Blockchains & Cryptocurrencies

Crypto Background

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*Many slides from NBFMG

This lecture

Crypto background

- hash functions
- random oracle model
- digital signatures
- … and applications

Cryptographic Hash Functions

Hash function

- takes a string of arbitrary length as input
- fixed-size output (i.e., hash function "compresses" the input)
- efficiently computable

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Security properties:

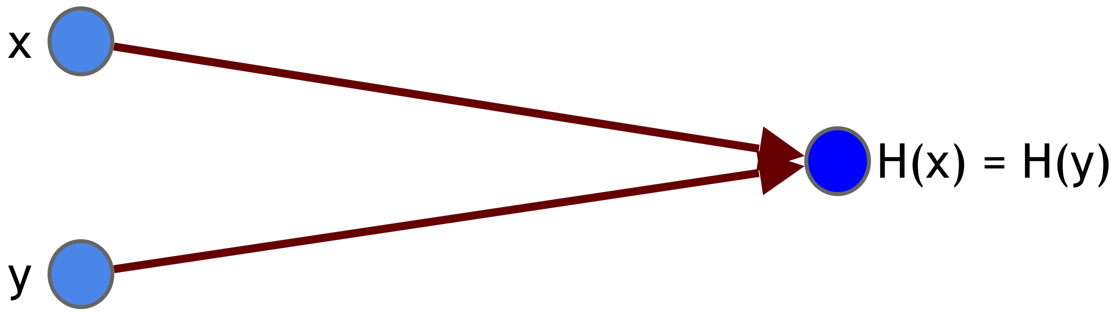
- Collision resistance
- Preimage resistance (one-way)

Property 1: Collision resistance

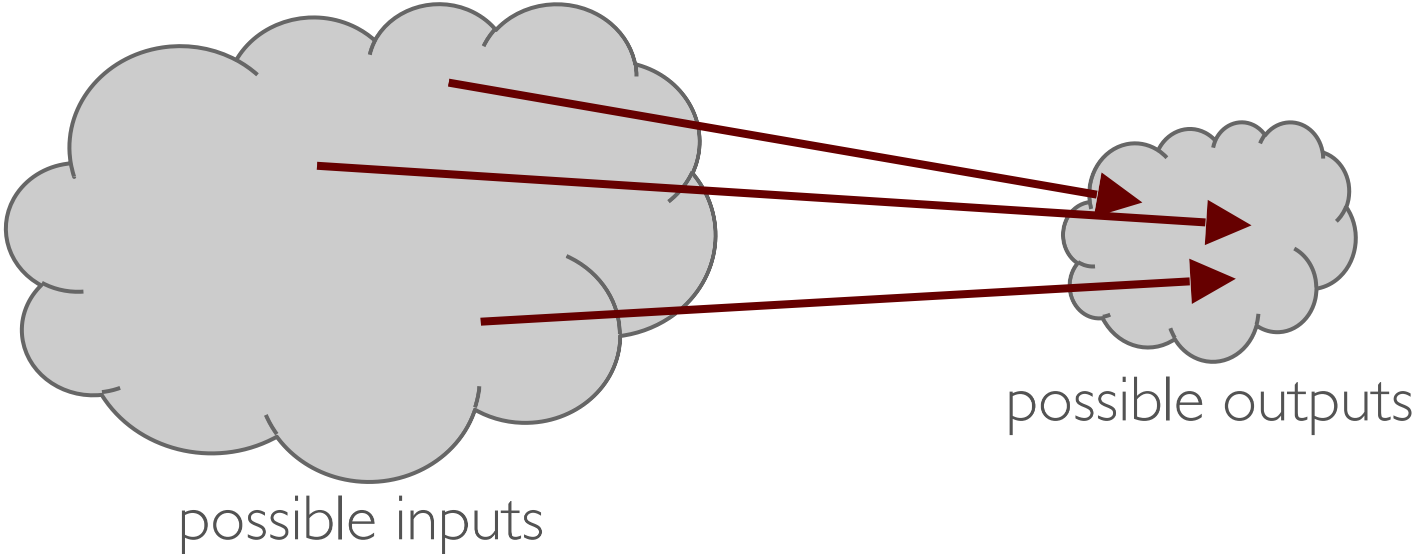
What's a collision?

Property 1: Collision resistance

Do collisions exist in common hash functions?



