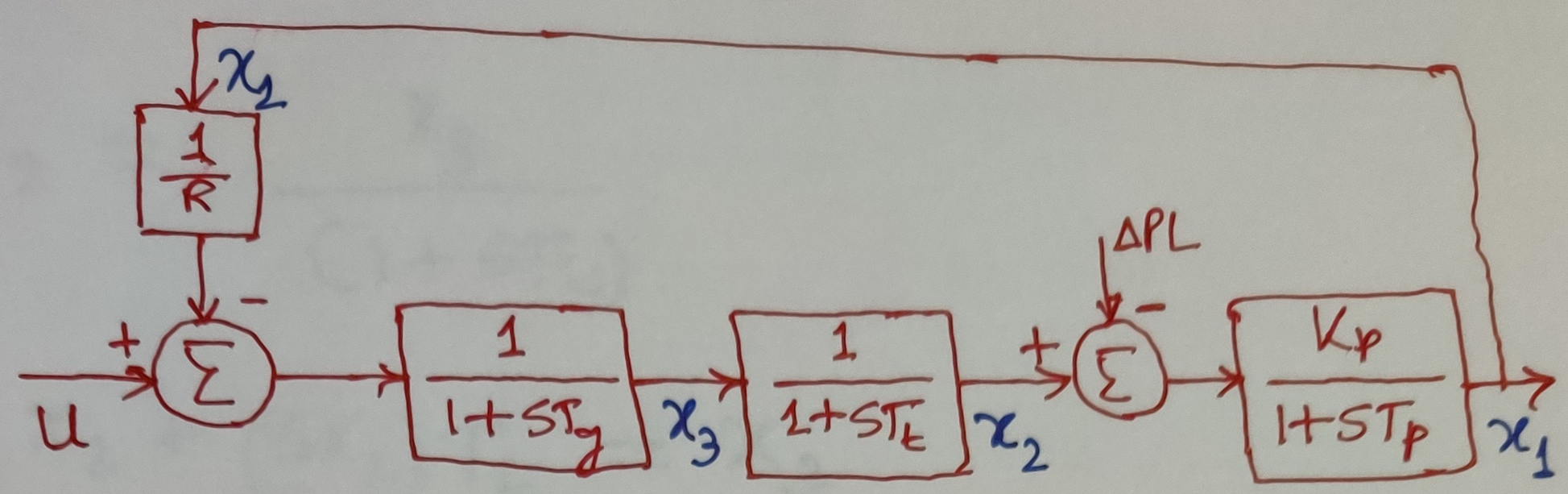


$$x_1 = (x_2 - \Delta PL) \cdot \frac{K_p}{(1+sT_p)}$$

$$\therefore (1+sT_p)x_1 = K_p x_2 - K_p \Delta PL$$

$$\therefore x_1 + (s x_1) T_p = K_p x_2 - K_p \Delta PL$$

$$\therefore x_1 + \dot{x}_1 T_p = K_p x_2 - K_p \cdot \Delta PL$$

$$\therefore \dot{x}_1 T_p = -x_1 + K_p x_2 - K_p \Delta PL$$

$$\boxed{\therefore \dot{x}_1 = -\frac{1}{T_p} x_1 + \frac{K_p}{T_p} x_2 - \frac{K_p}{T_p} \cdot \Delta PL \quad \text{--- (i)}}$$

$$x_2 = \frac{x_3}{(1+sT_t)}$$

$$\therefore \; x_2 + (sx_2)T_t = x_3$$

$$\therefore \; \dot{x}_2 T_t = -x_2 + x_3$$

$$\boxed{\therefore \; \dot{x}_2 = -\frac{1}{T_t}x_2 + \frac{1}{T_t}x_3 \quad \text{--(ii)}}$$

$$x_3 = \left(u - \frac{x_1}{R}\right) \cdot \frac{1}{(1+sT_g)}$$

$$\therefore \; x_3 + (sx_3)T_g = -\frac{x_1}{R} + u$$

$$\therefore \; \dot{x}_3 T_g = -\frac{x_1}{R} - x_3 + u$$

$$\boxed{\therefore \; \dot{x}_3 = \frac{-1}{RT_g}x_1 - \frac{1}{T_g}x_3 + \frac{1}{T_g}u \quad \text{--(iii)}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 \\ 0 & -\frac{1}{T_t} & \frac{1}{T_t} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix} u + \begin{bmatrix} -\frac{K_p}{T_p} \\ 0 \\ 0 \end{bmatrix} \Delta P_L \quad \text{--- (iv)}$$

$$\therefore \dot{X} = AX + BU + \Gamma p \quad \text{--- (v)}$$

$$X' = [x_1 \quad x_2 \quad x_3]$$

$$U = u, \quad p = \Delta P_L.$$

$$A = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 \\ 0 & -\frac{1}{T_t} & \frac{1}{T_t} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -\frac{K_p}{T_p} \\ 0 \\ 0 \end{bmatrix}$$

