Binomial, poission, geometric → discrete.

uniform, Exponential, Normal → continuous

Exponential distribution:

A continuous R.V X is said to follows exponential distribution if its PDF is given by.

$(0, \infty)$.

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

(ie) $0 \leq x \leq \infty$

Derive MGF, Mean & variance for the exponential Distribution.

soln:

Let X be a exponential R.V.

then its pdf is $f(x) = \lambda e^{-\lambda x}$,

$x \geq 0$ $[0, \infty)$

$$MGF = M_x(t) = E(e^{tx})$$ $e^{-\infty} = 0$

$e^{\infty} = \infty$

$$= \int_0^\infty e^{tx} f(x)\, dx$$

$\int e^{ax} dx = \frac{e^{ax}}{a}$

$$= \int_0^\infty e^{tx} \lambda e^{-\lambda x}\, dx.$$

$$= \lambda \int_0^\infty e^{tx} e^{-\lambda x}\, dx.$$

$$= \lambda \int_0^\infty e^{(t-\lambda)x}\, dx.$$

$$= \lambda \int_0^\infty e^{-(\lambda-t)x}\, dx.$$

$$= \lambda \left[\frac{e^{-(\lambda-t)x}}{-(\lambda-t)} \right]_0^\infty$$

$$= \frac{\lambda}{-(\lambda-t)} \left[e^{-\infty} - e^{0} \right]$$

$$MGF = \frac{\lambda}{\lambda - t}$$

$$\text{Mean} = E(x) = \left[\frac{d}{dt}\left(\frac{\lambda}{\lambda-t}\right)\right]_{t=0}$$

$$= \lambda\left[\frac{d}{dt}(\lambda-t)^{-1}\right]_{t=0}$$

$$= \lambda\left[-1(\lambda-t)^{-2}(0-1)\right]_{t=0}$$

$$= \lambda^{1}[\lambda^{-2}]$$

$$= \lambda^{-1}$$

$$\therefore E(x) = 1/\lambda$$

Variance :

$$E(x^2) = \left[\frac{d^2}{dt^2}(M_x(t))\right]_{t=0}$$

$$= \lambda\left[\frac{d}{dt}\left\{(\lambda-t)^{-2}\right\}\right]_{t=0}$$

$$= \lambda\left[(-2)\,(\lambda-t)^{-3}\,(-1)\right]_{t=0}$$

$$= \lambda\left[2\,(\lambda^{-3})\right]$$

$$= \lambda\left[2\lambda^{-3}\right]$$

$$= 2.\lambda^{-2}$$

$$= 2/\lambda^2$$

$$\text{var}(x) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$= 1/\lambda^2.$$

Q) Let X be an exponential R.V with $E(X^2) = \frac{1}{2}$. obtain. [parameter $-\lambda$]

Memory less
↓
exp.

i) $E(x)$ & $var(x)$.

ii) MGF, $M_X(t)$.

iii) $P(x > 3 \mid x > 1)$.

soln:

Let X be an exp. R.V.

then $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$

i) $MGF = \frac{\lambda}{\lambda - t}$

ii) $E(x) = 1/\lambda$

iii) $Var(x) = 1/\lambda^2$

given,

$$E(x^2) = \frac{1}{2}$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$\frac{1}{\lambda^2} = \frac{1}{2} - \left(\frac{1}{\lambda}\right)^2$$

$$\frac{2}{\lambda^2} = \frac{1}{2}$$

$$\frac{1}{\lambda^2} = \frac{1}{4}$$

$$\lambda^2 = 4$$

$$\lambda = \pm 2$$

$\lambda = -2$ (Not possible)

