$$= \bar{x} - \frac{1}{\pm \frac{1}{\sqrt{s^2 - \bar{x}^2}}}$$

$$\mu = \bar{x} \mp \sqrt{s^2 - \bar{x}^2} \quad \text{———} \quad 8$$

---

iii) $Y - \bar{Y} = r \frac{\sigma x}{\sigma y} (x - \bar{x})$.

$Y - 38 = -0.664 (30 - 32)$.

$Y - 38 = -0.064 \times -2$

$Y - 38 = 1.328$

$Y = 38 + 1.328$

$Y = 39.33$

4)

$$E(x) = \bar{x} = \frac{\Sigma x}{n} = \frac{320}{10} = 32$$

$$E(y) = \bar{y} = \frac{\Sigma y}{n} = \frac{380}{10} = 38$$

$$E(xy) = \frac{\Sigma xy}{n} = \frac{12067 \; \cancel{10380}}{10} = 1206.7 \; \cancel{1038.0}$$

$$E(x^2) = \frac{\Sigma x^2}{n} = \frac{10380}{10} = 1038$$

$$E(y^2) = \frac{\Sigma y^2}{n} = \frac{14.838}{10} = 1483.8$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$= 1038 - \cancel{[illegible]} \; 1024.$$

$$= \cancel{8936} \; \cancel{[illegible]} \; 14$$

$$\sigma x = \sqrt{var(x)} = \cancel{\sqrt{8936} = 94.5304}$$

$$= \sqrt{14}$$

$$= 3.74$$

$$var(y) = E(y^2) - [E(y)]^2$$
$$= 1483.8 - 1444$$
$$= 39.8$$

$$\sigma y = \sqrt{var(y)} = \sqrt{39.8} = 6.308$$

$$r(x, y) = \frac{Cov(x, y)}{\sigma x \, \sigma y}$$

$$= \frac{E(xy) - E(x).E(y)}{\sigma x \, \sigma y}$$

$$= \frac{1206.7 - \overbrace{32 \times 38}^{1216}}{3.74 \times 6.308}$$

$$= \frac{-9.3}{23.5919}$$

$$r = -0.39$$ //

Regression line //

y on x:

$$y - \bar{y} = r \frac{\sigma y}{\sigma x} (x - \bar{x})$$

$$y - 38 = (-0.39) \frac{6.308}{3.74} (x - 32)$$

$$y - 38 = (-0.39)(1.687)(x - 32)$$

$$y - 38 = -0.6578 (x - 32)$$

$$y - 38 = -0.6578x + 21.0496$$

$$y = -0.6578x + 59.0496$$

x on y:

$$x - \bar{x} = r \frac{\sigma x}{\sigma y} (y - \bar{y})$$

$$x - 32 = (-0.39) \frac{3.74}{6.308} (y - 38)$$

$$x - 32 = (-0.39)(0.5928)(y - 38)$$

$$x - 32 = -0.2313 (y - 38)$$

$$x - 32 = -0.2313y + 8.7867$$

$$x = -0.2313y + 40.787$$

Memoryless property of exponential distribution

$$p(x > s+t \mid x > s) = p(x > t)$$ for any $s,t > 0$.

proof.

$$p(x > k) = \int_k^{\infty} f(x)\, dx.$$

$$= \int_k^{\infty} \lambda e^{-\lambda x}$$

$$= \lambda \int_k^{\infty} e^{-\lambda x}.$$

$$= \cancel{\lambda} \left[\frac{e^{-\lambda x}}{-\cancel{\lambda}} \right]_k^{\infty}$$

$$= \frac{e^{-\infty}}{-1} + \frac{e^{-\lambda k}}{+1} = e^{-\lambda k}$$

$p(x > k)$

$\therefore\ p(x > s+t) = e^{-\lambda(s+t)}$

$p(x > s) = e^{-\lambda s}$

Now,

$p(x > s+t \mid x > s) =$

$p(A|B) = \frac{p(A \cdot B)}{p(B)}$

$$= \frac{p(x > s+t \ \& \ x > s)}{p(x > s)}$$

$$= \frac{p(x > s+t)}{p(x > s}$$

$$= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$$

$$= \frac{e^{-\lambda s - \lambda t}}{e^{-\lambda s}}$$

$$= \frac{e^{\cancel{-\lambda s}} \cdot e^{-\lambda t}}{e^{\cancel{-\lambda s}}}$$

$= e^{-\lambda t} \Rightarrow p(x > t)$ //

7)

step1 :

H0 ; $\mu = 158$

H1 ; $\mu \neq 158$.

step2 :

Ascending order :

given, 166, 141, 136, 153, 170, 162, 155

146, 183, 157, 148, 132, 160, 175

150

AO :

132, 136, 141, 146, 148, 150, 153,

155, 157, 160, 162, 166, 170, 175

183.

