Point Estimate :

A point estimate a single value calculated from a sample, that serves as the best guess or approximation of an unknown parameter of population.

Point estimates are often used in statistics when we want to make inference about a population based on a sample.

Confidence Interval :

In simple word, confidence interval is a range of values within which we expect a particular population parameter like a mean, SD to fall. It's a way to express the uncertainity around an estimate obtained from a sample data.

Confidence level :

Usally expressed as a percentage like 95%. It indicates how sure we are that that the true value lies within the interval.

$$\boxed{\text{Confidence interval} = \text{point estimate} \pm \text{Margin of Error}}$$

There are two way of finding the confidence interval –

(A) Z-procedure (B) T-procedure.

(A) Z-procedure (Population SD is known) :

Assumption :

(i) Random sampling (ii) population SD should be known to us.
(iii) Normal Distribution or large sample size.

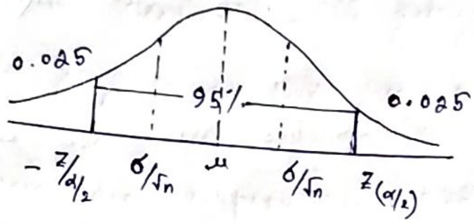
(Q) Let say we have 77K population in a XYZ state. We took a 100 sample and it is shown that the mean age = 48. (Pop s.D = 15) Construct a 95% confidence interval about the mean.

Ⓐ $\boxed{\text{point Estimate} \pm Z_{(\alpha/2)} \frac{\sigma}{\sqrt{n}}}$ n = sample size

(critical Value)

Ⓑ $\boxed{\text{Intution behind this formula}}$:

Actually we have to calculate the confidence interval with confidence level = 0.95.



Now, our sample data follows the normal distribution and the mean should be μ and s.d $\sigma/\sqrt{n}$ (as our SD of population = σ)

Now, mathematically we can ~~wright~~ write —

$$\cancel{P\left(-Z_{\alpha/2} < \bar{X} < Z_{\alpha/2}\right) = 0.95}$$ where $\bar{X}$ = point estim

let say our $\bar{X}$ is a random Variable. $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

$$Z = \left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}\right)$$

mathematically we can write —

$$P\left(-Z_{\alpha/2} < Z < +Z_{\alpha/2}\right) = 0.95$$

$$\Rightarrow P\left(-Z_{\alpha/2} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < +Z_{\alpha/2}\right) = 0.95$$

$$\Rightarrow P\left(-Z_{\alpha/2}\,\sigma/\sqrt{n} < \bar{X} - \mu < Z_{\alpha/2}\,\sigma/\sqrt{n}\right) = 0.95$$

$$\Rightarrow P\left(-Z_{\alpha/2}\,\sigma/\sqrt{n} - \bar{X} < -\mu < Z_{\alpha/2}\,\sigma/\sqrt{n} - \bar{X}\right) = 0.95$$

$$\Rightarrow P\left(\bar{X} - Z_{\alpha/2}\,\sigma/\sqrt{n} < \mu < \bar{X} + Z_{\alpha/2}\,\sigma/\sqrt{n}\right) = 0.95$$

This expression tell us that the population mean (μ) should fall into the interval $\left(\bar{x} - Z_{\alpha/2}\, \sigma/\sqrt{n}\ ,\ \bar{x} + Z_{\alpha/2}\, \sigma/\sqrt{n}\right)$, whose propability = 0.95.
and this is what we need to calculate.

So, here the confidence interval with confidence level 95% is –

$$\left(28 \pm \left(1.96 \times \frac{15}{10}\right)\right)$$

$$= \left[28 \pm 2.94\right]$$

$$= \left[25.06\ ,\ 30.94\right] \quad \text{(Ans)}$$

(Q) Same question with 85% confidence level.

Confidence interval $= \left(28 \pm Z_{0.075}\left(\frac{15}{10}\right)\right)$

$= \left[28 \pm \left(1.44 \times \frac{15}{10}\right)\right] = \left[25.84\ ,\ 30.16\right]$

$\boxed{\text{Factors Affecting Margin of Error}}$

① Confidence level (1 - Alpha) (ii) Sample Size (iii) Population SD.

- Margin of error $\propto$ population SD
- Margin of error $1/\propto$ sample size
- Margin of error $\propto$ Confidence level

(B) $\boxed{\text{T-} \cancel{\text{produre}} \text{ procedure}}$:

<u>Assumption</u> :

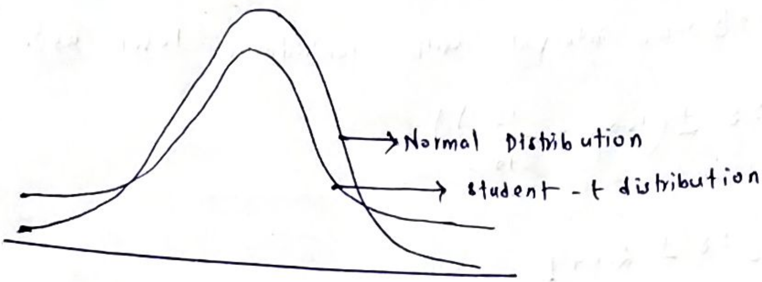
(i) Random sampling (ii) Sample SD is Known (iii) Approximately normal distribution (iv) Independent observations.

Formula : $CI = \boxed{\text{point estimate} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}}$

Here We don't have the population SD. So We use the sample SD but sample SD can vary over sample.

So,

$\bar{x} \sim$ a other distribution (student t distribution)



Student-t distribution has only one parameter, i.e degree of freedom. When degree of freedom ~~are~~ is getting increased, our student-t distribution are going to looks like Normal distribution.

(Q) Suppose there are a population. From that We are going to take a sample of 25. With s.d 8. Whose mean 28. Construct the confidence interval about the mean with 95% confidence level.

(A) Confidence interval (C.I) $= \bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\left(\begin{array}{l}\alpha = 0.05 \\ df = 24\end{array}\right)$

$= 28 \pm 2.064 \times \frac{8}{5}$

$= [24.6976, 31.3024]$ (Ans)

(Q) On the Varval season of the CAT exam, a sample of 25 test takers has mean of 520 with a sd 80. Construct a 99% C.I about the mean.

(A) Confidence interval (C.I) $= \bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\left(\begin{array}{l}\alpha = 0.01 \\ df = 24\end{array}\right)$

$= 520 \pm 2.797 \times \frac{80}{5}$

$= [520 - 44.752, 520 + 44.752]$

$= [475.248, 564.752]$ (Ans)