unit-4:

1) simplex Method:

Maximize $Z = 5x_1 + 7x_2$.

Subject to constraints $x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 24$

$10x_1 + 7x_2 \leq 35$,

$x_1, x_2 \geq 0$.

solution:

Let slack variables be s_1, s_2, s_3.

$x_1 + x_2 + s_1 = 4$.

$3x_1 + 8x_2 + s_2 = 24$

$10x_1 + 7x_2 + s_3 = 35$

The standard Linear Programming Problem is

Maximize $Z = 5x_1 + 7x_2 + 0s_1 + 0s_2 + 0s_3$.

step 1: Initial simplex Table

C_{Bj}	C_j	5	7	0	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3		
0	S_1	1	1	1	0	0	4	$\frac{4}{1} = 4$
0	S_2	3	$\boxed{8}$	0	1	0	24	$\frac{24}{8} = 3 \Leftarrow$
0	S_3	10	7	0	0	1	35.	$\frac{35}{7} = 5$
Z_j		0	0	0	0	0		
$Z_j - C_j$		-5	-7	0	0	0		

↑ most negative value

Key value = 8.
Entering variable = x_2
Leaving " = S_2

step 2: First Iteration Table:

C_{Bi}	C_j	5	7	0	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3		
0	S_1	$\boxed{5/8}$	0	1	$-1/8$	0	1	$\frac{1}{5/8} = \frac{8}{5}$ 1.6
7	x_2	$3/8$	1	0	$1/8$	0	$\frac{24}{8} = 3$	$\frac{3}{3/8} = 3 \times \frac{8}{3} = 8.$
0	S_3	$59/8$	0	0	$-7/8$	1	14	$\frac{14}{59/8} = \frac{14 \times 8}{59} = \frac{112}{59}$ 1.89
Z_j		$0 + \frac{21}{8} + 0$	7	0	$7/8$	0		
$Z_j - C_j$		$\frac{21}{8} - 5 = \frac{-19}{8}$	0	0	$7/8$	0		

pivot (or) key $= 5/8$

Entering variable $= x_1$

Leaving variable $= s'_1$

Calculation :

old value	$s_1 =$	1	1	1	0	0	4
New value	$x_2 =$	$3/8$	1	0	$1/8$	0	3
	(-)						
		$1-3/8 = \frac{5}{8}$	0	1	$-1/8$	0	1

old value	$s_3 =$	10	7	0	0	1	35
New value	$x_2 \times 7 =$	$21/8$	7	0	$7/8$	0	21
	(-)						
		$10-\frac{21}{8} = \frac{80-21}{8} = 59/8$	0	0	$-7/8$	1	14

step 3: second Iteration Table.

CBi	Cj	5	7	0	0	0.	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3		
5	x_1	1	0	8/5	-1/5	0	8/5	
7	x_2	0	1	-3/5	1/5	0	12/5	
0	S_3	0	0	$\frac{-59}{5}$	6/10	1	11/5	
z_j		5	7	$\frac{40}{5}-\frac{21}{5} = 19/5$	$-1/5+7/5 = 6/5$	0	$\boxed{\frac{40}{8}+\frac{84}{5}=\frac{124}{5}}$	
z_j-c_j		0	0	19/5	6/5	0.		

key =

Entering value =

Leaving variable =

$$\boxed{\begin{array}{l} Ans = Z = \frac{124}{5} \\ x_1 = 8/5 \\ x_2 = 12/5. \end{array}}$$

Calculations:

old x_2 =	3/8	1	0	1/8	0	$24/8 = 3$
New $\frac{3}{8} \times x_1$ =	3/8	0	$\frac{3}{8} \times 8/5$	$\frac{3}{8} \times -\frac{1}{5}$	0	$\frac{3}{8} \times 8/5$
	0	1	-3/5	1/5	0	$\frac{12}{5}$

old $S_3 = \frac{59}{8} \quad 0 \quad 0 \quad -7/8 \quad 1 \quad 14.$

New $\frac{59}{8} \times x_1 = \frac{59}{8} \quad 0 \quad 8/5 \times \frac{59}{8} \quad -1/5 \times \frac{59}{8} \quad 0 \quad 1 \quad 8/5 \times \frac{59}{8}$

$0 \quad 0 \quad -59/5 \quad \cancel{-7/8} \quad 1 \quad 11/5$

$6/10$

$$\boxed{\begin{aligned} z &= 5x_1 + 7x_2. \\ \frac{124}{5} &= \cancel{5}(8/\cancel{5}) + 7(12/5) \\ &= \frac{40}{5} + \frac{84}{5} \\ \frac{124}{5} &= \frac{124}{5}. \end{aligned}}$$

Check.

2) The stable Marriage. Problem :

> It is an important Algorithmic version of bipartite matching problem.

