CP241 Lab Assignment 4

October 30, 2023

- *Submission via Teams:* Assignments will be allocated to you via the Teams "Assignments" feature. You will have to upload your answer sheet via the same feature in Teams. **Answer sheets sent via e-mail will not be considered.**
- *File Logistics:* Write only the code in .mlx (MATLAB Livescript) with name "LA_4_'YOUR_SERIAL_NUMBER'" and submit in Teams.

Due Date: November 7, 2023
Maximum Marks: 30

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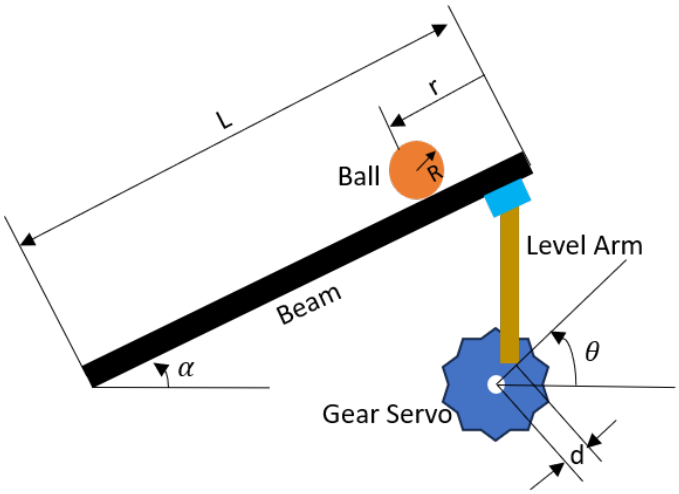
1. Consider a ball placed on a rod, allowed to roll along the length of the rod. One end of the rod is attached to a lever arm with a servo motor, which controls the angle of the rod, resulting in the ball rolling. Design a controller to balance the ball in a particular position along the rod. The equation of motion is given by:

$$(\frac{J}{R^2} + m)\ddot{r} - mg\sin\alpha - mr\dot{\alpha}^2 = 0, \tag{1}$$

$$\alpha = \frac{d}{L}\theta, \tag{2}$$

where, $m = 0.1$ kg, $R = 0.02$ m, and $J = 10^{-5}$ kg.m^2 are the mass, radius, and moment of inertial of the ball; $L = 1$m is the length of the rod; $d = 0.05$ m is the lever arm length; $g = 9.8$ m/sec^2. α is the rod angle which is linked to the control input: servo angle θ. r is the position of the ball on the rod.



(a) Write a simple MATLAB script file for controlling the position of the ball on the rod using PID control. Assume initial position $r = 0.25$m and initial velocity $\dot{r} = 0$ m/sec^2, and the reference position as $r_{ref} = 0.25\sin t + 0.25$ m. Take the simulation time span of $0 - 25$ sec.

(b) Mention your PID gains.

(c) Plot the position of the ball r and the control input $\theta(t)$.

[10 marks]

2. Consider the system given in question-1. Take the system parameters and initial conditions given in question-1.

   (a) Write a MATLAB script for controlling the position of the ball using full state feedback, with zero reference for all states, by placing the poles of the system linearized around the origin, to the following locations.

      i. $P = \{-1, -2\}$
      ii. $P = \{-2 + i, -2 - i\}$

   Plot the input and the position of the ball with respect to time in both cases. Take the simulation time span of 0-25s.

   (b) Write a MATLAB script for controlling the position of the ball using LQR for the system linearized around the origin, with the lqr gain matrices:tt $Q = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$ and $R = 2$. Plot the input and the position of the ball with respect to time. Take the simulation time span of 0-25s

[5+5 marks]

3. Consider a Permanent Magnet DC Motor(PMDC) whose differential equations are given as

$$L_a \frac{di_a}{dt} + R_a i_a + k_b w = V_{inp} \tag{3}$$

$$J \frac{dw}{dt} + Bw + T_{load} = k_t i_a \tag{4}$$

and the motor specifications given by $R_a = 0.1\Omega$, $L_a = 0.01H$, $J = 10kgm^2$, $B = 0.01Nms$, $k_b = 1Vs/rad$ and $k_t = 1Nm/A$

Write a simple MATLAB script file for controlling the speed of PMDC Motor using 2 cascaded PID controllers. If you have tuned the PID Controllers using any method please specify it in the .mlx file. Make sure that the outer loop is slower than the inner loop. Try to apply a torque disturbance to the system after it has reached the steady state and check if your PID is able to track the speed reference. Assume suitable details wherever necessary.

Hint:-

   (a) The general procedure to implement cascaded PID Control for PMDC motor is to have outer speed loop and inner current loop. (Students are expected to look at cascaded PID control. Plenty of resources are available online-One such resource can be found in this link)

   (b) Try to solve the problem with just one PID and then do the implement for the given question.

[10 marks]