NARAYANA IIT ACADEMY INDIA

SRAMAYEVA JAYATHE

40 YEARS OF EXCELLENCE

BASARA SARASWATHI BHAVAN_MDP N-120

Section: SR **CIRCLE-Synopsis** **Date: 02-05-2020**

Equation of circle,

1). Centre at (0,0) and radius "r" is $x^2+y^2=r^2$

2). Centre at (a,b) and radius "r" is $\left(x-a\right)^2+\left(y-b\right)^2=r^2$

3). Passing through origin and centre at (a, b) is, $\left(x-a\right)^2+\left(y-b\right)^2=a^2+b^2$

4). Centre at (a, b) & touching x – axis is $(x-a)^2+(y-b)^2=b^2 \Rightarrow$ (radius b = r)

5). Centre at (a, b) & touching y – axis is $(x-a)^2+(y-b)^2=a^2$ (radius a = r)

6). Circle, touching x – axis at origin and centre lies on y-axis is $\left(x-0\right)^2+\left(y\pm a\right)^2=a^2$

7). Circle, touching y-axis at origin and centre lies on x-axis $\left(x-a\right)^2+\left(y-0\right)^2=a^2$

8). Equation of a circle, which touches both x & y axis is $i.e.\ \left(x+a\right)^2+\left(y+a\right)^2=a^2.$

(both x & y coordinates are equal & equal to radius)

9). General equation of a circle.

$ax^2+2hxy+by^2+2gx+2fy+c=0$ represents equation of circle then $\Delta\neq 0,\ \ h=0,\ \ a=b.$

$x^2+y^2+2gx+2fy+c=0 \Rightarrow \left(x+g\right)^2+\left(y+f\right)^2=\left(\sqrt{g^2+f^2-c}\right)^2.$

represents equation of circle, then whose centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$

If $g^2+f^2-c=0 \Rightarrow$ point circle, ,

$g^2+f^2-c>0 \Rightarrow$ real circle and

$g^2+f^2-c<0 \Rightarrow$ Imaginary circle

10). Equation of circle $ax^2+ay^2+2gx+2fy+c=0$ has centre at $\left(\frac{-g}{a},\frac{-f}{a}\right)$ and radius $\frac{\sqrt{g^2+f^2-ca}}{|a|}.$

11). Image of circle $\left(x-a\right)^2+\left(y-b\right)^2=r^2$ with respect to the line $lx+my+n=0$ is $\left(x-a^1\right)^2+\left(y-b^1\right)^2=r^2$ $\left(a^1,b^1\right)$ is image of (a,b) with respect to the line $lx+my+n=0$

12). Equation of a circle, passing through origin and making x & y- intercepts as a & b units **(or)**

equation of the circle which passes through the points (0,0), (a,0) and (0,b) is

$x^2+y^2-ax-by=0$

(centre at $\left(\frac{a}{2},\frac{b}{2}\right)$, radius $\frac{\sqrt{a^2+b^2}}{2}$

13). Equation of a circle passing through two points A (a, b) & B (c, d) and having the smallest possible radius is
the circle, having A and B as end points of the diameters.

14). Equation of a circle passing through a given point A and touching the given circle externally at B, havingminimum radius is, the circle, drawn AB as diameter

Diameter form of a circle:

1).The equation of a circle, drawn on the line segment joining the points $(x, y_1)$and$(x_{2,} y_2)$ as diamete$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0 \Rightarrow x^2+y^2-x(x_1+x_2)-y(y_1+y_2)+(x_1x_2)+(y_1y_2)=0.$

Example: Equation of circle on the straight line joining the points of intersection of $ax^2+2hxy+by^2=0$ & $lx+my=1$as diameter is –(solve, given equations, to get x_1+x_2=---, $x_1x_2=$--

Solve again in terms of "y" to get $y_1+y_2=$---, $y_1y_2=$---,)

$$\Rightarrow x^2+y^2-\frac{2(bl-hm)}{am^2-2hlm+bl^2}x-\frac{2(am-hl)y}{am^2-2hlm+bl^2}+\frac{b}{am^2-2lm+bl^2}+\frac{a}{am^2-2hlm+bl^2}=0$$

Intercepts on the axes made by a circle;

1). Intercept on x-axis, made by the circle $x^2+y^2+2gx+2fy+c=0$ is $2\sqrt{g^2-c}$