$$\boxed{\lambda = +2}$$

i) $E(x) = \frac{1}{\lambda} = 1/2.$

ii) $Var(x) = \frac{1}{\lambda^2} = \frac{1}{4}.$

ii) $MGF = \frac{\lambda}{\lambda - t} = \frac{2}{2-t}$

iii) $p(x > 3 \mid x > 1) =$

$p(x > 2 + r \mid x > r).$

Memory less prop

$= p(x > 2).$

$p(x > m+n \mid x > m)$

$= p(x > n).$

$= 1 - p(x \le 2).$

$= 1 - \left[\int_0^2 f(x)\,dx \right].$

$= 1 - \left[2 \int_0^2 e^{-2x}\,dx \right]$

$= 1 - 2 \left[\frac{e^{-2x}}{-2} \right]_0^2$

$$= 1 + \left[e^{-4} - e^{0}\right]$$

$$= \cancel{1} + e^{-4} \cancel{-1}$$

$$= 1/e^{4}.$$

$f(x) = \lambda e^{-\lambda x}, x \geq 0,$
$x > 0$

Q) The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

i) what is the probability that the repair time exceeds 2hr. $P(x > 2) = ?$

ii) what is the conditional probability that a repair takes atleast 10 hr given that its duration min exceeds 9 hrs? $P(x \geq 10) / (x > 9)$)

Soln:

$$f(x) = \lambda e^{\lambda x}, \quad x \geq 0.$$

(i) $f(x) = \frac{1}{2} e^{-\frac{1}{2}x}, \quad x \geq 0$

i) p[the repair time exceeds 2 hrs] = $p(x > 2)$

$$= \int_2^0 f(x)\, dx.$$

$$= \int_2^\infty \frac{1}{2} e^{-1/2 x}\, dx.$$

$$= \frac{1}{2} \int_2^\infty e^{-1/2 x}\, dx.$$

$$= \cancel{\frac{1}{2}} \left[\frac{e^{-1/2 x}}{-1/\cancel{2}} \right]_2^\infty$$

$$= \frac{1}{2} \left[\frac{e^{-(\infty)}}{\cancel{-1/2}} - e^{\cancel{-2/2}} \right]$$

$$= -\left[0 - e^{-1} \right]$$

$$= e^{-1}$$

$$= 0.3679$$

ii) $p(x \geq 10 \mid x > 9) = p(x \geq 9+1 \mid x > 9)$

$p(x > \cancel{s}+t \mid x > \cancel{s})$

$= p(x > 1)$

$= \int_1^\infty f(x)\, dx$

$= \int_1^\infty \frac{1}{2} e^{-\frac{1}{2}x}\, dx$

$= \frac{1}{2} \int_1^\infty e^{-\frac{1}{2}x}\, dx$

$= \frac{1}{\cancel{2}} \left[\frac{e^{-\frac{1}{2}x}}{-\cancel{1/2}} \right]_1^\infty$

$= -\left[e^{(-\infty)} - e^{-(1)\frac{1}{2}} \right]$

$= e^{-1/2}$

$= 0.607$

2) The length of time a person speaks over phone follows exponential distribution with mean 6 min. What is the probability that the person will ~~talk~~ Talk for (i) More than 8 minutes (2) between 4 & 8 minutes?

soln :

$$f(x) = \lambda e^{-\lambda x}, \quad \lambda > 0; \quad x \geq 0$$

$$\text{Mean} = \frac{1}{\lambda} \qquad \text{mean} = 6.$$

$$\boxed{\lambda = \frac{1}{6}.}$$

i) $P(x > 8)$. ii) $P(4 < x < 8)$.

$$\int_8^\infty f(x)\, dx$$

$$= \int_{8}^{\infty} \frac{1}{6} e^{-\frac{1}{6}x} \, dx.$$

$$= \frac{1}{6} \int_{8}^{\infty} e^{-\frac{1}{6}x} \, dx.$$

$$= \cancel{\frac{1}{6}} \left[\frac{e^{-\frac{1}{6}x}}{-\cancel{1}/6} \right]_{8}^{\infty}$$

$$= \cancel{\frac{1}{6}} \left[e^{-(\infty)} - e^{-\frac{1}{6}(\cancel{8})^{4/3}} \right]$$

