MAT 1201 - Vector

Grades:

- Attendence - 10%
- Tutorial - 10%
- Open Book Test - 20%
- End Exam - 60%

100%

Vector algebra

Vector $\underset{\sim}{a}$, $\overrightarrow{AB}$

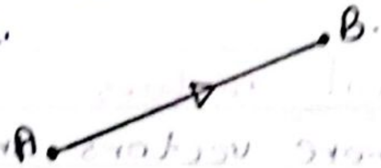
Vector has magnitude

$|\overrightarrow{AB}|$

Chapter 01: Vector

Vector quantities are there quantities that have magnitude and direction. It is generaly represented by directed line segment.

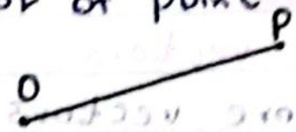
We represent a vecter as $\overrightarrow{AB}$, where initial point of vector is denoted by A and the terminal point by B. magnitude mean as of $\overrightarrow{AB}$ = $|\overrightarrow{AB}|$. But scalar has only magnitude.



position vector

Let us denoted by origin as "O" such that this is a fixed point.

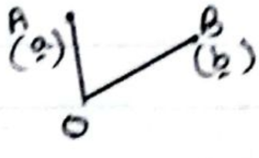
There is a point, say P at a distance from "O". Now the position vector of point P is given by the vector $\overrightarrow{OP}$



(a) $\underset{\sim}{a}$, $\vec{a}$, $\overrightarrow{AB}$, a (identify vector)

If A and B are position vectors to a and b which represent the position vectors of two points A and B. Then we can write

$\overrightarrow{AB} = (\underset{\sim}{b} - \underset{\sim}{a})$



Type of vectors

(I) zero vectors
ශූන්‍ය දෛශිකය

It has zero magnitude This means that vector has the same initial and terminal point. It is denoted by $\vec{0}$ or $\underset{\sim}{0}$. The direction of zero vector is indeterminate.

(II) Unit vectors $\frac{a}{|a|}$
ඒකක දෛශිකය

It has a unit magnitude and unit vector inderetion of a vector $\underset{\sim}{a}$ is denoted by $\hat{a}$ and symbolically as $\hat{a} = \frac{a}{|a|}$

(III) Co-initial vectors
ඒක ආරම්භක දෛශික

Two or more vectors are said to be co-initial if they have the same initial point

(IV) Equal vectors
සමාන දෛශික

Two vectors are said to be equal if they have the same magnitude and direction.

(V) Collinear vectors
ඒකරේඛීය දෛශික

Two or more vectors are said to be collinear if they are paralled to the same line irrespective of their direction. They are also called parallel vectors.

ඒකතල දෛශිකයෝ

(vi) Coplanar vectors

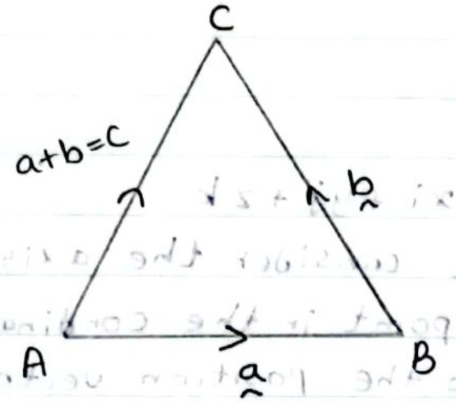
Those vectors which lie on same plane and they are all parallel to the same plane.

සෘණාත්මක දෛශිකයෝ

(vii) Negative vectors

A vector which has <u>same magnitude</u> but ප්‍රතිවිරුද්ධ <u>opposite direction</u> to another vector is called negative of that vectors.

Triangle law (ත්‍රිකෝණ නියමය)



$\overrightarrow{AC} = \underset{\frown}{a} + \underset{\frown}{b} = \underset{\frown}{c}$. considerd triangle ABC let the sum of two vectors

$\overrightarrow{AB} = \underset{\frown}{a}$

$\overrightarrow{BC} = \underset{\frown}{b}$

$\underset{\frown}{a}$ and $\underset{\frown}{b}$ reprecented by $\underset{\frown}{c}$

The position vectors are represented by $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{AC}$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

properties of vector addition

(a) Commutative properly

$$\underset{\frown}{a} + \underset{\frown}{b} = \underset{\frown}{b} + \underset{\frown}{a}$$

(b) Associative properly

$$(\underset{\frown}{a} + \underset{\frown}{b}) + c = \underset{\frown}{a} + (\underset{\frown}{b} + \underset{\frown}{c})$$

(c) Zero is the additive indentity

$$\underset{\frown}{a} + \underset{\frown}{0} = \underset{\frown}{0} + \underset{\frown}{a} = \underset{\frown}{a}$$

(d) $\underset{\frown}{a} + -\underset{\frown}{a} = 0$

$(-a) + \underset{\frown}{a} = 0$

Multiplication of vector by scalar

If $\underset{\sim}{a}$ is a vector and m is a scalar then the product is $m\underset{\sim}{a}$. If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are vectors and m and n are scaler, then

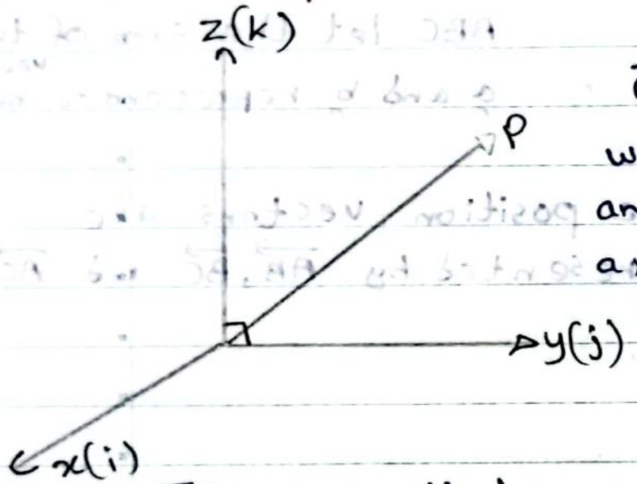
01) $m(\underset{\sim}{a}) = (\underset{\sim}{a})m = m\underset{\sim}{a}$

02) $m(n\underset{\sim}{a}) = n(m\underset{\sim}{a}) = mn\underset{\sim}{a}$

03) $m(\underset{\sim}{a}+\underset{\sim}{b}) = m\underset{\sim}{a} + m\underset{\sim}{b}$

04) $(m+n)\underset{\sim}{a} = m\underset{\sim}{a} + m\underset{\sim}{b}$

Componant from of vectors



$\overrightarrow{OP} = xi + yj + zk$

we have consider the axis x,y,z and a point in the cordinate axis; so the position vector for such a point would be written as, $\overrightarrow{OP} = xi + yj + zk$

* This is called componant vector scalar componance are x, y, z
* Vector componance are xi, yj, zk

Consider of two vectors

$\underset{\sim}{A} = ai + bj + ck$ and $B = pi + qi + rk$

(i). Sum of given by

$$A+B = (a+p)i + (b+q)j + (c+r)k$$

(ii). differnce is given by

$$A-B = (q-p)i + (b-q)j + (c-r)k$$

(iii) mutiplication by a scalar m is given by

$$mA = ma\underset{\sim}{i} + mbj + mck$$

(iv). The vectors are equal of

$$a = p \; , \; b = q \; , \; c = r$$

Test for collnearily

Three points A, B and O with position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are respectively are collinear if and only of there exist scalar x, y, z not all zero simul taneously such that,

$$x\underset{\sim}{a} + y\underset{\sim}{b} + \underset{\sim}{c} = 0$$

where $x + y + z = 0$

$$\text{Collinear} \overset{(iff)}{\Longleftrightarrow} x\underset{\sim}{a} + y\underset{\sim}{b} + zc = 0$$

$$x + y + z = 0$$

Test for coplaner points

For points A, B, C, D with position vectors $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}, \underset{\sim}{d}$ respectively are coplaner (iff) there exist scalars x, y, z, w not all zero simultaneously such that

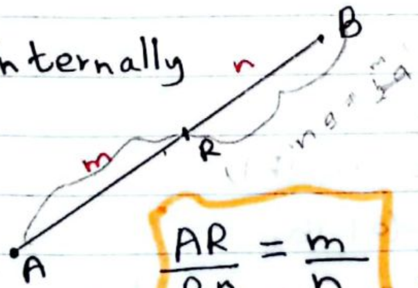
$$x\underset{\sim}{a} + y\underset{\sim}{b} + z\underset{\sim}{c} + w\underset{\sim}{d} = 0 \quad \text{and} \quad x + y + z + w = 0$$

$$\text{Coplonar} \Longleftrightarrow x\underset{\sim}{a} + y\underset{\sim}{b} + z\underset{\sim}{c} + wd = 0$$

$$x + y + z + w = 0$$

Section formular

Internally

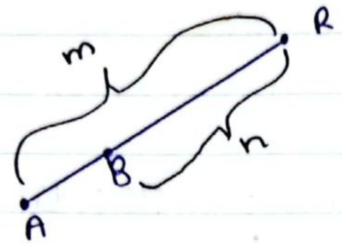


$$\boxed{\frac{AR}{RB} = \frac{m}{n}}$$

let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ position vectors of two point A and B. A Point R position vector $\underset{\sim}{r}$ divide AB such that $\frac{AR}{RB} = \frac{m}{n}$ and this denotes that AB is devided internaly in to ratio min then position vector of R.

$$\boxed{r = \frac{na + mb}{n+m}}$$

Externally



$$\boxed{r = \frac{mb - na}{m-n}}$$

Magnitude of vectors.

(01) For a vector $A = ai + bj + ck$ what is the magnitude of $|A| = \sqrt{a^2+b^2+c^2}$

(02) For vectors $A = ai + bj + ck$, $B = li + mj + nk$ the magnitude of

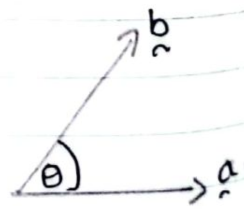
$$|\overrightarrow{AB}| = \sqrt{(l-a)^2 + (m-b)^2 + (n-c)^2}$$

product of vectors

Scale product :- For a two vector a and b the magnitude is the dot product can be represented as $a.b$

if is defined $a.b = |a|\,|b| \cos\theta$

$(0 \leq \theta \leq \pi)$



We have the bellow possibilities

(i) if θ is acule, then $a.b > 0$

(ii) if θ is abtuse, then $a.b < 0$

(iii) if θ is zero, then $a.b = |a|\,|b|$

(iv) if θ is π, then $a.b = -|a|\,|b|$

(v) if $a.b = 0$, then $a.b$ are perpendicular

(vi) Consider the unit vector i, j and k

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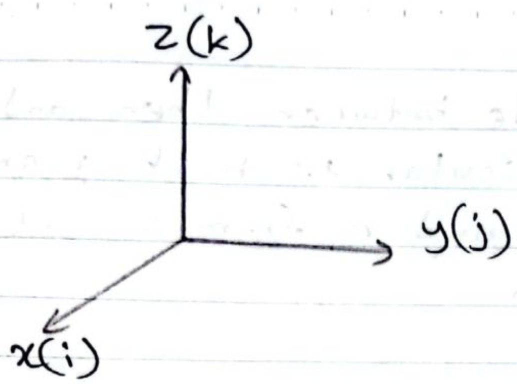


$$\boxed{\begin{array}{l} i \cdot j = j \cdot k = k \cdot i = 0 \\ i \cdot i = j \cdot j = k \cdot k = 1 \end{array}}$$

(vii) $\underset{\sim}{A} = ai + bj + ck$ and $\underset{\sim}{B} = li + mj + nk$ then
$A \cdot B = al + bm + cn$ //

Properties of scalar product

(i) $a \cdot a = |a|\,|a| \cos\theta = |a|^2$
$a \cdot b = b \cdot a$ (commutative)

(ii) $a(b + c) = a \cdot b + a \cdot c$ (Distributive)

(iii) $(m \cdot \underset{\sim}{a})\underset{\sim}{b} = \underset{\sim}{a}(m \cdot \underset{\sim}{b}) = m(\underset{\sim}{a} \cdot \underset{\sim}{b})$ (Associative) there
misscaler

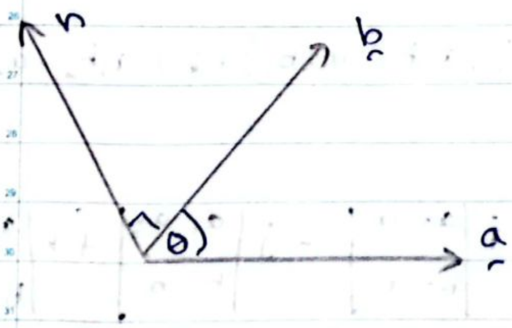
Projection of vector $\underset{\sim}{a}$ on $\underset{\sim}{b}$

$$\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|}$$

Maximum value of $\underset{\sim}{a} \cdot \underset{\sim}{b} = |a|\,|b|$
Minimum value of $\underset{\sim}{a} \cdot \underset{\sim}{b} = -|a|\,|b|$

Vector product

It is called cross product



$\underset{\sim}{a}$ and $\underset{\sim}{b}$ are two vectors

$$\boxed{\underset{\sim}{a} \wedge \underset{\sim}{b} = |a|\,|b| \sin\theta \; n}$$

Where θ is the angle between then and is the
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unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$
such that (s.t) $\underset{\sim}{a}.\underset{\sim}{b}$ and n from a right
handed screw system

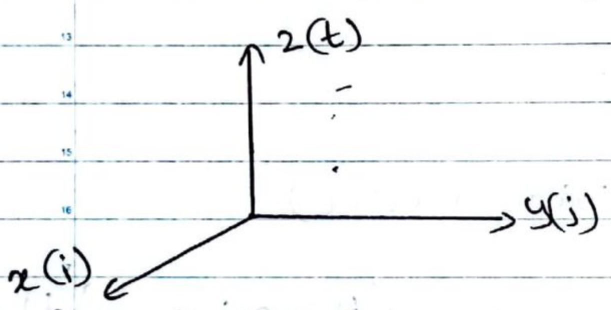
crose $\rightarrow$ $\underset{\sim}{a} \wedge \underset{\sim}{b} = 0$

$\underset{\sim}{a} \wedge \underset{\sim}{b} = |a|\,|b| \sin\theta\, n = 0$

$\underset{\sim}{a} \cdot \underset{\sim}{b} \neq 0 \qquad \sin\theta = 0$

$a \parallel b \qquad \theta = 2\pi$

this vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-zero and $\underset{\sim}{a} \wedge \underset{\sim}{b} = 0$
then it is the condition for them to be parallel vect



(i) $i \wedge i = j \wedge j = k \wedge k = 0$

(ii) $i \wedge j = k$
$j \wedge k = i$
$k \wedge i = j$

(iii) If $\underset{\sim}{a} = a_1 i + a_2 j + a_3 k$ and
$\underset{\sim}{b} = b_1 i + b_2 j + b_3 k$

then

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ \begin{array}{ccc} 1 & 2 & 3 \end{array} \end{array}$$

$$\underset{\sim}{a} \wedge \underset{\sim}{b} = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b$$

$$\underset{\sim}{a} \wedge \underset{\sim}{b} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{array} = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 \\ b_1 \end{vmatrix}$$

$$\underset{\sim}{a} \wedge \underset{\sim}{b} = i(a_2 b_3 - a_3 b_2) - a_1 (j b_3 - k b_2) + b_1 (j a_3 - k a$$

$$i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - b_1 a_3) + k(a_1 b_2 - b_1$$

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Prosection and Component vector

Prosection of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ $= \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|}$.