> It is a classic problem.

> Goal → Stable matching between two sets with various preference to each other.

set $y = \{m1, m2 \ldots mn\}$

set $x = \{w1, w2 \ldots wn\}$.

Mens preferences

	1st	2nd	3rd.
Bob	Lea	Ann	Sue
Jim	Lea	sue	Ann
Tom	sue	Lea	Ann

Womens preferences.

	1st	2nd	3rd.
Ann	Jim	Tom	Bob
Lea	Tom	Bob	Jim
sue	Jim	Tom	bob.

Ranking Matrix.

	Ann	Lea	Sue
Bob	2,3	1,2	3,3
Jim	3,1	1,3	2,1
Tom	3,2	2,1	1,2

Free Men
Bob
Jim
Tom.

	Ann	Lea	Sue.
Bob	2,3	$\boxed{1,2}$	3,3
Jim	3,1	1,3	2,1
Tom	3,2	2,1	1,2

Bob proposed to Lea, Lea Accepted.

Free Men.
Jim
Tom.

	Ann	Lea	Sue.
Bob	2,3	$\boxed{1,2}$	3,3
Jim	3,1	$\boxed{1,3}$	$\boxed{2,1}$
Tom	3,2	2,1	1,2

Jim proposed to Lea, but Lea rejected.

Jim proposed to Sue, Sue accepted

Free Men Tom.

	Ann	Lea	Sue
Bob	2,3	$\boxed{1,2}$	3,3
Jim	3,1	1,3	$\boxed{2,1}$
Tom	3,2	$\boxed{2,1}$	$\boxed{1,2}$

Tom proposed to Sue, Sue rejected.

Tom proposed to Lea. Lea Break up with Bob and accepted Tom

Free Men Bob.

	Ann	Lea	Sue
Bob	$\boxed{2,3}$	1,2	3,3
Jim	3,1	1,3	$\boxed{2,1}$
Tom	3,2	$\boxed{2,1}$	1,2

Bob proposed to Ann, Ann accepted

(Bob, Ann), (Jim, Sue), (Tom, Lea).

Algorithm :

step 1 : Initially all the men and all the women are free, but having their own preference list with them.

step 2 : Propose: one of the free man m proposes the woman w. this woman is normally the highest preferred one from his preference list.

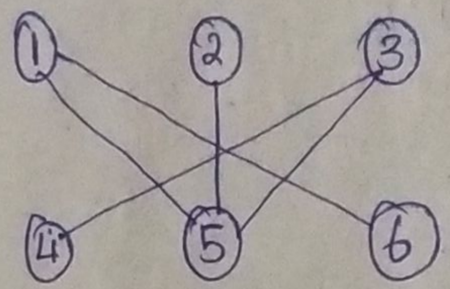
Responce : If the woman w is free then she accepts the proposal of matched m. If she is not free, she compares the m with her current mate. If she prefers m than the current mate then she accepts his proposal making former mate free otherwise simply rejects m's proposal.

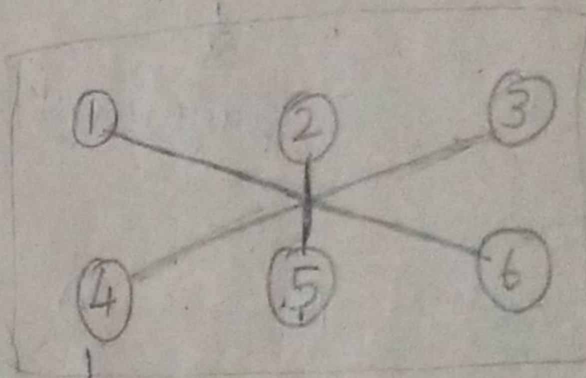
step 3 : Finally, returns the matching pairs of (m, w).

Time Efficiency class : $\Theta(n^2)$.

3) Maximum Matching in Bipartite graphs:

1)



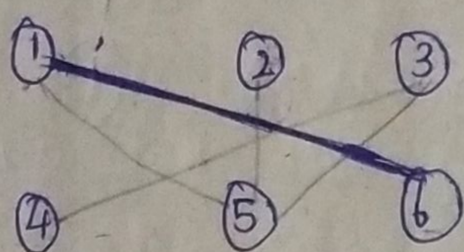


soln:

