27/04/21

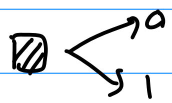
Quantum Computing 0-100

Agenda

* Bits to Qubits
  - → Qubits, Dirac notation
  - → Superposition, Measurement
* Multiple Qubits
  - → Multi qubit states
  - → Entanglement
  - → Bell pairs
  - → NO cloning
* Why Quantum
  - → Parallelism
  - → Deutsch algorithm
  - → QKD
* Road ahead

0/1

00110011 → classical



↓ Quantum

 → $\underbrace{0 \text{ and } 1}_{\text{Super position.}}$

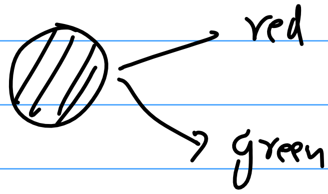
$0 \to \underbrace{|0\rangle}_{\text{Dirac braket}}$ $\qquad$ $1 \to \underbrace{|1\rangle}_{\text{Dirac braket}}$

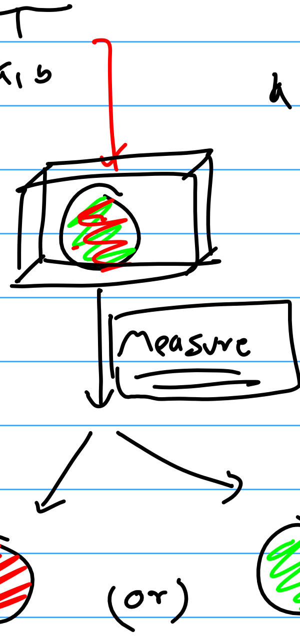
* Superposition }
* Entanglement }

Qubit (cbit) → $\underbrace{\underline{a}|\underline{0}\rangle + \underline{b}|\underline{1}\rangle}_{a,b}$ $\quad (a\underline{x} + b\underline{y})$

a, b - probabilities

$\underbrace{|a|^2}_{1-p} + \underbrace{|b|^2}_{p} = 1$



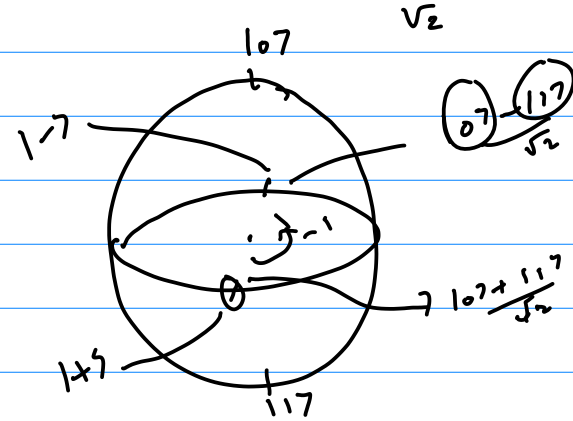


H

$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \rightarrow |+\rangle$

phase factor

$|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}} \rightarrow |-\rangle$



$$\begin{pmatrix} a \\ b \end{pmatrix} \xrightarrow{H} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow |0\rangle$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

<u>CNOT:-</u>

$$|0\rangle|0\rangle \xrightarrow{H} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle \xrightarrow{CNOT(1\rightarrow 2)} \boxed{\frac{|00\rangle + |11\rangle}{\sqrt{2}}}$$

Ball 1 Ball 2

$$\frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

$|11\rangle$

If B1 = $|0\rangle$
Does nothing
B1 = $|1\rangle$
Flips B2

$\cancel{|01\rangle \; |10\rangle}$

$|0\rangle|0\rangle \rightarrow |00\rangle$

Deutche Algorithm

Goal:- Given a function $f(\underset{\hookrightarrow 0/1}{\ }) = \begin{matrix} b(0) \\ b(1) \end{matrix}$

Classically, I have to calculate $f(0)$, $f(1)$

Then check $f(0) \overset{?}{=} f(1)$

$$|01\rangle \xrightarrow{H,H} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$(-)$

$\downarrow$ CNOT $(1 \to 2)$

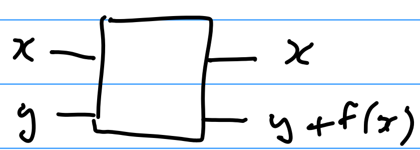
$$\left(|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + |1\rangle\left(\frac{|1\rangle - |0\rangle}{\sqrt{2}}\right)\right)$$

$(-)$ $(-)$

$$\hookrightarrow \boxed{\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)}$$

$f()$

$$y \xrightarrow{x} y + f(x)$$



$f(x) = x$

$\hookrightarrow$ CNOT

$x=0, \quad y=0 \qquad y+f(x) = 0$

$x=1, \quad y=0 \qquad y+f(x) = 1$

$\Rightarrow 1$

$x=1, \quad y=1 \qquad y+f(x) = 0$

$x=1, \quad y=0 \qquad y+f(x) = 1$

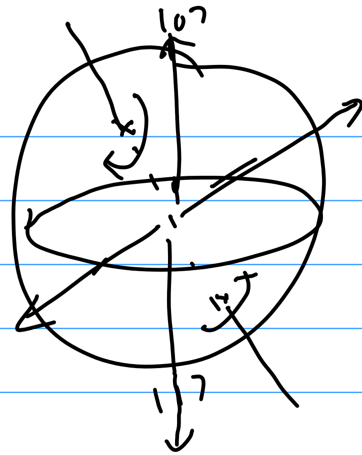
$$\Rightarrow \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \xrightarrow{H_1} |1\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$H\,(|1\rangle \longrightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}})$$

