The maximum flow problem:-

→ Maximum flow is a maximum feasible flow b/w source & sink.
(written above "flow": → rate)

The Ford-fulkerson method:-

Terminology.

→ Its an iterative method by using 3 method.

(i) Residual network

(ii) Augmenting path.

(iii) Cuts.

Residual Network. Graph.

→ Suppose we have flow Network $G=(V,E)$ with S and t. Let every edge u,v is having a pair flow/capacity. then called Residual N/w.

$$\boxed{c_f(u,v) = c(u,v) - f(u,v)}$$

current flow.

Source → No incoming Node and only outward Node.

Sink → No outward only inward Node.

The given graph which Represents the flow Network where every edge has capacity Also given two vertices S & T in the graph. Find out the maximum possible flow From s to t with following constraints

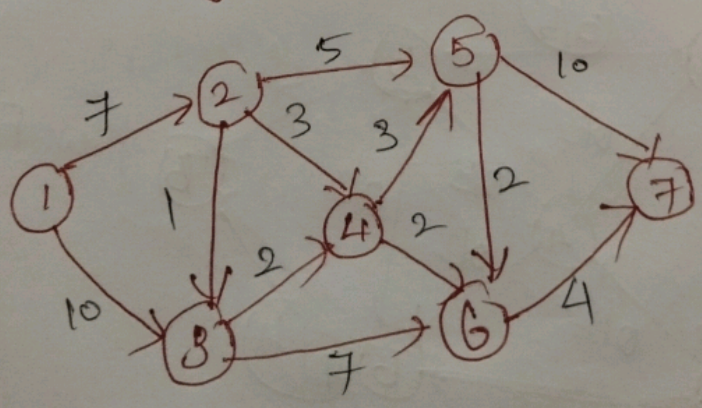
(i) Flow on an edge does not exit the given capacity of edge.

(ii) Inflow is equal to outflow for every vertex Except S & t.

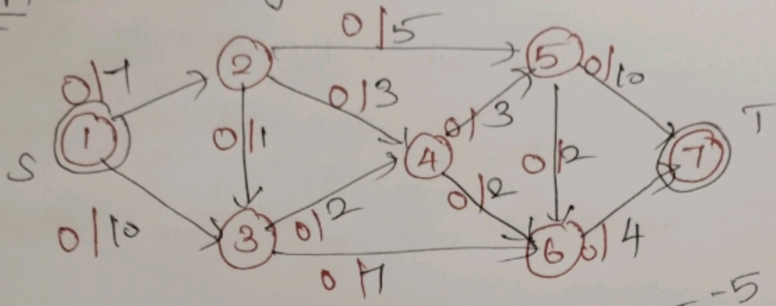
Algorithm:-

Start ~~with~~ a initial flow.

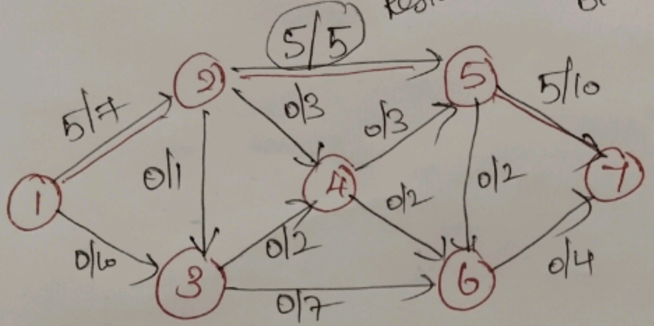
~~Both~~ Find the maximum flow through the given Network using ford fulkerson algorithm.



step1 Initial graph.



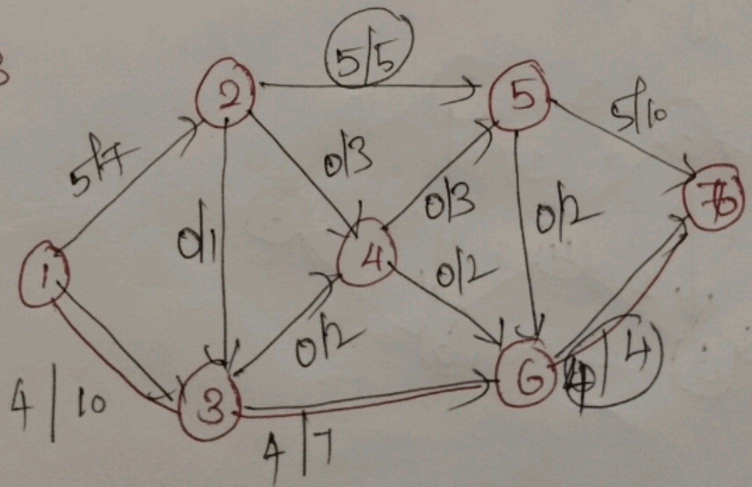
step2 choose path Residual graph 5-5 0 Block it.



