1||V_2|\sin(\delta_2-\delta_1)X}{R^2+X^2}\right.$$

$$+ j\left[\frac{|V_1||V_2|R\sin(\delta_2-\delta_1) - |V_1||V_2|X\cos(\delta_2-\delta_1) - |V_2|^2 X}{R^2+X^2}\right]$$

$$P_{tie\,2,1} = \frac{-|V_2|^2}{R^2+X^2}R + \frac{|V_1||V_2|}{R^2+X^2}\left[R\cos(\delta_1-\delta_2) + X\sin(\delta_1-\delta_2)\right]$$

$$Q_{tie\,2,1} = \frac{-|V_2|^2}{R^2+X^2}X + \frac{|V_1||V_2|}{R^2+X^2}\left[X\cos(\delta_1-\delta_2) - R\sin(\delta_1-\delta_2)\right].$$

$$\Delta P_{tie\,1,2}^{(pu)} = T_{12}(\Delta\delta_1 - \Delta\delta_2)$$

where $T_{12} = \frac{|V_1||V_2|}{P_{r1}(R^2+X^2)}\left[X\cos(\delta_1-\delta_2) + R\sin(\delta_1-\delta_2)\right]$

$\Rightarrow \Delta P_{tie\,1,2} = T_{12}\,2\pi.\left[\int \Delta f_1\,dt - \int \Delta f_2\,dt\right]$

$$\Delta P_{g1} - \Delta P_{L1} = \frac{2H_1}{f^0}\frac{d\,\Delta f_1}{dt} + D_1\Delta f_1 + \Delta P_{tie\,1,2}$$

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Take Laplace Transform

$$\Delta f_1(s) = \left[\Delta P_{g_1}(s) - \Delta P_{L_1}(s) - \Delta P_{tie_{1,2}}(s)\right] \frac{k_{P_1}}{1+sT_{P_1}}$$

where $k_{P_1} = \frac{1}{D_1}$, $T_{P_1} = \frac{2H_1}{D_1 f^0}$

Similarly,

$$\Delta P_{tie_{2,1}} \overset{(iv)}{=} 2\pi T_{21}\left(\int \Delta f_2 - \int \Delta f_1\right) dt$$

where $T_{21} = \frac{-|V_2||V_1|}{P_{T_2}(R^2+\alpha^2)}\left[\alpha \cos(\delta_1-\delta_2) + R\sin(\delta_1-\delta_2)\right]$

Also,

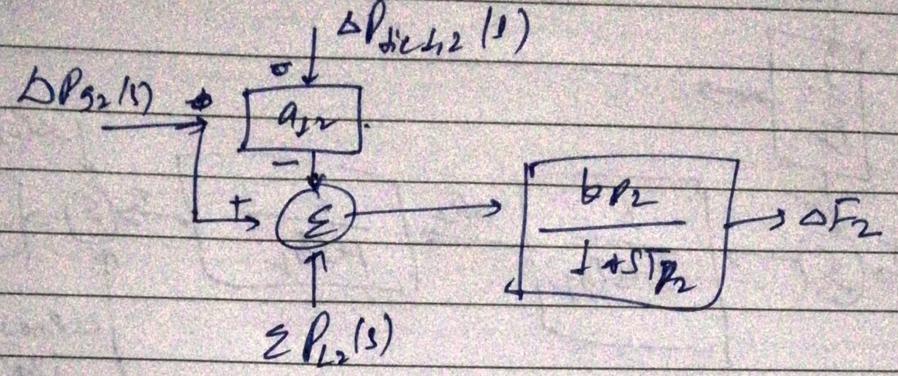
$$\Delta f_2(s) = \left[\Delta P_{g_2}(s) - \Delta P_{L_2}(s) - \Delta P_{tie_{2,1}}(s)\right] \frac{k_{P_2}}{1+sT_{P_2}}.$$

By method of equal area criterion,

$$\Delta P_{tie_{1,2}} = a_{12}\Delta P_{tie_{2,1}}.$$

$$\therefore a_{12} = \frac{-P_{T_2}\left(\alpha\cos(\delta_1-\delta_2) + R\sin(\delta_1-\delta_2)\right)}{P_{T_1}\left(\alpha\cos(\delta_2-\delta_1) + R\sin(\delta_2-\delta_1)\right)}$$

Block diagram becomes.

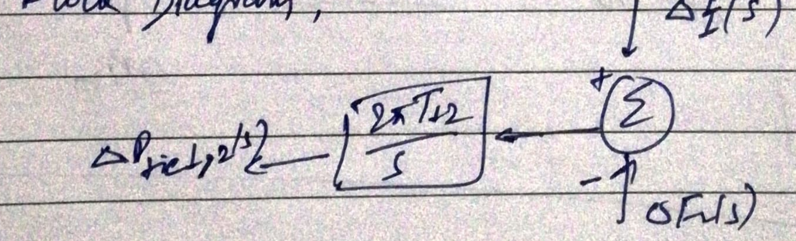


Applying laplace Transform to

$$\Delta P_{tie,1,2}^{(pu)} = 2\pi T_{12}\left[\int \Delta f_1 dt - \int \Delta f_2 dt\right]$$

$$\Delta P_{tie(1,2)} = \frac{2\pi T_{12}}{s}(\Delta F_1 - \Delta F_2)$$

Block Diagram,



However block diagram showing small perturbation transfer function remains the same.
The parameters of a_{12}, T_{12} are changed.

