Blockchains & Cryptocurrencies

Crypto Background

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Johns Hopkins University - Spring 2023

*Many slides from NBFMG

This lecture

Crypto background

- hash functions
- random oracle model
- digital signatures
- … and applications

Cryptographic Hash Functions

Hash function

- takes a string of arbitrary length as input
- fixed-size output (i.e., hash function "compresses" the input)
- efficiently computable

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Security properties:

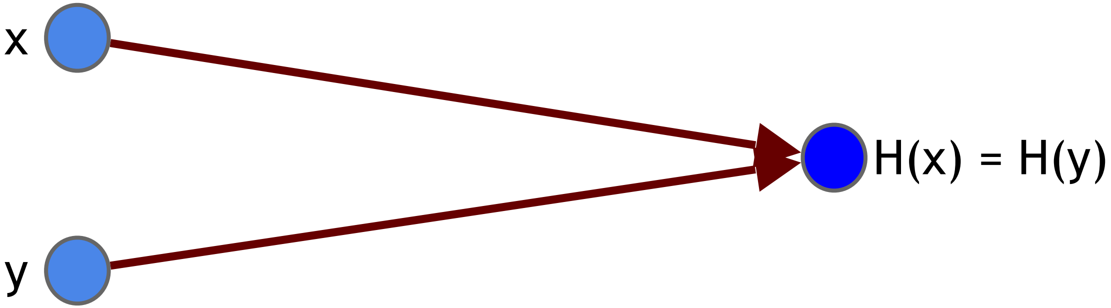
- Collision resistance
- Preimage resistance (one-way)

Property 1: Collision resistance

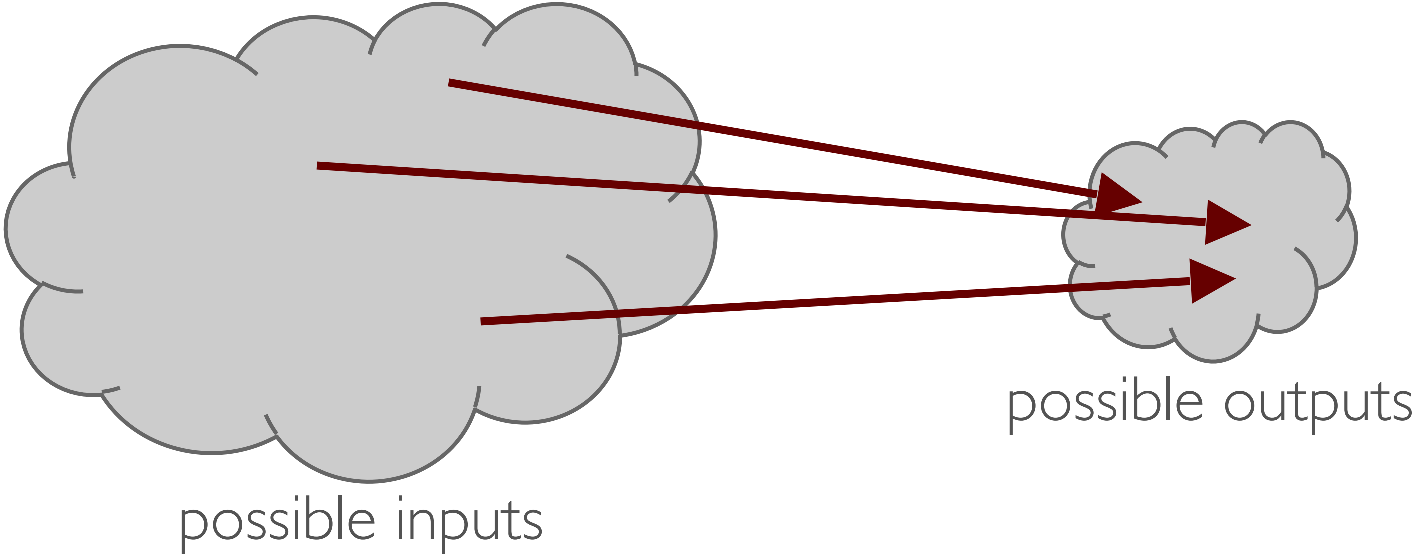
What's a collision?

Property 1: Collision resistance

Do collisions exist in common hash functions?



