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* Differential equations reducable to homogeneous form

Consider the DE ;

$$\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$$

where a, b, c, a', b', c' are constants.

observe that when $c = 0$ and $c' = 0$, the DE reduces to,

$$\frac{dy}{dx} = \frac{ax + by}{a'x + b'y} = \frac{a + b(y/x)}{a' + b'(y/x)}$$, which is a

homogeneous equation.

But, when atleast, one of c and c' is non zero, the DE is not homogeneous.

$c = 0$	and	$c' \neq 0$
$c \neq 0$	and	$c' = 0$
$c = 0$	and	$c' = 0$

* Suppose, that the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are not parallel.

to make the given DE homogeneous, let $x = X + h$ and $y = Y + k$ where X and Y are new variables and h and k are constants, yet to be chosen

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$$\left.\begin{aligned} x &= X + h \\ y &= Y + k \end{aligned}\right\} \rightarrow \begin{aligned} dx &= dX \\ dy &= dY \end{aligned}$$

Now the given DE reduces to

$$\frac{dY}{dX} = \frac{a(X+h) + b(Y+k) + c}{a'(X+h) + b'(Y+k) + c'}$$

$$= \frac{aX + bY + (ah + bk + c)}{a'X + b'Y + (a'h + b'k + c')}$$

In order to have eq - ② as a homogeneous DE, choose h and k such that the following equations are satisfied;

$$ah + bk + c = 0$$
$$a'h + b'k + c' = 0$$

Now, ② $\rightarrow$ $$\frac{dY}{dX} = \frac{aX + bY}{a'X + b'Y}$$

$$= \frac{a + b(Y/X)}{a' + b'(Y/X)}$$

This can be solve using, by $Y = VX$

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Example 01

Solve ; $(2x+y-3)\,dy = (x+2y-3)\,dx$

Solution

This DE can be written as,

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$$

$$\frac{dY}{dX} = \frac{X+h+2(Y+k)-3}{2(X+h)+Y+k-3}$$

$$\frac{dY}{dX} = \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$$

Let,

$x = X + h$

$y = Y + k$

where h and k are constants to be determined.

$$\frac{dY}{dX} = \frac{X+2Y}{2X+Y}$$

to find h and k solve, the equations.

$$h + 2k - 3 = 0 \quad \text{— ①}$$

$$2h + k - 3 = 0 \quad \text{— ②}$$

① − 2×② ⇒ $h - 3 - 4h + 6 = 0$ $\qquad$ $k = 3 - 2\times 1$

$h = 1$ $\qquad$ $k = 1$

then having solved, we have $h=1$ and $k=1$,

then therefore,

$\therefore X = x - 1$ and $Y = y - 1$

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and, $\frac{dy}{dx} = \frac{x + 2y}{2x + y} = \frac{1 + 2(y/x)}{2 + (y/x)}$

To solve this HDE, put $y = vx$

Then, $\frac{dy}{dx} = \frac{d(vx)}{dx}$

$$\frac{dy}{dx} = v\frac{dx}{dx} + x.\frac{dv}{dx} = v + x.\frac{dv}{dx}$$

Then we have,

$$v + x.\frac{dv}{dx} = \frac{1 + 2v}{2 + v}$$

$$x.\frac{dv}{dx} = \frac{1 + 2v}{2 + v} - v$$

$$x.\frac{dv}{dx} = \frac{1 + \cancel{2v} - \cancel{2v} - v^2}{2 + v} = \frac{1 - v^2}{2 + v}$$

seperating variables, we have,

$$\frac{v + 2}{1 - v^2}\, dv = \frac{dx}{x}$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int \frac{dx}{x} = \ln|x| + c$$

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Partial fractions

$$\frac{v+2}{1-v^2} = \frac{v+2}{(1-v)(1+v)} = \frac{A}{(1-v)} + \frac{B}{(1+v)}$$

