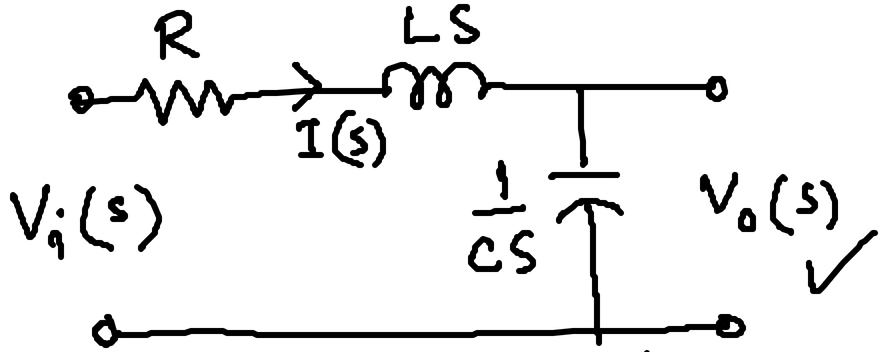
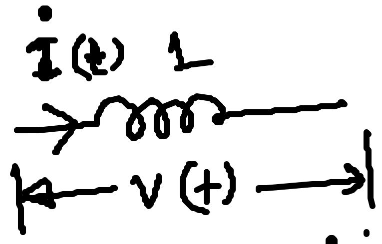
Ex ✓





$$v(t) = L\frac{di}{dt}$$

$$V(s) = Ls\,I(s)$$

(initially relaxed)

$$\frac{I(s)}{V(s)} = \frac{1}{Ls} \qquad \frac{V(s)}{I(s)} = Ls$$

proper improper

$$R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt = V_i(t)$$

Taking Laplace transform

$$R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s) = V_i(s)$$

[all initial conditions are zero]

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = V_i(s)$$

$$I(s) = \frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

output

$$V_o(s) = \frac{1}{Cs} \times I(s)$$

input

$$= \frac{1}{Cs}\,\frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

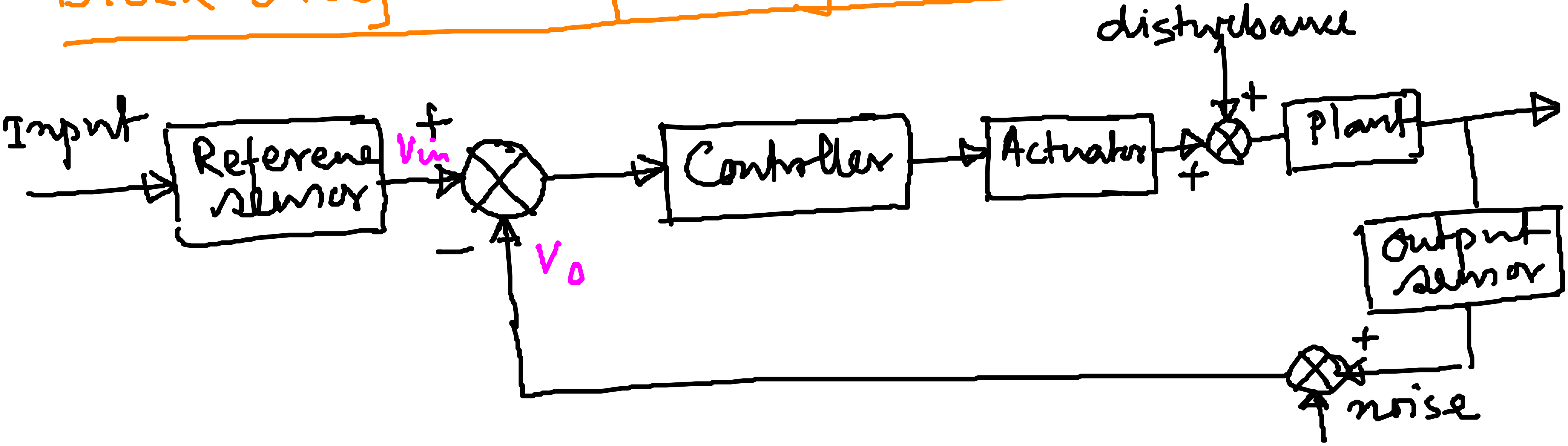
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