If we discretize Eqn (V), we can write,

$$x(k+1) = \Phi x(k) + \Psi u(k) + \gamma p(k) \quad \text{--(vi)}$$

$k \Rightarrow$ Sampling count.

$$\Phi = e^{AT}, \quad \Psi = (e^{AT} - I) A^{-1} B,$$

$$\gamma = (e^{AT} - I) A^{-1} \Gamma.$$

T = Sampling time.

I = Identity matrix.

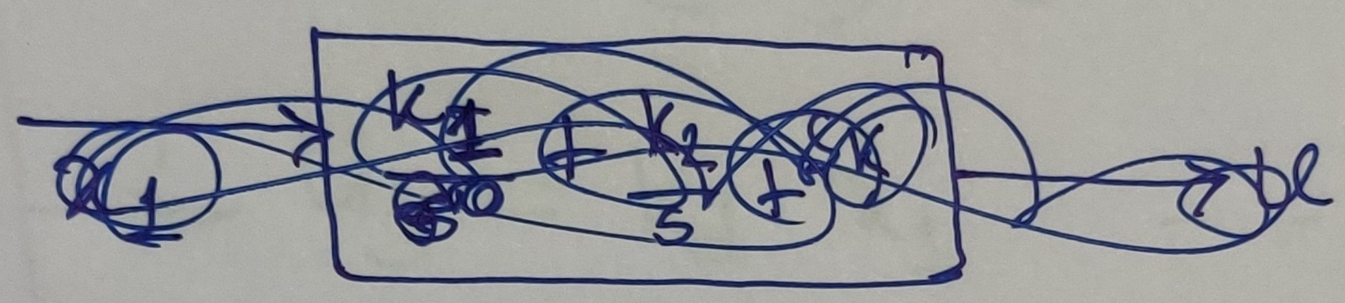
$p(k) = \Delta PL$ for all k

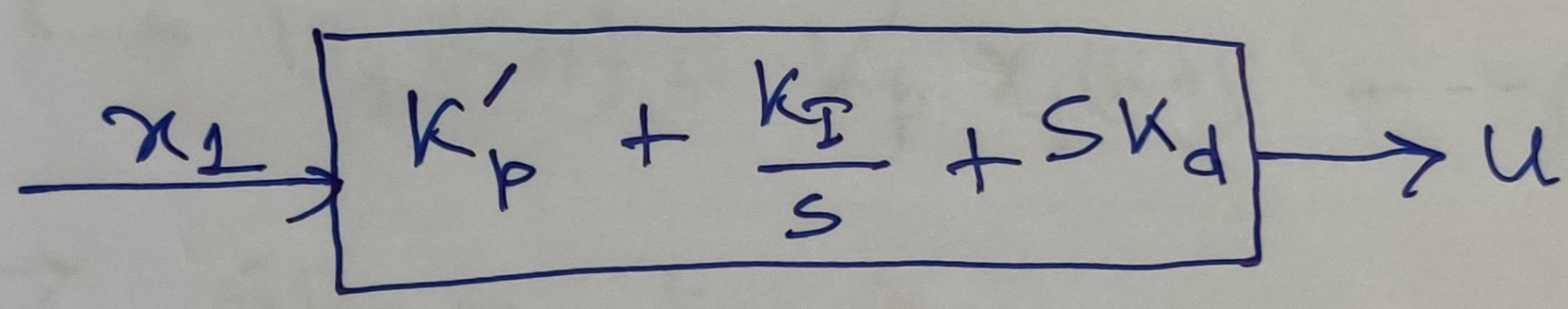
Φ = State transition matrix

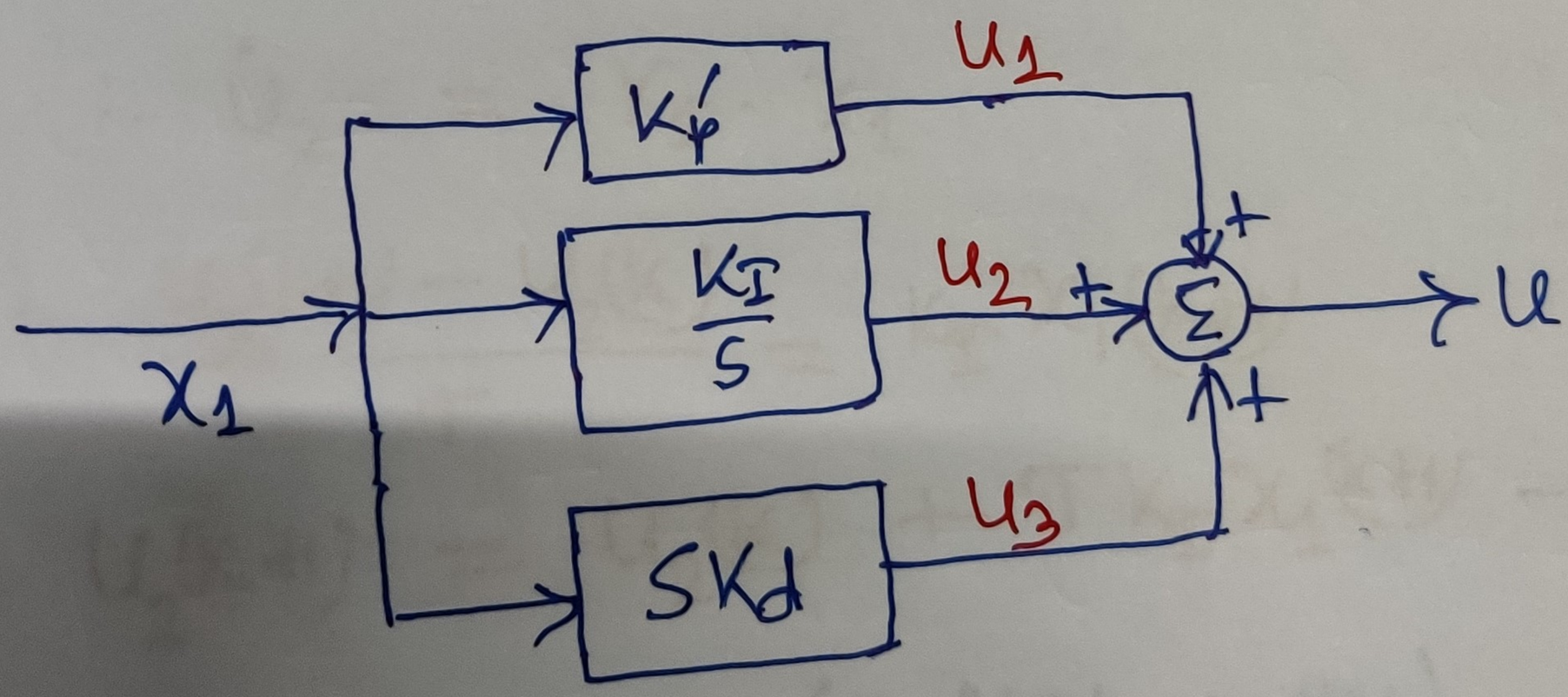
Ψ = Control transition matrix

γ = disturbance transition matrix.

Controller







$$u = u_1 + u_2 + u_3 \quad \text{---(vi)}$$

$$u_1 = K_p' \, x_1 \quad \text{----(vii)}$$

$$u_2 = \frac{K_I}{s} \cdot x_1 \quad \text{---(viii)}$$

$$u_3 = (s K_d) x_1 = (s x_1) K_d .$$

$$\dot{u}_3 = K_d \cdot \dot{x}_1 \quad \text{---(ix)}$$

$$u_1(k+1) = K_p' \, x_1(k+1) \quad \text{---(x)}$$