2). Intercept on y-axis, made by the circle $x^2+y^2+2gx+2fy+c=0$ is $2\sqrt{f^2-c}$

3). If $g^2-c<0$,then the circle does not touch or intersect x- axis

4). If $f^2-c<0$,then the circle does not touch or intersect y- axis

5). If $g^2-c=0 \Rightarrow$the circle touches x-axis (i.e. y-coordinate of the centre = radius of the circle) If $f^2-c=0 \Rightarrow$ the circle touches y-axis (i.e. x-coordinate of the centre = radius of the circle)

6). Equation of a Circle which touches x axis is $(x+a)^2+(y+b)^2=b^2$

similarly equation of a circle which touches the y – axis, is $(x+a)^2+(y+b)^2=a^2$.

Position of a point "P" with respect to given circle, whose centre is at C and radius is r

1) If $P(x_1, y_1)$ lies inside the circle $x^2+y^2+2gx+2fy+c=0$ then $x_1^2+y_1^2+2gx_1+2fy_1+c<0$ (CP < r)

2) If $P(x_1, y_1)$lies outside the circle $x^2+y^2+2gx+2fy+c=0$ then $x_1^2+y_1^2+2gx_1+2fy_1+c>0$ (CP < r)

3) If $P(x_1, y_1)$ lies on the circle $x^2+y^2+2gx+2fy+c=0$ then $x_1^2+y_1^2+2gx_1+2fy_1+c=0$(CP = r)

4). If $P(x_1, y_1)$ lies inside and larger segment of the circle $x^2+y^2+2gx+2fy+c=0$, made by the chord L=0, then centre of the circle & the given point "P" should lie on same side of the chord.

5). Given line L=0 lies between the two circles $S_1=0$ & $S_2=0$ with centresC_1 & C_2 and radii r_1 & r_2 ,without intersecting and touching either of the circles, is $\Rightarrow$

a).distance between C_1 & L $is > r_{1.}$ and distance between C_2 & L $is > r_{2.}$

b). $C_1 \& C_2$ lies on opposite side of the line L=0. i.e. $L(C_1).L(C_2)<0.$

6). The greatest and least distance of an external point A from a circle with centre C and radius r is **AC+r, AC-r**

(The line through A intersects the circle at P and Q The points P and Q are also obtained, using section formula)

Equation of a circle in parametric form:

1). General point on the circle $x^2+y^2=r^2$ is ($r\cos\theta$, $r\sin\theta$)

2). General point on the circle $(x-a)^2+(y-b)^2=r^2$ is ($a+r\cos\theta$, $b+r\sin\theta$), $\theta\in(-\pi,\pi]$

3). Equation of the chord, joining two points on the circle $x^2+y^2=r^2$ whose coordinates are $(r\cos\alpha, r\sin\alpha),(r\cos\beta, r\sin\beta)$, is $x\cos\left(\frac{\alpha+\beta}{2}\right)+y\sin\left(\frac{\alpha+\beta}{2}\right)=r\cos\left(\frac{\alpha-\beta}{2}\right)$

Equation of circumcircle:

1). Equation of circumcircle of a triangle, formed by the lines $L_1=0, L_2=0$ and $L_3=0$, is $L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$ and the values of $\lambda \& \mu$ are selected, using the conditions that the coefficient of x^2 is equal to coefficient of y^2 and coefficient of xy =0

2). Equation of circumcircle of a quadrilateral, formed by the lines $L_1=0, L_2=0$, $L_3=0$ and $L_4=0$, is $L_1L_3+\lambda L_2L_4=0$ and the values of $\lambda \& \mu$ are selected, using the conditions that the coefficient of x^2 and coefficient of y^2 and coefficient of xy =0

3). Lines $L_1: a_1x+b_1y+c_1=0$ & $L:=a_2x+b_2y+c_2=0$ cut the coordinate axes at four con cyclic points, then $a_1a_2=b_1b_{2.}$ and the equation of circle, through these con cyclic points is $L_1L_2+\lambda xy=0$

i.e. $(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=(a_1b_2+a_2b_1)xy$ is the equation of the required circle

4).The pair of lines $ax^2+2hxy+2gx+2fy+c=0$ cut the coordinate axes at four con cyclic points then a = b.

