Section: Senior **ELLIPSE** **Date:29-05-2020**

1. An ellipse is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distances from a fixed straight line (called directrix) is always constant which is always less than unity. (fixed ratio is eccentricity which is denoted by "e")
2. General second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents ellipse if $abc+2fgh-af^2-bg^2-ch^2\neq 0$ and $h^2<ab$
3.

	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a^2>b^2)$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (b^2>a^2)$
Equation of the major axis	Y = 0	X = 0
Equation of the minor axis	X = 0	Y = 0
Centre	(0,0)	(0,0)
Length of the major axis	2a	2b
Length of the minor axis	2b	2a
Coordinates of the foci	$(\pm ae,0)$	$x^2+y^2=0$
Equation of the directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{b}{e}$
Length of the latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Coordinates of the end points of the latus rectum	$\left(\pm ae,\pm\frac{b^2}{a}\right)$	$\left(\pm\frac{a^2}{b},\pm be\right)$
Eccentricity	$e^2=1-\frac{b^2}{a^2}$	$e^2=1-\frac{a^2}{b^2}$
Coordinates of the vertices	$(\pm a,0)$	$(0,\pm b)$
Focal distance	$a\pm ex$	$b\pm ey.$

4. Equation of ellipse, when focus S (a, b) and directrix $lx+my+n=0$ and its eccentricity "e" is given by $(x-a)^2+(y-b)^2=e^2\left(\frac{lx+my+n}{\sqrt{l^2+m^2}}\right)^{2.}$

$(SP=e\,PM \Rightarrow \quad SP^2=e^2(PM)^{2.})$

5. Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{(\text{distance between any point } P(x,y) \text{ \& minor axis})^2}{(\text{length of the semi}-\text{major axis})^2}+\frac{(\text{distance between any point } P(x,y) \text{ \& major axis})^2}{(\text{length of the semi}-\text{major axis})^2}=1.$$

i.e. Equation of the minor axis is $a_1x+b_1y+c_1=0$ and equation of the major axis is $a_2x+b_2y+c_2=0$ & Length of major axis 2p and length of minor axis is 2q then equation of the ellipse is $\dfrac{\left(\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{p^2}+\dfrac{\left(\dfrac{b_1x-a_1y+c_2}{\sqrt{a_1^2+b_1^2}}\right)^2}{q^2}=1$.

Standard Locus:

1. An ellipse is the locus of a point, which moves in a plane, such a way that the sum of its distances from two fixed points in the same plane, is constant (greater than, the distance between the two fixed points).
   ($i.e.\ PA+PB=k,\quad k>AB.$) (If $PA+PB=k,\ k=AB$ locus of P is a line segment AB and k <AB, no such P exists)
   (Here the two points A and B are foci SS' =2ae, PS + PS' = 2a)
2. A rod of given length, slides between two perpendicular lines. Then, locus of a point, which lies on the rod, and divides the distance between the lines in the ratio 1: λ , is ellipse and its eccentricity is $\sqrt{1-\lambda^2}$.
3. A point moves so that the sum of the squares of its distances from two intersecting lines is, constant then its locus is an ellipse.
   (Let the two lines $y=x\tan\theta$ and $y=-x\tan\theta$ the point P(h,k)
   $\Rightarrow\left(\dfrac{h\tan\theta-k}{\sqrt{2}}\right)^2+\left(\dfrac{-h\tan\theta-k}{\sqrt{2}}\right)^2=\lambda^2$.which is the equation of ellipse)

Position of a point with respect to ellipse:

1. $P(x_1,y_1)$ lies outside, on or inside the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ if $\dfrac{x_1^2}{a^2}+\dfrac{y_1^2}{b^2}-1>0,\ =0,\ <0.$

Eccentric angle:

1. General point on ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is $(a\cos\theta, b\sin\theta)$.
2. Equation of the chord, joining the points $(a\cos\theta_1, b\sin\theta_1)$.and $(a\cos\theta_2, b\sin\theta_2)$.on the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, is $\dfrac{x}{a}\cos\left(\dfrac{\theta_1+\theta_2}{2}\right)+\dfrac{y}{b}\sin\left(\dfrac{\theta_1+\theta_2}{2}\right)=\cos\left(\dfrac{\theta_1-\theta_2}{2}\right)$
3. Focal chord, joining the points $(a\cos\theta_1, b\sin\theta_1)$.and $(a\cos\theta_2, b\sin\theta_2)$, then
   $\tan\left(\dfrac{\theta_1}{2}\right)\tan\left(\dfrac{\theta_2}{2}\right)+\left(\dfrac{1\mp e}{1\pm e}\right)=0$

4. If the eccentricity angles of the extremities of a focal chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

are $¯_1$ & $¯_2$ then eccentricity of the ellipse is $\frac{\sin ¯_1+\sin ¯_2}{\sin(¯_1+¯_2)}=\frac{\cos\left(\frac{¯_1-¯_2}{2}\right)}{\cos\left(\frac{¯_1+¯_2}{2}\right)}$ =e

5. Let $A_i\left(a\cos ¯_i, b\sin ¯_i\right).\ (i=1,2,3\ldots,n)$ be n points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci S and S' such that $A_1SA_2, A_2S'A_3, A_3SA_4, A_4S'A_5\ldots(\ )$ are focal chords then,

$$\tan\left(\frac{¯_1}{2}\right)\tan\left(\frac{¯_2}{2}\right)=\cot\left(\frac{¯_2}{2}\right)\cot\left(\frac{¯_3}{2}\right)=\tan\left(\frac{¯_3}{2}\right)\tan\left(\frac{¯_4}{2}\right)=\cot\left(\frac{¯_4}{2}\right)\cot\left(\frac{¯_5}{2}\right)=\ldots.$$

6. If S and S' are the foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentricity is **e** and P is a point on it, then $\tan\left(\frac{\angle PSS'}{2}\right)\tan\left(\frac{\angle PS'S}{2}\right)=\left(\frac{1-e}{1+e}\right)$

7a) Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ and P be any point on it and S and S' are the foci, then ,Area of triangle SPS**'** = $abe\sin ¯$

7b). Area of triangle SPS**'** is maximum when $¯=\frac{®}{2}$ and its maximum area is **abe**

And "P" becomes end point of minor axis when the area is maximum

8). Triangle ABC, inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ where A$(a\cos\sim, b\sin\sim)$.; B $(a\cos\ \ , b\sin\ \ )$.

