Bayesian Neural Network (BNNs)

We implement a Bayesian Neural Network (BNN) using torchbnn to predict Spectral Acceleration (SA) values based on seismic parameters. The model accounts for uncertainty by learning distributions over weights, making it suitable for high-risk applications like earthquake engineering.

Unlike other models that deal with deterministic data, this model deals with probabilistic data. For very uncertain data like EQ data, such a model can truly prove to have more accuracy compared to other models.

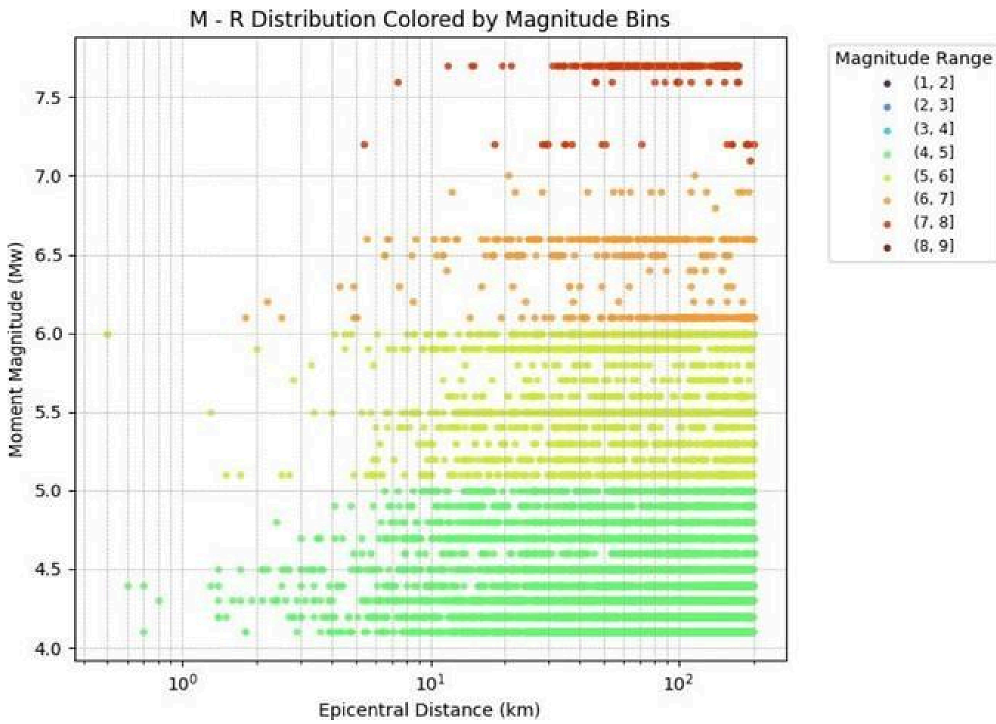
However, the disadvantage of this model is that the number of unknowns is more than twice as large as a normal ANN. Therefore, the run time also increases significantly.