$$= -\left[0 - e^{-4/3} \right]$$

$$= e^{-4/3} = 0.264.$$

ii) $\Rightarrow$

$$= \int_4^8 f(x)\, dx$$

$$= \int_4^8 \frac{1}{6} e^{-1/6x}\, dx$$

$$= \frac{1}{6} \int_4^8 e^{-1/6x}\, dx$$

$$= \frac{1}{6} \left[\frac{e^{-1/6x}}{-1/6} \right]_4^8$$

$$= -\left[e^{-8/6} - e^{-4/6} \right]$$

$$= \left[-e^{-4/3} + e^{-2/3} \right]$$

$$= 0.25$$

a) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probability that one of these tires will last

i) atleast 20,000 km . ii) atmost 30,000 km

maximum.

soln:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

$$\text{Mean} = \frac{1}{\lambda}.$$

$$40000 = \frac{1}{\lambda}$$

$$\lambda = \frac{1}{40000}$$

i) $P(x \leq 20{,}000)$ ii) $P(x \leq 30{,}000)$.

$$= \int_{20{,}000}^{\infty} f(x)\, dx.$$

$$= \int_{20{,}000}^{\infty} \frac{1}{40000} e^{-1/40000\, x}\, dx.$$

$$= \frac{1}{40000} \int_{20000}^{\infty} e^{-1/40000\, x} \, dx$$

$$= \frac{1}{40000} \left[\frac{e^{-1/40000\, x}}{-1/40000} \right]_{20000}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-1/\cancel{40000}\; 20000} \right]$$

$$= e^{-1/2\cancel{0000}}$$

$$= 0.6065.$$

ii) $P(x \leq 30000)$

$$\int_{0}^{30000} f(x)\, dx.$$

$$= \int_{0}^{30000} \frac{1}{40000} e^{-1/40000\, x} \, dx.$$

$$= \frac{1}{40000} \int_{0}^{30000} e^{-1/40000\, x} \, dx.$$

$$= \frac{1}{40000}\left[\frac{e^{-1/40000\, x}}{-1/40000}\right]_0^{30000}$$

$$= -\left[e^{-1/40000 \cdot 30000} - e^0\right]$$

$$= -\left[e^{-3/4} - e^0\right]$$

$$= \left[-e^{-3/4} + e^0\right]$$

$$= -e^{-3/4} + 1$$

$$= 0.5276$$

Uniform Distribution :

The pdf of uniform distribution is $f(x) = \frac{1}{b-a}, a < x < b$.

Find Mean, Variance & MGF of uniform distribution.

soln :

$$\text{Mean } (x) = E(x) = \int_a^b x . f(x) \, dx.$$

$$= \int_a^b x . \left(\frac{1}{b-a}\right) . \, dx$$

$$= \frac{1}{b-a} \int_a^b x^1 . dx.$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b dx.$$

$$= \frac{1}{b-a} \left[\frac{b^2}{2} - \frac{a^2}{2} \right].$$

$$= \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right].$$

$$= \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

Mean(x) $\frac{b+a}{2}$ (or) $\frac{a+b}{2}$

$$\text{var}(x) = E(x^2) - [E(x)]^2 \quad \text{---} \; ①$$

$$E(x^2) = \int_a^b x^2 \cdot f(x)\, dx$$

$$= \int_a^b x^2 \cdot \frac{1}{b-a}\, dx.$$

$$= \frac{1}{b-a}\int_a^b x^2\, dx.$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{a^3}{3}\right]$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right].$$