$158 > = +$

$158 < = -$

+ − − − + + −

− + − − − + +

−

$x = 6$, $n = 15$. $\alpha = 0.05$.

Table value $= 0.3036$

$0.3036 > 0.05$
(T V)

~~H~~ Accepts H0.

11)

S	1	2	3	4	5	6	7	8	9	10
deffrus	4	3	2	3	4					

Samples :	1	2	3	4	5	6	7	8	9	10
defectives :	2	1	1	2	3	5	5	1	2	3
fraction	0.04	0.02	0.02	0.04	0.06	0.1	0.1	0.02	0.04	0.06

$$\bar{p} = \frac{\Sigma p_i}{N} = \frac{0.5}{10} = 0.05$$

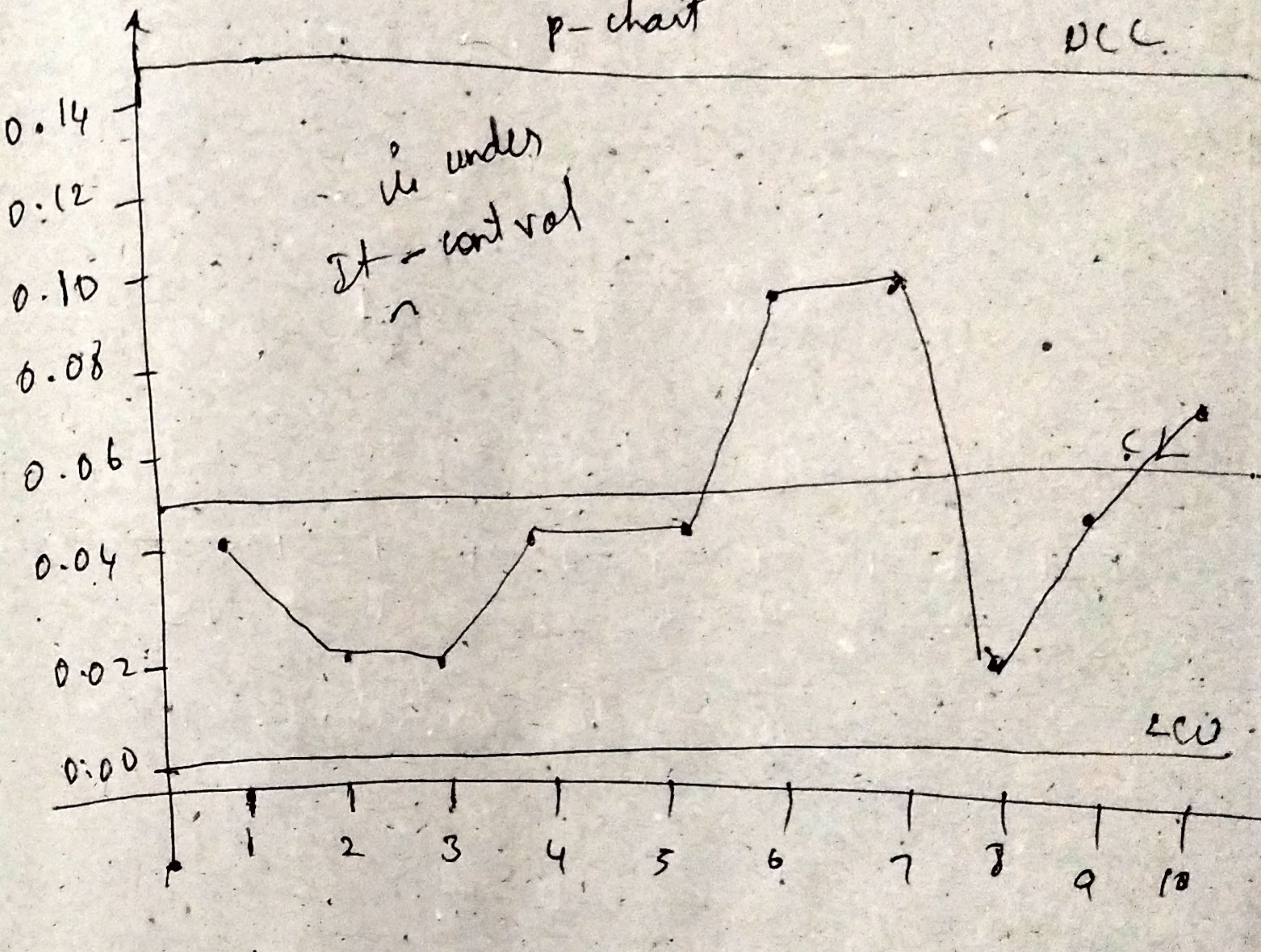
step 2 :

