

1) function $x = f(n)$

$x = 1;$

for $i = 1 : n$

for $j = 1 : n$

$x = x + 1;$

2) find the runtime of the algorithm mathematically?

function $x = f(x)$

$x = 1;$

for $i = 1 : n$

for $j = 1 : n$

$$\sum_{i=1}^{n+1} \sum_{j=1}^{n+1} 1$$

$x = x + 1;$

$$\sum_{i=1}^n \sum_{j=1}^{n+1} 1 = 1$$

$$T(n) = 1 + (n+1) + (n^2 + n) + n^2$$

\Rightarrow Simplify the eqn we get.

$$T(n) = 2 + 3n + 3n^2$$

The dominant term here is n^2 , so runtime is determined by n^2

\Rightarrow Runtime is $O(n^2)$.

2)

For calculating the time taken for the function $f(n)$, we used values of n from small to large. The values plotted with n on x-axis & time taken on y-axis.

Interpretation of Plotting:

- From the plot we can see a clear trend in quadratic as the values of n increases. showing $O(n^2)$ complexity.
- The fitted curve is polynomial which is quadratic that closely matches the timing points.

3) Upper Bound: (Big-O): The upper bound on the graph is shown by the blue dashed line which is slightly above the fitted curve. This indicates that the time complexity is $O(n^2)$.

Lower Bound: (Big Omega): The lower bound is represented by orange dotted line which is below fitted curve, indicating the time complexity is $\Omega(n^2)$.

Tight bound: (Big Theta):

Since both upper & lower bound grow at the same rate

$T(n)$ is $\Theta(n^2)$

4) In the code no ~~is~~ is a vertical dashed line (yellow) at 200.

This is a value of n where the function starts to follow the quadratic trend. This can be visualized by zooming in on the plot & identifying the point where trend is minimal.

4) $x = f(n)$

$x = 1$;

$y = 1$;

for $i = 1:n$

for $j = 1:n$

$x = x + 1$;

$y = i + j$;

In the above Pseudocode $y = H[j]$ is added in inner loop. This is a constant-time operation $O(1)$. So it will increase the Overall time taken Per iteration. But the Overall time Complexity remains same which is $O(n^2)$.

- 5) No, adding $y = H[j]$ will not affect the results from the runtime analysis. Since operation is $O(1)$, runtime will be same as its $O(n^2)$.