$\because (a-b)(a^2+ab+b^2)$

$$= \frac{1}{\cancel{b-a}}\left[\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3}\right]$$

$$E(x^2) = \frac{b^2+ab+a^2}{3}.$$

$$① \Rightarrow \frac{b^2+ab+a^2}{3} - \left[\frac{a+b}{2}\right]^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{a^2+2ab+b^2}{4}.$$

$$= \frac{4(b^2+ab+a^2) - 3(a^2+2ab+b^2}{12}$$

$$= \frac{4b^2 + 4ab + 4a^2 - (3a^2 + 6ab + 3b^2)}{12}$$

$$= \frac{b^2 + a^2 - 2ab}{12}$$

$$\mathrm{Var}(x) = \frac{(a-b)^2}{12}$$

$$\text{M.G.F} = E(e^{tx}) = \int e^{tx} \cdot f(x)\, dx$$

$$= \int_a^b e^{tx} \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b e^{tx}\, dx$$

$$= \frac{1}{b-a} \left[\frac{e^{tx}}{t} \right]_a^b$$

$$= \frac{1}{b-a} \left[\frac{e^{tb}}{t} - \frac{e^{ta}}{t} \right]$$

$$M_x(t) = \frac{1}{b-a}\left[\frac{e^{bt}-e^{at}}{t}\right]$$

Interval given $(0,30)$ $\quad n, p$ × not given

Q) Electric trains on a certain line run every half an hour b/w midnight and sixe in the morning. What is the probability that a man entering the station at a random time during this period will have to wait atleast 20 minutes? $(P(X \geq 20))$

minimum

soln:

$$f(x) = \frac{1}{b-a} \qquad a < x < b$$

Let the random variable x denote the waiting time in minutes for the next train.

here x is uniformly distributed in $(0, 30)$.

$$\therefore f(x) = \frac{1}{30-0} = \frac{1}{30}$$

$$p(x \geq 20) = \int_a^b f(x)\,dx$$

$$= \int_{20}^{30} \cancel{f(x)\,dx} \; \cancel{\tfrac{1}{b-a}} \; \tfrac{1}{30}\,dx$$

$$= \cancel{\frac{1}{b-a}\left[\right.}$$

$$= \frac{1}{30}\Big[x\Big]_{20}^{30}$$

$$= \frac{1}{30}\left[30-20\right]$$

$$= \frac{10}{30} = \frac{1}{3}$$

Q) A random variable x has a uniform distribution over $(-3, 3)$. compute

i) $p(x < 2)$, ii) $p(|x| < 2)$, iii) $p(|x-2| < 2)$,

iv) Find k for which $p(x > k) = \frac{1}{3}$

soln:

$$f(x) = \frac{1}{b-a}, \quad a < x < b.$$

soln:

$$f(x) = \frac{1}{3-(-3)}, \quad (-3, 3)$$

$$f(x) = \frac{1}{6} \quad -3 < x < 3.$$

i) $$p(x < 2) = \int_{-3}^{2} f(x)\, dx.$$

$$= \int_{-3}^{2} \frac{1}{6}\, dx.$$

$$= \frac{1}{6} \left[x \right]_{-3}^{2}$$

$= \frac{1}{6}\left[2-(-3)\right]$

$= \frac{5}{6}$

$|x| < a \Rightarrow$
$-a < x < a$

ii) $p(|x| < 2) = p(-2 < x < 2)$

$$= \int_{-2}^{2} f(x)\, dx.$$

$$= \int_{-2}^{2} \frac{1}{6}\, dx.$$

$$= \frac{1}{6}\left[x\right]_{-2}^{2}$$

$$= \frac{1}{6}\left[2-(-2)\right]$$

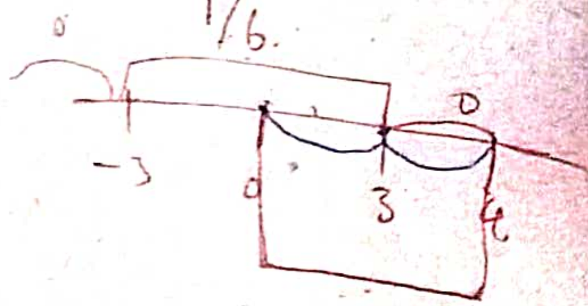
$$= \frac{4}{6} = \frac{2}{3}$$

Q) iii) $P(|x-2| < 2) =$

$+2 \Rightarrow P(-2 < x-2 < 2)$

$P(0 < x < 4).$

$\int_0^4 f(x)\,dx$ $\quad \because f(x) = \begin{cases} \frac{1}{6}, & -3 < x < 3 \\ 0, & \text{o.w.} \end{cases}$