5). If two curves whose equations are $ax^2+2hxy+by^2+2gx+2fy+c=0$ and $a'x^2+2h'xy+b'y^2+2g'x+2f'y+c'=0$ intersect at four con cyclic points then $\frac{a-b}{h}=\frac{a'-b'}{h'}$

Position of two circles

1). If two circles are disjoint, then $c_1c_2>r_1+r_2$ and they have four common tangents (two direct and two transverse)

2). If two circles touch each other externally, then $c_1c_2=r_1+r_2$ and they have three common tangents (two direct and one transverse)

if $c_1(x_1,y_1), c_2(x_2,y_2)$ then P (point of contact) is given by $\left(\frac{r_1x_2+r_2x_1}{r_1+r_2}, \frac{r_1y_2-r_2y_1}{r_1+r_2.}\right)$

3). If two circles touch each other internally, then $c_1c_2=|r_1-r_2|$ and they have one common tangent and P is the point of contact, then P$\left(\frac{r_1x_2-r_2x_1}{r_1-r_2}, \frac{r_1y_2-r_2y_1}{r_1-r_2}\right)$.

4). Two circles, touch each other $\Rightarrow c_1c_2=r_1+r_2\,(or)\,|r_1-r_2|$.

5). If two circles intersect, then $|r_1 - r_2| < c_1c_2 < r_1 + r_2$ and they have two common tangents (two direct)

6). If one circle completely lies in side the other, then $c_1c_2 < |r_1 - r_2|$ and they do not have any common tangent.

7). If two circles are concentric, then their centres are same and they do not have any common tangent.

Intersection of a line and a circle:

Consider the liney = mx + c and the circle $x^2 + y^2 = 1$

$\Rightarrow x^2 + (mx + c)^2 = a^2 \Rightarrow x^2(1 + m^2) + 2mcx + (c^2 - a^2) = 0. \rightarrow 1$

Discriminant of equation 1 is "D"

i) $D = 0 \Rightarrow c^2 = a^2m^2 + a^2. \Rightarrow$ the line touches the circle

ii) $D > 0 \Rightarrow$ the line intersects the circle at two distinct points.

iii) $D < 0 \Rightarrow$ the line & circle are disjoint.

iv) Length of the intercept cut off from a line y = mx + c by a circle $x^2 + y^2 = a^2$

P and Q are the points on the circle, at which the line intersects the circle

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = |x_1 - x_2|\sqrt{1 + m^2} = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}.\sqrt{1 + m^2} = \frac{2\sqrt{a^2(1 + m^2) - c^2}}{\sqrt{1 + m^2}}$$

i) If the line does not intersect or touch the circle then $a^2(1 + m^2) < c^2$

ii) If the line is a tangent to the circle then $PQ = 0$

Equation of tangent:

1). Equation of the tangent, to the circle $x^2 + y^2 = r^2$ at (x_1, y_1) is $xx_1 + yy_1 = r^2$

2). Equation of the tangent, to the circle $(x - a)^2 + (y - b)^2 = r^2$ at (x_1, y_1) is

$(x - a)(x_1 - a) + (y - b)(y_1 - b) = r^2$

3). Equation of the tangent, to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

4). Equation of the tangent, to the circle $x^2 + y^2 = a^2$ at $(a\cos\theta, a\sin\theta)$ is $x\cos\theta + y\sin\theta = a$

5). Equation of the tangent, to the circle $(x - a)^2 + (y - b)^2 = r^2$ at $(a + r\cos\theta, b + r\sin\theta)$ is

$(x - a)\cos\theta + (y - b)\sin\theta = r$

6). The point of intersection of the tangents at the points $P(\alpha)$ and $Q(\beta)$ on the circle

$x^2 + y^2 = r^2$,is $\left(\dfrac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \dfrac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{a\cos\left(\frac{\alpha+\beta}{2}\right)} \right)$

7). $y = mx \pm \sqrt{a^2m^2 + a^2}$ are tangent lines to the circle $x^2 + y^2 = a^2$ at the points A and B are $\left(\dfrac{\pm am}{\sqrt{1 + m^2}}, \dfrac{\mp a}{\sqrt{1 + m^2}} \right)$

