

The maximum flow problem:-

→ Maximum flow $\xrightarrow{\text{rate}}$ is a maximum feasible flow b/w source & sink.

The Ford-Fulkerson method :-

Terminology.

→ It's an iterative method by using 3 methods.

(i) Residual network

(ii) Augmenting Path.

(iii) Cuts.

Residual Network Graph.

→ Suppose we have flow network $G = (V, E)$ with s and t . Let every edge u, v is having a pair flow/capacity, then called Residual NW.

$$c_f(u, v) = c(u, v) - f(u, v)$$

↳ current flow.

Source \rightarrow No incoming node and only outward node.

Sink \rightarrow No outward only inward node.

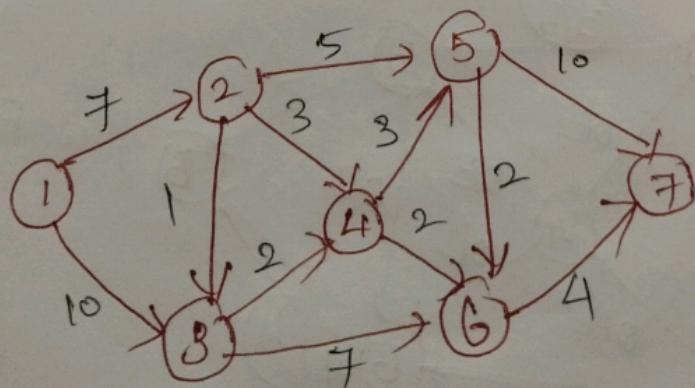
The given graph which represents the flow network where every edge has capacity. Also given two vertices s & t in the graph. Find flow from s to t out the maximum possible with following constraints

(i) Flow on an edge does not exceed the given capacity of edge.

(ii) Inflow is equal to outflow for every vertex except s & t .

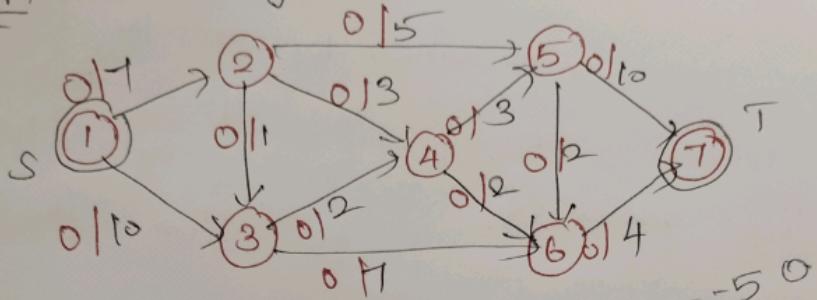
Algorithm:-
Start with a initial flow

Find the maximum flow through the given network using ford fulkerson algorithm.

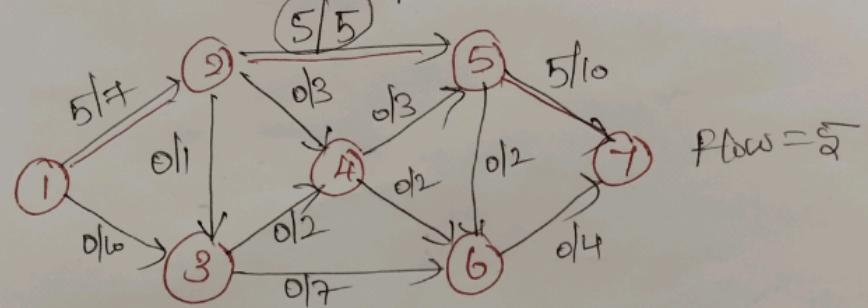


BTS

Step 1 Initial graph.



Step 2 choose path Residual graph Block it.