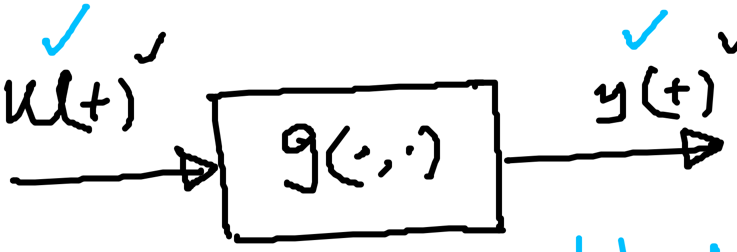
Block-diagram representation

The common elements in block-diagram of most control systems include

- comparator
- blocks representing individual TFs
  - Reference sensor
  - Output sensor
  - Actuator
  - Controller
  - Plant
- Input signals
- Output signals
- Disturbance, noise
- Feedback loops.

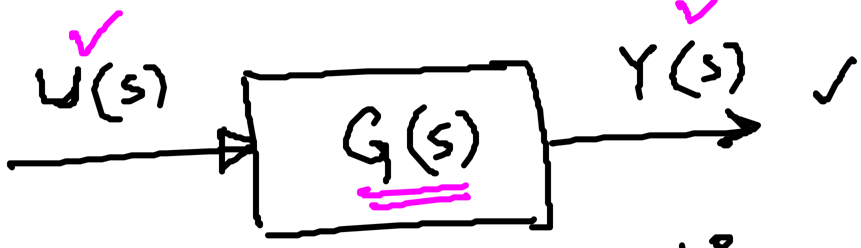
Block diagram of a general control system