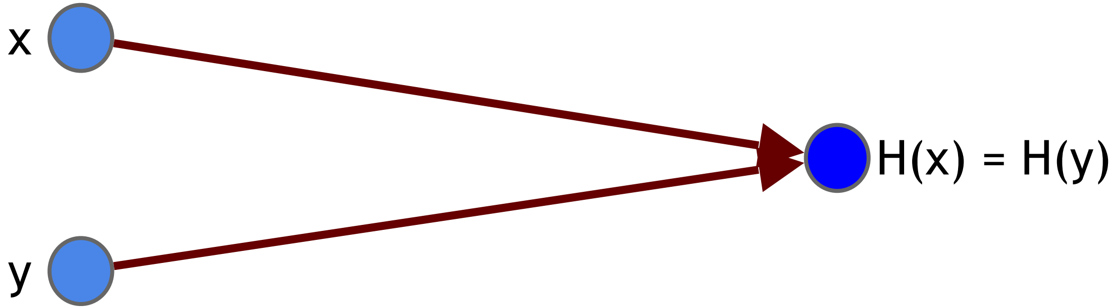
Collisions do exist ...



… but can a real-world adversary find them?

Property 1: Collision resistance

No real-world adversary can find x and y such that
x != y and H(x)=H(y)



How to find a collision (for 256 bit output)

- try 2^{130} randomly chosen inputs
- 99.8% chance that two of them will collide

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Is there a faster way to find collisions?

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Provably secure collision-resistant hash functions can be constructed based on “hard” number-theoretic problems.

Defining Collision Resistance

- Real-world adversaries
  - In practice, everyone has finite resources
  - Therefore, reasonable to model a real-world adversary as such an entity
  - However, we do not make any assumptions about the adversarial strategy. They can use their resources in any way

Formally:

A probabilistic polynomial-time (PPT) algorithm

Defining Collision Resistance…

- Collision Resistance (informal): A hash function H is collision-resistant if for all PPT adversaries A,

Pr[A outputs x,y s.t. x!=y and H(x)=H(y)]
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- "Very small" captured via a function that tends to 0.
- Formal definition: Modern Cryptography (601.441/641)

Application: Hash as message digest

If we know $H(x) = H(y)$, and H is collision resistant
it's safe to assume that $x = y$.

To recognize a file that we saw before,
just remember its hash.

Useful because the hash is <u>small</u>.

Property 2: Pre-image Resistance

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<u>Problem</u>: What if input space of x is very small, or some inputs are much more likely than others?

H("heads")

H("tails")

easy to find x!

Property 2: Pre-im

This definition is useless in this setting. How can we specify a meaningful version of the definition?

Intuition: Given H(x), no … adversary can find x, except with very small prob… ity

Problem: What if input space of x is very small, or some inputs are much more likely than others?

H("heads")

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Defining Preimage Resistance

- **Preimage Resistance:** A hash function H is preimage-resistant if for all PPT adversaries A,

$$\Pr[x \leftarrow \{0,1\}^k, A(H(x)) \text{ outputs } x' \text{ s.t. } H(x')=H(x)] = \text{small}$$

x is drawn from uniform distribution over $\{0,1\}^k$ for some sufficiently large k

Preimage Resistance (contd.)

- If x is drawn from the uniform distribution, then inverting $H(x)$ is hard
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Collision Resistance vs Preimage Resistance

Theorem: Collision resistance implies preimage resistance if the hash function is sufficiently compressing

Application: Commitment

Want to "seal a value in an envelope", and
"open the envelope" later.

Commit to a value, reveal it later.

Commitment Schemes

(***com***, ***key***) := commit(***msg***)

match := verify(*com*, *key*, *msg*)

To seal ***msg*** in envelope:

(***com***, ***key***) := commit(***msg***) -- then publish ***com***

To open envelope:

publish ***key***, ***msg***

anyone can use verify() to check validity

Commitment Schemes

(***com***) ← commit(***msg***; ***key***)

match ← **verify(*com*, *key*, *msg*)**

Security properties:

- Hiding: Given ***com***, no PPT adversary can find* ***msg***

* Except with very small probability

Commitment Schemes

(***com***) ← commit(***msg***; ***key***)

match ← **verify(*com*, *key*, *msg*)**

Security properties:

- Hiding: Given ***com***, no PPT adversary can find* ***msg***
- Binding: No PPT adversary can find* (***msg*, *key*) != (*msg'*,*key'*)**
  such that verify(commit(***msg***; ***key***), ***key'***,***msg'***) == true

* Except with very small probability

Commitment Schemes from Hash Functions

commit(***msg; key)*** → (H(***key*** | ***msg***))

where ***key*** is a random 256-bit value

verify(***com, key, msg***) → (H(***key*** | ***msg) == com***)

Commitment Schemes from Hash Functions

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Security properties:

- Binding: Collision-resistance → Hard to find ***(key,msg) !=*** ***(key',msg')*** such that H(***key*** | ***msg)*** == H(***key*** | ***msg')***
- Hiding: If H is a ***random oracle***, given H(***key*** | ***msg)***, hard to find ***msg***.