Augment path

1-2-5-7

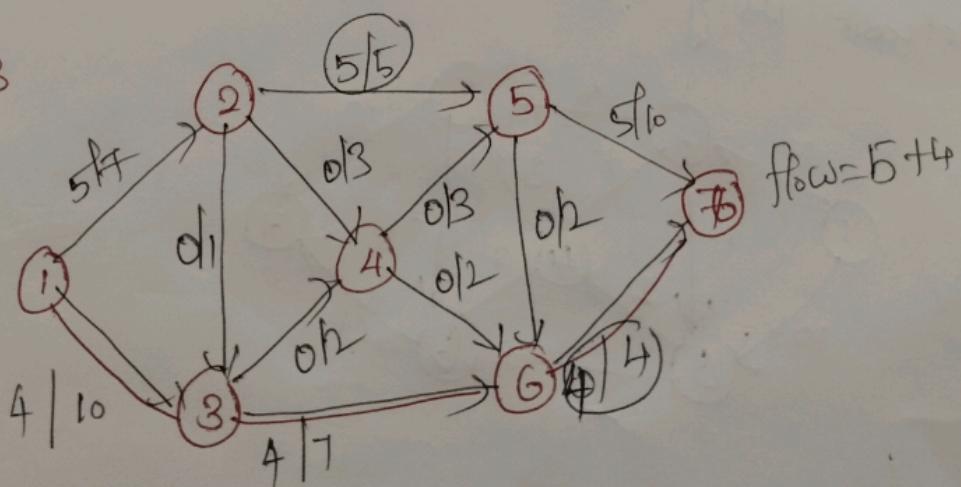
1 → 3-6-7

Bottle Neck capacity

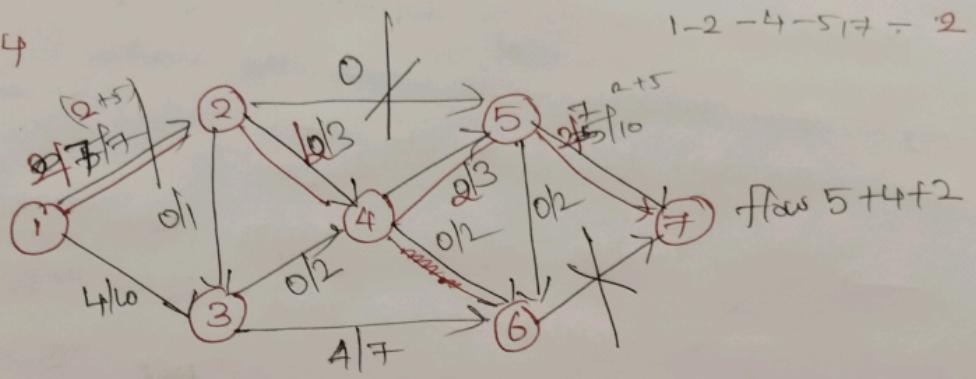
5

4

Step 3



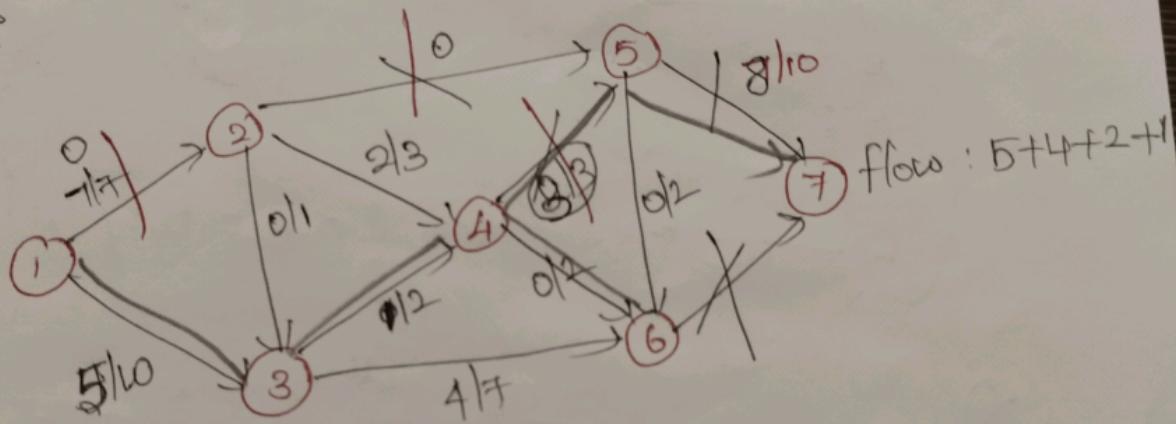
Step 4



$$1-2-4-5-7 = 2$$

$$\text{flow } 5+4+2$$

Step 5

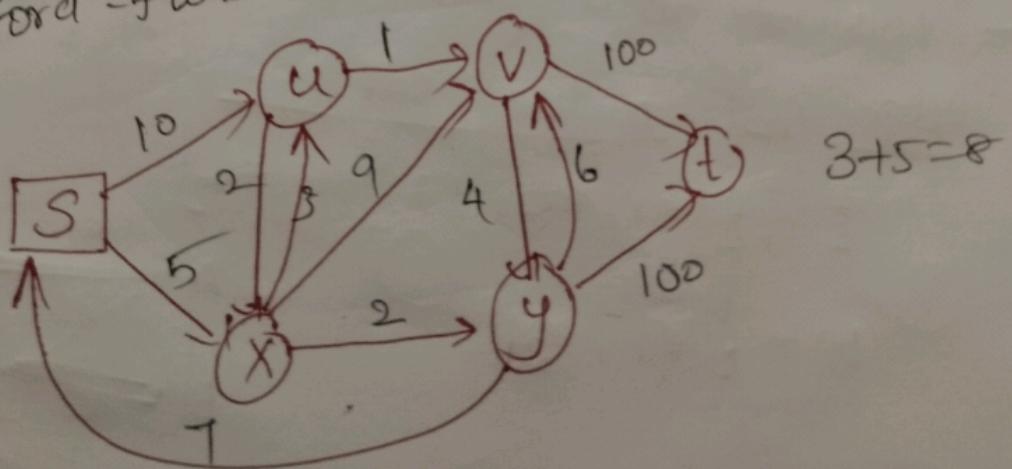


$$1-3-4-5-7 = 12$$

maximum flow through the given NW using
Ford - fulkerson method is $5+4+2+1 = 12$

Ford - fulkerson

Ex-2

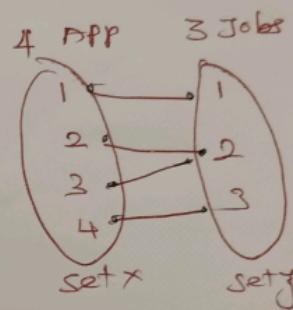