Prosection of $\underset{\sim}{b}$ on $\underset{\sim}{a}$ $= \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|}$

work done by a force

* $\underset{\sim}{F} \cdot \underset{\sim}{S}$ = dot product of force and displacement
* suppose $F_1, F_2 \dots F_n$ are n forces acted on a practicale then during the displecement S of the particle, the separate forces do quatities of work

$a \cdot b = |a|\,|b| \cos\theta$

$\underset{\sim}{a} \wedge \underset{\sim}{b} = |a|\,|b| \sin\theta \; n$

Total work done is

$$\sum_{n=1}^{n} \underset{\sim}{F} \cdot \underset{\sim}{S}$$

Triple product

* The scalar Triple product
* It is a means of combining three vectors via cross product and a dot product

Given the vectors

$\underset{\sim}{A} = A_1 i + A_2 j + A_3 k$

$\underset{\sim}{B} = B_1 i + B_2 j + B_3 k$

$\underset{\sim}{C} = C_1 i + C_2 j + C_3 k$

Scalar triple product

$A \cdot (\underset{\sim}{B} \wedge \underset{\sim}{C})$

$$\underset{\sim}{B} \wedge \underset{\sim}{C} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = i \begin{vmatrix} B_2 & B_3 \\ C_2 & C_3 \end{vmatrix} - j \begin{vmatrix} B_1 & B_3 \\ C_1 & C_3 \end{vmatrix} + k \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix}$$

$$\underset{\sim}{A} \cdot \underset{\sim}{B} \wedge \underset{\sim}{C} = [A_1 i + A_2 j + A_3 k] - \left[i \begin{vmatrix} B_2 & B_3 \\ C_2 & C_3 \end{vmatrix} - j \begin{vmatrix} B_1 & B_3 \\ C_1 & C_3 \end{vmatrix} + k \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix}\right.$$

$$= A_1 \begin{vmatrix} B_2 & B_3 \\ C_2 & C_3 \end{vmatrix} - A_2 \begin{vmatrix} B_1 & B_3 \\ C_1 & C_3 \end{vmatrix} + A_3 \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix}$$

$$\underline{A}\cdot\underline{B}\wedge\underline{C} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$\underline{A}\cdot\underline{B}\wedge\underline{C} = \underline{A}\wedge\underline{B}\cdot\underline{C} = \underline{C}\cdot\underline{A}\wedge\underline{B} = \underline{C}\wedge\underline{A}\cdot\underline{B} = \underline{B}\cdot\underline{C}\wedge\underline{A}$$

$$\underline{A}\cdot\underline{B}\wedge\underline{C} = A\cdot C\wedge B = A\wedge C\cdot B = B\cdot\underline{A}\wedge\underline{C} = B\cdot C\wedge A$$
$$= \underline{C}\wedge A\cdot B = C\cdot A\wedge B = C\cdot B\wedge A$$

$\underline{A} = 2i + 3j - k$

$\underline{B} = -i + j$

$\underline{C} = 2i + 2j$ Find $A\cdot B\wedge\underline{C}$

$\cancel{(2i+3j-k)}\ \cancel{(-i+j)}\ \cancel{(2i+2j)}$

$$A\cdot B\wedge C = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{vmatrix} 2 & +3 & -1 \\ -1 & 1 & 0 \\ 2 & 2 & 0 \end{vmatrix} \end{array}$$

$A\cdot B\wedge C = A\wedge B\cdot C$

$= 2(1\times 0 - 2\times 0) - 3(-1\times 0 - 2\times 0) + -1(-1\times 2 - 2\times 1)$

$= +4$ //

$A\cdot B\wedge C = A\wedge B\cdot C$

$A = A_1 i + A_2 j + A_3 k$

$B = B_1 i + B_2 j + B_3 k$

$C = C_1 i + C_2 j + C_3 k$

$$\underline{A}\cdot\underline{B}\wedge\underline{C} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = (-)\begin{vmatrix} C_1 & C_2 & C_3 \\ B_1 & B_2 & B_3 \\ A_1 & A_2 & A_3 \end{vmatrix} = (-1)(-1)\begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$\underline{C}\cdot\underline{A}\wedge\underline{B} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ \ldots \end{vmatrix} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & \ldots & A_3 \end{vmatrix}$$

The vector Triple product

$$\underset{\sim}{A} \wedge (\underset{\sim}{B} \wedge \underset{\sim}{C}) = (\underset{\sim}{A} \cdot \underset{\sim}{C})B - (\underset{\sim}{A} \cdot \underset{\sim}{B})\underset{\sim}{C}$$

$A = A_1 i + A_2 j + A_3 k$

$B = B_1 i + B_2 j + B_3 k$

$C = C_1 i + C_2 j + C_3 k$

$$L.H.S = \underset{\sim}{A} \wedge (\underset{\sim}{B} \wedge \underset{\sim}{C})$$

$$\underset{\sim}{B} \wedge \underset{\sim}{C} = \begin{vmatrix} i & j & k \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$= i(B_2C_3 - B_3C_2) - j(B_1C_3 - B_3C_1) + k(B_1C_2 - B_2C_1)$$

$$A \wedge (B \wedge C) = (A_1 i + A_2 j + A_3 k) \wedge [i(B_2C_3 - B_3C_2) - j(B_1C_3 - B_3C_1) + k(B_1C_2 - B_2C_1)]$$

$$\boxed{\begin{array}{l} i \wedge i = j \wedge j = k \wedge k = 0 \\ i \wedge j = k \\ j \wedge k = i \\ k \wedge i = j \end{array}}$$

$$A \wedge (B \wedge C) = -A_1 k(B_1C_3 - B_3C_1) - A_1 j(B_1C_2 - B_2C_1) \mp k A_2 (B_2C_3 - B_3C_2)$$
$$+ i A_2 (B_1C_2 - B_3C_1) + A_3 j (B_2C_3 - B_3C_2) + i A_3 (B_1C_3 - B_3C_1)$$

$$= i(A_2B_1C_2 - A_2B_2C_1 + A_3B_1C_3 \mp A_3B_3C_1)$$
$$+ j[-A_1B_1C_2 + A_1B_2C_1 + A_3B_2C_3 - A_3B_3C_2]$$
$$+ k[\mp A_1B_1C_3 \mp A_1B_3C_1 \mp A_2B_2C_3 \mp A_2B_3C_2]$$

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$$R.H.S = (\underset{\sim}{A} \cdot \underset{\sim}{C})\underset{\sim}{B} - (A \cdot B)C$$

$$(A \cdot C) = (A_1 i + A_2 j + A_3 k) \cdot (C_1 i + C_2 j + C_3 k)$$
$$= (A_1 C_1 + A_2 C_2 + A_3 C_3) //$$

$$(A \cdot B) = (A_1 i + A_2 j + A_3 k) \cdot (B_1 i + B_2 j + B_3 k)$$
$$= (A_1 B_1 + A_2 B_2 + A_3 B_3) //$$

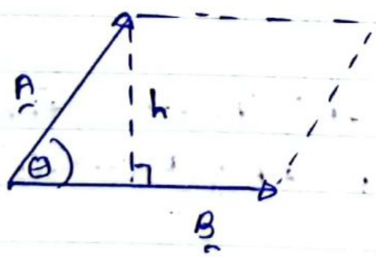
$$(A \cdot C)B - (A \cdot B)C = (A_1C_1 + A_2C_2 + A_3C_3) \cdot (B_1 i + B_2 j + B_3 k)$$
$$- (A_1B_1 + A_2B_2 + A_3B_3)(C_1 i + C_2 j + C_3 k)$$

$$= i(\cancel{A_1C_1B_1} + A_2C_2B_1 + A_3C_3B_1 - \cancel{A_1B_1C_1} - A_2B_2C_1 - A$$
$$+ j(A_1C_1B_2 + \cancel{A_2C_2B_2} + A_3C_3B_2 - A_1B_1C_2 - \cancel{A_2B_2C_2} - A$$
$$+ k(A_1C_1B_3 + A_2C_2B_3 + \cancel{A_3C_3B_3} - A_1B_1C_3 - A_2B_2C_3 - \cancel{A_3}$$

$$= i(A_2B_1C_2 - A_2B_2C_1 + A_3B_1C_3 - A_3B_3C_1)$$
$$j \cdot (-A_1B_1C_2 + A_1B_2C_1 + A_3B_2C_3 - A_3B_3C_2)$$
$$k(-A_1B_1C_3 + A_1B_3C_1 - A_2B_2C_3 + A_2B_3C_2) //$$

Area and Volum using cross Product

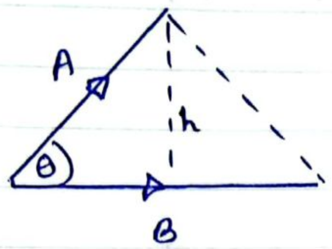
The area of a parallelogram



Area of a parallelogram $= |B| h$

$$= |A \sin\theta|\,|B|$$
$$= |BA \sin\theta|$$
$$= \boxed{|\underset{\sim}{A} \wedge \underset{\sim}{B}|}$$

The area of trangle



area of trangle $= \frac{1}{2} h \, |\underset{\sim}{B}|$

$= \frac{1}{2} \left| AB \sin\theta \right|$

$= \boxed{\frac{1}{2} \left| \underset{\sim}{A} \wedge \underset{\sim}{B} \right|}$ //

prove that

(i) $(\underset{\sim}{A} \wedge B) \wedge (C \wedge D) = \underset{\sim}{C} (\underset{\sim}{A} \cdot \underset{\sim}{B} \wedge D) - D (A \cdot B \wedge \underset{\sim}{C})$

(ii) $(A \cdot B) \wedge (C \wedge D) = \underset{\sim}{B} (A \cdot C \wedge D) - A (B \cdot C \wedge D)$

(i) $(A \wedge B) \wedge (C \wedge D) = \underset{\sim}{C} (A \cdot B \wedge D) - D (\underset{\sim}{A} \cdot \underset{\sim}{B} \wedge \underset{\sim}{C})$

$u = (A \wedge B)$

$(A \wedge B) \wedge (C \wedge D) = \underset{\sim}{C} (u \cdot D) - D (u \cdot C)$

$= \underset{\sim}{C} (A \wedge B \cdot D) - \underset{\sim}{D} (A \wedge B \cdot C)$

$= \underset{\sim}{C} (A \cdot B \wedge D) - \underset{\sim}{D} (A \cdot B \wedge C)$ //

(ii) $(A \cdot B) \wedge (C \wedge D) = \underset{\sim}{B} (A \cdot C \wedge D) - A (B \cdot C \wedge D)$

$v = C \wedge D$

$(A \cdot B) \wedge (C \wedge D) = \underset{\sim}{B} (\underset{\sim}{A} \cdot \underset{\sim}{v}) - \underset{\sim}{A} (B \cdot v)$

$\underset{\sim}{B} (A \cdot C \wedge D) - \underset{\sim}{A} (B \cdot C \wedge D)$ //

eg:- f a, b, c and $\underset{\sim}{p}, q, r$ are any two systems of three vectors and it

$\underset{\sim}{p} = x_1 \underset{\sim}{a} + y_1 \underset{\sim}{b} + z_1 \underset{\sim}{c}$

$\underset{\sim}{q} = x_2 \underset{\sim}{a} + y_2 \underset{\sim}{b} + z_2 \underset{\sim}{c}$

$\underset{\sim}{r} = x_3 a + y_3 \underset{\sim}{b} + z_3 \underset{\sim}{c}$ then

