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PS - Asg - 2

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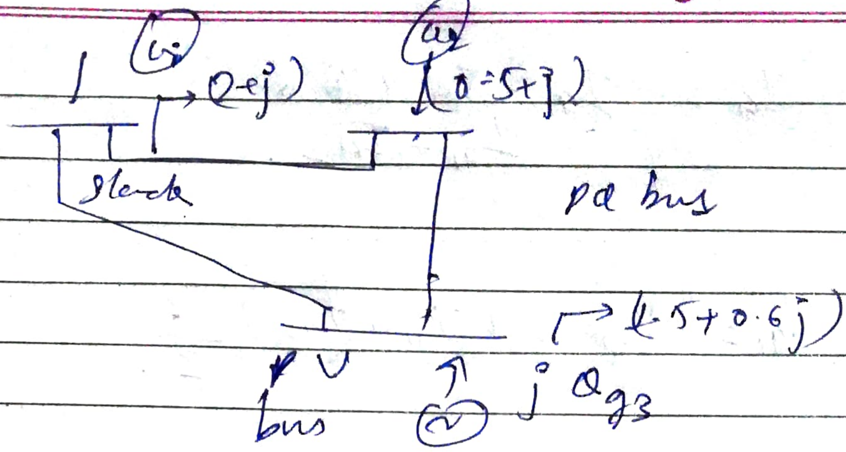

$$Y_{bus} = \begin{bmatrix} 29.0925\angle -68.2^\circ & 11.18\angle 116.6^\circ & 15.81\angle 108.4^\circ \\ 11.18\angle 116.6^\circ & 29.0625\angle -63.4^\circ & 17.885\angle 116.6^\circ \\ 15.81\angle 108.4^\circ & 17.885\angle 116.6^\circ & 33.615\angle -62.2^\circ \end{bmatrix}$$

$V_1 = 1\angle 0^\circ$ (slack)

$|V_2|^{0} = 1.0 \qquad \delta_2^{(0)} = 0.0$

$|V_3| = 1$ PV bus

$\delta_3^{(0)} = (0.0)$.

We know,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(P)} = \begin{bmatrix} \partial P_2/\partial \delta_2 & \partial P_2/\partial \delta_3 & \partial P_2/\partial |V_2| \\ \partial P_3/\partial \delta_2 & \partial P_3/\partial \delta_3 & \partial P_3/\partial |V_3| \\ \partial Q_2/\partial \delta_2 & \partial Q_2/\partial \delta_2 & \partial Q_2/\partial |V_2| \end{bmatrix}^{P} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}^{(P)}$$

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We know that,

$$P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \rightarrow A$$

$$Q_i = - \sum_{k=1}^{n} |V_i||V_k||Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \rightarrow B.$$

$\therefore$

$$P_2 = |V_1||V_2||Y_{12}| \cos(\theta_{12} - \delta_2 + \delta_1)$$
$$+ |V_2|^2 |Y_{22}| \cos\theta_{22}$$
$$+ |V_3||V_2||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

For the Jacobian Matrix Current

$$\frac{\partial P_2}{\partial \delta_2} = |V_1||V_2||Y_{12}| \sin(\theta_{12} - \delta_2 + \delta_1)$$
$$+ |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{12} - \delta_2 + \delta_1)$$
$$+ 2|V_2||Y_{22}| \cos\theta_{22}$$
$$+ |V_3||Y_{32}| \cos(\theta_{32} - \delta_2 + \delta_1)$$

replacing i with 3.

$$\frac{\partial P_3}{\partial \delta_2} = -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_1||V_3||Y_{13}| \sin(\theta_{13} - \delta_3 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_2 = -|V_1||V_2||Y_{12}| \sin(\theta_{12} - \delta_2 + \delta_1) - |V_2|^2 |Y_{22}| \sin\theta_{22} - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_1||V_2||Y_{12}| \cos(\theta_{12} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

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$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{12}| \sin(\theta_{12} - \delta_2 + \delta_1)$$
$$-2|V_2||Y_{22}| \sin \theta_{22}$$
$$-|V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$P_2^{scheduled} = P_{g_2} - P_{l_2} = 0.5\ pu$

$Q_2^{scheduled} = Q_{g_2} - Q_{l_2} = 1\ pu$

$P_3^{scheduled} = P_{g_3} - P_{l_3} = -1.5\ pu$

Putting these values in (A) & (B)

$P_2^{(0)} \simeq 0 \quad P_3^{(0)} \simeq 0 \quad Q_2^{(0)} = 0.$

$\Delta P_2^{0} = P_2^{scheduled} - P_2^{calculated}$

$= 0.5\ pu.$

$\Delta P_3^{0} = -1.5\ pu . \quad \Delta Q_2^{0} = 1\ pu$

Putting the values in the derivative eqn,

$$\frac{\partial P_2}{\partial \delta_2} = 11.18 \sin 116^\circ + 17.885 \sin(116.6^\circ)$$
$$\simeq 26.$$

$$\frac{\partial P_2}{\partial \delta_3} = \cancel{-1.59} \; -15.99 \simeq 16$$

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$$\frac{\partial P_2}{\partial |V_2|} = 12.014$$
$$\simeq 13.$$

