Assignment-3

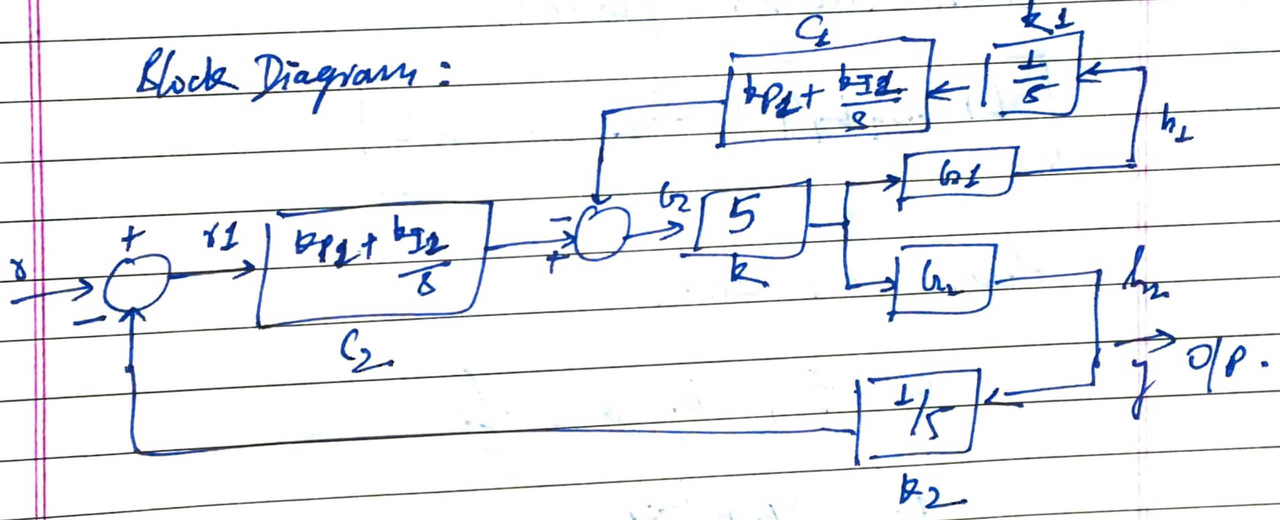
Pratyush Jaiswal
18EE35014

Q1. from Assignment 2,

$$G_1 = \frac{0.0625s + 0.0107}{s^2 + 0.258s + 0.0074}$$

$$G_2 = \frac{0.0053}{s^2 + 0.258s + 0.0074}$$

Block Diagram:



Here, $C_2 r_1 - k G_1 k_1 C_1 G = G$.

$$\boxed{G = \frac{C_2 r_1}{1 + k G_1 k_1 C_1}}$$

$h_2 = y = k G G_2$

$r - y k_2 = r_1$

$r - y k_2 = (1 + k G_1 k_1 C_1) \frac{G}{C_2}$

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$$\Rightarrow r - k_2 y = \left(1 + kG_1 k_1 C_1\right)\frac{y}{kG_2C_2}$$

$$\Rightarrow r = \left(k_2 + \frac{1 + kG_1k_1C_1}{kG_2C_2}\right) y$$

$$\Rightarrow \boxed{\frac{y}{r} = \frac{kG_2C_2}{1 + kG_1k_1C_1 + kG_2C_2k_2}}$$

Characteristics Equation,

$$\boxed{1 + kG_1k_1C_1 + kG_2C_2k_2 = 0}$$

Here, we have,

$$k = 5, \quad k_1 = k_2 = \frac{1}{5}$$

$$C_1 = k_{P1} + \frac{k_{I1}}{s}$$

$$C_2 = k_{P2} + \frac{k_{I2}}{s}$$

$$G_1 = \frac{0.0625s + 0.0107}{s^2 + 0.258s + 0.0074}$$

$$G_2 = \frac{0.0053}{s^2 + 0.258s + 0.0074}$$

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Putting these into the characteristics Equation,

$$1 + k_{c1}G_{v1}G_1 + k_{c2}G_{v2}G_2 = 0$$

$$1 + 5 \times \frac{1}{5} \times \frac{\left(k_{P1} + \frac{k_{I1}}{s}\right)(0.0625s + 0.0107)}{(s^2 + 0.258s + 0.0074)}$$

$$+ \frac{0.005375\left(k_{P2} + \frac{k_{I2}}{s}\right)}{(s^2 + 0.258s + 0.0074)} \times 5 \times \frac{1}{5} = 0$$

$$\Rightarrow s(s^2 + 0.258s + 0.0074) + (k_{P1}s + k_{I1})(0.0625s + 0.0107) + (0.005375)(k_{P2}s + k_{I2}) = 0$$

$$\Rightarrow \boxed{s^3 + \cancel{0.258}(0.258 + 0.0625k_{P1})s^2 + (0.0074 + 0.0107k_{P1} + 0.0625k_{I1} + 0.005375k_{P2})s + (0.0107k_{I1} + 0.005375k_{I2}) = 0.}$$

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~~Here, PI controllers are used, steady state error will be zero;~~

Desired Characteristics equation properties given in question :-

Peak Overshoot (P.O) $\leq 10\%$

Settling time (2% tolerance band) $\leq 20s$

Steady State Error $= 0$

Peak Overshoot (P.O) $= 10\%$

$$e^{\frac{-\rho\pi}{\sqrt{1-\rho^2}}} = 0.1 \Rightarrow \rho = 0.5911$$

T_s (Settling Time) $= 20s$

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\rho^2}\right)}{\rho T_s} = 0.3491$$

$$s^2 + 2\omega_n \xi s + \omega_n^2 = 0.$$

$$\boxed{s^2 + 0.4127s + 0.1219 = 0}$$

$$\Rightarrow \boxed{s = -0.206 \pm j\,0.2816}$$

Third pole

$p \approx 10 \times$ Real part of first pole

By pole placement of third pole,
I am considering it as 5 times the real part of dominant pole,

$$\& \; P_3 = -0.206 \times 5$$
$$= -1.030$$
$$\simeq -1$$

The characteristics Equation can be written as.