Magnitude vs Joyner-Boore Distance

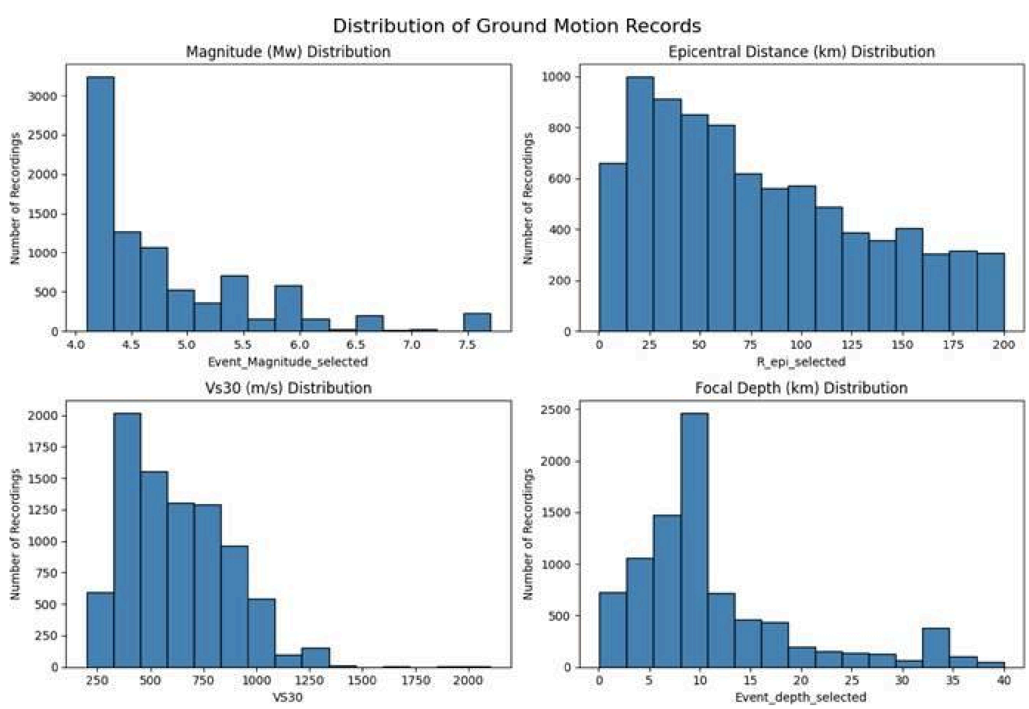
Earthquake magnitudes in the dataset range from about 4 to 8 Mw, with a higher concentration of events between magnitudes 4 and 7.

There is no clear trend or correlation between magnitude and R epi distance; earthquakes of all magnitudes occur across the full range of distances up to 1000 km.

The data points are densely clustered at higher R epi distances (100 - 1000 km), indicating most recorded events are relatively away to the reference point, but significant events are also observed at closer distances.



Histogram of each input parameter



Magnitude (Mw):

Most earthquake records have magnitudes between 4.0 and 5.0, with a sharp peak around 4 Mw. There are fewer records for both lower (>6.5) and higher (>7.5) magnitudes, indicating the dataset is dominated by slight to moderate earthquakes.

R epi distance:

The majority of records are at distances between 25 km and 75 km, peaking near 20 km. The number of records decreases steadily at larger distances, showing that most recordings are made relatively close to the earthquake source.

Shear Velocity (VS30):

Most records correspond to sites with VS30 values between 200 m/s and 600 m/s, peaking around 400 m/s. This suggests that the data predominantly represent sites with soft to moderately stiff soil conditions, with fewer records from very stiff or rock sites (VS30 > 800 m/s).

Table 01 - Input Parameters:

	Event_Magnitude_selected	R_epi_selected	VS30	Event_depth_selected
min	4.1000	0.5000	198.0000	0.1000
max	7.7000	199.9000	2104.2000	40.0000
mean	4.8315	79.5987	623.9816	11.1482
std	0.8117	53.8051	242.8279	8.2397
skewness	1.5924	0.5351	0.6112	1.5129
kurtosis	2.4603	-0.8112	0.0107	1.7629

Table 01 - Output Parameters:

	U_target_selected_1	U_target_selected_2	U_target_selected_3
min	0.0000	0.0000	0.0000
max	4836.5170	4302.4922	3097.6693
mean	10.0283	25.3087	30.2395
std	93.1098	162.5251	154.4057
skewness	24.2859	12.0879	9.2424
kurtosis	974.0350	185.9719	108.9522

	U_target_selected_4	U_target_selected_5	U_target_selected_6
min	0.0000	0.0001	0.0002
max	4299.3010	4713.7618	4196.1082
mean	32.3795	36.3600	37.8630
std	162.0937	175.5214	171.9430
skewness	11.3707	10.8141	10.1882
kurtosis	196.7534	180.0279	151.6814

	U_target_selected_7	U_target_selected_8	U_target_selected_9
min	0.0003	0.0004	0.0006
max	4130.3150	6415.0268	6400.9357
mean	40.1494	42.6016	45.3880
std	172.5549	188.0650	200.8263
skewness	8.3731	11.1739	12.4133
kurtosis	97.7752	218.8951	271.2209

Input Parameters:

The input variables (mw, r_epi, vs30, depth) show a range of values, with means and standard deviations indicating moderate spread.

