unit : 4

1. T-Test For one sample :

degree of freedom $= n - 1$

Formula : $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$

$$S^2 = \frac{\sum (x - \bar{x})^2}{n}$$

$S = \sqrt{}$ ← Answer

Eg :

A library system lends books for period of 21 days. This policy is being reevaluated in view of a possible new loan period that could be either longer or shorter

13.5 (*) given, $n = 8$, $\mu = 21$

soln :

($\mu = 21$) H0 : The loan period is 21 days

($\mu \neq 21$) H1 : The loan period is not 21 days.

$$\bar{x} = \frac{21 + 15 + 12 + 24 + 20 + 21 + 13 + 16}{8 \cancel{-1}}$$

$$= \frac{142}{8} = 17.75.$$

$$s_1^2 = \frac{\Sigma (x - \bar{x})^2}{n-1}$$

$$= \frac{(21 - 17.75)^2 + (15 - 17.75)^2 + (12 - 17.75)^2 + (24 - 17.75)^2 + (20 - 17.75)^2 + (21 - 17.75)^2 + (13 - 17.75)^2 + (16 - 17.75)^2}{8 - 1}$$

$$= \frac{(3.25)^2 + (-2.75)^2 + (-5.75)^2 + (6.25)^2 + (2.25)^2 + (3.25)^2 + (-4.75)^2 + (-1.75)^2}{8 - 1}$$

$$= \frac{10.56 + 7.56 + 33.06 + 39.06 + 5.06 + 10.56 + 22.56 + 3.06}{8}$$

$$s^2 = \frac{131.48}{8-1} = \cancel{16.435}. = 18.782$$

$$s = \sqrt{\cancel{16.435}\ 18.782}$$

$$\cancel{s = 4.0540.}$$

$$s = 4.33.$$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$t = \frac{17.75 - 21}{4.33 / \sqrt{8}} \qquad \sqrt{8} = 2.828.$$

$$= \frac{-3.25}{1.53}$$

$$\boxed{t = -2.124.}$$

dof $= n-1$

$= 8-1$

$= 7$

Table value :

LOS : 0.05 $= 2.365$.

Calculated value		Table value.
$\|-2.214\|$	$<$	± 2.365.

Accept HO.

13.6)

soln :

Null hypothesis (Ho) : The mean of no. of trails required is 32. ($\mu = 32$)

Alternative hypothesis (H_1) : the mean of no. of trails required is different from 32. ($\mu \neq 32$).

1.

$\bar{x} = 34.89$, $s = 3.02$, $n = 7$.

$\mu = 32$.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$= \frac{34.89 - 32}{3.02 / \sqrt{7}}$$ $\therefore \sqrt{7} = 2.645$

$$= \frac{2.89}{1.141}$$

$t = 2.5328$.

dof $= n - 1$

$= 7 - 1 = 6$.

Table value at 0.05 LOS $= 2.447$

C.V. > T.V.

2.53 > 2.447

Reject H0.

b) Confidence Interval :

Margin of Error = $t_{\alpha/2} \times \frac{s}{\sqrt{n}}$.

$= 2.447 \times 1.141$

$ME = 2.79$.

construct the confidence interval :

$\bar{X} \pm ME$.

$(34.89 - 2.79, 34.89 + 2.79) \approx$

$(32.10, 37.68)$ $\bar{X} \pm ME$

c) Interpretation :-

95% of confidence interval (32.10 – 37.68) indicates that the true average no. of trials needed with shock likely falls within this range. This suggests that the mild electrical shock might increase the no. of trials required for the rats to learn the maze.

13.7 (*).

a).

Ho : The mean deviation is zero ($\mu = 0$)

H1 : the mean " " not zero ($\mu > 0$).

$\bar{x} = 1.14$, $\mu = 0$, $S = 0.10$,

$n = 10$.

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

$$= \frac{1.14 - 0}{0.10/\sqrt{10}} \quad \sqrt{10} = 3.162$$

$$= \frac{1.14}{0.031}$$

$$t = 36.774$$

dof = n - 1

= 10 - 1

= 9.

Table Value at 0.1 LOS = 3.250.

2.821

T.V C.V

2.821 < 36.774

It reject the HO.

b) Confidence interval:

Margin of error (ME).

ME = 2.821 X 0.031.

= 0.087451

Confindende Interval : $\bar{x} \pm ME$

1.14 ± 0.087.

$1.053 \leq \mu \leq 1.227$

↓

annual global temperature for

(their recent years is greater than 1.053 F°

for the entire twentieth century.

t-test For Two Independent samples:

a) The difference means for experiment B is more likely to be viewed as real because of its smaller variability.

b) Ho : $\mu_B - \mu_B = 0$

$\mu_B - \mu_B \neq 0$

14.14)

a)

HO : There is no difference in the mean no. of errors between the treatment & control groups ($\mu_1 = \mu_2$).