$[\underset{\sim}{a}, \underset{\sim}{b}, c] = \underset{\sim}{a} \cdot \underset{\sim}{b} \wedge c = \underset{\sim}{a} \wedge b \cdot c$

$$(p \cdot q \cdot r) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \quad [\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}]$$

$$(p \cdot q \cdot r) = (p \wedge q \cdot r)$$

$$(p \wedge q) = (x_1 a + y_1 b + z_1 c) \wedge (x_2 a + y_2 b + z_2 c)$$
$$= x_1 y_2 a \wedge b + x_1 z_2 a \wedge c + y_1 x_2 \underset{\sim}{b} \wedge a + y_1 z_2 b \wedge c + z_1 x_2 \underset{\sim}{c} \wedge \underset{\sim}{a} + z_1 y_2 c \wedge b$$

$$= (x_1 y_2 - y_1 x_2) a \wedge b + (y_1 z_2 - z_1 y_2) b \wedge c + (z_1 x_2 - x_1 z_2) c \wedge a$$

$$= \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} a \wedge b + \begin{vmatrix} y_1 & y_2 \\ z_1 & z_2 \end{vmatrix} b \wedge c + \begin{vmatrix} z_1 & z_2 \\ x_1 & x_2 \end{vmatrix} \underset{\sim}{c} \wedge \underset{\sim}{a}$$

$$p \wedge q \cdot r = \left[\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} a \wedge b + \begin{vmatrix} y_1 & y_2 \\ z_1 & z_2 \end{vmatrix} b \wedge c + \begin{vmatrix} z_1 & z_2 \\ x_1 & x_2 \end{vmatrix} c \wedge a \right] \cdot [x_3 a + y_3 b + z_3 c]$$

$$= x_3 \begin{vmatrix} y_1 & y_2 \\ z_1 & z_2 \end{vmatrix} a \cdot b \wedge c + y_3 \begin{vmatrix} z_1 & z_2 \\ x_1 & x_2 \end{vmatrix} b \cdot c \wedge a + z_3 \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} c \cdot a \wedge b$$

$$= \left[x_3 \begin{vmatrix} y_1 & y_2 \\ z_1 & z_2 \end{vmatrix} + y_3 \begin{vmatrix} z_1 & z_2 \\ x_1 & x_2 \end{vmatrix} + z_3 \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \right] a \cdot b \wedge \underset{\sim}{c}$$

$$= \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} (\underset{\sim}{a}, \underset{\sim}{b}, c) \quad //$$

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Note:- a, b, c are non-coplanar and

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \neq 0$$ then the three vectors

p, q, r also non coplanar

If $a = -3i - j + 5k$

$b = i - 2j + k$

$c = 4j - 5k$ find $a \cdot (b \wedge c)$

$$(b \wedge c) = (i - 2j + k) \wedge (4j - 5k)$$

$$= 1 \times 4 \, i \wedge j + 1 \times -5 \, i \wedge k + 2 \times 5 j \cdot k + 1 \times 4 \, j \cdot k$$

$$= 4k - 5j + 10i + 4i$$

$$= 4k - 5j + 14i$$

$$a \cdot (b \wedge c) = (-3i - j + 5k) \cdot (4k - 5j + 14i)$$

$$= -42 \, i \cdot i + 5 j \cdot j + 20 k \cdot k$$

$$= -17$$

$$a \cdot b \wedge c = \begin{vmatrix} -3 & -1 & 5 \\ 1 & -2 & 1 \\ 0 & 4 & -5 \end{vmatrix} (i \cdot j \wedge k)$$

$$= -3(10 - 4) - (-1)(-5 - 0) + 5(4 - 0)$$

$$= -3 \times 6 - 5 + 20$$

$$= -18 - 5 + 20$$

$$= -23 + 20$$

$$= -3$$

If $2i - j + 3k$, $3i + 2j + k$ and $i + mj + 4k$ are coplanar find the value m?

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0$$

$[2(8-m) - -1(12-1) + 3(3m-2)] = 0$

$16 - 2m + 11 + 9m - 6 = 0$

$7m = -21$

$\underline{\underline{m = -3}}$

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Vector joining two point

If $P_1 = (x_1, y_1, z_1)$.

$P_2 = (x_2, y_2, z_2)$

Then

$\overrightarrow{P_1P_2} = \overrightarrow{P_1O} + \overrightarrow{OP_2}$

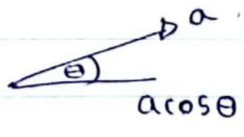
$= -(x_1 i + y_1 j + z_1 k) + x_2 i + y_2 j + z_2 k$

$= (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

projection of a along b

is $\dfrac{a \cdot b}{|b|}$ and the projection

vector of a along b is $\left(\dfrac{a \cdot b}{|b|}\right) b$



$a \cos\theta = \dfrac{|a|\,|b|}{|b|} \cos\theta = \dfrac{a \cdot b}{|b|}$

$\dfrac{a \cdot b}{|b|} = \dfrac{|a|\,|b| \cos\theta}{|b|} = |a| \cos\theta$

If $\underset{\frown}{a} = a_1 i + a_2 j + a_3 k$ and $\underset{\frown}{b} = b_1 i + b_2 j + b_3 k$ are two vectors and λ is any scaler then

$$\underset{\frown}{a} + \underset{\frown}{b} = (a_1 + b_1) i + (a_2 + b_2) j + (a_3 + b_3) k$$

$$\lambda \underset{\frown}{a} = \lambda (a_1 i + a_2 j + a_3 k)$$
$$= \lambda a_1 i + \lambda a_2 j + \lambda a_3 k \;//$$

$i.i = j.j = k.k = 1$

$i.j = i.k = j.k = 0$

$$a \cdot b = (a_1 i + a_2 j + a_3 k) \cdot (b_1 i + b_2 j + b_3 k)$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\underset{\frown}{a} \wedge \underset{\frown}{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

angle between two vectors.

$$\underset{\frown}{a} \cdot \underset{\frown}{b} = |a|\,|b| \cos\theta$$

$$\cos\theta = \frac{\underset{\frown}{a} \cdot \underset{\frown}{b}}{|a|\,|b|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} + \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$a \cdot a = a_1^2 + a_2^2 + a_3^2$$
$$|a|^2 = a_1^2 + a_2^2 + a_3^2$$
$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Find the unit Vector in the direction sum of the vectors

$$\underset{\frown}{a} = 2i - j + 2k \qquad \underset{\frown}{b} = -i + j + 3k$$

$$\underset{\frown}{c} = \underset{\frown}{a} + \underset{\frown}{b} = 2i - j + 2k + -i + j + 3k$$
$$= i + 5k \;//$$

$$|c| = |\underset{\frown}{a} + \underset{\frown}{b}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$\text{unit vector of} = \frac{\underset{\frown}{c}}{|\underset{\frown}{c}|} = \frac{i + 5k}{\sqrt{26}} \;//$$

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Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the pts (1,3,2) and (-1,0,8) respectively.

$$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$$
$$= (i+3j+2k) + -(-i+8k)$$
$$= -(2i-3j+6k)$$ //

$$\overrightarrow{QP} = P - Q$$
$$= i+3j+2k-(-i+8k)$$
$$= 2i+3j-6k$$ //

Unit vector of $\overrightarrow{QP} = \dfrac{2i+3j-6k}{\sqrt{2^2+3^2+6^2}} = \dfrac{2i+3j-6k}{7}$

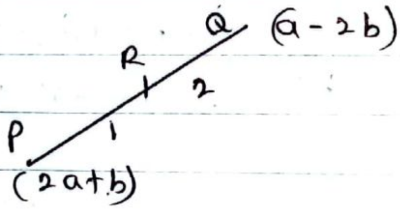
Vector of magnatude 11 in the $\overrightarrow{QP}$ direction

$$\frac{11}{7}(2i+3j-6k)$$ //

(01) Find the position vector of a point R which divides the line joining the two pts P and Q with position vectors $\overrightarrow{OP} = 2\underset{\sim}{a}+\underset{\sim}{b}$ and $\overrightarrow{OQ} = \underset{\sim}{a}-2\underset{\sim}{b}$ respectively in the radio 1:2

(i) internally

(ii) externally



(i) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$
$$= -2a-b+a-2b$$
$$= -a-3b$$ //

$$\overrightarrow{PR} = \frac{1}{3}\overrightarrow{PQ}$$

$$= \frac{1}{3}(-a-3b)$$ //

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$
$$= 2a+b+\frac{1}{3}(-a-3b)$$

$$= \frac{1}{3}(6a+3b-a-3b) = \frac{1}{3}(5a)$$ //

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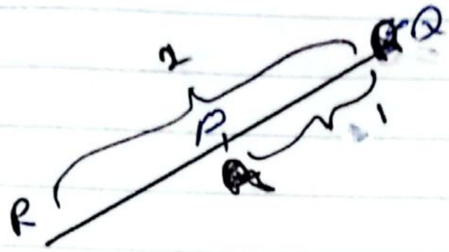
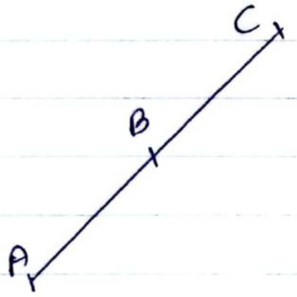


$$\vec{PR} = \vec{PQ} + \vec{QR}$$
$$= \vec{PQ} + \cancel{\vec{OQ}} + \frac{1}{2}\vec{PQ}$$
$$= \frac{1}{2}(3\vec{PQ})$$
$$= \frac{3}{2}($$

(02) If the points $(-1,-1,2)$, $(2,m,5)$ and $(3,11,6)$ are collinear, find the value of m



$$\vec{AB} = \vec{AO} + \vec{OB}$$
$$= -(-i-j+2k)+(2i+mj+5k)$$
$$= 3i+(m+1)j+7k \,//$$

$$\vec{BC} = \vec{BO} + \vec{OC}$$
$$= -2i+mj-5k+3i+11j+6k$$
$$= i+(11-m)j+k \,//$$

$\vec{AB} \parallel \vec{BC}$

$\therefore \vec{AB} = \lambda(\vec{BC})$

$3i+(m+1)j+3k = \lambda\,(i+(11-m)j+k)$

$i \neq 0 \quad j \neq 0 \quad i \neq j$

$\therefore$

$i \Rightarrow \quad \underline{\underline{3 = \lambda}}$

$j \Rightarrow \quad m+1 = \lambda(11-m)$

$m+1 = 33-3m$

$4m = 32$

$\underline{\underline{m = 8}}$

(03) If $a = 2i-j+k$, $b = i+j-2k$ and $c = i+3j-k$ find λ sush that a is perpendicular ලම්බකයි $\lambda b + c$

$(\lambda b + c)\cdot a = 0 \quad (\because a$ is perpendicular$)$

$[\lambda(i+j-2k)+i+3j-k]\cdot(2i-j+k) = 0$

$((1+\lambda)i \cancel{+4j-3k})\cdot(2i-j+k) = 0$

$+(3+\lambda)j + -(2\lambda+1)]$

No

$$2(\lambda+1) - (\lambda+3) - (2\lambda+1) = 0$$
$$2\lambda + 2 - \lambda - 3 - 2\lambda - 1 = 0$$
$$\underline{\underline{\lambda = -2}}$$

**

(04). Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the place of $i+2j+k$ and $-i+3j+4k$

$\underset{\sim}{a} = i+2j+k$ $\quad$ $\underset{\sim}{b} = -i+3j+4k$

$$|a| = \frac{\underset{\sim}{a}}{\hat{a}}$$

$$a \wedge b = \begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} i & j & k \\ i & 2j & k \\ -i & 3j & 4k \end{vmatrix}$$

$$= 5i - 5j + 5k \quad //$$

Unit vector of direction $\underset{\sim}{a} \cdot \underset{\sim}{b}$

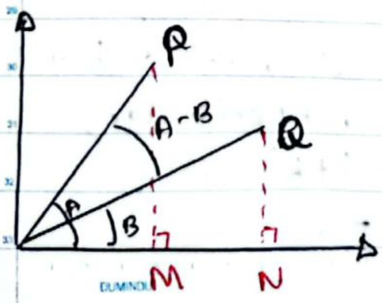
$$n = \frac{5i - 5j + 5k}{5\sqrt{3}} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a} \wedge \underset{\sim}{b}|}$$

magnitude $10\sqrt{3}$ that are perpendicular to the place

$$= 10\sqrt{3}\left(\frac{5i - 5j + 5k}{5\sqrt{3}}\right) = 2(5i - 5j + 5k) \quad //$$

* Using vectors, prove that

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$



Let $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ unit vectors

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making angle between A and B

Then $Q\hat{O}P = A - B$

$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$

$= \cos A\, i + \sin A\, j$ - i ඒකකයක් නිසා $\overrightarrow{OP}$ cos A [illegible]

$\overrightarrow{OQ} = \overrightarrow{ON} + \overrightarrow{NP}$

$= \cos B\, i + \sin B\, j$

unit vectors

$\overrightarrow{OP} \cdot \overrightarrow{OQ} = |\overrightarrow{OP}|\,|\overrightarrow{OQ}| \cos(A - B)$

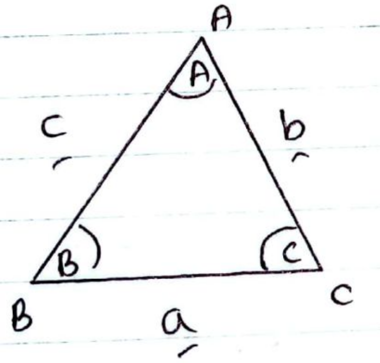
$(\cos A\, i + \sin A\, j)\,(\cos B\, i + \sin B\, j) = \cos(A - B)$

$\cos A \cos B + \sin A \sin B = \cos(A - B)$ //

$i \cdot i = 1$
$j \cdot j = 1$

prove that in a Δ ABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

where a, b, c represent the magnitude of the sides opposite to vertices A, B, C respectirely.