Time-domain

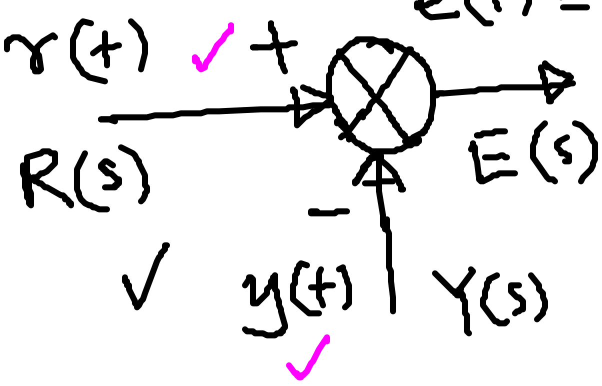
for nonlinear block



Transfer function

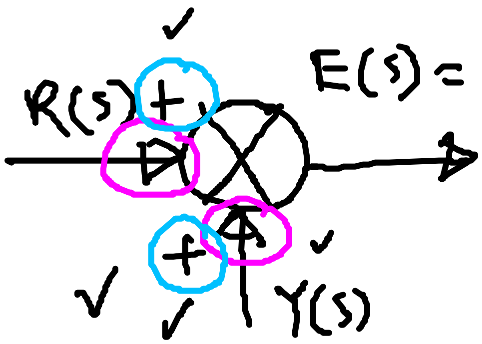
Comparator

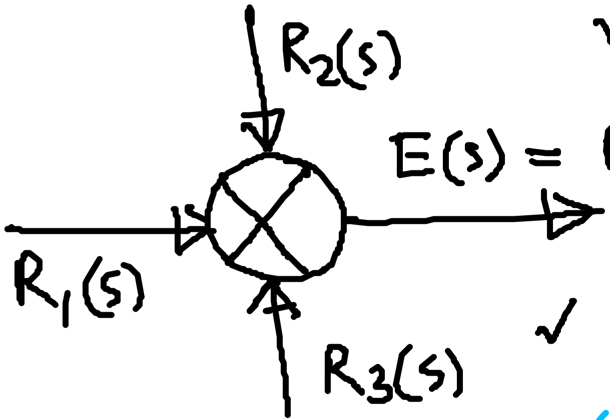
$e(t) = r(t) - y(t)$



$E(s) = R(s) - Y(s)$

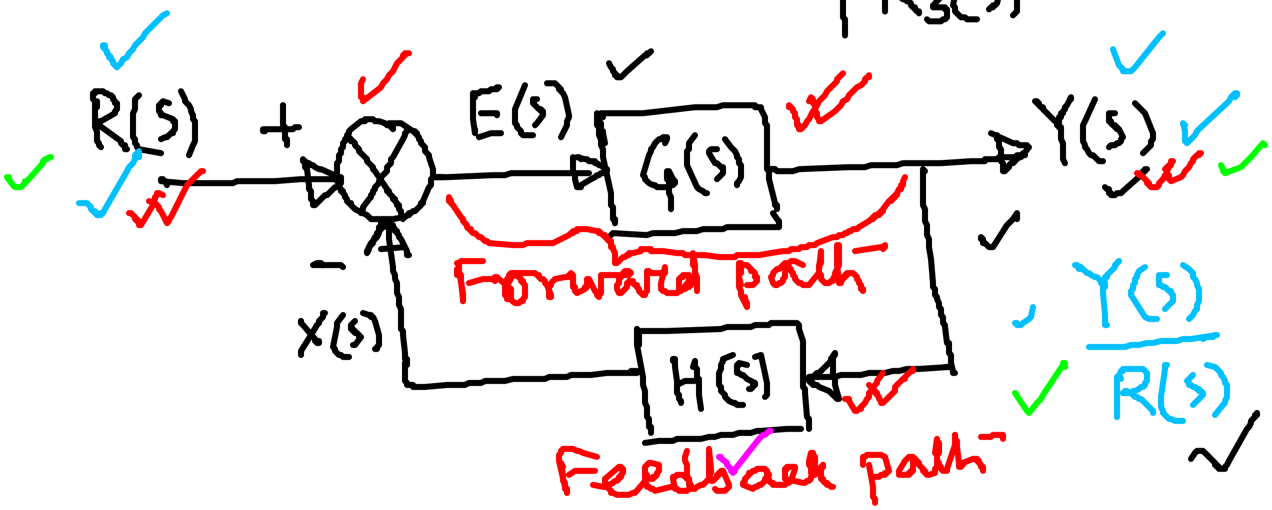
$E(s) = R(s) + Y(s)$





$E(s) = R_1(s) + R_2(s) + R_3(s)$

Ex



$\dfrac{Y(s)}{R(s)}$?