8). $(y-b)=m(x-a)\pm\sqrt{a^2m^2+a^2}$ are tangent lines to the circle $(x-a)^2+(y-b)^2=r^2$ at the points A and B are $\left(a+\frac{\pm rm}{\sqrt{1+m^2}}, b+\frac{\mp r}{\sqrt{1+m^2}}\right)$

9). The line $lx+my+1=0$ is a tangent to the circle $(x-a)^2+(y-b)^2=r^2$ then

$$\left|\frac{a.l+bm+1}{\sqrt{l^2+m^2}}\right|=r.\ (d=r)$$

(distance between the centre& the line is equal to radius).

example:

a). If $7l^2-9m^2+8l+1=0$, then the line $lx+my+1=0$ touches the circle with centre (4,0) and radius 3 $\left(7l^2-9m^2+8l+1=0 \Rightarrow (4l+1)^2=9(l^2+m^2) \Rightarrow \frac{4l+0.m+1}{\sqrt{l^2+m^2}}=3\right)$

b). If $16l^2+9m^2-24lm-6l-8m-1=0$, then the line $lx+my+1=0$ touches the circle with centre (3,4) and radius 5.

, $\left(16l^2+9m^2-24lm-6l-8m-1=0 \Rightarrow (3l+4m+1)^2=25(l^2+m^2) \Rightarrow \left|\frac{3l+4m+1}{\sqrt{r^2+m^2}}\right|=5\right)$

c). If $4l^2-5m^2+6l+1=0$, then the line $lx+my+1=0$ touches the circle with centre (3,4) and radius $\sqrt{5}$.

$$\left(4l^2-5m^2+6l+1=0 \Rightarrow (3l+1)^2=5(l^2+m^2) \Rightarrow \frac{3l+1}{\sqrt{l^2+m^2}}=5 \Rightarrow centre\ (3.0)\ radius\ \sqrt{5}.\right)$$

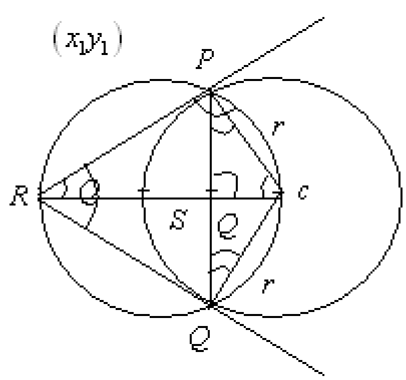
Pair of tangents drawn to a circle and their properties:

$R(x_1,y_1)$ is a point, lies out side the given circle $x^2+y^2=r^2$, then there are two tangents RP & RQ drawn from it, to the circle and C–centre of the circle, then

i) $PR=RQ=\sqrt{x_1^2+y_1^2-r^2}$

(or) $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$ if equation of the circle is $x^2+y^2+2gx+2fy+c=0$

ii). CR is the angle bisector of $\angle QRP$ & $\angle QCP$



iii). $\angle QRP$ & $\angle QCP$ are supplementary.

iv). Equation of PQ (chord of contact) $\Rightarrow xx_1+yy_1-r^2=0$

(or) $xx_1+yy_1+gx+gx_1+f(y+y_1)+c=0$.if equation of the circle is $x^2+y^2+2gx+2fy+c=0$

v). Triangles PSC & QSC are congruent. $\Rightarrow PS = SQ$ & $\angle PSC = \angle QSC = 90^0$.
(S is the point of intersection of CR and PQ)
vi). If equation of the circle is $x^2 + y^2 = r^2$ then.

$$RS = \frac{x_1^2 + y_1^2 - r^2}{\sqrt{x_1^2 + y_1^2}} \quad CS = \frac{r^2}{\sqrt{x_1^2 + y_1^2}}$$

Length of the chord of contact PQ drawn from (x_1, y_1) to the circle $x^2 + y^2 = r^2$ is

$$PQ = \frac{2r\sqrt{x_1^2 + y_1^2 - r^2}}{\sqrt{x_1^2 + y_1^2}}$$

vii). Area of the triangle, formed by the pair of tangents RP & RQ drawn from (x_1, y_1) to the circle