Let the 3 side of trianglar BC, CA, AB be represent by a, b, c repectirely

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\underset{\sim}{c} = -\underset{\sim}{b} + -\underset{\sim}{a}$

$a + b = -c \longleftarrow (1)$

$(1) \wedge a$

$a \wedge a + a \wedge b = (-c) \wedge a$

$(1) \times b$ $a \wedge a = 0$

$\underset{\sim}{a} \wedge \underset{\sim}{b} + b \cdot b + = -c \wedge b$

$a \wedge b = -cb$

(1) හා (2) න්

$a \wedge b = b \wedge c = c \wedge a$

$|a \wedge b| = |b \wedge c| = |c \wedge a| \longleftarrow (3)$

No

Then ② an ③

$a \wedge b = b \wedge c = c \wedge a$

$|a \wedge b| = |b \wedge c| = |c \wedge a|$

$|a||b| \sin(\pi - C) = |b||c| \sin(\pi - A) = |c||a| \sin(\pi - B)$

$|a||b| \sin C = |b||c| \sin A = |c||a| \sin(B)$

$\div |a||b||c|$

$$\boxed{\frac{\sin C}{|c|} = \frac{\sin B}{|B|} = \frac{\sin A}{|a|}}$$

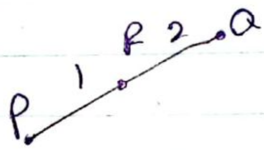
(01) The magnitude of the vector $6i + 2j + 3k$ is

(A) 5 (B) 7 (C) 12 (D) 1 $\sqrt{36 + 4 + 9}$

(02) The position vector of the point which divise the join with propertion vector $a+b$, and $2a-b$ in the ratio 1 : 2 is

(A) $\frac{3a+2b}{3}$ B) $B = a$

(C) $\frac{5a-b}{3}$ (d) $\frac{4a+b}{3}$



(03) The angle ~~vectors~~ between the vectors $i-j$ and $j-k$ is

a) $\pi/3$ B) $\frac{2\pi}{3}$ c) $-\frac{\pi}{3}$ d) $\frac{5\pi}{6}$

$\frac{5 \times 180}{6} = 150$

(04) $(i-j)(j-k) = |i-j||j-k| \cos\theta$

$1-1 = \cos\theta$

$0 = \cos\theta$

The value of λ for which the two vectors $2i - j + 2k$ and $3i + \lambda j + k$

a) 2 B) 4 c) 6 d) 8

$(2i - j + 2k)(3i + \lambda j + k) = 0$

$\sqrt{2^2 + 1 + 2^2} =$

$\lambda = 8$

Example:- ① If $|a| = 8$, $|b| = 3$, and $|a \wedge b| = 12$ then Value of $a \cdot b$

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) None of these

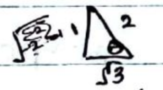
$|\underset{\frown}{a} \wedge \underset{\frown}{b}| = 12$

$\boxed{a \cdot b = |a|\,|b| \cos\theta}$

$= 8 \times 3 \times \frac{\sqrt{3}}{2}$

$\boxed{\underset{\frown}{a} \wedge \underset{\frown}{b} = |a|\,|b| \sin\theta\, n}$

$|\underset{\frown}{a} \wedge \underset{\frown}{b}| = |a|\,|b| \sin\theta$



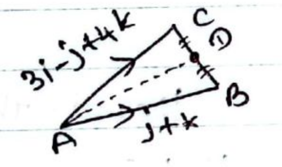
$= 12\sqrt{3}$

$12 = 8 \times 3 \cdot \sin\theta$

$\sin\theta = \frac{12}{24} = \frac{1}{2}$ //

② The 2 vectors $j+k$ and $3i-j+4k$ represent the two sides AB and AC, respectively of a $\triangle ABC$. The length of the midian through A is

(a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$ (c) $\sqrt{18}$ (d) None of these



$\vec{AD} = \vec{AB} + \vec{BD}$

$= j+k + \frac{1}{2}\vec{BC}$

$= j+k + \frac{1}{2}(\vec{BA} + \vec{AC})$

$= j+k + \frac{1}{2}(-j-k+3i-j+4k)$

$= \frac{1}{2}[2j+2k-2j+3i+3k]$

$= \frac{1}{2}(3i+5k)$ //

$|\vec{AD}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{25}{4}}$

$= \frac{1}{2}\sqrt{34}$ //

(3) The projection of vector $a = 2i - j + k$ along $b = i + 2j + 2k$ is (a) $2/3$ (b) $1/3$ (c) 2 d) $\sqrt{6}$

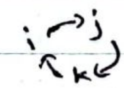
$$\frac{a \cdot b}{|b|} = \frac{(2i - j + k) \cdot (i + 2j + 2k)}{\sqrt{1^2 + 2^2 + 2^2}}$$

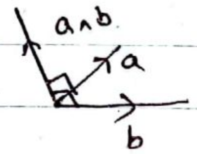
$$= \frac{2 + 2 - 2}{\sqrt{9}} = \frac{2}{3}$$

(4) The unit vector perpendicular to the vectors $i-j$ and $i+j$ forming a right handed system is

(a) k (b) $-k$ (c) $\frac{i-j}{\sqrt{2}}$ (d) $\frac{i+j}{\sqrt{2}}$

$\hat{a} = \frac{a}{|a|}$ $\qquad \frac{(i-j) \wedge (i+j)}{|(i-j) \wedge (i+j)|}$





$i \wedge j = k$ $\qquad \frac{k + k}{\sqrt{2^2}} = \frac{2k}{2} = \underline{\underline{k}}$

(5) If $|a| = 3$ and $-1 < k < 2$ then $|ka|$ lies in the interval

(A) $(0,6)$ (b) $(-3,6)$ (c) $(3,6)$ (d) $1,2$

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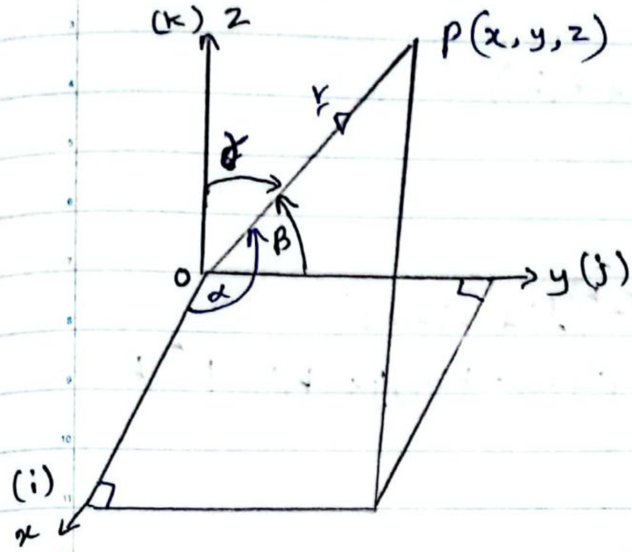
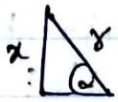
Direction Cosines Vectors



Suppose that vector $r = xi + yj + zk$ makes angles α, β and γ with i, j, k respectively



$\cos\alpha = \frac{x}{r}$

$\cos\beta = \frac{y}{r}$

$\cos\gamma = \frac{z}{r}$

The unit vector of the direction of $\underset{\sim}{r}$ can be written as,

$r = xi + yj + zk$

$r = r(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$

$\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosmes of

Further $\underset{\sim}{r} = r(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$

and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

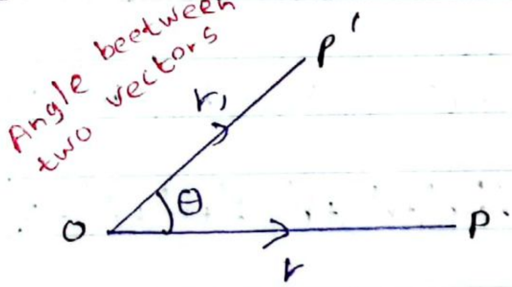
Thedirection cos sines α a vector are usually dendcled by l, m, n

Thus $l^2 + m^2 + n^2 = 1$

$l = \cos\alpha$

$m = \cos\beta$

$n = \cos\gamma$



Position vectors of the points P and P' relative to O are r_1 and r and direction cosines of OP and OP' are (l, m, n) and (l_1, m_1, n_1) find $|PP'|$

If θ is the angele beetween $\overrightarrow{OP}$ and $\overrightarrow{OP'}$ then usms cosine rude for a tringular show that

$\cos\theta = ll_1 + mm_1 + nn_1$

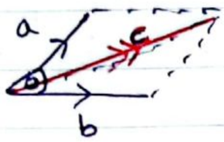
Further $r = r(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$

$\overrightarrow{OP} = r = r(li + mj + nk)$

$\overrightarrow{OP'} = r_1 = r_1(l_1 i + m_1 j + n_1 k)$

$$\overrightarrow{PP'} = \overrightarrow{PO} + \overrightarrow{OP'}$$
$$= r_1 - r$$
$$= r_1(l_1 i + m_1 j + n_1 k) - r(li + mj + nk)$$
$$= (r_1 l_1 - rl)i + (r_1 m_1 - rm)j + (r_1 n_1 - rn)k$$

$\overrightarrow{OP} \cdot \overrightarrow{OP'} =$



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|PP'|^2 = |OP'|^2 + |OP|^2 - 2|OP'||OP|\cos\theta$$

$$|r_1 l_1 - rl|^2 + |r_1 m_1 - rm|^2 + |r_1 n_1 - rn|^2 = (r_1 l_1)^2 + (r_1 m_1)^2 + (r_1 n_1)^2$$
$$+ (rl)^2 + (rm)^2 + (rn)^2 - 2\sqrt{(r_1 l_1)^2 + (r_1 m_1)^2 + (r_1 n_1)^2}$$
$$\sqrt{(rl)^2 + (rm)^2 + (rn)^2}\cos\theta$$

$$r_1^2 l_1^2 + r^2 l^2 - 2rr_1 ll_1 + r_1^2 m_1^2 + r^2 m^2 - 2rr_1 mm_1 + r_1^2 n_1^2 + r^2 n^2 - 2r_1 n_1 rn$$
$$r_1^2(\underbrace{l_1^2 + m_1^2 + n_1^2}_{1}) + r^2(\underbrace{l^2 + m^2 + n^2}_{1}) - 2(rr_1 ll_1 + rr_1 mm_1 + rr_1 nn_1)$$

$$r_1^2 + r^2 - 2rr_1(ll_1 + mm_1 + nn_1)$$

$$r_1^2(l_1^2 + m_1^2 + n_1^2) + r^2(l^2 + m^2 + n^2) - 2\sqrt{r_1^2(l_1^2 + m_1^2 + n_1^2)}\sqrt{r^2(l^2 + m^2 + n^2)}\cos\theta$$

$$\cancel{r_1^2} + \cancel{r^2} - 2\sqrt{r_1^2}\sqrt{r^2}\cos\theta = r_1^2 + r^2 - 2rr_1(ll_1 + mm_1 + nn_1)$$
$$-2r_1 r\cos\theta = -2rr_1(ll_1 + mm_1 + nn_1)$$

$$\cos\theta = ll_1 + mm_1 + nn_1 \quad //$$

Vector Equations

Make $\underset{\sim}{a}$ the subject of the eq^n $6a-3b+\frac{1}{2}c=d$

$$6\underset{\sim}{a}-3\underset{\sim}{b}+\frac{\underset{\sim}{c}}{2}=d$$

$$6a = d+3b-\frac{c}{2}$$

$$a = \frac{d}{6}+\frac{b}{2}-\frac{c}{12} //$$

Solve the vector eq^n (what is the p)

$$p+(6i+j-k) = (4i+2j-3k)+3(-i+k)$$

$$p = i+2j-6i-j+k$$

$$p = -5i+j+k //$$

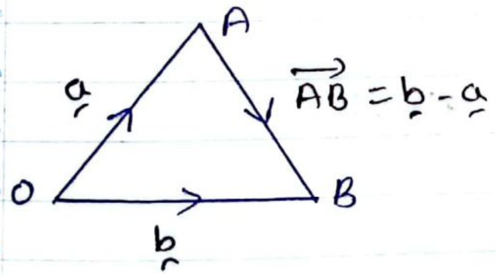
$$p+\begin{pmatrix}6\\1\\-1\end{pmatrix} = \begin{pmatrix}4\\2\\-3\end{pmatrix}+3\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$p = \begin{pmatrix}1\\2\\0\end{pmatrix}-\begin{pmatrix}6\\1\\-1\end{pmatrix}$$

$$p = \begin{pmatrix}-5\\1\\1\end{pmatrix} //$$

Vector equation of the straight line.

(1) Suppose $\underset{\sim}{a} = \overrightarrow{OA}$ is the vector joing the origin O to the point A and $\underset{\sim}{b} = \overrightarrow{OB}$ the vector joins O to B.