$$s u_2 = K_I \cdot x_1$$

$$\therefore \dot{u}_2 = K_I \cdot x_1$$

$$\therefore \frac{u_2(k+1) - u_2(k)}{T} = K_I \, x_1(k+1)$$

$$\therefore u_2(k+1) = u_2(k) + T K_I \, x_1(k+1) \quad \text{--(xi)}$$

$$u_3(k+1) = K_d \cdot \frac{\left(x_1(k+1) - x_1(k) \right)}{T} \quad \text{--(xii)}$$

$$\therefore\ U(k+1) = u_1(k+1) + u_2(k+1) + u_3(k+1) \quad \text{--(xiii)}$$

$K_p' =$ proportional gain

$K_I =$ integral gain

$K_d =$ derivative gain

OBJECTIVE FUNCTION

$$J = \int_0^t x_1^2 \, dt$$ [Quadratic Objective function]

$$\therefore\ \dot{J} = x_1^2$$

$$\therefore\ \frac{J(k+1) - J(k)}{T} = x_1^2(k+1)$$

$$\therefore\ J(k+1) = J(k) + T \cdot x_1(k+1) \cdot x_1(k+1).$$

$$\underset{\text{current value}}{J} \rightarrow \underset{\text{previous value}}{J} + T \cdot \underbrace{x_1 \cdot x_1}_{\text{current value}}$$