$$(s+1)(s^2 + 0.4127s + 0.122) = 0$$

$$\Rightarrow \boxed{s^3 + 1.4127s^2 + \cancel{49} \; 0.5347s + 0.122 = 0}$$

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After comparing,

$$k_{P1} = \frac{1.4127 - 0.258}{0.0625} = 18.4752$$

Taking $k_{I1} = \cancel{10}\ 5$

$$0.0107 \times \cancel{10}\ 5 + 0.005375\, k_{I2} = 0.122$$

$$\Rightarrow \quad k_{I2} = \cancel{2.79}\ 12.744$$

$$k_{P2} = \frac{0.5347 - 0.0074 - 0.0107 \times 18.4752 - 0.0625 \times \cancel{10}\ 5}{0.005375}$$

$$= 3.184.$$

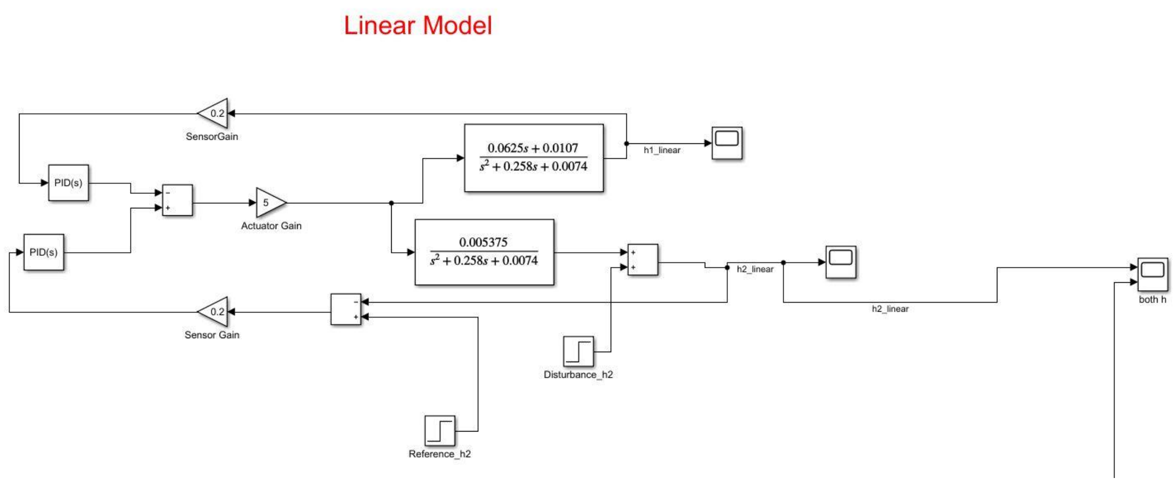
$\therefore$ $$\boxed{\begin{array}{ll} k_{P1} = 18.4752 & k_{I1} = 5 \\ k_{P2} = 3.184 & k_{I2} = 12.744 \end{array}}$$

Also roots are:

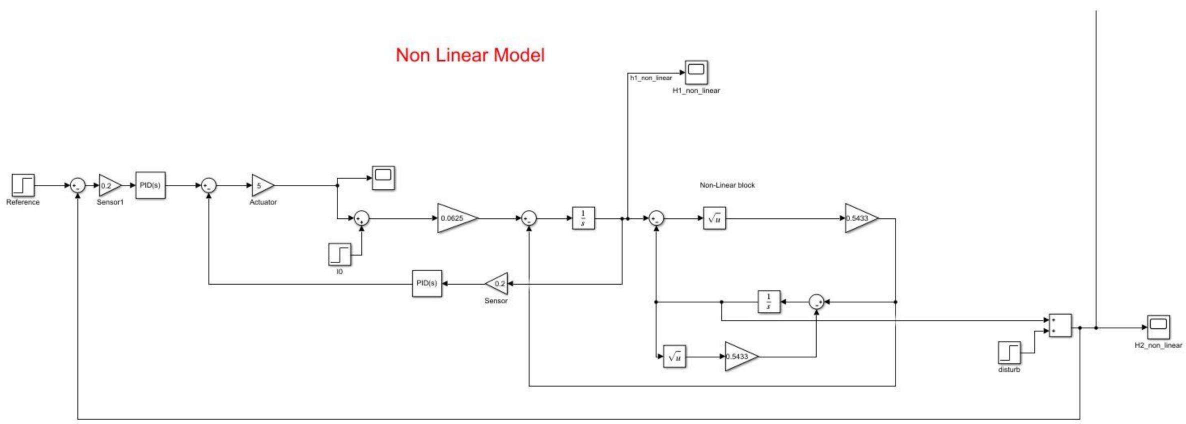
$$\boxed{\begin{array}{l} P_{1,2} = -0.206 \pm j0.281 \\ P_3 = -1 \end{array}}$$