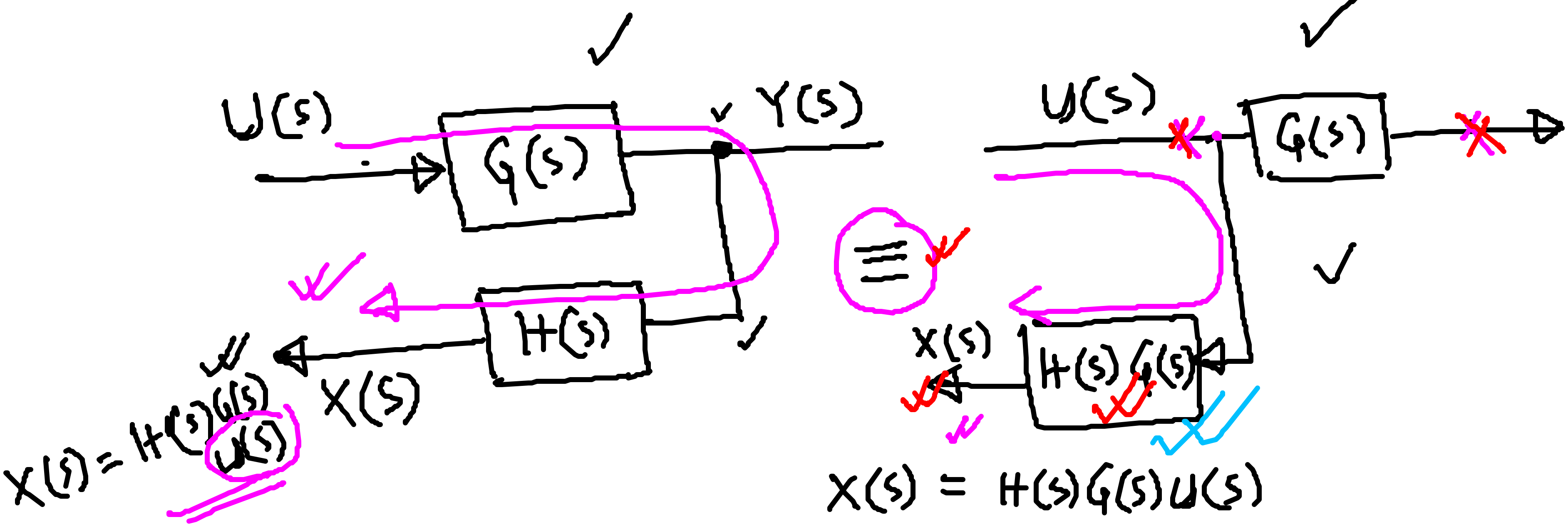
$$E(s) = R(s) - X(s) \qquad X(s) = H(s)\,Y(s)$$
$$= R(s) - H(s)\,Y(s)$$
$$= R(s) - \underline{H(s)\,G(s)\,E(s)} \qquad \underline{Y(s) = G(s)\,E(s)}$$

$$\big(1 + H(s)\,G(s)\big)\,E(s) = R(s)$$
$$\Rightarrow E(s) = \frac{1}{1 + H(s)\,G(s)}\,R(s)$$

$$Y(s) = G(s)\,\frac{1}{1 + H(s)\,G(s)}\,R(s)$$

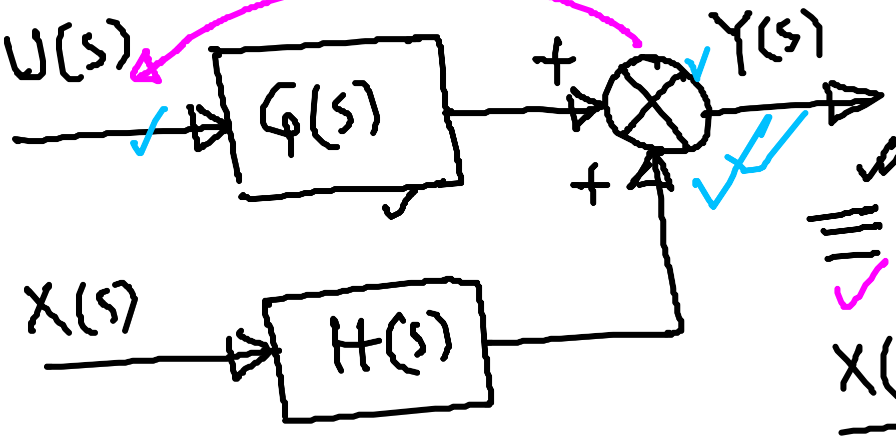
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + H(s)\,G(s)}}$$