Random Oracle (RO)

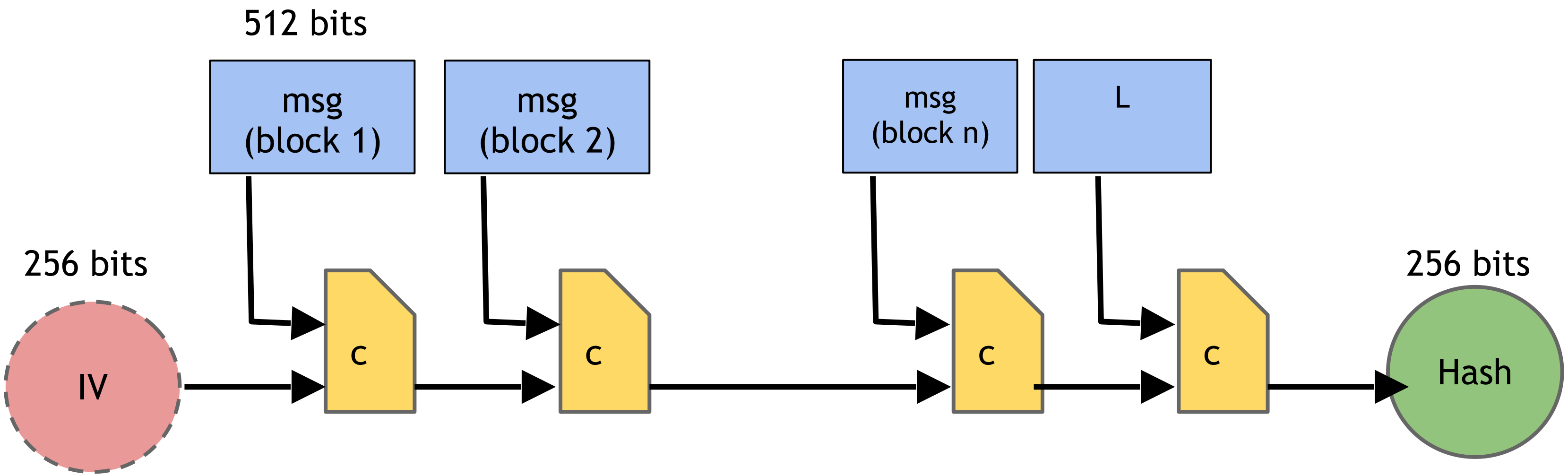
- Imagine an elf in a box with an infinite writing scroll
- Upon receiving an input x, the elf checks the scroll if there is an entry y corresponding to x. If yes, it returns y.
- Otherwise, elf chooses a random value y (from the output space) and returns it. It adds an entry (x,y) to the scroll.

Random Oracle (RO)

- In practice-oriented provable security, hash functions are often modeled as a random oracle
- Each party (including adversary) is given black-box access to the random oracle. They can query the random oracle any polynomial number of times
- By definition, the answers of random oracle answers are unpredictable
- Random oracle captures many security properties such as one-wayness, collision-resistance .

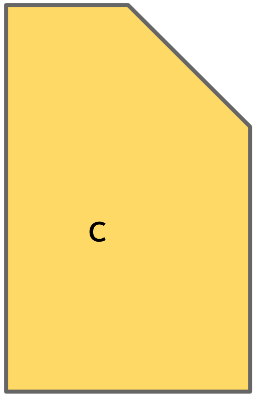
SHA-256 hash function

Suppose msg is of length L s.t. L is a multiple of 512 (pad with 0s otherwise)



Theorem [Merkle-Damgard]: If c is collision-resistant, then SHA-256 is collision-resistant.

SHA-256 hash function



Q: What the heck is inside of c?