Question 1 Circuits and Waveforms

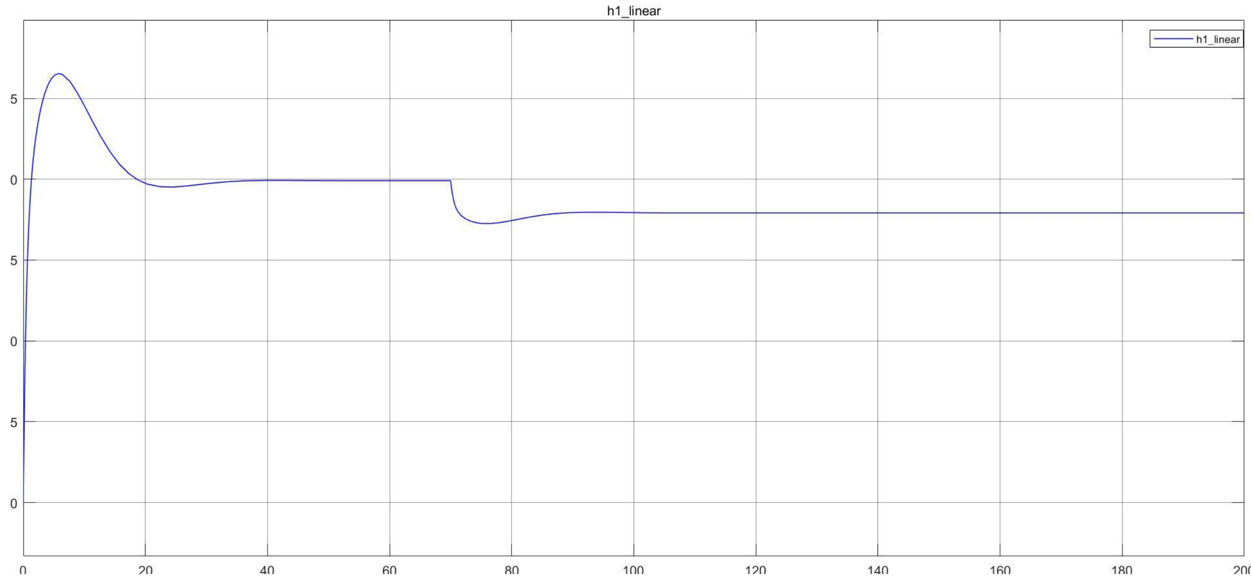
Linear Circuit



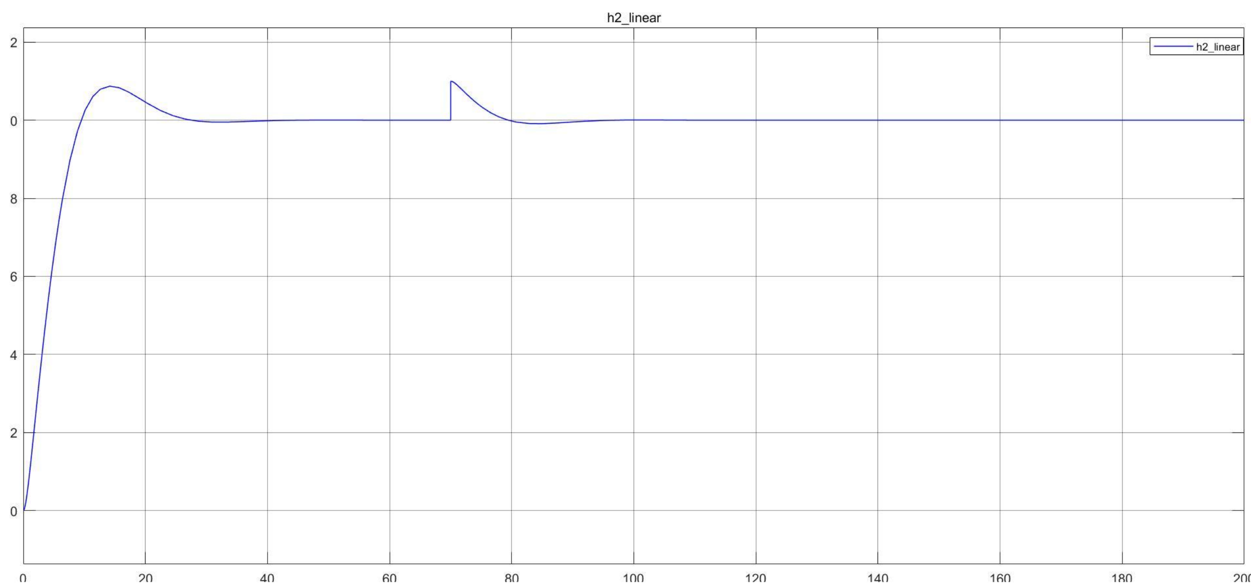
Non-Linear Circuit



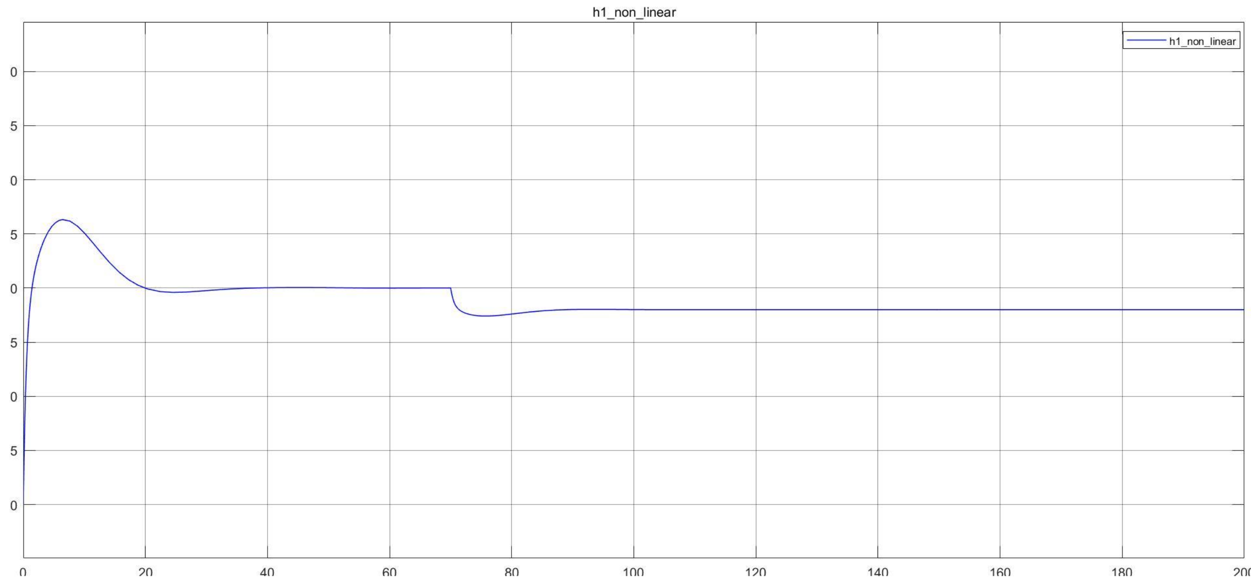
H1 Linear



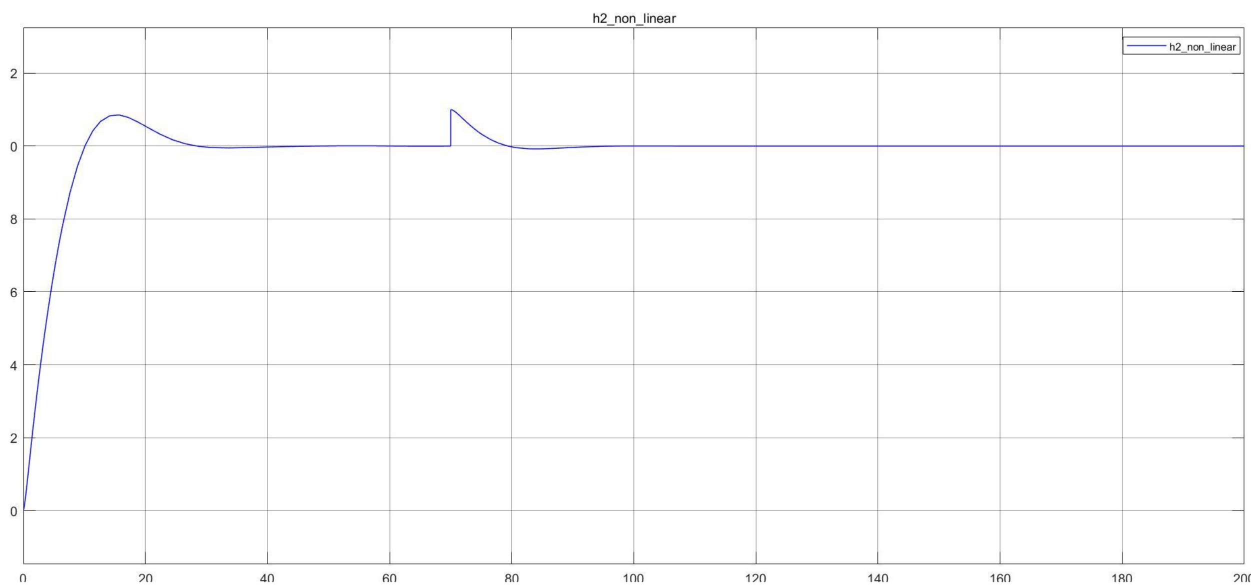