$A + B = 2$

$A - B = 1$

$2A = 3$

$\boxed{A = \frac{3}{2}}$

$2B = 1$

$\boxed{B = 1/2}$

$$\frac{v+2}{1-v^2} = \frac{3}{2(1-v)} + \frac{1}{2(1+v)}$$

$$\frac{3}{2(1-v)} + \frac{1}{2(1+v)} = \frac{dx}{x}$$

$$\frac{3}{2}\int\left(\frac{1}{1-v}\right)dv + \frac{1}{2}\int\left(\frac{1}{1+v}\right)dv = \int\frac{dx}{x}$$

$$-\frac{3}{2}\ln|1-v| + \frac{1}{2}\ln|1+v| = \ln|x| + \ln c$$

$$-\frac{3}{2}\ln|1-v| = -\ln|1-v|^{3/2}$$

$$\frac{1}{2}\ln|1+v| = \ln|1+v|^{1/2}$$

$$\ln\frac{a}{b} = \ln a - \ln b$$

$$\ln ab = \ln a + \ln b$$

$$\ln\left|\frac{(1+v)^{1/2}}{(1-v)^{3/2}}\right| = \ln|xc|$$

$$\frac{(1+v)^{1/2}}{(1-v)^{3/2}} = xc$$

$$\frac{(1+v)}{(1-v)^3} = x^2c^2$$

$$1 + v = (1-v)^3 x^2 c^2$$

$$1 + \frac{y-1}{x-1} = \left(1 - \frac{y-1}{x-1}\right)^3 (x-1)^2 c^2$$

$$x-1+y-1 = \cancel{x-1-(y-1)^3} (x-1-y+1)^3 c^2$$

$$x+y-2 = (x-y)^3 c^2$$

Solve

i) $(2x - y - 1)\,dx + (2y - x - 1)\,dy = 0$

ii) $\dfrac{dy}{dx} = \dfrac{y - x + 1}{y + x + 5}$

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* Suppose that $ax + by + c = 0$ and $a'x + b'x + c' = 0$ are parrallel,

Then we have,

$$\frac{a}{a'} = \frac{b}{b'} = t \quad (\quad)$$

$$\frac{a}{a'} = t \qquad \frac{b}{b'} = t$$

$$a = a't \qquad b = b't$$

$ax + by + c = 0$

$y = -\frac{a}{b}x + c$

$a'x + b'y + c' = 0$

$y = -\frac{a'}{b'}x + c'$

$-\frac{a}{b} = -\frac{a'}{b'}$

$\frac{a}{b} = \frac{a'}{b'}$

Then, $$\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$$

$$\frac{dy}{dx} = \frac{a'tx + b'ty + c}{a'x + b'y + c'}$$

let $z = a'x + b'y$

$$\Rightarrow \frac{dz}{dx} = a' + b'\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{b'}\left(\frac{dz}{dx} - a'\right)$$

Then we have,

$$\frac{1}{b'}\left(\frac{dz}{dx} - a'\right) = \frac{zt + c}{z + c'}$$

$$\frac{dz}{dx} = b'\left(\frac{zt + c}{z + c'}\right) + a'$$

Example

Solve : $\frac{dy}{dx} = \frac{y - x + 1}{y - x + 5}$

Note that the lines , $y - x + 1 = 0$ and $y - x + 5 = 0$ are parallel,

Let, $y - x = z$

$\frac{dz}{dx} = \frac{dy}{dx} - 1$

Then we have,

$$\frac{dz}{dx} + 1 = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1 - z - 5}{z+5} = -\frac{4}{z+5}$$

Seperating the ~~gives,~~ variables gives,

$$(z+5)\,dz = \cancel{-4(z+5)}\ -4 \cdot dx$$

integrating gives,

$$\int (z+5)\,dz = -4\int dx$$

$$\frac{z^2}{2} + 5z = -4x + c$$

when c is a arbitary constant.

$$z^2 + 10z = -8x + 2c$$

$$(y-x)^2 + 10(y-x) + 8x = A \qquad (A = 2c)$$

H.W. $(2x + 4y + 3)\dfrac{dy}{dx} = x + 2y + 1$

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Linear Differential Equations