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.05 - 3\sqrt{\frac{0.05(1-0.05)}{50}} = 0$$

$$UCL = 0.05 + 3\sqrt{\frac{0.05(1-0.05)}{50}}$$

$$= 0.1425.$$

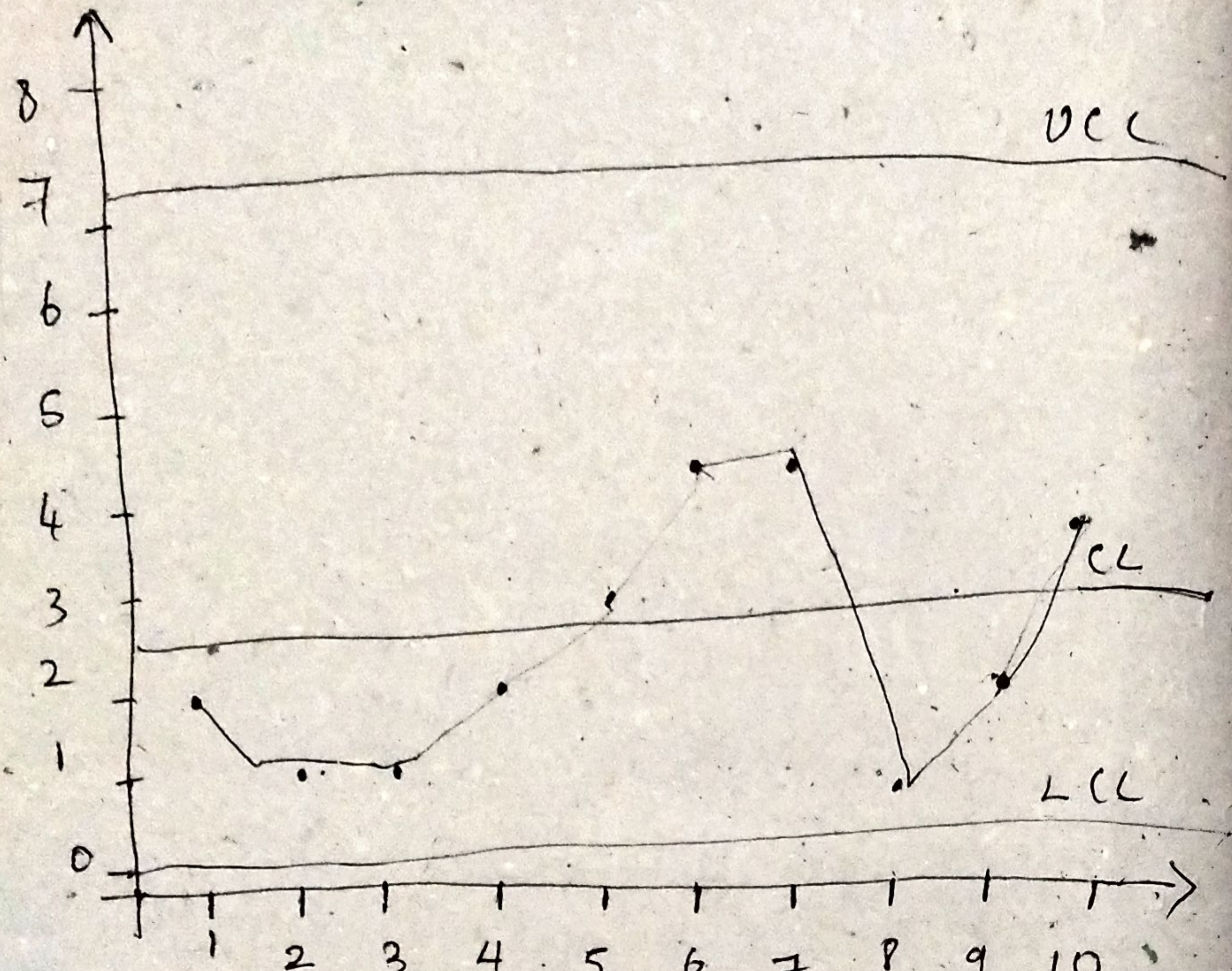


np chart:

