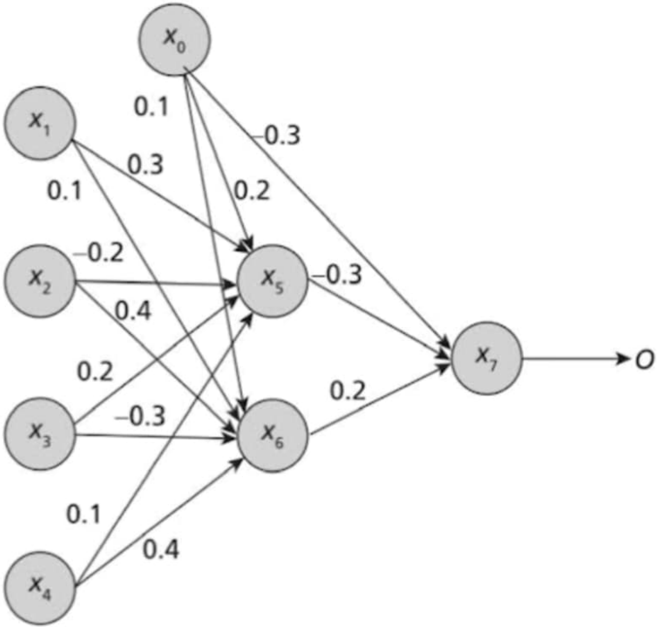
Multi-Layer Perceptron Learning – Solved Example

- The given MLP consists of an Input layer, one Hidden layer and an Output layer.
- The input layer has 4 neurons, the hidden layer has 2 neurons and the output layer has a single neuron.
- Train the MLP by updating the weights and biases in the network.
- Learning Rate = 0.8

x_1	x_2	x_3	x_4	$O_{Desired}$
1	1	0	1	1



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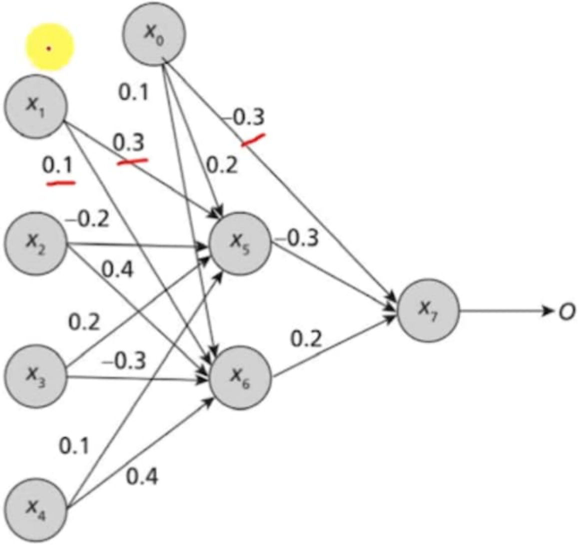
x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

1. Calculate Input and Output in the Input Layer

Net Input and Output Calculation

Input layer	I_j	O_j
x_1	1	1
x_2	1	1
x_3	0	0
x_4	1	1



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1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

2. Calculate Net Input and Output in the Hidden Layer and Output Layer as

Unit$_j$	Net Input I_j	Output O_j
x_5	$I_5 = x_1 \times w_{15} + x_2 \times w_{25} + x_3 \times w_{35} + x_4 \times w_{45} + x_0 \times \theta_5$ $I_5 = 1 \times 0.3 + 1 \times -0.2 + 0 \times 0.2 + 1 \times 0.1 + 1 \times 0.2 = 0.4$	$O_5 = \frac{1}{1+e^{-I_5}} \quad \frac{1}{1+e^{-0.4}} = 0.599$
x_6	$I_6 = x_1 \times w_{15} + x_2 \times w_{26} + x_3 \times w_{36} + x_4 \times w_{46} + x_0 \times \theta_6$ $I_6 = 1 \times 0.3 + 1 \times 0.4 + 0 \times -0.3 + 1 \times 0.4 + 1 \times 0.1 = 1.2$	$O_6 = \frac{1}{1+e^{-I_6}} \quad \frac{1}{1+e^{-1.2}} = 0.769$
x_7	$I_7 = O_5 \times w_{57} + O_6 \times w_{67} + x_0 \times \theta_7$ $I_7 = 0.599 \times -0.3 + 0.769 \times 0.2 + 1 \times -0.3 = -0.326$	$O_7 = \frac{1}{1+e^{-I_7}} \quad \frac{1}{1+e^{0.326}} = 0.419$

