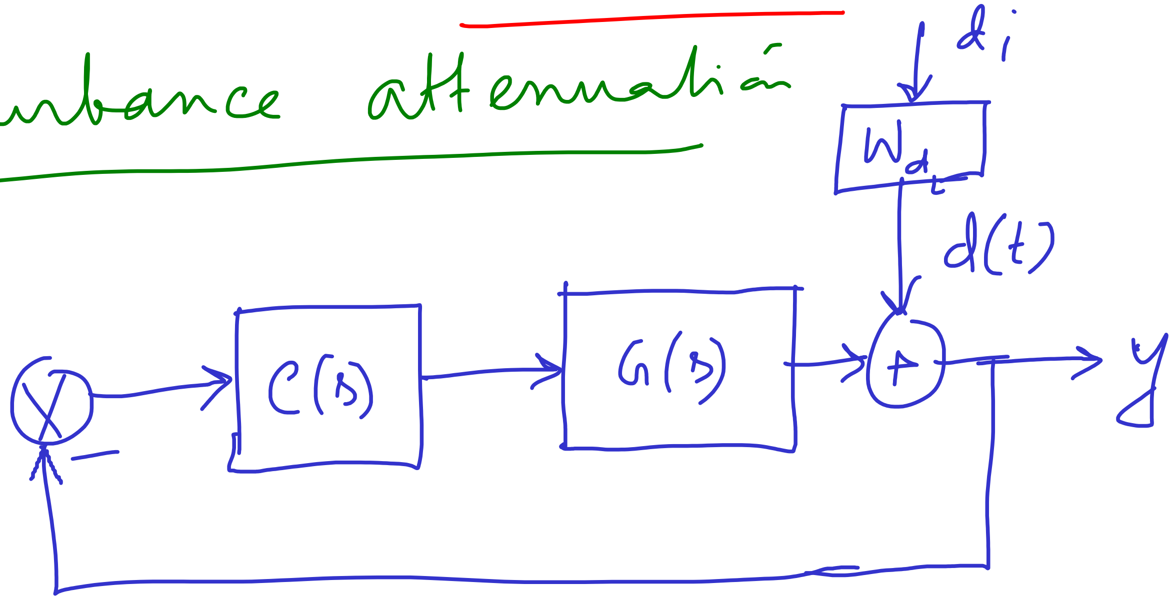
Lecture 4

Disturbance attenuation



W_d = disturbance weighting function that characterises the freq spectrum of $d(t)$

W_d is a low-pass filter

Disturbance attenuation problem:

To design controller $C(s)$ such that

$$\left|\frac{y(j\omega)}{d_i(j\omega)}\right| < 1 \quad \forall\ \omega \le \omega_d$$

$$\left[\, |W_d(j\omega)| < 1 \quad \forall\ \omega > \omega_d \,\right]$$

Condition for disturbance attenuation

$$|W_d(j\omega)\, S(j\omega)| < 1 \quad \forall\ \omega \le \omega_d$$

$$\Rightarrow |W_d(j\omega)| < |1 + L(j\omega)| \quad \forall\, \omega \le \omega_d$$

The above is satisfied if

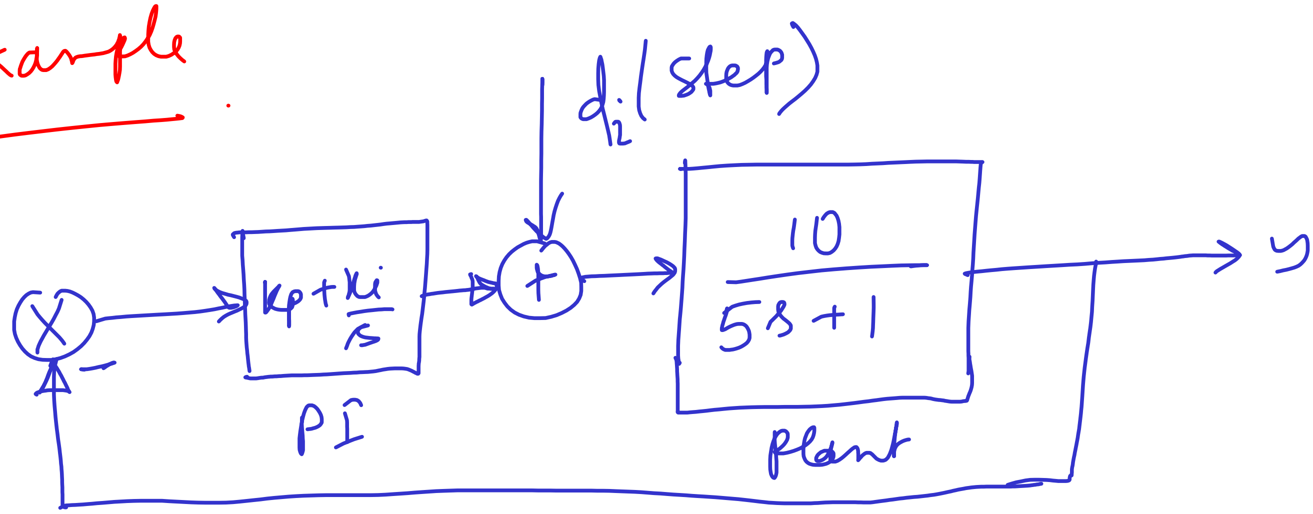
$$|W_d(j\omega)| < |L(j\omega)| \quad \left[\because |W_d(j\omega)| > 1 \;\; \forall\, \omega \le \omega_d\right]$$

