

Mobile Robotics

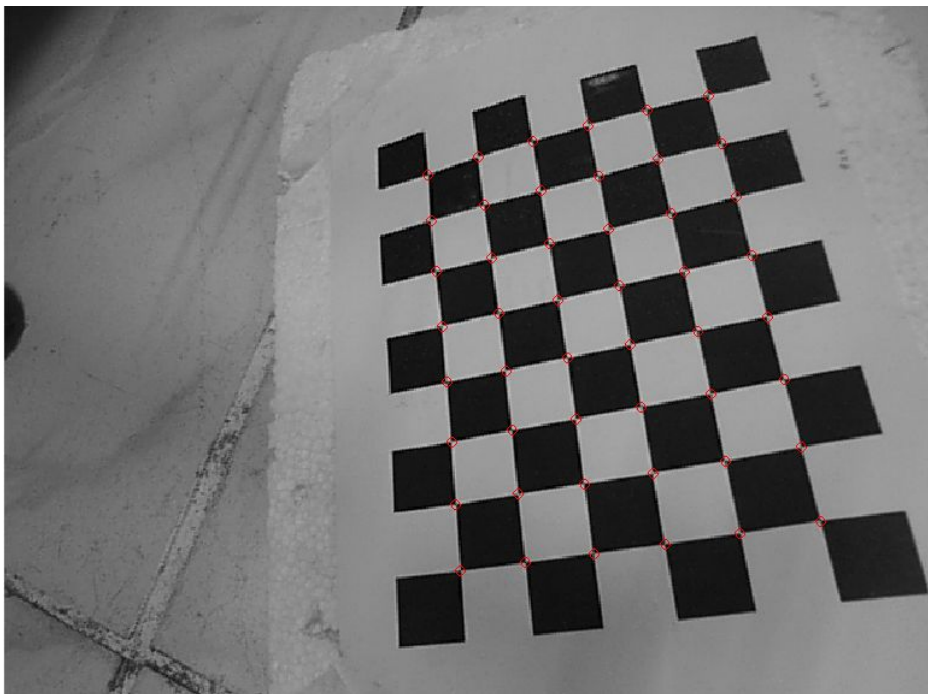
Camera Calibration

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The Matlab code in the folder, finds out the intrinsic parameters of a camera using Zang's method.

Code Explanation -

In the code 3 images `Img1`, `Img2`, `Img3` were read (minimum number of images required to find out intrinsic parameters). After it since we knew cube size, i.e., 2.4 cm and number of cubes, so ground truth was generated by considering first point as origin and then moving 2.4 cm in X and Y direction. Since all points are on same plane, so Z coordinate will be equal and were assumed to be equal to zero.



M matrix was generated by the given formula -

$$\begin{aligned}
 \mathbf{p} &= (p_k) = \text{vec}(\mathbf{P}^\top) \\
 \mathbf{a}_{x_i}^\top &= (-\mathbf{X}_i^\top, \mathbf{0}^\top, x_i \mathbf{X}_i^\top) \\
 &= (-X_i, -Y_i, -Z_i, -1, 0, 0, 0, 0, x_i X_i, x_i Y_i, x_i Z_i, x_i) \\
 \mathbf{a}_{y_i}^\top &= (\mathbf{0}^\top, -\mathbf{X}_i^\top, y_i \mathbf{X}_i^\top) \\
 &= (0, 0, 0, 0, -X_i, -Y_i, -Z_i, -1, y_i X_i, y_i Y_i, y_i Z_i, y_i)
 \end{aligned}$$

V matrix was calculated was applying SVD on M matrix -

▪ Singular value decomposition (SVD)

$$\mathbf{M}_{2I \times 12} = \mathbf{U}_{2I \times 12} \mathbf{S}_{12 \times 12} \mathbf{V}^\top_{12 \times 12} = \sum_{i=1}^{12} s_i \mathbf{u}_i \mathbf{v}_i^\top$$

with properties $\mathbf{U}^\top \mathbf{U} = \mathbf{I}_{12}$, $\mathbf{V}^\top \mathbf{V} = \mathbf{I}_{12}$
and $s_1 \geq s_2 \geq \dots \geq s_{12}$

Last row of V was chosen, matrix which was corresponding lowest eigenvalue of S matrix.

Solution

$$\Omega = \mathbf{p}^T \left(\sum_{i=1}^{12} s_i^2 \mathbf{v}_i \mathbf{v}_i^T \right) \mathbf{p}$$

- Due to orthogonality of V

$$\mathbf{v}_i \mathbf{v}_j^T = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

- If we choose $\mathbf{p} = \mathbf{v}_i$

$$\Omega = \mathbf{v}_i^T (s_i^2 \mathbf{v}_i \mathbf{v}_i^T) \mathbf{v}_i = s_i^2 \mathbf{v}_i^T \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_i = s_i^2$$

- Choosing $\mathbf{p} = \mathbf{v}_{12}$ (the singular vector belonging to the smallest singular value s_{12}) minimizes Ω

Similar process was applied on all three images.

Then V matrix was calculated by the below formula -

- The matrix V is given as

$$V = \begin{pmatrix} \mathbf{v}_{11}^T & \mathbf{v}_{12}^T \\ \mathbf{v}_{21}^T & \mathbf{v}_{22}^T \end{pmatrix} \quad \text{with} \quad \mathbf{v}_{ij} = \begin{bmatrix} h_{1i}h_{1j} \\ h_{1i}h_{2j} + h_{2i}h_{1j} \\ h_{3i}h_{1j} + h_{1i}h_{3j} \\ h_{2i}h_{2j} \\ h_{3i}h_{2j} + h_{2i}h_{3j} \\ h_{3i}h_{3j} \end{bmatrix}$$

- For one image, we obtain

$$\begin{pmatrix} \mathbf{v}_{11}^T & \mathbf{v}_{12}^T \\ \mathbf{v}_{21}^T & \mathbf{v}_{22}^T \end{pmatrix} \mathbf{b} = 0$$

elements of H

B matrix was obtained by applying SVD on the V matrix,

Then Cholesky decomposition was used on B matrix to obtain the desired K matrix.

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix} \quad \begin{aligned} \text{chol}(B) &= AA^T \\ A &= K^{-T} \end{aligned}$$

Bonus -

Rotation and translation of each checkerboard image w.r.t. camera was calculated, and checkerboard points were also plotted.

Inbuilt function of matlab was used to evaluate intrinsic parameters of camera as reference.