$f(0) \neq f(1)$

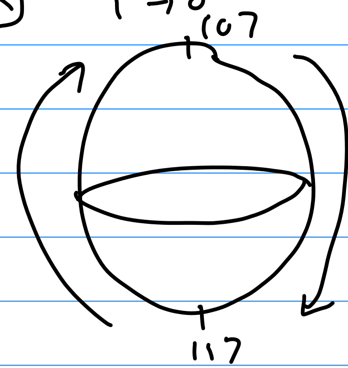
$f(0) = f(1) \rightarrow |0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$

H



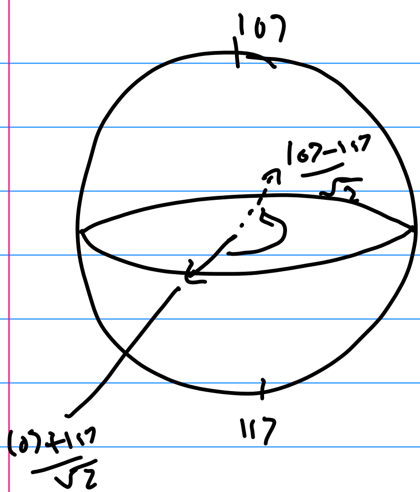
(NOT)

$\boxed{X}$ $0 \to 1$, $1 \to 0$

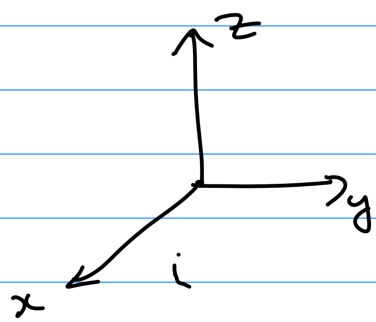


$\boxed{Z}$ $0 \to 0$, $1 \to -1$

$a|0\rangle + b|1\rangle \xrightarrow{Z} a|0\rangle - b|1\rangle$

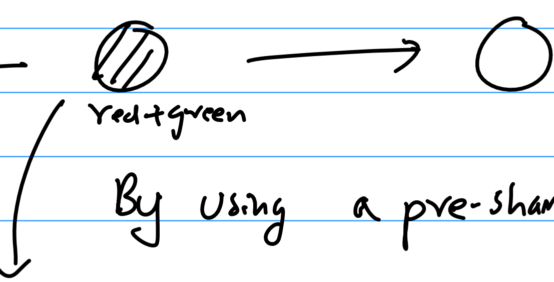


$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \xrightarrow{Z} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

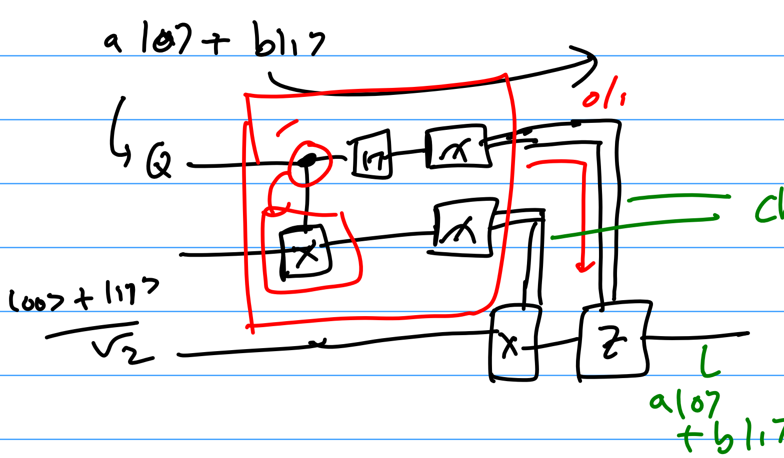


Quantum Teleportation

Goal —



By using a pre-shared entangled pair $\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$



Classical information

By sending 2 classical bits, I can share 1 qubit

Quantum Key Distribution

$A \xrightarrow{m} B$

E

$m \xrightarrow{Enc} C$

requires : key

need not be secure

$A \xrightarrow{key} B$

* Measurement in different basis

$|+\rangle$, $|-\rangle$

$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\quad$ $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$

, $|0\rangle$, $|1\rangle$

$\rightarrow |0\rangle$

$\rightarrow |1\rangle$

$$Q_1 \rightarrow |0\rangle \rightarrow \frac{1}{2}\left(\underbrace{|0\rangle + |1\rangle}_{|+\rangle} + \underbrace{|0\rangle - |1\rangle}_{|-\rangle}\right)$$

$$\frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|0\rangle \xrightarrow{(0,1)} 0 \quad (\text{no } 1)$$

$$\xrightarrow{(+,-)} |+\rangle \text{ (or) } |-\rangle \ (50\%)$$

$|0\rangle \longrightarrow$

$b_1 \longrightarrow$ decides measurement in $(|0\rangle, |1\rangle)$ (or) $(|+\rangle, |-\rangle)$

$b_1 = 0$

$b_1 = 1$

A

B

$|0\rangle$

Chooses b_1

$b_1 = 0$

$|0\rangle$

$\times n$

E

$|0\rangle \to |+\rangle$

$E \to b_3$

$|0\rangle$

Does not know b_1

Guess b_1 as b_2

$c_1 \ldots c_n$

$\times$ if $b_2 = 0$

$|0\rangle \xrightarrow{X} 0$ (100%)

$b_2 = 1$

$|0\rangle \xrightarrow[(+,-)]{X} 0/1$ (50%)

$b_1 \stackrel{?}{=} b_2$

b_2

Yes / No

$\cancel{0}$ (100%)