Unit-5

$\bar{x} = (\ )+(\ )+(\ )+(\ )+(\ )+(\ )$

Control chart : ($\bar{\bar{x}}$ - R chart)

Q.) The following table gives the sample mean & range for 10 samples, each of size (6) in the production of certain component construct the control charts for mean and range and comment on the nature of control.

Sample No	1	2	3	4	5	6	7	8	9	10
Mean $\bar{x}$	(37.3)	49.8	51.5	59.2	54.7	34.7	51.4	61.4	70.7	75.3
Range R	7.5	12.8	10	9.1	7.8	5.8	14.5	2.8	3.7	8

How to find $\bar{\bar{x}}$ & R chart ?
$\bar{\bar{x}}$ & R by value given

soln :

step 1 :

here N = 10

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{37.3 + 49.8 + 51.5 + 59.2 + 54.7 + 34.7 + 51.4 + 61.4 + 70.7 + 75.3}{10}$$

$\bar{\bar{x}} = 54.6$

$\bar{R} = \frac{\Sigma R}{N} = \frac{9.5 + 12.8 + 10 + 9.1 + 7.8 + 5.8 + 14.5 + 22 + 3.7 + 8}{10}$

$= 8.4$

from the table of control chart for sample size 6,

$A_2 = 0.483$, $D_3 = 0$, $D_4 = 2.004$

step2: control limits for $\bar{x}$ chart:

CL (central line) $= \bar{\bar{x}} = 54.6$

LCL (Lower control Limit) $= \bar{\bar{x}} - A_2\bar{R}$

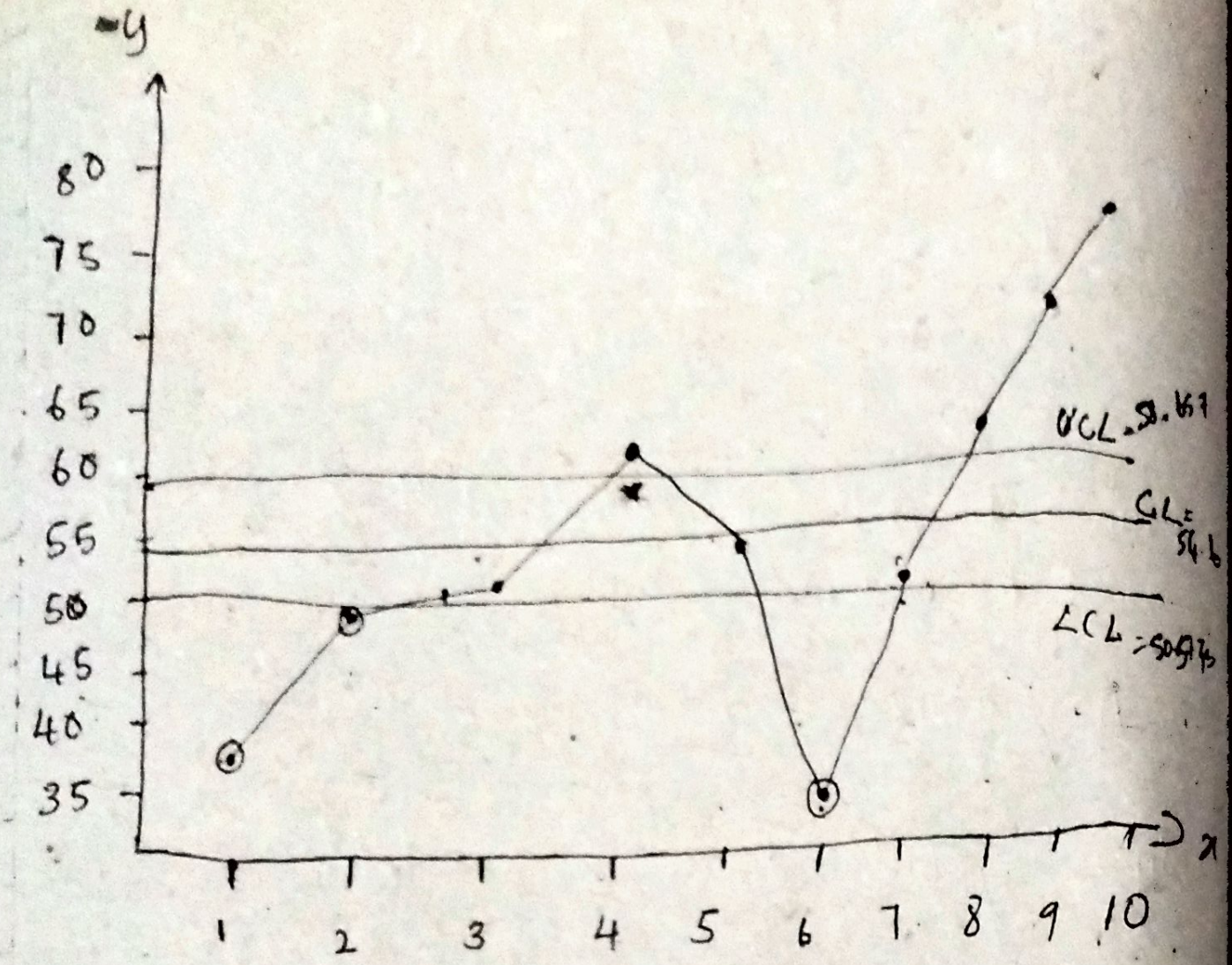
$= 54.6 - (0.483)(8.4)$

$= 50.543$

UCL (upper control Limit) $= \bar{\bar{x}} + A_2\bar{R}$

$= 54.6 + (0.483)(84)$

$= 58.657$



Conclusion:

Since 1st, 2nd, 4th, 6th, 8th, 9th and 10th sample means fall outside the control limits.

∴ The statistical process is out of control according to $\bar{x}$ chart.

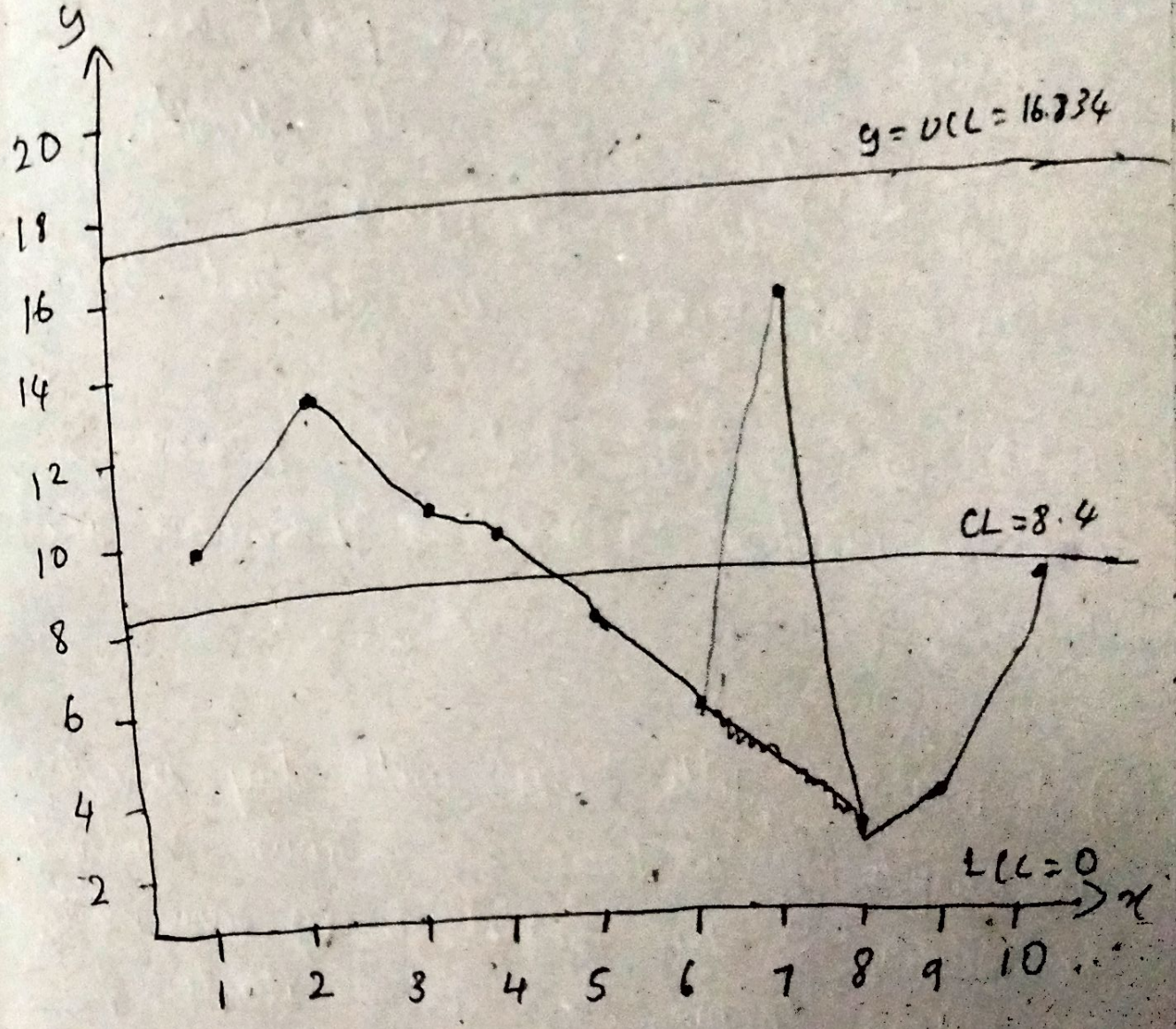
step 3:

control limits of R-chart:

$$CL = \bar{R} = 8.4$$

$$UCL = D_4 \bar{R} = 2.004 \times 8.4 = 16.834$$

$$LCL = D_3 \bar{R} = 0$$



conclusion:

Here all the sample mean falls within the control lines.

