Fractals and Chaos

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Today's Presentation

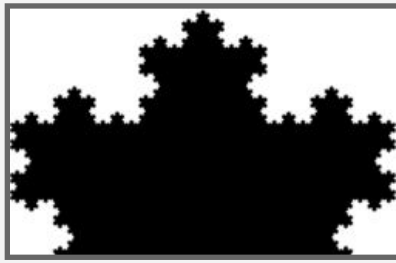
Today we will be presenting the article:

Chaos and Fractals in Human Physiology
Author(s): Ary L. Goldberger, David R. Rigney and Bruce J. West
Source: Scientific American

The Article talks about how fractals and chaos are an important part of the human body and how chaos in bodily functioning signals health and periodic behavior can often foreshadow disease, it uses several examples along the way to achieve this.

INTRODUCTION TO FRACTALS

Introduction to Fractals



A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales created by repeating a simple process over and over in an ongoing feedback loop.

Fractal patterns are extremely familiar, since nature is full of fractals. For instance: trees, rivers etc.

Self Similarity

In mathematics, a self-similar object is exactly or approximately similar to a part of itself. Many objects in the real world, such as coastlines, are statistically self-similar, i.e. parts of them show the same statistical properties at many scales.

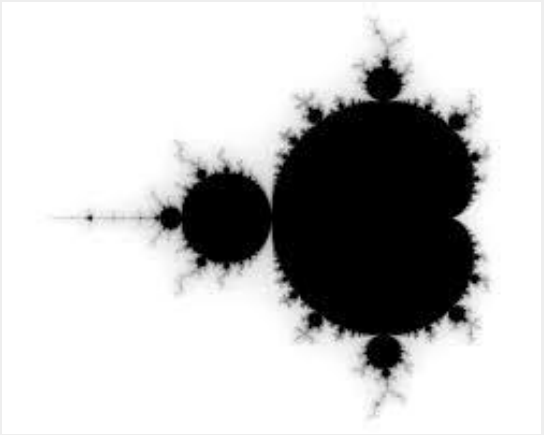
Self-similarity has important consequences for the design of computer networks, stock markets, nature and music.

Mandelbrot Set

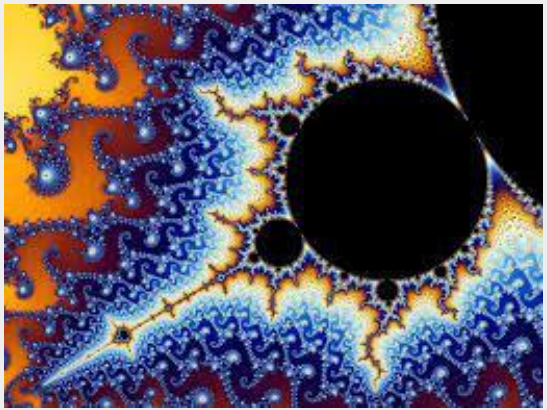
–The term "fractal" was first used by mathematician Benoit Mandelbrot in 1975. The Mandelbrot set is a fractal with many mathematical implications. We shall introduce the Mandelbrot set to you:

-Consider the equation:

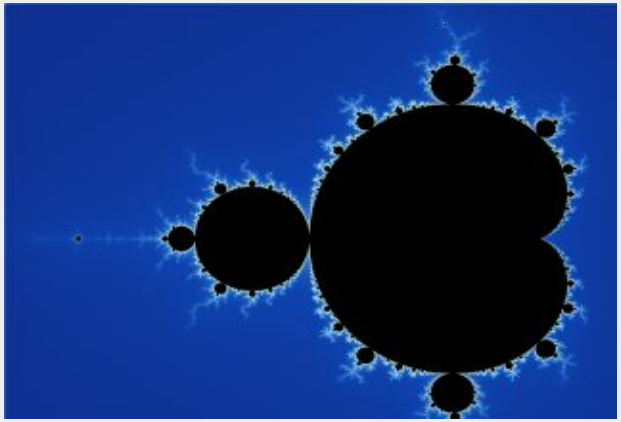
$$z_{n+1} = z_n^2 + c$$



The Mandelbrot set is self-similar under magnification in various neighbourhood points, making it one of the most awe-inspiring mathematical discoveries.



$$z_{n+1} = z_n^2 + c$$



For our result, we assign z_0 =0.