Flow = 5

Augment path	Bottle neck capacity.
1-2-5-7	5
1→3-6-7	4

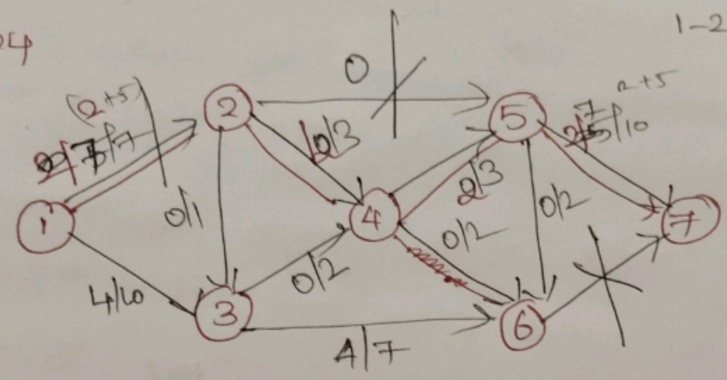
step3



flow = 5+4

Step4

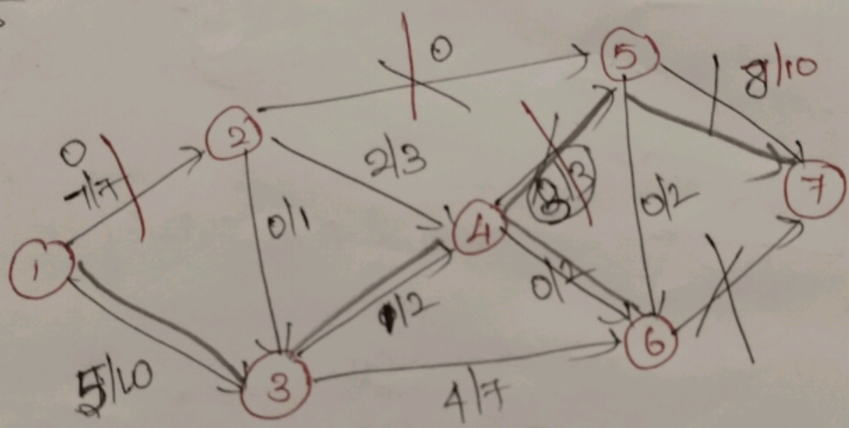
1-2-4-5-7 = 2



flow 5+4+2

Step5

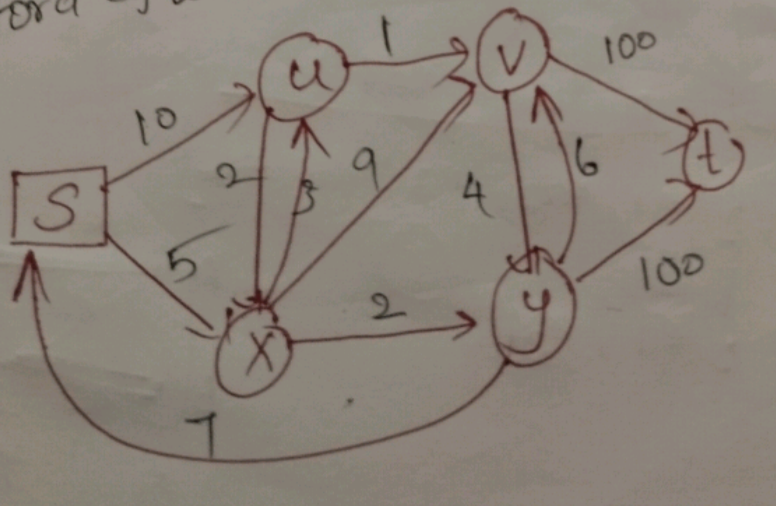
1-3-4-5-7 = 1



flow : 5+4+2+1

maximum flow through the given N/w using Ford-fulkerson method is $\boxed{5+4+2+1=12}$