$x^2 + y^2 = r^2$ and the chord of contact of tangents, $\dfrac{r\left(x_1^2 + y_1^2 - r^2\right)^{\frac{3}{2}}}{x_1^2 + y_1^2.}$

viii). $\tan\theta = \dfrac{r}{RP} = \dfrac{r}{\sqrt{x_1^2 + y_1^2 - r^2}}$ and

if $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ then $\tan\theta = \dfrac{\sqrt{g^2 + f^2 - c}}{\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}}$ and $\cos 2\theta = \dfrac{s_1 + c - g^2 - f^2}{s_1 - c + g^2 + f^2}$

ix). Quadrilateral RPCQ is a cyclic quadrilateral $\left(\because \angle P + \angle Q = 180^0 \;\&\; \angle PRQ + \angle QCP = 180^0\right)$.
$\therefore$ circum circle for the triangles RPC , PQC, RQC & $quadrilateral\ RPCQ$ are the same

x). Centre of the circumcircle drawn to the triangle RPC is the midpoint of CR.

xi) Circumcircle drawn to the RPCQ, passes through the centre of the given circle (i.e.) C & R are the end points of the diameter of the circum circle.

xii) equation of pair of tangents drawn to the circle from R (x_1, y_1) is

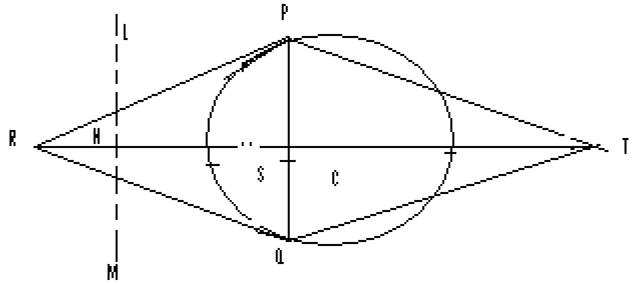
$$T^2 = SS_1 \Rightarrow \left(xx_1 + yy_1 - r^2\right)^2 = \left(x^2 + y^2 - r^2\right)\left(x_1^2 + y_1^2 - r^2\right)$$

(or) $\left(xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c\right)^2 = \left(x^2 + y^2 + 2gx + 2fy + c\right)\left(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c\right)$

xiv) angle between pair of lines.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ or $ax^2 + 2hxy + by^2 = 0$ is $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a + b}$

xv) taking RP and RQ (tangents) as adjacent sides, complete a parallelogram then RPQT is a rhombus.
R,C,T are collinear points
Image of R with respect to PQ (chord of contact) is T. and Image of P with respect to RT is Q.



xvi) L,M are the mid points of RP and RQ then "N" is the mid point of RS.

Equation of normal

Equation of normal drawn to the circle $x^2+y^2+2gx+2fy+c=0$ at (x_1,y_1) is,

$(y_1+f)x-(x_1+g)y=fx_1-gy_1$ (or) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ -g & -f & 1 \end{vmatrix}=0$ and it always passes through its centre.

Length of the tangent:

Length of the tangent drawn to the circle $x^2+y^2+2gx+2fy+c=0$ from an external point (x_1,y_1) is $PT=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}.$

$PT^2=$ power of the point $P(x_1,y_1)$ with respect to the circle

PAB is a secant and PT is a tangent, drawn to a circle then $PT^2=PA.PB$

Director circle:-

1). Locus of the point of intersection of perpendicular tangents to a given circle is known as **director circle**.

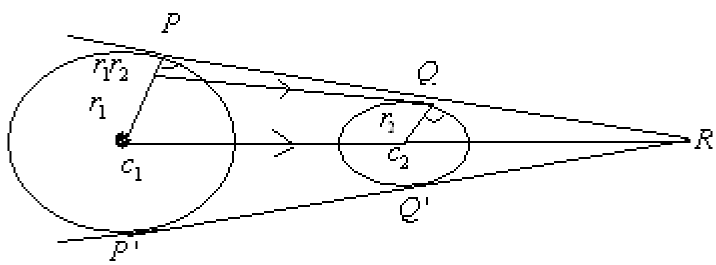
circle and its director circle are concentric circles having radius equal to $\sqrt{2}$ times that of the original circle

2). Director circle, for the circle $x^2+y^2=a^2$, is $x^2+y^2=2a^2$.

3). Director circle, for the circle $(x-a)^2+(y-b)^2=r^2$, is $(x-a)^2+(y-b)^2=2r^2$

Direct common tangents and transverse common tangents

a) Consider two circles with centre C_1 & C_2 and radii r_1, r_2. $(r_1>r_2)$ and they are disjoint.