Block-diagram reduction

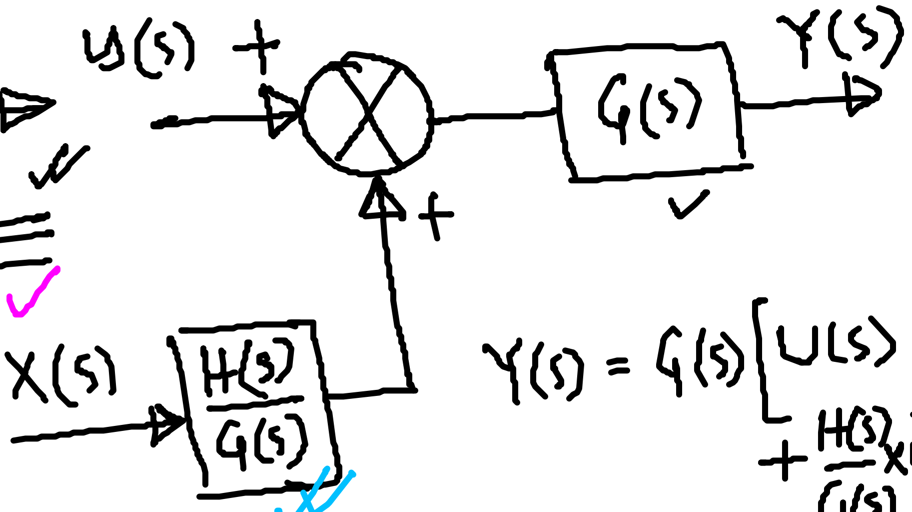


$$X(s) = H(s)\,G(s)\,U(s)$$

$$Y(s) = G(s)U(s) + H(s)X(s)$$



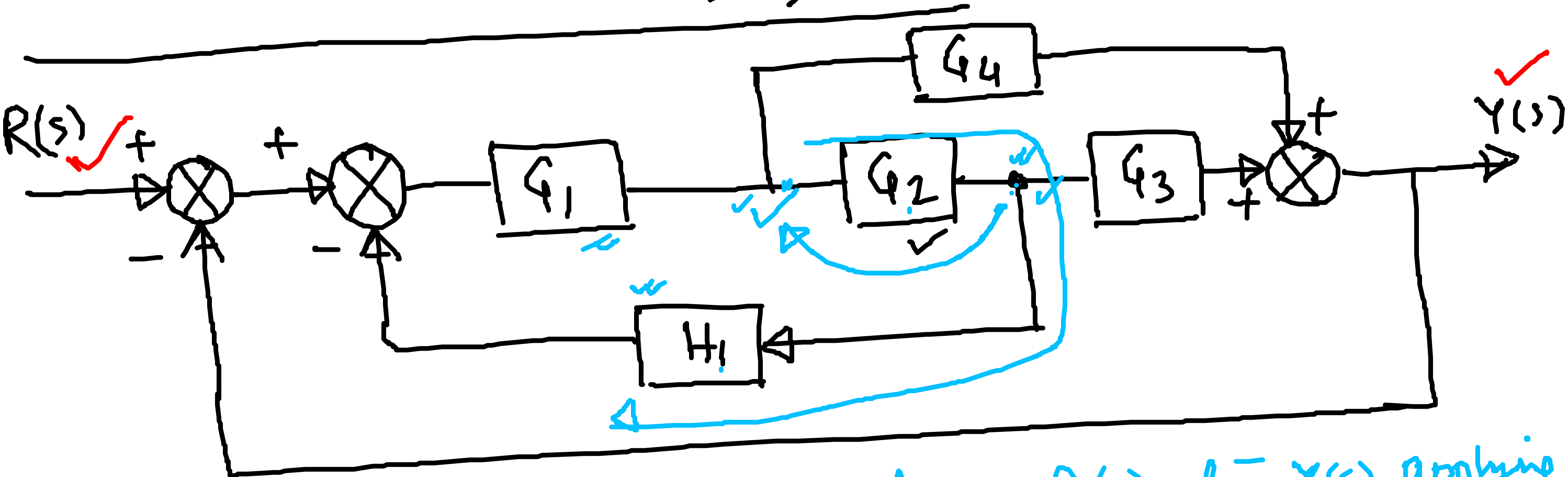
$\equiv$