Ex2



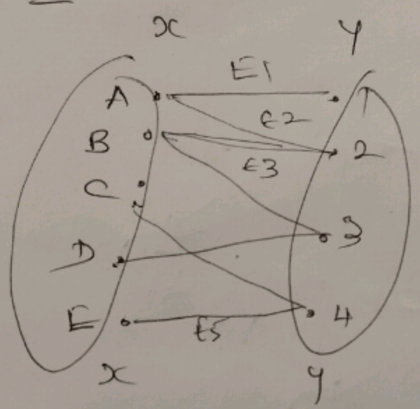
3+5=8

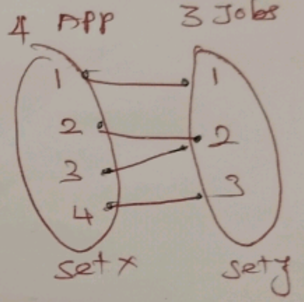
Maximum matching Bipartite graph:

→ Graph $G(V,E)$, in which the vertex set V is divided into 2 ~~distingue~~ Sets X & Y.
disjoint sets

→ Every edge of the graph has one end point in X and other end point in Y

Ex





$$V = \{\overset{X}{A,B,C,D,E} \,/\, \overset{Y}{1,2,3,4}\}$$

$$x \cup y = \{v\}$$
$$x \cap y = \text{Null.}$$

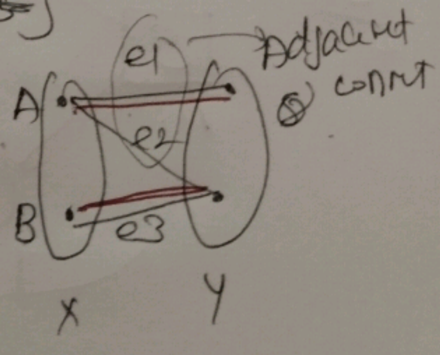
(*) Vertices of the same set are not Connected

Matching :- (M)

→ Matching in a graph is a Subsets of edge that no two edges share a Vertex

ex $E = \{e1, e2, e3, \cancel{e4, e5}\}$

$M = \{e1, e3\}$



Two Colorable graph

→ A graph that can be Colored only with 2 Colors

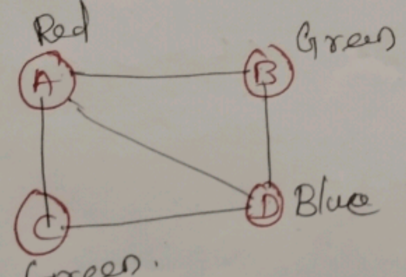
→ No two edges Connects the same colors.

→ Bipartite Graph is a 2 color graph.

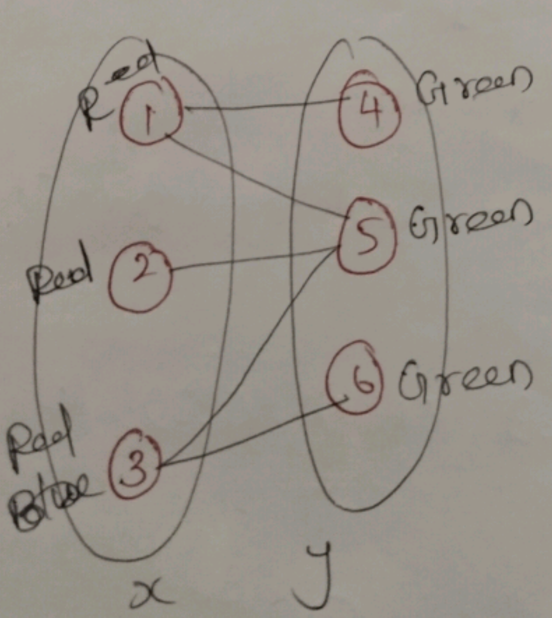
Graph Coloring:-

→ No 2 adjacent states will be of same Color.

Ex



→ minimum color.



only 2 two color

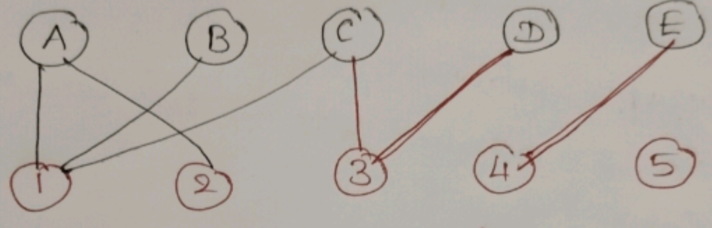
x y

Rule 1 → color → X
1 → color → Y.

Problem

Maxim matching problem.

Ex1 Let M be the matching in bipartite graph G.