1) Put c=+1;

the sequence is 0, 1, 2, 5, 26,…., which tends to infinity.

2) Put c=-1

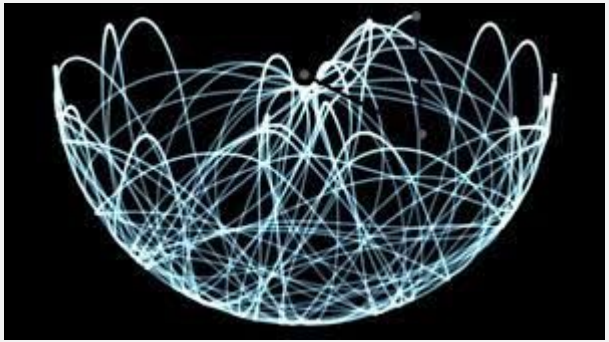
the sequence is 0, –1, 0, –1, 0, ..., which is bounded.

- A complex number c is a member of the Mandelbrot set if, when starting with z_0 = 0 and applying the iteration repeatedly, the absolute value of z_n remains bounded for all $n > 0$.
- The different shades in the image signify the rate at which the iteration tends to infinity.

Chaos

Chaos is the science of surprises, of the nonlinear and the unpredictable. Under some circumstances deterministic nonlinear systems-those that have only a few simple elements-behave erratically, a state called chaos.

The deterministic chaos of nonlinear dynamics is not the same as chaos in the dictionary sense of complete disorganization or randomness.



Chaos and Fractals

Fractal structures are often the remnants of chaotic nonlinear dynamics. Wherever a chaotic process has shaped an environment, fractals are likely to be left behind as seen in the case of rock formation and coastline.

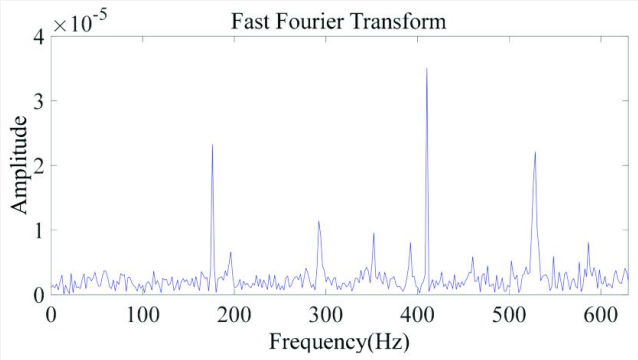
Chaos and fractals are subjects associated with the discipline of nonlinear dynamics: the study of systems that respond disproportionately to stimuli.

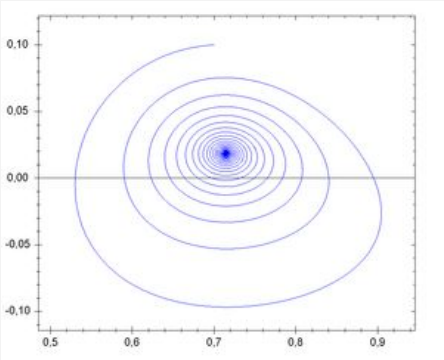
Recognizing the chaotic, fractal nature of our world can give us new insight, power, and wisdom.

Chaotic Vs Periodic

There are two major ways to test whether a system is chaotic or periodic in nature:

1. Fourier Spectrum Analysis
2. Phase-Space Representation

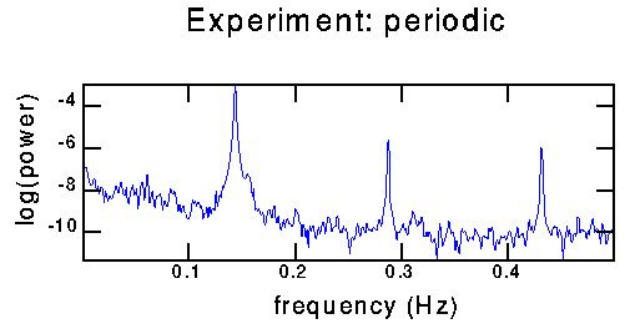


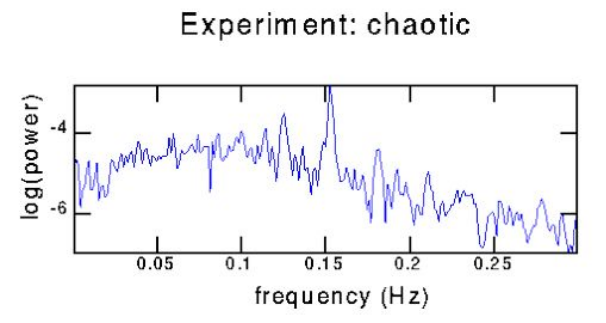


Fourier Spectrum Analysis

The Fourier spectrum of any waveform (such as the time series plot) reveals the presence of periodic components. If there is periodicity, the spectrum would show a sharp spike at the frequency.

