# Numerical Methods

## Assignment - 5

Q1 Central Difference Relation

$$\text{TP}: y(x_0)'' = y_0'' = \frac{y_1 - 2y_0 + y_{-1}}{h^2} - \frac{h^2}{12} y^{(4)}(\xi) \quad -①$$

We have the following Taylor Series expansion:

$$y_1(x_0 + h) = y_0 + h y_0' + \frac{h^2}{2} y_0'' + \frac{h^3}{6} y_0''' + \frac{h^4}{24} y_0^{(4)}$$

$$y_{-1}(x - h) = y_0 - h y_0' + \frac{h^2}{2} y_0'' - \frac{h^3}{6} y_0''' + \frac{h^4}{24} y_0^{(4)}$$

~~Add $y_1 - 2y_0 +$~~ Use the ~~of~~ above equations in eq ①:

$$\frac{y_1 - 2y_0 + y_{-1}}{h^2} = \frac{1}{h^2}\Big[ y_0 + h y_0' + \frac{h^2}{2} y_0'' + \frac{h^3}{6} y_0''' + \frac{h^4}{24} y_0^{(4)} - 2y_0 + y_0 - h y_0' + \frac{h^2}{2} y_0'' - \frac{h^3}{6} y_0''' + \frac{h^4}{24} y^{(4)} \Big]$$

$$= \frac{1}{h^2}\Big[ h^2 y_0'' + \frac{h^4}{12} y^{(4)} \Big]$$

$$\therefore \frac{y_1 - 2y_0 + y_{-1}}{2} = y_0'' + \frac{h^2}{12} y^{(4)}$$

$$\rightarrow \boxed{y_0'' = \frac{y_{-1} - 2y_0 + y_{-1}}{2} - \frac{h^2}{12} y^{(4)}}$$

Hence, proved!

Here, error term of $O(h^2) = \frac{-h^2}{12} y^{(4)}$