H_1: there is a difference " "

($\mu_1 \neq \mu_2$)

$\bar{x}_1 = 26.4$ $\quad \bar{x}_2 = 18.6$

(standard error?)

$n_1 = n_2 = 60.$ $\quad SE = 2.4$

$t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE}$ $\quad \therefore SE = \sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}$

$= \dfrac{26.4 - 18.6}{2.4}$ $\quad \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$= 3.25.$

$dof = n_1 + n_2 - 2$ $\quad \dfrac{"}{SE}$

$= 60 + 60 - 2$

$= 118$

Table Value $= 1.98$ (LOS 0.05)

T.V. $\quad$ C.V.

$1.98 < 3.28$

It reject H0.

t - distribution with 118

b) $p < \cancel{0.001}$ dof.

$p = 2 \times (1 - CDF(t, df)) \approx$

$= 2 \times (1 - 0.9994)$

$= 2 \times 0.0006$

$= 0.0012.$

$p < 0.001$

c) 95% confidence interval.

$ME = t_{\alpha/2} \times SE$

$= 1.98 \times 2.4$

$= 4.7521$

$(\bar{X}_1 - \bar{X}_2) \pm ME$

$(26.4 - 18.6) \pm 4.75$

$7.8 \pm 4.75.$

$(3.05, 12.55.)$

$3.06 \leq \mu \leq 12.55.$

d) Cohen's d.

$$d = \frac{\bar{x_1} - \bar{x_2}}{S_{pooled.}}$$

$$S_{pooled} = \sqrt{\frac{(s_1^2 + s_2^2)}{2}}$$

$$= \sqrt{\frac{(13.99)^2 + (12.15)^2}{2}}$$

$$= \sqrt{\frac{195.72 + 147.62}{2}} = \frac{343.34}{2}$$

$$= \sqrt{171.67}$$

$$S_p = 13.11$$

$$d_2 = \frac{7.8}{13.11}$$

$$= 0.59$$

e) Mean errors on a driving simulator are significantly greater when alcohol is consumed ($\bar{x} = 26.4$, $s = 13.99$) than when no alcohol is consumed ($\bar{x} = 18.6$, $s = 12.15$), according to a t-test.

[$t(118) = 3.25$, $p < .001$, $d = 0.59$]

14.13)

a) H_0 : There is no difference b/w the delta grade student & pass/fail student. ($\mu_1 = \mu_2$).

H_1 : There is a difference "

($\mu_1 \neq \mu_2$).

$\overline{x_1} = 86.2$ $\overline{x_2} = 81.6.$ $SE = 1.50$

$n_1 = n_2 = 20.$

$$t = \frac{\overline{x_1} - \overline{x_2}}{SE}$$

$$= \frac{86.2 - 81.6}{1.50}$$

$$= \frac{4.6}{1.50}$$

$$t = 3.07$$

$$dof = n_1 + n_2 - 2$$

$$= 20 + 20 - 2$$

$$= 38$$

Table value = 2.042 (at 0.05 LOS).

T.V   C.V

$2.042 < 3.07$

Reject H0.

b) Reversing roles of X1 & X2.

$\overline{X1} = 81.6$, $\overline{X2} = 86.2$, $SE = 1.50$.

$$t = \frac{81.6 - 86.2}{1.50}$$

$$\simeq \frac{-4.6}{1.50}$$

$$= -3.07.$$

Reject the H0. (No difference b/w the groups).

c). self-selection could introduce bias,

because of self-selection, groups might differ with respect to any one or several uncontrolled. variables, such as motivation, aptitude, and so on, in addition to the ~~diff~~ difference in grading policy. Hence any observed. difference b/w the mean achievement scores for these two groups would not be attributed solely to the difference in grading policy.

d). $p < 0.01$.

e) $s_p = 5.$

$$d = \frac{\bar{X_1} - \bar{X_2}}{s_p}$$

$$= \frac{86.2 - 81.6}{5}$$

$$= 0.92.$$

f). Introductory biology students have higher mean achievement scores when awarded letter grades ($\bar{x} = 86.2$, $S = 5.39$) rather than a simple pass/fail ($\bar{x} = 81.6$, $S = 4.58$), according to the t test [$t(38) = 3.07$, $p < 0.01$, $d = 0.92$).