On the other hand, the time series plot of a chaotic system would generate a spectrum that showed either broad peaks or no well defined peaks.



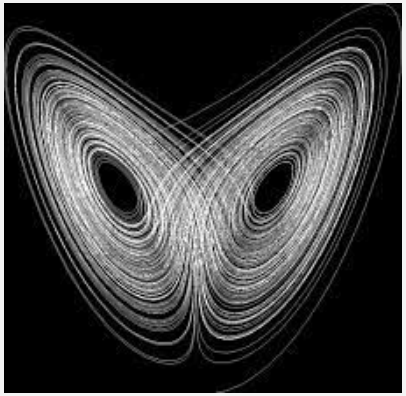


Phase Space Representation

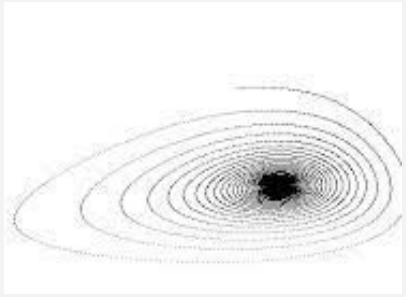
This technique tracks the values of independent variables that change with time. A phase space is essentially a space in which all possible states of a system are represented, with each possible state corresponding to one unique point in the phase space.

To identify the type of system dynamics (chaotic or periodic), one determines the trajectories for many different initial conditions. Then one searches for an attractor: a region of phase space that attracts trajectories.

Phase Space Representation (Cont.)



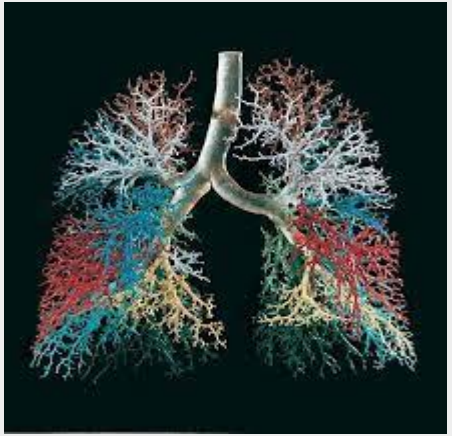
Chaotic systems are defined by strange attractors. They describe systems that are neither static nor periodic. In the phase space near a strange attractor, two trajectories that started under almost identical conditions will diverge over the short term and become very different over the long term.



For periodic systems one could see simpler attractors like the point attractor.

FRACTALS IN HUMANS

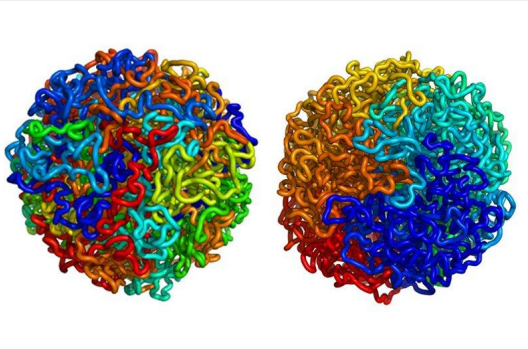
Fractals In Humans:



–In the human body, fractal like structures abound in networks of blood vessels, nerves and ducts, and various organ systems, providing tremendous advantages to the body.

-On analyzing various mammals, despite subtle interspecies differences, their dimensions represent a fractal.