$$CL = n.\bar{p} = 50 \times 0.05 = 2.5$$

$$LCL = n.\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0$$

$$UCL = n.\bar{p} + 3 \quad '' \quad = 7.1233$$



It is under control.

1)

i) Find a.

$\Sigma p_i = 1$.

$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$81a = 1 \Rightarrow a = \frac{1}{81}$.

ii) $P(X < 3)$, $= P(X=0) + P(X=1) + P(X=2)$

$= a + 3a + 5a$.

$= 9a$.

$= 9 \times \frac{1}{81} = \frac{9}{81} = 1/9$

iii)

$P(0 < X < 3) = P(X=1) + P(X=2)$

$= 3a + 5a$.

$= 8a$

$= 8 \times \frac{1}{81} = \frac{8}{81}$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= a + 3a + 5a + 7a$$

$$= 16a$$

$$= 16 \times \frac{1}{81} = \frac{16}{81}$$

iii) Determine the distribution :

X	0	1	2	3	4	5	6	7	8
F(x)	a	4a	9a	16a	25a	36a	49a	64a	81a
$P(X \leq x)$	1/81	4/81	9/81	16/81	25/81	36/81	49/81	64/81	1

3) $f(x) = \begin{cases} c\,(4x - 2x^2), & 0 < x < 2 \\ 0, & \end{cases}$

$$\int_0^2 c\,(4x - 2x^2)\,dx = 1$$

$$c\int_0^2 (4x - 2x^2)\,dx = 1$$

$$\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$c\left[\frac{4(2)^2}{2} - \frac{2(2)^3}{3} - 0\right] = 1$$

$$c\left(\frac{\cancel{16}^{\,8}}{\cancel{2}} - \frac{16}{3}\right) = 1$$

$$c\left(8 - \frac{16}{3}\right) = 1$$

$$c\left[\frac{24}{3}-\frac{16}{3}\right]=1$$

$$c\left(\frac{8}{3}\right]=1$$

$$c=\frac{3}{8}$$

ii) $$E(x)=\int_0^2 x\cdot\frac{3}{8}(4x-2x^2)\,dx$$

$$=\frac{3}{8}\int_0^2(4x^2-2x^3)\,dx.$$

$$=\frac{3}{8}\left[\frac{4x^3}{3}-\frac{2x^4}{4}\right]_0^2$$

$$=\frac{3}{8}\left(\frac{4(8)}{3}-\frac{2(16)}{4}-0\right]$$

$$=\frac{3}{8}\left(\frac{32}{3}-8\right]$$

$$= \frac{3}{8}\left[\frac{32-24}{3}\right]$$

$$= \frac{\cancel{3}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{3}}$$

$f(x) = 1$

iii) $P(X>1)$: $\int_1^2 f(x)\,dx$.

$$= \frac{3}{8}\int_1^2 (4x - 2x^2)\,dx.$$

$$= \frac{3}{8}\left[\frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{2x^3}{3}\right]_1^2$$

$$= \frac{3}{8}\left[2x^2 - \frac{2x^3}{3}\right]_1^2$$

$$= \frac{3}{8}\left[2(4) - \frac{2(8)}{3} - 2(1) - \frac{2(1)}{3}\right.$$

$$= \frac{3}{8}\left[\frac{8-16}{3} - \frac{6-2}{3}\right]$$

$$= \frac{3}{8}\left[-\frac{8}{3} + \frac{4}{3}\right]$$

$$= \frac{\cancel{3}}{8} \times -\frac{4}{\cancel{3}}$$

$$= -\frac{\cancel{4}^{1}}{\cancel{8}_{2}}$$

$$= -1/2$$ //

13) a)