$A \Rightarrow 3 \times 3$ matrix

See page — 41

$$A(1,1) = \frac{-1}{T_p}$$

$$A(1,2) = \frac{K_p}{T_p}$$

$$A(1,3) = 0.$$

$$A(2,1) = 0$$

$$A(2,2) = \frac{-1}{T_t}$$

$$A(2,3) = \frac{1}{T_t}$$

$$A(3,1) = \frac{-1}{RT_g}$$

$$A(3,2) = 0$$

$$A(3,3) = \frac{-1}{T_g}$$

Parameters

$R = 2.40$

$T_p = 20$

$K_p = 120$

$T_t = 0.30$

$T_g = 0.08$

$B(1,1) = 0, \quad B(2,1) = 0, \quad B(3,1) = \frac{1}{T_g}$

$\Gamma(1,1) = \frac{-K_p}{T_p}, \quad \Gamma(2,1) = 0, \quad \Gamma(3,1) = 0.$

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$k \Rightarrow$ Sampling count.

T = Sampling time,

$t(k) = k.T$

Read all the data.

Set $u_2 = 0.0$ (initial values)

~~$x(i) = 0.0$~~

$x_1 = 0,\ x_2 = 0,\ x_3 = 0.$

$d = 0.0,\ J = 0.0$

~~for k = 1, 4000~~

~~k~~ Compute ϕ, ψ and Γ

for $k = 1, 4000$

$t = k*T$

$X = \phi x + \psi u + \Gamma p.$

$u_1 = k_p \cdot x_1$

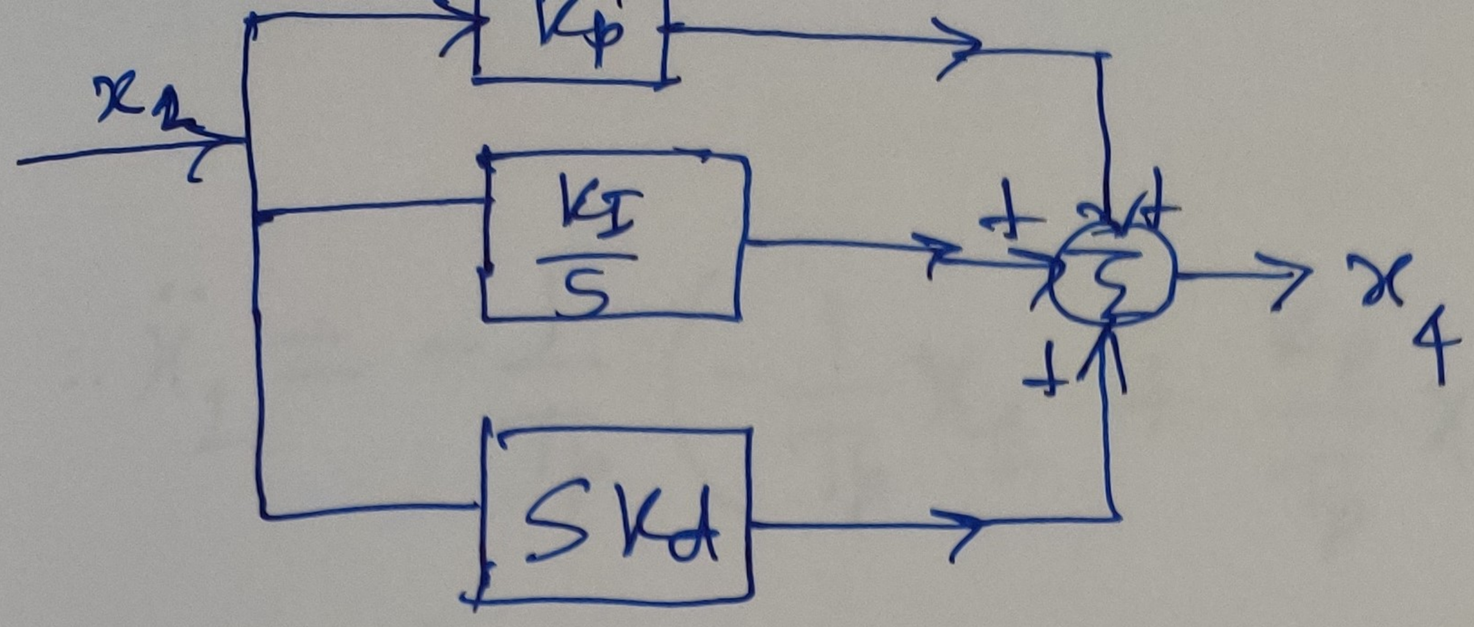
$u_2 = u_2 + T.k_I x_1.$

$u_3 = k_d \dfrac{(x_1 - d)}{T}$

$d = x_1$

$J = J + T.*x_1*x_1$

end.