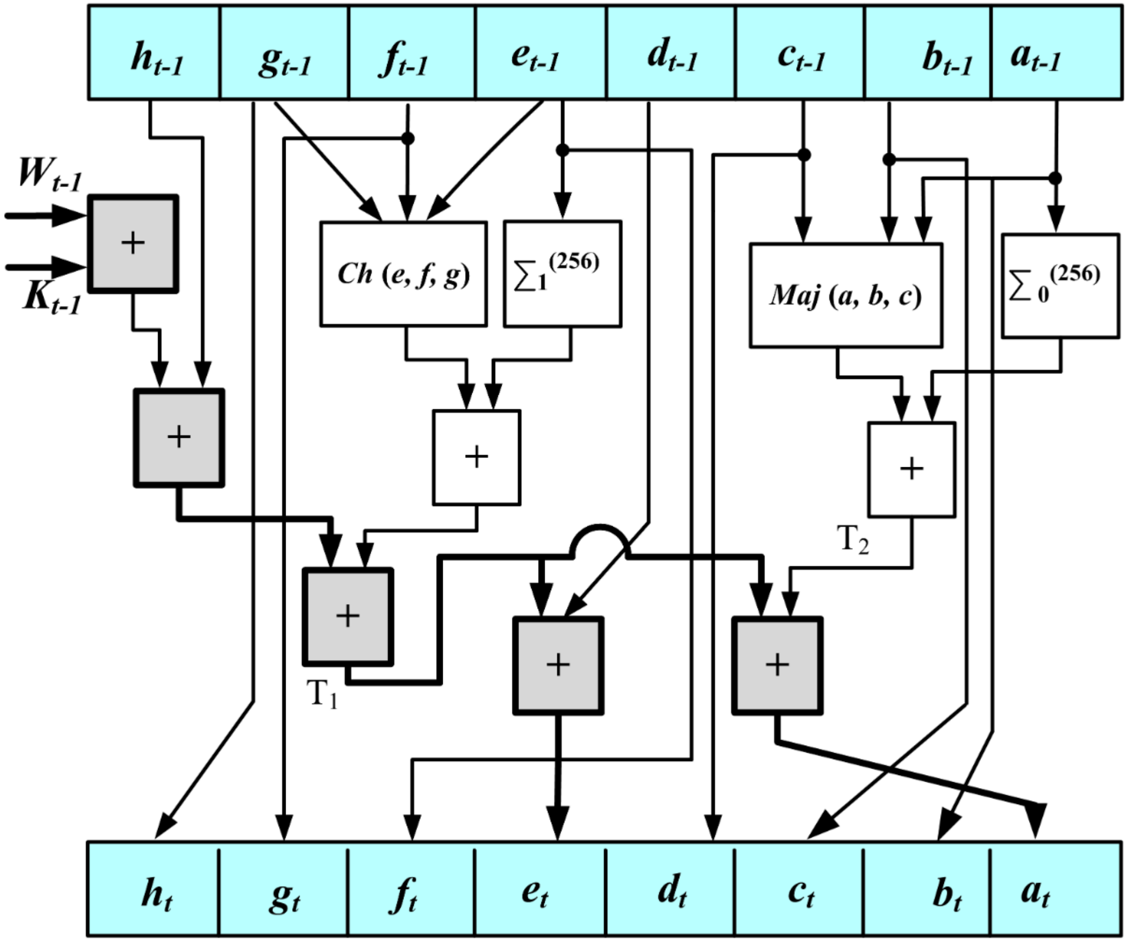


Fig. 3. SHA-256 hash function. Base transformation round

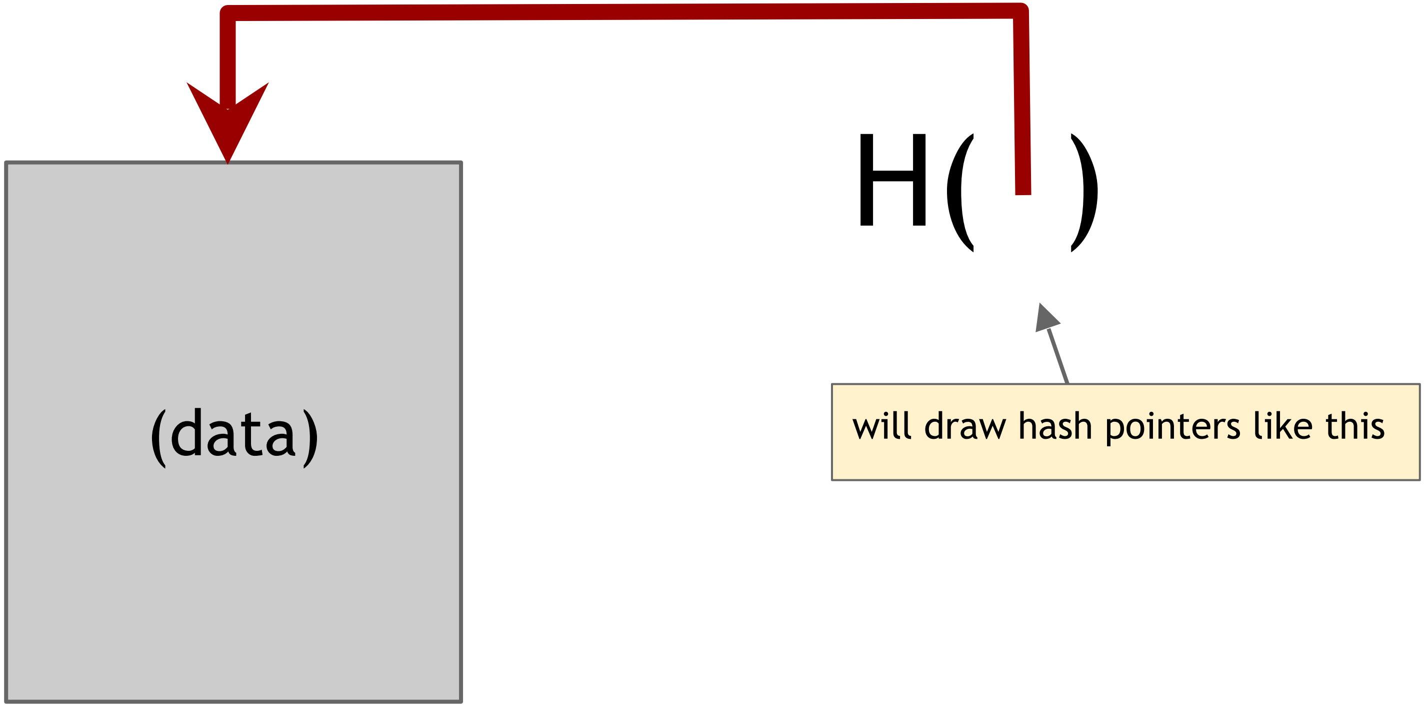
Hash Pointers and Data Structures

Hash pointer