$M_a = \{(D,3), (E,4)\}$

Soln

Find free vertex

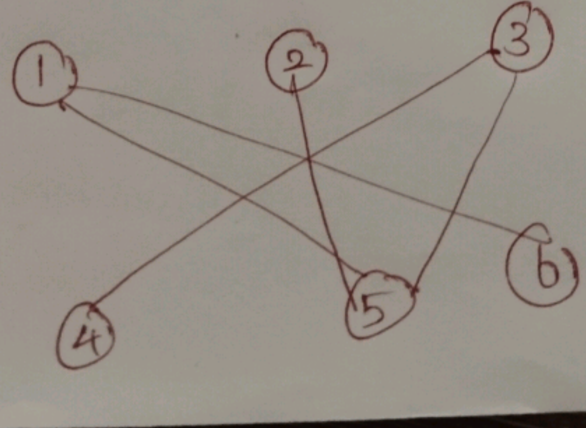
$x = \{A, B, C, D, E\}$

$y = \{1, 2, 3, 4, 5\}$

Consider vertex a; Mb:

Thus the maximum matching for the Bipartite graph is

$M = \{(A,2), (B,1), (C,3), (D,5), E,4)\}$.



UNIT - IV

Iterative improvement

Steps:-

1. Start with some feasible solutions
2. Repeat the following step until No improvement is found
3. Change the current feasible solution to a feasible solution with better value of objective function.
4. Return the last feasible solution as the optimal solution.

Applications:

1. Simplex Method
2. Ford - flukerson.
3. Maximum matching
4. Stable Marriage pbm.

1 Simplex Method:-

2m. Linear Programming Problem (LP):- is a pbm in which we have to find minimum or max values of linear objective function

Ex

$$\underbrace{x+y+z}_{\text{Decision variable}}=P$$

Ex Maximize $P = 2x + 3y + z$ subject to

$x + y + z \le 40,$ $\quad x \ge 0,\ y \ge 0,\ z \ge 0$

$2x + y - z \ge 10;$

$-y + z \ge 10$

Solution:-

Convert the given eqn into linear eqn. So

$\boxed{\text{Eliminate } \le \ \ge}$ introduce some slack variable.

$\ge \rightarrow (-)$

$\le \rightarrow (+)$ (add)

$s, t, u \rightarrow$ slack variable

$$x + y + z + s = 40 \longrightarrow (1)$$

$$2x + y - z - t = 10 \longrightarrow (2)$$

$$-y + z - u = 10 \longrightarrow (3)$$

Objective function is $P = 2x + 3y + z$

bring it left side of P

$$\boxed{-2x - 3y - z + P = 0}$$

(2)

Build initial table. (Based on what are call the variable which used us)

	x	y	z	s	t	u	p	RHS value
eq 1	1	1	1	1	0	0	0	40
eq 2	2	1	-1	0	-1	0	0	10
eq 3	0	-1	1	0	0	-1	0	10

For objective function.

$$-2x - 3y - z + p = 0$$

x	y	z	s	t	u	p	RHS.
-2	-3	-1	0	0	0	1	0

Find basic solution:

check which columns are cleared (i.e) all (should contain only one value in col)

s, t, u, p $\rightarrow$ active variables (clear col$_n$)

x, y, z $\rightarrow$ Inactive variables (

So write s, t, u, p one Left size of table.

So

	x	y	z	s	t	u	p	RHS
s —	1	1	1	1	0	0	0	40
* t —	2	1	-1	0	-1	0	0	10
* u —	0	-1	1	0	0	-1	0	10
* p —	-2	-3	-1	0	0	0	1	0

For active variable check <u>Test ratio</u>

$s \rightarrow \frac{\text{RHS}}{\text{clear colm value}} = \frac{40}{1} = 40$ +ve value (consider least +ve value)

$u \rightarrow \frac{10}{-1} = -10$

$t = \frac{10}{-1} = -10$

$p = \frac{0}{1} = 0$

Other than positive values, mark (*) for the variable in the above table. [check on previous tbl]

Step3: Eliminate stars:-

condition.

* In t, u, Find largest postive value to } Except RHS.
* In P find Least negative value.

Choose pivot column.

	x	y	z	s	t	u	p	RHS	
s	1	1	1	0	0	0	0	40	R_1
* t	2	1	-1	0	-1	0	0	10	R_2
* u	0	-1	1	0	0	-1	0	10	R_3
* p	-2	-3	-1	0	0	0	1	0	R_4

pivot also it may varry

In row 2, 't' x is the maximum value,

So column x is the pivot column.

choose pivot element from pivot column using test ratio-

So

$\frac{40}{1} = 40$, $\frac{10}{2} = 5$, $\frac{10}{0} = \infty$, $\frac{0}{-2} = 0$ choose. [Least +ve value)

The pivot element is 2, so other than the pivot element, all values called pivot column. then made '0'

pivot

$R_1 = 2R_1 - R_2$ ⟹ $R_1 = 2R_1 + (+R_2)$

$R_4 = R_2 + R_4$

+2 (−2)

−2
1
2 −4

	x	y	z	s	t	u	P	RHS
S	0	1	3	2	1	0	0	
t	2							
* u	0							
* P	0							

eliminate *

x → entering variable (active variable).

t → departing variable.

