Date / /
Page

Bionomial distribution :

W.K.T

$$p(x) = nC_x \, p^x q^{n-x} \quad , x = 0,1,2..n$$
$$\& \; q = 1-p$$

$$m.g.f = M_x(t) = E(e^{tx})$$

$$= \sum_{x=0}^{n} e^{tx} \cdot p(x)$$

$q^{n-x} = q^n \cdot q^{-x}$

$e^{tx} = (e^t)^x$

$$= \sum_{x=0}^{n} e^{tx} \cdot nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} (e^t)^x \, nC_x \, p^x \, \frac{q^n}{q^x}$$

$$= \sum_{x=0}^{n} nC_x \left(\frac{e^t p}{q}\right)^x q^n$$

$$= q^n \sum_{x=0}^{n} nC_x \left(\frac{e^t p}{q}\right)^x$$

$$= q^n \left[nC_0 \left(\frac{e^t p}{q}\right)^0 + nC_1 \left(\frac{e^t p}{q}\right)^1 + nC_2 \left(\frac{e^t p}{q}\right)^2 + \dots + nC_n \left(\frac{e^t p}{q}\right)^n \right]$$

Date $\therefore {}^nC_n = 1$
Page $\therefore {}^nC_0 = 1$

$$= q^n\left[1 + {}^nC_1\left(\frac{e^t p}{q}\right)^1 + {}^nC_2\left(\frac{e^t p}{q}\right)^2 + \cdots\cdots + \left(\frac{e^t p}{q}\right)^n\right]$$

$(\because 1 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + x^n) = (1+x)^n$.

$$= q^n\left(1 + \frac{pe^t}{q}\right)^n$$

$$= q^n\left[\frac{q+pe^t}{q}\right)^n$$

$$= \cancel{q^n}\left[\frac{(q+pe^t)^n}{\cancel{q^n}}\right]$$

$\frac{d}{dx}e^x = e^x$

$$M_x(t) = (q + pe^t)^n$$

Mean : $E(x) = \left\{\frac{d}{dt}(M_x(t))\right\}_{t=0}$

$$= \left\{\frac{d}{dt}[q+pe^t]^n\right\}_{t=0}$$

$\frac{d(x^n)}{dx} = nx^{n-1}$

$$= \left\{n(q+pe^t)^{n-1} \times (0 + pe^t)\right\}_{t=0}$$

$$= \left[np\ e^t\ (q+pe^t)^{n-1}\right]_{t=0}$$

$$= np\ \underbrace{e^0}_{1}\ (\underbrace{q+pe^0}_{1})^{n-1}$$

$$E(x) = np$$

$\frac{d}{}(uv) = u \cdot v' + v\, u'$

Date

Page

$Var(x) = E(x^2) - (E(x))^2$

$E(x^2) = \left\{ \frac{d^2}{dt^2} M_x(t) \right\}_{t=0}$

$= \left\{ \frac{d}{dt} \left[\frac{dt}{dt} M_x(t) \right] \right\}_{t=0}$

$= \left\{ \frac{d}{dt} \left[np\, e^t (q + pe^t)^{n-1} \right] \right\}_{t=0}$

$= \left\{ np \frac{d}{dt} \left(\underbrace{e^t}_{u} \underbrace{(q + pe^t)^{n-1}}_{v} \right) \right\}_{t=0}$

$= \left\{ np \left[e^t (n-1 (q + pe^t)^{n-2} * (pe^t) + (q+pe^t)^{n-1} \times e^t \right] \right\}_{t=0}$

$= \left\{ np \left[e^0 (n-1)(q + pe^0)^{n-2} (pe^0) + (q+pe^0)^{n-1} \times e^0 \right] \right\}_{t=0}$

$= np \left[(n-1)(q+p)^{n-2}\, p + (q+p)^{n-1} \right)$

$= np\, (n-1)p + 1)$ $\quad p+q=1$

$q = 1-p.$

$= np\, (np - p + 1)$

$= np\, [np + q]$

$E(x^2) = n^2p^2 + npq$

$Var(x) = E(x^2) - [E(x)]^2$

$= \cancel{n^2p^2} + npq - \cancel{n^2p^2}$

$= npq$

Eg:

1) A machine Manufacturing screws is Known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are
i) exactly 3 defectives
ii) not more than 3 defectives.

Soln:

$p = 5\% = \frac{5}{100} = 0.05$

$\therefore q = 1 - p = 1 - 0.05 = 0.95$

$n = 15$.

$p(x) = nC_x \, p^x q^{n-x} = 15C_x (0.05)^x (0.95)^{15-x}$

i) p[exactly 3 defectives] $= p[x=3]$

$= 15C_3 \, (0.05)^3 \, (0.95)^{12}$

$= 0.0307$

15 0.445

Date
Page

ii) $p(x \leq 3)$

$= p(x=0) + p(x=1) + p(x=2) + p(x=3)$.