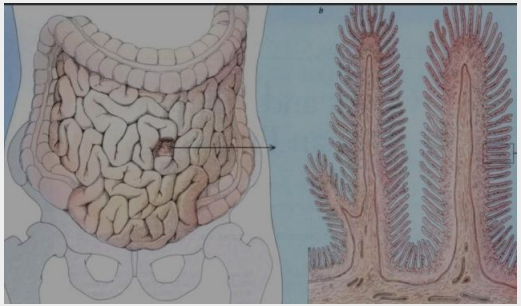
Fractal anatomy



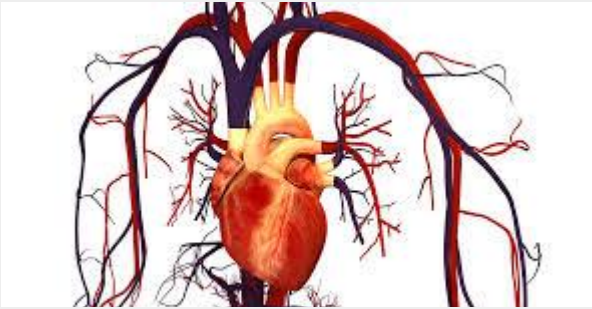
–Fractal branches or folds greatly amplify the surface area available for absorption (as in the intestine), distribution or collection (by the blood vessels, bile ducts and bronchial tubes) and information processing (by the nerves).

-It was recently discovered that Chromatid is a fractal, which keeps DNA from getting tangled.

Fractal structures, partly by virtue of their redundancy and irregularity, are robust and resistant to injury.



HEART



1) Fractal-like network of coronary arteries and veins conveys blood to and from the heart muscles.

2) A fractal like canopy of connective tissue fibers within the heart tethers valves to the underlying muscles.

3) Fractal architecture is also evident in the branching pattern of certain cardiac muscles, as well as in the His-Purkinje system, which conducts electrical signals from the atria to the cardiac muscles of the ventricles.

Hence, the heart may continue to pump with relatively minimal mechanical dysfunction, despite extensive damage to the His-Purkinje system.

Fractals and cognition

–There is proof for fractal visual processing in us; we may have evolved to be skilled decoders of the fractals that surround us in nature.

–This means that we are built to process fractals easily and efficiently while at the same time, being subconsciously drawn to the patterns. Nature's most common fractal dimensions fall between 1.3 and 1.5.

–A similar study found that looking at mid dimension fractals reduces stress by as much as 60 percent. This may also be why patients recover quickly when they have a window with a natural view, as they produce a strong alpha wave response which corresponds to a wakeful relaxed state, and a strong beta wave response indicating a high ability to focus.

CHAOS IS HEALTH

Ideology of Medicine

The conventional wisdom in medicine holds that disease and aging arise from stress on an otherwise orderly and machine-like system, the stress decreases order by provoking erratic responses or by upsetting the body's normal periodic rhythms, thus suggesting chaos to be a signal of disease.

However in recent years it has been discovered that the heart and other physiological systems may behave most erratically when they are young and healthy.Irregularity and unpredictability, then, are important features of health. On the other hand, decreased variability and accentuated periodicities are associated with disease.

The Case of Heart Rate

In the early 1980' s, when investigators began to apply chaos theory to physiological systems, they expected that chaos would be most apparent in diseased or aging systems.

Indeed, intuition and medical tradition gave them good reason to think so. If one listens to the heart through a stethoscope or feels the pulse at the wrist, the rhythm of the heart seems to be regular. For an individual at rest the pulse strength and the interval between heartbeats seem roughly constant.

The Case of Heart Rate (Cont.)

Homeostasis-

Physicians have interpreted fluctuations in heart rate in terms of **the principle of homeostasis:**

Physiological systems normally operate to reduce variability and to maintain a constancy of internal function.

According to this theory, any physiological variable, including heart rate, should return to its "normal" steady state after it has been perturbed. The principle of homeostasis suggests that variations of the heart rate are merely responses to a fluctuating environment.

The Case of Heart Rate (Cont.)

More careful analysis reveals that healthy individuals have heart rates that fluctuate considerably even at rest on a time series of plot over a day.

If one concentrates on a few hours of the time series, one finds more rapid fluctuations whose range and sequence look somewhat like the original, longer time·series plot.

At even shorter time scales (minutes), one finds even more rapid fluctuations that again appear to be similar to the original plot.

The Case of Heart Rate (Cont.)

The beat to beat fluctuations on different time scales appear to be self similar, just like the branches of a geometric fractal. This finding suggests that the mechanism that controls heart rate may be intrinsically chaotic.