Thus $\overrightarrow{AB} = \underset{\sim}{b}-\underset{\sim}{a}$ is the vector from A to B

$$\overrightarrow{AB} = \underset{\sim}{b}-\underset{\sim}{a}$$

$$\underset{\sim}{b} = \underset{\sim}{a}+\overrightarrow{AB}$$

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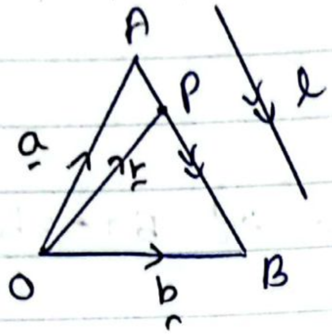

$\overrightarrow{AP} = \lambda \overrightarrow{AB}$

$r - a = \lambda (b - a)$

$r = a + \lambda (b - a)$ //

$\overrightarrow{AP} = \mu l$ (∵ l රේඛාවට සමාන්තරයි AB)

$r - a = \mu l$

$r = \mu l + a$ //

(2) The position vector of the points A and B are given by $a = 6i + 2j - k$ $\quad b = -3i + 2j + k$

a) Find the vector $\overrightarrow{AB}$

b) Find the vector $\overrightarrow{BA}$

c) Find the vector $p = b + 2(\overrightarrow{BA})$

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d) Discribe in terms of geometry the vector $r = b + s(\overrightarrow{BA})$ where s is any real number

$s > 0,\ s = 0,\ s < 0$

a) $\overrightarrow{AB} = b - a$

$= -3i + 2j + k - (6i + 2j - k)$

$= -9i + 2k$ //

(b) $\overrightarrow{BA} = a - b$

$= 6i + 2j - k - (-3i + 2j + k)$

$= 9i - 2k$ //

(c) $\cancel{\overrightarrow{AP}}$ $p = b + 2\overrightarrow{BA}$

$= -3i + 2j + k + 2(9i - 2k)$

$= 15i + 2j - 3k$ //

(d) $r = b + s(\overrightarrow{BA})$

$= -3i + 2j + k + s(9i - 2k)$

$= (9s - 3)i + 2j + (1 - 2s)k$ //

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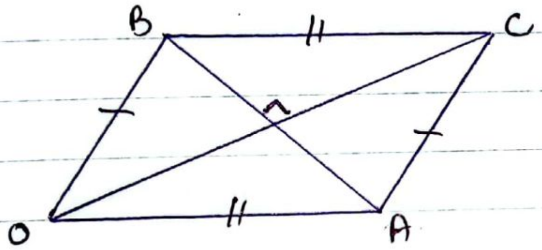
$s > 0$,

* S is various or this position various. Therefore it is the vector equation of the straight Line joining සරල රේඛාව B to A.

* S can take negative value. we get the vector equation of the Lining jaining A to B

* geometricaly this are the same line what traveling opposite way

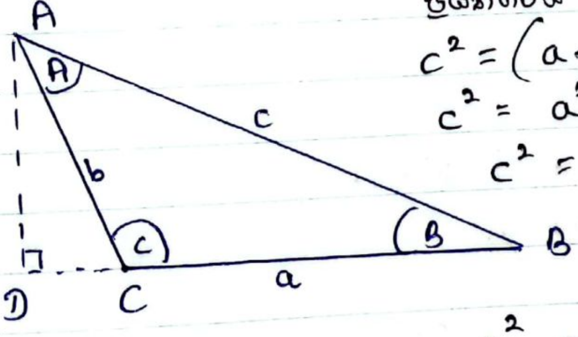
H.W. (1) The diasonal of a rhombus Mlersect at right anstes (an angle $90°$)



$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$

(2) The angle subtended by a diameter at the circumference of a semi-cricle is a right angle

(3) Show that $c^2 = a^2 + b^2 - 2ab\cos C$



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$c^2 = (a + b\cos(180 - C))^2 + (b\sin(180 - C))^2$

$c^2 = a^2 + b^2\cos^2(C) - 2ab\cos C + b^2\sin^2 C$

$c^2 = a^2 + b^2(\cos^2 C + \sin^2 C) - 2ab\cos C$

$c^2 = a^2 + b^2 - 2ab\cos C$ //

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(01) $|\overrightarrow{OA}| = |\overrightarrow{AC}| = |\overrightarrow{CB}| = |\overrightarrow{BO}|$

$\overrightarrow{OA} = \overrightarrow{BC}$; $|\overrightarrow{AC}| = |\overrightarrow{OB}|$

$$\begin{aligned}
\overrightarrow{OC} \cdot \overrightarrow{AB} &= (\overrightarrow{OA} + \overrightarrow{AC}) \cdot (\overrightarrow{AC} + \overrightarrow{CB}) \\
&= (\overrightarrow{OA} + \overrightarrow{AC}) \cdot (\overrightarrow{AC} + -\overrightarrow{OA}) \\
&= \underline{\overrightarrow{OA} \cdot \overrightarrow{AC}} - \overrightarrow{OA} \cdot \overrightarrow{OA} + \overrightarrow{AC}\,\overrightarrow{AC} - \underline{\overrightarrow{AC} \cdot \overrightarrow{OA}} \\
&= \overrightarrow{AC} \cdot \overrightarrow{AC} - \overrightarrow{OA} \cdot \overrightarrow{OA} \\
&= |\overrightarrow{AC}|^2 - |\overrightarrow{OA}|^2 \\
&= \underline{\underline{0}}
\end{aligned}$$

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① The position vectors of points A and B relativ to the orisin O are $4i+4j+7k$ and $5i-2j+6k$ respectirely, find the derection cosines of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ and determine the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$

$\overrightarrow{OA} = 4i+4j+7k$

$\overrightarrow{OB} = 5i-2j+6k$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -4i-4j-7k + 5i-2j+6k$

$\overrightarrow{AB} = +i-6j+\cancel{13k}\ -k$ //

$|\overrightarrow{OA}| = \sqrt{4^2+4^2+7^2} = \sqrt{81} = \underline{\underline{9}}$

$|\overrightarrow{OB}| = \sqrt{5^2+(-2)^2+6^2} = \sqrt{65}$

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$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-6)^2 + (13)^2}$

$\dfrac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \dfrac{4i+4j-7k}{9}$

Direction cosne of $\overrightarrow{OA} = \frac{4}{9}, \frac{4}{9}, \frac{-7}{9}$

Direction cosne of $\overrightarrow{OB} = \frac{5}{\sqrt{65}}, \frac{-2}{\sqrt{65}}, \frac{6}{\sqrt{65}}$

(2) Given Points $A(5,-1,3)$ and B with position vector $\overrightarrow{OB} = i+2j-4k$ find the vector $\overrightarrow{AB}$ It is given further that c has coordinate$(13, y, 17)$ Given A, B. and C are collinear find y A, B and C are colinear $\Longleftrightarrow \overrightarrow{AB} = k\overrightarrow{AC}$ for some $k \neq 0$

A, B, C are collinear, $\overrightarrow{AB} = k\overrightarrow{AC}$ for some $k \neq 0$

$\overrightarrow{OA} = 5i-j+3k$

$\overrightarrow{OB} = i+2j-4k$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -5i+j-3k+i+2j-4k$

$= -4i+3j-7k$

$\overrightarrow{OC} = 13i+yj+17k$

$\overrightarrow{AB} = k\,\overrightarrow{AC}$

$-4i+3j-7k = k(\overrightarrow{AO}+\overrightarrow{OC})$

$-4i+3j-7k = k(-5i+j-3k+13i+yj+17k)$

$-4i+3j-7k = k(8i+(y+1)j+14k)$

$i \neq 0 \quad j \neq 0 \quad k \neq 0 \quad i \nparallel j \quad j \nparallel k \quad i \nparallel k$

$i \Rightarrow -4 = k8$

$-\frac{1}{2} = k$ //

$j \Rightarrow 3 = (y+1)k$

$3 = (y+1)(-\frac{1}{2})$

$-6 = y+1$

$\underline{-7 = y}$

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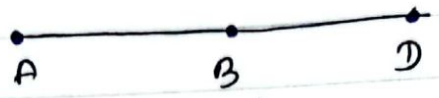
(3) If D has cardinate $(-3, 5, -11)$ show that $A(5, -1, 3)$ $B(1, 2, -4)$ and D lie on the same line

$$\overrightarrow{AB} = \lambda \overrightarrow{AD}$$



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -5i + 1j - 3k + i + 2j - 4k$$
$$= -4i + 3j - 7k \,//$$

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$$
$$= -5i + 1j - 3k + -3i + 5j - 11k$$
$$= -8i + 6j - 14k \,//$$

$$\overrightarrow{AB} = \tfrac{1}{2} \overrightarrow{AD}$$

Therefore $\overrightarrow{AB} \parallel \overrightarrow{AD}$

A, B, D same line.

(4) Determine whether $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $z = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are coplanar vectors

$\underset{\sim}{x} \cdot \underset{\sim}{y} \wedge \underset{\sim}{z}$

$$\begin{array}{ccc} i & -j & k \end{array}$$
$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1(1\times1 - 2\times1) - 1(2\times1 - 3\times2) + 1($$
$$= -1 - (-4) - 1$$
$$= \underline{\underline{2}} \neq 0$$

We have to calcular scalar triple product $x \cdot y \wedge z$

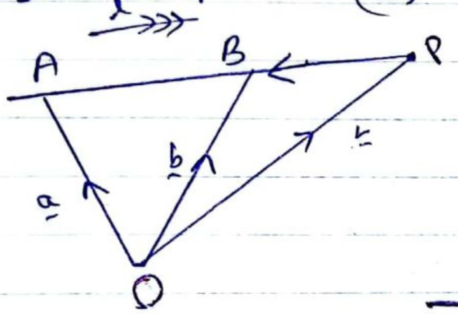
$$y \wedge z = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} = i(1-2) - j(1-1) + k(2-1)$$
$$= -i + k \,//$$

$$x \cdot y \wedge z = (i + 2j + 3k) \cdot (-i + k)$$
$$= -1 + 3 = 2 \neq 0$$

Therefore non coplanar.

(5) We can see, Scalar triple product is not equal to zero, hence vector x, y, and z are not coplanar.

Write down the eqation of the line that posses through the points $(2,-1,3)$ and $(1,4,-3)$



$$\overrightarrow{AP} = \lambda \overrightarrow{AB}$$
$$r - a = \lambda(b - a)$$
$$r = a + \lambda(b - a) \,//$$

$$\overrightarrow{AP} = m l$$
$$r - a = m l$$
$$r = a + m l \,//$$

$$\overrightarrow{OA} = a = (2,-1,3)$$
$$\overrightarrow{OB} = b = (1,4,-3)$$

$$r = (2,-1,3) + \lambda[(1+4-3) - (2,-1+3)]$$
$$(x,y,z) = (2,-1,3) + \lambda(-1,5,-6)$$

$$x = 2 - \lambda$$
$$y = -1 + 5\lambda$$
$$z = 3 - 6\lambda$$

$$\frac{x-2}{-1} = \lambda$$

$$\frac{y+1}{5} = \lambda$$

$$\frac{z-3}{-6} = \lambda$$

$$\frac{x-2}{-1} = \frac{y+1}{5} = \frac{z-3}{-6} \,//$$

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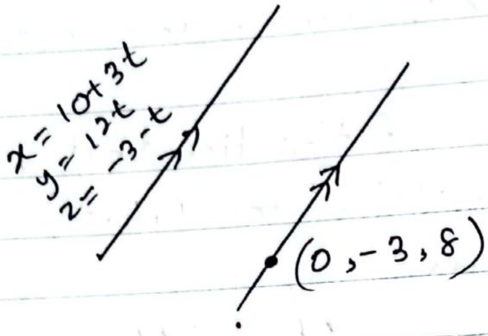
(6) Determine if the line that passes through the point $(0, -3, 8)$ and is paralled to the line given by $x = 10 + 3t$, $y = 12t$ and $z = -3 - t$ passes through the xz plane If does given the Cootinales of the point.



$r = a + \mu l$

$10 + 3t = 12t + (-3 - t)(0, -3, 8)$

$10 + 3t = 12t + 9 + 24 + 3t - 8t$

$10 + 3t = 7t + 33$

$-23 = 4t$

$\frac{-23}{4} = t$

$\frac{x - 10}{3} = t$

$\frac{y - 0}{12} = t$

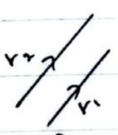
$\frac{z + 3}{-1} = t$

$r = a + \mu l$

$= (0, -3, 8) + \mu(3, 12, -1)$

$\frac{x - 10}{3} = \frac{y}{12} = \frac{z + 3}{-1}$

(7) Let a, b and c be three non-coplanar vectors. Show that the lines $r_1 = 8a - 9b + 10c + \lambda(3a - 16b + 7c)$, $r_2 = 15a + 29b + 5c + m(3a + 8b - 5c)$ are non coplaner.



$r_1 = 8a - 9b + 10c + \lambda(3a - 16b + 7c)$

$r_1 = (8 + 3\lambda)a + (-16\lambda - 9)b + (7\lambda + 10)c$

$r_2 = 15a + 29b + 5c + m(3a + 8b - 5c)$

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$r = a + \lambda \hat{l}$

$r = b + \mu m$

Assume that the lines have a common point

$8a - 9b + 10c + \lambda(3a - 16b + 7c) = 15a + 29b + 5c + m(3a + 8b - 5c)$

$a(8 + 3\lambda - 15 - 3m) + b(-9 - 16\lambda - 29 - 8m) + c(10 + 7\lambda - 5 + 5m)$

$a(-7 + 3\lambda - 3m) + b(-38 - 16\lambda - 8m) + c(5 + 7\lambda + 5m) = 0$

$-7 + 3\lambda - 3m = 0 \leftarrow (1)$

$-38 - 16\lambda - 8m = 0 \leftarrow (2)$

$5 + 7\lambda + 5m = 0 \leftarrow (3)$

$3\lambda - 3m = 7 \leftarrow (4)$

$7\lambda + 5m = -5 \leftarrow (5)$

$(4) \times 5 + (5) \times 3$

$15\lambda - \cancel{15m} + 21\lambda + \cancel{15m} = 35 - 15$

$36\lambda = 20$

$\lambda = \frac{10}{18}$

$\lambda = \frac{5}{9}$ //

$\cancel{3} \times \frac{5}{\cancel{9}_3} - 7 = 3m$

$\frac{5 - 21}{3} = 3m$

$\frac{16}{9} = m$ //

$r = 8a - 9b + 10c + \frac{5}{9}(3a - 16b + 7c)$

$r = 15a + 29b + 5c + \frac{16}{9}(3a + 8b - 5c)$

these equations are not consistant two lines have no common pts Here are not coplanar.

(8) A traffic control is tracking two planes invicinity of their airport At a given moment one plane is at a location 45 km east and 120 km north of the airport at an alltitude height of 7.5 km The second plane is located 63 km east and 96 km south of the airport at an altetude 06 6.0 km. The 1^{st} plane is flgins directly toward the airport while the 2^{nd} plane is continuns at a constant alltitude with a headins defined by the vector $\vec{h_2} = (3,4,0)$ to land eventually at another airport to the northwest of our air traffic controllers Do the paths of these two aircraft cross?

