Computer Science 271
Project 0001
Due Wednesday, February 1

Type your answers in LaTeX. Please hand in a PDF of your solutions named `proj1_yourname.pdf`.

1. Consider the searching problem:

   Input: a sequence $A = \langle a_1, a_2, \ldots, a_n \rangle$ and a value v
   Output: an index i such that $v = A[i]$ or NIL is v is not found

   The linear search algorithm scans through the sequence one element at a time until an element $A[i] = v$ is found, at which point it stops and returns i.

   (a) Write pseudocode for a linear search.

   (b) Write a loop invariant for the loop in your algorithm.

   (c) Prove that the loop invariant is correct.

   (d) Use the termination condition of your loop invariant to prove that your algorithm is correct.

   (e) What are the best case, worst case, and average case time complexities for linear search, using Θ notation? For the average case, assume v is equally likely to match any element in the array. Justify your answers.

2. Consider the bubble sort algorithm:

   ```
   BubbleSort(A, n)
       for i = 1 to n − 1
           for j = n downto i + 1
               if A[j] < A[j − 1] then
                   exchange A[j] and A[j − 1]
   ```

   (a) State precisely a loop invariant for the inner for loop, and prove that this loop invariant is correct.

   (b) State a loop invariant for the outer for loop, and use the termination condition of the inner loop invariant to prove that the outer loop invariant is correct.

   (c) Use the termination condition of the outer loop invariant to prove that the algorithm is correct.

   (d) What are the best case and worst case time complexities for BubbleSort, in Θ notation? Justify your answers.

3. Use mathematical induction to prove that, for all integers $n \geq 0$,

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1.$$

4. Use mathematical induction to prove that, for all integers $n \geq 1$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$