∴ The statistical process is under control according to R-chart.

Step 4: Inference:

Though all the sample point in R-chart lies within control limits, some of the sample points in $\bar{X}$ chart lie outside the control limits.

Hence we conclude that the process is out of control, corrective measures are necessary.

Q) A machine fills boxes with dry cereal. 15 samples of 4 boxes are drawn randomly. The weights of the sampled boxes are shown as follows. Draw the control charts for the sample mean and sample range and determine whether the process is in a state of control.

sample Number	1	2	3	4	5	6	7
weights of boxes (x)	10.0	10.3	11.5	11.0	11.3	10.7	11.3
	10.2	10.9	10.7	11.1	11.6	11.4	11.4
	11.3	10.7	11.4	10.7	11.9	10.7	11.1
	12.4	11.7	12.4	11.4	12.1	11.0	10.3

sample Numbes	8	9	10	11	12	13	14	15
weights of boxes (x)	12.3	11.0	11.3	12.5	11.9	12.1	11.9	10.6
	12.1	13.1	12.1	11.9	12.1	11.1	12.1	11.9
	12.7	13.1	10.7	11.8	11.6	12.1	13.1	11.7
	10.7	12.4	11.5	11.3	11.4	11.7	12.0	12.1

Sample NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\bar{x}$	10.975	10.9	11.5	11.05	11.725	10.95	11.025	11.95	12.4	11.4	11.875	11.75	11.75	12.275	11.575
R	2.4	1.4	1.7	0.7	0.8	0.7	1.1	2	2.1	1.4	1.2	0.7	1	1.2	1.5

step 2 : here $N = 15$, sample size $n = 4$

$$\overline{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{173.1}{15} = 11.54$$

$$\bar{R} = \frac{\Sigma R}{N} = \frac{19.9}{15} = 1.3267$$

from the table for $n = 4$, $A_2 = 0.729$,

$D_3 = 0$ & $D_4 = 2.282$.

step 3 : control limit for $\bar{x}$ - chart.

