Statistical Signal Processing (EE60102)

End-semester Examination, Spring 2021-22

Time: 2 hours **Total Marks: 30**

Q1. **(a)** Let $x(n)$ be a real-valued random process generated by the system

$$x(n) = \alpha x(n-1) + w(n) \qquad n \geq 0 \qquad x(-1) = 0$$

where $w(n)$ is stationary random process with mean μ_w and $r_w(l) = \sigma_w^2 \delta(l)$.

The process $x(n)$ generated above is known as a first order auto-regressive AR(1) process and $w(n)$ is known as a white noise process.

(i) Determine the mean $\mu_x(n)$ of $x(n)$ and comment about its stationarity.

(ii) What happens if it is assumed that the system is BIBO stable such that $|\alpha| < 1$ and $n \to \infty$?

(3 marks)

(b) Let $\mathbf{u}^B(n) = [u(n-M+1), u(n-M+2), \ldots u(n)]^T$ be the observation vector of a discrete-time stochastic process re-arranged backwards.

Prove that the correlation matrix obtained by using $\mathbf{u}^B(n)$ is the transpose of that obtained by using the vector $\mathbf{u}(n) = [u(n), u(n-1), \ldots u(n-M+1)]^T$.

(2 marks)

Q2. Consider an auto-regressive process $u(n)$ of order two described by the difference equation

$$u(n) = u(n-1) - 0.5u(n-2) + v(n)$$

where $v(n)$ is white noise with zero mean and variance 0.5.

(a) Write the Yule-Walker equations for the process.

(b) Solve these equations for the auto-correlation function values $r(1)$ and $r(2)$.

(c) Find the variance of $u(n)$.

(4 marks)

Q3. **(a)** Mathematically specify the principle of orthogonality for the Wiener filtering problem. Draw the figure corresponding to the Statistician's Pythagoras' Theorem.

(2 marks)

(b) Consider a Wiener filtering problem characterized by the following auto-correlation matrix R of the tap input vector $\mathbf{u}(n)$ and the cross-correlation vector $\mathbf{p}$ between $\mathbf{u}(n)$ and the desired response $d(n)$

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \qquad \mathbf{p} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

(i) Suggest a suitable value for the step-size parameter μ that would ensure convergence of the method of steepest descent

(ii) Using the value proposed in (i) determine the recursions for computing the tap weights $w_1(n)$ and $w_2(n)$ of the tap-weight vector $\mathbf{w}(n)$ of the Wiener filter. You may assume the initial values $w_1(0) = w_2(0) = 0$.

(2 + 2 marks)

(c) What is the advantage of the normalized LMS algorithm over the basic LMS method?

(2 marks)

Q4. A real-valued random signal is observed in white noise which is uncorrelated with the signal. The observed sequence is given by

$$x(n) = s(n) + w(n)$$

where $R_s(l) = 2(0.8)^{|l|}$ and $R_w(l) = 2\delta(l)$.

(a) It is desired to estimate $s(n)$ using the present and previous two observations. Obtain the optimal Wiener filter for this task. Also, calculate the minimum mean square error.

(b) It is desired to estimate $s(n+2)$ using the same set of three observations as in **(a)**. Obtain the optimal Wiener filter for this purpose and again compute the minimum mean square error.

Comment about the difference in the minimum mean square values for **(a)** and **(b)**.

(3 + 3 marks)

Q5. Consider a wide-sense stationary process $u(n)$ whose auto-correlation function has the following values for different lags:

$r(0) = 1, r(1) = 0.8, r(2) = 0.6, r(3) = 0.4$.

(i) Use the Levinson-Durbin recursion to evaluate the reflection coefficients κ_1, κ_2 and κ_3.

(ii) Draw a three-stage lattice predictor for this process, using the values for the reflection coefficients found in part **(i)**.

(iii) Evaluate the average power of the prediction error produced at the output of each of the three stages in this lattice predictor. Hence, make a plot of prediction-error power versus prediction order. Comment on your results.

(3 + 2 + 2 marks)