$= 15C_0 (0.05)^0 (0.95)^{15} +$
$15C_1 (0.05)^1 (0.95)^{14} +$
$15C_2 (0.05)^2 (0.95)^{13} +$
$15C_3 (0.05)^3 (0.95)^{12}$

$= 0.9945$

Q) If x follows $B(3, \frac{1}{3})$ and Y follows $B(5, \frac{1}{3})$. find $p(x+Y \geq 1)$

$B(n,p)$

soln :

given, $n_1 = 3$, $n_2 = 5$

$p = \frac{1}{3}$ $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore X + Y$ follows $B(n_1 + n_2, p)$

Date
Page

$x = 0, 1, 2 \ldots 8$

Let $X = x + y$

$\therefore$ X follows $B(8, \frac{1}{3})$

$$p(x) = nCx \; p^x \; q^{n-x}$$

$$p(x) = 8Cx \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{8-x}$$

$$\therefore p(x+y \geq 1) = p(X \geq 1)$$

$$= 1 - p(X < 1)$$

$$= 1 - p(X = 0)$$

$$= 1 - 8C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{8-0}$$

$$= 1 - \left(\frac{2}{3}\right)^8 = 0.961$$

Q) The mean of a Binomial distribution is 20 & S.D is 4. Determine the parameters of the distribution.

soln:

given, Mean $= 20$ & S.D $= 4$

$\Downarrow$ $\Downarrow$

np $\sqrt{var} = S.D$

npq

$np = 20$ —— ①

Square on both sides

Date / /
Page

$Var = 16$

(ie) $npq = 16$ — ②

$20q = 16$ (∵ by ①)

$q = \frac{16}{20} = \frac{4}{5}$

$p = 1 - q$

$= 1 - 4/5$

$= 1/5$

put p in ① $\Rightarrow$

$n \times \frac{1}{5} = 20$

$n = 20 \times 5$

$n = 100$

Q) If the probability of success is $\frac{1}{4}$, how many trials are necessary in order that the probability of atleast one success is greater than $\frac{2}{3}$?

Date
Page

soln :

given , $p = 1/4 \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$

Find n value given that p[atleast one success] $> \frac{2}{3}$

$p(x) = nc_x\, p^x q^{n-x}$

$$p(x \geq 1) > \frac{2}{3}$$

$$[1 - p(x < 1)] > \frac{2}{3}$$

$$(i.e) \{1 - p(x = 0)\} > \frac{2}{3}$$

$$\Rightarrow 1 - nC_0\, p^0 q^{n-0} => \frac{2}{3}$$

$$\Rightarrow 1 - \left(\frac{3}{4}\right)^n > \frac{2}{3}$$

$$\Rightarrow 1 - \frac{2}{3} > \left(\frac{3}{4}\right)^n =$$

$$\Rightarrow \frac{1}{3} > \left(\frac{3}{4}\right)^n$$

$$(ii) \quad \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$0.3164 < 0.333$$

$n = 4$

Date / /
Page

Q.) The probability of a bomb hitting a target is 1/5. two bombs enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed?

soln :

$$p = \frac{1}{5}, \quad n = 6 \quad q = 1 - \frac{1}{5} = \frac{4}{5}$$

$$p(x) = nC_x \, p^x \, q^{n-x}$$

$$= 6C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{6-x}$$

$$p(x=2) = 6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{6-2}$$

$$= 6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$$

$$= 0.2458$$

Date
Page

Poisson Distribution :

proof :

W.K.T $p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2 \ldots \infty$

$$M_x(t) = E(e^{tx})$$

$$= \sum e^{tx} \cdot p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots = e^x$

$$= e^{-\lambda} \sum_{x=0}^{\infty} e^{tx} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} \left[\frac{(\lambda e^t)^0}{0!} + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \ldots \infty \right]$$

$$= e^{-\lambda} \left[1 + \frac{(\lambda e^t)}{1!} + \ldots \right]$$

$$M_x(t) = e^{-\lambda} \cdot e^{(\lambda e^t)}$$

Date $\frac{d}{dx}(e^{ax})$

Page

$e \times a$

$$\text{Mean} = E(x) = \left\{\frac{d}{dt}[M_x(t)]\right\}_{t=0}$$

$$= \left\{\frac{d}{dt}(e^{-\lambda} e^{\lambda e^t})\right\}_{t=0}$$

$$= \left\{e^{-\lambda} \frac{d}{dt}(e^{\lambda e^t})\right\}_{t=0}$$

$$= \left\{e^{-\lambda}\left[e^{\lambda e^t} \times \lambda \cdot e^t\right]\right\}_{t=0}$$

