# Transfer function of linear time-invariant system

- relation between input - output variables
- An LTI system can be characterised by its output response when the input is unit impulse.
- input is unit impulse $\left( \mathcal{L}[\delta(t)] = 1 \right)$
- Once the impulse response of a linear system is known, the output of the system $y(t)$ with any input $u(t)$ can be found by using the transfer function.
- Transfer function is the Laplace transform of the impulse response of an LTI system with zero initial condition.
- $$G(s) = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} = \frac{Y(s)}{U(s)}$$

  (in case of SISO)

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

2 X 2

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$Y(s) = G(s)\, U(s) \quad \text{(MIMO)}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

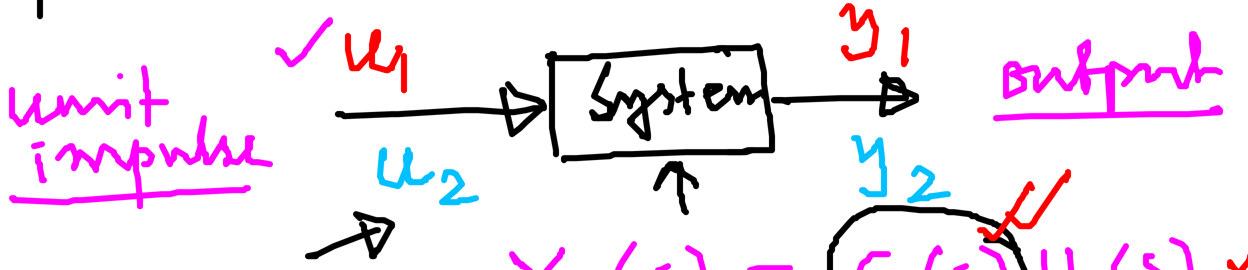
$$\boxed{\text{Not } \cancel{U(s)\, G(s)}}$$

$AB \neq BA$

Matrix Algebra

- only for LTI system

– independent of input of the system




$$Y_1(s) = G(s)\,U_1(s)$$
$$Y_2(s) = G(s)\,U_2(s)$$

– $G(s)$; It is a function of a complex variable 's' [not function of real variable, or time]

Ex
$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2\,y(t) = \omega_n^2\,u(t)$$

Input is $u(t)$ and output $y(t)$.

Taking Laplace transform

→ $s^2 Y(s) + 2\zeta\omega_n s\,Y(s) + \omega_n^2 Y(s) = \omega_n^2 U(s)$

[All initial conditions are zero]

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

Let
$$G(s) = \frac{b_m s^{m} + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^{n} + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$0 = s^{n} + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

$b_i, a_i \rightarrow$ real co-efficients.

(1) Proper or bi-proper TF: $m = n$  $\quad G(s) = \dfrac{s^2 + 2s + 3}{5s^2 + s + 6}$

(2) Improper TF: $m > n$  $\quad G(s) = \dfrac{s+1}{1}$

[physically not realizable]

(3) Strictly proper TF: $m < n$  $\quad G(s) = \dfrac{s+1}{s^2 + 2s + 3}$

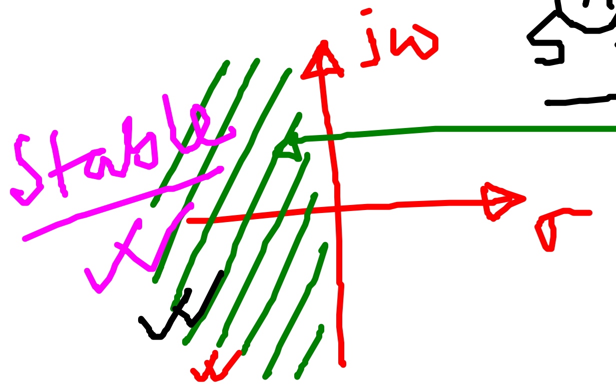
## Characteristic equation:

- Ch. eq$^n$ of a linear system is defined as the eq$^n$ obtained by setting the the denominator polynomial of the TF to zero.

$$s^{n} + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

roots

degree of ch. eq$^n$



- Stability of a system (LTI) depends on the location of the roots of the Ch. eqn.
- Order of the system = degree of Ch. eqn.