- pointer to where some info is stored, ***and***
- cryptographic hash of the info

If we have a hash pointer, we can

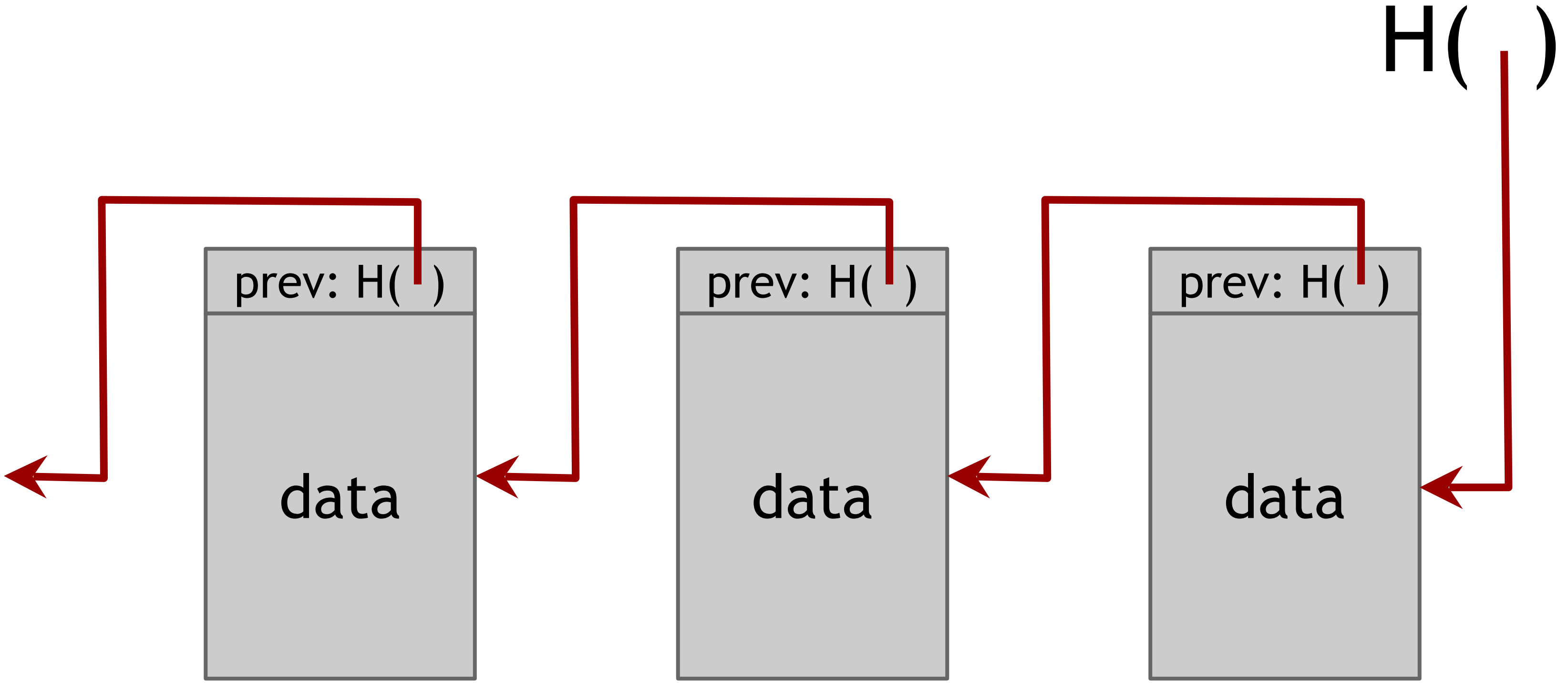
- ask to get the info back, and
- verify that it hasn't changed