The 1^{st} thins we need to do is to determine the position vectors of the two planes Define our airport as the orisin of cordinates and define

x = east

y = north

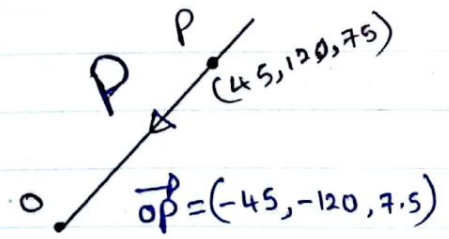
z = upward

Position vector of plane (1)

$p = (45, 120, 75)$

position vector of plane (2)

$\varepsilon = (63, -96, 6)$



$\vec{op} = 45, 120, 75$

$\vec{op} = \lambda [\vec{op})$

$r = (63, 96, 6)$

plane 1 is heading directly toword the airport. The vector from the position of plane 1 to the origin is givenly

$$\underset{\sim}{v} = \underset{\sim}{o} - \underset{\sim}{p} = (0-45, 0-120, 0-7.5)$$
$$= (-45, -120, -7.5)$$

The equation of the line representry plane 1 mention there becoms $r_1 = (45, 175, 7.5) + k_1(-45, -120, -7.5)$

plane 2 continuous at a constant altitude with headly $\vec{h}_2 = (3, 4, 0)$ The equation of the line representions plane metion the becomes.

$r_2 = (-63, 96, 66) + k_2(3, 4, 0)$

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* $(m+n)\underset{\sim}{a} = m\underset{\sim}{a} + na$

case I $\quad m=0 \quad n=0 \quad \underset{\sim}{a}=0$ ✓

$0 \times 0 = 0 + 0$

case II $\quad m>0 \quad n>0 \quad a \neq 0$

$|(m+n)\underset{\sim}{a}| = |m\underset{\sim}{a} + n\underset{\sim}{a}|$

$(m+n)|\underset{\sim}{a}| = |(m+n)\underset{\sim}{a}|$

$(m+n)|\underset{\sim}{a}| = (m+n)|a|$

R.H.S = L.H.S

$(m+n) > 0$

$(m+n)a$ has the same director since $(m+n) > 0$

$ma + na$ has the same director since both have only vector $\underset{\sim}{a}$

Case III $\quad m<0, n<0, a \neq 0$

Let us take $\quad p = -m$ and $q = -n$

$\quad -p = m \quad -q = n$

$(p+q)a = pa + qa$ (using case II)

$-(p+q)a = -(pa+qa)$

$(-p-q)a = -pa - qa$

$(m+n)a = ma + na$ //

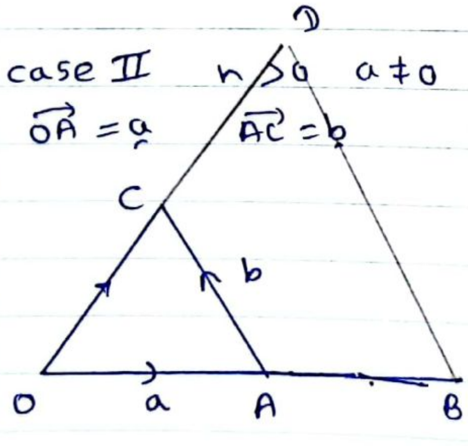
* $n(\underset{\sim}{a}+b) = n\underset{\sim}{a} + n\underset{\sim}{b}$

case I $\quad n=0, a=0, b=0$

$0 \times 0 = 0 + 0$

case II $\quad n>0 \quad a \neq 0$ and $b \neq 0$

$\overrightarrow{OA} = \underset{\sim}{a} \quad \overrightarrow{AC} = b$



Extend OA to B and dr[illegible] a parallel line to AC which is named as D

$\overset{\frown}{AC} \parallel BD$

$\overrightarrow{OC} = a+b$

$\overrightarrow{OD} = n(a+b)$

$\overrightarrow{OD} = n\overrightarrow{AC}$

$\overrightarrow{OD} = n\overrightarrow{OC}$

$\overrightarrow{OD} = n(a+b) \leftarrow ①$

$OB = n\,\overrightarrow{OA}$

$BD = n\,\overrightarrow{AC}$

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$\overrightarrow{OD} = na + nb \quad \leftarrow (2)$

(1), (2) $\Rightarrow$

$$n(a+b) = na + nb \quad //$$

Case III $\quad n<0, \ a\neq 0, \ b\neq 0$

Let us take $\quad n = -m \quad$ where $\quad m>0$

using the result form case II :

$$m(a+b) = ma + mb$$

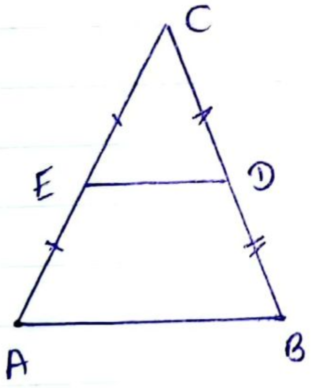
$$-m(a+b) = -(ma+mb)$$

$$-m(a+b) = -(ma+mb)$$

$$-m(a+b) = -ma - mb$$

$$n(a+b) = na + nb \quad //$$

(5) Prove that the line joining the midpoints of the two sides of a triangle is parallel to the third side and equal to half of it's length.



$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$ $\qquad \overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\quad = 2\overrightarrow{EA} + 2\overrightarrow{BD}$

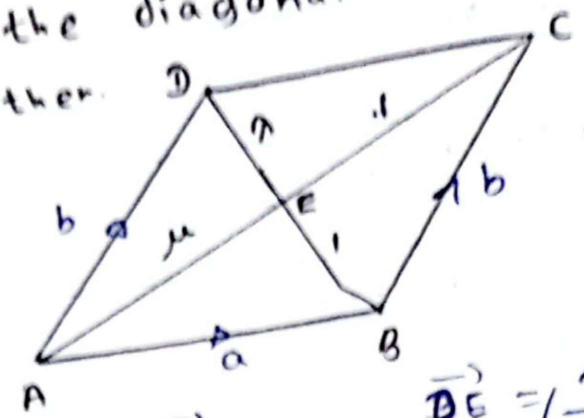
$\overrightarrow{AB} = 2(\overrightarrow{EC} + \overrightarrow{CD})$

$\quad = 2\,\overrightarrow{ED}$

$\therefore \overrightarrow{ED} = \frac{1}{2}\overrightarrow{AB}$

Since $\overrightarrow{ED}$ and $\overrightarrow{AB}$ have linear relation multiply by a sular we ~~are~~ can say there are parallel to each other.

⑥ Prove the diagonals of a parallelogram bisect each other.



$\overrightarrow{DE} = \left(\frac{\lambda}{1+\lambda}\right) \overrightarrow{DB}$

$\overrightarrow{AE} = \left(\frac{\mu}{\mu+1}\right) \overrightarrow{AC} \leftarrow ①$

$$\begin{aligned}\overrightarrow{AE} &= \overrightarrow{AD} + \overrightarrow{DE} \\ &= \underset{\sim}{b} + \left(\frac{\lambda}{1+\lambda}\right) \overrightarrow{DB} \\ &= \underset{\sim}{b} + \left(\frac{\lambda}{1+\lambda}\right) (\overrightarrow{DA} + \overrightarrow{AB}) \\ &= \underset{\sim}{b} + \left(\frac{\lambda}{1+\lambda}\right) (-b + a) \\ &= \underset{\sim}{b} - \left(\frac{\lambda}{1+\lambda}\right) b + \frac{\lambda a}{1+\lambda} \\ &= \frac{(1+\lambda) b - \lambda b + \lambda a}{1+\lambda}\end{aligned}$$

$\overrightarrow{AE} = \frac{1+\lambda a}{1+\lambda} \leftarrow ②$

①ல்

$\overrightarrow{AE} = \left(\frac{\mu}{1+\mu}\right) (\overrightarrow{AB} + \overrightarrow{BC})$

$\overrightarrow{AE} = \left(\frac{\mu}{1+\mu}\right) (\underset{\sim}{a} + \underset{\sim}{b}) \leftarrow ①$

① = ②

$$\frac{\mu}{1+\mu}(a+b) = \frac{1+\lambda a}{1+\lambda}$$

$a \neq 0 \quad b \neq 0 \quad a \not\parallel b$

$a \Rightarrow$

$$\frac{\mu}{1+\mu} = \frac{\lambda}{1+\lambda}$$

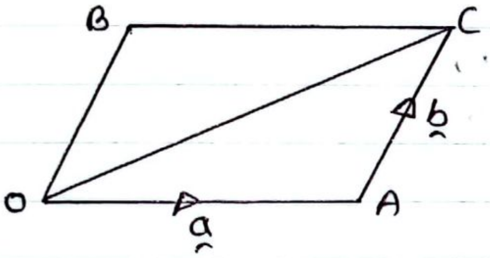
$b \Rightarrow$

$$\frac{\mu}{1+\mu} = \frac{1}{1+\lambda}$$

$\lambda = 1 \quad \therefore \quad \mu = 1$

therefore parallelogram biset each other.

(01) $a+b = b+a$



Traingalar rule for $OAC\ \Delta$

$\vec{OC} = \vec{OA} + \vec{AC}$

$\vec{OC} = a + b \leftarrow$ ①

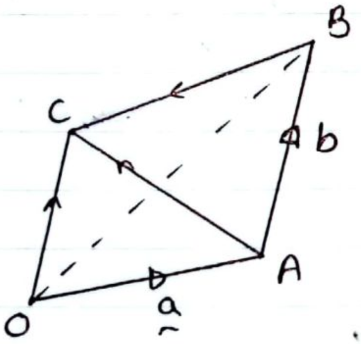
similary $\Delta\ OBC$

$\vec{OC} = \vec{OB} + \vec{BC}$

$\vec{OC} = b + a \leftarrow$ ②

①, ② Hence $a+b = b+a$ //

(02)



$a+(b+c) = (a+b)+c$

let us assume

$\vec{OA} = a \qquad \vec{AB} = b \qquad \vec{BC} = c$

$\vec{AC} = \vec{AB} + \vec{BC}$

$= b + c$ //

$\vec{OB} = \vec{OA} + \vec{AB}$

$= a + b$

$\vec{OC} = \vec{OA} + \vec{AC}$

$= a + (b+c) \leftarrow$ ①

$\vec{OC} = \vec{OB} + \vec{BC}$

$= (a+b) + c$

①, ② Hence $a+(b+c) = (a+b)+c$ //

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Example

(01) In physics, the motion of an object travelus at a constant speed is described by the equation $\vec{s}_t = \vec{s}_i + \vec{v}t$ where s_i is the initial position, s is the position at same later time t, and v is the object write the vector equation which returns the set of position vector $\vec{s}$ for an object havins an initial position $\vec{s}_i = (2,3,4)$ and a velocity of $\vec{v} = (1,1,-2)$ and determine the objects location at $t = 10s$

$$\vec{S}_t = \vec{S}_i + \vec{V}t$$
$$= (2,3,4) + (1,1,-2)t$$

At $t = 10s$

$$\vec{S}_{10} = (2,3,4) + (1,1,-2)\,10$$
$$= (2,3,4) + (10,10,-20)$$
$$= (12,13,-16)\,//$$

An object has a position of $\vec{s}_i = (3,3,6)$ at $t=0$ and a velocity of $\vec{v} = (10,7,3)$ use the vector equation $\vec{s} = \vec{s}_i + \vec{v}t$ to determine the distance traveled by the object between $t = 3s$ and $t = 5$ Distance measned in meters

$$\vec{S}_t = \vec{S}_i + \vec{V}t$$
$$= (3,3,6) + (10,7,3)t$$

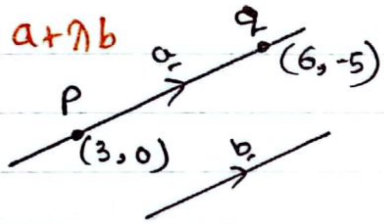
$$S_3 = (3,3,6) + (10,7,3)\,3$$
$$S_5 = (3,3,6) + (10,7,3)\,5$$

$$S_3 = (3,3,6) + (30,21,9) = (33,24,15)$$
$$S_5 = (3,3,6) + (50,35,15) = (53,38,21)$$

$$|s_5 - s_3| = |(53, 38, 21) - (33, 24, 15)|$$
$$= |(20, 14, 6)|$$
$$= \sqrt{(20)^2 + (14)^2 + 6^2}$$
$$= \sqrt{400 + 196 + 36}$$
$$= \sqrt{632}$$

Write the equation of $y = -\frac{5}{3}x + 5$ as a vector equation
Start by choosing two pts on the line, $x = 3$, $y = 0$
$x = 6$; $y = -\frac{5}{3} \times 6 + 5 = -5$

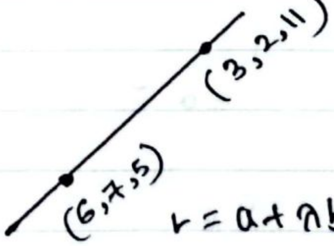
$r = a + \lambda b$



$$PQ = q - p$$
$$= (6, -5) - (3, 0)$$
$$= (3, -5)$$

Finally vector equation of the line
$$r = a + \lambda b$$
$$= (3, 0) + \lambda(3, -5)$$
$$= (3 + 3\lambda, -5\lambda)$$

Determine the equation for the line define by the pts $P = (6, 7, 5)$ and $q = (3, 2, 11)$ Then find the position vector for a point R, half - was between these two pts.
~~Finally vector equation of the line~~
~~$r = a + \lambda b$~~
~~$= (\ \ ) + \lambda(\ \ )$~~
~~$= (\ \ )$~~