$$\Rightarrow \boxed{|L(j\omega)| > |W_d(j\omega)| \quad \forall\, \omega \le \omega_d}$$

↳ condn for disturbance attenuation

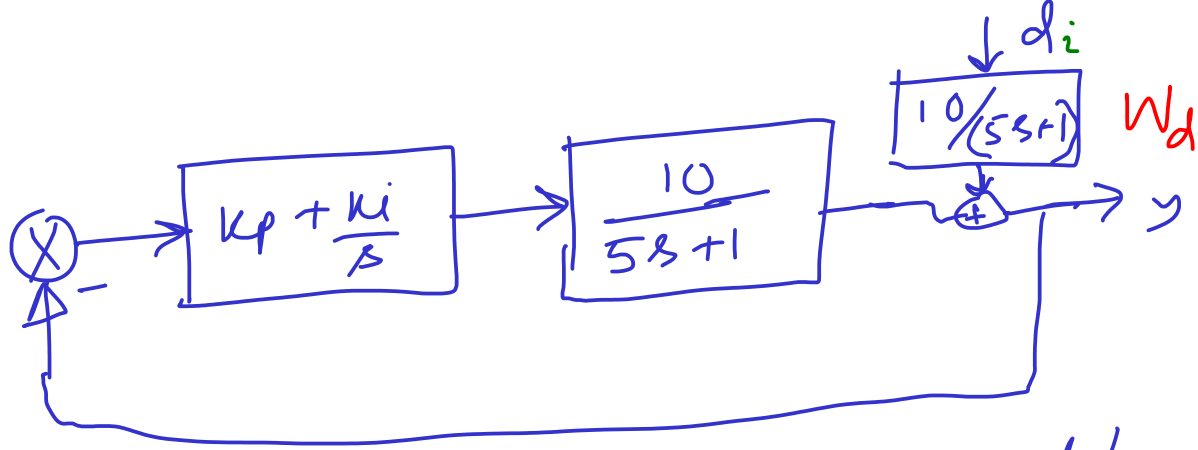
- $|L(j\omega)| \gg 1$ at low freq or $|S(j\omega)|$ should be as less as possible at low freq

Example



Determine k_p, k_i for disturbance attenuation.

Soln :



$$\left| \frac{y(j\omega)}{d_i(j\omega)} \right| < 1 \quad \forall \ \omega < 2 \text{ rad/s}$$

$$\Rightarrow \quad |L(j\omega)| > |W_d(j\omega)| \quad \forall \ \omega < 2$$

$$\Rightarrow \quad \left| \left(k_p + \frac{k_i}{j\omega}\right)\left(\frac{10}{5j\omega+1}\right) \right| > \left| \frac{10}{5j\omega+1} \right| \quad \forall \ \omega < 2$$

$$\Rightarrow \quad \left| \left(k_p + \frac{k_i}{j\omega}\right) \right| > 1 \quad \forall \ \omega < 2$$

Let $\boxed{k_p = 5k_i}$

then $\left| \frac{k_i(5j\omega+1)}{j\omega} \right| > 1 \quad \forall \ \omega < 2$

$$\Rightarrow \quad k_i > \left| \frac{\omega}{5j\omega+1} \right| \quad \forall \ \omega < 2$$

$$\Rightarrow \quad k_i > \frac{1}{\sqrt{25 + \frac{1}{\omega^2}}} \quad \forall\, \omega < 2$$