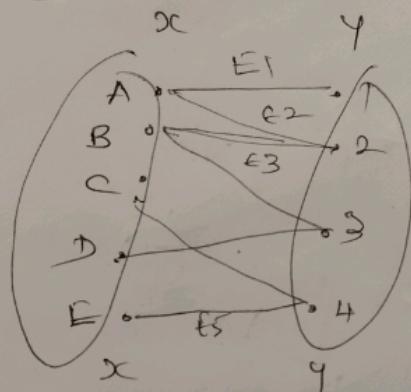


Maximum matching Bipartite graph:

→ Graph $G(V, E)$, in which the vertex set V is divided into 2 distinct sets $X \& Y$.

→ Every edge of the graph has one end point in X and other end point in Y .

Ex



$$V = \{A, B, C, D, E / 1, 2, 3, 4\}$$

$$x \cap y = \{\emptyset\}$$

$$x \cap y = \text{null.}$$

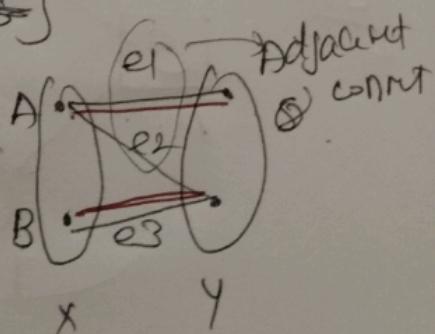
Vertices of the same set are not connected

Matching: - (n)