The linear DE of the first order takes the form;

$$\boxed{\frac{dy}{dx} + Py = Q,} \quad \text{—} \; \circledast$$

where P and Q are functions of x alone.

$$P = P(x) \qquad Q = Q(x)$$

A simple example is,

$$\frac{dy}{dx} + \frac{1}{x}y = x^2 \quad \text{—} \; (1)$$

Here $P = P(x) = \frac{1}{x}$ and $Q = Q(x) = x^2$

Multiplying (1) by x gives,

$$\underbrace{x \cdot \frac{dy}{dx} + y}_{\downarrow} = x^3$$

$$\frac{d(yx)}{dx} = x^3$$

$$d(yx) = x^3 . dx$$

$$\int d(yx) = \int x^3 dx$$

$$yx = \frac{x^4}{4} + C$$

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$$y = \frac{x^3}{4} + \frac{c}{x}$$

where c is an arbitary constant provided $x \neq 0$

Here x is called the integrating factor

Now let us try to find an integrating factor R in general sense, such that

$$\frac{d}{dx}(yR) = R\left(\frac{dy}{dx} + Py\right)$$

$$= R\frac{dy}{dx} + RPy$$

Now, multiplying ⊛ by R gives

$$R.\frac{dy}{dx} + RPy = RQ$$

$$\frac{d}{dx}(Ry) = RQ$$

$\int^{ing}$ gives

$$Ry = \int RQ.dx + c$$

$$y = \frac{1}{R}\left[\int RQ.dx + c\right]$$

① → $R.\cancel{\frac{dy}{dx}} + y\frac{dR}{dx} = R\cancel{\frac{dy}{dx}} + RPy$

② ⟶ $\frac{dR}{dx} = RP$ $\quad y \neq 0$

$\int^{ing}$ (2) w.r.t. x gives

$$\frac{dR}{dx} = RP$$

$$\frac{dR}{R} = P.dx$$

$$\int \frac{dR}{R} = \int P\,dx$$

$$\boxed{\ln R = \int P.dx \qquad R = e^{\int P.dx}}$$

The factor $R = e^{\int P\,dx}$ is called the integrating factor of the DE ; $\frac{dy}{dx} + Py = Q$

$$\boxed{\frac{dy}{dx} + Py = Q \qquad \text{I.F.} = R = e^{\int P\,dx}}$$

* To solve the DE multiply the DE by R (step 1)

~~Ex:~~ $\frac{dy}{dx} + \frac{1}{x} y = x^2$

$$\boxed{e^{\ln z} = z}$$

$p = P(x) = \frac{1}{x}$

$$\text{I.F.} = R = e^{\int P.dx} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

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Example 01

Solve $\frac{dy}{dx} + 2xy = 2e^{-x^2}$

$P = P(x) = 2x$

I.F. $\Rightarrow$ $R = e^{\int P.dx} = e^{\int 2x.dx} = e^{\frac{2x^2}{2}} = e^{x^2}$
(Integrating factor)

Now, multiplying the D.E by $R = e^{x^2}$ gives

$$e^{x^2}\frac{dy}{dx} + 2xy.e^{x^2} = 2.e^{-x^2}.e^{x^2}$$

$$\frac{d}{dx}(ye^{x^2}) = 2$$

$\int^{ing}$ gives,

$$y.e^{x^2} = 2x + c$$

$$y = e^{-x^2}(2x+c)$$ where c is an arbitary constant.

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Example 02

Solve ; $(x+1)\dfrac{dy}{dx} - xy = 1-x$

the given DE can be written as,

$$\frac{dy}{dx} - \frac{x}{(x+1)}\cdot y = \frac{1-x}{x+1} \quad \text{if } x \neq 1$$

$$P = P(x) = -\frac{x}{(x+1)} = -1 + \frac{1}{x+1} \quad \Big| \quad Q = Q(x) = \frac{1-x}{1+x}$$

$$\text{I.F} = R = e^{\int P\,dx} = e^{\int \frac{-x}{(x+1)}dx} = e^{\int\left(-1+\frac{1}{x+1}\right)dx}$$