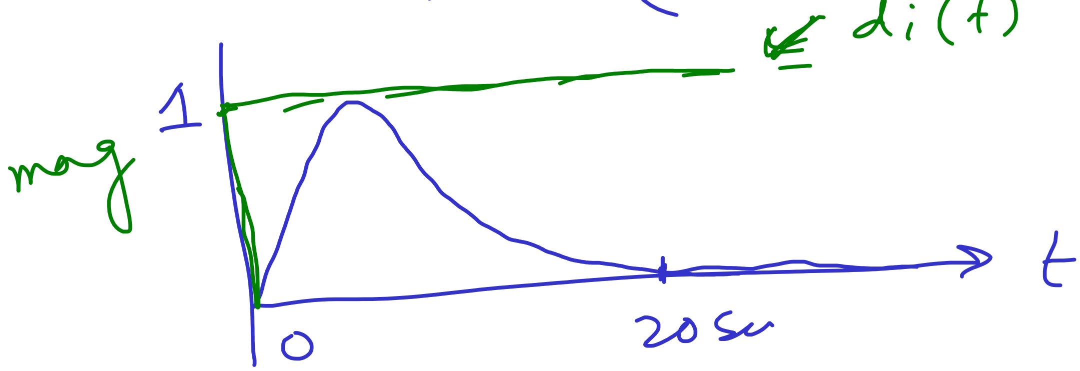
$$\Rightarrow \quad \boxed{k_i > \frac{1}{5}}$$

Here $\quad \dfrac{y(s)}{d(s)} = \dfrac{G}{1+GC} = \dfrac{\frac{10}{5s+1}}{1 + \frac{10}{\cancel{5s+1}} \cdot \frac{k_i \cancel{(5s+1)}}{s}}$

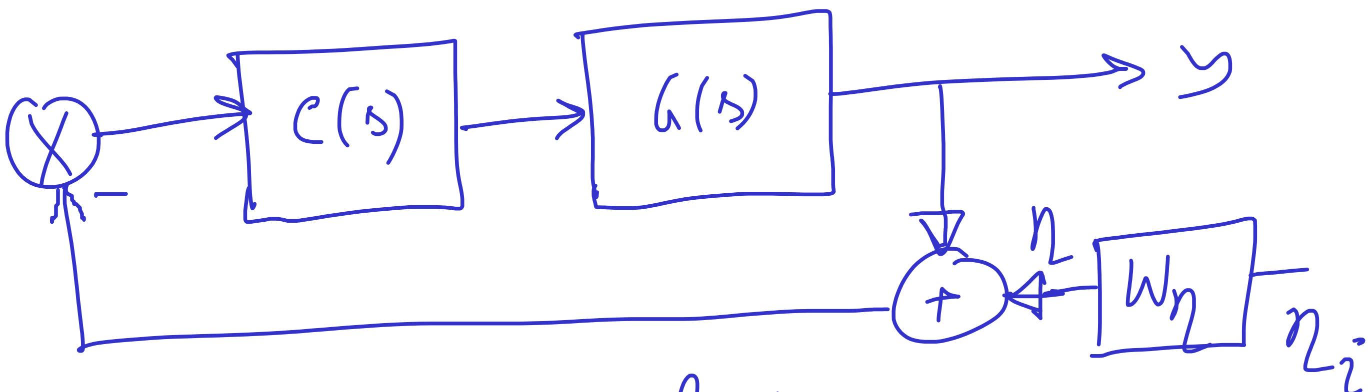
$$= \frac{10s}{(5s+1)(s+10k_i)}$$

Let $k_i = 0.3$

$$\Rightarrow \quad \frac{y(s)}{d_i(s)} = \frac{10s}{(5s+1)(s+3)}$$



Noise attenuation



$W_n \rightarrow$ high pass filter

Problem: To design $C(s)$ such that

$$\left|\frac{y(j\omega)}{n_i(j\omega)}\right| < 1 \quad \forall\ \omega \geq \omega_n$$

$$\left[\because \ |W_n(j\omega)| << 1 \quad \forall\ \omega < \omega_n\right]$$

Condition for noise attenuation

$$\left| W_n(j\omega)\, T(j\omega) \right| < 1 \quad \forall\ \omega \geq \omega_n$$

$$\Rightarrow \quad |W_n(j\omega)| < \left| \frac{1 + L(j\omega)}{L(j\omega)} \right|$$

The above is satisfied $\forall\ \omega \geq \omega_n$

if $|W_\eta(j\omega)| < \frac{1}{|L(j\omega)|} \quad \forall \omega \geq \omega_\eta$

[$\because |W_\eta(j\omega)| > 1$ for $\omega \geq \omega_\eta$]

$$\Rightarrow \boxed{|L(j\omega)| < \frac{1}{|W_\eta(j\omega)|} \quad \forall \omega \geq \omega_\eta}$$

↳ condn for noise attenuation

- For noise attenuation $|L(j\omega)| << 1$ at high freq or $|T(j\omega)|$ much less at high freq → for noise attenuation