$\frac{1}{6}\int_0^4 dx.$

$\frac{1}{6}[x]_0^4 = \int_0^3 \frac{1}{6}\,dx + \int_3^4 0\,dx.$

$\frac{1}{6}[4]$ $\quad \frac{1}{6}[x]_0^3$

$\frac{4}{6} \;\; \frac{2}{3}$ $\quad \frac{3}{6}$

$= \frac{2}{3}.$ $\quad = \frac{1}{2}$

(iv) $p(x > k) = \frac{1}{3}$

$$\int_{k}^{3} f(x)\,dx = \frac{1}{3}$$

$$\int_{K}^{3} \frac{1}{6}\,dx = \frac{1}{3}$$

$$\frac{1}{6}\left[x\right]_{k}^{3} = \frac{1}{3}$$

$$\frac{1}{6}\left[3-k\right] = \frac{1}{3}$$

$$3-k = \frac{\cancel{6}\,2}{\cancel{3}}$$

$\cancel{-k}$

$$K = 3-2$$

$$\boxed{k = 1}$$

Normal distribution:

We have m.g.f. of a continuous r.v. X is $M_X(t) = E(e^{tx})$

$$= \int_{-\infty}^{\infty} e^{tx}. f(x)\, dx \quad \left[\because f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \right.$$

$-\infty < x < \infty$.

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2\left[\frac{x-\mu}{\sigma}\right]^2} dx \qquad \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{tx} e^{-1/2\left[\frac{x-\mu}{\sigma}\right]^2} dx \quad \text{—} ①$$

put $\frac{x-\mu}{\sigma} = y$ — ② $\quad \frac{x}{\sigma} - \frac{\mu}{\sigma}$ 0

$$\frac{1}{\sigma} dx = dy$$

when $x = -\infty$, then by eqn ② $y = -\infty$

when $x = \infty$, then by eqn ② $y = \infty$

$$x - \mu = \sigma y$$

$$x = \sigma y + \mu$$

$$e^{t\sigma y + t\mu}$$

$$① \Rightarrow M_x(t) = \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{-\infty}^{\infty} e^{t(\sigma y + \mu)}\, e^{-\frac{1}{2} y^2} (\cancel{\sigma}\, dy)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma y} \cdot e^{t\mu}\, e^{-\frac{1}{2} y^2}\, dy$$

$$e^{x+y} = e^x \cdot e^y$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{t\sigma y - \frac{1}{2} y^2}\, dy$$

$$\frac{2t\sigma y - y^2}{2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(y^2 - 2 (t\sigma) y\right)}\, dy$$

b, a, $a^2 - 2$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{tu} \int_{-\infty}^{\infty} e^{-1/2(y^2 - 2t\sigma y + t^2\sigma^2 - t^2\sigma^2)} dy$$

$$= \frac{1}{\sqrt{2\pi}} e^{tu} \int_{-\infty}^{\infty} e^{-1/2[(y - t\sigma)^2 - t^2\sigma^2)} dy$$

$$= \frac{1}{\sqrt{2\pi}} e^{tu} \int_{-\infty}^{\infty} e^{-1/2(y - t\sigma)^2 + \frac{t^2\sigma^2}{2}} dy$$

$$= \frac{1}{\sqrt{2\pi}} e^{tu} \int_{-\infty}^{\infty} e^{-1/2(y - t\sigma)^2} \cdot e^{\frac{t^2\sigma^2}{2}} dy$$

$$= \frac{1}{\sqrt{2\pi}} e^{tu} \cdot e^{\frac{t^2\sigma^2}{2}} \int_{-\infty}^{\infty} e^{-1/2 (y - t\sigma)^2} dy \quad \text{--- (3)}$$

put $y - t\sigma = z$ --- (4)

diff.