$$a_{12} = \frac{-P_{r2}}{P_{r1}}\frac{[\alpha\cos(\delta_1-\delta_2)+\beta\sin(\delta_1-\delta_2)]}{[\alpha\cos(\delta_1-\delta_2)+\beta\sin(\delta_1-\delta_2)]}$$

$$T_{12} = \frac{|V_1||V_2|}{P_{r1}(r^2+\alpha^2)}\left[\alpha\cos(\delta_1-\delta_2)+\beta\sin(\delta_1-\delta_2)\right]$$

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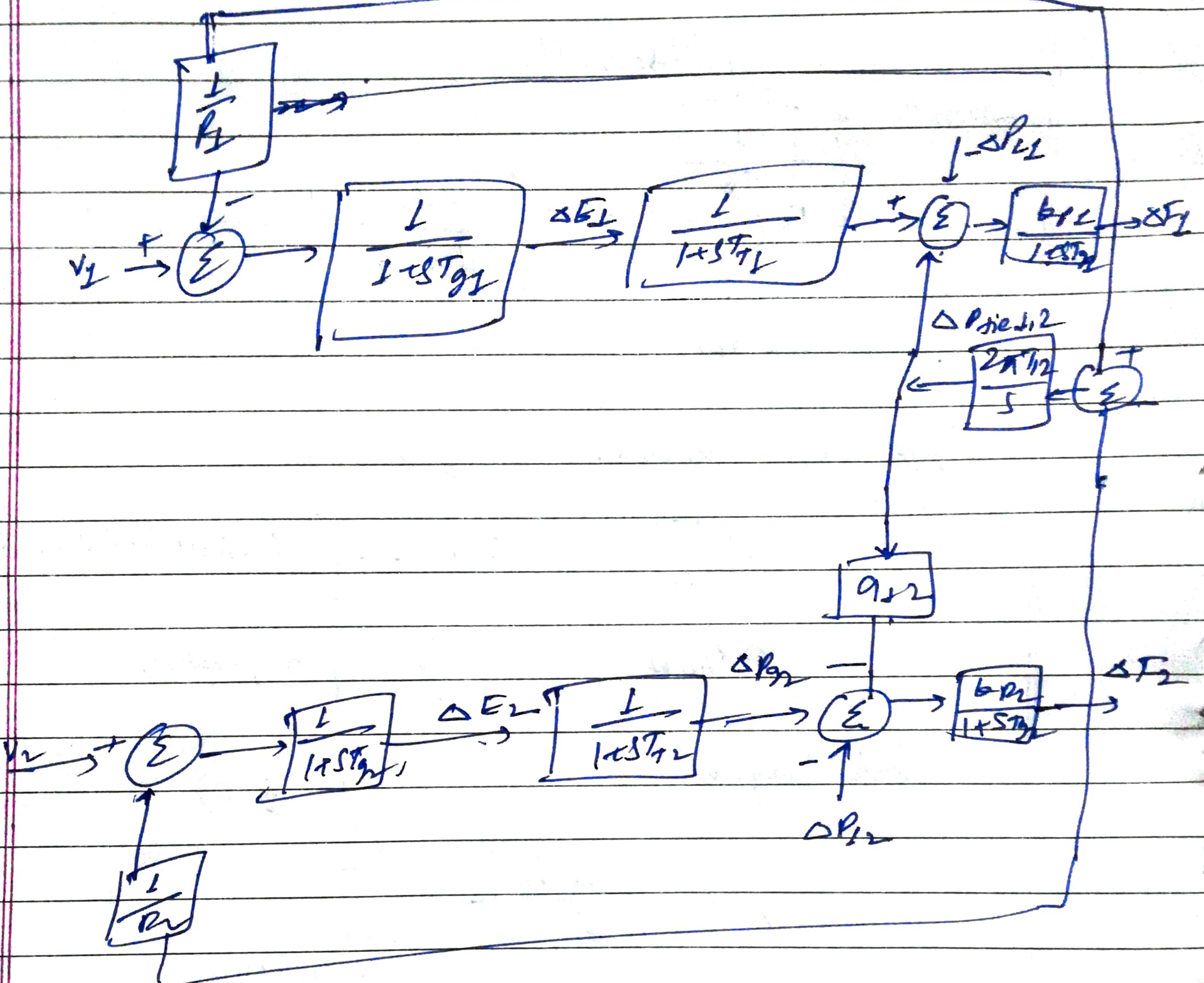

1/R1
v1
+
−
Σ
1/(1+sTg1)
ΔE1
1/(1+sTT1)
−ΔPL1
Kp1/(1+sTp1)
ΔF1
ΔPtie,1,2
2πT12/s
a12
ΔPg2
ΔF2
v2
ΔE2
1/(1+sTg2)
1/(1+sTT2)
Kp2/(1+sTp2)
ΔPL2
1/R2