From Page-43

$$x_4 = \left(K_p' + \frac{K_I}{s} + sK_d\right) x_1$$

$$\therefore sx_4 = K_p'(sx_1) + K_I x_1 + K_d(s^2 x_1)$$

$$\boxed{\therefore \dot{x}_4 = K_p' \dot{x}_1 + K_I x_1 + K_d \ddot{x}_1}$$

From Page-39, (Eqn (i)).

$$\Rightarrow \dot{x}_1 = -\frac{1}{T_p} x_1 + \frac{K_p}{T_p} x_2 - \frac{K_p}{T_p} \Delta P_L.$$

$$\Rightarrow \boxed{\therefore \ddot{x}_1 = -\frac{1}{T_p} \dot{x}_1 + \frac{K_p}{T_p} \dot{x}_2}$$

From Page-40, [Eqn (ii)]

$$\Rightarrow \dot{x}_2 = -\frac{1}{T_t} x_2 + \frac{1}{T_t} x_3 \text{ ------}$$

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$$\therefore \ddot{x}_1 = -\frac{1}{T_p}\left(-\frac{1}{T_p}x_1 + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_L\right)$$

$$+ \frac{K_p}{T_p}\left(-\frac{1}{T_t}x_2 + \frac{1}{T_t}x_3\right)$$

$$\boxed{\therefore \ddot{x}_1 = \frac{1}{T_p^2}x_1 - \frac{K_p}{T_p^2}x_2 + \frac{K_p}{T_p^2}\Delta P_L - \frac{K_p}{T_p T_t}x_2 + \frac{K_p}{T_p T_t}x_3}$$

$$\therefore \dot{x}_4 = K_p'\left(-\frac{1}{T_p}x_1 + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_L\right)$$

$$+ K_I x_2$$

$$+ K_d\left(\frac{1}{T_p^2}x_1 - \frac{K_p}{T_p^2}x_2 + \frac{K_p}{T_p^2}\Delta P_L\right.$$

$$\left. - \frac{K_p}{T_p T_t}x_2 + \frac{K_p}{T_p T_t}x_3\right)$$

$$\boxed{\begin{aligned} \dot{x}_4 = &\left(\frac{-K_p'}{T_p} + K_I + \frac{K_d}{T_p^2}\right)x_1 \\ &+ \left(\frac{K_p' K_p}{T_p} - \frac{K_d K_p}{T_p^2} - \frac{K_d K_p}{T_p T_t}\right)x_2 \\ &+ \frac{K_d K_p}{T_p T_t}x_3 + \left(-\frac{K_p' K_p}{T_p} + \frac{K_p}{T_p^2}\right)\Delta P_L \end{aligned}}$$

Dom - Page-39

$$u = x_4.$$

$$x_3 = \left(x_4 - \frac{x_1}{R}\right)\frac{1}{(1+sT_g)}$$

$$\therefore x_3 + \dot{x}_3 T_g = -\frac{x_1}{R} + x_4$$

$$\therefore \dot{x}_3 T_g = -\frac{x_1}{R} - x_3 + x_4$$

$$\therefore \boxed{\dot{x}_3 = -\frac{x_1}{RT_g} - \frac{1}{T_g}x_3 + \frac{1}{T_g}x_4}$$

$$A \Rightarrow \underline{\underline{4 \times 4}}$$

$$A(1,4) = 0.$$

$$A(2,4) = 0$$

$$A(3,4) = \frac{1}{T_g}$$

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$$A(4,1) = \left(-\frac{K_p'}{T_p} + K_I + \frac{K_d}{T_p^2} \right)$$

$$A(4,2) = \left(\frac{K_p' K_p}{T_p} - \frac{K_d K_p}{T_p^2} - \frac{K_d K_p}{T_p T_t} \right)$$

$$A(4,3) = \frac{K_d K_p}{T_p T_t}$$

$$A(4,4) = 0$$