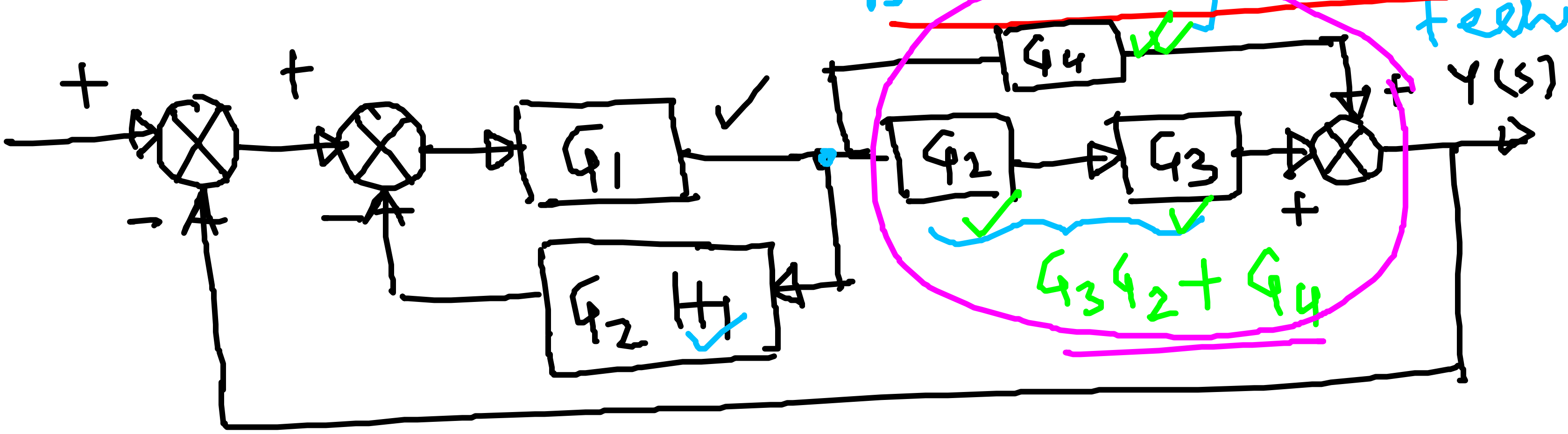
$$Y(s) = G(s)\left[U(s) + \frac{H(s)}{G(s)}X(s)\right]$$

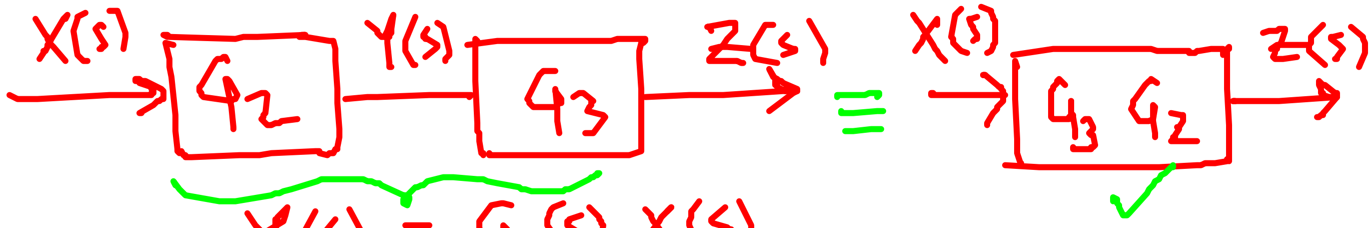
$$= G(s)U(s) + H(s)X(s)$$

Example (Block-diagram reduction technique)