$$= \lambda e^{-\lambda}(e^{\lambda e^0} \cdot e^0)$$

$e^x \cdot e^y = e^{x+y}$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$E(x) = \lambda$$

$$\mathrm{Var}(x) = E(x^2) - [E(x)]^2 \quad \text{--- } (1)$$

$$E(x^2) = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0}$$

$$\left\{\frac{d}{dt}\left[\frac{d}{dt} M_x(t)\right]\right\}_{t=0}$$

$$\left\{\frac{d}{dt}\left[\cancel{e^{\lambda}} \cancel{[} e\right.\right.$$

$d(UV) = UV' + VU'$

$$= \left\{ \frac{d}{dt} \left[\lambda e^{-\lambda} e^{t} \; e^{\lambda e^{t}} \right] \right\}_{t=0}$$

$$= \left\{ \lambda e^{-\lambda} \frac{d}{dt} \left[\underset{u}{e^{t}} \; \underset{v}{e^{\lambda e^{t}}} \right] \right\}_{t=0}$$

$$= \lambda e^{-\lambda} \left[e^{t} \cdot (e^{\lambda e^{t}}) \times \lambda e^{t} + e^{\lambda e^{t}} \cdot e^{t} \right]_{t=0}$$

$$= \lambda e^{-\lambda} \left[e^{0} (e^{\lambda e^{0}}) \times \lambda e^{0} + e^{\lambda e^{0}} \times e^{0} \right\}$$

$$= \lambda e^{-\lambda} \left\{ e^{\lambda} * \lambda + e^{\lambda} \right\}$$

$$E(x^2) = \lambda^2 + \lambda$$

① $\Rightarrow$

$$= \lambda^2 + \lambda - (\lambda)^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$Var(a) = \lambda$$

Date
Page

A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day follows a poisson distribution with Mean 1.5 λ. calculate the proportion of days on which

i) Neither car is used and $p(x=0)$
ii) some demand is not fulfilled. $p(x>2)$

soln:

Let $x \to$ no. of demands for a car.

given mean $\lambda = 1.5$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

(ie) $$p(x) = \frac{e^{-1.5}(1.5)^x}{x!}$$

i) $p(x=0)$

$$= \frac{e^{-1.5}(1.5)^0}{0!} = 0.2231$$

Date / /
Page

ii) $p(x>2)$

$= 1 - p(x \le 2)$

$= 1 - [p(x=0) + p(x=1) + p(x=2)]$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - e^{-1.5}\left[1 + 1.5 + \frac{(1.5)^2}{2}\right]$$

$= 0.1912$

6) The number of monthly breakdown of a computer is a random variable having a poisson distribution with mean equal to (1.8). Find the probability that this computer will function for a month

1) without a breakdown
2) with only one breakdown
3) with atleast one breakdown
min

soln:

given $\lambda = 1.8$

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

Date / /
Page

(b) $p(x) = \frac{e^{-1.8} (1.8)^x}{x!}$

i) $p(x=0)$

$= \frac{e^{-1.8} (1.8)^0}{0!}$

$= 0.1652$

ii) $p(x=1)$

$= \frac{e^{-1.8} (1.8)^1}{1!}$

$= 0.1652 (1.8)$

$= 0.2975$

iii) $p(x \geq 1)$

$= 1 - p(x < 1)$

$= 1 - p(x=0)$

$= 1 - 0.1653 \quad = 0.8347$

Date / /
Page

Geometric Distribution:-

a) A & B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are $\frac{3}{5}$ P1 and P2 5/7 respectively. Find the probability that B will require more shots than A.

[soln :]

Let $X \rightarrow A$

$Y \rightarrow B.$

given,

$$P1 = \frac{3}{5} \quad \& \quad P2 = \frac{5}{7}$$

$$q_1 = 1 - P1 = 1 - \frac{3}{5} = \frac{2}{5}$$

$$q_2 = 1 - P2 = 1 - \frac{5}{7} = \frac{2}{7}$$

By,

$$p(x) = P_1 \, q_1^{x-1}, \quad x = 1, 2, \ldots \infty$$

$$p(y) = P_2 \, q_2^{y-1}, \quad y = 1, 2, \ldots \infty$$

Date $x^{m+n} = x^m \cdot x$
Page

(ii) $p(x) = \left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^{x-1}, \; x = 1, \ldots \infty$

$p(y) = \left(\frac{5}{7}\right)\left(\frac{2}{7}\right)^{y-1}, \; x = 1, \ldots \infty$

$\therefore$ p [B requires more trials to get his first success than A requires to get his first success]