A summary on role of S, T or L

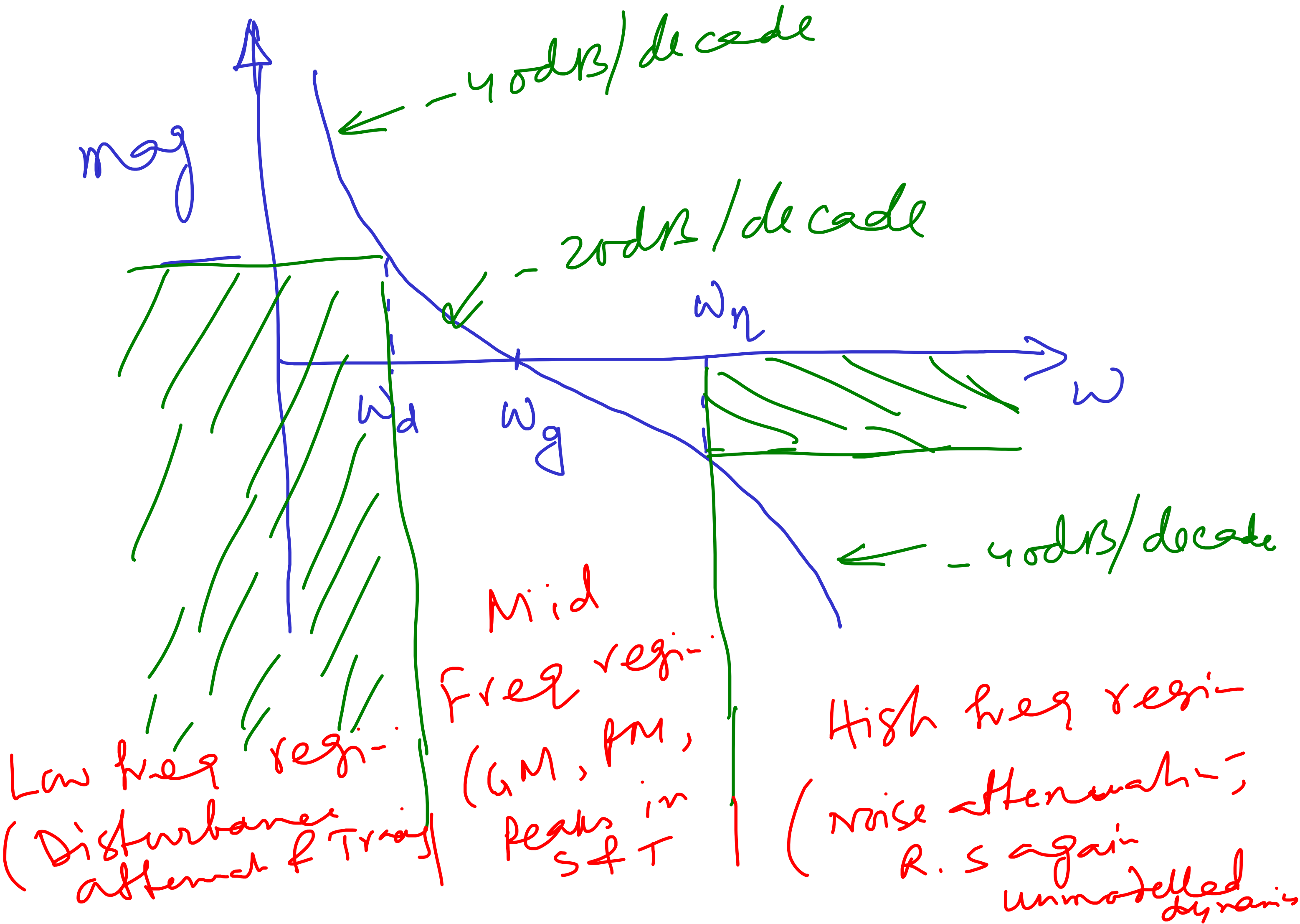
- $|L(j\omega)| > |W_d(j\omega)| \quad \forall \omega < \omega_d$
  or $|S(j\omega)| < 1/|W_d(j\omega)|$ - - -
  ↳ for disturbance attenuation
- $M_S < 2$ (at ~~least~~ least) for R.S against parameter variation
  $M_T < 2$ - - - R.S against unmodelled dynamics

- $|L(j\omega)| < \dfrac{1}{|W_2(j\omega)|} \quad \forall\ \omega > \omega_2$

or $|T(j\omega)| < \dfrac{1}{|W_2(j\omega)|} \quad \forall\ \text{- - - -}$

— for noise attenuation & robustness against unmodelled dynamics

Loop gain characteristics for a good design



S & T plots