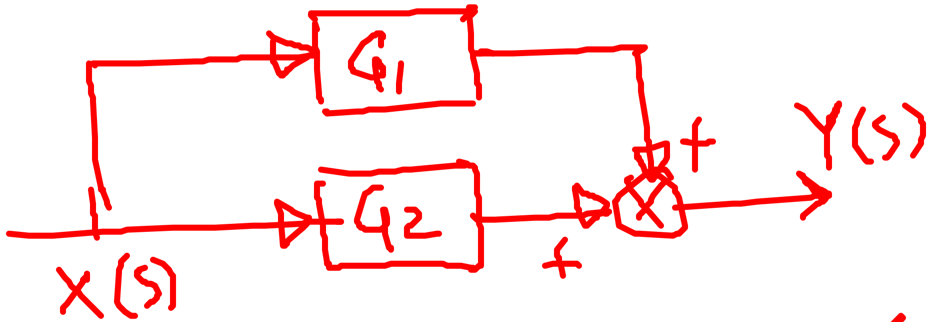
Find TF from R(s) to Y(s) applying Block-diagram reduction technique.





$$Y(s) = G_2(s)\,X(s)$$

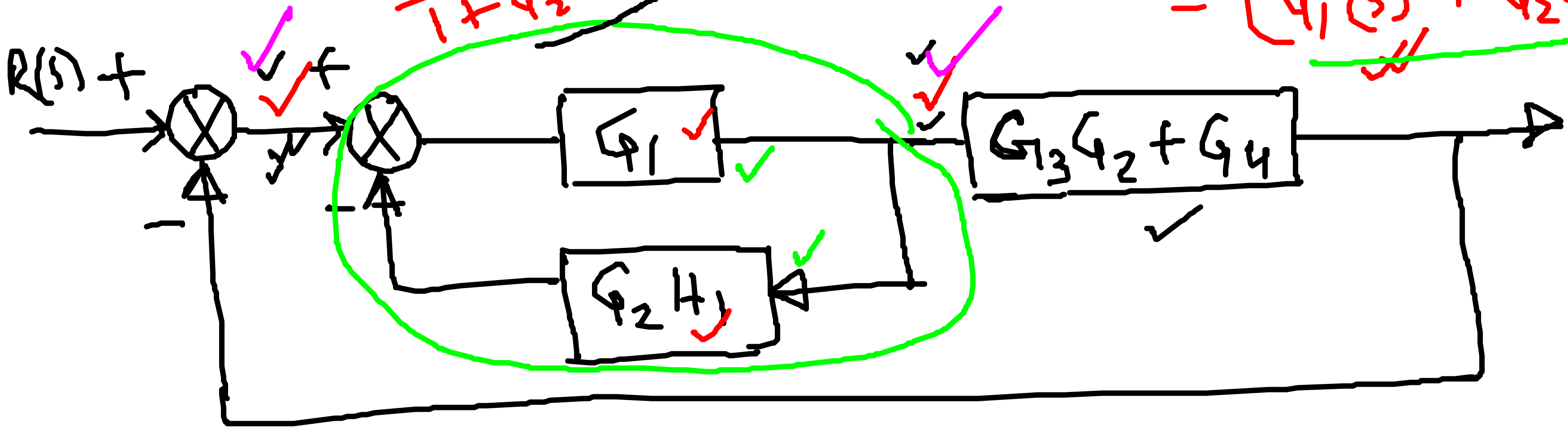
$$Z(s) = G_3(s)\,Y(s) = G_3(s)\,G_2(s)\,X(s)$$