H2 Linear



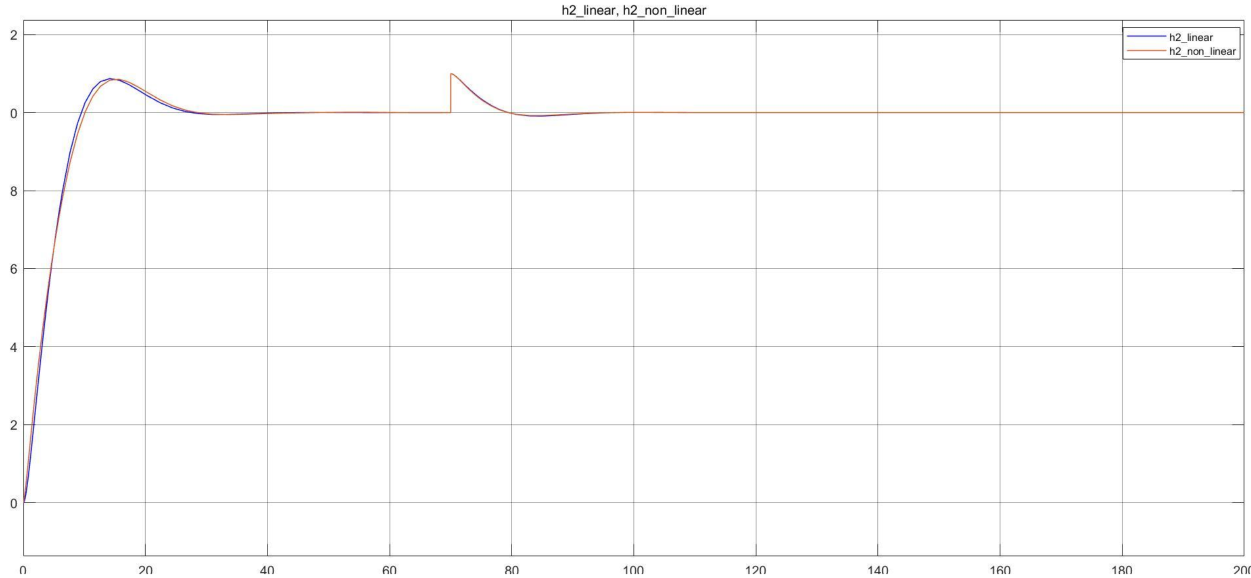
H1 Non-Linear



H2 Non-Linear



H2 Linear and Non-Linear Comparision



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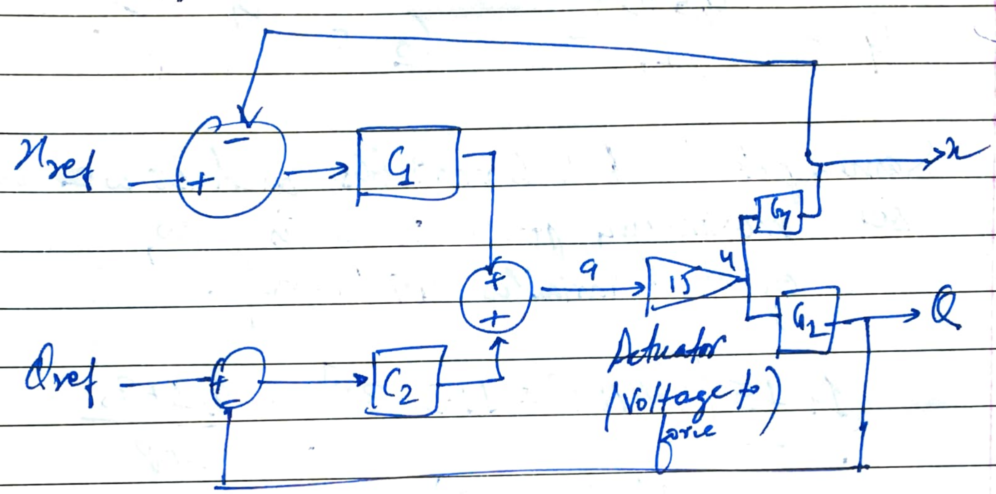
Q2.