polynomial $\to u$
expont $\to v$

$$f(x) = \theta e^{-\theta(x-\mu)} \quad , \quad x > \mu$$

$e^{-\infty} = 0$

$\mu_1' = m_1'$ — ①

$\mu_2' = m_2'$ — ②

$\theta = ?$
$\mu = ?$

$$\mu_1' = E(x) = \int x \cdot f(x)\, dx.$$

$$= \int_{\mu}^{\infty} x \cdot \theta e^{-\theta(x-\mu)}\, dx.$$

$$= \int_{\mu}^{\infty} x \cdot \theta e^{-\theta x + \theta \mu}\, dx.$$

$$= \int_{\mu}^{\infty} x \cdot \theta\, e^{-\theta x} \cdot e^{+\theta\mu}\, dx$$

$$= \theta e^{\theta \mu} \int_{\mu}^{\infty} x \cdot e^{-\theta x}\, dx.$$

$$= \theta e^{\theta u}\left[(x)\left(\frac{e^{-\theta x}}{\theta}\right) - (1)\left(\frac{e^{-\theta x}}{\theta^2}\right) + 0 \right]$$

$$= \theta e^{\theta u}\left[0 - \left[\frac{\mu e^{\theta \mu}}{\theta} - \frac{e^{-\theta u}}{\theta^2} \right]\right]$$

$$= \cancel{\theta} e^{\theta \mu} \frac{\mu e^{-\theta u}}{\cancel{\theta}} + \cancel{\theta} e^{\theta u} \frac{e^{-\theta u}}{\theta \cancel{2}}$$

$$= \left[e^{\theta u - \theta \mu} \mu + \frac{e^{\theta u - \theta u}}{\theta} \right]$$

$$\mu_1' = \mu + \frac{1}{\theta} \quad —① $$

$$\mu_2' = E(x^2) = \int x^2 \cdot f(x)\, dx.$$

$$= \int_{\mu}^{\infty} x^2 \cdot \theta e^{-\theta x} \cdot e^{\theta u}\, dx.$$

$$= \theta e^{\theta u} \int_{\mu}^{\infty} \underset{u}{x^2} \cdot \underset{v}{e^{-\theta x}}\, dx$$

$$= \theta e^{\theta u} \left((x^2)\left(\frac{e^{-\theta x}}{-\theta}\right) - (2x)\left(\frac{e^{-\theta x}}{\theta^2}\right) + (2)\left(\frac{e^{-\theta x}}{-\theta^3}\right) \right.$$

$$= \theta e^{\theta u}\left[(0) - \frac{\mu^2 e^{-\theta u}}{-\theta} - \frac{2\mu e^{-\theta u}}{\theta^2} - \frac{2 e^{-\theta u}}{\theta^3}\right]$$

$$= \cancel{\theta} e^{\theta u} \frac{\mu^2 e^{-\theta u}}{\cancel{\theta}} \cancel{+} \cancel{\theta} e^{\theta u} \frac{2\mu e^{-\theta u}}{\theta^{\cancel{2}}} + \cancel{\theta} e^{\theta u} \cdot \frac{2 e^{-\theta u}}{\theta^{\cancel{3}2}}$$

$$= \mu^2 e^{\theta u - \theta u} + \frac{2\mu e^{\theta u - \theta u}}{\theta} + \frac{2 e^{\theta u - \theta u}}{\theta^2}$$

$$\mu_2' = \mu^2 + \frac{2\mu}{\theta} + \frac{2}{\theta^2} \quad \text{--- (2)}$$

Now $\mu + \frac{1}{\theta} = \bar{x}$ --- (4)

$$\mu^2 + \frac{2\mu}{\theta} + \frac{2}{\theta^2} = s^2 \quad \text{--- (5)}$$

Now squaring eqn (4),

$$\left(\mu + \frac{1}{\theta}\right)^2 = \bar{x}^2$$

$$\mu^2 + 2\mu\frac{1}{\theta} + \left(\frac{1}{\theta}\right)^2 = \bar{x}^2 \quad \text{---} \quad (6)$$