$$= e^{-x+\ln|x+1|}$$

$$= e^{-x} * . e^{\ln|x+1|} = e^{-x}(x+1) \qquad = \cancel{e^{-x*(x+1)}}$$

Now multipling the DE by $R = e^{-x} + \ln|x+1|$ gives,

$$e^{-x}(x+1)\frac{dy}{dx} - e^{-x}\cancel{(x+1)}\frac{x}{\cancel{(x+1)}}y = \frac{(1-x)}{(1+x)} \times e^{-x}(x+1)$$

$$e^{-x}(x+1)\frac{dy}{dx} - e^{-x}.x\,y = (1-x)e^{-x}$$

$$\frac{d}{dx}\left[y.\,e^{-x}(x+1)\right] = (1-x)e^{x}$$

$\int$ ing gives,

$$y\,e^{-x}(x+1) = \int (1-x)e^{-x}\,dx$$

$$= \int e^{-x}dx - \int x.e^{-x}\,dx \rightarrow (*)$$

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$$\int u.dv = uv - \int v.du \qquad \frac{d[e^{-x}]}{dx} = -e^{-x}$$

$$\int x.e^{-x}dx \qquad \boxed{d[e^{-x}] = -e^{-x}.dx}$$

$$\int x[-d(e^{-x})]$$

$$-\int x.d(e^{-x}) = -\left[x.e^{-x} - \int e^{-x}.dx\right.$$

$$= -(x.e^{-x} + e^{-x})$$

$$= -x.e^{-x} - e^{-x}$$

Now (*) gives,

$$y.e^{-x}(x+1) = \int e^{-x}dx - \int x.e^{-x}dx.$$

$$y.e^{-x}(x+1) = (\cancel{-e^{-x}}) - (-x.e^{-x} \cancel{-e^{-x}})$$

$$= xe^{-x} + c$$

i.e., $y.e^{-x}(x+1) = x.e^{-x} + c$

$$y(x+1) = x + ce^{x}$$

$$y = \frac{x}{(x+1)} + \frac{ce^{x}}{(x+1)}$$

where c is a arbitary constant. $(x \neq -1)$

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H.W.

Solve.

$$y \cdot dx - x \cdot dy + \ln x \, dx = 0$$

$$y - x \cdot \frac{dy}{dx} + \ln x = 0$$

$$\frac{dy}{dx} - \frac{y}{x} = \frac{\ln x}{x}$$

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Bernoulli Equation

A first order DE of the form;

$$\frac{dy}{dx} + P(xy) = Q(x)y^n \quad \text{where } n \in \mathbb{R}$$

is called a Bernoulli eq$^{\underline{n}}$

Note

i) When $n = 0$

The Bernoulli Equation (BE) becomes

$$\frac{dy}{dx} + P(x)y = Q(x)$$

, which is a linear eq$^{\underline{n}}$

ii) When $n = 1$

The BE becomes

$$\frac{dy}{dx} + P(x)y = Qy,$$

which is also a linear eq$^{\underline{n}}$

iii) When $n \neq 0, 1$ ~~or~~ ~~not~~ the BE is a non linear eq$^{\underline{n}}$

To solve BDE we use the ~~su~~ transformation $v = y^{1-n}$, where $n \neq 0, 1$

Multiplying the BDE by $v = y^{1-n}$ gives

$$y^{-n}\frac{dy}{dx} + p(x)y^{1-n} = Q(x) \quad \text{—— } ①$$

$$v = y^{1-n} \Rightarrow \frac{dv}{dx} = \frac{d}{dy}(y^{1-n})\frac{dy}{dx}$$

$$\frac{dy}{dx} = (1-n)\,y^{-n}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y^n}{1-n}\cdot\frac{dy}{dx}$$

Substituing in ① gives

$$\frac{1}{1-n}\frac{dv}{dx} + p(x)\,v = Q(x)$$

$$\frac{dv}{dx} + (1-n)\,P(x)v = (1-n)\,Q(x)$$

is a linear eq^n in the form

$$\frac{dy}{dx} + P(y) = Q(x)$$

To solve we may use the integrating factor method.