$$CL = \overline{\bar{x}} = 11.54$$

$$LCL = \overline{\bar{x}} - A_2 \bar{R}$$

$$= 11.54 - (0.729 \times 1.3267)$$

$$= 10.5728$$

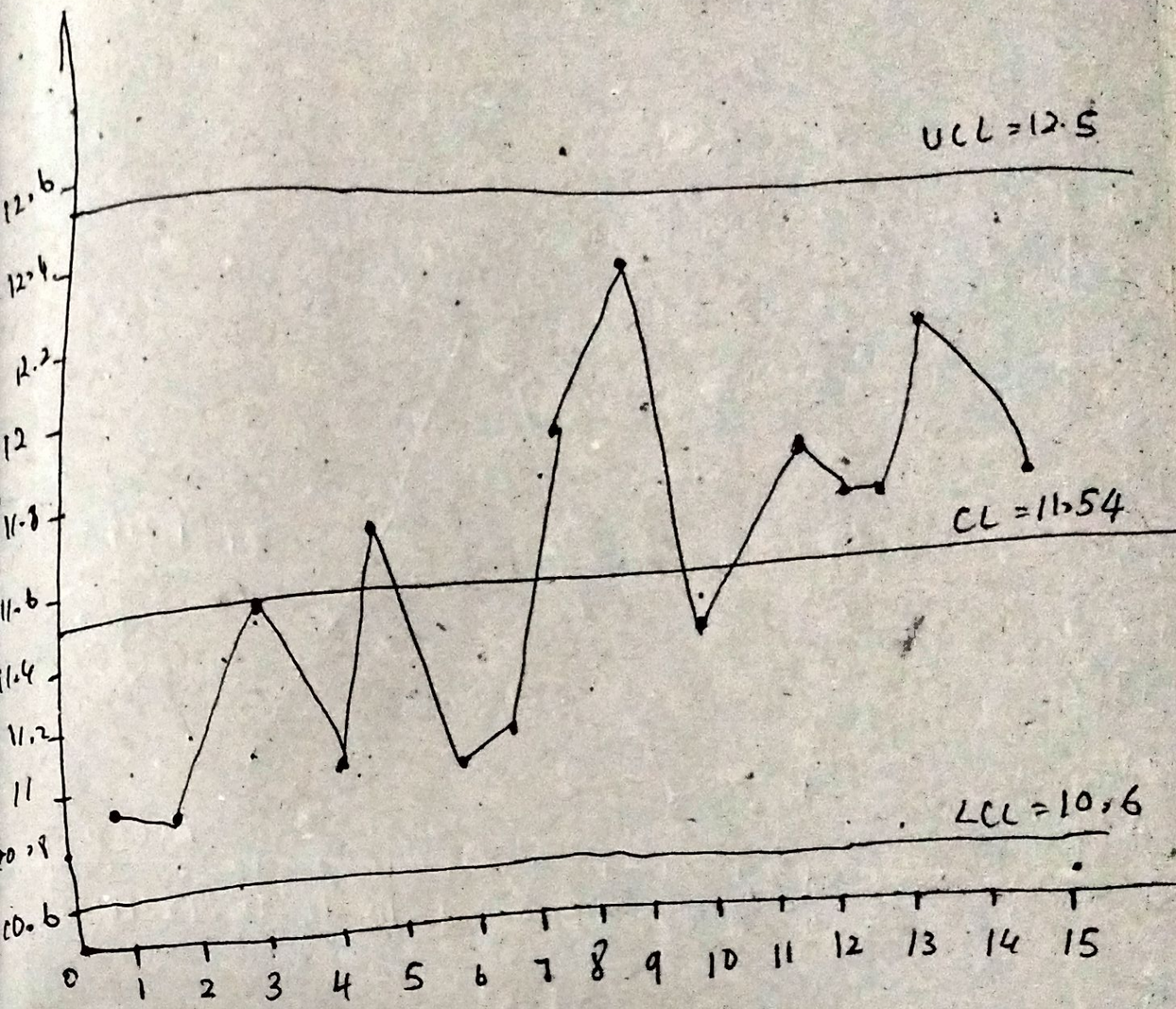
$$\approx 10.6$$

$$UCL = \overline{\bar{x}} + A_2 \bar{R}$$

$$= 11.54 + (0.729 \times 1.3267)$$

$$= 12.5072$$

$$\approx 12.5$$



It is in control

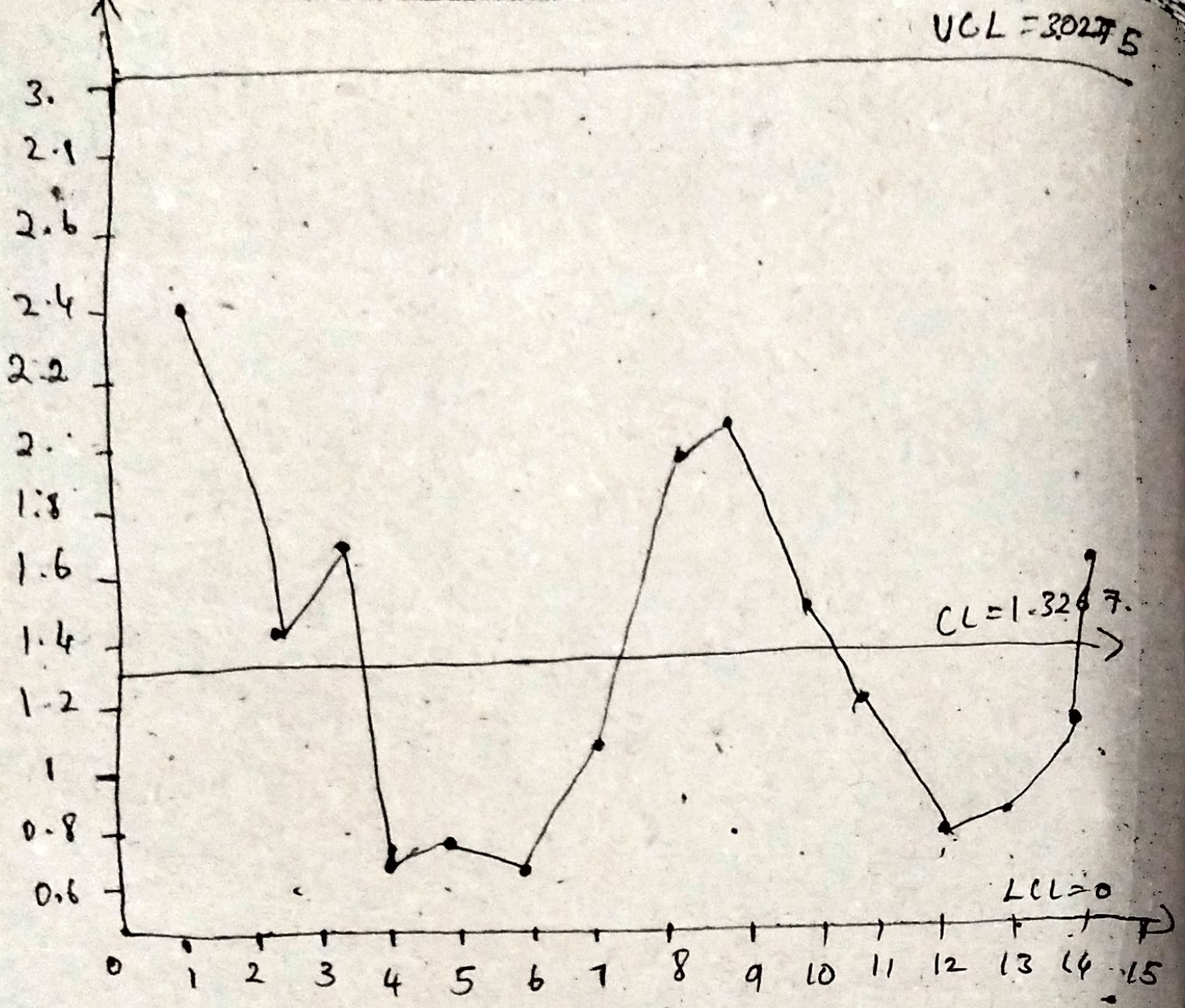
step 4: control limit for R-chart:

$CL = \bar{R} = 1.3267$

$LCL = D_3 \bar{R} = 0$

$UCL = D_4 \bar{R} = 2.282 \times 1.3267$

$= 3.0275$



It is in control

Step 5: Inference

From both $\bar{x}$ & $\bar{R}$ - chart, we can conclude that the process is under statistical control.

Q) Given below are the values of sample mean $\bar{x}$ and sample range R for 10 samples, each of size 5. Construct the control chart for mean and range. Find the revised control limit if necessary.

sample No.	1	2	3	4	5	6	7	8	9	10
mean ($\bar{x}$)	43	49	37	44	45	37	51	46	43	47
range (R)	5	6	5	7	7	4	8	6	4	6

soln:

Here $N = 10$ & $n = 5$

step 1

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{442}{10} = 44.2$$

$$\bar{R} = \frac{\Sigma R}{N} = \frac{58}{10} = 5.8$$

From the table, For sample size $n = 5$

We have $A_2 = 0.577$, $D_3 = 0$ & $D_4 = 2.114$

step 2: control limits for $\bar{x}$- chart:

$$CL = \bar{\bar{x}} = 44.2$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

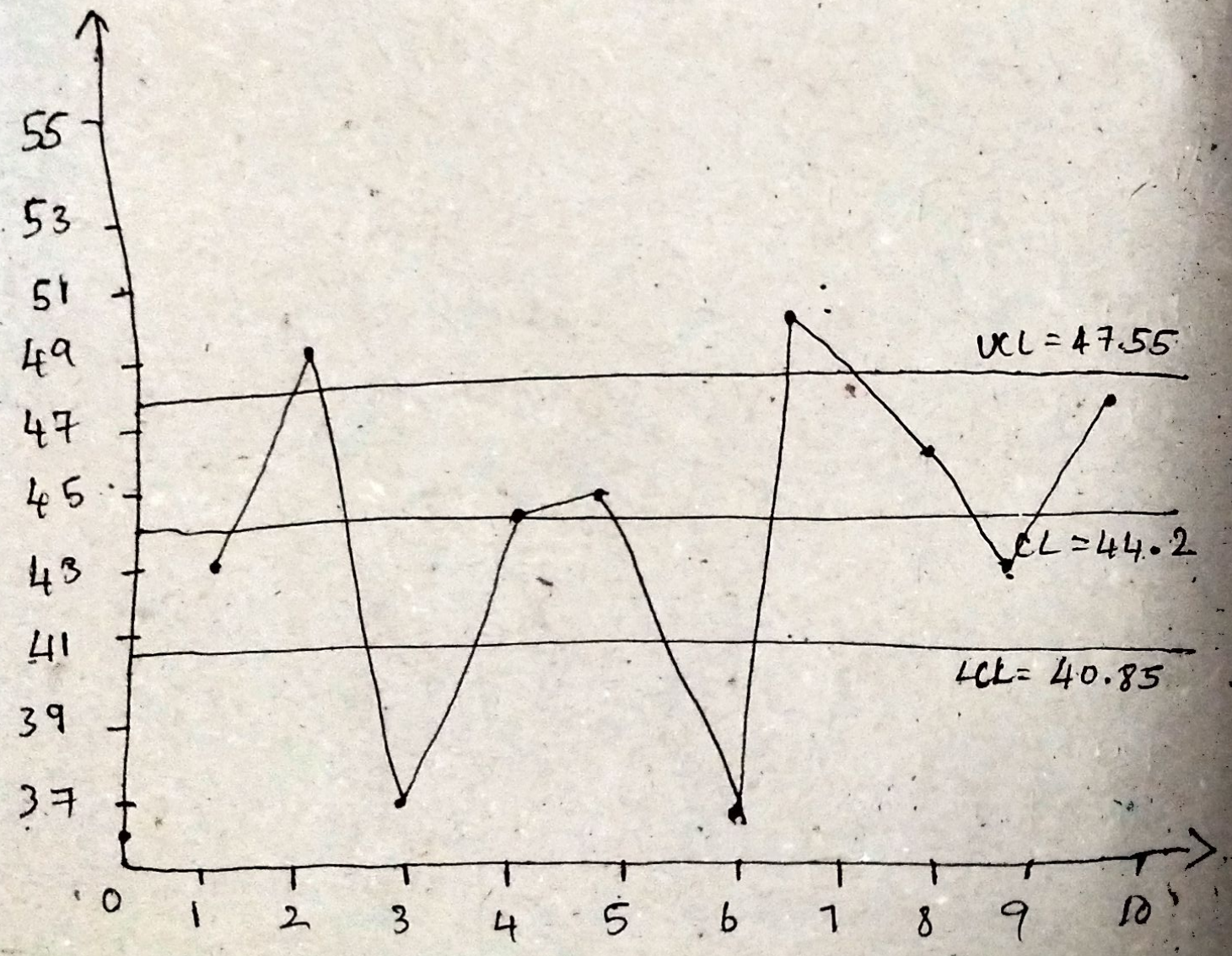
$$= 44.2 - (0.577 \times 5.8)$$

$$= 40.85$$

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$= 44.2 + (0.577 \times 5.8)$$

$$= 47.55$$



2nd, 3rd, 6th & 7th sample means fall outside the control limits.