Skewness and kurtosis values suggest that most input parameters are moderately skewed (either positive or negative) and have distributions close to normal.

Output Parameters:

All output parameters have minimum values of 0 or close, indicating possible zero or censored data.

The means are generally low compared to their maximums, suggesting that most data points are clustered near the lower end of the range.

High skewness and kurtosis values across almost all output parameters indicate highly skewed distributions with heavy tails. For example, U_target_selected_8 has skewness of 11.79 and kurtosis of 218.19, showing extreme outliers or rare large values.

Standard deviations are often close to or larger than the mean, reinforcing the presence of outliers or a wide range of values.

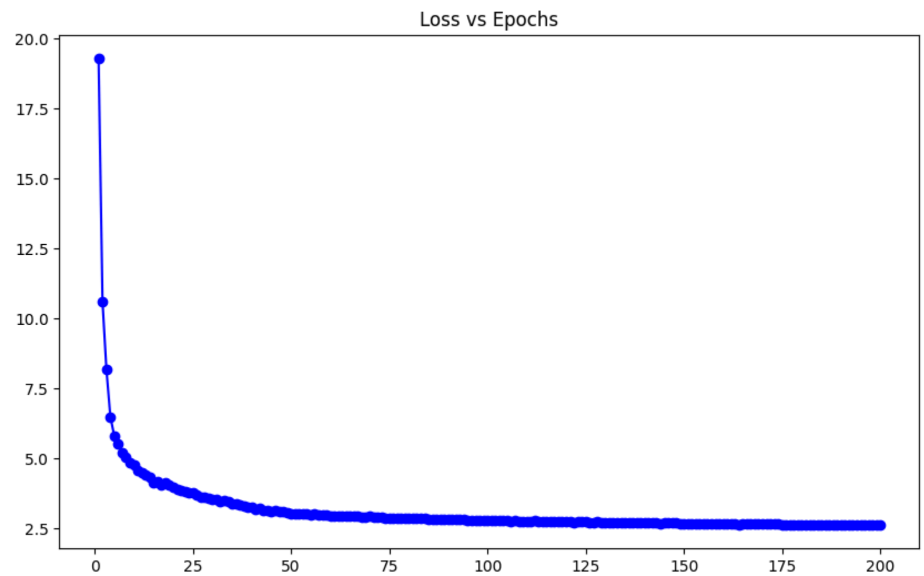
Model Summary

This model comprises of three hidden layers

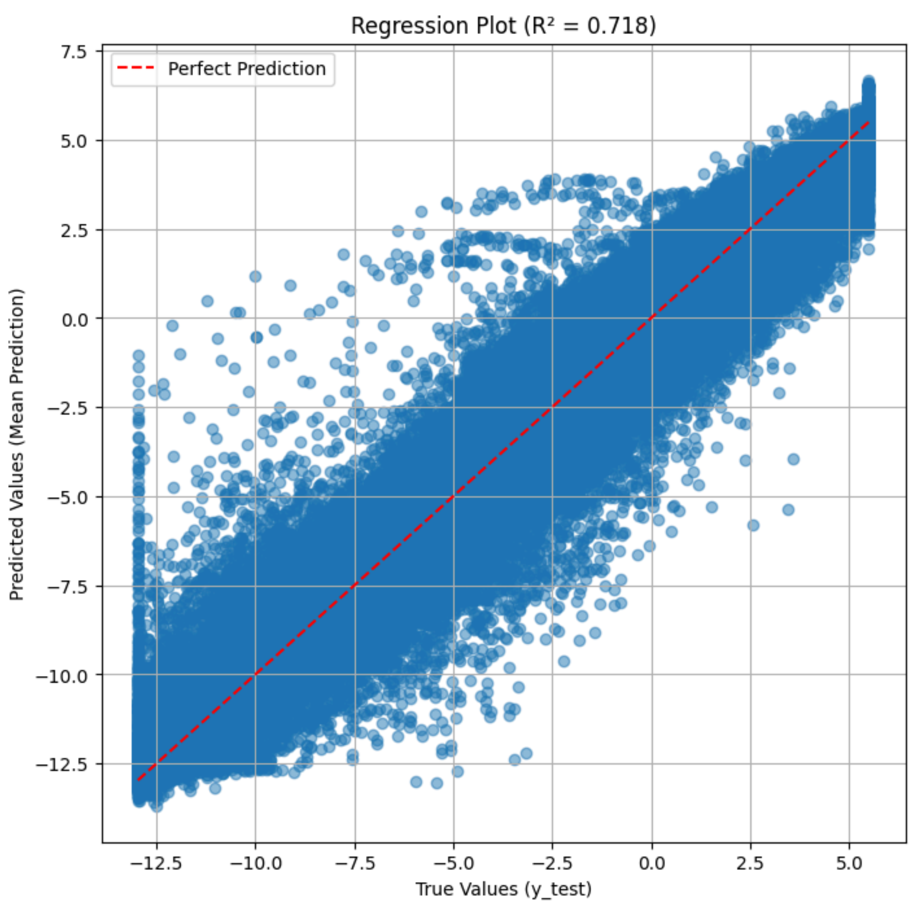
- Input Layer - 6 neurons
- Hidden layers - 20, 30, 40 neurons
- Output Layer - 56 neurons
- Number of epochs - 200
- Learning rate = 3e-4

Results

The model was stable as the average loss was mostly decreasing with every epoch as shown in the figure below.



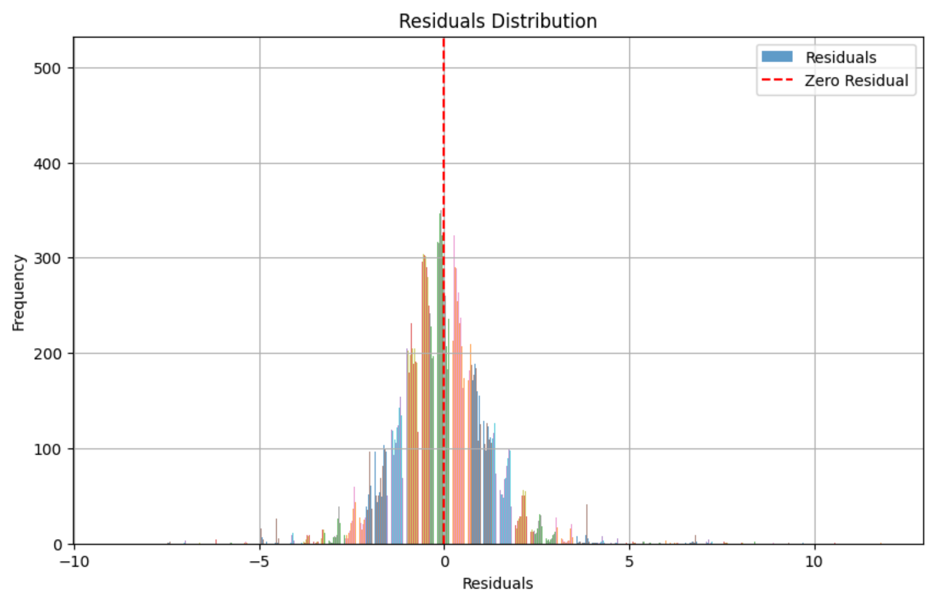
Regression Plot



The model was able to achieve an R^2 value of 0.718.