Pair of tangents are drawn to the circle from the point R.

i). PQR & P'Q'R are direct common tangents $PR=P'R$ & $RQ=RQ' \Rightarrow PQ=P'Q'$

ii). C_1, C_2 and R are collinear points.

iii). Triangles C_1PR & C_2RQ are similar and hence $\frac{r_1}{r_2}=\frac{C_1R}{C_2R}$

iv). If $C_1(a,b)$ and $C_2(c,d)$ then $R\left(\frac{r_1c+r_2a}{r_1+r_2},\frac{r_1d+r_2b}{r_1+r_2}\right)$

v). Length of the direct common tangent is $PQ=\sqrt{d^2-(r_1-r_2)^2}$ (d = distance between the centres)

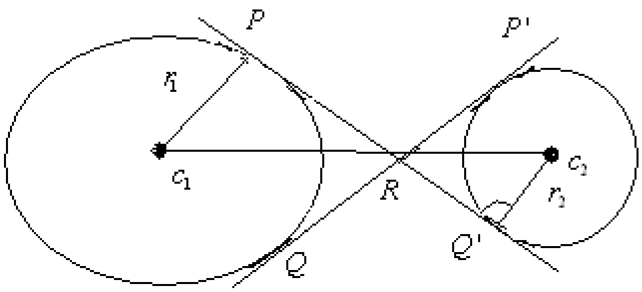
vi). To find the equation of the direct common tangent let $y=mx+c$:

(it passes through the point R and make use of tangency condition i.e d = r)

$(or)\, use\; T^2 = SS_{1.}$

---

b) Consider two circles with centre $C_1\, \&\, C_2$ and radii $r_1, r_{2.}$ $(r_1 > r_2)$ and they are disjoint



Pair of tangents are drawn to the circles from R.

i). PRQ′ & P′ RQ are transverse common tangents and $RP = RQ\ \&\ RP' = RQ' \Rightarrow PQ' = P'Q$

ii). C_1, C_2 *and* R are collinear points.

iii). Triangles $C_1PR\ \&\ C_2RQ^1$ are similar and hence $\frac{r_1}{r_2}=\frac{C_1R}{C_2R}$

iv). If $C_1(a,b)$ and $C_2(c,d)$ then $R\left(\frac{r_1c+r_2a}{r_1+r_2}, \frac{r_1d+r_2b}{r_1+r_2}\right)$

v). Length of transverse common tangents $PQ' = P'Q = \sqrt{d^2-(r_1+r_2)^2}$ (d = distance between the centres)

vi). To find the equation of the transverse common tangent, let $y=mx+c$.

(it passes through the point R and make use of tangency condition i.e d = r)

$(or)\, use\; T^2 = SS_{1.}$)

Angle of intersection of two circles:

The angle of intersection of two circles is defined as the angle between the tangents drawn to the circles

at thepoint of intersection.

1). Angle between the two circles at the point of intersection is given by

$$\cos\theta = \frac{r_1^2+r_2^2-(c_1c_2)^2}{2r_1.r_2}$$

(c_1, c_2 are centres and r_1, r_2 are the radii of the two given circles)

2). If the two circles $S_1 : x^2+y^2+2g_1x+2f_1y+c_1=0$ and $S_2 : x^2+y^2+2g_2x+2f_2y+c_2=0$ intersect orthogonally, then $(c_1c_2)^2 = r_1^2+r_2^2$ (or) $2g_1g_2+2f_1f_2=c_1+c_2$

Common chord:

Chord joining the points of intersection of two given circles is called their common chord and its equation is $S_1 - S_2 = 0.$ Equation of two circles

ar $S_1 : x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ & $S_2 : x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

andtheir equation of common chord is $2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0.$

1). If P and Q are the points of intersection of the two given circles
PQ is prerpendicular to the line joining the points $C_1 \& C_2$

M is the midpoint point of PQ and $PQ = 2PM = 2\sqrt{r_1^2 - (C_1M)^2}$

2). If the length of the common chord is zero, then the circles touch each other and common chord becomes the common tangent to the circles at the common point of contact.

3). The common chord of two circles will be of maximum length, if it is a diameter of the smaller of the two circles,

4). Circle $S_1 = 0$ bisects the circumference of the circle $S_2 = 0$ then common chord of $S_1 = 0 \& S_2 = 0$ passes through centreof $S_2 = 0$.

5). Circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ then $2g'(g - g') + 2f'(f - f') = c - c'.$

Radical axis:

The radical axis of two circles is the locus of a point which moves in such a way that the lengths of the tangents, drawn from it to the two circles are equal.

1). Radical axis of two circles

$S_1 : x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ & $S_2 : x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is given by $S_1 - S_2 = 0$

2). When two circles intersect each other, then radical axis exist and part of it, is the common chord.

(i.e. points on the common chord do not lie on the radical axis)

3). When two circles touch each other internally/externally, then radical axis exists and length of the common chord becomes zero.

4). When two circles do not intersect or touch each other, then only radical axis exists.

5). When one circle lies completely in the other circle, then no common chord / no radical axis exist.

6). Radical axis of two circles is always perpendicular to the line joining the centres of the circles

7). Radical axis bisects a common tangent between the two circles

8). Radical axis of three circles whose centres are non collinear, taken in pairs are concurrent.

9). Centre of the circle, cutting two given circles orthogonally, lies on their radical axis (or) Locus of the centres of the circles, cutting two given circles orthogonally, is their radical axis.

10). Radical centre:- Point of intersection of radical axes of three circles whose centres are non collinear taken in pairs, in called their radical centre.

11). The circle with centre at the radical centre of the three given circles and radius equal to the

length of the tangent from it to one of the circles intersects all the three circles orthogonally.

12). Radical centre of the three circles described on the sides of a triangle, as diameters, is the orthocenter of the triangle.

13). The line $Ax+By+c=0$ cuts the circle $x^2+y^2+ax+by+d=0$ at P and Q and the line $A'x+B'y+c'=0$ cuts the circles $x^2+y^2+a'x+b'y+d'=0$ at R & S and if P,Q,R,S are con cyclic points, then $\begin{vmatrix} a-a' & b-b' & d-d' \\ A & B & C \\ A & B' & C' \end{vmatrix}=0$

(family of circles passing through the points of intersection of $S_1=0$ & $L_1=0$ is $S_1+\lambda L_1=0 \to 1$ and family of circles passing through the points of intersection of $S_2=0$ & $L_2=0$ is $S_2+\mu L_2=0 \to 2$ and 1&2 must represent the same circle)

Equation of a chord whose midpoint is given

1). Equation of a chord, drawn to the circle $x^2+y^2=r^2$ whose mid point (x_1,y_1) is given**,** is $xx_1+yy_1=x_1^2+y_1^2$

2). Equation of a chord, drawn to the circle $x^2+y^2+2gx+2fy+c=0$ whose mid point (x_1,y_1) is given, is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=x_1^2+y_1^2+2gx_1+2fy_1+c.$

3). The shortest chord of a circle passing through a point M, inside the circle, is one chord whose middle point is M

Some examples:

1). Locus of the mid points of the chords of the circle $x^2+y^2=r^2$ which subtend right angle at the centre, is $x^2+y^2=\frac{r^2}{2}$

2). Tangents are drawn to the circle $x^2+y^2=a^2$ from a point on the line $lx+my=1.$ Then locus of the mid point of the chord of contact is $x^2+y^2-a^2(lx+my)=0.$

3). Condition on a and b, if two chords drawn to the circle $2x^2-2ax+2y^2-2by=0\ (a\neq 0, b\neq 0)$ from a point (a, b/2) are bisected by x – axis, is $a^2>2b^2.$

Family of circles:

1). Equation of family of circles, passing through the intersection of a circle S : $x^2+y^2+2gx+2fy+c=0$ and the line L: $lx+my+n=0$ is $S+\lambda L=0.$

2). Equation of the family of circles passing through the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+\lambda\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0.$$ (i.e. S+ λ (equation of line AB) = 0)

(taking A & B as end points of a diameter, draw a circle and line L, passing through two points A & B.)

