Opinion Dynamics

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Introduction

According to classical opinion dynamics models in which social interactions add constructively to opinion formation, the increasing interaction rates of modern societies would eventually lead to a global consensus, even on controversial issues. This classical prediction has been recently challenged by the empirical observation of opinion polarization, i.e., the presence of two well- separated peaks in the opinion distribution. Here we shall justify a model of agent based opinion dynamics that takes into consideration, multiple opinions, and try to quantify the consensus and polarization of opinions based on the model parameters.

A model for opinion dynamics

Opinions on a particular topic is a real number. When there are multiple topics, it makes sense to model each topic as an axis on an euclidean space. Thus, if we have T topics, then we have a T dimensional euclidean space, where the angle between each axis indicates the topic overlap. Let N be the number of agents on the social network, then opinion of each agent i, is a vector $\mathbf{x}_i = (x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(T)})$. We are interested in the time evolution of each of the N vectors. Empirical evidences suggests that

$$\frac{\mathrm{d}x_i^{(v)}}{\mathrm{d}t} = -x_i^{(v)} + K\sum_j A_{ij}(t)\tanh\left(\alpha[\phi x_j]^{(v)}\right)$$

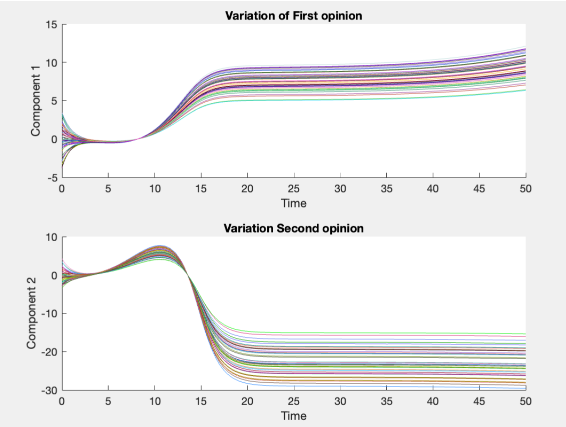
Where K is the connectivity of the network, A is the temporal adjacency matrix of the network, α is the controversialness of the topics which for simplicity is taken as same, and ϕ is the matrix that embeds the information about topic overlap. $(i,j)^{th}$ entry of ϕ the cosine of angle between topics i and j. The initial opinions are distributed normally with some mean and variance, and then the opinions at later time can be captured by solving the system of $N \times T$ coupled differential equations. But the adjacency matrix is not constant at all the times. So the entries of the matrix are governed by the Activity Driven(AD) model.

