JEE Mains 2019 Chapter wise Question Bank

Circle – Questions

Q1

Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :

(1) $2\sqrt{3}y = 12x + 1$ (2) $\sqrt{3}y = x + 3$

(3) $2\sqrt{3}y = -x - 12$ (4) $\sqrt{3}y = 3x + 1$

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Q2

Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then:

(1) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(2) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

(3) a, b, c are in A.P

(4) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

9 Jan Morning

Q3

If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x-4)^2 + (y-7)^2 = 36$ intersect at two distinct points, then:

(1) $r > 11$ (2) $0 < r < 1$

(3) $r = 11$ (4) $1 < r < 11$

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Q4

If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, – 1), then the radius of C is:

(1) $2\sqrt{5}$ (2) 4

(3) 5 (4) $\sqrt{57}$

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Q5

If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to:

(1) 13 (2) 20

(3) -25 (4) 25

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Q6

A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is :

(1) 6 (2) $\sqrt{137}$

(3) $\sqrt{41}$ (4) 13

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Q7

Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) 2

(3) $2\sqrt{2}$ (4) $\sqrt{2}$

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Q8

A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :

(1) a hyperbola (2) an ellipse
(3) a straight line (4) a parabola

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Q9

Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2=0$ and $x^2+y^2-6x-6y+14=0$ respectively. If P and Q are the points of intersection of these circles then, the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6
(3) 9 (4) 4

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Q10

If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval :

(1) $(2,17)$ (2) $[13,23]$
(3) $[12,21]$ (4) $(23,31)$

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Q11

If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $\left(x^2+y^2\right)^2=4R^2x^2y^2$

(2) $\left(x^2+y^2\right)^3=4R^2x^2y^2$

(3) $\left(x^2+y^2\right)^2=4Rx^2y^2$

(4) $\left(x^2+y^2\right)(x+y)=R^2xy$

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Q12

The sum of the squares of the lengths of the chords intercepted on the circle, $x^2+y^2=16$, by the lines, $x+y=n$, $n\in N$, where N is the set of all natural numbers, is :

(1) 320 (2) 105 (3) 160 (4) 210

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Q13

The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2+y^2=4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{1}{3}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{3}}$

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Q14

If a tangent to the circle $x^2+y^2=1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is:

(1) $x^2+y^2-4x^2y^2=0$ (2) $x^2+y^2-2xy=0$
(3) $x^2+y^2-16x^2y^2=0$ (4) $x^2+y^2-2x^2y^2=0$

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Q15

The common tangent to the circles $x^2+y^2=4$ and $x^2+y^2+6x+8y-24=0$ also passes through the point:

(1) $(4,-2)$ (2) $(-6,4)$
(3) $(6,-2)$ (4) $(-4,6)$

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Q16

If the circles $x^2+y^2+5Kx+2y+K=0$ and $2(x^2+y^2)+2Kx+3y-1=0$, $(K\in\mathbf{R})$, intersect at the points P and Q, then the line $4x+5y-K=0$ passes through P and Q, for :

(1) infinitely many values of K
(2) no value of K.
(3) exactly two values of K
(4) exactly one value of K

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Q17

The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is:

(1) 3 (2) $2\sqrt{2}$ (3) 2 (4) $3\sqrt{2}$

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Q18

The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is:

(1) $x = \sqrt{1+4y}, y \geq 0$ (2) $y = \sqrt{1+2x}, x \geq 0$

(3) $y = \sqrt{1+4x}, x \geq 0$ (4) $x = \sqrt{1+2y}, y \geq 0$

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Q19

If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{13}{5}$ (2) $\frac{120}{13}$ (3) $\frac{60}{13}$ (4) $\frac{13}{2}$

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Q20

A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (3, 10) (2) (3, 5) (3) (2, 3) (4) (1, 5)

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Circle - Solutions

Q1

(2) Since, the equation of tangent to parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m} \quad ...(1)$$

The line (1) is also the tangent to circle

$x^2 + y^2 - 6x = 0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

$$\therefore \quad \frac{\left|3m + \frac{1}{m}\right|}{\sqrt{1+m^2}} = 3 \Rightarrow \left(3m + \frac{1}{m}\right)^2 = 9\left(1+m^2\right)$$

$$\Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$

Then, from equation (1): $y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{3}$

Hence, $\sqrt{3}y = x + 3$ is one of the required common tangent.