first order Controllers



$$G_1 = \frac{X(s)}{U(s)} = \frac{0.466\,(s^2 - 27.22)}{s^4 - 30.17s^2}$$

$$G_2 = \frac{Q(s)}{U(s)} = \frac{-1.1574}{s^2 - 30.17129}$$

$$u = 15a$$

$$= 15\left[C_1 x_{ref} + C_2 Q_{ref} - C_1 x - C_2 Q\right]$$

$$u = 15\left[C_1 x_{ref} + C_2 Q_{ref}\right] - 15\left[C_1 G_1 u + C_2 G_2 u\right]$$

$$u = \frac{15\left[C_1 x_{ref} + C_2 Q_{ref}\right]}{1 + 15C_1G_1 + 15C_2G_2}$$

Characteristics Equation $= 1 + 15C_1G_1 + 15C_2G_2 = 0$

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Let

$$G_1 = \frac{a_1 s + a_0}{s+p} \qquad G_2 = \frac{b_1 s + b_0}{s+p}$$

Same pole (Total 6 variables and ~~so~~ as we will see maximum power of s is 6 so, 6 eqn & 6 variables will work].

$$1 + \frac{15\left[0.416 s^2 - 11.3235\right]}{s^2(s^2 - 30.17)} \times \frac{a_1 s + a_0}{s+p}$$

$$+ \frac{15 \times (\cancel{15} - 1.1574)}{(s^2 - 30.17)} \times \frac{b_1 s + b_0}{s+p} = 0.$$

$$\Rightarrow s^2(s^2 - 30.17)(s+p) + 15\left[0.416 s^2 - 11.323\right]\left[a_1 s + a_0\right]$$

$$- 15(1.1574 s^2)(b_1 s + b_0) = 0$$

$$\Rightarrow s^5 + p s^4 + (-30.17 + 6.24 a_1 - 17.36 b_1) s^3$$

$$+ (-30.17 p + 6.24 a_0 - 17.36 b_0) s^2$$

$$+ (-169.84591) s + (-169.84591) = 0$$

Characteristic Equation

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$PM = 60, \quad T_S = 5 \text{ sec}$

$$\frac{2\rho}{\sqrt{-2\rho^2 + \sqrt{1+4\rho^4}}} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow 4\rho^2 = 3\left(-2\rho^2 + \sqrt{1+4\rho^4}\right)$$

$$\Rightarrow 10\rho^2 = 3\sqrt{1+4\rho^4}$$

$$\Rightarrow 100\rho^4 = 9 + 36\rho^4$$

$$\Rightarrow \rho^4 = \frac{9}{64} \Rightarrow \boxed{\rho = 0.6124}$$

$T_S = 5 \text{ sec}$

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\rho^2}\right)}{\rho T_s} = 1.35436$$

$\therefore$ 2nd Order Eq^n is

$$s^2 + 2 \times 0.6124 \times 1.35436\, s + 1.35436^2 = 0.$$

$$\Rightarrow s^2 + 1.6588 s + 1.843 = 0$$

$$s = -0.8294 \pm \sqrt{1.0707}$$

Dominant poles $P_1, P_2, P_3 \Rightarrow 10 \times 0.3274$

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Finding characteristics Equation,

$$(s^2 + 1.6588s + 1.8344)(s+P_1)(s+P_2)(s+P_3) = 0$$

$$s^5 + (1.6588 + (P_1+P_2+P_3))s^4 + (1.8344 + 1.6588(P_1+P_2+P_3) + (P_1P_2 + P_2P_3 + P_1P_3))s^3 +$$

$$(1.8344(P_1+P_2+P_3) + 1.6588(P_1P_2 + P_2P_3 + P_3P_1) + (P_1P_2P_3))s^2 + (1.8344(P_1P_2 + P_2P_3 + P_3P_1) + 1.6588P_1P_2P_3)s$$

$$+ 1.8344P_1P_2P_3 = 0.$$

$\Rightarrow$

$$s^5 + k_4s^4 + k_3s^3 + k_2s^2 + k_1s + k_0 = 0.$$

Now for $|GM| \geq 6dB$ & $|PM| \geq 40°$, on different iterations,

$P_1 = 30$, $P_2 = 40$, $P_3 = 50$ has $GM = 19.4dB$.

& $PM = 56°$ which is good enough here.

So, the final concluded characteristics $Eq^n \Rightarrow$

$$(s^2 + 1.6588s + 1.8343)(s+30)(s+40)(s+50) = 0$$

Pratyush Talwar
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Comparing coefficients,

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 6.2491 & 0 & -17.36 & 0 & 0 \\ 0 & 6.2491 & 0 & -17.36 & -30.17 \\ -169.84591 & 0 & 0 & 0 & 0 \\ 0 & -169.84 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \\ b_1 \\ b_0 \\ p \end{bmatrix} = \begin{bmatrix} k_4 \\ k_3 \\ k_2 \\ k_1 \\ k_0 \end{bmatrix} - \begin{bmatrix} 0 \\ -30.17 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

from Matlab,

$a_1 = -665.7117$ $\qquad$ $b_1 = -523.38$

$a_0 = -1282$ $\qquad$ $b_0 = -4586.7$

$$C_1 = \frac{a_1 s + a_0}{s+p} \qquad C_2 = \frac{b_1 s + b_0}{s+p}.$$

<u>Non Linear Model :</u>

Moment of Inertia $I \approx 0$

$$u = (M+m)\ddot{x} + ml\frac{d^2}{dt^2}(\sin\theta)$$

$$\boxed{u = (M+m)\ddot{x} + (ml\cos\theta)\ddot{\theta} - (ml\sin\theta)(\dot{\theta})^2} \quad -(1)$$