→ Matching in a graph is a subset of edges that no two edges share a vertex

Ex $E = \{e_1, e_2, e_3, \{e_4, e_5\}\}$

$$M = \{e_1, e_2\}$$

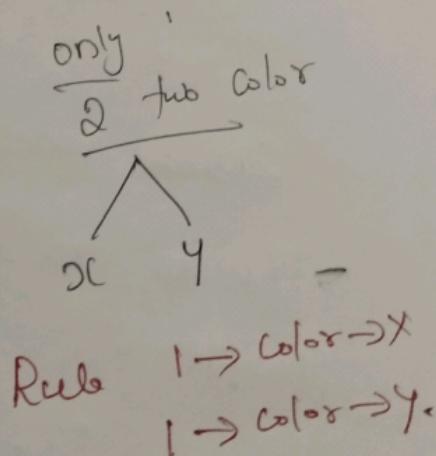
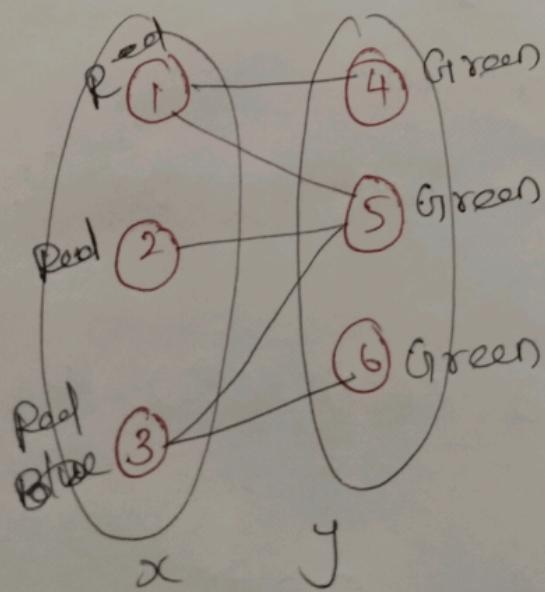
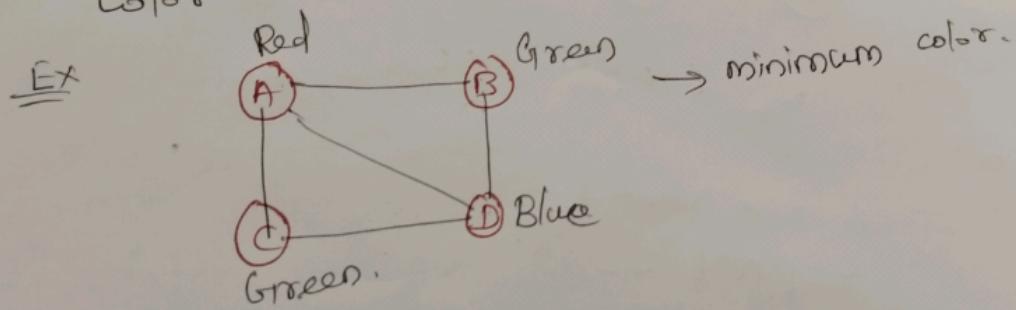


Two Colorable graph

- A graph that can be colored only with 2 colors
- No two edges connects the same colors.
- Bipartite Graph is a 2 color graph.

Graph coloring:-

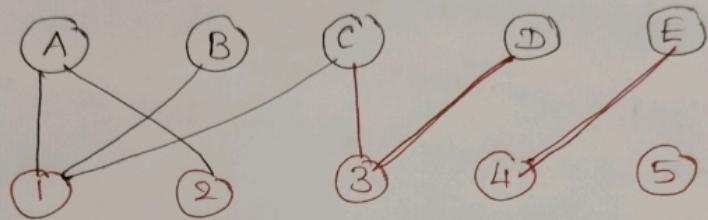
- No 2 adjacent states will be of same color.



Problem

Maxim matching problem.

Ex1 Let M be the matching in bipartite graph G .



$$M_{\text{a}} = \{(D, 3), (E, 4)\}$$

Soln

Find free vertex

$$x = \{A, B, C, D, E\}$$

$$y = \{1, 2, 3, 4, 5\}$$

Consider vertex a ; M_b :

Thus the maximum matching for the Bipartite

graph is

$$M = \{(A, 2), (B, 1), (C, 3), (D, 5), (E, 4)\}$$