15.7).

soln:

H0: There is no difference b/w the physical and no physical exercise group. (

H1: There is a "

$d_i = x_1 - x_2$

$d_1 = 4.00 - 3.75 = 0.25$

$d_2 = 2.67 - 2.74 = -0.07$

$d_3 = 3.65 - 3.42 = 0.23$

$d_4 = 2.11 - 1.67 = 0.44$

$d_5 = 3.21 - 3.00 = 0.21$

$d_6 = 3.60 - 3.25 = 0.35$

$d_7 = 2.80 - 2.65 = 0.15$

$$\bar{d} = \frac{\sum d_i}{n}$$

$$= \frac{0.25 - 0.07 + 0.23 + 0.44 + 0.21 + 0.35 + 0.15}{7}$$

$$= \frac{1.56}{7}$$

$$\boxed{\bar{d} = 0.223.}$$

$1.56^2 = 2.4337$

$$SSD = \sum d_i^2 - \left(\frac{(\sum d_i)^2}{n} \right).$$

$$= 0.49 - 0.34$$

$$= 0.15$$

$$sd = \sqrt{\frac{SSD}{n-1}}$$

$$= \sqrt{\frac{0.15}{6}}.$$

$$= \sqrt{0.025} \doteq 0.158$$

$$S_{\bar{D}} = \frac{sd}{\sqrt{n}}$$

$$= \frac{0.158}{\sqrt{7}}$$

$$= \frac{0.158}{2.645}$$

$$S_{\bar{D}} = 0.059$$

$$t = \frac{\bar{d}}{S_{\bar{D}}}$$

$$= \frac{0.22}{0.05}$$

$$\boxed{t = 4.40}$$

$$dof = n-1 = 7-1 = 6$$

Table value $= 3.143$.

TV $<$ CV

$3.143 < 4.40$

Reject Ho.

b) $p < 0.01$

c) $d = \frac{\bar{d}}{sd}$

$= \frac{0.223}{0.158}$

$= 1.4\cancel{17}$

d) there was no significant diff in GPAS between the exercise group. ($\bar{D} = 0.22$; $S_D = 0.15$), according to a t test [$t(6) = 4.40$, $p < 0.01$; $d = 1.57$].

F-Test :

It is a statistical test that is used in hypothesis testing to check whether the variance of two populations of two samples are equal or not.

Formula :

$$F = \frac{S_1^2}{S_2^2}$$

where $S_1^2 = \frac{\Sigma x_1^2}{n_1 - 1}$

$S_2^2 = \frac{\Sigma x_2^2}{n_2 - 1}$

where n_1 = no. of samples in 1st population.

n_2 = No. of samples in 2nd population.

significance :

Variance $v_1 = n_{high} - 1$

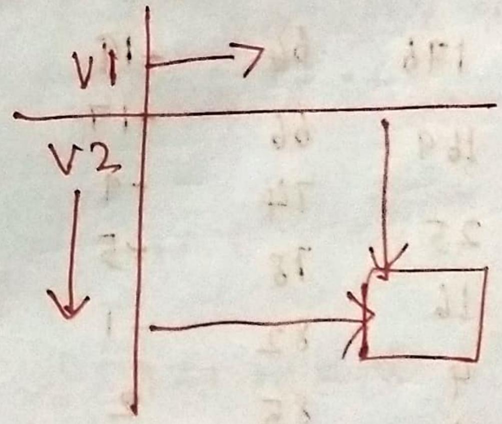
$v_2 = n_{low} - 1$

there n value of v_1 is always the sample with higher population.

Table value is calculated using F-Test

If CV < TV (Accept)

CV > TV (Reject)



Q)

Two random samples were draw from two normal population and their Values are :

A : 66 67 75 76 82 84 88 90 92

B : 64 66 74 78 82 85 87 92 93 95 97

Test whether the two population have same variance at 5% level of significance for $v_1 = 10$ & $v_2 = 8$.

soln:

A	$x_1 = X_1 - \bar{X}$	x_1^2 (913)	B	$x_2 = X_2 - \bar{X_2}$	x_2^2 (1298)
66	-14	196	64	-19	361
67	-13	169	66	-17	289
75	-5	25	74	-9	81
76	-4	16	78	-5	25
82	2	4	82	-1	1
84	4	16	85	2	4
88	8	64	87	4	16
90	10	100	92	9	81
92	12	144	93	10	100
			95	12	144
720		734	97	14	196

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{720}{9} = 80$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{913}{11} = 83$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1 - 1} = \frac{734}{\cancel{99}\ 9-1} = 91.75$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2 - 1} = \frac{1298}{11-1} = 129.8$$

$$F = \frac{s_1^2}{s_2^2} = \cancel{0.707}$$

$$= \frac{91.75}{129.8} = \cancel{0.707} = 1.415$$

$$v_1 = 11 - 1 = 10 \quad (x \text{ axis})$$

$$v_2 = 9 - 1 = 8. \quad (y \text{ axis})$$

Table Value = 3.347.

C.V   T.V.

$\cancel{0.707} < 3.347$ Accept H0.

1.415