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$$V = mg + ml\frac{d^2}{dt^2}(\cos\theta) \quad ; \quad H = m\ddot{x} + ml\frac{d^2}{dt^2}(\sin\theta);$$

$$V\sin\theta = H\cos\theta.$$

$$\Rightarrow \sin\theta\left[mg - ml\frac{d}{dt}[\sin\theta\,\dot{\theta}]\right] = \cos\theta\left[m\ddot{x} + ml\frac{d}{dt}[\cos\theta\,\dot{\theta}]\right]$$

$$\Rightarrow \sin\theta\left[g - l\sin\theta\,\ddot{\theta} - \cancel{l\cos\theta(\dot{\theta})^2}\right]$$
$$= \cos\theta\left[\ddot{x} + l\cos\theta\,\ddot{\theta} - \cancel{l\sin\theta(\dot{\theta})^2}\right]$$

$$\Rightarrow g\sin\theta = \ddot{x}\cos\theta + l\ddot{\theta}\left[\cos^2\theta + \sin^2\theta\right]$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{g\sin\theta}{l} - \frac{\ddot{x}}{l}\cos\theta}$$

$$\boxed{\ddot{x} = \frac{1}{M+m}\left[u + ml\left[-\cos\theta\,\ddot{\theta} + \sin\theta(\dot{\theta})^2\right]\right]}$$

```
clear all;
zeta=0.42;
wn=1.905;
p1=30;
p2=40;
p3=50;

z1=2*zeta*wn;
z2=wn*wn;

k4=z1+p1+p2+p3;
k3=z2+z1*(p1+p2+p3)+(p1*p2+p2*p3+p1*p3);
k2=z2*(p1+p2+p3)+z1*(p1*p2+p2*p3+p1*p3)+p1*p2*p3;
k1=z2*(p1*p2+p2*p3+p1*p3)+z1*(p1*p2*p3);
k0=z2*p1*p2*p3;
```

```
M=[k4;k3;k2;k1;k0];
A=[0 0 0 0 1;6.2491 0 -17.36 0 0;0 6.24 0 -17.36 -30.17;-169.84591 0 0 0 0;0 -169.84591 0 0 0];
B=[0;-30.17;0;0;0];
D=(A)^(-1);
TF=D*(M-B);
```

```
G1=tf([0.4167 0 -11.3426],[1 0 -30.17 0 0]);
G2=tf(-1.157,[1 0 -30.17]);
a1=TF(1)
```

```
a1 = -665.7117
```

```
a0=TF(2)
```

```
a0 = -1.2820e+03
```

```
b1=TF(3)
```

```
b1 = -523.3826
```

```
b0=TF(4)
```

```
b0 = -4.5867e+03
```

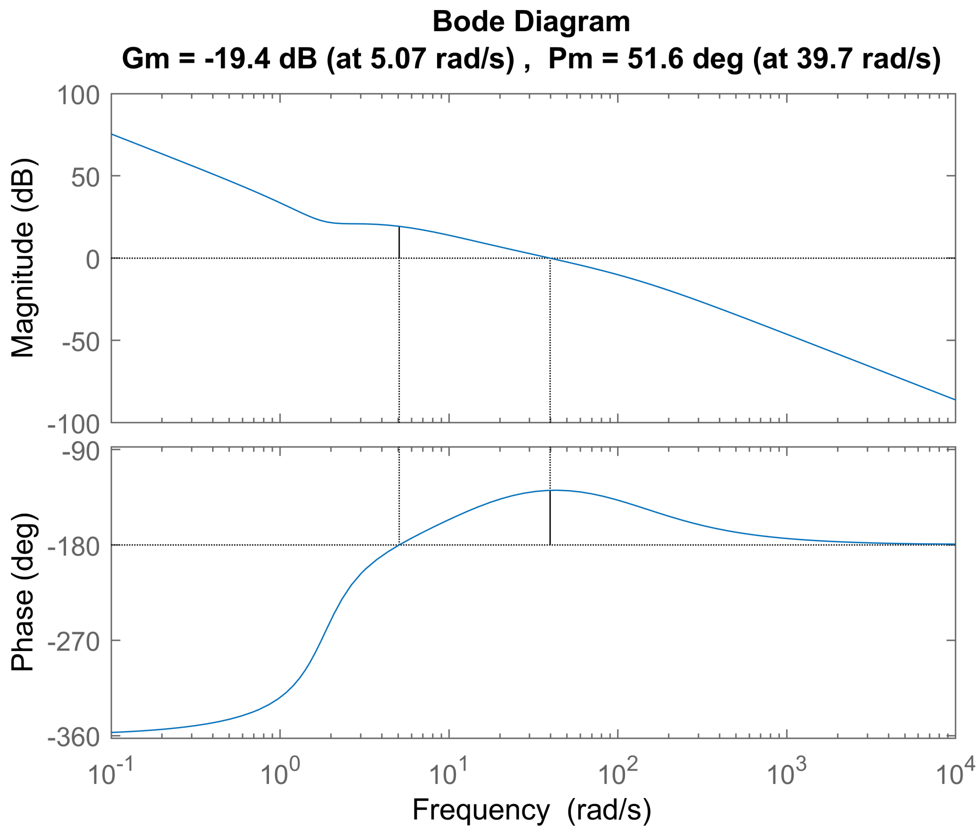
```
p=TF(5)
```

```
p = 121.6002
```

```
C1=tf([TF(1),TF(2)],[1 TF(5)]);
C2=tf([TF(3),TF(4)],[1 TF(5)]);
P=G1*C1;
Q=G2*C2;
b=minreal(P+Q);
cltf=15*(b);
allmargin(cltf)
```