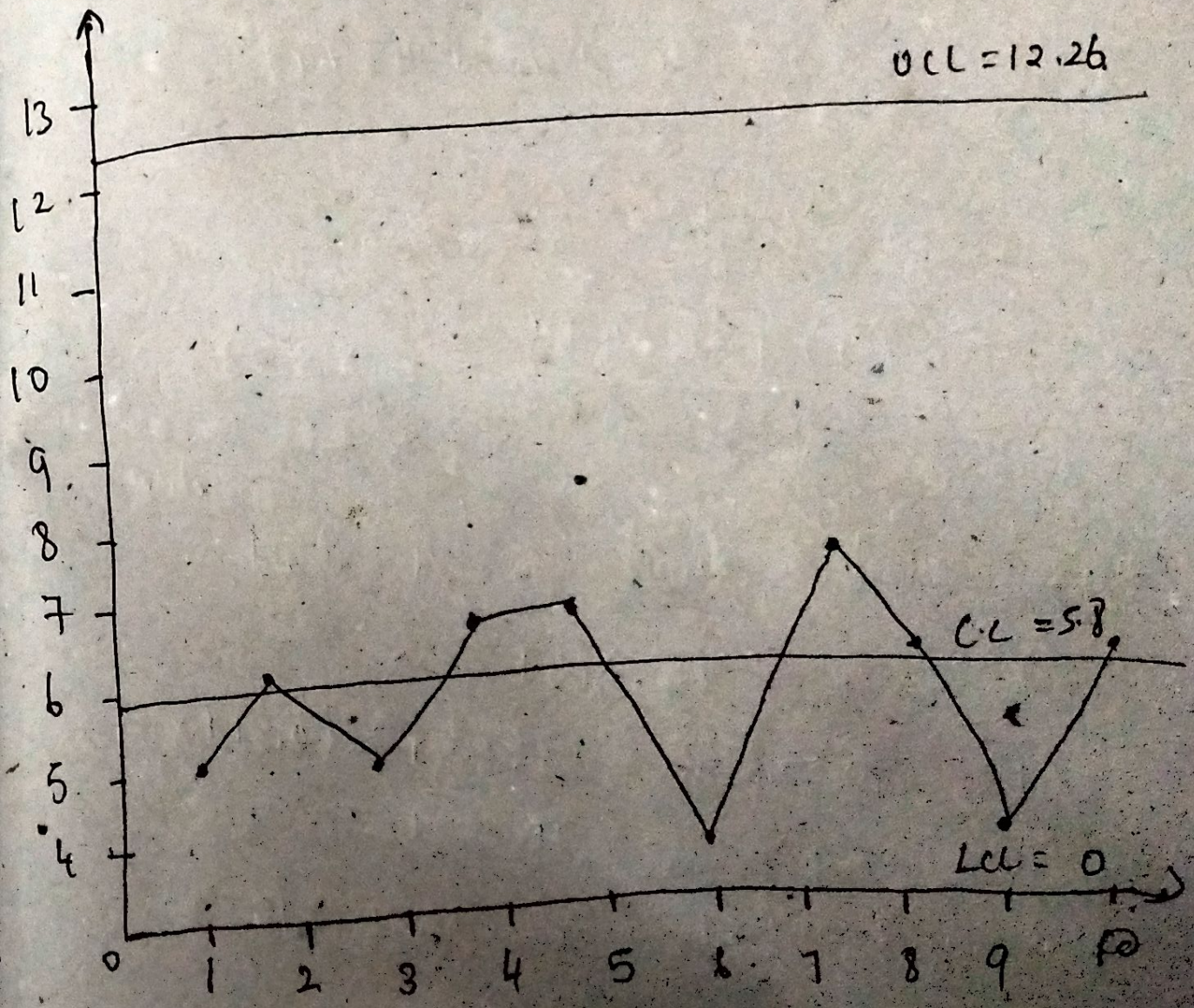
∴ It is out of control.

step 3: control limits for R-chart:

here $CL = \bar{R} = 5.8$

$LCL = D_3 \bar{R} = 0$

$UCL = D_4 \bar{R} = 2.114 \times 5.8 = 12.26$



It is in the control.

Step 4 : Conclusion :

From both $\bar{\bar{x}}$ & R-chart we can conclude that the statistical process is out of control.

Step 5 : Revised control Limit.

$$\bar{\bar{x}} = \frac{43+44+45+46+43+47}{6}$$

$$= 44.6667$$

$$\bar{R} = \frac{5+7+7+6+4+6}{6} = 5.8333$$

Revised control limit for $\bar{x}$ - chart

$$CL = \bar{\bar{x}} = 44.6667$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 44.6667 - (0.577 \times 5.8333)$$

$$= 41.3009$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 44.6667 + (0.577 \times 5.8333)$$

$$= 48.0325$$

Revised control limit for R-chart:

$$CL = \bar{R} = 5.8333$$

$$LCL = D_3 \bar{R} = 0$$

$$UCL = D_4 \bar{R} = 2.114 \times 5.8333 = 12.3316$$

Control charts for Attributes (C, P, np-chart)

no. of defect.

Q) i) construct a control chart for defectives for the following data:

Sample NO:	1	2	3	4	5	6	7	8	9	10
NO inputed.	90	65	85	70	80	80	70	95	95	75
NO of defectives.	9	7	3	2	9	5	3	9	6	7

ii) 15 tape-recorders were examined for Quality control test. the no. of defects in each tape-recorder is recorded below. Draw the appropriate control chart and comment on the state of control.

No of units :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No of defects :	2	4	3	1	1	2	5	3	6	7	3	1	4	2	1

soln: (ii) here $N = 15$

step 1 : Control limits for c-chart.

(Here the no. of defects per sample containing only one item is given. so use c-chart).

$$\bar{c} = \frac{\sum c_i}{N} = \frac{45}{15} = 3$$

$CL = \bar{c} = 3$ ($\because$ LCL cannot be negative)

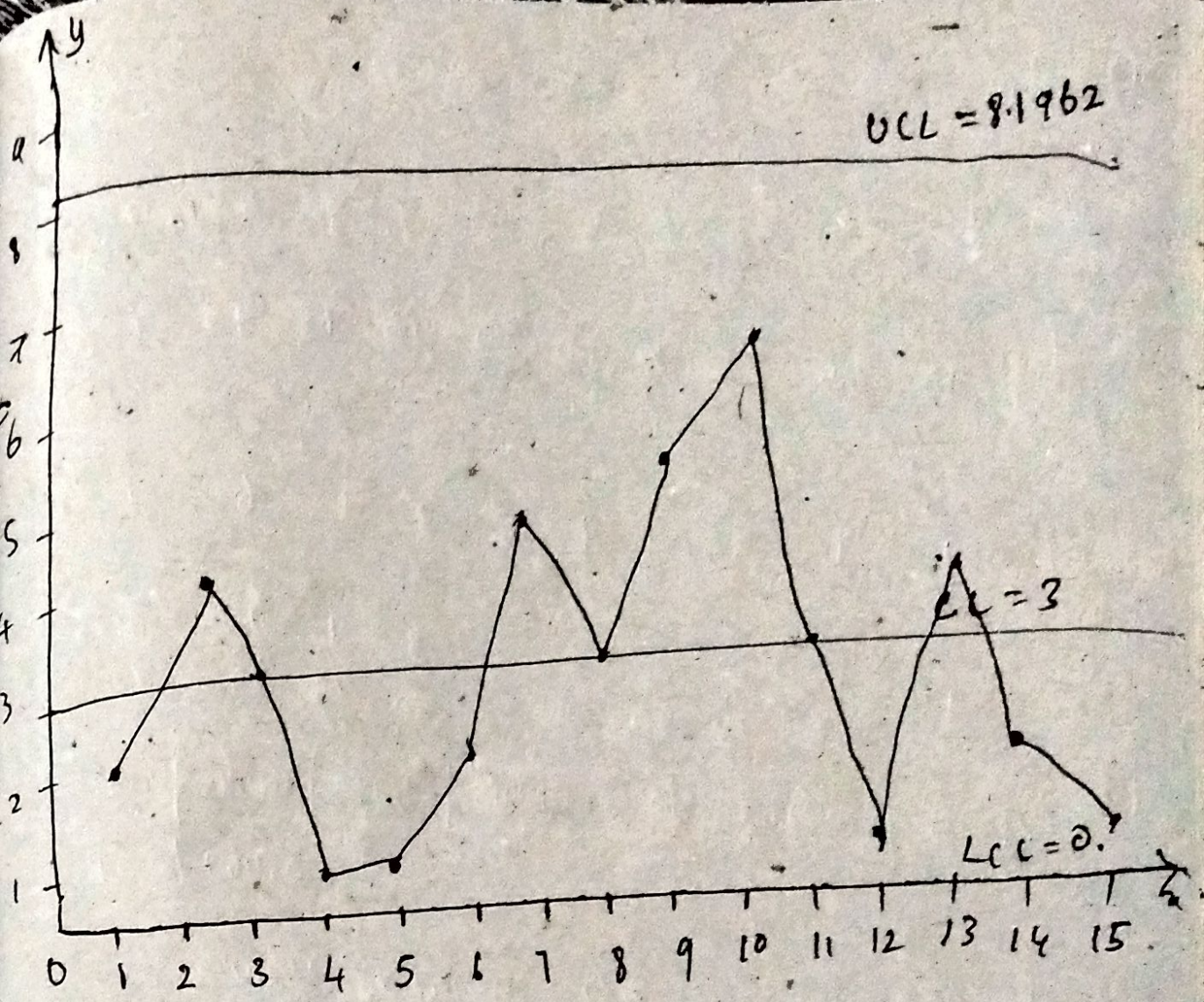
$LCL = \bar{c} - 3\sqrt{\bar{c}}$

$= 3 - 3\sqrt{3}$

$= -2.1962 \Rightarrow LCL = 0$

$UCL = \bar{c} + 3\sqrt{\bar{c}}$

$= 3 + 3\sqrt{3} = 8.1962$



Step 2: Conclusion:

Here all the sample points lie within the control limits.