3). Equation of family of circles passing through the intersection of two circles $S_1 = 0$ & $S_2 = 0$ is $S_1+\lambda S_2=0\ (or)\ S_1+\lambda(S_1-S_2)=0.$ (or) $S_1+\lambda$ (common chord) = 0 $(\lambda \neq -1)$

4). Equation of family of circles touching the circle $S: x^2+y^2+2gx+2fy+c=0$ at the point (x_1, y_1) is $S+\lambda L=0$ where L is the tangent line drawn to the circle at (x_1, y_1)

(i.e. $x^2+y^2+2gx+2fy+c+\lambda(xx_1+yy_1+g(x+x_1)+f(y+y_1)+c)=0.$)

5). Equation of family of circles touching the given line $L=0$ at the point $L=0$ is $S+\lambda L=0$ (i.e. $(x-x_1)^2+(y-y_1)^2+\lambda L=0$) (here S = 0 is the point circle taking (x_1, y_1) as centre and "0" as radius)

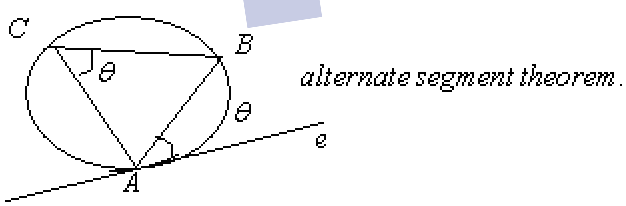
6). Equation of a line, passing through two points A and B on the given circle which are at a distance of "a" units, from a point "c" given on the circle, is nothing but the common chord of the given circle, and the circle centered at C, with "a" as radius.

Miscellaneous:

1). Area of an equilateral triangle inscribed in a circle of radius r is $\frac{\sqrt{3}}{4}\left(\sqrt{3}r\right)^2$

2). Image of orthocenter of a triangle with respect to any side of the triangle lies on its circum circle.

3)



4). If a secant, drawn from a point P intersects a circle at A and B, then product of the distances PA and PB is constant (i.e. $PT^2 = PA.PB$)

5). The points $(a,b),(a,-b),(-a,b)\&(-a,-b)$ lie on a circle with centre (0, 0) and its equation is $x^2+y^2=a^2+b^2.$

6). If the centre of a circle is not a rational point then the maximum number of rational points on
its circumference is 2

7). Number of lattice points (integral coordinates) lying in the interior of the circle $x^2+y^2=25\ \ is\ \ 69.$

8). Area of an equilateral triangle. inscribed in the circle $x^2+y^2+2gx+2fy+c=0\ is\ \frac{3\sqrt{3}}{4}\left(g^2+f^2-c\right).$

(area = $\frac{\sqrt{3}}{4}$.(side)$^{\mathbf{2}}$ and side = $\sqrt{3}r$)

9). Chord of contact of tangents, drawn from points, lie on a line, to a circle, are concurrent

(Chord of contact of tangents, drawn from three collinear points, to a circle are passing through a fixed point)

10). Locus of image of a point (x_1, y_1) to the family of lines $L_1 + m L_2 = 0$, is a circle, with centre at the point of intersection of L_1 and L_2 and passing through the point (x_1, y_1)

11). Shortest distance between the line ax+by+c=0 and the given circle "c" with centre (x_1, y_1) radius r is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}-r\right|$

Some of the standard locus.

1). A point moves in such a manner that the sum of the squares of its distances from the vertices of a triangle isconstant then the locus of the point is a circle having its centre at the centroid of the triangle.

2). "O" is a fixed point and a point R moves along a fixed line L, not passing through "O". If S is a point on OR suchthat $OR.OS=\lambda^2,(\lambda\in R)$ then locus of S is a circle.

3). Locus of the point P such that $PA^2+PB^2=AB^2$ where A and B are two given points, is circle, has center at mid point of AB.

4). A circle of constant radius of "3r" passes through the origin and cuts the coordinate axes at A and B. Then locus ofthe centroid of triangle OAB is the circle $x^2+y^2=4r^2$

5). Locus of a point which moves in such away that the sum of the squares of its distances from the four sides of asquare is constant, is a circle.

6). If $A=(a,0), B(-a,0)$ are two fixed points $\forall\ a\in(-\infty,0)$ and P is a moving point such that $PA=n\,PB\,(n\neq o)$.then

6a). if $|n|=1,$ then locus of P is a line.

6b). if $|n|\neq 1$ the locus of P is a circle, which never pass through A & B.

6c). if $0<n<1$ then locus of P is a circle and A lies inside and B lies outside the circle

6d). if $n>1$ then locus of P is a circle and A lies outside and B lies inside the circle