(5) - (6)

$$\cancel{\mu^2} + \cancel{\frac{2\mu}{\theta}} + \frac{2}{\theta^2} - \cancel{\mu^2} - \cancel{\frac{2\mu}{\theta}} - \frac{1}{\theta^2} = s^2 - \bar{x}^2$$

$$\left(\frac{2}{\theta^2} - \frac{1}{\theta^2}\right) = s^2 - \bar{x}^2$$

$$\frac{1}{\theta^2} = s^2 - \bar{x}^2$$

$$\theta^2 = \frac{1}{s^2 - \bar{x}^2}$$

$$\theta = \pm\sqrt{\frac{1}{s^2 - \bar{x}^2}}$$

$$\theta = \pm\frac{1}{\sqrt{s^2 - \bar{x}^2}} \quad \text{---} \quad (7)$$

$$(4) \Rightarrow \quad \mu = \bar{x} - \frac{1}{\theta}$$

14) b) ii)

$n = 6 + 7 + 5$
$n = 18$

H-test

FM : 94, 88, 91, 74, 87, 97

SM : 85, 82, 79, 84, 61, 72, 80.

TM : 89, 67, 72, 76, 69.

Ascending order:

61, 67, 69, 72, 72, 74, 76, 79,

80, 82, 84, 85, 87, 88, 89.

91, 94, 97

~~Rank~~

Rank 1 (FM): $17 + 14 + 16 + 6 + 13 + 18 = 84$

$\boxed{n_1 = 6}$

Rank 2 (SM): $12 + 10 + 8 + 11 + 1 + 4.5 + 9 = 55.5$

$\boxed{n_2 = 7}$

Rank 3 (TM): $15 + 2 + 4.5 + 7 + 3 = 31.5$

$\boxed{n_3 = 5}$

$$H = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

$$= \frac{12}{18(19)}\left(\frac{84^2}{6} + \frac{55.5^2}{7} + \frac{31.5^2}{5}\right] - 3(19)$$

$$= \frac{12}{342}\left(\frac{7056}{6} + \frac{3080}{7} + \frac{992}{5}\right] - 57$$

$$= \frac{12}{342}\left[1{,}176 + 440 + \underset{198.4}{\cancel{992}}\right] - 57$$

$$= \frac{12}{342}\left(\underset{1814.4}{\cancel{2608}}\right) - 57.$$

$$= 12\left(\underset{5.3052}{\cancel{7.6257}}\right) - 57$$

$$= \underset{63.6884}{\cancel{41.508}} - 57$$

$$H = \cancel{34.508}\ 6.663.$$

$$dof = k-1$$
$$= 3-1$$
$$= 2$$

$$TV = 5.991$$

$$H > TV$$

so It reject H_0

14) b) i)

29, 25,

H0: Arrange random

H1: 〃 not random

Ascending order:

25, 26, 26, 27, 28, 28, 28, 28, 29, 29, 30

30, 30, 31, 31, 31, 31, 33, 33, 33, 35, 35, 36,

38

$\frac{60}{2} = 30$

12 12

12 + 13

30 < a

30 > b

~~bbbbbbbbbbbb aaaaaaaa~~

bb a b a b aaa b aaa

b aaa b a b aaa b

step 2 :

$n_1 \rightarrow b = 9$

$n_2 \rightarrow a = 15$

$r \rightarrow 15$

step 3 :

$$z = \frac{r - \mu}{\sigma}.$$

$$\mu = \frac{2n_1 n_2}{n_1 + n_2} + 1$$

$$= \frac{2 \times 9 \times 15}{24} + 1$$

$$= 12.25$$

$$\sigma = \sqrt{\frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1+n_2)^2 (n_1 + n_2 - 1)}}$$

246

$$= \sqrt{\frac{270 (270 - 24)}{(24)^2 (23)}}$$

$$= \sqrt{\frac{66420}{13,248}}$$

$$= \sqrt{5.0135}$$

$$\sigma = 2.239$$

$$= \frac{15 - 12.25}{2.239}$$

$$= \frac{2.75}{2.239}$$

$$\boxed{z = 1.23}$$

TAble Value = 2.576

$z = 1.23 < 2.576$ (TV)

It accepts H0.

0.01 0.05 0.10

Model Question paper:

15) a) i)

$N = 5$, $n = 5$.