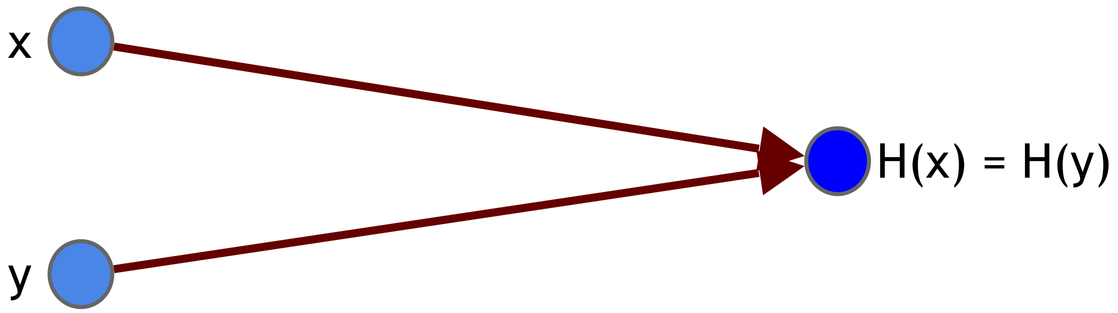
Collisions do exist ...



… but can a real-world adversary find them?

Property 1: Collision resistance

No <u>*efficient* adversary</u> can find x and y such that x != y and H(x)=H(y)



How to find a collision (for 256 bit output)

- try 2^{130} randomly chosen inputs
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This works no matter what H is, but it takes too long to matter

- If a computer calculates 10,000 hashes/sec, it would take 10^{27} years to compute 2^{128} hashes

How to find a collision (for 256 bit output)

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Q: How many hashes/sec does the Bitcoin network compute?

- If a computer calculates 10,000 hashes/sec, it would take 10^{27} years to compute 2^{128} hashes

Is there a faster way to find collisions?

- For some possible H's, yes.
- For others (like SHA-256), we don't know of one.

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Provably secure collision-resistant hash functions can be constructed based on "hard" number-theoretic problems.

Defining Collision Resistance

- Real-world adversaries
  - In practice, everyone has bounded resources
  - Therefore, reasonable to model a real-world adversary as such an entity
  - However, we do not make any assumptions about the adversarial strategy. He can use its (bounded) resources in any possible way

Cryptographic adversary: A probabilistic polynomial-time (PPT) algorithm

Defining Collision Resistance…

- Collision Resistance (informal): A hash function H is collision-resistant if for all PPT adversaries A,

Pr[A outputs x,y s.t. x!=y and H(x)=H(y)]
= "very small"

Defining Collision Resistance…

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  = "very small"

- "Very small" captured via a function that tends to 0. Formal definition: Modern Cryptography

Application: Hash as message digest

If we know $H(x) = H(y)$, and H is collision resistant

it's safe to assume that $x = y$.

To recognize a file that we saw before,

just remember its hash.

Useful because the hash is <u>small</u>.

Property 2: Pre-image Resistance

Intuition: Given H(x), no efficient adversary can find x, except with very small probability

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Problem: What if input space of x is very small, or some inputs are much more likely than others?

H("heads")

H("tails")

easy to find x!

Property 2: Pre-im

This definition is useless in this setting. How can we specify a meaningful version of the definition?

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Problem: What if input space of x is very small, or some inputs are much more likely than others?

H("heads")

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Defining Preimage Resistance

- **Preimage Resistance:** A hash function H is preimage-resistant if for all PPT adversaries A,

$$\Pr[x \leftarrow \{0,1\}^k, A(H(x)) \text{ outputs } x' \text{ s.t. } H(x')=H(x)] = \text{small}$$

x is drawn from uniform distribution over $\{0,1\}^k$ for some sufficiently large k

Preimage Resistance (contd.)

- If x is drawn from the uniform distribution, then inverting H(x) is hard
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Preimage Resistance (contd.)

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Preimage Resistance (contd.)

- If x is drawn from the uniform distribution, then inverting H(x) is hard
- But what if x is drawn from low-entropy distribution?
- Can append a random string r to x and then compute H(r | x) to prevent enumeration attacks

Theorem: Collision resistance implies preimage resistance if the hash function is sufficiently compressing

Application: Commitment

Want to “seal a value in an envelope”, and
“open the envelope” later.

Commit to a value, reveal it later.