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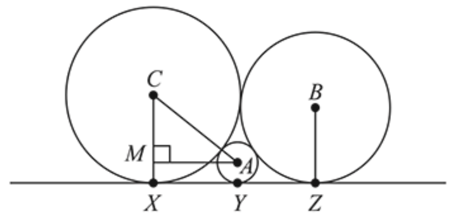
Q2

(1)



$AM^2 = AC^2 - MC^2$

$= (a + c)^2 - (a - c)^2 = 4ac$

$\Rightarrow AM^2 = XY^2 = 4ac$

$\Rightarrow XY = 2\sqrt{ac}$

Similarly, $YZ = 2\sqrt{ba}$ and $XZ = 2\sqrt{bc}$

Then, $XZ = XY + YZ$

$\Rightarrow 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$

$$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$$

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Q3

(4) Consider the equation of circles as,

$x^2 + y^2 - 16x - 20y + 164 = r^2$

i.e. $(x - 8)^2 + (y - 10)^2 = r^2 \quad ...(1)$

and $(x - 4)^2 + (y - 7)^2 = 36 \quad ...(2)$

Both the circles intersect each other at two distinct points.

Distance between centres

$$= \sqrt{(8-4)^2 + (10-7)^2} = 5$$

$\therefore \quad |r - 6| < 5 < |r + 6|$

$\therefore \quad$ If $|r - 6| < 5 \Rightarrow r \in (1, 11) \quad ...(3)$

and $|r + 6| > 5 \Rightarrow r \in (-\infty, -11) \cup (-1, \infty) \quad ...(4)$

From (3) and (4),

$r \in (1, 11)$

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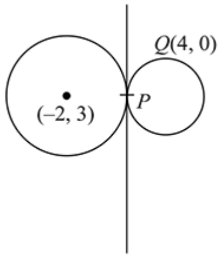
Q4

(3) The equation of circle $x^2 + y^2 + 4x - 6y = 12$ can be written as $(x+2)^2 + (y-3)^2 = 25$



Let $P = (1, -1)$ & $Q = (4, 0)$

Equation of tangent at $P(1, -1)$ to the given circle :

$x(1) + y(-1) + 2(x+1) - 3(y-1) - 12 = 0$

$$3x - 4y - 7 = 0 \quad ...(1)$$

The required circle is tangent to (1) at $(1, -1)$.

$$\therefore \quad (x-1)^2 + (y+1)^2 + \lambda(3x - 4y - 7) = 0 \quad ...(2)$$

Equation (2) passes through $Q(4, 0)$

$\Rightarrow 3^2 + 1^2 + \lambda(12 - 7) = 0 \Rightarrow 5\lambda + 10 = 0$

$\Rightarrow \lambda = -2$

Equation (2) becomes $x^2 + y^2 - 8x + 10y + 16 = 0$

$$\text{radius} = \sqrt{(-4)^2 + (5)^2 - 16} = 5$$

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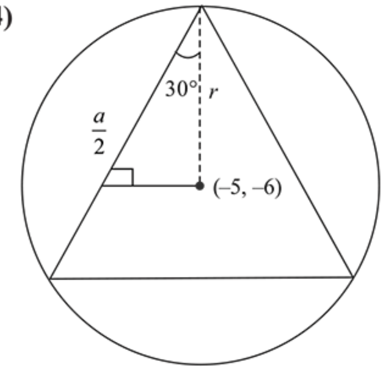
Q5

(4)



Let the sides of equilateral Δ inscribed in the circle be a, then

$$\cos 30° = \frac{a}{2r}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{2r}$$

$$a = \sqrt{3}r$$

Then, area of the equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4}\left(\sqrt{3}r\right)^2$$

$$= \frac{3\sqrt{3}}{4}r^2$$

But it is given that area of equilateral triangle $= 27\sqrt{3}$

Then, $27\sqrt{3} = \frac{3\sqrt{3}}{4}r^2$

$$r^2 = 36$$

$$\Rightarrow \quad r = 6$$

But $\left(-\frac{1}{2}\text{coeff. of } x\right)^2 + \left(-\frac{1}{2}\text{coeff. of } y\right)^2$ – constant term $= r^2$