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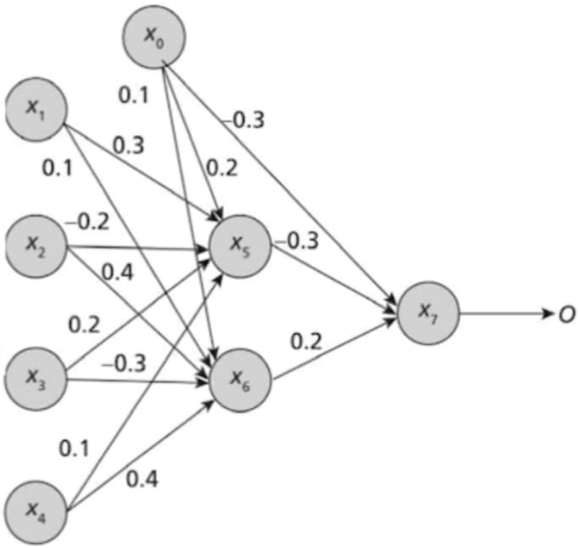
x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

3. Calculate Error $= O_{desired} - O_{Estimated}$

So, error for this network is:

Error $= O_{desired} - O_7 = 1 - 0.419 = 0.581$

So, we need to back propagate to reduce the error.



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x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

Step 2: Backward Propagation

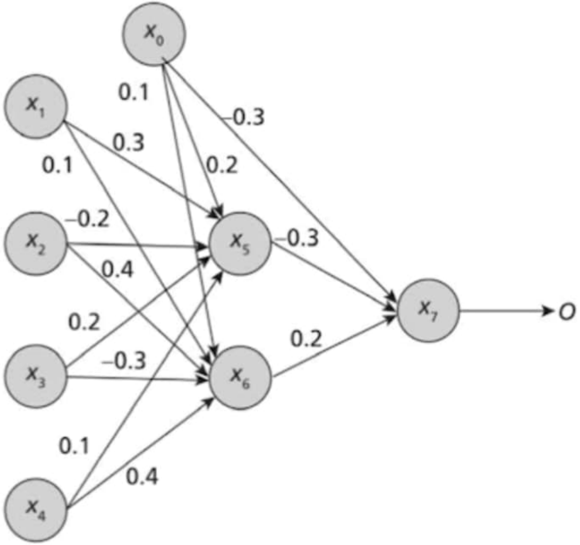
1. Calculate Error at each node

   For each unit k in the output layer, calculate:

   $\text{Error}_k = O_k(1 - O_k)(O_{desired} - O_k)$

   For each unit j in the hidden layer, calculate:

   $\text{Error}_j = O_j(1 - O_j)\sum_k \text{Error}_k\, w_{jk}$



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x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

Error Calculation for Each Unit in the Output Layer and Hidden Layer

For Output Layer Unit$_k$	Error$_k$
x_7	$\text{Error}_7 = O_7(1 - O_7)(Y_n - O_7)$ $= 0.419 \times (1 - 0.419) \times (1 - 0.419) = 0.141$
For Hidden Layer Unit$_j$	**Error$_j$**
x_6	$\text{Error}_6 = O_6(1 - O_6)\sum_k \text{Error}_k\, w_{jk} = O_6(1 - O_6)\,\text{Error}_7\, w_{67}$ $= 0.769\,(1 - 0.769) \times 0.2 \times 0.141 = 0.005$
x_5	$\text{Error}_5 = O_5(1 - O_5)\sum_k \text{Error}_k\, w_{jk} = O_5(1 - O_5)\,\text{Error}_7\, w_{57}$ $= 0.599(1 - 0.599) \times 0.141 \times -0.3 = -0.0101$

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x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

2. Update weight using the below formula:

Learning rate $\alpha = 0.8$.

$w_{ij} = w_{ij} + \propto \times \text{Error}_j \times O_i$

The updated weights and bias

w_{ij}	$w_{ij} = w_{ij} + \propto \times \text{Error}_j \times O_i$	New Weight
w_{15}	$w_{15} = w_{15} + 0.8 \times \text{Error}_5 \times O_1$ $= 0.3 + 0.8 \times -0.0101 \times 1$	0.292
w_{16}	$w_{16} = w_{16} + 0.8 \times \text{Error}_6 \times O_1$ $= 0.1 + 0.8 \times 0.005 \times 1$	0.104
w_{25}	$w_{25} = w_{25} + 0.8 \times \text{Error}_5 \times O_2$ $= -0.2 + 0.8 \times -0.0101 \times 1$	–0.208
w_{26}	$w_{26} = w_{26} + 0.8 \times \text{Error}_6 \times O_2$ $= 0.4 + 0.8 \times 0.005 \times 1$	0.404

Multi-Layer Perceptron Learning – Solved Example

2. Update weight using the below formula:

   Learning rate $\alpha = 0.8$.