$\Rightarrow dy = dz$

when $y = -\infty$, then by ④ $z = -\infty$

" " $= \infty$, " " " $z = \infty$

③ $\Rightarrow M_x(t) = \frac{1}{\sqrt{2\pi}} e^{t\mu + \frac{t^2\sigma^2}{2}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$

$f(x) = f(-x)$
$\Rightarrow$ even function.

put $\frac{z^2}{2} = s$ — ⑥. $\left(\because \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \right.$ if $f(x)$ even$\left.\right)$

$z^2 = 2s$
$z = \sqrt{2}\sqrt{s}$.

$M_x(t) = \frac{1}{\sqrt{2\pi}} e^{t\mu + \frac{t^2\sigma^2}{2}} \cdot 2\int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$ — ⑤.

$\frac{2z\,dz}{2} = ds \Rightarrow dz = \frac{1}{z} ds.$

$= \frac{1}{\sqrt{2}\sqrt{s}} ds$

when $z = 0$, then by (5), $s = 0$
" " ∞ " " " $s = \infty$

$$(5) \Rightarrow M_x(t) = \frac{2}{\sqrt{2\pi}} e^{tu + \frac{t^2\sigma^2}{2}} \int_0^\infty e^{-s} \left(\frac{1}{\sqrt{2}\sqrt{s}} ds \right.$$

$$= \frac{1}{\sqrt{\pi}} e^{tu + \frac{t^2\sigma^2}{2}} \int_0^\infty e^{-s} s^{-1/2} \, ds$$

$$= \frac{1}{\sqrt{\pi}} e^{ut + \frac{t^2\sigma^2}{2}} \int_0^\infty e^{-s} s^{1/2 - 1} \, ds$$

$$\because \Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$$

$$= \frac{1}{\cancel{\sqrt{\pi}}} e^{tu + \frac{t^2\sigma^2}{2}} \left[\cancel{\Gamma \tfrac{1}{2}} \right] \qquad \because \Gamma \tfrac{1}{2} = \sqrt{\pi}$$

$$M_x(t) = e^{tu + \frac{t^2\sigma^2}{2}}.$$

2) The savings bank account of a customer showed an average balance of Rs. 150 and a standard deviation of Rs. 50. Assuming that the account balances are normally distributed. $p(x > 200)$

i) what percentage of account is over Rs. 200?

ii) what percentage of account is b/w Rs. 120 & Rs. 170? $p(170 > x > 120)$

iii) what percentage of account is less than Rs. 75? $p(x < 75)$

soln :

given $\mu = 150$ & $\sigma = 50$.

we have $z = \frac{x-\mu}{\sigma} = \frac{x-150}{50}$ — ①

i) $p(x > 200) = p(z > 1)$

(a)

$= 0.5 - p(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$\therefore$ If $x = 200$ then by ① $z = \frac{200-150}{50} = 1$

$\therefore$ percentage $= 100 \times 0.1587$

$= 15.87\%$

ii) $p(120 < x < 170) = p(-0.6 < z < 0.4)$

$= p(-0.6 < z < 0) + p(0 < z < 0.4)$

If $x = 120$ then by ① $z = \frac{120-150}{50} = -0.6$

$= p(0 < z < 0.6) + p(0 < z < 0.4)$

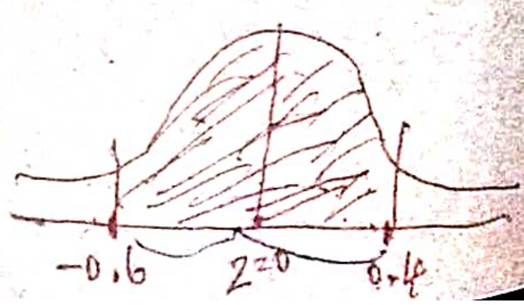
If $x = 170$, then $z = \frac{170-150}{50} = 0.4$

