Contents

0	Prov	ve It!	1		
1	Log	arithms	3		
2	Not	Just For Right Triangles	10		
	2.1	Trigonometric Functions	10		
	2.2	Graphing Trigonometric Functions	12		
	2.3	Going Backwards	14		
	2.4	Tying It All Together	15		
	2.5	Solving Problems Using Trigonometric Identities	20		
3	Moı	More Triangles! 24			
	3.1	Triangle Laws	24		
	3.2	Areas, Areas	27		
	3.3	More Important Lines	29		
4	Cyclic Quadrilaterals 3				
	4.1	Properties of Cyclic Quadrilaterals	33		
	4.2	Finding Cyclic Quadrilaterals	33		
	4.3	Ptolemy's Theorem	35		
5	Con	ics and Polar Coordinates	38		
	5.1	Parabolas	38		
	5.2	Ellipses	40		
	5.3	Hyperbolas	44		
	5.4	Polar Coordinates Revisited	47		
	5.5	That Pesky <i>xy</i> Term	49		
6	Poly	Polynomials 5			
	6.1	What is a Polynomial?	52		
	6.2	Multiplying and Dividing Polynomials	52		
	6.3	Finding Roots of Polynomials			
	6.4	Coefficients and Roots	60		

<u>-4</u>	>		CONTENTS			
	(F	Transforming Polynomials	62			
	6.5 6.6	Newton's Sums				
7	Fun	ctions	69			
•	7.1	The Inverse of a Function				
	7.2	Functional Identities				
	7.2	Solving Functional Identities				
	7.5	7.3.1 Isolation				
		7.3.2 Substituting in Values				
		7.3.3 Using Cyclic Functions				
		7.3.4 Arbitrary Functions				
8	Taki	ing it to the Limit	76			
	8.1	What is a Limit?				
	8.2	Tricky				
	8.3	Working with Limits				
	8.4	Continuity				
	8.5	Asymptotes				
	8.6	Trig Limits				
	8.7	e				
9	Complex Numbers 88					
	9.1	Drawing the Complex Numbers	88			
	9.2	The Complex Absolute Value				
	9.3	Complex Multiplication and Coordinates				
	9.4	Complex Powers and Geometry				
	9.5	DeMoivre's Theorem	92			
	9.6	Exponential Form	94			
	9.7	Two for One	96			
	9.8	The Roots of Unity				
10	Vect	tors and Matrices	100			
	10.1	What is a Vector?	100			
	10.2	The Dot Product	101			
	10.3	Coordinate Representation of Vectors	102			
		What is a Matrix?				
	10.5	Matrix Multiplication	104			
	10.6	Matrices in Higher Dimensions	107			
		Better Matrix Notation				
11	Cros	ss Products and Determinants	112			

tn	e ART of PROBLEM SOLVING: volume 2	4 -3
	11.1 The Cross Product	. 112
	11.2 The Cross Product in Coordinates	. 113
	11.3 The Determinant	. 113
	11.4 Determinants in Higher Dimensions	. 115
	11.5 Minors	. 116
	11.6 Row and Column Operations	. 118
	11.7 The Inverse of a Matrix	. 120
12	Analytic Geometry	124
14	12.1 Lines, Angles, and Distances	
	12.2 Parameters	
	12.3 Vectors	
	12.4 Points, Lines, and Planes	
	12.5 Curved Surfaces	
	12.6 Using Analytic Geometry	
	12.7 Vectors and Geometry Problems	
13	Equations and Expressions	143
	13.1 Linear Equations	
	13.2 Convenient Systems	
	13.3 Symmetric Expressions and Advanced Factorizations	
	13.4 More Polynomials	
	13.5 Squares and Cubes	
	13.6 Using Graphing	. 155
14	Inequalities	159
	14.1 Trivial Inequality Revisited	. 159
	14.2 Arithmetic Mean-Geometric Mean Inequality	
	14.3 Cauchy's Inequality	
	14.4 Maximization and Minimization	. 164
	14.5 Geometry and Inequalities	. 165
	14.6 Wrap-Up and Parting Hints	. 166
15	Combinatorics	170
13	15.1 Identities	
	15.2 Pascal's Identity	
	15.3 More Identities	
	15.4 Block Walking	
	15.5 The Binomial Theorem	
	20.0 21.0 21.001.01. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	. 170
16	Sequences and Series	180

<u>-2</u>	>		CONTEN	<i>ITS</i>
	171	F. C. Col. P.		100
		Fractions in Other Bases		
		\sum ome \sum pecial \sum eries		
		The Fibonacci Numbers		
		Dealing with Recurrences		
		Dealing with Sums		
	16.6	The Binomial Theorem Revisited		188
	16.7	Harmonic Sequences		191
17	Cou	nting in the Twilight Zone		196
	17.1	One to One		196
	17.2	Clever Correspondences		196
	17.3	Easy as		198
		Generating Functions		
	17.5	Partitions		203
	17.6	Counting on Graphs		204
		Counting Infinite Sets		
18	Agai	in and Again		211
	_	Repeats		211
		Off to Infinity		
		Rational Continued Fractions		
		Real Continued Fractions		
19	Prob	pability		216
1,		Review, Definitions, and Notation		
		Going a Step Further		
		Geometry and Probability		
		Conditional Probability		
	17.1	Conditional Probability		21)
20	Find	l It and Make It		224
	20.1	Locus		224
		Construction		
21	Coll	linearity and Concurrency		233
		Three Points and a Line		233
		Three Lines and a Point		
22	Geo	metry Tidbits		241
		Projections		241
		Inversion		
		Homothecy		
		Geometric Continuity		246

<u>th</u>	e ART of PROBLEM SOLVING: Volume 2	4	-1			
	22.5 Given a Finite Number of		247			
23	Number Theory		252			
	23.1 Divisibility		252			
	23.2 Division in Congruences		253			
	23.3 Solving Linear Congruences		254			
	23.4 Solving Quadratic Congruences		256			
	23.5 The Sum of the Divisors		257			
	23.6 Fermat's Theorem		258			
	23.7 The ϕ Function		260			
	23.8 Wilson's Theorem		262			
24	Diophantine Equations		266			
	$24.1 \ ax + by = c \dots \dots$		266			
	$24.1.1 c = 0 \dots \dots \dots \dots \dots \dots \dots \dots \dots $					
	$24.1.2 c \neq 0 \dots \dots$					
	24.2 $x^2 + y^2 = z^2$					
	24.3 $x^4 + y^4 = z^2$					
	24.4 The Pell Equation					
	24.5 General Methods					
25	5 Graph Theory 2					
	25.1 Points and Lines					
	25.2 Planar Graphs					
	25.3 Example: The Platonic Solids					
	25.4 Walking Around on Graphs					
	25.5 Euler Trails					
	25.6 Colorings					
26	Parting Shots		284			