∴ the process is under control.

i) Soln:

Sample No.	1	2	3	4	5	6	7	8	9	10
No of inspected n_i	90	65	85	70	80	80	70	95	90	75
No. of defective c_i	9	7	3	2	9	5	3	9	6	7
p_i	$\frac{9}{90}$	$\frac{7}{65}$	$\frac{3}{85}$	$\frac{2}{70}$	$\frac{9}{80}$	$\frac{5}{80}$	$\frac{3}{70}$	$\frac{9}{95}$	$\frac{6}{90}$	$\frac{7}{75}$
	$=0.1$	$=0.108$	$=0.035$	0.029	$=0.113$	$=0.063$	$=0.043$	$=0.095$	$=0.067$	$=0.093$

Here the size of the samples varies. so we can construct p-chart, provided

$0.75\bar{n} < n_i < 1.25\bar{n}$, for all i

$$\bar{n} = \frac{\sum n_i}{N} = \frac{800}{10} = 80$$

$\therefore 0.75\bar{n} = 60$ & $1.25\bar{n} = 100$

Here all the values of n_i lies between 60 & 100, Hence we can draw p-chart.

step 2: control limits for p-chart:

$$\bar{p} = \frac{\text{Total no. of defectives}}{\text{Total no. of items inspected}}$$

$$= \frac{\Sigma ci}{\Sigma ni}$$

$$= \frac{60}{800}$$

$$= 0.075$$

$$CL = \bar{p} = 0.075$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

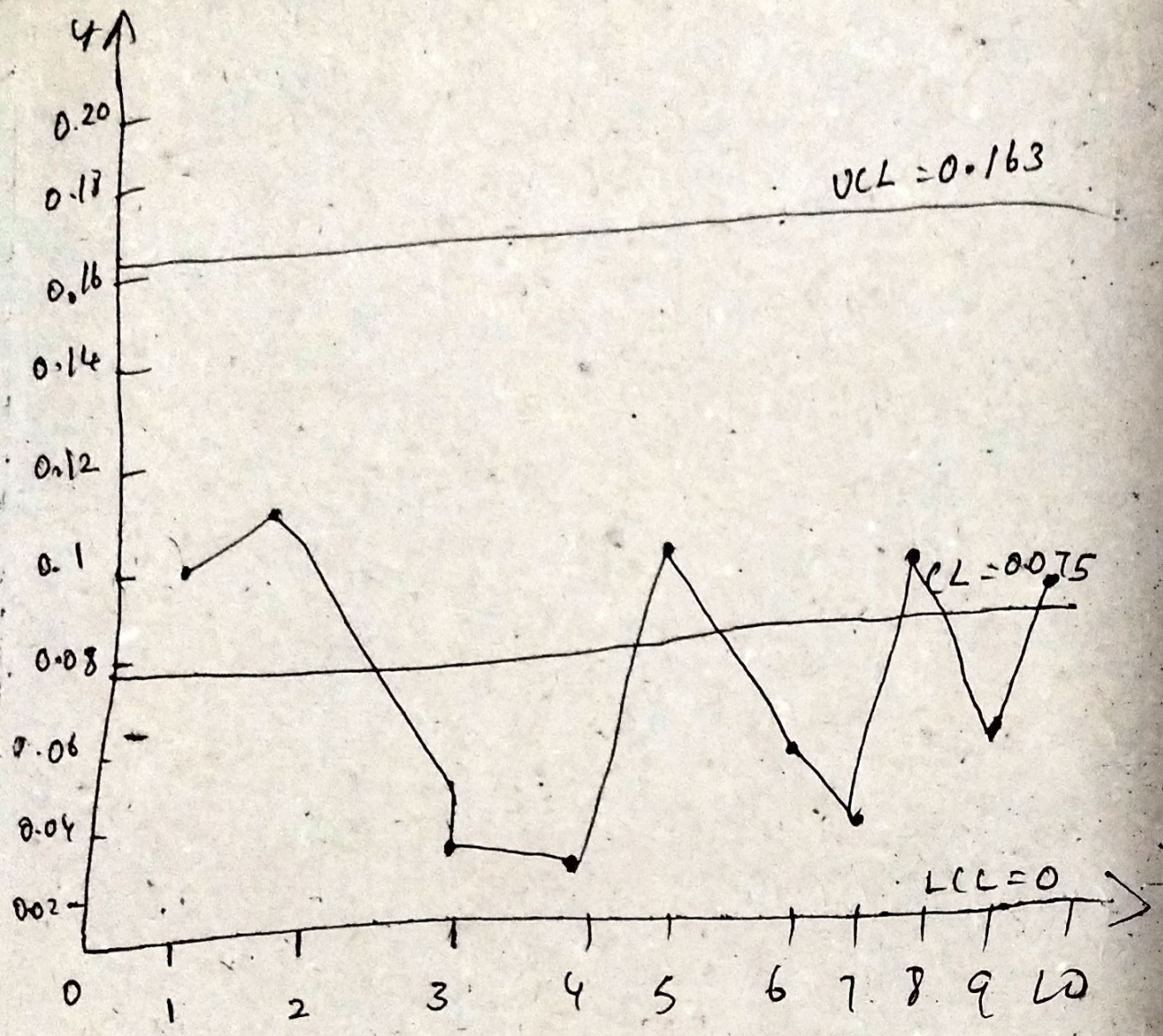
$$= 0.075 - 3\sqrt{\frac{0.075(1-0.075)}{80}}$$

$$= -0.013$$

$$= 0$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.075 + 3\sqrt{\frac{0.075 \times 0.925}{80}}$$

$$= 0.163$$



step 3: here all the sample points lies within the control limits.

Hence the process is under the control.

Q2) The data given below are the number of defectives in 10 samples of 100 items each. Construct a p-chart and an np-chart and comment on the results.

sample no	1	2	3	4	5	6	7	9	9	10
no. of defectives	6	16	7	3	8	12	7	11	11	4.

soln:

step 1:

Here $N = 10$ & the sample size is same for all samples $\therefore$ sample size $n = 100$

sample size	100	100	100	100	100	100	100	100	100	100
proportion of defectives.	6/100	0.16	0.07	0.03	0.08	0.12	0.07	0.11	0.11	0.04
	0.06									

$$\bar{p} = 0.85$$

step 2 : construction of p chart

$$\bar{p} = \frac{\Sigma p_i}{N} = \frac{0.85}{10} = 0.085$$

$$CL = \bar{p} = 0.085$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

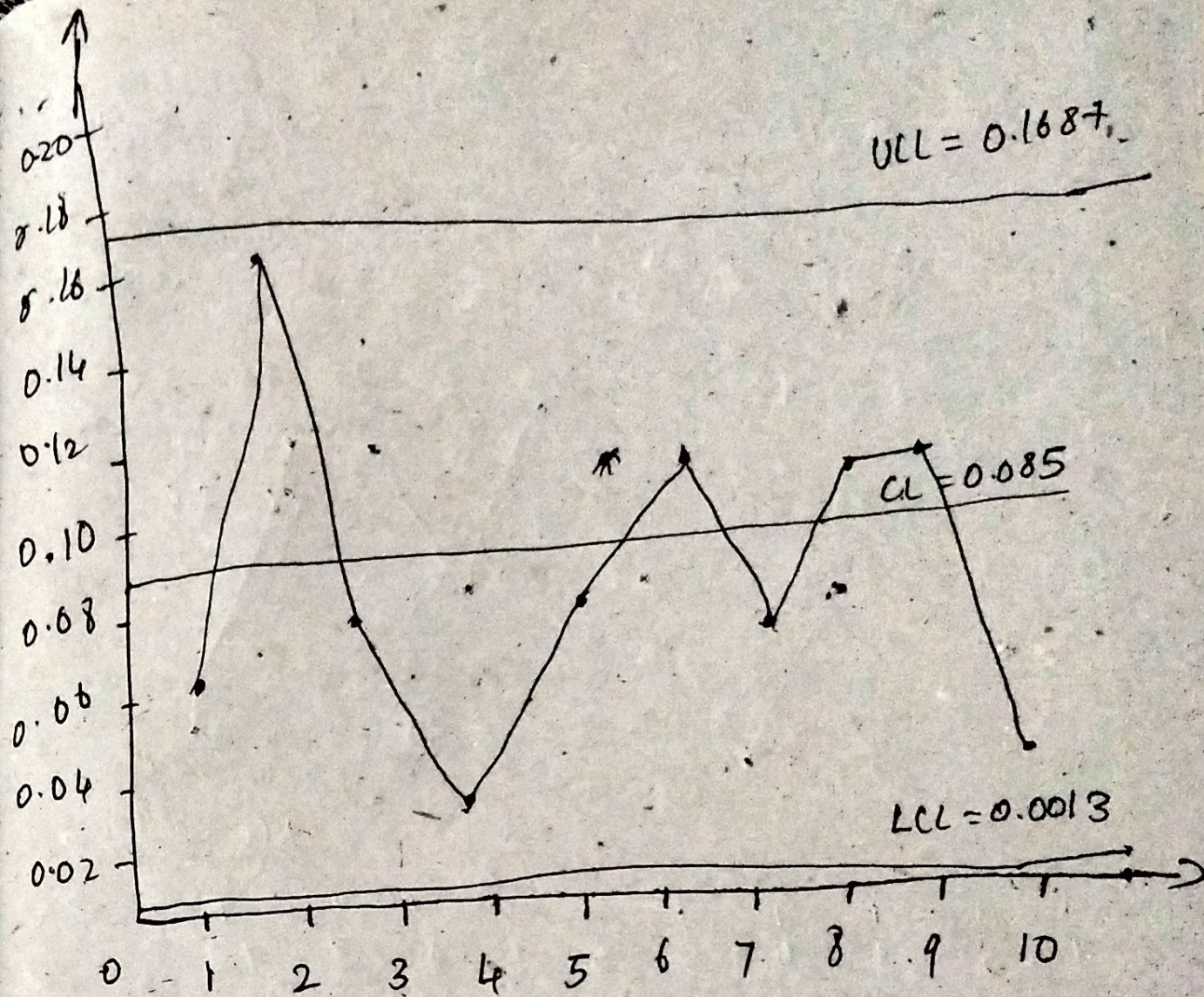
$$= 0.085 - 3\sqrt{\frac{(0.085)(1-0.085)}{100}}$$