$= 0.2257 + 0.1554$

$= 0.3811$

$\% = 0.3811 \times 100$

$= 38.11\%$



iii) $p(x < 75) = p(z < -1.5)$

$= 0.5 - p(-1.5 < z < 0)$

$= 0.5 - p(0 < z < 1.5)$

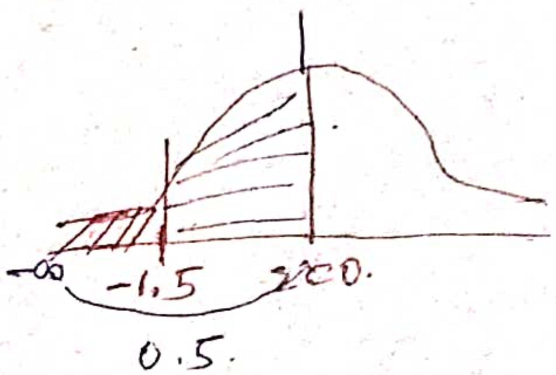
$= 0.5 - 0.4332$

$= 0.0668$

If $x = 75$, then by ①

$z = \frac{75 - 150}{50}$

$= -1.5$



$\therefore \% = 0.0668 \times 100$

$= 6.68\%$

Q.) In a test on 2000 electric bulbs, it was found that the life of a philips bulb was normally distributed with an average life of 2040 hrs and S.D of 60 hrs. Estimate the number of bulbs likely to burn for (i) More than 2150 hrs, ii) less than 1950 hrs & iii) More than 1920 hrs but less than 2160 hrs.

i) $p(x > 2150)$
ii) $p(x < 1950)$
iii) $p(1920 < x < 2160)$

soln:

$\mu = 2040$ hrs

$SD = \sigma = 60$ hrs

we have $z = \dfrac{x-\mu}{\sigma}$ — (1)

∵ If $x = 2150$, then by (1)

$z = \dfrac{2150-2040}{60}$

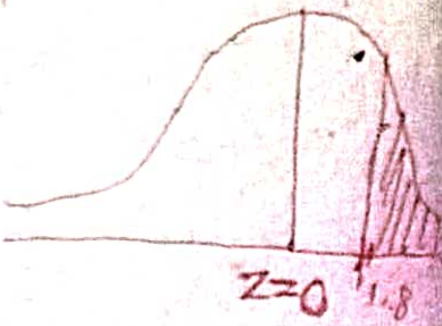
$= 1.833$

i) $p(x > 2150) = p(z > 1.833)$

$= 0.5 - p(0 < z < 1.833)$

$= 0.5 - 0.4664$

$= 0.0336$



∴ No. of bulbs expected to burn for more than

2150 hrs $= 0.0331 \times 2000$

$= 67.2$

≈ 67 bulbs.

ii) $p(x < 1950) = p(z > -1.5)$

$= 0.5 - p(-1.5 < z < 0)$

$= 0.5 - (0 < z < 1.5)$

$= 0.5 - 0.4332$

$= 0.0.668.$

$\therefore$ 1950 hrs $= 0.0668 \times 2000$

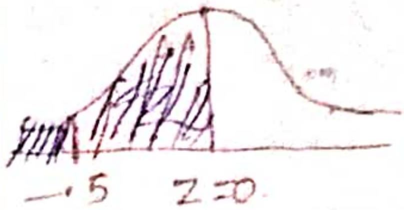
$= 133.6$

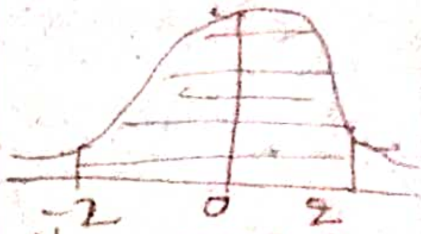
$= 134$ bulbs.

