Computer Science 271
Project `0010`
Due Wednesday, February 8

Type your answers in LaTeX.

1. Prove Theorem 3.1 on page 48:

   For any two functions $f(n)$ and $g(n)$, $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

2. Prove the following using the definitions of O, Ω, and Θ.

   (a) $n^2 + 3n - 20 = O(n^2)$

   (b) $n - 2 = \Omega(n)$

   (c) $\log_{10} n + 4 = \Theta(\log_2 n)$

   (d) $2^{n+1} = O(2^n)$

   (e) $\ln n = \Theta(\log_2 n)$

   (f) $n^\epsilon = \Omega(\lg n)$ for any $\epsilon > 0$

3. For each of the following recurrences, find a tight upper bound for $T(n)$. It will be useful to remember that

   $$\sum_{j=0}^{\infty} a^i = \frac{1}{1-a}, \text{ if } |a| < 1.$$

   Prove that each is correct using induction. In each case, assume that $T(n)$ is constant for $n \leq 2$.

   (a) $T(n) = 2T(n/2) + n^3$

   (b) $T(n) = T(9n/10) + n$

   (c) $T(n) = 7T(n/3) + n^2$

   (d) $T(n) = T(\sqrt{n}) + 1$

   (e) $T(n) = T(n-1) + \lg n$

4. *Carefully* prove by induction that the i^{th} Fibonacci number satisfies the equality

   $$F_i = \frac{\phi^i - \widehat{\phi}^i}{\sqrt{5}},$$

   where $\phi = (1 + \sqrt{5})/2$ is the golden ratio and $\widehat{\phi} = (1 - \sqrt{5})/2$ is its conjugate.