CP241 Lab Assignment 3

September 27, 2023

- *Submission via Teams:* Assignments will be allocated to you via the Teams "Assignments" feature. You will have to upload your answer sheet via the same feature in Teams. **Answer sheets sent via e-mail will not be considered.**
- *File Logistics:* Write only the code in .mlx (MATLAB Livescript) with name "LA_3_'YOUR_SERIAL_NUMBER'" and submit in Teams.
- Do not use the *phasePortrait.p* file provided in GitHub link for the class materials for submission. If founded then no mark will be given for that answer.

Due Date: October 13, 2023
Maximum Marks: 45

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1. Consider a Permanent Magnet DC Motor(PMDC) whose differential equations are given as

$$L_a \frac{di_a}{dt} + R_a i_a + k_b w = V_{inp} \tag{1}$$
$$J \frac{dw}{dt} + Bw + T_{load} = k_t i_a \tag{2}$$

and the motor specifications given by $R_a = 0.1\Omega$, $L_a = 0.01H$, $J = 10kgm^2$, $B = 0.01Nms$, $k_b = 1Vs/rad$ and $k_t = 1Nm/A$

(a) Solve the differential equation system using ODE45 for a time span of $90s$, when the load torque is zero for the entire duration and the input voltage is kept constant at 20V . Plot the response of current and speed with respect to time. Assume zero initial conditions.

[3 marks]

(b) Plot the response of the current and speed of the PMDC motor for a time span of $90s$ when voltage of magnitude $20V$ is applied as step at $t = 2s$ and load torque of magnitude $5Nm$ is applied as step at $t = 30s$. Assume zero initial conditions. [5 marks]

(c) Find the equilibrium points and eigen values of the system under zero excitation. [2 marks]

(d) Plot the phase trajectories of the system and identify what kind of equilibrium point(s) it(they) is(are).

[5 marks]

2. Consider a system

$$\dot{x}_1 = x_2 \tag{3}$$
$$\dot{x}_2 = -x_1 + \epsilon x_2(1 - x_1^2) \tag{4}$$

which is a Vanderpol's oscillator. Attack the following questions for 2 cases when ϵ is 0 and 1.

(a) Identify the equilibrium point(s) of the system and using phase portrait identify if it is a stable focus/unstable focus/stable node/unstable node/center/saddle. Take the horizontal axis corresponds to x_1 and the vertical axis corresponds to x_2. [3+3 marks]

(b) What do you infer about the system's behaviour after identifying what kind of equilibrium point it is?

[2+2 marks]

(c) Construct the phase plane trajectory for two random initial conditions $x_0 = [x_1(0),\ x_2(0)]^T$ using **ginput** (a MATLAB command for taking inputs from GUI) command from figure window, on top of the phase portrait constructed earlier. [2.5+2.5 marks]

3. Consider the interacting tank in series system which contains some fluid as shown in the Figure 1

$$\begin{aligned}\frac{dh_1}{dt} &= \frac{q_i}{A_1} - \frac{a_1}{A_1}\sqrt{2g(h_1-h_2)} \\ \frac{dh_2}{dt} &= \frac{a_1}{A_2}\sqrt{2g(h_1-h_2)} - \frac{a_2}{A_2}\sqrt{2gh_2};\ h_1>0,\ h_2>0\end{aligned} \tag{5}$$

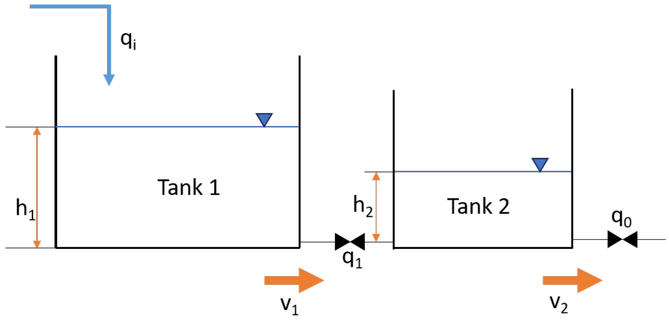

Figure 1: Two tanks in series

Where, a_1 & a_2 are cross-section of flow valves of Tank 1 and Tank 2 respectively, A_1 & A_2 are cross-section of tank 1 and tank 2 respectively, h_1 & h_2 are height of fluid levels in tank 1 and tank 2 respectively. Here q_i is fluid inflow to the tank 1 and q_1 & q_0 are flow rates through the tank 1 and tank 2 respectively. Answer the following questions
(Assume that tanks can store infinite amount of liquid)

(a) Construct the phase portrait for the system in 5. [5 Marks]
Take the following constants $a_1 = 2.0 \times 10^{-3} m^2$, $a_2 = 2.0 \times 10^{-3} m^2$, $A_1 = 0.25 \times m^2$, $A_2 = 0.10 \times m^2$, $g = 9.8 m/s^2$ and $q_i = 15 \times 10^{-3} m^3/s$ is control input.

(b) Given $q_i = 15 \times 10^{-3} m^3/s$ and initial values for height of the fluid $h_1 = 3$ m and $h_2 = 1.5$ m in tank 1 and tank 2 respectively. For what values of a_1 & a_2 the system becomes stable and unstable? [5 + 5 Marks]