Process control

- control charts
  - Variables
    - $\bar{X}$-chart
    - σ-chart
    - R-chart
  - Attributes
    - P-chart
    - np-chart
    - c-chart

control charts for variables

upper control limit (UCL) $= \mu + 3\left[\frac{\sigma}{\sqrt{n}}\right]$

lower control limit (LCL) $= \mu - 3\left[\frac{\sigma}{\sqrt{n}}\right]$

Procedure to draw $\bar{X}$-R chart

1) Find $\bar{X} = \frac{\Sigma X_i}{N}$

2) Find $\bar{R} = \frac{\Sigma R_i}{N}$

3) Find sample size $= n$.

4) Find A_2, D_3, D_4 using table of size n.

5) For $\bar{X}$:

$$CL = \frac{\Sigma X_i}{N}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

$$UCL = \bar{\bar{X}} + A_2\bar{R}$$

$\bar{x} \to$ mean

$R_i \to$ range

6) For R

$$CL = \bar{R}$$

$$LCL = D_3 \bar{R}$$

$$UCL = D_4 \bar{R}$$

7) Draw the graph for $\bar{X}$ & R chart

8) Conclusion

→ It is not in control

(or)

→ It is in control

Attributes

P-chart, np-chart, c-chart

i) P-chart for proportion of defectives - for sample of varying size.

ii) nP-chart for number of defectives - for constant sample size.

iii) C-chart for number of defects per unit.

Procedure to draw c-chart

C-chart

$$\bar{c} = \frac{\text{Number of defects in all units inspected}}{\text{Total number of units inspected}}$$

$\therefore \sigma^2 = \bar{c}$

&

$\sigma = \sqrt{c}$

$\therefore UCL_c = \bar{c} + 3\sigma$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} \text{ where } \bar{c} = \frac{c_1 + c_2 + \cdots + c_R}{R}.$$

$\therefore c_i$ is the no. of defects in i^{th} unit.

i) Find $\bar{c} = \frac{\sum c_i}{N}$

ii) Find $CL = \bar{c}$

iii) Find $UCL_c = \bar{c} + 3\sigma$ (or) $UCL_c = \bar{c} + 3\sqrt{\bar{c}}$

iii) Find $LCL_c = \bar{c} - 3\sigma$ (or) $LCL_c = \bar{c} - 3\sqrt{\bar{c}}$

iv) Draw the graph using UCL, CL & LCL

v) Conclusion

→ It is not in control.

(or)

→ It is in control.

P-Chart

$$P = \frac{\text{Number of defectives in the sample}}{\text{Sample size}}$$

$$\bar{P} = \frac{\sum p}{\text{Number of samples}}$$

(or)

$$\bar{P} = \frac{\text{Total no. of defectives in all samples}}{\text{Total no. of items inspected in all samples}}$$

$$UCL_p = \bar{P} + 3\sigma_p = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$$

$$LCL_p = \bar{P} - 3\sigma_p = \bar{P} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$$

∴ n is the sample size (if constant)

p-chart when sample size is varying:-

Method 1:-

$$UCL_p = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$LCL_p = p - 3\sqrt{\frac{p(1-p)}{n}}.$$

Method 2:-

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

Procedure to draw p-chart

i) Find p

ii) Find $\bar{p}$ $\quad \therefore CL_p = \bar{p}$

iii) Find $UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

iv) Find $LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

v) Draw the graph using UCL_p, CL_p & LCL_p

vi) Conclusion.

-> It is not in control.

(or)

-> It is in control.

np - chart

$$Y = n\bar{p} = \frac{\text{Total no. of defectives of all samples}}{\text{Total no. of samples inspected.}}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}.$$