Commitment Schemes

(*com*, *key*) := commit(*msg*)

match := verify(*com*, *key*, *msg*)

To seal *msg* in envelope:

(*com*, *key*) := commit(*msg*) -- then publish *com*

To open envelope:

publish *key*, *msg*

anyone can use verify() to check validity

Commitment Schemes

(***com***) ← commit(***msg***; ***key***)

match ← verify(***com***, ***key***, ***msg***)

Security properties:

- Hiding: Given ***com***, no PPT adversary can find* ***msg***
- Binding: No PPT adversary can find* (***msg***, ***key***) ***!=*** (***msg'***,***key'***) such that verify(commit(***msg***; ***key***), ***key'***,***msg'***) == true

* Except with very small probability

Commitment Schemes

commit(***msg; key)*** → (H(***key*** | ***msg***))

where ***key*** is a random 256-bit value

verify(***com***, ***key***, ***msg***) → (H(***key*** | ***msg***) == ***com***)

Security properties:

- Hiding: If H is a ***random oracle***, given H(***key*** | ***msg***), hard to find ***msg***.
- Binding: Collision-resistance → Hard to find ***(key,msg) !=*** ***(key',msg')*** such that H(***key*** | ***msg***) == H(***key*** | ***msg'***)

Random Oracle (RO)

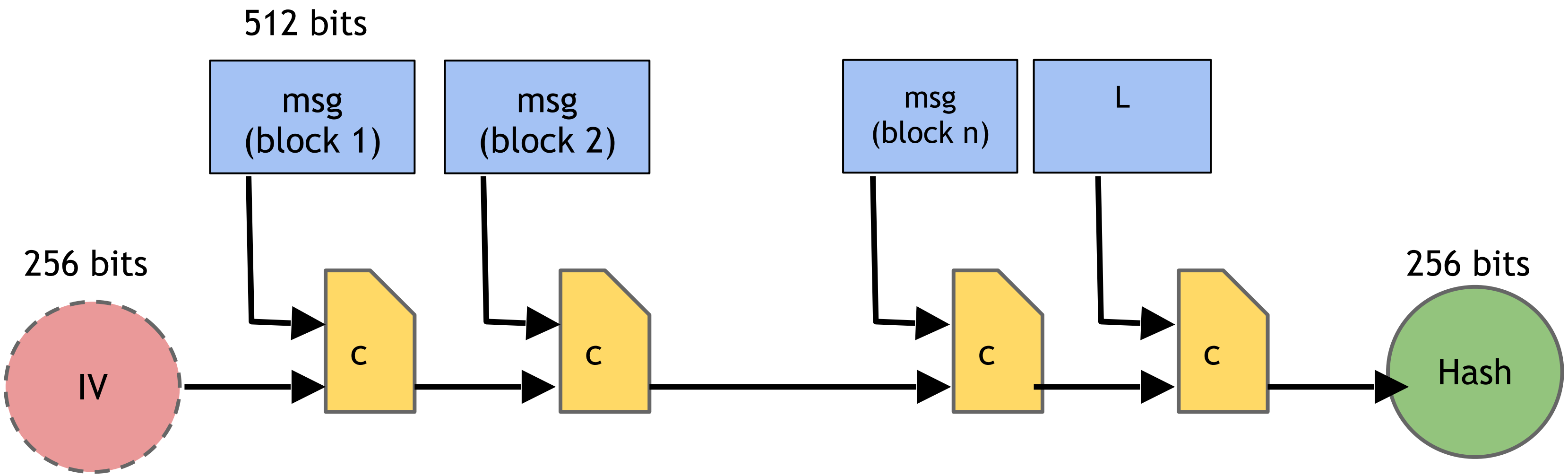
- Imagine an elf in a box with an infinite writing scroll
- Upon receiving an input x, the elf checks the scroll if there is an entry y corresponding to x. If yes, it returns y.
- Otherwise, elf chooses a random value y (from the output space) and returns it. It adds an entry (x,y) to the scroll.

Random Oracle (RO)

- In practice-oriented provable security, hash functions are often modeled as a random oracle
- Each party (including adversary) is given black-box access to the random oracle. They can query the random oracle any polynomial number of times
- By definition, the answers of random oracle answers are unpredictable
- Random oracle captures many security properties such as one-wayness, collision-resistance .