$$(-5)^2 + (-6)^2 - c = 36$$

$$c = 25$$

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Q6

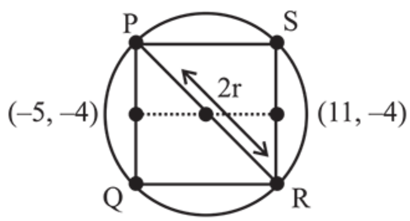
(3) The equation of circle is,

$x^2+y^2-6x+8y-103=0$

$\Rightarrow$ $(x-3)^2+(y+4)^2=(8\sqrt{2})^2$

$C(3,-4),\ r=8\sqrt{2}$

$\Rightarrow$ Length of side of square $=\sqrt{2}r=16$



$\Rightarrow$ $P(-5,4),\ Q(-5,-12)$

$R(11,-12),\ S(11,4)$

$\Rightarrow$ Required distance $=OP$

$=\sqrt{(-5-0)^2+(-4-0)^2}=\sqrt{25+16}=\sqrt{41}$

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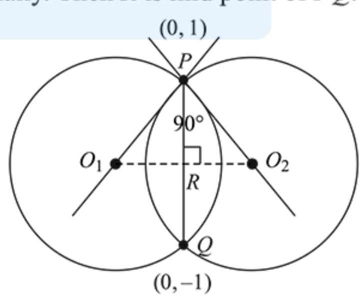
Q7

(2) $\because$ Two circles of equal radii intersect each other orthogonally. Then R is mid point of PQ.



and $PR=O_1R=O_2R$

$$PR=\frac{1}{2}\sqrt{(0-0)^2+(1+1)^2}=1$$

$\therefore$ Distance between centres $=1+1=2$.

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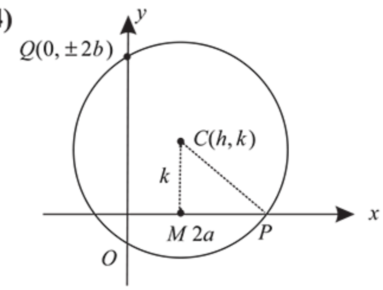
Q8

(4)



Let centre be $C(h,k)$

$CQ=CP=r$

$\Rightarrow$ $CQ^2=CP^2$

$(h-0)^2+(k\pm 0)^2=CM^2+MP^2$

$h^2+(k\pm 2b)^2=k^2+4a^2$

$h^2+k^2+4b^2\pm 4bk=k^2+4a^2$

Then, the locus of centre $C(h,k)$

$x^2+4b^2\pm 4by=4a^2$

Hence, the above locus of the centre of circle is a parabola.

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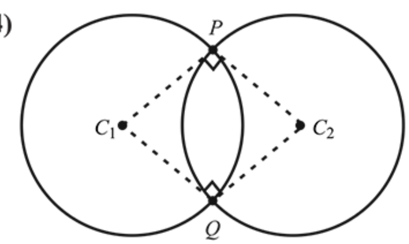
Q9

(4)



$2g_1g_2+2f_1f_2=2(-1)(-3)+2(-1)(-3)=12$

$c_1+c_2=14-2=12$

Since, $2g_1g_2+2f_1f_2=c_1+c_2$

Hence, circles intersect orthogonally

$\therefore$ Area of the quadrilateral PC_1QC_1

$=\ 2\left(\frac{1}{2}(C_1P)(C_2P)\right)$

$=2\times\frac{1}{2}r_1r_2=(2)(2)=4$ sq, units

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Q10

(3) Condition 1: The centre of the two circles are (1, 1) and (9, 1). The circles are on opposite sides of the line $3x + 4y - \lambda = 0$.

Put $x = 1, y = 1$ in the equation of line,

$$3(1) + 4(1) - \lambda = 0$$

$$7 - \lambda = 0$$

Now, put $x = 9, y = 1$ in the equation of line,

$3(9) + 4(1) - \lambda = 0$

Then, $(7 - \lambda)(27 + 4 - \lambda) < 0$

$\Rightarrow \quad (\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)\ldots(1)$

Condition 2: Perpendicular distance from centre on line $\geq$ radius of circle.

For $x^2 + y^2 - 2x - 2y = 1$,

$\Rightarrow \quad \dfrac{|3+4-\lambda|}{5} \geq 1$

$\Rightarrow \quad |\lambda - 7| \geq 5$

$\Rightarrow \quad \lambda \geq 12 \quad$ or $\quad \lambda \Rightarrow 2$ ...(2)

For $x^2 + y^2 - 18x - 2y + 78 = 0$

$\dfrac{|27+4-\lambda|}{5} \geq 2$

$\Rightarrow \quad \lambda \geq 41 \quad$ or $\quad \lambda \leq 21$...(3)

Intersection of (1), (2) and (3) gives $\lambda \in [12, 21]$.