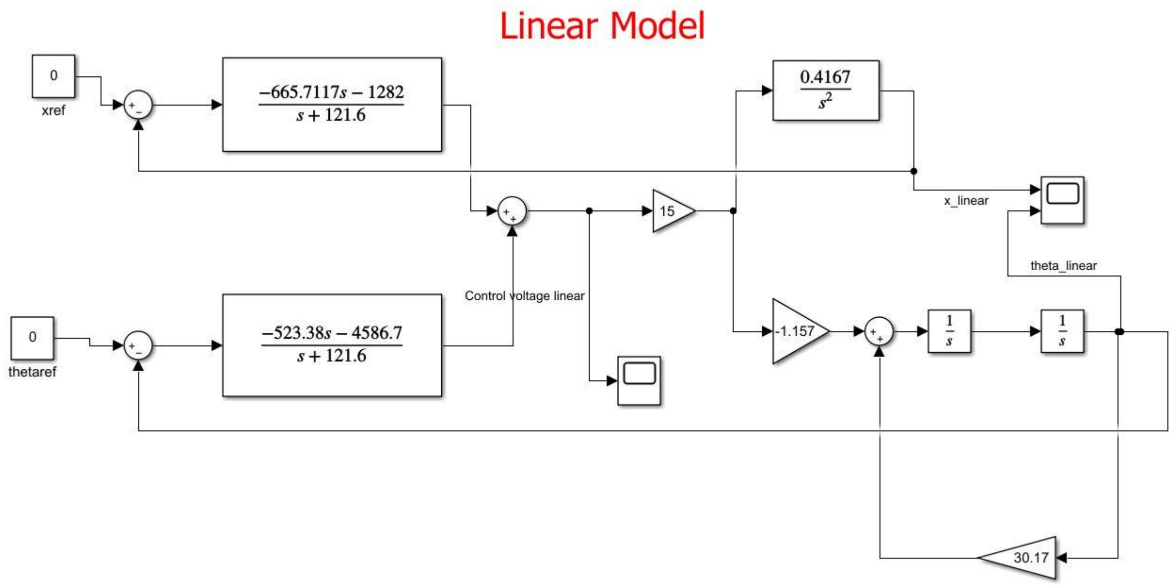
```
ans = struct with fields:
      GainMargin: [0.1077 Inf]
     GMFrequency: [5.0726 Inf]
     PhaseMargin: 51.5609
     PMFrequency: 39.6862
     DelayMargin: 0.0227
     DMFrequency: 39.6862
          Stable: 1
```

```
margin(cltf)
```