Simplex Method

Maximize Minimize Method.

Maximize Method:

Ex) maximize $Z = 5x_1 + 7x_2$ Object to

Constraints
$$\begin{cases} x_1 + x_2 \leq 4 \\ 3x_1 + 8x_2 \leq 24 \\ 10x_1 + 7x_2 \leq 35, \quad x_1, x_2 \geq 0 \end{cases}$$

Soln

Let slack variable be S_1, S_2, S_3

$x_1, x_2, S_1, S_2 \geq 0$

$x_1 + x_2 + S_1 = 4$

$3x_1 + 8x_2 + S_2 = 24$

$10x_1 + 7x_2 + S_3 = 35$

$\leq \Rightarrow +$

$\geq \Rightarrow (-)$

slack vs S_1, S_2

The standard Linear programming Problem is

Maximize $Z = \underbrace{5x_1 + 7x_2}_{Cj} + \underbrace{0S_1 + 0S_2 + 0S_3}_{CBi}$

Step 1: Initial table

$\boxed{\text{key Pivot} = 8}$

coeff of obj → LPB

Pivot column

CBi	Cj	5	7	0	0	0	Solution (RHS)	Ratio RHS / PC value
	BV	x_1	x_2	S_1	S_2	S_3		
0	S_1	1	1	1	0	0	4 = 4	$\frac{4}{1} = 4$
0	S_2	3	8	0	1	0	24	$\frac{24}{8} = 3$ min +ve value
0	S_3	10	7	0	0	1	35	$35/7 = 5$
	Zj	0	0	0	0	0	0	
	$Zj - Cj$	-5	-7	0	0	0		

slack variable coeff

(-ve) Max

Choose CBi most -ve

$x_1, x_2 \geq 0$

key value

change

Max

0 should not select

$z_j = (0 \times 0) + 0 \times$

$x_2 \rightarrow$ Entering variable

$S_2 \rightarrow$ Leaving variable.

Step 2 : First iterative table :- [check previous table Based on)

$k = 5/8$ Pivot

CB_i	C_j	5	7	0	0	0	Soln.	Ratio
	BV	x_1	x_2	S_1	S_2	S_3	RHS	Min.
0	S_1	$5/8$	0	1	$-1/8$	0	1	$\frac{1}{5/8} = 8/5$ 1.6
7	x_2	$3/8$	1	0	$1/8$	0	$\frac{24}{8} = 3$	$\frac{3}{3/8} \Rightarrow 8$
0	S_3	$59/8$	0	0	$-7/8$	1	14	$\frac{14}{59/8}$ $14 \times \frac{8}{59} = \frac{112}{59}$
	z_j	$0 + \frac{21}{8} + 0$ $21/8$	$0 + 7 + 0$ 7	0	$7/8$	0		
	$z_j - c_j$	$\frac{21}{8} - 5$ $\boxed{-19/8}$	0	0	$7/8$	0		if points case

$\frac{21-5}{8/40}$ $\frac{21-40}{8}$

Calculations take all the values from previous table.

Old value $S_1 = 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 4$

$(-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-)$ $\Big\}$ $S_1 - x_2$

New value $x_2 = \frac{3}{8} \quad 1 \quad 0 \quad \frac{1}{8} \quad 0 \quad 3$

$S_1 = 1 - \frac{3}{8} \quad 0 \quad 1 \quad \frac{-1}{8} \quad 0 \quad 1$

$5/8$

0 so multiply $\boxed{7 \times x_2}$

$S_3 = 10 \quad 7 \quad 0 \quad 0 \quad 1 \quad 35$

$(-) \quad (-) \quad (-) \quad (-) \quad (-) \quad (-)$ $\Big\}$ $S_3 - x_2$

$x_2 = \frac{21}{8} \quad 7 \quad 0 \quad 7/8 \quad 0 \quad 21$

$10 - 21/8 \quad 0 \quad 0 \quad -7/8 \quad 1 \quad 14.$

Pivot = Entering Leaving calc

Pivot = 5/8

Entering variable = x_1

Leaving variable = s_1.

calculations:-

Step 8: Second iteration table.