$$= 0.0013$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.085 - 3\sqrt{\frac{(0.085)(1-0.085)}{100}}$$

$$= 0.1687$$



It is in control.

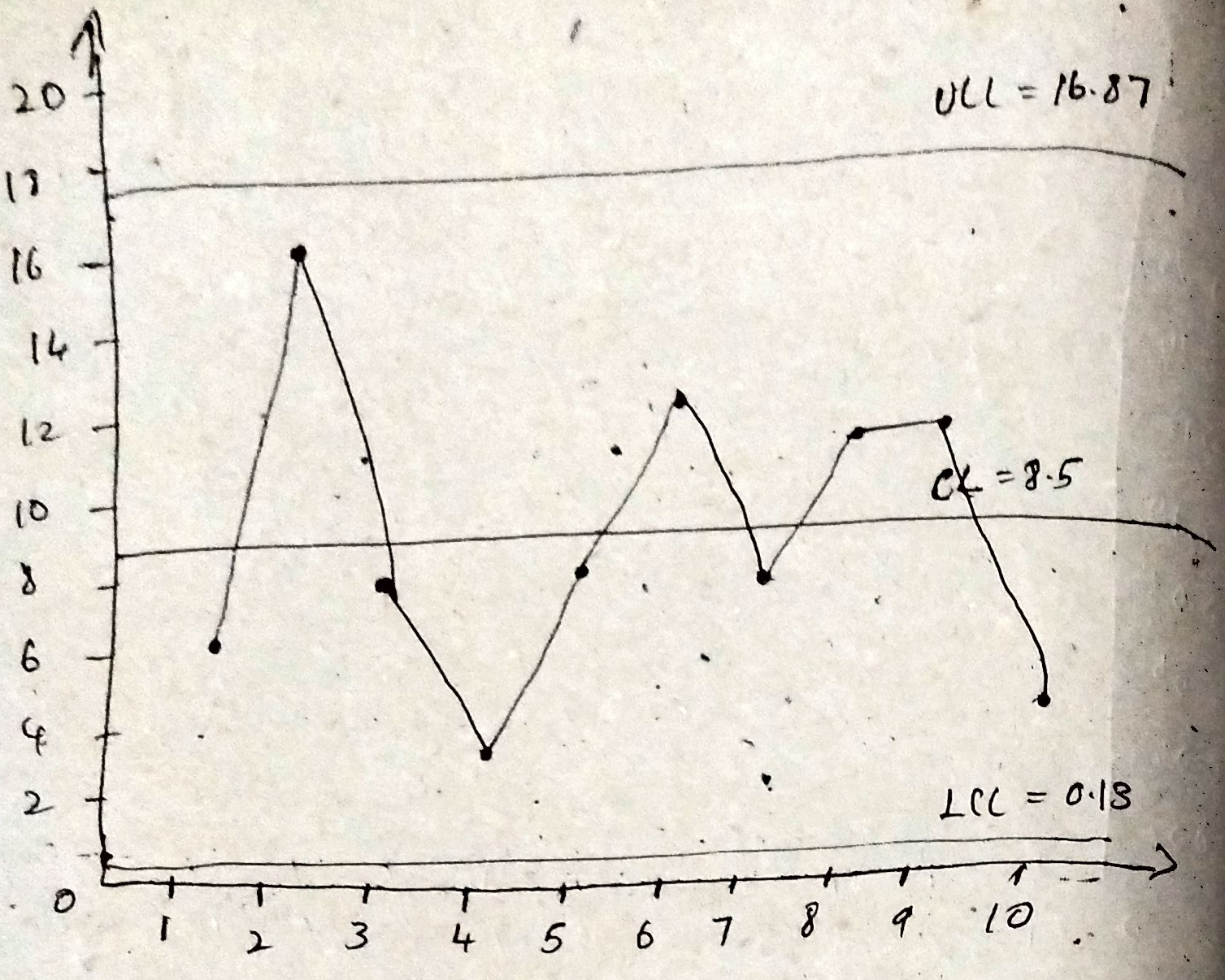
step 3 : construction of np-chart

$$CL = n\bar{p} = 100 \times 0.085 = 8.5$$

$$LCL = n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$$

$$= 100 \times 0.0013 = 0.13$$

$$UCL = 100 \times 0.1687 = 16.87$$



It is under control.

Tolerance limit :

$\bar{x}$ & R - chart → under control → $\bar{\bar{x}} \mp \frac{3\bar{R}}{d_2}$

$\left(\bar{\bar{x}} - \frac{3\bar{R}}{d_2}\right), \left(\bar{\bar{x}} + \frac{3\bar{R}}{d_2}\right)$

LL UL

out control

↓

Revised control limit → tolerance.

Q) The specifications for a certain quantity characteristic are (60 ± 24) in coded values. The table given below gives the measurements obtained in 10 samples. Find the tolerance limits for the process and test if the process meets the specifications.

sample no.	1	2	3	4	5	6	7	8	9	10
Measurements (x)	75	41	57	61	55	49	74	67	66	62
	66	79	55	71	68	98	13	70	65	68
	50	53	58	66	58	65	62	68	58	66
	62	61	61	69	62	64	57	56	52	68
	52	49	72	77	75	66	62	61	58	73
	70	56	63	53	63	64	64	66	50	68

soln : sample size.

here $N = 10$, $n = 6$

$\bar{x}$	62.5	57.1	60.2	66.2	63.5	67.7	63.7	64.7	58.2	67.3
R	25	31	19	24	20	49	17	14	16	11

$\Sigma \bar{x} = 631.9$ $\Sigma R = 226$

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{631.9}{10} = 63.19$$

$$\bar{R} = \frac{\Sigma R}{N} = \frac{226}{10} = 22.6$$

For sample size 6, $A_2 = 0.483$ $D_3 = 0$ &

$D_4 = 2.004$

step 2 : constructing $\bar{x}$ - chart :