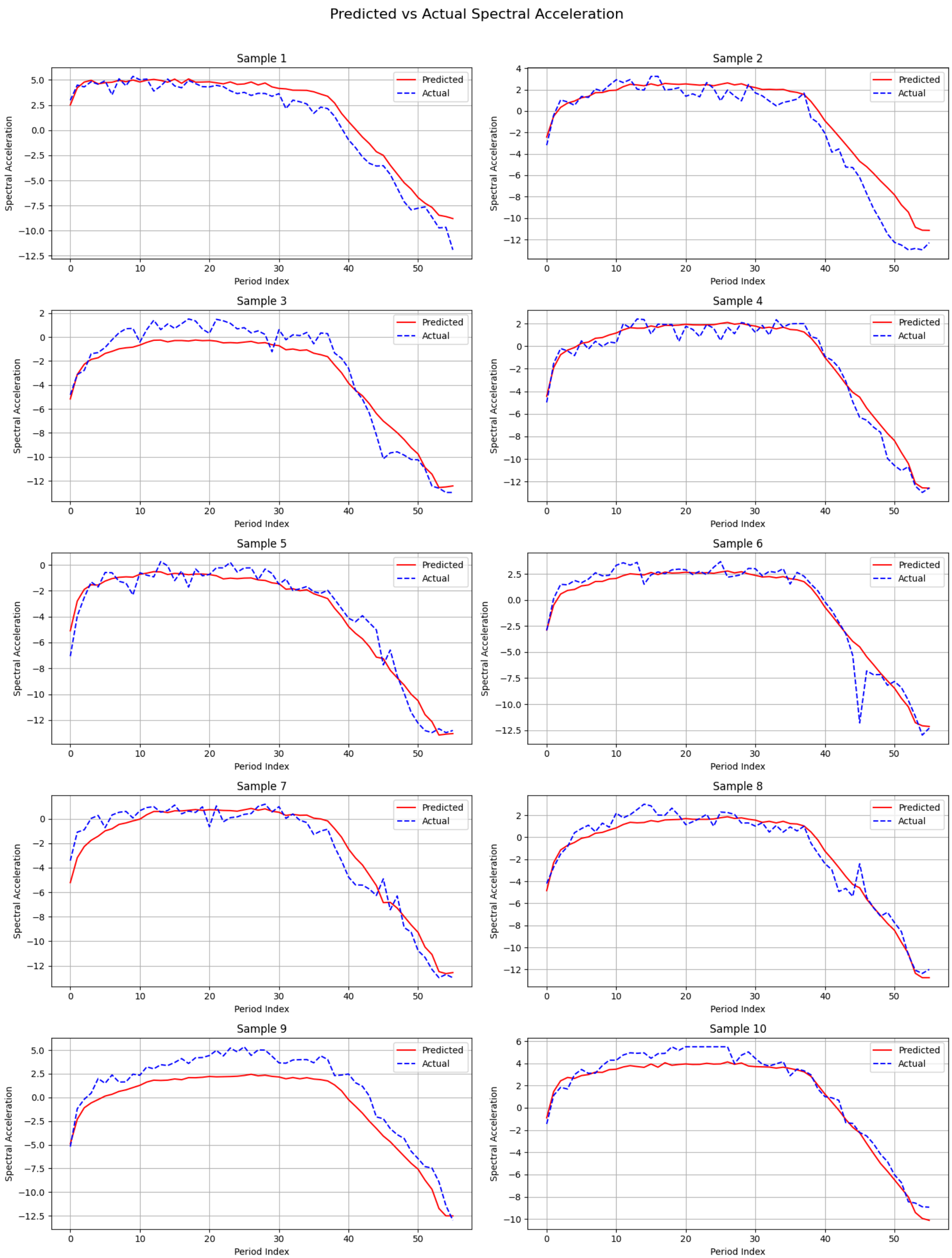
The points were mostly present near the centerline indicating that there isn't too much distortion from the original data.