	1	2	3	4	5
$\bar{x}$	~~42.4~~ 44	~~42.6~~ 41.6	~~44~~ 40.8	~~42.8~~ 43	~~42.8~~ 45.2
$\bar{R}$	4	4	2	3	4

step 1:

$$\bar{\bar{X}} = \frac{\bar{X}}{n} = \frac{214.6}{5} = 42.92.$$

$$\bar{R} = \frac{R}{n} = \frac{17}{5} = 3.4$$

step 2: control limit for $\bar{\bar{X}}$

$\boxed{A2 = 0.577}$

$$CL = \bar{\bar{X}} = 42.92.$$

$$LCL = \bar{\bar{X}} - A_2\bar{R}$$

$$= 42.92 - (0.577)(3.4)$$

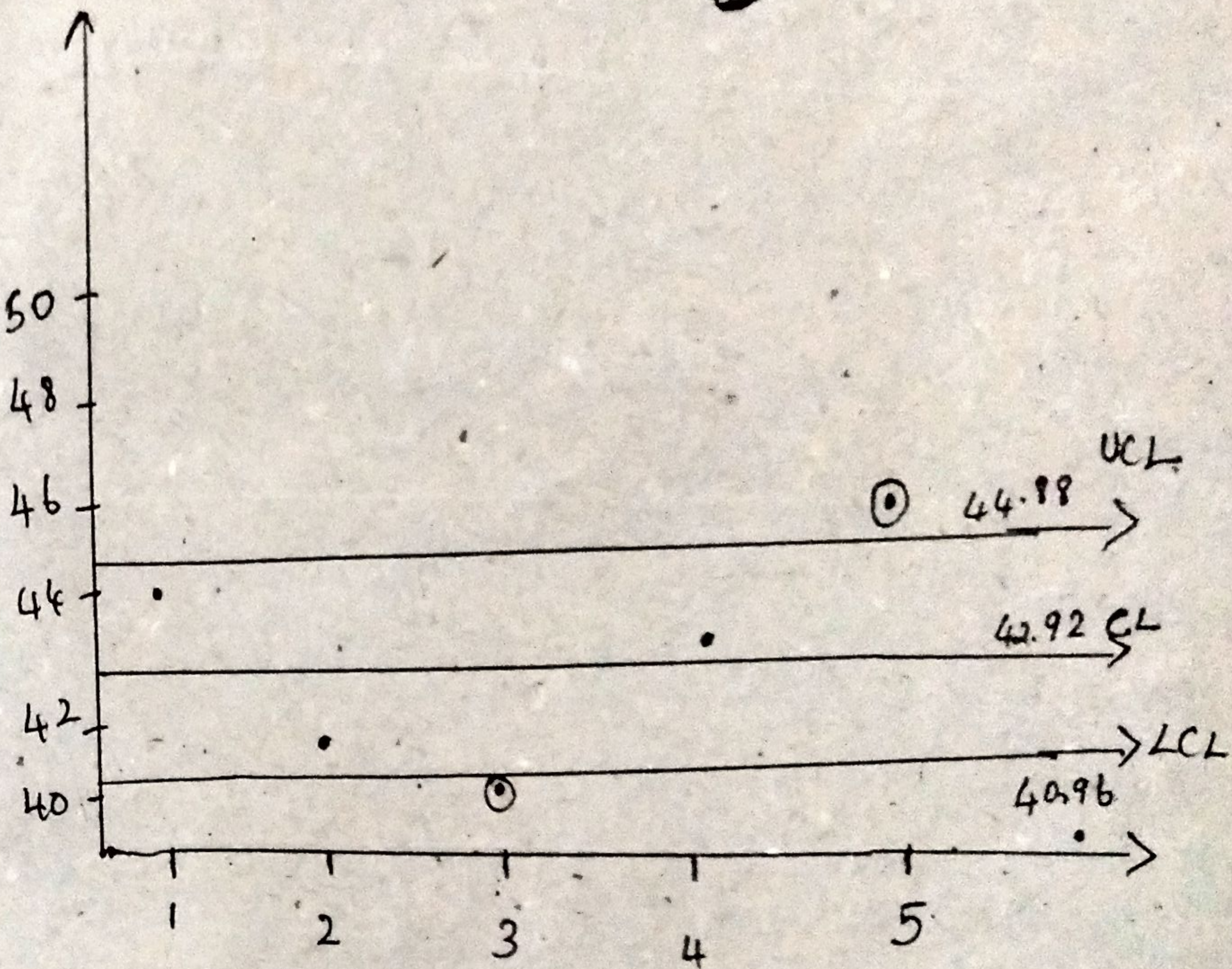
$$= 42.92 - 1.961 = 40.96$$

$UCL = \overline{\overline{X}} + A_2\overline{R}$

$= 42.92 + 1.96$

$= 44.88$

$\boxed{9788954798}$



It is out of control.

step 3 : control limit for R.