   $w_{ij} = w_{ij} + \propto \times \text{Error}_j \times O_i$

   The updated weights and bias

w_{ij}	$w_{ij} = w_{ij} + \propto \times \text{Error}_j \times O_i$	New Weight
w_{35}	$w_{35} = w_{35} + 0.8 \times \text{Error}_5 \times O_3$ $= 0.2 + 0.8 \times -0.0101 \times 0$	0.2
w_{36}	$w_{36} = w_{36} + 0.8 \times \text{Error}_6 \times O_3$ $= -0.3 + 0.8 \times 0.005 \times 0$	–0.3
w_{45}	$w_{45} = w_{45} + 0.8 \times \text{Error}_5 \times O_4$ $= 0.1 + 0.8 \times -0.0101 \times 1$	0.092
w_{46}	$w_{46} = w_{46} + 0.8 \times \text{Error}_6 \times O_4$ $= 0.4 + 0.8 \times 0.005 \times 1$	0.404
w_{57}	$w_{57} = w_{57} + 0.8 \times \text{Error}_7 \times O_5$ $= -0.3 + 0.8 \times 0.141 \times 0.599$	–0.232
w_{67}	$w_{67} = w_{67} + 0.8 \times \text{Error}_7 \times O_6$ $= 0.2 + 0.8 \times 0.141 \times 0.769$	0.287

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x_1	x_2	x_3	x_4	w_{15}	w_{16}	w_{25}	w_{26}	w_{35}	w_{36}	w_{45}	w_{46}	w_{57}	w_{67}	θ_5	θ_6	θ_7
1	1	0	1	0.3	0.1	–0.2	0.4	0.2	–0.3	0.1	0.4	–0.3	0.2	0.2	0.1	–0.3

Update bias using the below formula:

$$\theta_j = \theta_j + \propto \times \text{Error}_j$$

θ_j	$\theta_j = \theta_j + \propto \times \text{Error}_j$	New Bias
θ_5	$\theta_5 = \theta_5 + \propto \times \text{Error}_5$ $= 0.2 + 0.8 \times -0.0101$	0.192
θ_6	$\theta_6 = \theta_6 + \propto \times \text{Error}_6$ $= 0.1 + 0.8 \times 0.005$	0.104
θ_7	$\theta_7 = \theta_7 + \propto \times \text{Error}_7$ $= -0.3 + 0.8 \times 0.141$	–0.187

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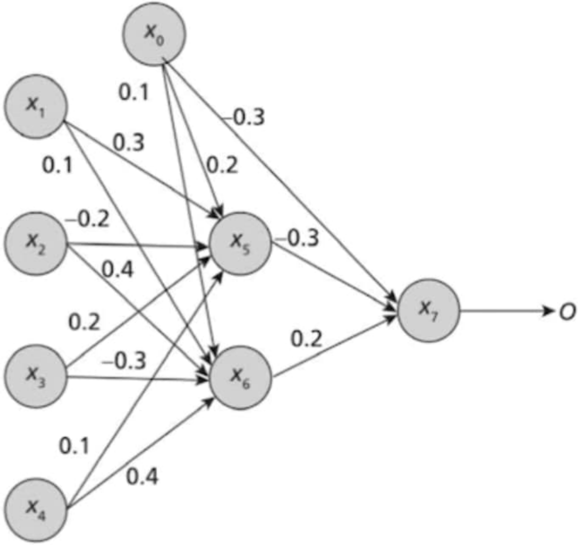
Iteration 2

Now, with the updated weights and biases:

1. Calculate Input and Output in the Input Layer

Net Input and Output Calculation

Input Layer	I_j	O_j
x_1	1	1
x_2	1	1
x_3	0	0
x_4	1	1



Multi-Layer Perceptron Learning – Solved Example

2. Calculate Net Input and Output in the Hidden Layer and Output Layer

Net Input and Output Calculation in the Hidden Layer and Output Layer

Unit j	Net Input I_j	Output O_j
x_5	$I_5 = x_1 \times w_{15} + x_2 \times w_{25} + x_3 \times w_{35} + x_4 \times w_{45} + x_0 \times \theta_5$ $I_5 = 1 \times 0.292 + 1 \times -0.208 + 0 \times 0.2 + 1 \times 0.092 + 1 \times 0.192 = 0.368$	$O_5 = \frac{1}{1+e^{-I_5}} = \frac{1}{1+e^{-0.368}} = 0.591$
x_6	$I_6 = x_1 \times w_{15} + x_2 \times w_{26} + x_3 \times w_{36} + x_4 \times w_{46} + x_0 \times \theta_6$ $I_6 = 1 \times 0.292 + 1 \times 0.404 + 0 \times -0.3 + 1 \times 0.404 + 1 \times 0.104 = 1.204$	$O_6 = \frac{1}{1+e^{-I_6}} = \frac{1}{1+e^{-1.204}} = 0.7692$
x_7	$I_7 = O_5 \times w_{57} + O_6 \times w_{67} + x_0 \times \theta_7$ $I_7 = 0.591 \times -0.232 + 0.7692 \times 0.287 + 1 \times -0.187 = -0.326$	$O_7 = \frac{1}{1+e^{-I_7}} = \frac{1}{1+e^{0.1034}} = 0.474$

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- The output we receive in the network at node 7 is 0.474

Error= 1 - 0.474 = 0.526

- Now, when we compare the error, we get in the previous iteration and in the current iteration,

Error reduced is 0.581 - 0.526 = 0.055

- It is visible that the network has learnt and reduced the error by 0.055.
- Thus, the training is continued for a predefined number of epochs or until the training error is reduced below a threshold value.