Residuals Plot



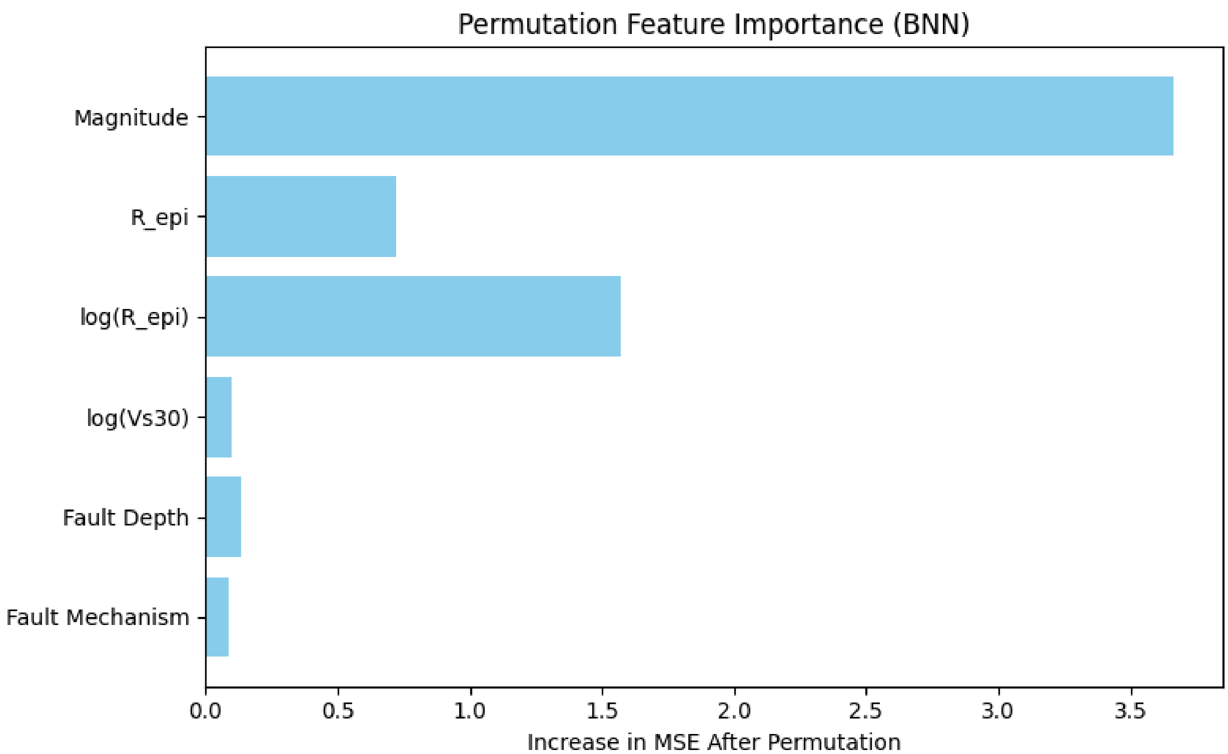
The residual plot is normally distributed with the mean being near zero. This indicates that the predictions of the model are mostly unbiased with equal chance of positive and negative errors. Also, the frequency of high residuals is quite low.

Sample Results



- The predicted plots seem quite close to the actual plot, however the model is not able to predict sudden uncertainties.
- However, such uncertainties in actual data may be due to instrument error, etc.
- Therefore, this model reduces the risk of overfitting the training data.