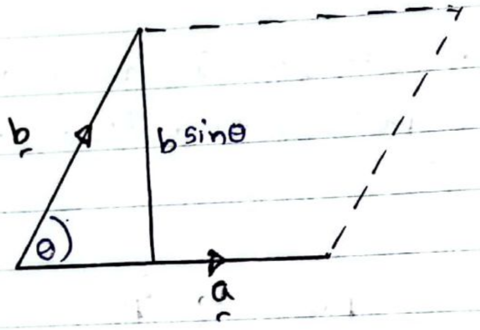


$r = a + \lambda b$

$$l = (3-6\ ,\ 2-7\ ,\ 11-5)$$
$$= (-3\ ,\ -5\ ,\ 6)$$

$$r = a + \lambda b$$
$$= (6,7,5) + \lambda\ (-3,-5,6)$$
$$= (6,7,5) + \frac{1}{2}\ (-3,-5,6)$$
$$= \left(\frac{9}{2}, \frac{5}{2}, 8\right)$$

Geometrical intopretation of vector produc

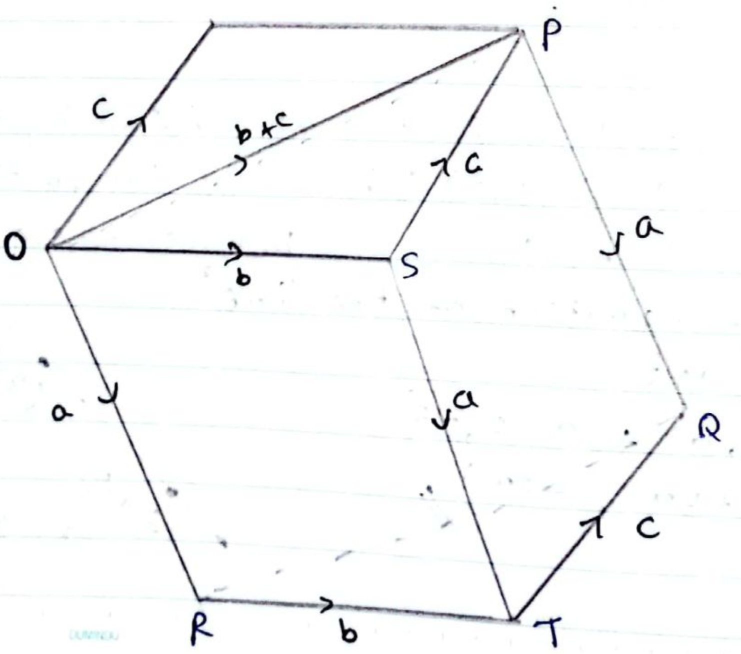


$$A = \underset{\sim}{a} \wedge \underset{\sim}{b} = |a|\ |b| \sin\theta\ n$$
$$|A| = |\underset{\sim}{a} \wedge \underset{\sim}{b}| = |a|\ |b| \sin\theta = \text{area of the paralle}$$

Distribution law

(i) $\underset{\sim}{a} \wedge (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \wedge \underset{\sim}{b} + \underset{\sim}{a} \wedge \underset{\sim}{c}$

(ii) $(\underset{\sim}{a} + \underset{\sim}{b}) \wedge \underset{\sim}{c} = \underset{\sim}{a} \wedge \underset{\sim}{c} + \underset{\sim}{b} \wedge \underset{\sim}{c}$



$\underset{\sim}{a} = \overrightarrow{OR}$, $b = \overrightarrow{OS}$, $c = \overrightarrow{TQ}$ The triangles OSP and RTQ are made up of the same vectors $\overrightarrow{RT} = \underset{\sim}{b}$, $\overrightarrow{TQ} = c$, $\overrightarrow{RQ} = \underset{\sim}{b} + \underset{\sim}{c}$ shows the three parallelograms OSTR, SPQT and OPQR in the same plane Their areas are equal.

OSTR SPQT OPQR

$|\underset{\sim}{a} \wedge b|$, $|a \wedge c|$ and $|(b+c) \wedge a|$ respectidy

$$|a \wedge (b+c)| = |(b \wedge c) \wedge a|$$

$$|a \wedge (b+c)| = \text{area of OPQR}$$

$$= \text{area of polygon OSPQTR}$$

$$= \text{area of OSTR} + \text{area of STQP}$$

$$|a \wedge (b+c)| = |\underset{\sim}{a} \wedge \underset{\sim}{b}| + |\underset{\sim}{a} \wedge \underset{\sim}{c}| \longleftarrow (1)$$

Next consider the directions. The direction of each of the three vector, $|a \wedge b|$ and $|\underset{\sim}{a} \wedge \underset{\sim}{c}|$ is the same and let n be the unit vector in that direction

From (1) sves $|a \wedge (b+c)| \underset{\sim}{n} = |a \wedge b| \underset{\sim}{n} + |a \wedge c| \underset{\sim}{n}$

Prove that $(\underset{\sim}{a} + \underset{\sim}{b}) \wedge (\underset{\sim}{a} - b) = -2 \underset{\sim}{a} \wedge b$

$$(\underset{\sim}{a} + \underset{\sim}{b}) \wedge (\underset{\sim}{a} - \underset{\sim}{b})$$

$$(\underset{\sim}{a} + \underset{\sim}{b}) \underset{\sim}{a} \wedge (\underset{\sim}{a} + \underset{\sim}{b}) \underset{\sim}{b}$$

$$\underbrace{\underset{\sim}{a} \wedge \underset{\sim}{a}}_{= 0} + \underset{\sim}{b} \wedge \underset{\sim}{a} - \underset{\sim}{a} \wedge \underset{\sim}{b} - \underbrace{\underset{\sim}{b} \wedge \underset{\sim}{b}}_{= 0}$$

$$\underset{\sim}{b} \wedge \underset{\sim}{a} - a \wedge b$$

$$= a \wedge b - a \wedge b$$

$$-2 \, a \wedge b$$

Simplity

(i) $(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}) \wedge (\underset{\sim}{b}-\underset{\sim}{c})$

(ii) $x \wedge y$ where $x = 2u - v$, $y = u + 2v$ and y and v being any two vectors

$(a+b+c)\wedge b - (a+b+c)\wedge c$

$a\wedge b + \cancel{*} c\wedge b - a\wedge c - b\wedge c$

$a\wedge b - a\wedge c + 2c\wedge b$ //

$v \wedge y = (2u-v) \wedge (u+2v)$

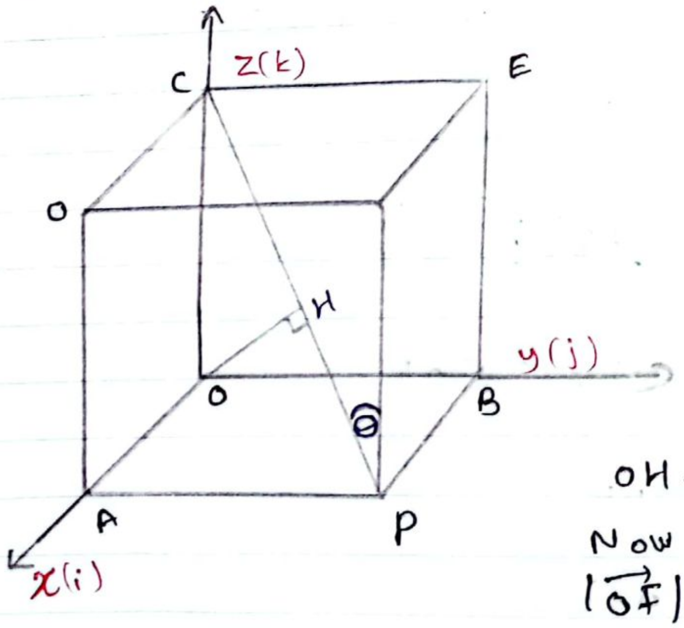
$=(2u-v)\wedge u + (2u-v)$

$= 2u\wedge u - v\wedge u + 4u\wedge v - 2v\wedge v$

$= -v\wedge u + 4u\wedge v$

$= 5u\wedge v$ //

Find the perpendicular distance of a Corner of unit cube from a diasand which does not pass thranshit



$OH = OF \sin\theta$ ← ①

Now $|\vec{OF} \wedge \vec{FC}|$

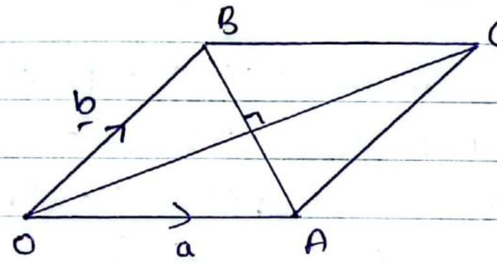
$|\vec{OF}|\,|\vec{FC}| \sin\theta$

$$\sin\theta = \frac{|\vec{OF} \wedge \vec{FC}|}{|\vec{OF}|\,|\vec{FC}|}$$

From ① $\Rightarrow$ $\quad OH = OF \dfrac{|\vec{OF} \wedge \vec{FC}|}{|\vec{OF}|\,|\vec{FC}|}$

$$= \frac{|\vec{OF} \wedge \vec{FC}|}{|\vec{FC}|} \quad //$$

The diagonal of a rhombus intersect at right angles (an angle 90°)



$\vec{OC} \cdot \vec{AB} = 0$

Suppose that $\vec{OA} = \underset{\sim}{a}$ and $\vec{OB} = \underset{\sim}{b}$

$\vec{OC} = \vec{OA} + \vec{AC}$

$= a + c$

$\vec{AB} = \vec{AO} + \vec{OB}$

$= b - a$

Since OACB is a rhomlus then $|a| = |b|$

Now prove that

$\vec{OC} \cdot \vec{AB} = (a+b) \cdot (b-a)$

$= a \cdot b - a \cdot a + b \cdot b - \underset{\sim}{b} - \underset{\sim}{a}$

$= b \cdot b - a \cdot a$

$= |b|^2 - |a|^2$

$= 0 \quad (\because |a| = |b|)$

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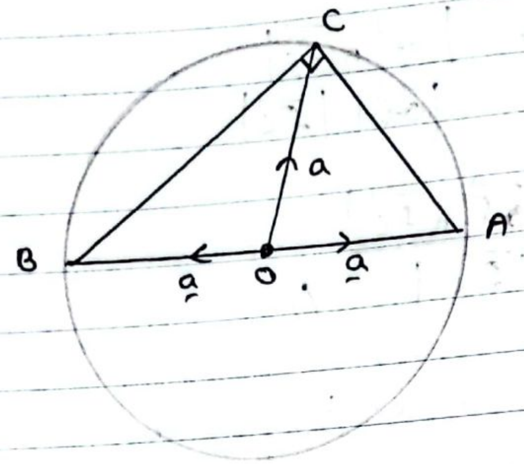
The angle subtended by a diameter at the Circumference of a sime cricle is a right angle



Taking o as the orism, let the position vector A, B, C be $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}$ repections The $|c| = |b| = |c|$ teradins To prove BC is perpendicula to CA consider the dot product $\overrightarrow{CA}$ and $\overrightarrow{CB}$

$$\overrightarrow{CA} \cdot \overrightarrow{CB} = (a-c)(b-c)$$

$$= a \cdot b - a \cdot c - c \cdot b - c \cdot c$$

If there non-zero a, b, c are coplanar, Then $(a \wedge b) \cdot \underset{\sim}{c} - 0$ and conversity, if the scalar Product $\underset{\sim}{a} \wedge \underset{\sim}{b} \cdot \underset{\sim}{c}$ of there non-zero vectors is zero, then thes are coplaner.

a, b, c are coplaner $\Leftrightarrow$ $a \wedge b \cdot c = 0$

proof

Assume that the vectors are coplanar

The c can be expressed as a linear combination

$$c = \lambda a + \mu b$$

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$$a \wedge b \cdot c = a \wedge b \cdot (\lambda a + \mu b)$$
$$= \lambda \, a \wedge b \cdot a + \mu \, a \wedge b \cdot b$$

let assume that $a \wedge b \cdot c = 0$

$a \wedge b \cdot c = 0$ ⇒(implies) that the volume of the parallelpiped formd by the vector vanishes.

Thus three vector are coplaner.

Some properties of the scalar triple product

(1) Scalar triple product $\{a, b, c\}$ where the vectors are s in terms of three non coplanar vectors l, m, n

let
$$a = a_1 l + a_2 m + a_3 n$$
$$b = b_1 l + b_2 m + b_3 n$$
$$c = c_1 l + c_2 m + c_3 n$$

Show that
$$a \wedge b \cdot c = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} (l, m, n)$$

If a, b, c are three vectors then

$$a \wedge (b \wedge c) = (a \cdot c) b - (a \cdot b) c$$

Proof let
$$a = a_1 i + a_2 j + a_3 k$$
$$b =$$
$$c =$$

$$a \wedge b \wedge c = (a_1 i + a_2 j + a_3 k) \wedge \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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$$= (a_1 i + a_2 j + a_3 k) \wedge \left(i (b_2 c_3 - b_3 c_2) - j (b_1 c_3 - c_1 b_3) + k (b_1 c_2 - c_1 b_2)\right)$$

$$= k\, a_1 (b_1 c_3 - c_1 b_3) - a_1 j\, (b_1 c_2 - c_1 b_2) - a_2 k (b_2 c_3 - b \ldots + a_2 (b_1 c_2 - c_1 b_2) + a_3 j (b_2 c_3 - b_3 c_2) - a_3 i\, (b_1 c_3 - c_1 \ldots$$

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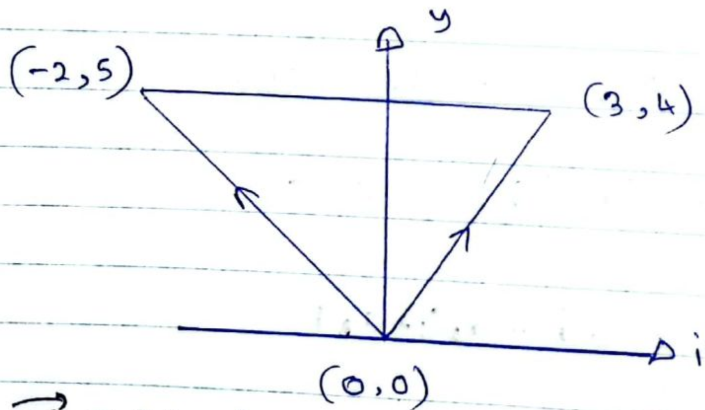
Example

(01) (i) Represent the vector joinins the points A (3,4), B (-2,5) to the origin (0,0) on a cartesian diagram.