1st order system $\quad s+1=0$

$$G_1(s) = \frac{2}{s+1}$$

$$G_2(s) = \frac{s+2}{s+3}$$

2nd order system

$$G_1(s) = \frac{s+1}{s^2+2s+3}$$

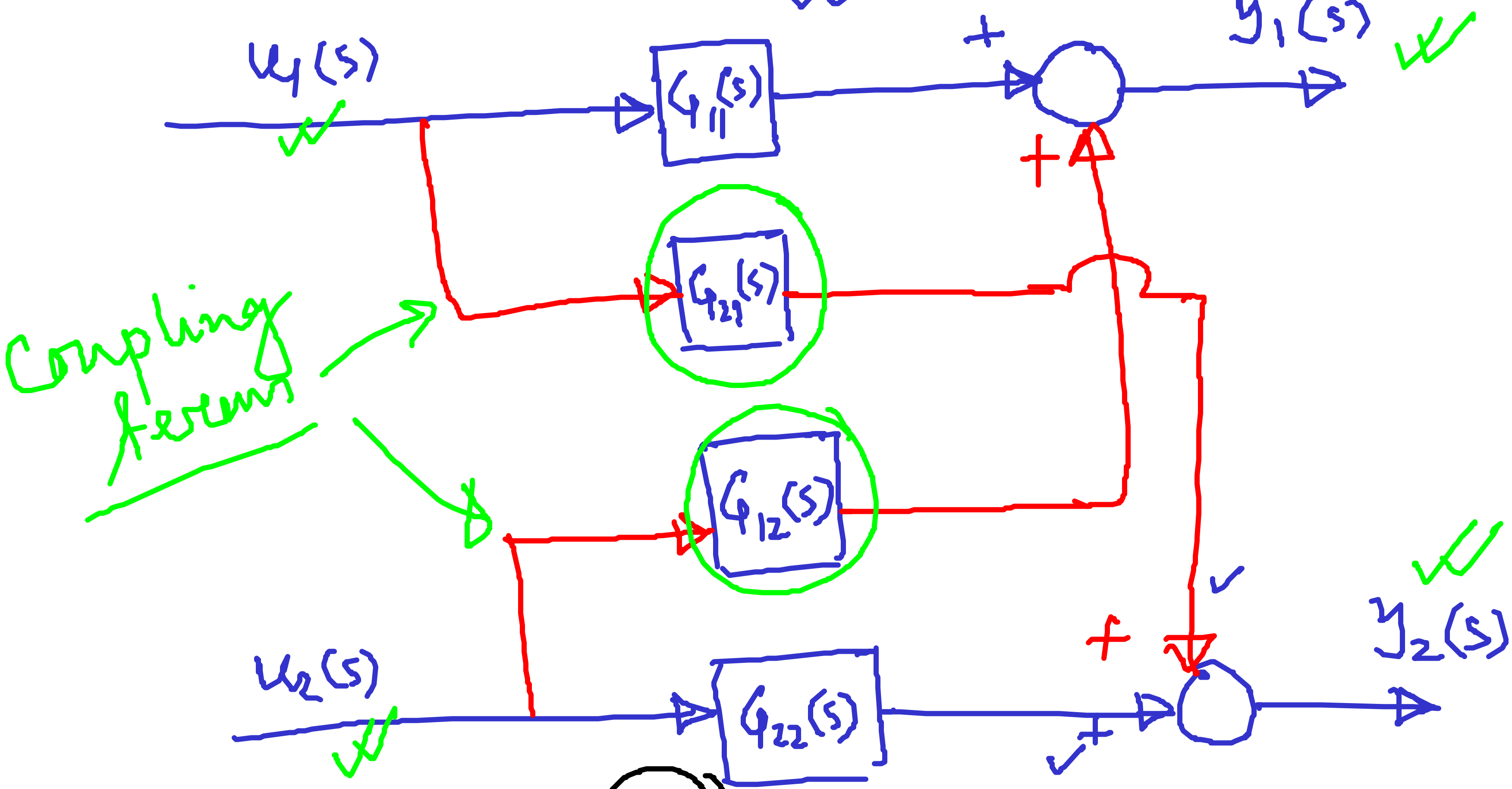
## TF of multi-input multi-output (MIMO) system:

$$\underset{2\times 1}{\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}}$$

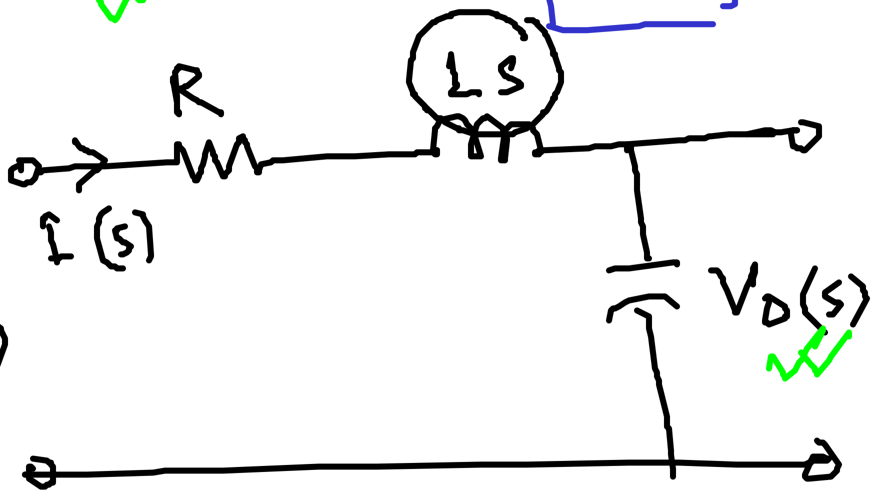
$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \underset{2\times 2}{\begin{bmatrix} \frac{s}{s+1} & 2 \\ \frac{1}{s^2+2s+3} & \frac{s+2}{s^2+5s+3} \end{bmatrix}} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

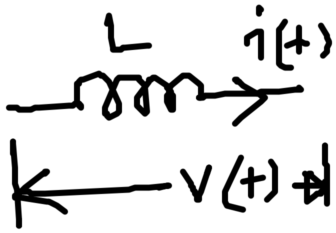
$$Y_1(s) = G_{11}(s)\,U_1(s) + G_{12}(s)\,U_2(s)$$

$$Y_2(s) = G_{21}(s)\,U_1(s) + G_{22}(s)\,U_2(s)$$




Ex




$$V(t) = L\,\frac{di(t)}{dt}$$

$$V(s) = LS\,I(s)$$

$$\frac{I(s)}{V(s)} = \frac{1}{LS}$$