Feature Importance Analysis



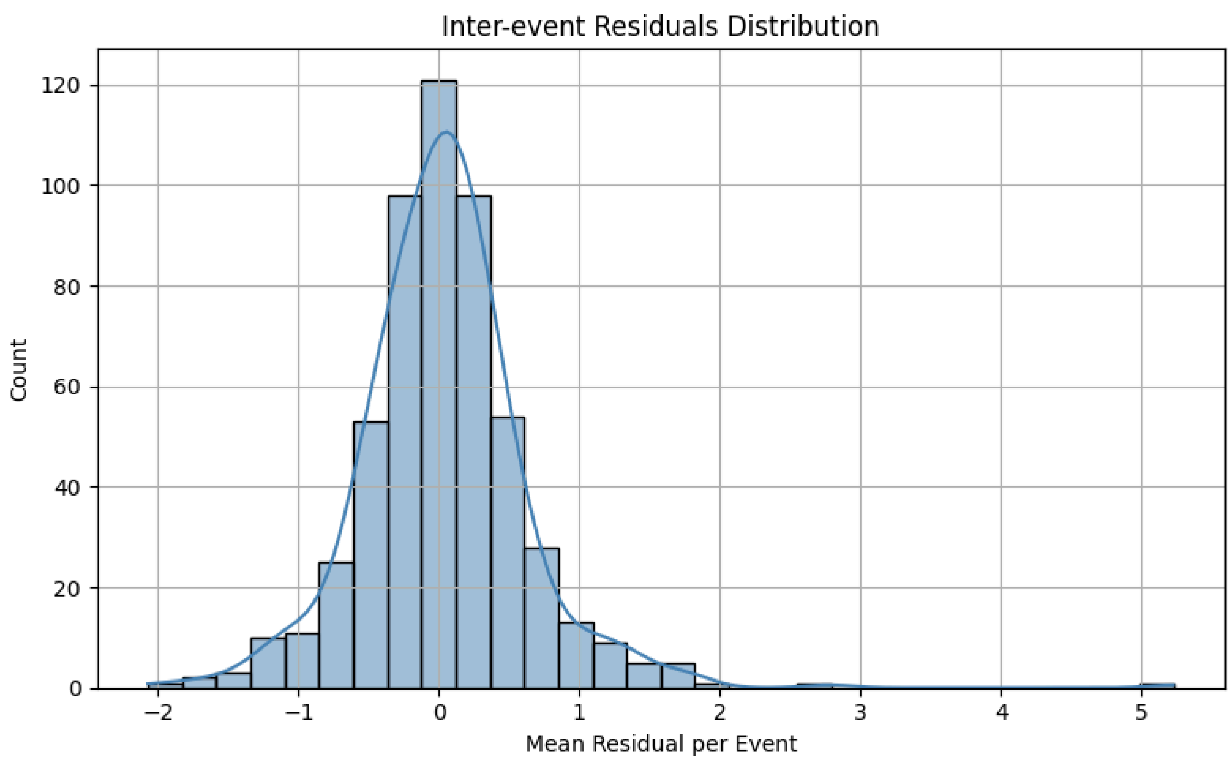
1. **Magnitude (mag)**: This is the single most important parameter in predicting earthquake behaviour.
2. **Repi**: Both normal and log distance parameters account for nearly half of the model's predictive power. This indicates the significant impact of distance on ground motion.

3. **log(Vs30)**: Has the lowest importance, suggesting that site condition (Vs30) has a smaller impact compared to magnitude and distance.

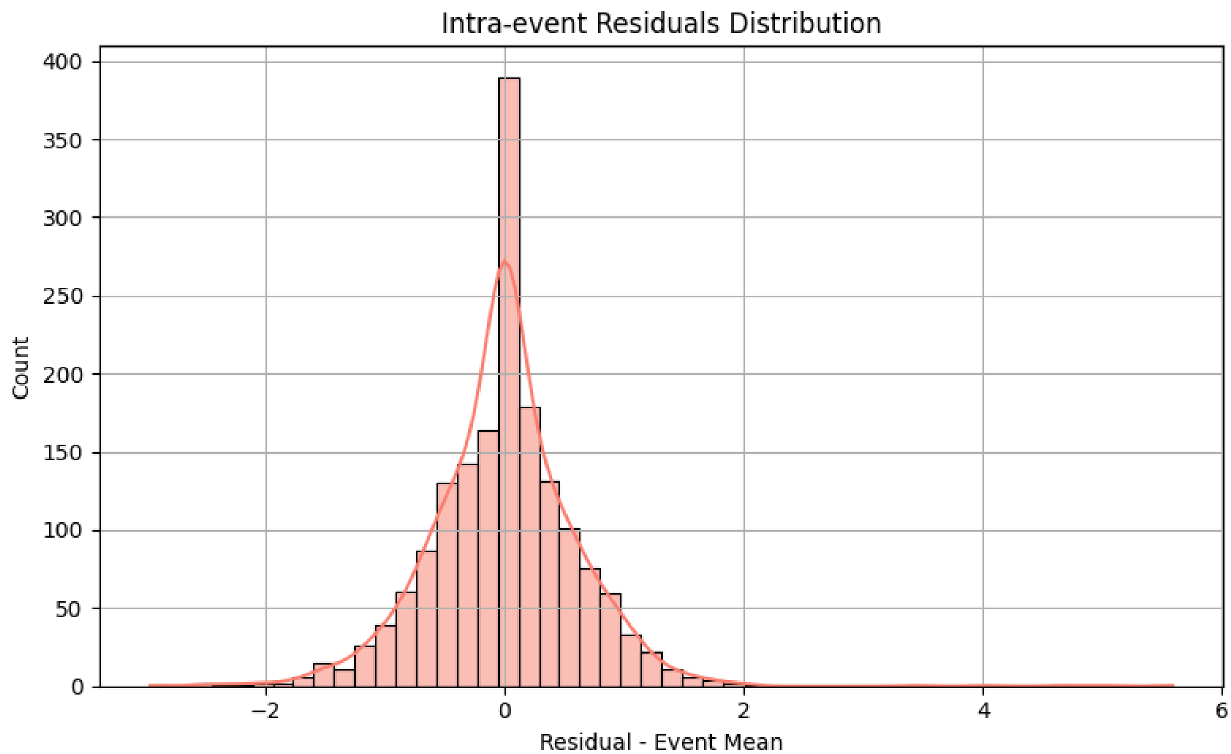
Seismic Design Implications:

