MAT 1204-Differential Equations I
Assignment # 01

Solve all the problems and submit the solutions for the following problems: **1**(i,ii,iii,v), **2**(i,iii,v), **3**(i,iii,v),**4**(i,ii,iv),**5**(i,iv,v),**6**(i,iii,v) on or before 9th May, 2023.

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1. Solve each of the following differential equations:
   i. $(x+1)\frac{dy}{dx} = x(y^2+1)$
   ii. $(xy^2+x)dx + (yx^2+y)dy = 0$
   iii. $x^4\frac{dy}{dx} + x^3y = -\sec(xy)$
   iv. $x\frac{dy}{dx} + \cot y = 0$
   v. $(x+y)^2\left(x\frac{dy}{dx}+y\right) = xy\left(1+\frac{dy}{dx}\right)$
   vi. $\cos(x+y)dy = dx$
   vii. $\frac{dy}{dx} - x\tan(y-x) = 1$

2. Solve the following homogeneous differential equations:
   i. $\frac{dy}{dx} = \frac{3y^2+2xy}{2xy+x^2}$
   ii. $(x^2+y^2)\frac{dy}{dx} = 2xy$
   iii. $(y^2-xy)dx + x^2dy = 0$
   iv. $(x^2-y^2)dx + 2xydy = 0$
   v. $\frac{dy}{dx} = \frac{3xy+y^2}{3x^2}$

   Note that a function $f = f(x,y)$ is homogeneous of degree n if $f(tx,ty) = t^n f(x,y)$ for all x and y.

3. Solve each of the following differential equations by reducing to the corresponding homogeneous forms:
   i. $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$
   ii. $\frac{dy}{dx} = \frac{2x+9y-20}{6x+2y-10}$
   iii. $\frac{dy}{dx} = \frac{x+2y-1}{x+2y+1}$
   iv. $\frac{dy}{dx} = -\frac{3y-7x+7}{7y-3x+3}$
   v. $(6x-4y+1)dy - (3x-2y+1)dx = 0$

4. Solve the following linear differential equations using an integrating factor:
   i. $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$
   ii. $x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$
   iii. $\frac{dy}{dx} + 2y\tan x = \sin x$

iv. $\sec x \frac{dy}{dx} = y + \sin x$

v. $(2y - 3x)dx + xdy = 0$

5. Solve the following Bernoulli equations by transforming into linear equations:

i. $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

ii. $\frac{dy}{dx} + y = xy^3$

iii. $\frac{dy}{dx} + 2xy = 4y$

iv. $\frac{dy}{dx} = y^4 \cos x + y \tan x$

v. $x\frac{dy}{dx} + 2y + x^5y^3e^x = 0$

6. Determine if the following differential equations are exact and solve.

i. $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0$

ii. $(e^y + 1)\cos x\ dx + e^y \sin x\ dy = 0$

iii. $\frac{y}{x} + (y^3 + \ln x)\frac{dy}{dx} = 0$

iv. $2x\left(1 + \sqrt{x^2 - y}\right) - \sqrt{x^2 - y}\frac{dy}{dx} = 0$

v. $\frac{3x^2+y^2}{y^2} - \frac{2x^3+5y}{y^3}\frac{dy}{dx} = 0$