$\because$ If $x = 1950$ then by ①

$z = \frac{1950 - 2040}{60}$

$= -1.5.$





Ill $x = 1920$ then

$z = \frac{1920 - 2040}{60}$

$= -2$

$x = 2160$, then

$z = \frac{2160 - 2040}{60}$

$= 2$

iii) $p(1920 < x < 2160)$

$= p(-2 < z < 2)$

$= p(-2 < z < 0) + p(0 < z < 2)$

$= p(0 < z < 2) + p(0 < z < 2)$

Q)

$= 0.4772 + 0.4772$

$= 0.9544$

$\therefore$ No of bulls $= 0.9544 \times 2000$

$= 1908.8$

$\simeq 1909$ bulls.

Q) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean & variance of the distribution.

soln.

i) $p(x < 45) = 31\%$

ii) $p(x > 64) = 8\%$

given,

μ & $\sigma^2 = ?$

$$p(x < 45) = 31\% = \frac{31}{100} = 0.31$$

$$p(x > 64) = 8\% = \frac{8}{100} = 0.08$$

ie $p(x < 45) = 0.31$ — ① &

$p(x > 64) = 0.08$ — ②

we have, $Z = \dfrac{x-\mu}{\sigma}$

If $x = 45$, $Z = \dfrac{45-\mu}{\sigma} = Z1$

If $x = 64$, $Z = \dfrac{64-\mu}{\sigma} = Z_2$

ie $Z1 = \dfrac{45-\mu}{\sigma}$ — ③ & $Z_2 = \dfrac{64-\mu}{\sigma}$ — ④

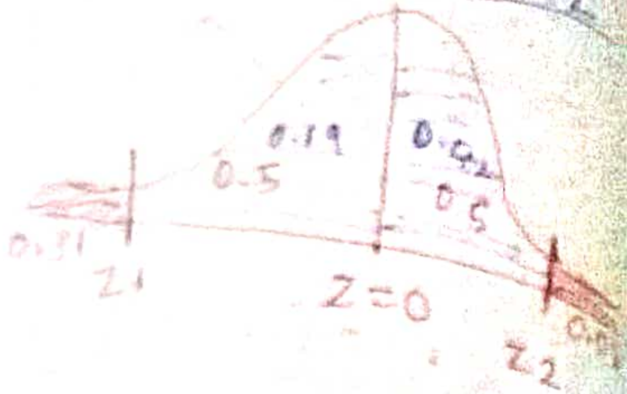
Now ① ⇒ $p(Z < Z1) = 0.31$ &

② ⇒ $p(Z > Z2) = 0.08$

$P(z_1 < z < 0) = 0.19$ &

$P(0 < z < z_2) = 0.42$

$$\begin{array}{r} 0.50 \\ 0.31 \\ \hline 0.19 \end{array} \qquad \begin{array}{r} 0.50 \\ 0.08 \\ \hline 0.42 \end{array}$$



$z_1 = -0.5$

$z_2 = 1.4$

~~① ⇒ -0.5 =~~

put $z_1 = -0.5$ in ③,

$$-0.5 = \frac{45 - \mu}{\sigma}$$

8

put $z_2 = 1.4$ in ④,

$$1.4 = \frac{64 - \mu}{\sigma}$$

$45 - \mu = -0.5\sigma$ & $64 - \mu = 1.4\sigma$

$45 = \mu - 0.5\sigma$ — (5) & $64 = \mu + 1.4\sigma$ — (6)

(5) & (6) $\Rightarrow$ (6) − (5)

$(64 - 45) = (\mu + 1.4\sigma) - (\mu - 0.5\sigma)$

$19 = 1.9\sigma$

$\sigma = \frac{19}{1.9}$

$\boxed{\sigma = 10}$

sub $\sigma = 10$ in (6)

$64 = \mu + 1.4(10)$

$64 = \mu + 14$

$\mu = 64 - 14$

$\boxed{\mu = 50}$

Mean $= \mu = 50$

Variance $= \sigma^2$

$= 10^2 = 100$