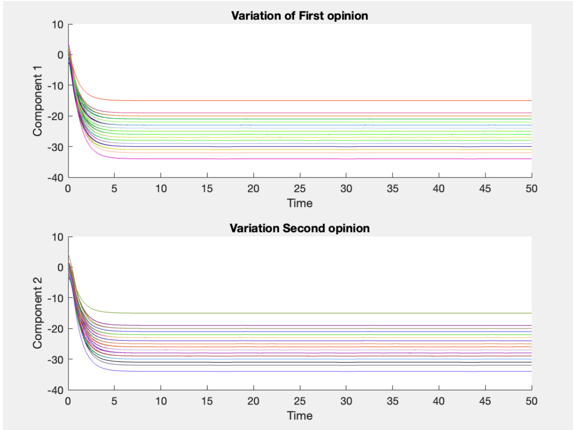
AD Model

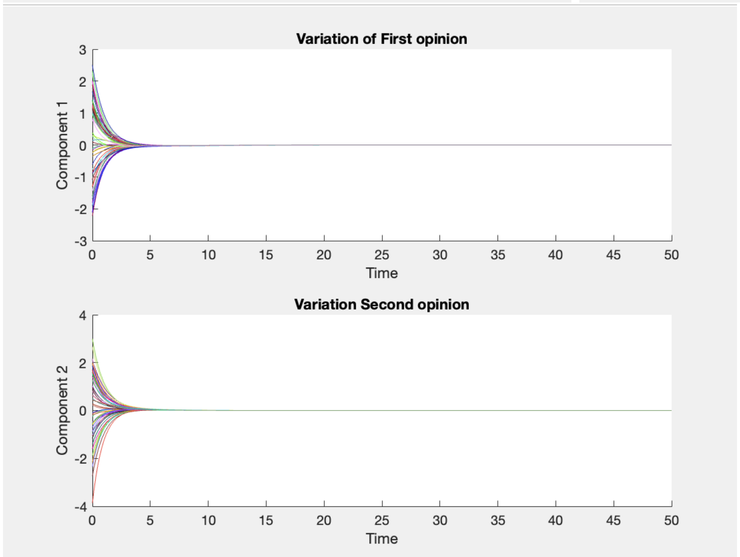
The contact pattern among the agents follow AD dynamics. It suggests that each agent i is characterized by an activity $a_i \in [\epsilon, 1]$, representing his or her propensity to become active at a given time step.Upon activation, the agent contacts m other distinct agents chosen at random. Activities are extracted from the distribution $\frac{1-\gamma}{1-e^{1-\gamma}} \cdot a^{-\gamma}$. On top of this, we assume that social interactions are governed by homophily, a well observed phenomenon. Thus, p_{ij}, the probability that an active agent i will contact a peer j is modelled as a decreasing function of the distance between their opinions. $p_{ij} = \frac{d(\mathbf{x}_i,\mathbf{x}_j)^{-\beta}}{\sum_j d(\mathbf{x}_i,\mathbf{x}_j)^{-\beta}}$.

Consensus and Polarization

We observe that for a given set of model parameters, lower value of α gives us a consensus in opinion, while a larger value of α gives us a diverging/ polarizing opinions.







We will try to calculate the maximum value of α for which the opinions converge using mean field approximation. When N is very large, and there is a strong homophily in the network (β is large), an agent's opinions are closer to the opinions of it's interacting partners. Thus, we have $x_i^{(u)} \approx x_j^{(v)} = x^{(u)}$. With this approximation, the dynamics of a single agent is then described solely by interaction with neighbours holding the same opinion. For faster interaction, the mean number of interaction received by an agent at each time step can be approximated by $2m\langle a\rangle$, which is a sum of two terms. First, the average number of links an agent generates upon activation is $m\langle a\rangle$, and the second term which stems from the links expected to be received by agent i from all other agents. Hence our model boils down to

$$\frac{\mathrm{d}x^{(v)}}{\mathrm{d}t} = -x^{(v)} + 2Km\langle a\rangle \tanh\left(\alpha[\phi x]^{(v)}\right)$$

Let us make an assumption that all pairwise topic overlaps are equal, and angle between any two of them be δ. Then we can write the Jacobian of the above differential equation as

$$J(\mathbf{0}) = \begin{pmatrix} -1+\Lambda\alpha & \Lambda\alpha\cos(\delta) & \cdots & \Lambda\alpha\cos(\alpha) \\ \Lambda\alpha\cos\delta & -1+\Lambda\alpha & \cdots & \Lambda\alpha\cos(\delta) \\ \vdots & \vdots & \vdots & \vdots \\ \Lambda\alpha\cos(\delta) & \Lambda\alpha\cos(\delta) & \cdots & -1+\Lambda\alpha \end{pmatrix}$$

where $\Lambda = 2Km\langle a\rangle$. The maximum eigenvalue of the above Jacobian is given by

$$\lambda_{max} = (T-1)(-1+\Lambda\alpha) + \Lambda\alpha\cos\delta$$

. If $\lambda_{max} < 0$, then we will have stable consensus. Thus for critical value of the controversialness putting $\lambda_{max} = 0$, yields $\alpha_c = \frac{T-1}{2Km\langle a\rangle[T-1+\cos(\delta)]}$.