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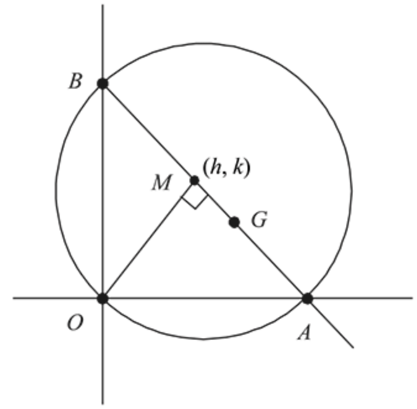
Q11

(2) As $\angle AOB = 90°$



Let AB diameter and $M(h, k)$ be foot of perpendicular, then

$$M_{AB} = —$$

Then, equation of AB

$$(y - k) = \frac{-h}{k}(x - h)$$

$$\Rightarrow \quad hx + ky = h^2 + k^2$$

Then, $A\left(\dfrac{h^2+k^2}{h}, 0\right)$ and $B\left(0, \dfrac{h^2+k^2}{k}\right)$

$\because AB$ is the diameter, then

$AB = 2R$

$\Rightarrow \quad AB^2 = 4R^2$

$$\Rightarrow \quad \left(\frac{h^2+k^2}{h}\right)^2 + \left(\frac{h^2+k^2}{k}\right) = 4R^2$$

Hence, required locus is $(x^2 + y^2)^3 = 4R^2 x^2 y^2$

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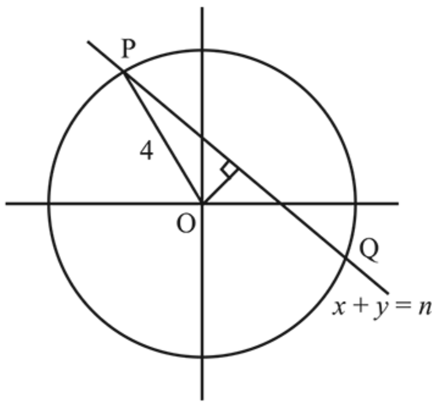
Q12

(4) Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at P and Q Length of perpendicular from O on PQ

$$= \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$$



Then, length of chord $PQ = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2}$

$$= 2\sqrt{16 - \frac{n^2}{2}}$$

Thus only possible values of n are 1, 2, 3, 4, 5.
Hence, the sum of squares of lengths of chords

$$= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right)$$

$$= 64 \times 5 - 2.\frac{5\times6\times11}{6} = 210$$

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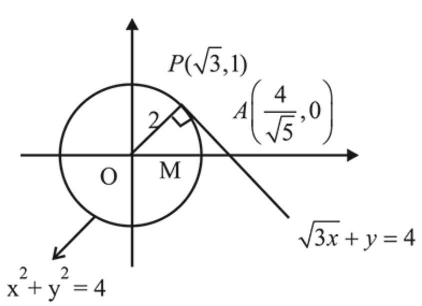
Q13

(3) Equation of tangent to circle at point $(\sqrt{3}, 1)$ is $\sqrt{3}x + y = 4$



$\therefore$ coordinates of the point $\left(\sqrt{\ }, 0\right)$

$$\text{Area} = \frac{1}{2}\times OA\times PM = \frac{1}{2}\times\frac{4}{\sqrt{3}}\times1 = \frac{2}{\sqrt{3}} \text{ sq. units}$$

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Q14

(1) Let any tangent to circle $x^2 + y^2 = 1$ is
$x\cos\theta + y\sin\theta = 1$
Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P \equiv \left(\frac{1}{\cos\theta}, 0\right)$ & $Q \equiv \left(0, \frac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be $M \equiv \left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) \equiv (h, k)$

$$\Rightarrow \cos\theta = \frac{1}{2h} \quad ...(1)$$

$$\sin\theta = \frac{1}{2k} \quad ...(2)$$

Now squaring and adding equation (1) and (2)

$$\frac{1}{h^2} + \frac{1}{k^2} = 4$$

$\Rightarrow h^2 + k^2 = 4h^2k^2$
$\therefore$ locus of M is : $x^2 + y^2 = 4x^2y^2$

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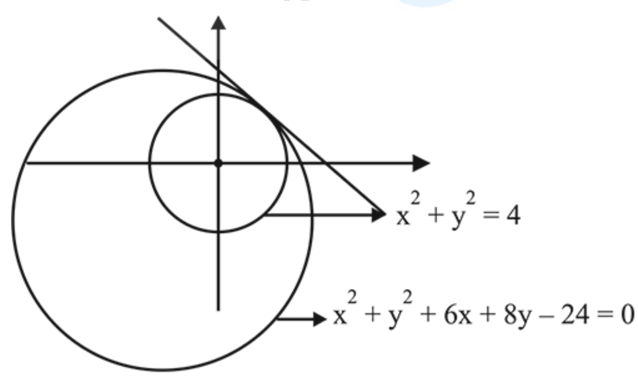
Q15

(3) By the diagram, $d_{c_1c_2} = |r_1 - r_2|$



Equation of common tangent is,
$S_1 - S_2 = 0$
$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$
Hence $(6, -2)$ lies on it.