(ii) Express the position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of vector i and $\sqrt{2}$

(iii) Write down the unit vectors in the direction of $\overrightarrow{OA}$ and $\overrightarrow{OB}$

(iv) Find $\overrightarrow{AB}$ in terms of i, j, z and herne find the magnitude of $\overrightarrow{AB}$



A (3,4)

B (-2,5)

$\overrightarrow{OA} = 3i + 4j$

$\overrightarrow{OB} = -2i + 5j$

(iii) $\dfrac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \dfrac{3i + 4j}{\sqrt{3^2 + 4^2}}$ $\quad |b| = \dfrac{\overrightarrow{OB}}{|\overrightarrow{OB}|} = \dfrac{-2i + 5j}{\sqrt{(-2)^2 + 5^2}}$

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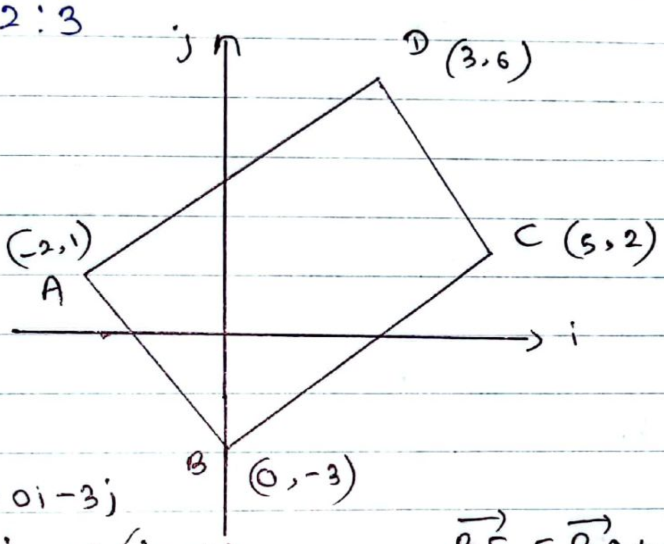
(iv). $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3i + 4j + -2i + 5j$

$= -5i + j$

$|\overrightarrow{AB}| = \sqrt{5^2 + 1^2} = \sqrt{26}$ //

(02). Prove that A (-2,1) , B (0,-3) , C (5,2) and D(3,6) are the vectors of a parallelogram. If E is the point (3,-9) Show that A, B and E are collinear and that B divides AE internally in the ratioo AB : BE = 2 : 3



$\overrightarrow{OA} = -2i + j$

$\overrightarrow{OB} = -3j$

$\overrightarrow{OC} = 5i + 2j$

$\overrightarrow{OD} = 3i + 6j$

$\overrightarrow{OE} = 3i - 9j$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= 2i - j + 0i - 3j$

$= 2i - 4j$ // $= 2(i - 2j)$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= -5i - 2j + 3i + 6j$

$= -2i + 4j$ //

$|\overrightarrow{AB}| = \sqrt{2^2 + (-4)^2}$

$= \sqrt{20}$

$|\overrightarrow{CD}| = \sqrt{20}$

AB = CD ∴ ABCD is parallelogram.

$\overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$

$= 3j + 3i - 9j$

$= 3i - 6j$ // $= 3(i - 2j)$

$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{BE}$

∴ A, B, and E are collinear

AB : BE = 2 : 3

$|\overrightarrow{AB}| = \sqrt{20}$

$|\overrightarrow{BE}| = \sqrt{3^2 + (-6)^2} = \sqrt{9 + 36}$

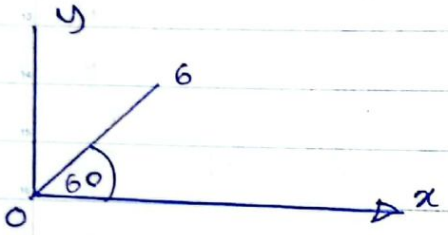
$= \sqrt{45} = 3\sqrt{5}$

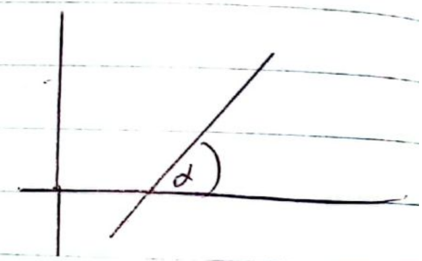
$$\frac{|AB|}{|BE|} = \frac{AB}{BE} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

$AB : BE = 2 : 3$ //

(3) a) Find the components along the x and y axis a vector of magnative b which makes an angle of 60 with OX

b) Find the magnative of the vector $a = -i - \sqrt{3}j$ and the made by a with the +v axies





a) $a = 6\cos 60^\circ i + 6\sin 60^\circ j$

$= 6\left(\frac{1}{2}i + \frac{\sqrt{3}}{2}j\right)$

$= 3i + 3\sqrt{3}j$ //

b) $a = -i - \sqrt{3}j$

$|a| = \sqrt{(-1)^2 + (\sqrt{3})^2}$

$= \sqrt{4}$

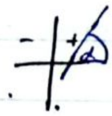
$= \underline{\underline{2}}$

$$\hat{a} = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$$

$$= \frac{-i - \sqrt{3}j}{2}$$ //

If α is the angle made by the vector with the position direction of x then

$\cos\alpha = -\frac{1}{2}$ $\qquad$ $\sin\alpha = \frac{\sqrt{3}}{2}$



$\alpha = \cos^{-1} -\frac{1}{2}$

$\cancel{\alpha = \frac{3\pi}{2}}$

$\alpha = \pi + \frac{\pi}{3}$ //

(4) If $A(-1,2,0)$, $B(1,3,2)$, $C(7,1,5)$ find the point D such that $ABCD$ is a parallelogram.

$\vec{AB} = \vec{AO} + \vec{OB}$

$= i - 2j + i + 3j + 2k$

$= 2i + j + 2k$ //

$\vec{CD} = \vec{CO} + \vec{OD}$

$= -7i - j - 5z + xi + yj + z'z$

$\cancel{= (x-7)i + (y-1)j + (z-5)z}$

$= -7i - j - 5k + d$ //

$\cancel{\vec{AB} \parallel \vec{CD}}$

$\cancel{\vec{AB} = \lambda \vec{CD}}$

$\cancel{2i + j + 2z = \lambda\left((x-7)i + (y-1)j + (z-5)z\right)}$

$\cancel{i \neq 0 \quad j \neq 0 \quad z \neq 0 \quad i \nparallel j \quad i \nparallel z}$

$\vec{AB} = \vec{CD}$

$2i + j + 2k = -7i - j - 5k + d$

$d = 2i + j + 2k + 7i + j + 5k$

$d = 9i + 2j + 7k$ //

$5i + 3k$ //

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(5) Find a vector eq for the straight line that passes through the points A (2,1,1), B (-1,2,-1) show that point (5,0,3) also lies on the line.

$\vec{AB} = \vec{AO} + \vec{OB}$

$= -2i - j - k + -i + 2j - k$

$= -3i + j - 2k$ //

$r = a + \lambda l$

$= (2, 1, 1) + \lambda (-3, 1, -2)$

$x, y, z = (2, 1, 1) + \lambda (-3, 1, -2)$

$(x-2), (y-1), (z-1) = \lambda (-3, 1, -2)$

$x - 2 = -3\lambda$ $\quad$ $y - 1 = \lambda$ $\quad$ $z - 1 = -2\lambda$

$\frac{x-2}{-3} = \lambda$ $\quad$ $\frac{z-1}{-2} = \lambda$

$\left(\frac{x-2}{-3}, \frac{y-1}{1}, \frac{z-1}{-2}\right)$ //

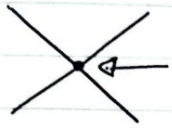
$r = a + \lambda l$

$(5, 0, 3)$

$\frac{5-2}{-3} = \lambda$ $\quad$ $0 - 1 = \lambda$ $\quad$ $\frac{3-1}{-2} = \lambda$

$\underline{\underline{-1 = \lambda}}$ $\quad$ $\underline{\underline{-1 = \lambda}}$ $\quad$ $\underline{\underline{-1 = \lambda}}$

(6) Show that lines $r = 7i - 3j + 5k + \lambda(3i - 2j + k)$ and $r = 7i - 2j + 4k + \mu(-2i + j - k)$ intersect and find the possition vector of the poin intersection



Intersection point

$$7i - 3j + 5k + \lambda(3i - 2j + k) = 7i - 2j + 4k + \mu(-2i + j - k)$$

$$i(7 + 3\lambda) + j(-3 - 2\lambda) + k(5 + \lambda) = i(7 - 2\mu) + j(-2 + \mu) + k(4 - \mu)$$

$$i(3\lambda + 2\mu) + j(-1 - 2\lambda - \mu) + k(1 + \lambda + \mu) = 0$$

$i \neq 0 \quad j \neq 0 \quad k \neq 0$

E q

$3\lambda + 2\mu = 0 \leftarrow (1)$ $\qquad 3\lambda = -2\mu$

$-1 - 2\lambda - \mu = 0 \leftarrow (2)$ $\qquad \lambda = \dfrac{-2\mu}{3}$

$1 + \lambda + \mu = 0 \leftarrow (3)$

(2) $\Rightarrow$

$$-1 - 2\left(\frac{-2\mu}{3}\right) - \mu = 0$$

$$-1 + \frac{4\mu}{3} = \mu$$

$$\cancel{\frac{4\mu - 3}{3} = \mu}$$

$$+\frac{4\mu}{3} - \mu = 1$$

$$\frac{\mu}{3} = 1$$

$$\underline{\underline{\mu = 3}}$$

$$\lambda = -\frac{2 \times 3}{3} = -2$$

$$\underline{\underline{\lambda = -2}}$$

(7) Let a, b and c be three non-coplanar vectors. Show that the lines

$r = 8a - 9b + 10c + \lambda(3a - 16b + 7c)$

$r = 15a + 29b + 5c + \mu(3a + 8b - 5c)$

are non coplanar.

Assume that lines have a point incommon Then

$8a - 9b + 10c + \lambda(3a - 16b + 7c) = 15a + 29b + 5c + \mu(3a + 8b - 5c)$

$a(8 + 3\lambda - 15 - 3\mu) + b(-9 - 16\lambda - 29 - 8\mu) + c(10 + 7\lambda - 5 + 5\mu) =$

$a(3\lambda - 3\mu - 7) + b(-38 - 16\lambda - 8\mu) + c(5 + 7\lambda + 5\mu) = 0$

a, b, c are non coplanar

$3\lambda - 3\mu - 7 = 0 \longleftarrow (1)$

$-38 - 16\lambda - 8\mu = 0 \longleftarrow (2)$

$5 + 7\lambda + 5\mu = 0 \longleftarrow (3)$

$7\lambda = -5\mu - 5$

$\lambda = \frac{-5}{7}(\mu + 1)$ //

(1) =>

$3\left(\frac{-5}{7}(\mu+1)\right) - 3\mu - 7 = 0$

$-15\mu - 15 - 21\mu - 49 = 0$

$36\mu = 64$

$\mu = \frac{64}{36}$

$\mu = \frac{11}{36}$

$\mu = \frac{29}{36}$

$\mu = \frac{16}{9}$ //

$\lambda = \frac{-5}{7}\left(\frac{16}{9} + 1\right)$

$\lambda = \frac{-5}{7}\left(\frac{25}{9}\right)$

$\lambda =$

$$\cancel{\lambda = \frac{-5}{7}\mu - \frac{5}{7}} = \cancel{\frac{-5}{7}\left(\frac{16}{9}\right) - \frac{5}{7}} =$$

Vector triple product

(a, b, c) is defined as the cross product of vector a with the cross product of vectors b and c ie $a \wedge (b \wedge c)$. Here $a \wedge (b \wedge c)$ is coplanar with the vectors b and c and preperdicular to a. Hence we can write,

$a \wedge (b \wedge c)$ as linear combmation of vector band c
That is $a \wedge (b \wedge c) = xb + yc$ vector triple product formula

$$a \wedge (b \wedge c) = (a \cdot c)b - (a \cdot b)c$$

and $(a \wedge b) \wedge c = (a \cdot c)b - (b \cdot c)a$

$\Rightarrow$ $a \wedge (b \wedge c) + (a \wedge b) \wedge c$

Vector triple product proof ~~$(a \wedge b) \wedge c = (a \cdot c)b - (b \cdot c)a$~~

$$(a \wedge b) \wedge c = (a \cdot c)b - (b \cdot c)a$$

$(a \wedge b) \wedge c$ as a linear combination of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

$$(a \wedge b) \wedge c = xa + yb \leftarrow (1)$$

$c \cdot (1)$ $\qquad\qquad a \cdot b \wedge c = a \wedge b \cdot c$

$$c \cdot \underbrace{[(a \wedge b) \wedge c]}_{0} = c \cdot [xa + yb]$$

$$= x\, a \cdot c + y\, b \cdot c$$

$$0 = x\, a \cdot c + y\, b \cdot c$$