$CL = \bar{\bar{x}} = 63.19$

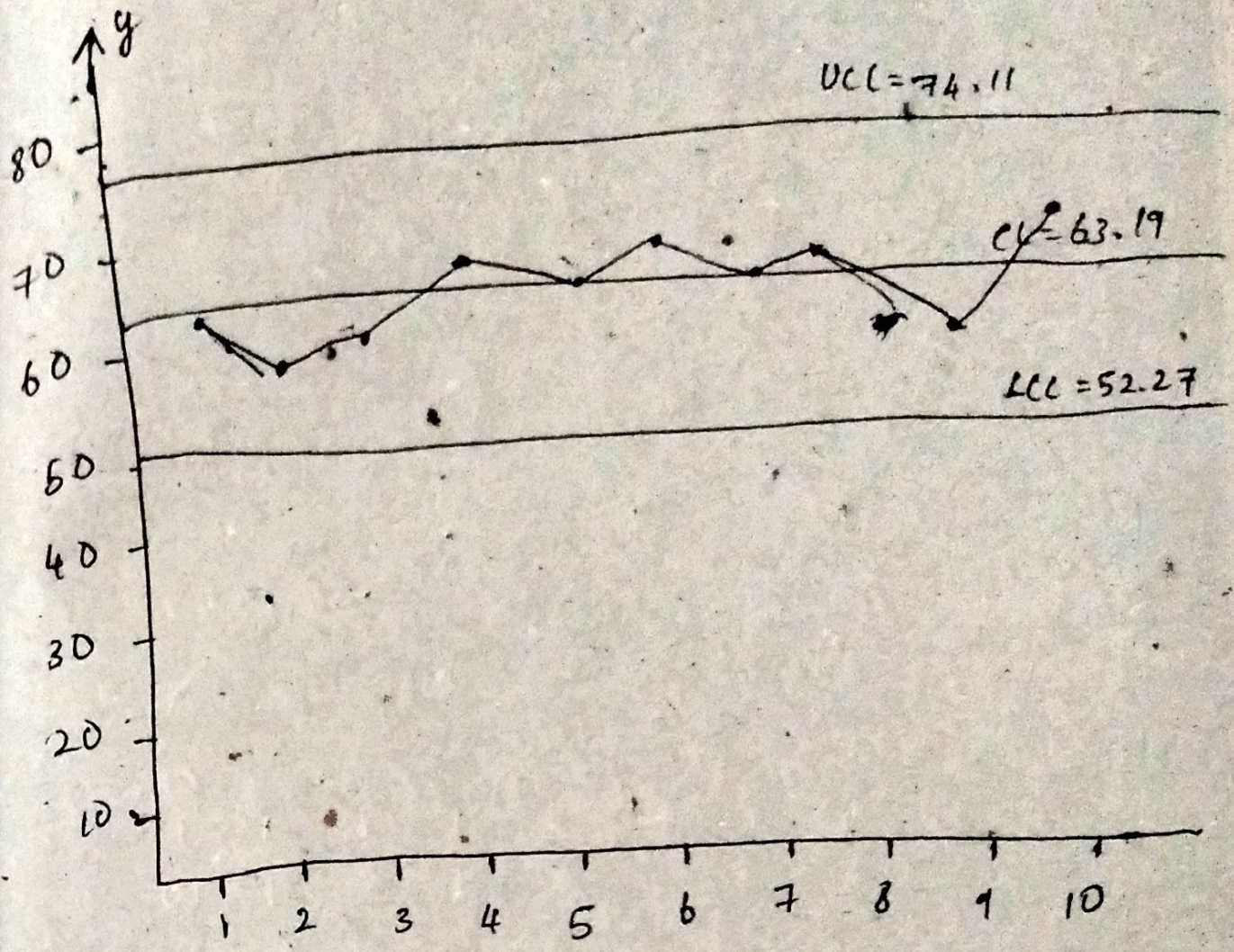
$LCL = \bar{\bar{x}} - A_2 \bar{R} = 63.19 - (0.483 \times 22.6)$

$= 52.27.$

$UCL = \bar{\bar{x}} + A_2 \bar{R} = 63.19 + (0.483 \times 22.6)$

$= 74.11$

$\bar{x}$ -chart :



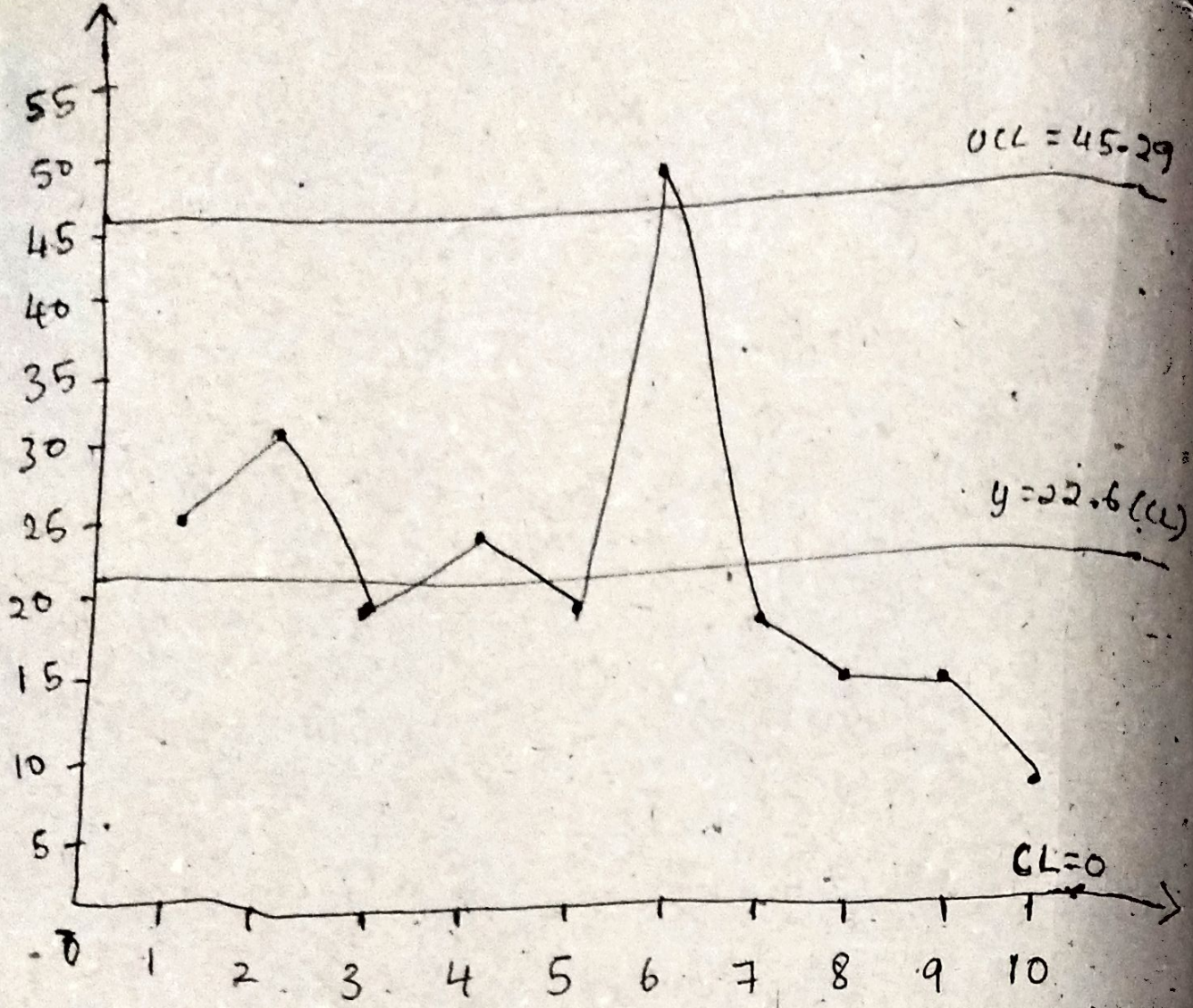
It is under control.

step3: constructing R-chart :

$CL = \bar{R} = 22.6$

$LCL = D_3\bar{R} = 0$

$UCL = D_4\bar{R} = 2.004 \times 22.6 = 45.29$



It is not under control.

Find the revised control Limit:

Now remove the sample number 6 from the given data then compute $\bar{\bar{x}}$ & $\bar{R}$ based on the remaining 9 samples.

NOW,

$\Sigma \bar{x} = 631.9 - 67.7 = 564.2$ &

$\Sigma R = 226 - 49 = 177$

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{n} = \frac{564.2}{9} = 62.69$$

$$\bar{R} = \frac{\Sigma R}{9} = \frac{177}{9} = 19.67$$

Revised control limits for $\bar{x}$ - chart:

$$CL = \bar{\bar{x}} = 62.69$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 62.69 - (0.483 \times 19.67)$$

$$= 53.19$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 62.69 + (0.483 \times 19.67)$$

$$= 72.19.$$

Revised control limits for R-chart:

$$CL = \bar{R} = 19.67$$

$$LCL = D_3\bar{R} = 0$$

$$UCL = D_4\bar{R} = 2.004 \times 19.67 = 39.42$$

Now we see that the process is under control with respect to the 9 samples considered.

Find the Tolerance limit: (here we use the revised values of $\bar{\bar{x}}$ & $\bar{R}$).

The tolerance limits are given by,

$$\bar{\bar{x}} \mp \frac{3\bar{R}}{d_2}.$$

Tolerance limits $= 62.69 \mp \frac{3(19.67)}{2.534}$

$= 62.69 \mp 23.2873$

$= (39.40, 85.98)$

These tolerance limit are outside its specification limit. (36, 84).

Hence the process does not meet the required specification.

$\bar{x}$ - S chart:

Q) The following data give the coded measurements of 10 samples each of size 5. drawn from a production process at intervals of 1 hour. Calculate the sample means and S.D.'s and draw the control charts for $\bar{x}$ and S.

Sample no.	1	2	3	4	5	6	7	8	9	10
Measurement (x)	9	10	10	8	7	12	9	15	10	11
	15	11	13	13	9	15	9	15	13	14
	14	13	8	11	10	7	9	10	14	12
	9	6	12	10	4	16	13	13	7	14
	13	10	7	13	5	12	5	17	11	14

soln:

step 1:

Here, $N = 10$ & $n = 5$

sample no.	1	2	3	4	5	6	7	8	9	10
$\bar{x}$	12	10	10	11	7	12	9	14	11	14
$\Sigma(x-\bar{x})^2$	32	26	26	18	26	54	32	28	30	8
$S =$	2.5	2.3	2.3	2.1	2.3	3.5	2.5	2.4	2.6	1.3

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{N} = \frac{12+10+10+11+7+12+9+14+11+14}{10}$$

$$\bar{S} = \frac{\Sigma S}{N} = \frac{23.2}{10} = 2.32$$

from the control chart table, for sample size $n = 5$,

$A1 = 1.596$, $B3 = 0$, $B4 = 2.089$

step 2: constructing $\bar{x}$ - chart.