(data)
H( )
will draw hash pointers like this

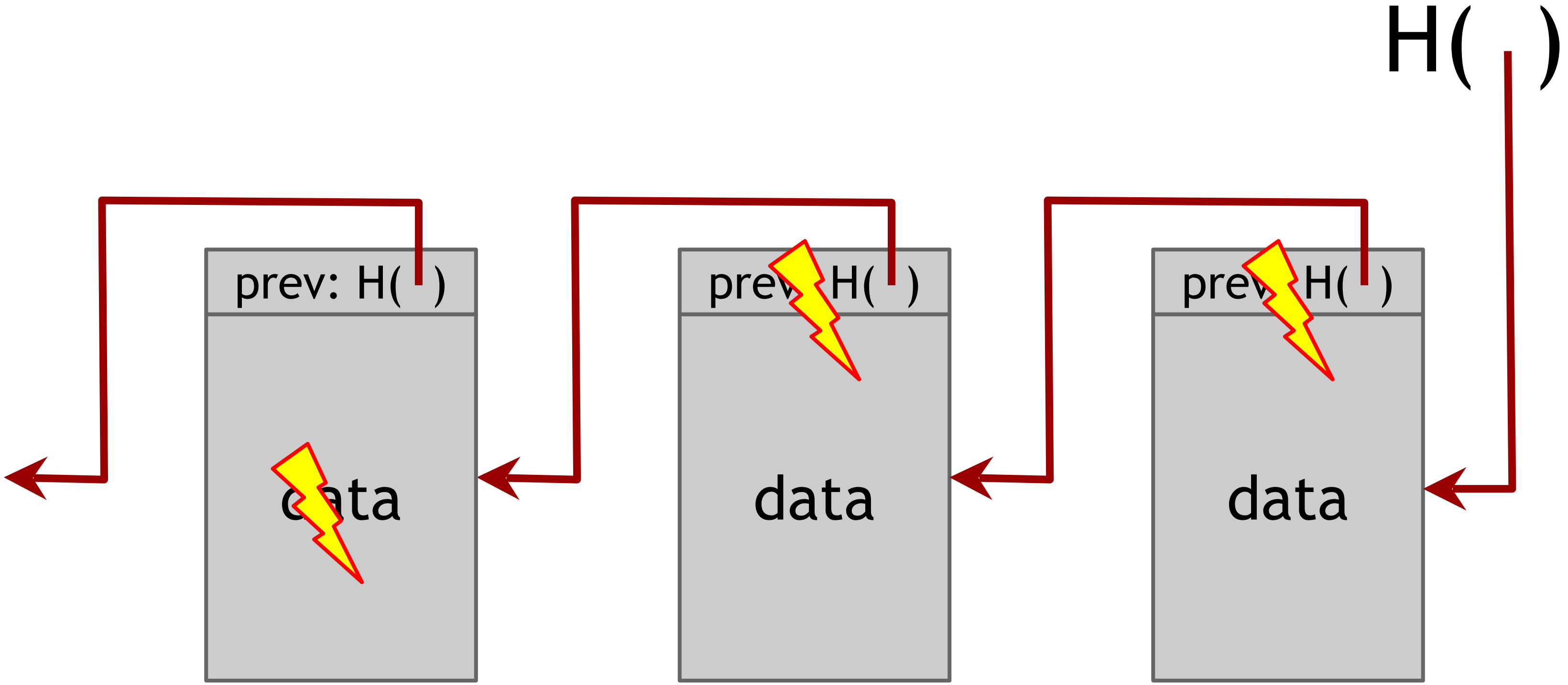
Building data structures with hash pointers