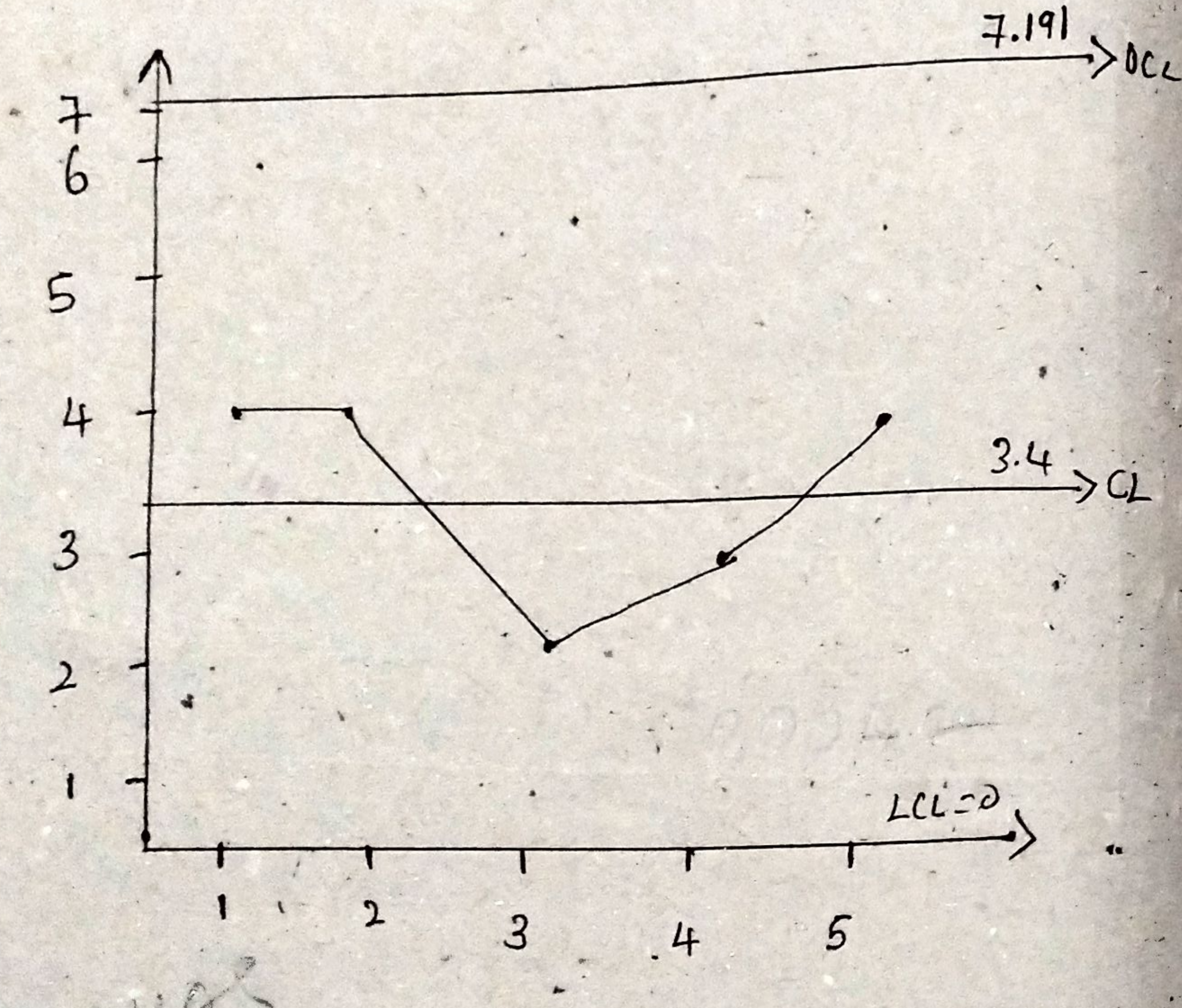
$CL = \bar{R} = 3.4$

$D3 = 0$

$D4 = 2.115$

$LCL = D3\bar{R} = 0$

$UCL = D4\bar{R} = 7.191$



It is under control.