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Q16

(2) $S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$S_2 \equiv x^2 + y^2 + Kx + \frac{3}{2}y - \frac{1}{2} = 0$

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 4Kx + \frac{y}{2} + K + \frac{1}{2} = 0$ ...(1)

Equation of the line passing through the intersection points P & Q is,

$4x + 5y - K = 0$ ...(2)

Comparing (1) and (2),

$\frac{4K}{4} = \frac{1}{10} = \frac{2K+1}{-2K}$ ...(3)

$\Rightarrow K = \frac{1}{10}$ and $-2K = 20K + 10$

$\Rightarrow 22K = -10 \Rightarrow K = \frac{-5}{11}$

$\because K = \frac{1}{10}$ or $\frac{-5}{11}$ is not satisfying equation (3)

$\therefore$ No value of K exists.

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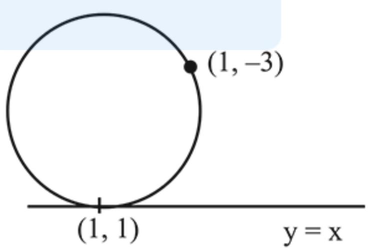
Q17

(2) Equation of circle which touches the line $y = x$ at (1, 1) is, $(x - 1)^2 + (y - 1)^2 + \lambda (y - x) = 0$

This circle passes through (1, – 3)



$\therefore 0 + 16 + \lambda(-3 - 1) = 0$

$\Rightarrow 16 + \lambda\ (-4) = 0 \Rightarrow \lambda = 4$

Hence, equation of circle will be,

$(x - 1)^2 + (y - 1)^2 + 4y - 4x = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\therefore$ Radius $= \sqrt{9+1-2} = 2\sqrt{2}$

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Q18

(2) Let centre of required circle is (h, k).

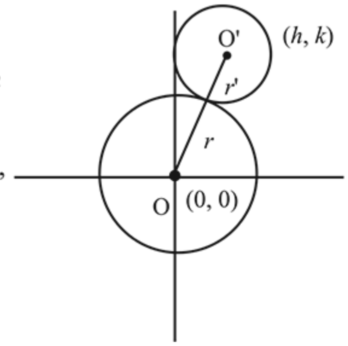
$\therefore OO' = r + r'$ [By the diagram]

$\Rightarrow \sqrt{h^2 + k^2} = 1 + h$

$\Rightarrow h^2 + k^2 = 1 + h^2 + 2h$

$\Rightarrow k^2 = 1 + 2h$

$\therefore$ locus is $y = \sqrt{1+2x}$, $x \geq 0$



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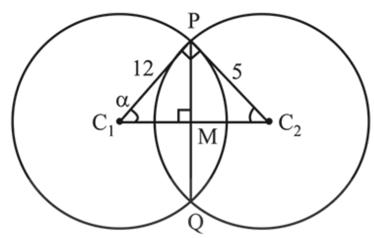
Q19

(2)



According to the diagram,

In ΔPC_1C_2, $\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$

In ΔPC_1M, $\sin\alpha = \frac{PM}{12} \Rightarrow \frac{5}{13} = \frac{PM}{12} \Rightarrow PM = \frac{60}{13}$

Hence, length of common chord $(PQ) = \frac{120}{13}$

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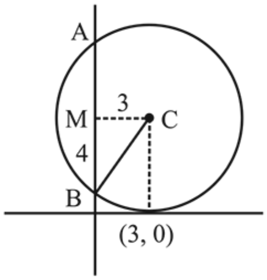
Q20

(1) Let centre of circle is C and circle cuts the y-axis at B and A. Let mid-point of chord BA is M.



$CB = \sqrt{MC^2 + MB^2}$

$\sqrt{3^2 + 4^2} = 5$ = radius of circle

$\therefore$ equation of circle is,

$(x-3)^2 + (y-5)^2 = 5^2$

(3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation $(x-3)^2 + (y-5)^2 = 5^2$

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