Linked list with hash pointers = "Blockchain"



use case: tamper-evident log

Detecting Tampering



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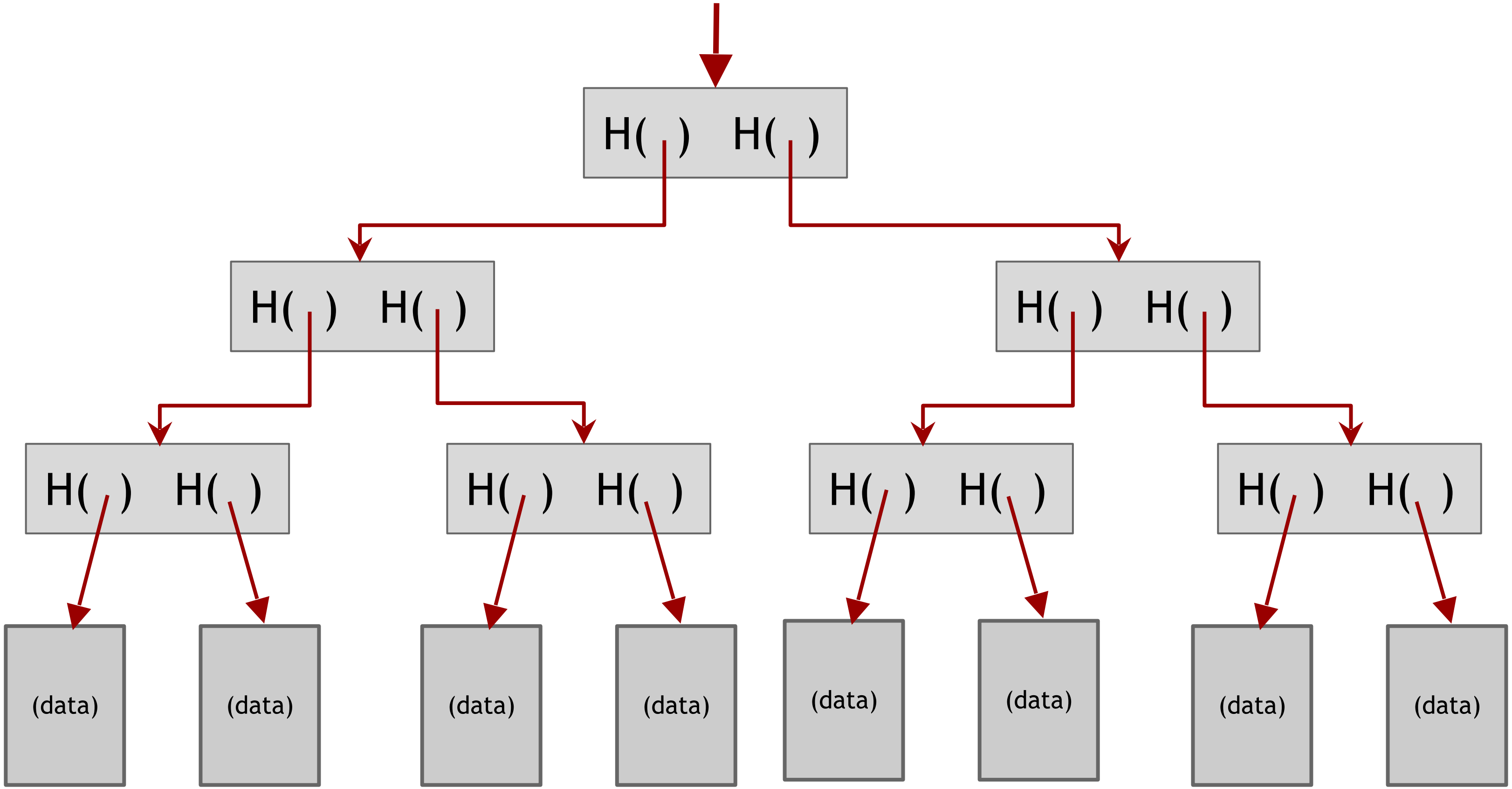
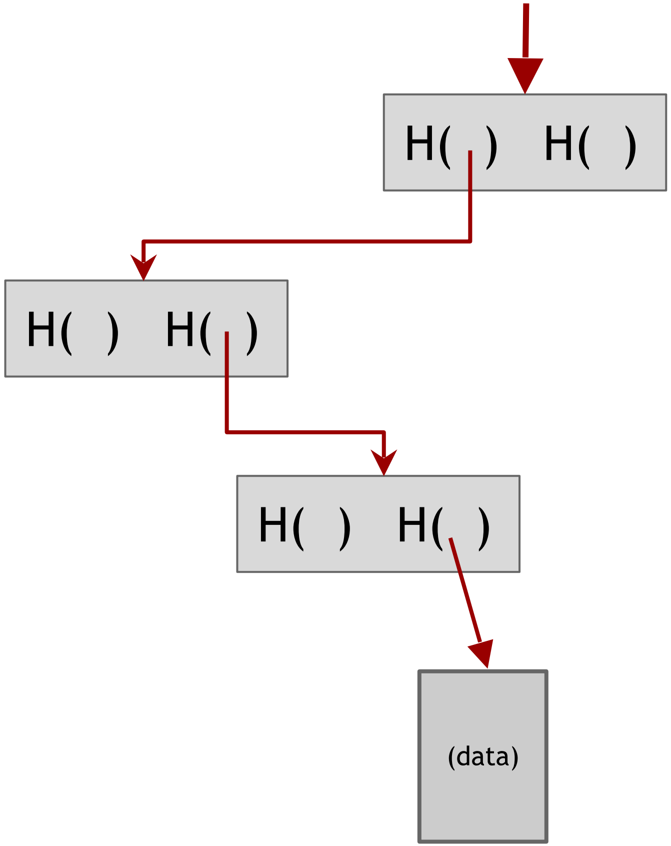
Binary tree with Hash pointers = "Merkle tree"