$p[x = r$ and $y = r+1$ (or) $r+2, \ldots \infty]$

$= \sum_{r=1}^{\infty} \left(p(x=r) \cdot p(y = r+1 \text{ (or) } r+2 \text{ (or)} \ldots \infty) \right]$

$= \sum_{r=1}^{\infty} p(x=r) \cdot \sum_{k=1}^{\infty} p(y = r+k)$

$= \sum_{r=1}^{\infty} \left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^{r-1} \sum_{k=1}^{\infty} \left(\frac{5}{7}\right)\left(\frac{2}{7}\right)^{r+k-1}$

$= \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{7} \left[\sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \right] \left[\sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{r-1} \cdot \left(\frac{2}{7}\right)^{k} \right]$

$x^m \cdot y^m = (xy)^m$

$= \frac{3}{7} \sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \cdot \left(\frac{2}{7}\right)^{r-1} \sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{k}$

$= \frac{3}{7} \left[\sum_{r=1}^{\infty} \left(\frac{4}{35}\right)^{r-1} \right] \left[\frac{2}{7} + \left(\frac{2}{7}\right)^2 + \left(\frac{2}{7}\right)^3 \right]$

$\because \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

Date $(1-x)^{-1} = 1+x+x^2 + x^3 + \ldots$

Page

$$= \frac{3}{7}\left\{\left[1+\frac{4}{35}+\left(\frac{4}{35}\right)^2+\cdots\right]\left\{\left(\frac{2}{7}\right)\left[1+\left(\frac{2}{7}\right)+\left(\frac{2}{7}\right)^2+\cdots\right]\right\}\right\}$$

$$= \frac{3}{7}\times\frac{2}{7}\left\{\left(1-\frac{4}{35}\right)^{-1}\left(1-\frac{2}{7}\right)^{-1}\right\}$$

$$= \frac{6}{49}\left[\left(\frac{35-4}{35}\right)^{-1}\left(\frac{7-2}{7}\right)^{-1}\right]$$

$$= \frac{6}{\cancel{49}}\left[\frac{\cancel{35}^{7}}{31}\times\frac{\cancel{7}}{\cancel{5}}\right]$$

$$= \frac{6}{31}$$

Q) If the probability that an applicant for a drivers licence will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test

a) on the 4th trial and

b) less than 4 trails?

soln:

Date
Page

Let x denote the no. of trials required to achieve the first success.

$p = 0.8$, $q = 1 - p$
$= 1 - 0.8$
$= 0.2$

$$p(x) = pq^{x-1}, \quad x = 1, 2, \ldots \infty$$

(ii) $p(x) = (0.8)(0.2)^{x-1}$

a) $p(x = 4) = (0.8)(0.2)^{4-1}$
$= (0.8)(0.2)^3 = 0.0064$

b) $p(x < 4) = p(x = 1) + p(x = 2) + p(x = 3)$

$= (0.8)(0.2)^0 + (0.8)(0.2)^1 + (0.7)(0.2)^2$

$= 0.8 [1 + 0.2 + 0.04]$

$= 0.992$

Date
Page

Q) suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shoot on any one shot is 0.8.

i) what is the probability that the target would be hit on 6th attempt?

ii) What is the probability that it takes him less than 5 shots?

iii) What is the probability that it takes him an even number

given,

$$p = 0.8$$

$$q = 1 - p = 1 - 0.8 = 0.2$$

$$p(x) = pq^{x-1}$$

$$(i) \quad (0.8)(0.2)^{x-1}$$

$$\text{i) } P(x=6) = (0.8)(0.2)^{6-1}$$

$$= (0.8)(0.2)^5$$

$$= 0.000256$$

$(1-x)^{-1} = 1 + x + x^2 + \dots$

ii) $p(x < 5) = p(x=1) + p(x=2) + p(x=3) + p(x=4)$.

$= (0.8)(0.2)^0 + (0.8)(0.2)^1 + (0.8)(0.2)^2 + (0.8)(0.2)^3$

$= 0.9984$

iii) $p(x = 2, 4, 6, \dots)$

$= p(x=2) + p(x=4) + p(x=6) + \dots$

$= (0.8)(0.2)^1 + (0.8)(0.2)^3 + (0.8)(0.2)^5 + \dots$

$= (0.8)(0.2)\left[1 + (0.2)^2 + (0.2)^4 + \dots\right]$

$= (0.8)(0.2)\left[1 + (0.04) + (0.04)^2 + \dots\right]$

$= (0.8)(0.2)\left[1 - (0.04)\right]^{-1}$

$= \dfrac{0.16}{1 - 0.04}$

$= 0.1667$

Date __/__/__

Page ____