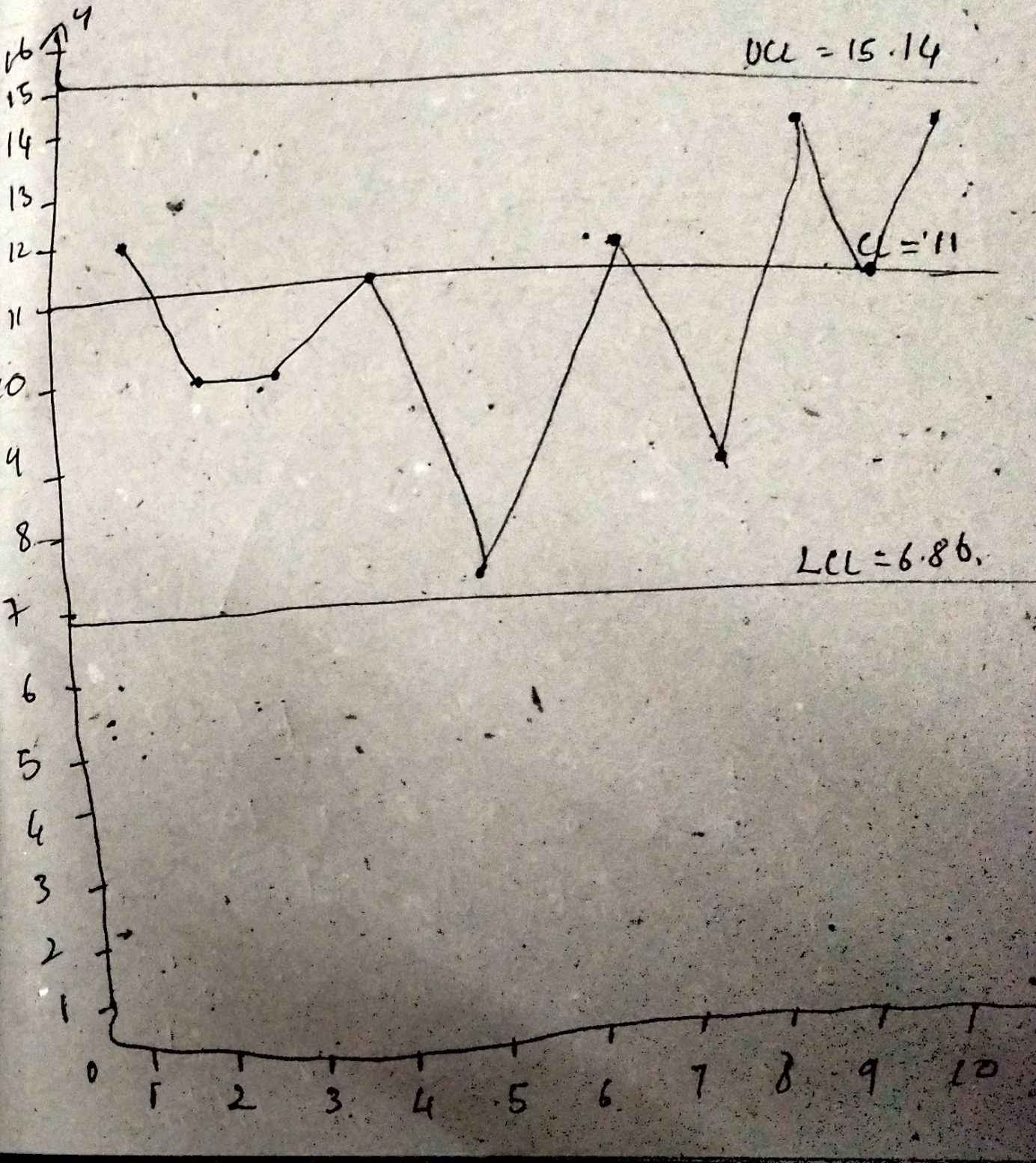
$CL = \bar{\bar{x}} = 11$

$$LCL = \bar{\bar{x}} - A_1 \sqrt{\frac{n}{n-1}}\,\bar{s}$$

$$= 11 - (1.596 \times \sqrt{\frac{5}{4}} \times 2.32) = 6.86$$

$$UCL = \bar{\bar{x}} + A_1 \sqrt{\frac{n}{n-1}}\,\bar{s}$$

$$= 11 + (1.596 \times \sqrt{\frac{5}{4}} \times 2.32) = 15.14$$

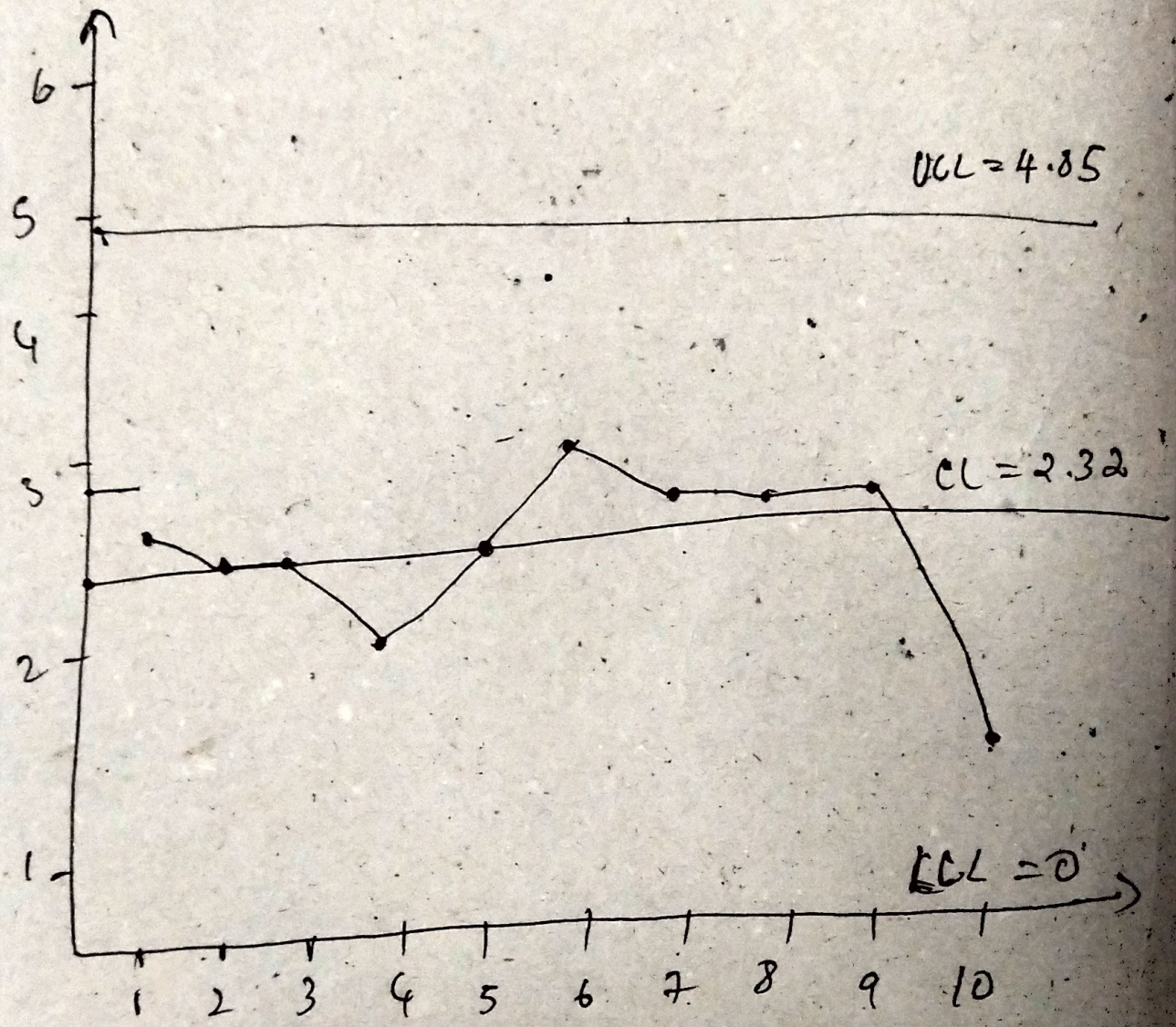


step 3 : constructing S-chart :

$$CL = \bar{S} = 2.32$$

$$LCL = B_3 \times \bar{S} = 0$$

$$UCL = B_4 \times \bar{S} = 2.089 \times 2.32 = 4.85$$



It is under control.

step 4 : conclusion : The process is under control with respect to average and variability.