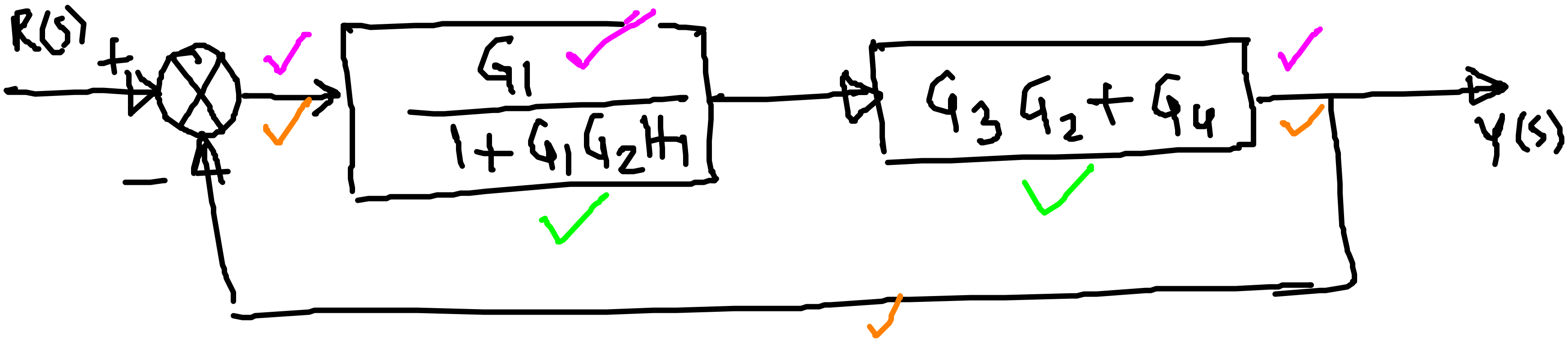


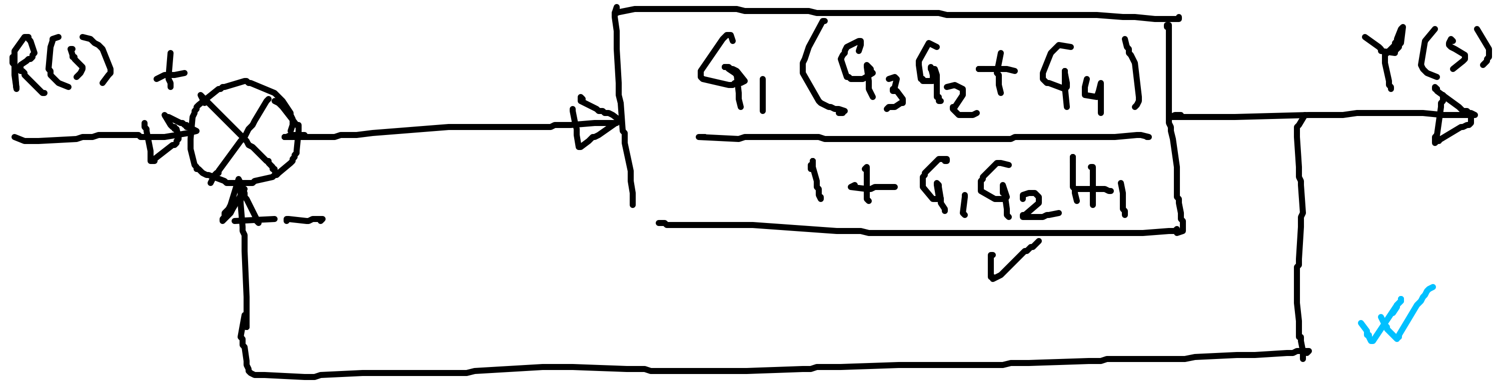
$$Y(s) = G_1(s)\,X(s) + G_2(s)\,X(s)$$

$$= \left(G_1(s) + G_2(s)\right)X(s)$$

$$\frac{G_1}{1 + G_2 H_1 G_1}$$







$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1(G_3G_2+G_4)}{1+G_1G_2H_1}}{1 + 1 \times \dfrac{G_1(G_3G_2+G_4)}{1+G_1G_2H_1}}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1(G_3G_2+G_4)}{1+G_1G_2H_1+G_1G_3G_2+G_1G_4}}$$