Similarly.

$$\frac{\partial P_3}{\partial \delta_2} = -17.885 \sin 116.6^\circ$$
$$\simeq -16.$$

$$\frac{\partial P_3}{\partial \delta_3} = 30.97$$
$$\simeq 31$$

$$\frac{\partial P_3}{\partial |V_2|} = 17.85 \cos 116.6^\circ$$
$$\simeq -8.$$

Similarly.

$$\frac{\partial Q_2}{\partial \delta_2} = 11.8 \cos 116.6^\circ + 11.885 \cos 116.6^\circ$$
$$= -13.$$

$$\frac{\partial Q_2}{\partial \delta_3} = -17.885 \cos 116^\circ$$
$$= 8$$

$$\frac{\partial Q_2}{\partial |V_2|} = 26.$$

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$$\therefore J = \begin{bmatrix} 26 & -16 & 13 \\ -16 & 31 & 8 \\ -13 & 8 & 26 \end{bmatrix}.$$

$$J^{-1} = \begin{bmatrix} 792 & 526 & -331 \\ 312 & 845 & -416 \\ 275 & 0 & 550 \end{bmatrix} \frac{1}{17875}.$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix}^{P} = \frac{1}{17875} \begin{bmatrix} 792 & 520 & -331 \\ 312 & 845 & -416 \\ 275 & 0 & 550 \end{bmatrix} \begin{bmatrix} 0.5 \\ -1.5 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix} = \begin{bmatrix} -3.014 \\ -4.894 \\ 0.0385 \end{bmatrix}$$

$\Delta\delta_2^{(0)} = -3.014^\circ$ $\Delta\delta_3^{(0)} = -4.894^\circ$

$\Delta|V_2| = 0.638$

Updating Values.

$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)}$

$= -3.014^\circ$

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$\delta_3^{(1)} = -4.894^\circ$

$|V_2| = 1.0385$ pu

New P_2, P_3, Q_3 from A & B after putting updated values,

$P_{2(calc)}^{(1)} = 0.743$ pu.

$P_{3(calc)}^{(1)} = -2.083$ pu.

$Q_{3(calc)}^{(1)} = 1.069$ pu.

$\Delta P_2^{(1)} = P_2^{scheduled} - P_{2\,calc}^{(1)}$

$= -0.243$ pu

$\Delta P_3 = -1.5 + 2.083 = 0.583$ pu

$\Delta Q_3 = 1 - 1.069$ pu $= -0.069$ pu

$P = 1$ 2^{nd} Iteration

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$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}^{(1)} = \frac{1}{12875} \begin{bmatrix} 742 & 520 & -131 \\ 312 & 849 & -44 \\ 225 & 0 & 555 \end{bmatrix} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(1)}$$

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}^{1} = \begin{bmatrix} 0.51^\circ \\ 1.375^\circ \\ -0.0058 \end{bmatrix}$$

$$\Delta \delta_2^{(1)} = 0.51^\circ$$

$$\Delta \delta_3^{(1)} = 1.375^\circ$$

$$\Delta |V_2|^1 = -0.006 \text{ pu.}$$

Updating,

$$\delta_2^{(2)} = \delta_2^{(1)} + \Delta \delta_2^{(1)}.$$

$$= -2.514^\circ.$$

$$\delta_3^{(2)} = -3.519^\circ.$$

$$|V_2| = 1.0235 \text{ pu}$$

New P_2, P_3, Q_3 from (A) & (B)

$P_2^{(2)}(calc) = 0.281$ pu $P_3^{(2)} calc = -1.433$ pu.

$Q_3^{(2)}(calc) = 0.966$ pu.

$$\Delta P_2^{(2)} = 0.219 \text{ pu}$$

$$\Delta P_3^{(2)} = 0.067 \text{ pu}.$$

$$\Delta Q_3^{(2)} = 0.034 \text{ pu}.$$

3^{rd} Iteration, $p = 2$.

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}^{(2)} = J^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^{(2)}.$$

$$= \frac{1}{17825} \begin{bmatrix} 792 & 520 & -521 \\ 312 & 845 & -416 \\ 325 & 0 & 750 \end{bmatrix} \begin{bmatrix} 0.219 \\ -0.067 \\ 0.034 \end{bmatrix}$$

$$= \begin{bmatrix} 0.349^\circ \\ 0.007^\circ \\ 0.004^\circ \end{bmatrix}.$$

$$\delta_2^{(3)} = \delta_2^{(2)} + \Delta \delta_2^{(2)}$$

$$= -2.108^\circ.$$

$$\delta_3^{(3)} = -3.512^\circ$$

$$|V_2|^{(3)} = 1.0865.$$

It can be seen that the values are already started to converge after 3^{rd} iteration.