User	A	B	C	D	E
①	4	5	3	?	2
②	5	3	4	2	1
③	2	?	5	4	3
④	?	4	3	5	4

computing similarity for.

user 1 → user 2.

① ⇒ 4 5 3 2

② ⇒ 5 3 4 1

$$\frac{A \cdot B}{||A|| \times ||B||}$$

(cosine similarity)

$$\frac{\Sigma(AB)}{\sqrt{\Sigma A^2} \cdot \sqrt{\Sigma B^2}}$$

$$\frac{(4\times5) + (5\times3) + (3\times4) + (2\times1)}{\sqrt{4^2+5^2+3^2+2^2} \times \sqrt{5^2+3^2+4^2+1^2}}$$

$$\frac{20 + 15 + 12 + 2}{\sqrt{16+25+9+4} \times \sqrt{25+9+16+1}} = \frac{49}{52.49} = 0.933$$

$\sqrt{54} \times \sqrt{51}$

user 1 → user 3

① ⇒ 4, 3, 2

③ ⇒ 2, 5, 3

$$\frac{(4\times2)+(3\times5)+(2\times3)}{\sqrt{(4^2+3^2+2^2)} * \sqrt{(2^2+5^2+3^2)}}$$

$$= \frac{29}{\sqrt{29}\times\sqrt{38}} \quad (5.38 \times 6.16)$$

$$= \frac{29}{33.10} \approx 0.876$$

user 1 → user 4.

common movies : B, C, E

① ⇒ [5, 3, 2]

② ⇒ [4, 3, 4]

$$\frac{(5\times4)+(3\times3)+(2\times4)}{\sqrt{(5^2+3^2+2^2)} \times \sqrt{(4^2+3^2+4^2}} = \frac{37}{39.42}$$

$$= 0.939$$

User 3 — User 2

Common movies : A, C, D, E

(3) => 2, 5, 4, 3

(2) => 5, 4, 2, 1

$$\frac{(2\times5)+(5\times4)+(4\times2)+(3\times1)}{\sqrt{(2^2+5^2+4^2+3^2)}\times\sqrt{5^2+4^2+2^2+1^2}}$$

$$\frac{41}{49.85} \approx 0.822$$

User 3 — User 4

Common movies : C, D, E

(3) => 5 4 3

(4) => 3 5 4

$$\frac{(5\times3)+(4\times5)+(3\times4)}{\sqrt{(25+16+9)}\times\sqrt{9+25+16}} = \frac{47}{50} \approx 0.94$$

User 2 - User 4

Common : B, C, D, E

② = 3, 4, 2, 1

④ = 4, 3, 5, 4

$$\frac{(3\times4)+(4\times3)+(2\times5)+(1\times4)}{\sqrt{9+16+4+1}\times\sqrt{16+9+25+16}}$$

$$\frac{38}{44.5} = 0.854$$

---

step 2: predict Missing values:

1. User-1 (D)

User 2: 2 (0.933)

User 3: 4 (0.876)

User 4: 5 (0.939)

$$\hat{r} = \frac{(0.933 \times 2) + (0.876 \times 4) + (0.939 \times 5)}{0.933 + 0.876 + 0.939}$$

$$= \frac{1.866 + 3.504 + 4.695}{2.748}$$

$$= \frac{10.065}{2.748}$$

$$= 3.66$$

2. user 3 (B)

$$\hat{r} = 4.02$$

3. user 4 (A)

$$\hat{r} = 3.63$$

---

* Cosine similarity
* weighted average formula. to predict missing values
* Final matrix is recommend.

2)

part 1 : Mean centering.

(user A)

Ratings = [4, 5, 6]

1 : Mean

$$\mu = \frac{4+5+6}{3} = \frac{15}{3} = 5$$

2 : Subtract the Mean

Normalized rating = $r_i - \mu$.

1	4	4 − 5 = −1
2	5	5 − 5 = 0
3	6	6 − 5 = +1

(−1, 0, +1)

Part 2 : Z- score Normalization.

(Ans B)

Ratings = [2, 4, 6]

Mean (μ) = 4

So (σ) = 2

$$z = \frac{vi - \mu}{\sigma}$$

1	2	$\frac{2-4}{2} = -1$
2	4	$\frac{4-4}{2} = 0$
3	6	$\frac{6-4}{2} = +1$

[-1, 0, +1]