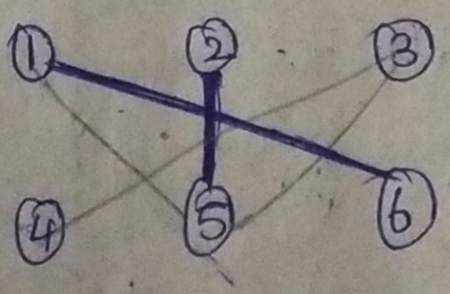
Queue : 1, 2

↓

augmented with 6



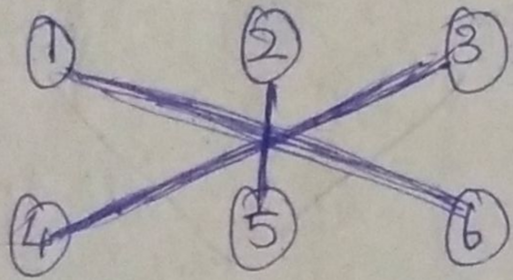
Queue : 2, 3. → Augmented with 5



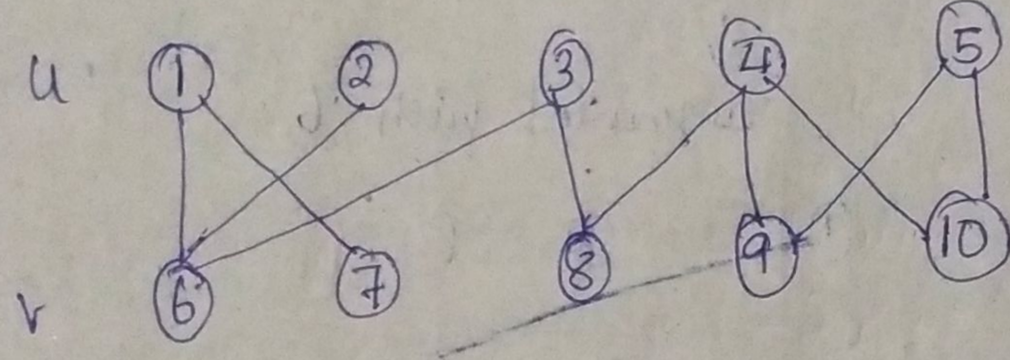
Queue : 3, 4

↓

augument with 4.



2)

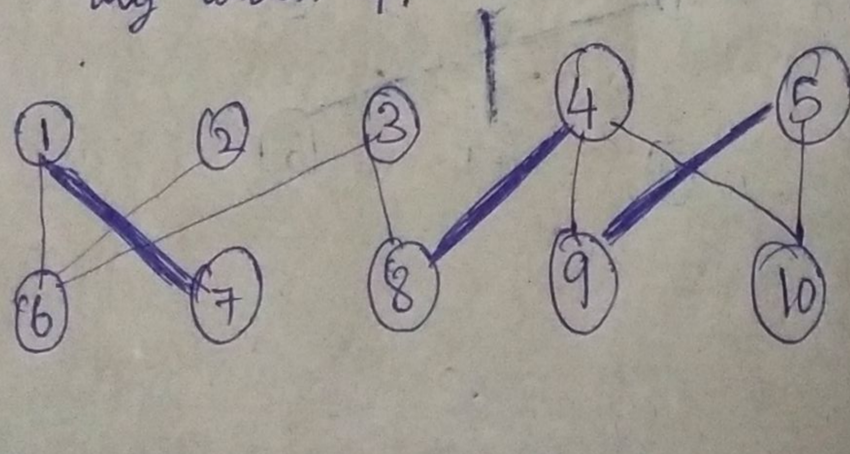


soln:

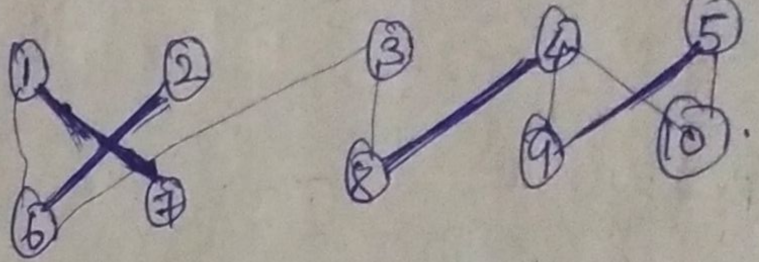
Queue : 1, 2.

↓

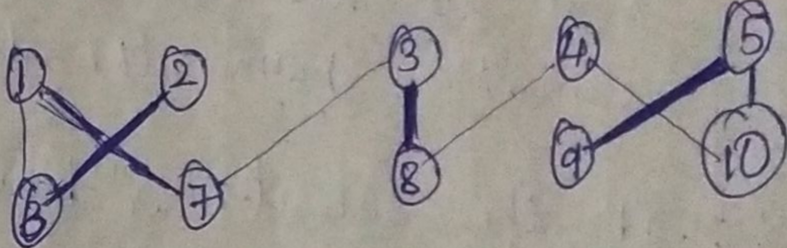
aug with 7.



Queue : 2, 3
↓
aug with 6.



Queue : 3, 4.
↓
aug 8.



Queue : 4, 5. ⇒ aug with 9.
↓
aug with 10