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Example

Solve $\frac{dy}{dx} + y = xy^3$

$$\frac{dy}{dx} + p(x)y = Q(x)y^n$$

Transformation $v = y^{1-n}$

Let $v = y^{1-n} = y^{1-3} = y^{-2}$ (Since $n=3$)

$$\frac{dv}{dx} = \frac{dv}{dy}\frac{dy}{dx} \quad \text{(chain rule)}$$

$$= \frac{dv}{dy}(y^{-2}) \cdot \frac{dy}{dx}.$$

$$= -2\boxed{y^{-3}\frac{dy}{dx}}$$

Mutplying The given BDE ~~becomes~~ by y^{-3} gives

$$y^{-3}\frac{dy}{dx} + y^{-2} = x$$

$$-\frac{\cancel{2}1}{2}\frac{dv}{dx} + y^{-2} = x$$

$$\frac{dv}{dx} - 2v = -2x$$

This is a linear eq^n with integrating factor,

$$I.F = e^{\int p\,dx} = e^{\int -2\,dx} = e^{-2x}$$

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Multiplying ② by e^{-2x} gives

$$e^{-2x} \cdot \frac{dv}{dx} - 2e^{-2x} v = -2x \cdot e^{-2x}$$

$$\frac{d}{dx}\left(v e^{-2x}\right) = -2x e^{-2x}$$

$$\Rightarrow \quad v = e^{2x} \int -2x \cdot e^{-2x} \, dx \quad \text{(By integrating)}$$

Product Rule

$$\int u \, dv = uv - \int v \cdot du$$

$$\int u \cdot \frac{dv}{dx} \cdot dx = uv - \int v \cdot \frac{du}{dx} \cdot dx$$

$$\frac{d}{dx}\left(e^{-2x}\right) = -2e^{-2x}$$

$$d\left[e^{-2x}\right] = -2e^{-2x} \, dx$$

$$\int x \cdot e^{-2x} \cdot dx = \int x \cdot d\left[e^{-2x}\right]$$

$$\Rightarrow \quad v = e^{2x} \int x \underbrace{\left(-2e^{-2x} \, dx\right)}_{d\left[e^{-2x}\right]}$$

Now integrating by parts gives

$$v = e^{2x}\left(x e^{-2x} + \frac{1}{2} e^{-2x} + c\right)$$

$$= x + \frac{1}{2} + c \cdot e^{2x}$$

$$\Rightarrow \quad y^{-2} = x + \frac{1}{2} + ce^{2x} \quad (\text{since } v = y^{-2})$$

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$$y^2 = \frac{1}{\left(x + \frac{1}{2} + ce^{2x}\right)}$$

$$y = \pm\sqrt{\left(x + \frac{1}{2} + ce^{2x}\right)^{-1}}$$

this valids if $\left(x + \frac{1}{2} + ce^{2x}\right) \neq 0$

H.W. <u>Solve</u>

$$(1 - x^2)\frac{dy}{dx} + xy = xy^2$$

$$\frac{dy}{dx} + \frac{x}{(1-x^2)} y = \frac{x}{(1-x^2)} y^2 \qquad (x \neq \pm 1)$$

$$v = y^{1-n} = y^{1-2} = y^{-1}$$

Exact Differential Equations

Let $F(x,y)$ be a function of two real variables such that F has continuous first order partial derivatives in a domain D. The total differential, dF, of F is defined by

$$\boxed{dF = \frac{\partial F}{\partial x} \cdot dx + \frac{\partial F}{\partial y} \cdot dy}$$

for all, $(x,y) \in D$

Let $F(x,y) = xy^2 + 2x^3y$

$$\frac{\partial F}{\partial x} = y^2 + 6x^2y$$

$$\frac{\partial F}{\partial y} = 2xy + 2x^3$$

Then, $dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy$

$$= (y^2 + 6x^2y)\,dx + (2xy + 2x^3)\,dy$$

- Total differntial of F

Definition

The differential form $M(x,y)\,dx + N(x,y)\,dy$ is said to be exact in a domain D if there is a function $F(x,y)$ such that .