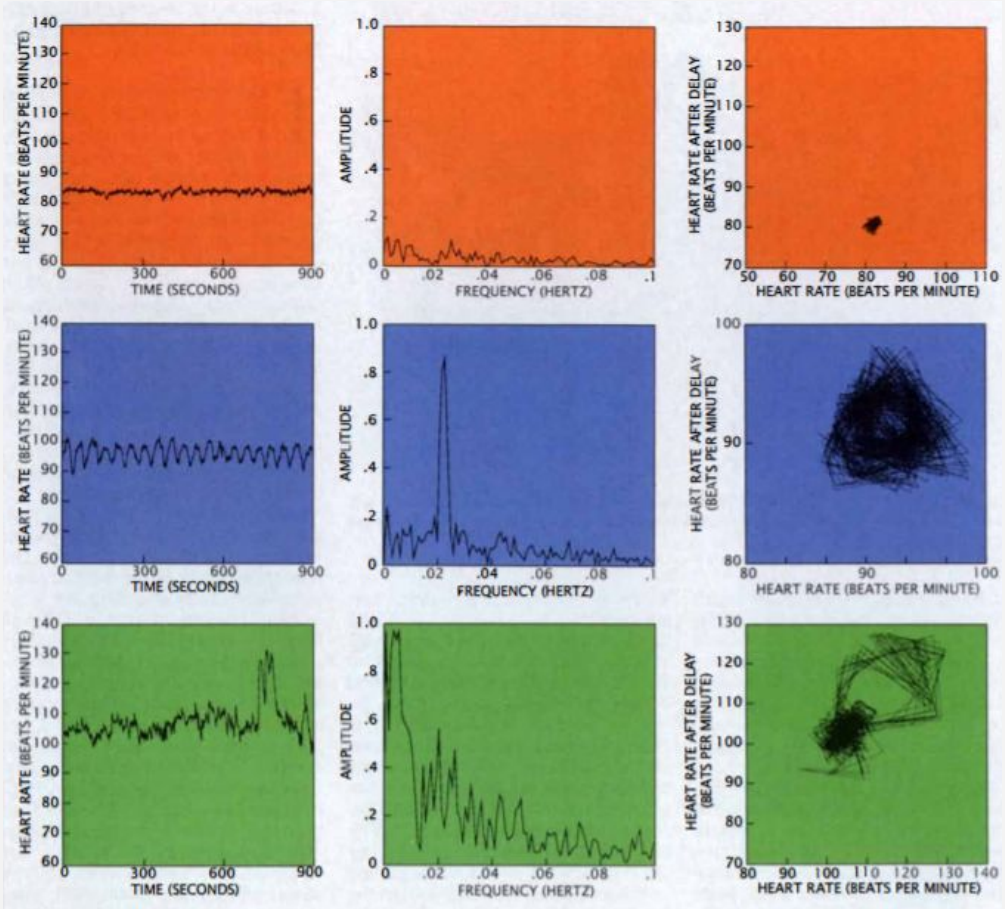
To investigate whether beat to beat heart rate variations are indeed chaotic or periodic:

1. We can compute the Fourier spectrum of the time series plot for heart rate, if the heart rate is periodic we would see a peak.
2. We can plot a phase space plot and look at the kind of attractor to determine whether heart rate is chaotic or periodic.

The Case of Heart Rate (Cont.)

HEART RATE is shown as time-series plots (left), Fourier spectra (center) and phase-space plots (right).

- A heart rate 13 hours before cardiac arrest (top).
- A heart rate eight days before sudden cardiac death (middle).
- A healthy heart rate (bottom)



The Case of Heart Rate (Cont.)

The mechanism for chaos in the beat-to-beat variability of the healthy heart probably arises from the nervous system. The sinus node (the heart's natural pacemaker) receives signals from the involuntary portion of the nervous system. The nervous system in turn has two major branches:

- Parasympathetic stimulation decreases the firing rate of sinus-node cells
- Sympathetic stimulation increases the firing rate of sinus-node cells

The influence of these two branches results in a constant tug-of-war on the pacemaker. The result of this continuous buffeting is fluctuations in the heart rate of healthy subjects causing it to show chaotic behaviour.

Conclusion

So many natural systems previously thought off limits to mathematicians can now be explained in terms of fractals, and by applying nature's best practices, we can then solve real world problems.

Fractals are a physical representation of what lies between order and chaos. Its philosophical parallelisms, and mathematical implications, still astound scientists. It’s a beautiful and complex subject of research, and we have tried to give a glimpse of that, through our presentation.

Thank You!