Proving membership in a Merkle tree



show O(log n) items

Advantages of Merkle trees

- Tree holds many items, but just need to remember the root hash
- Can verify membership in O(log n) time/space

<u>Variant</u>: ***sorted*** Merkle tree

- can verify non-membership in O(log n)
- show items before, after the missing one

More generally ...

Can use hash pointers in any pointer-based data structure that has no cycles

Digital Signatures

What we want from signatures

- Only you can sign, but anyone can verify
- Signature is tied to a particular document

  (***can't be cut-and-pasted to another doc)***
- Even if one can see your signature on some documents, he cannot "forge" it

Digital signatures

randomness

- (sk, pk) ← keygen(r)

  sk: secret signing key

  pk: public verification key

- sig ← sign(sk, message)

- isValid ← verify(pk, message, sig)

randomized algorithm

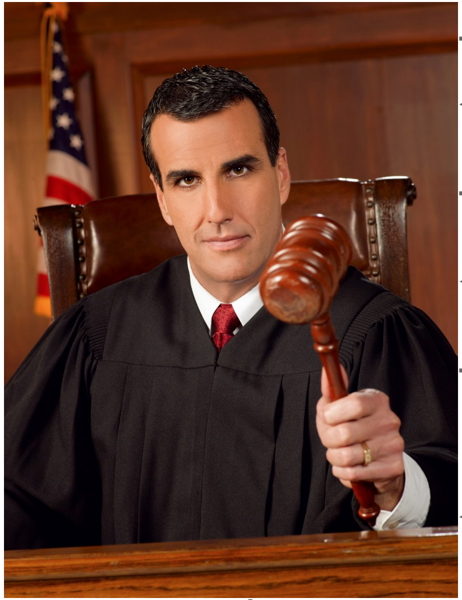
Typically randomized

Requirements for signatures

- Correctness: "valid signatures verify"
  - verify(pk, message, sign(sk, message)) == true
- Unforgeability under chosen-message attacks (UF-CMA): "can't forge signatures"
  - adversary who knows pk, and gets to see signatures on messages of his choice, can't produce a verifiable signature on another message

UF-CMA Security

(sk, pk) ← keygen(1^k)



pk

m_0

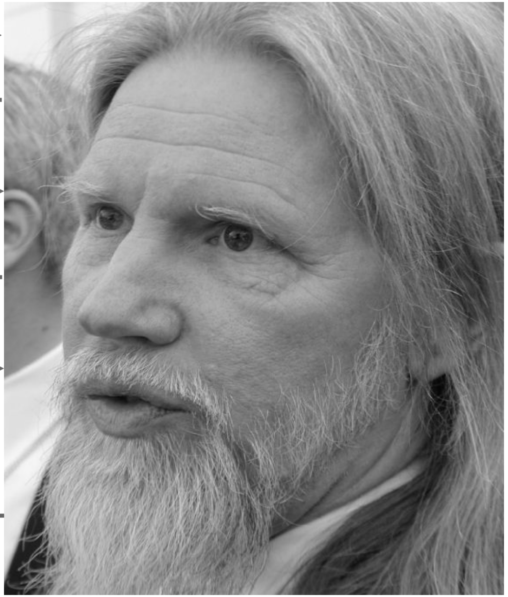
sign(sk, m_0)

m_1

sign(sk, m_1)

• • •

M, sig



M not in { m_0, m_1, ... }

Challenger

verify(pk, M, sig)

Adversary

ifValid, attacker wins

Definition: A signature scheme (keygen,sign,verify) is UF-CMA secure if for every PPT adversary A, Pr[A wins in above game] = very small

Notes

- Algorithms are randomized: need good source of randomness. Bad randomness may reveal the secret key
- fun trick: sign a hash pointer. signature "covers" the whole structure
- Bitcoin uses Elliptic Curve Digital Signature Algorithm (ECDSA), a close variant of Schnorr over Elliptic curves