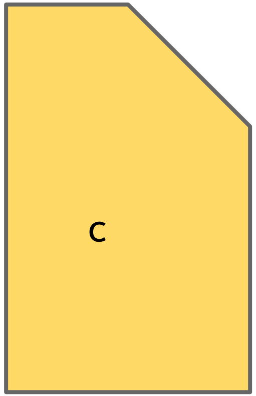
SHA-256 hash function

Suppose msg is of length L s.t. L is a multiple of 512 (pad with 0s otherwise)



Theorem [Merkle-Damgard]: If c is collision-resistant, then SHA-256 is collision-resistant.

SHA-256 hash function



Q: What the heck is inside of c?

<u>Theorem [Merkle-Damgard]</u>: If c is collision-resistant, then SHA-256 is collision-resistant.

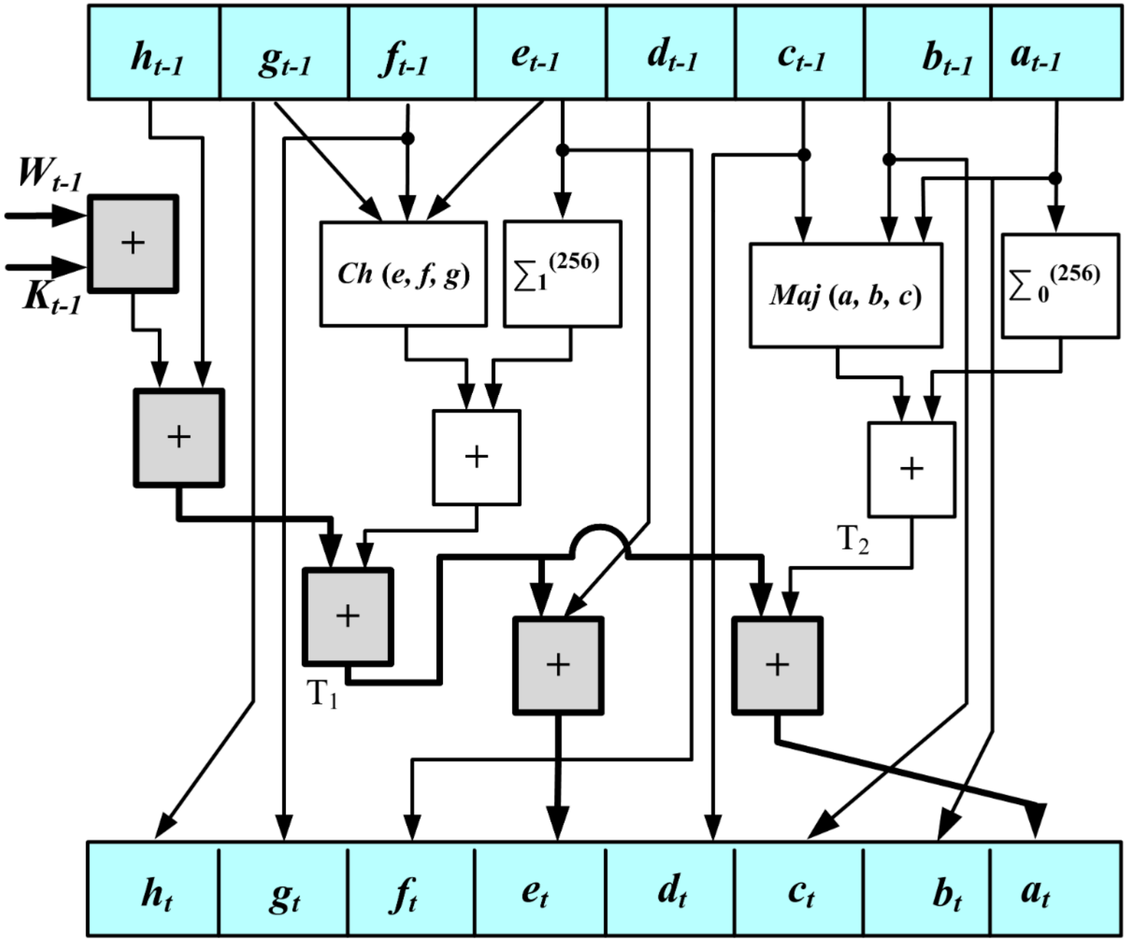


Fig. 3. SHA-256 hash function. Base transformation round