- Magnitude and distance (Rjb) are the primary drivers of ground motion predictions.
- Site condition (Vs30) is relatively less critical in this model.

Inter and Intra Event Residuals:



The distribution is normally distributed and peaked around zero showing that there are no biases.

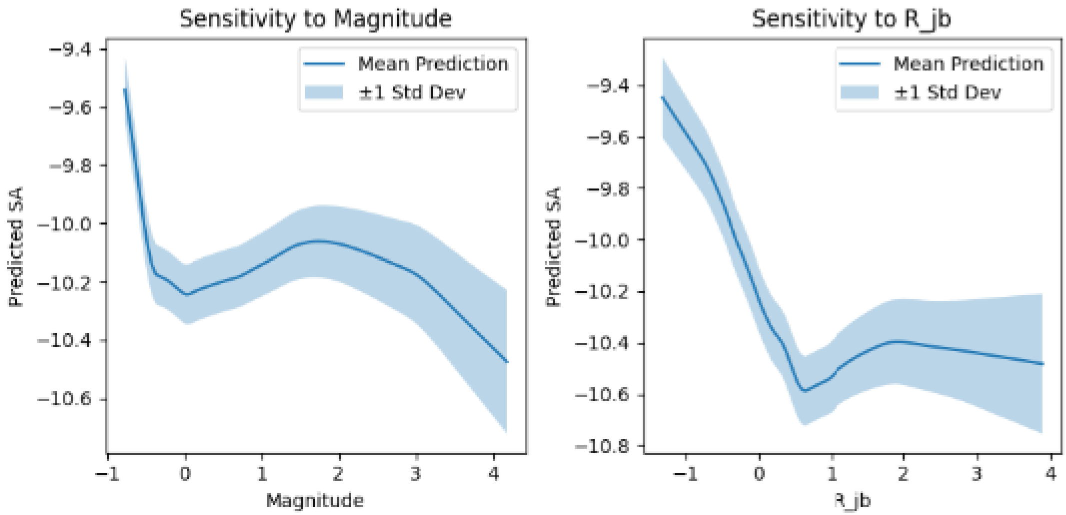


The intra-event residuals also are centered around zero and follow a normal distribution.

The distribution appears slightly right-skewed, which could point to some systematic underprediction in certain within-event samples.

The shape roughly follows a Laplacian-like distribution, often expected in ground motion modeling where intra-event variability dominates noise.

Checking Sensitivity to Parameters:



Both have sharp slopes indicating dependence of each variables to be high.

Non-linear graph shows complex interdependence between y and the parameters.

Conclusion

The BNN model accurately predicted SA while quantifying epistemic uncertainty. Residuals and sensitivity plots were consistent with ground motion theory. This approach offers interpretable, uncertainty-aware predictions essential for seismic risk modeling.