CB_i	C_j / BV	5 / x_1	7 / x_2	0 / s_1	0 / s_2	0 / s_3	Soln (RHS)	Ratio
5	x_1	1	0	8/5	$-\frac{1}{8}\times\frac{8}{5}$ = -1/5	0	8/5	x_1
7	x_2	0	1	-3/5	1/5	0	12/5	x_2
0	s_3	0	0	$\frac{-59}{5}$	$\frac{6}{10}$	1	11/5	
z_j		5	7	$\frac{40}{5}-\frac{21}{5}$ = 19/5	6/5	0	$40/5+\frac{84}{5}$ $\boxed{124/5}$	
$z_j - c_j$		0	0	19/5	6/5	0		

Key =

No Negative value with 0

No Need to find Ratio

Calculation

old x_2 = 3/8   1   0   1/8   0 $\frac{24}{8}$ 3 } x_2 ...

(−)   (−)   (−)   (−)   (−)   (−)

New $\frac{3}{8} \times x_1$ = 3/8   0 $\frac{3}{8}\times\frac{8}{5}$ $\frac{3}{8}\times -\frac{1}{5}$ 0 $\frac{3}{8}\times\frac{8}{5}$

0   1   -3/5 $\frac{1}{8}+\frac{8}{40}$ 0 $3-\frac{3}{5}=\frac{15-3}{5}$

$=\frac{5+3}{40}=8/40$ $\frac{12}{5}$

1/5.

Simplex Method [Minimize] First choose leaving variable. (1)

Convert into maximize form. (ie) the given Objective function multiply with (−)

Ex

Min. $z = 3x_1 + 2x_2 + x_3$

subject to $3x_1 + x_2 + x_3 \geq 3$

$-3x_1 + 3x_2 + x_3 \geq 6$, $x_1 + x_2 + x_3 \leq 3$

$x_1, x_2, x_3 \geq 0$

Soln

multiply (−) maximize $z = -3x_1 - 2x_2 - x_3$

$\boxed{\geq \Rightarrow \leq}$

$-3x_1 - x_2 - x_3 \leq -3$

$3x_1 - 3x_2 - x_3 \leq -6$ } equality form

$x_1 + x_2 + x_3 \leq 3$ → As it is because it already $\leq$

then as it is proceed with slack variables.

$z = -3x_1 - 2x_2 - x_3 + 0s_1 + 0s_2 + 0s_3$

Step 1: Initial table

Find Ratio only for ≤ 0 value

① modulus ≥ |$\frac{z_j - c_j}{\text{pivot row value}}$|

	C_j	−3	−2	−1	0	0	0	Soln RHS
C_{Bi}	BV	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	−3	−1	−1	1	0	0	−3
0	s_2	3	−3	−1	0	1	0	−6 ← max (−ve) → Pivot Row
0	s_3	1	1	1	0	0	1	3
z_j	z_j	0	0	0	0	0	0	
	$z_j - c_j$	+3	+2	1	0	0	0	
	Find +ve value Ratio	−	$\left\lvert\frac{2}{-3}\right\rvert = 2/3$	$\left\lvert\frac{1}{-1}\right\rvert = 1$	−	−	−	{ mini +ve value

Entering Variable = x_2
Leaving Variable = S_2
Pivot or key = -3

Step 2

CB_i	C_j / BV	-3 / x_1	-2 / x_2	-1 / x_3	0 / S_1	0 / S_2	0 / S_3	Soln RHS
0	S_1	-4	0	$-2/3$	1	$-1/3$	0	-1
-2	x_2 ~~-3~~	-1	1	$+1/3$	0	$-1/3$	0	2
0	S_3	2	0	$2/3$	0	$1/3$	1	1
Z_j		2	-2	$-2/3$	0	$2/3$	0	
$Z_j - C_j$		$2-(-3)$ 5	$-2+2$ 0	$-2/3+1$ $=1/3$	0	$2/3$	0	
Ratio		$\frac{5}{-4}=-5/4$	$\frac{1}{-}$	$\frac{1/3}{-2/3}=-1/2$	$-$	$\frac{2/3}{-1/3}=-2$	$-$	

min -ve ← Ratio

To find S_1

Old S_1 ⇒ -3 -1 -1 1 0 0 -3
(+)
new x_2 ⇒ -1 1 $1/3$ 0 $-1/3$ 0 2

S_1 = -4 0 $\frac{-2}{3}$ 1 $-1/3$ 0 -1

To find S_3

S_3 ⇒ 1 1 1 0 0 1 3
(−)
x_2 ⇒ -1 1 $1/3$ 0 $-1/3$ 0 2

S_3 2 0 $2/3$ 0 $1/3$ 1 1

Entering Variable = x_3

Leaving Variable = S_1

Pivot (or) key = $-2/3$

Step3 :- Second iteration.

