MAT 1201 – MATHEMATICAL METHODS I
Assignment 2

Instructions: Answer all the questions and summit solutions for **1, 2, 5, 7,8,9, 11,13, 15, 19,21** on or before 21st May, 2023

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1. Find the scalar product of $\vec{a} = 2\vec{i} - 3\vec{j} + 5\vec{k}$ and $\vec{b} = \vec{i} - 3\vec{j} + \vec{k}$ and hence find the cosine of the angle between $\vec{a}$ and $\vec{b}$.
2. If $\vec{a} = 10\vec{i} - 3\vec{j} + 5\vec{k}, \vec{b} = 2\vec{i} + 6\vec{j} - 3\vec{k}$ and $\vec{c} = \vec{i} + 10\vec{j} - 2\vec{k}$, verify that $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = \vec{a} \cdot (\vec{b} + \vec{c})$.
3. The resultant of two vectors $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{a}$. If $\left|\vec{b}\right| = \sqrt{2}\,|\vec{a}|$, show that the resultant of $2\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{b}$.
4. Let $\vec{a} = 2\vec{i} + 3\vec{j} - 4\vec{k}$ and $\vec{b} = \vec{i} + \vec{j} + 2\vec{k}$. Find the component of $\vec{a}$ on $\vec{b}$ and the component of $\vec{b}$ on $\vec{b}$.
5. Find the projection of $\vec{a} = 4\vec{i} + \vec{j}$ onto the vector $\vec{b} = 2\vec{i} + 3\vec{j}$.
6. Find the work done by a constant force $\vec{F} = 2\vec{i} + 4\vec{j}$ if its point of application to a block moves from $P_1(1, 1)$ to $P_2(4, 6)$. Assume that $\left|\vec{F}\right|$ is measured in newtons and the distance between P_1 and P_2 measured in meters.
7. If two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, find the angle between and $\vec{a}$ and $\vec{b}$.
8. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \mathbf{0}$. $|\vec{a}| = 3, \left|\vec{b}\right| = 5$, and $|\vec{c}| = 7$, find the abgle between $\vec{a}$ and $\vec{b}$.
9. If $\vec{u}$ and $\vec{v}$ are unit vectors making an angle $\frac{\pi}{3}$ with each other, find $|\vec{u} - \vec{v}|$.
10. Find the scalar m so that the vector $2\vec{i} + \vec{j} - m\vec{k}$ is perpendicular to the sum of the vectors $\vec{i} - \vec{j} + 2\vec{k}$ and $3\vec{i} + 2\vec{j} + \vec{k}$.
11. Find the value of λ such that the scalar product of the vectors $\vec{i} + \vec{j} + \vec{k}$ with unite vector parallel to the sum of the vectors $\lambda\vec{i} + 2\vec{j} + 3\vec{k}$ and $2veci + 4\vec{j} - 5\vec{k}$.
12. Given two vectors $\vec{a}$ and $\vec{b}$ ($\vec{a} \neq \mathbf{0}$, $\vec{b} \neq \mathbf{0}$), show that
    (a) if $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular, then $|\vec{a}| = \left|\vec{b}\right|$;
    (b) if $\left|\vec{a} + \vec{b}\right| = \left|\vec{a}\vec{b}\right|$, then $\vec{a}$ and $\vec{b}$ are perpendicular.
13. Find $\vec{a} \times \vec{b}$ in each of the following cases:
    (a) $\vec{a} = \vec{i} - \vec{j}$, $\vec{b} = 3\vec{j} + 5\vec{k}$
    (b) $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = 4\vec{i} - \vec{k}$

(c) $\vec{a} = \vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + 4\vec{k}$

(d) $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}, \vec{b} == \vec{i} + 3\vec{j} - \vec{k}$

(e) $\vec{a} = 8\vec{i} + \vec{j} - 6\vec{k}, \vec{b} = \vec{i} - 2\vec{j} + 10\vec{k}$

14. If $P_1\,(2,1,3)$, $P_2\,(0,3,-1)$ and $P_3\,(-1,2,4)$,$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$.

15. Find the volume of the parallelepiped whose three edge are $\vec{a} = 3\vec{i} + \vec{j} + \vec{k}, \vec{b} = \vec{i} + 4\vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{j} + 5\vec{k}$.

16. Find the unit vector that is perpendicular to both $\vec{a} = -\vec{i} - 2\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} - \vec{j}$.

17. Determine whether the four points $P_1\,(1,1,-2)$, $P_2\,(4,0,-3)$, $P_3\,(1,-5,10)$ and $P_4\,(-7,2,4)$ lie in the same plane.

    Determine whether the vectors $\vec{a} = 4\vec{i} + 6\vec{j}$, $\vec{b} = -2\vec{i} + 6\vec{j} - 6\vec{k}$, and $vecc = \frac{5}{2}\vec{i} + 3\vec{j} - \frac{1}{2}\vec{k}$ are coplanar.

18. Two vectors $\vec{a}$ and $\vec{b}$ lie in the xy-plane so that the angle between them is $\frac{2\pi}{3}$. If $|\vec{a}| = \sqrt{27}$ and $\left|\vec{b}\right| = 8$, find all posssible value of $\vec{a} \times \vec{b}$.

19. Determine λ such that the following vectors are coplanar:

$$\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = 2\vec{i} - 4\vec{k} \text{ and } \vec{c} = \vec{i} + \lambda\vec{j} + 3\vec{k}.$$

20. Find the constant a such that the vectors

$$2\vec{i} - \vec{j} + \vec{k}, \vec{i} + 2\vec{j} - 3\vec{k} \text{ and } 3\vec{i} + a\vec{j} + 5\vec{k}$$

    are coplanar.

21. If $\vec{a} = \vec{i} + \vec{j} - \vec{k}, \vec{b} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} - \vec{j} - \vec{k}$, find the vector $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$.