$$\frac{\partial F}{\partial x} = M(x,y) \quad \text{and} \quad \frac{\partial F}{\partial y} = N(x,y)$$

for all $(x,y) \in D$

$$M(x,y)\,dx + N(x,y)\,dy$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{\partial F}{\partial x} \qquad\qquad \frac{\partial F}{\partial y}$$

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = dF$$

That is, the total differential of F satisfies,

$$dF(x,y) = M(x,y)\,dx + N(x,y)\,dy$$

If $M(x,y)\,dx + N(x,y)\,dy$ is an exact form, then the DE

$M(x,y)\,dx + N(x,y)\,dy = 0$ is called an exact eq^n.

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Consider the DE

$$y^2 + 2xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad y^2\,dx + 2xy\,dy = 0$$

If this is an exact DE, then there must be function $F(x,y)$ such that

$$\frac{\partial F}{\partial x} = y^2 = M(x,y)$$

$$\frac{\partial F}{\partial y} = 2xy = N(x,y)$$

Let $F(x,y) = xy^2$

$$\frac{\partial F}{\partial x} = y^2 \quad \text{and} \quad \frac{\partial F}{\partial y} = 2xy$$

$$= M(x,y) \qquad\qquad = N(x,y)$$

$\therefore$ The DE is exact

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Theorem 1

Suppose that the first partial derivatives of $M(x,y)$ and $N(x,y)$ give continuous in a domain D,

Then the DE

$$M(x,y)\,dx + N(x,y)\,dx = 0$$

is <u>exact</u> in D if and only if (iff)

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ for all (x,y) in D.

$$\underbrace{M(x,y)\,dx}_{\frac{\partial M}{\partial y}} + \underbrace{N(x,y)\,dy}_{\frac{\partial N}{\partial x}} = 0$$

If $M(x,y)\,dx + N(x,y)\,dy = 0$ if exact,

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$$

(F needs to be determined)

$\Rightarrow$ $dF = 0$

$\Rightarrow$ $F = F(x,y) = c$, where

c is an arbitrary constant

This is the general solution of the exact DE

$M(x,y)\,dx + N(x,y)\,dy = 0$

$F(x,y) = c$

a one parameter formation of ...

Example

Show that the DE

$(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$

is exact and solve it

$M = 3x^2 + 4xy$ and $N = 2x^2 + 2y$

~~So~~ Then, we have

$\frac{\partial M}{\partial y} = 4x$ $\frac{\partial N}{\partial x} = 4x$

Thus, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Here, the DE is exact

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To solve the DE, we will find an $F(x,y)$ such that,

$$\frac{\partial F}{\partial x} = M = 3x^2 + 4xy \longrightarrow (1) \quad \text{and}$$

$$\frac{\partial F}{\partial y} = N = 2x^2 + 2y \longrightarrow (2)$$

Integrating partially (1) w.r.t x gives

$$F = x^3 + 2x^2y + \phi(y) \longrightarrow (3)$$

Differetiating partially (3) w.r.t. y gives

$$\frac{\partial F}{\partial y} = 2x^2 + \phi'(y)$$

$$= 2x^2 + 2y \quad (\text{by } (2))$$

$$\Rightarrow \phi'(y) = 2y$$

$$\frac{d\phi}{dy} = 2y$$

$$\Rightarrow \phi = y^2 + c_1$$

Then from (3), we have

$$F = x^3 + 2x^2y + y^2 + c_1$$

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The sol^n of the DE is given implicitery

$$x^3 + 2x^2y + y^2 = c_1$$

one parameter family of solution

H.W. Solve

1) $(2x \cos y + 3x^2)\,dx + (x^3 - x^2 \sin y - y)\,dy = 0$
subject to the initial condition $y(0) = 2$

Solve

i) $(2xy + \sec^2 x)\,dx + (x^2 + 2y)\,dy = 0$

ii) $(1 + e^x y + xe^x y)\,dx + (xe^x + 2)\,dy = 0$