C_{B_1}	C_j / BV	-3 / x_1	-2 / x_2	-1 / x_3	0 / S_1	0 / S_2	0 / S_3	Soln (RHS)
-1	$\cancel{x_3}$ x_3 $\frac{-2}{3}$	6	0	1	$\frac{-3}{2}$	$1/2$	0	$3/2$
-2	x_2	-3	1	0	$1/2$	$-1/2$	0	$3/2$
0	S_3	-2	0	0	1	0	1	0
Z_J		$-6+6$ $=0$	-2	-1	$3/2-1$ $=1/2$	$-1/2+1$ $=1/2$	0	$-3/2+-3+0$ $=\frac{-9}{2}$
$Z_J - C_J$		3	0	0	$1/2$	$1/2$	0	
Ratio								

NO (-ve) value

to max

To find x_2

$x_2 \Rightarrow$ -1 1 $1/3$ 0 $-1/3$ 0 2

$x_3 \Rightarrow$ $6\times 1/3$ 0 $1\times 1/3$ $-3/2\times 1/3$ $1/2$ 0 $3/2$

$1/3 \times x_3$ -3 1 0 $1/2$ $-1/2$ 0 $3/2$

To find S3:-

S3 :	2	0	2/3	0	1/3	1	1
$(2/3 \times x_3)$ x_3	6	0	1	-3/2	1/2	0	3/2
	-2	0	0	1	0	1	0

$x_1 \Rightarrow 3/2 \quad x_2 = 3/2 \quad x_3 = 0$

Sub in Z.

$Z = -3x_1 \cancel{+} 2x_2 \cancel{+} x_3$

$= 3(3/2) \bar{+} 2(3/2) \bar{+} 0$

$= \frac{-9}{2} - 6/$

$-3x_1 - 2x_2 - x_3$

$= -3(3/2) - 2(3/2)$

$= \frac{-9}{2} - 6/2$

$= \frac{-9-6}{2} = -15/2$

(x)

$\boxed{\text{minimize} = \text{maximize} = (-z)}$

$= -(-9/2)$

$\boxed{z = 9/2}$ $x_1 = 0, \ x_2 = 3/2, \ x_3 = 3/2$

$z = 3x_1 + 2x_2 + x_3$

$= 3(3/2) + 2(3/2)$

$= \frac{9}{2} + 6/2$

The stable marrige problem:

→ find stable matching between two sets (Men & women) with various preferense to each other.

→ Consider two sets

$M = \{m_1, m_2, \cdots m_n)$ of n men.

$W = \{w_1, w_2, \ldots w_n\}$ of n women.

→ Each man has preferena list ordering the women as potential Marriage partner with no ties. same for women.

→ find out marriage ~~or~~ matching pair (m, w).

Ex

Mens Preferense

Bob:	Lea	Ann	Sue
Jim:	Lea	Sue	Ann
TOM:	Sue	Lea	Ann

woman's preferens.

Ann:	Jim	TOM	Bob
Lea:	TOM	bob	Jim
Sue:	Jim	TOM	Bob

Man optimal.

	Ann	Lea	Sue
Bob:	2,3	$\boxed{1,2}$	3,3
JIM:	3,1	1,3	2,1
TOM:	3,2	2,1	1,2

Bob proposed to Lea, accepted.

	Ann	Lea	Sue
Bob :	2,3	$\boxed{1,2}$	3,3
Jim :	3,1	$\boxed{1,3}$	2,1
Tom :	3,2	2,1	1,2

Jim proporsal to Lea.
rejects Jim. proposal

	Ann	Lea	Sue
Bob :	2,3	$\boxed{1,2}$	3,3
Jim :	3,1	1,3	$\boxed{2,1}$
Tom :	3,2	2,1	1,2

→ Jim proposed to Sue, She accepted

	Ann	Lea	Sue
Bob	2,3	$\boxed{1,2}$	3,3
Jim	3,1	1,3	$\boxed{2,1}$
Tom	3,2	2,1	1,2

TOM proposal to Sue, But rejected.