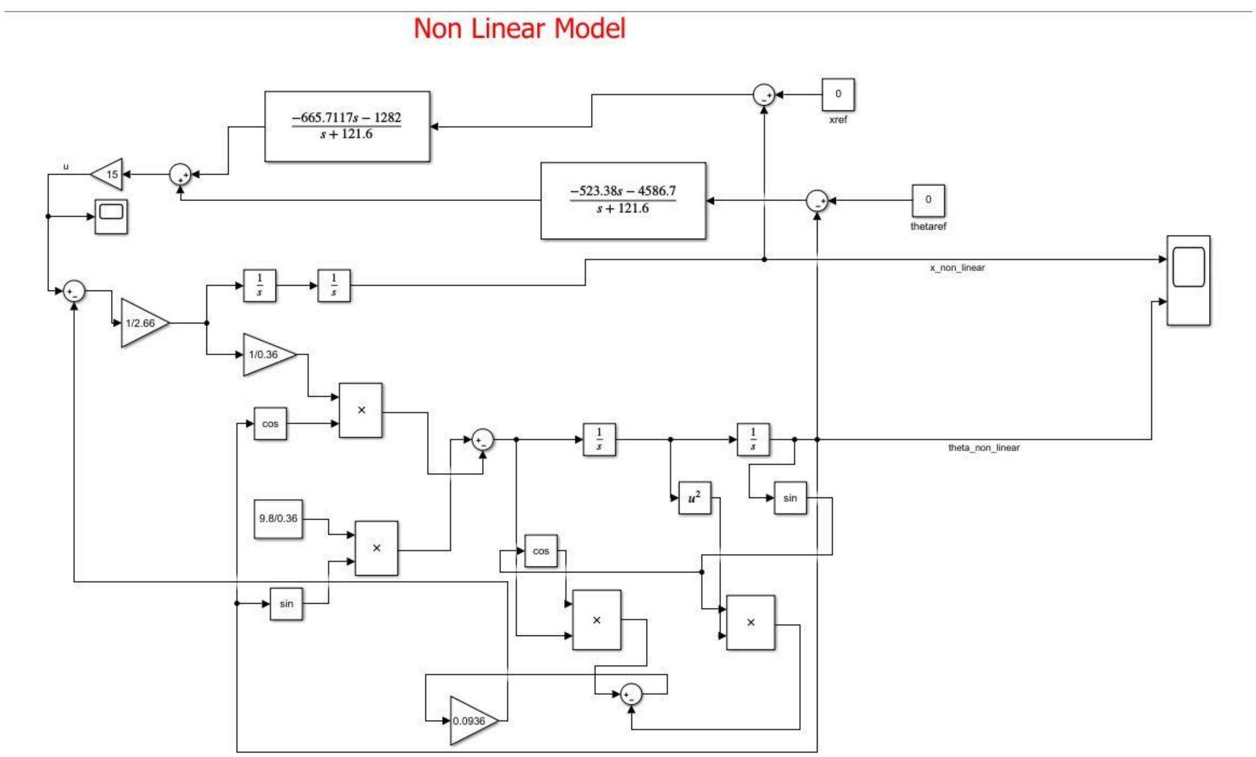


Question 2 Circuits and Waveforms

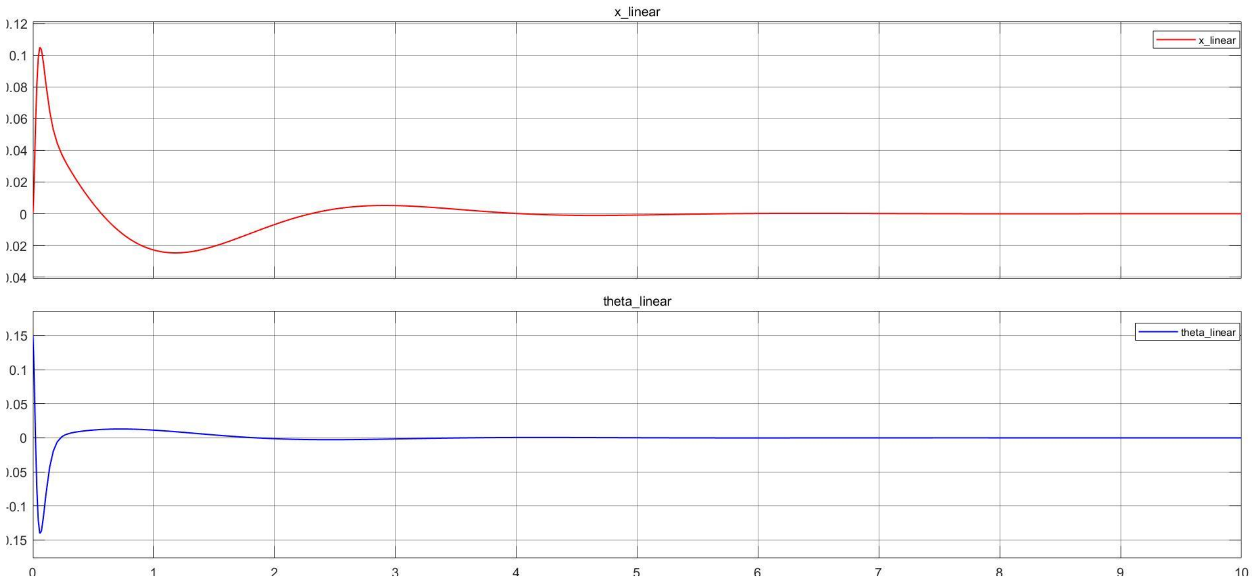
Linear Circuit



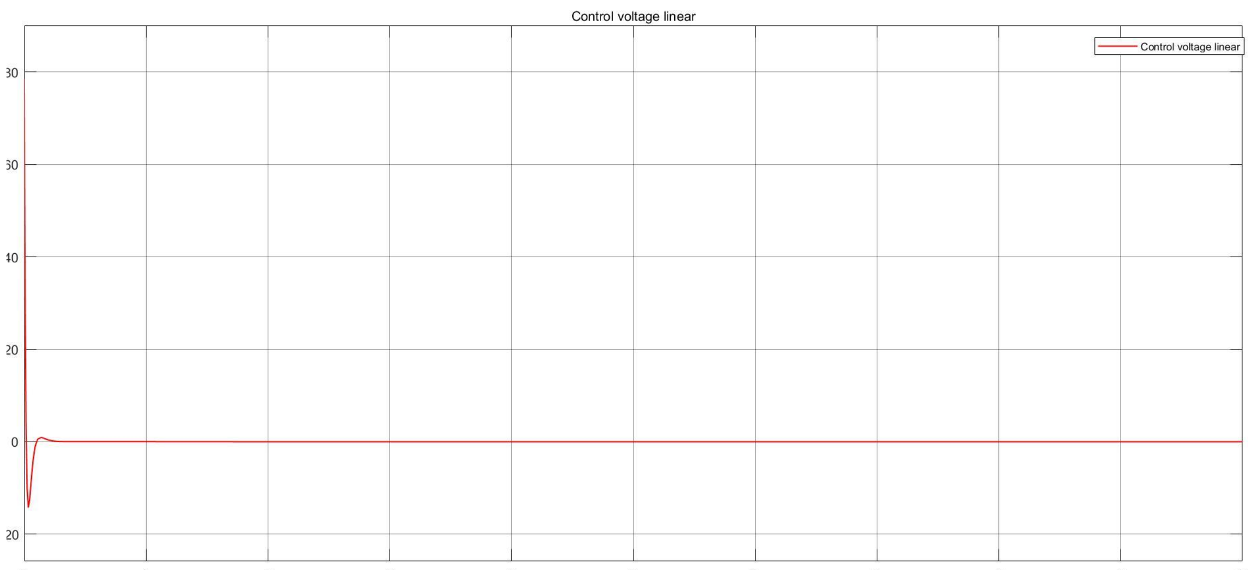
Non-Linear Circuit



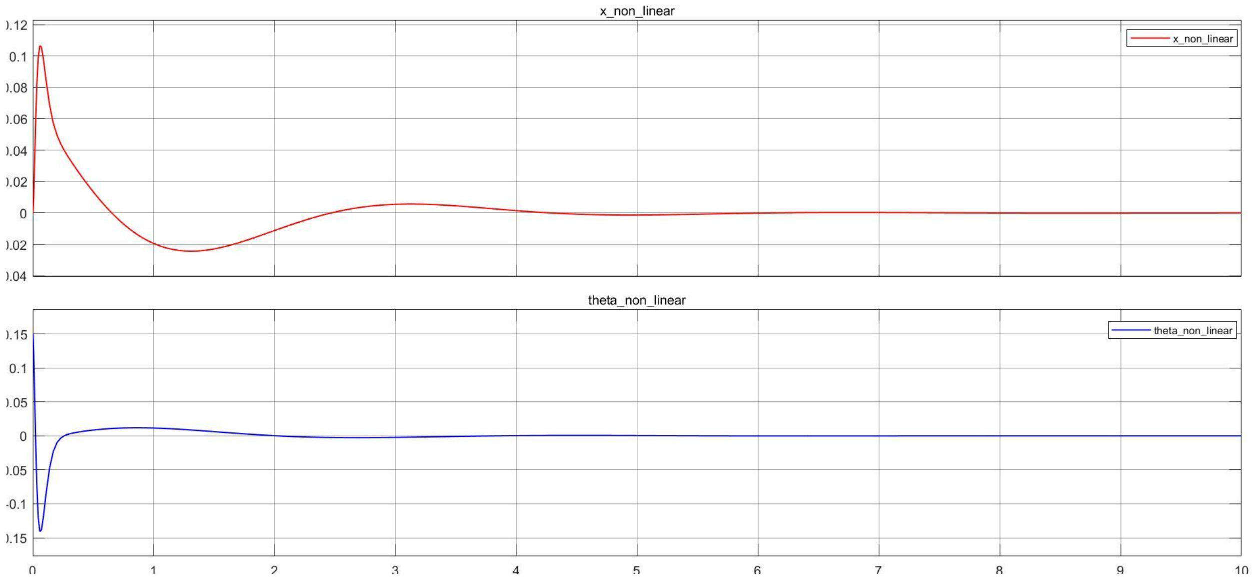
x and theta Linear



Linear Constant Voltage



x and theta Non-Linear



Non-Linear Constant Voltage