	Ann	Lea	Sue
Bob	2,3	$\boxed{1,2}$ ×	3,3
JIM	3,1	1,3	$\boxed{2,1}$
Tom	3,2	$\boxed{2.1}$	1.2

TOM proposal to Lea
Lea accepted, replaced

(2)

	Ann	Lea	Sue
Bob	$\boxed{2,3}$	1,2	3,3
Jim	3,1	1,3	$\boxed{2,1}$
Tom	3,2	$\boxed{2,1}$	1,2

→ bob proposed to Ann. She accepted

{(Bob, Ann), (Jim, Sue), (Tom, Lea)}

Unstable Marriage – Blocking pair (Bob, Lea)

Women Optimal

	Ann	Lea	Sue
Bob :	2,3	1,2	3,3
Jim :	$\boxed{3,1}$	1,3	2,1
Tom :	3,2	2,1	1,2

→ Ann Propose to Jim accepted.

	Ann	Lea	Sue
Bob :	2,3	1,2	3,3
Jim :	$\boxed{3,1}$	1,3	2,1
Tom :	3,2	$\boxed{2,1}$	1,2

Accepted.

	Ann	Lea	Sue
Bob	2,3	1,2	3,3
Jim	$\boxed{3,1}$	1,3	$\boxed{2,1}$
TOM	3,2	$\boxed{2,1}$	1,2

Sue proposing to Jim. Jim accepted

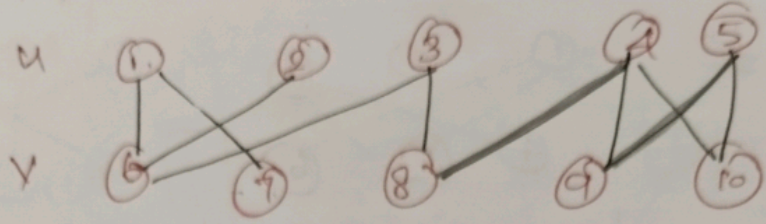
	Ann	Lea	Sue
Bob	$\boxed{2,3}$	1,2	3,3
Jim	3,1	1,3	$\boxed{2,1}$
TOM	3,2	$\boxed{2,1}$	1,2

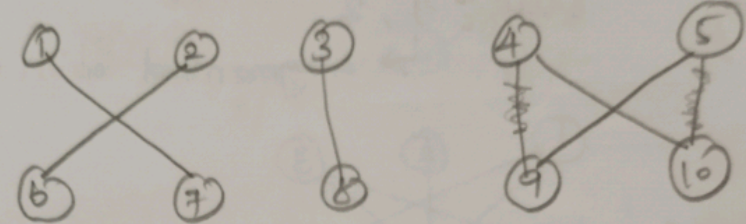
Ann proposing to Bob accepted.

Blocking pair { JIM, ann }

pair - {(Bob, Ann), (Jim, Sue) (TOM, Lea)}

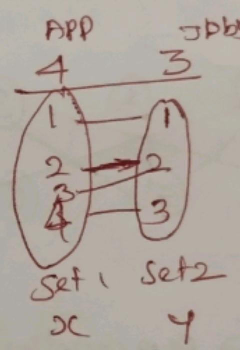
Ex2





Maximum matching

Pair of 2 vertices from 2 diff sets m app to n jobs.



Bipartite Graph:

* $G = (V, E)$, in which the Vertex set V is not same divided into 2 disjoint sets X & Y $V = \{(X), (Y)\}$ $E =$

Every edge of the Graph has one end point in X & other end point in Y

Property

$X \cup Y \to \{V\}$ Ex

$X \cap Y \to \phi$

disjoint set of vertices.

X	Y
A	1
B	2
C	3
D	
E	4

$V = \{\underbrace{(A\ B\ C\ D\ E)}_{X}, \underbrace{(1, 2, 3, 4)}_{Y}\}$

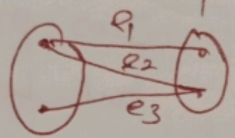
$E =$

* Vertices of the same set are not connected

X (A, B

(M) <u>Matching</u> in a graph is a "subset of edges" that no two edges share a vertex.

~~Ex~~= $E = \{e_1, e_2, e_3, e_4, e_5\}$
$M = \{e_1, e_3\}$



$E = \{e_1, e_2, e_3\}$
$M = \{e_1, e_3\}$

~~TODO~~ .

———— x ————

Stable Marriage Problem:-

Consider an instance of the stable marriage Problem. Consider there ~~a~~ ~~4~~ 3 men & 3 women. men name as ( ) & women named as.

Mens pref				Women's pref			
Bob:	Lea	An	Sue	Ann:	Jim	Tom	Bob
Jim:	Lea	Sue	Ann	Lea:	Tom	bob	Jim
Tom:	Sue	Lea	Ann	Sue:	Jim	Tom	Bob