C $(a\cos¥, b\sin¥)$.and the corresponding points on its auxiliary circle are P $(a\cos\sim, a\sin\sim)$. Q$(a\cos\ \ , b\sin\ \ )$. R$(a\cos¥, a\sin¥)$.

8a) Area of the triangle ABC = $\frac{ab}{2}\begin{vmatrix}\cos\sim & \sin\sim & 1\\ \cos & \sin & 1\\ \cos¥ & \sin¥ & 1\end{vmatrix}$ & Area of the triangle PQR =

$$\frac{a^2}{2}\begin{vmatrix}\cos\sim & \sin\sim & 1\\ \cos & \sin & 1\\ \cos¥ & \sin¥ & 1\end{vmatrix}$$

8b) Area of the triangle ABC = (b/a) Area of the triangle PQR

8c) Area of the triangle ABC is maximum when area of the triangle PQR is maximum and eccentric angles of the vertices of the triangle of maximum area inscribed in the ellipse, differ by $\frac{2\text{®}}{3}$

8d). Three points A, B, C are taken on the ellipse with the eccentric angles $\text{¯}, \text{¯}+\sim, \text{¯}+2\sim$ respectively, then

i) area of the triangle ABC is $2ab\sin^2\left(\frac{\sim}{2}\right)\sin\sim$

ii) area of the triangle is maximum when $\sim = \frac{2\text{®}}{3}$

iii) maximum area of the triangle is $\frac{3\sqrt{3}}{4}ab$

9). PSQ is a focal chord drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ then $\frac{2SP.SQ}{SP+SQ}=\frac{b^2}{a}$

(i. e. Harmonic mean of the segments of focal chord semi latus rectum).

10). Angle between the lines, joining the end points of minor axis of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ with its focus is $\frac{\text{®}}{2}$, then eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

11). The line $lx+my+n=0$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in two points whose eccentric angles differ by $\frac{\text{®}}{2}$ then $l^2a^2+b^2m^2=2n^2.$

12). AB and CD are the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and if the points A,B,C,D are concyclic points, then sum of eccentric angles of the points is $2n\text{®},\ n\in z.$

13). Radius of the circle, passing through the points of intersection of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2-y^2=0$ is $\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

14). Eccentric angles of three points P, Q, R on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $\sim, \frac{\text{®}}{2}+\sim, \text{®}+\sim.$

A circle through P, Q, R cuts the ellipse again at S, then the eccentric angle of S is $\frac{\text{®}}{2}-3\sim$

15). If two concentric ellipses be such that the foci of one be on the other and if e and e' are their eccentricities then the angle between their axes is $\cos^{-1}\left(\frac{\sqrt{e^2+(e')^2-1}}{e.e'}\right)$

16). A variable point P on an ellipse of eccentricity e is joined to its foci S and S' then

i). the coordinates of its incentre is $\left(ae\cos\text{¯}, \frac{be\sin\text{¯}}{e+1}\right)$

ii). locus of the incentre of the triangle SPS' is also an ellipse with eccentricity $\sqrt{\frac{2e}{1+e}}$

Equation of Tangent:

1a). The line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow c^2 = a^2m^2 + b^2$

1b). The line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the points

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

1c). Condition for the line $y = mx + c$, to intersect the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,at two distinct points is $c^2 > a^2m^2 + b^2$

1d). Condition for the line $y = mx + c$ does not intersect or touch the ellipse

$y = mx \pm \sqrt{a^2m^2+b^2}$

1e). Condition for the line $lx + my + n = 0$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $a^2l^2 + b^2m^2 = n^2$

1f). The line $lx + my = n$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(\frac{a^2l}{n}, \frac{b^2m}{n}\right)$

(compare $lx + my = n$ and $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$)

1g). The line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is, $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$

2a). Equation of the tangent at drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_1, y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

2b). Equation of the tangent drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $(a\cos\theta, b\sin\theta)$ is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1.$

2c). Equation of the tangent drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y = mx \pm \sqrt{a^2m^2+b^2}$

3a). Slope of the tangent, drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_1, y_1) *is* $\frac{-b^2x_1}{a^2y_1}$ and

3b). Slope of the tangent, drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at at$(a\cos\theta, b\sin\theta)$ is $\frac{-b}{a}\cot\theta$.

4). Point of intersection of tangents, drawn to the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at

$(a\cos\theta_1, b\sin\theta_1)$ & $(a\cos\theta_2, b\sin\theta_2)$ is $\left(\frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right)$

5). Equation of the tangent at P$(a\cos\theta, b\sin\theta)$, to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$meets its major and minor axis at A and B, and $(AB = a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta)$ then,

a). the locus of the midpoint of AB is $a^2y^2 + b^2x^2 = 4x^2y^2$

b). Minimum length of AB is a+b

c). Minimum length of AB is, obtained when $\tan^2\theta = \frac{b}{a}$

d). Minimum area of the triangle OAB, is ab

6). Product of the perpendiculars, calculated from the foci upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is b^2

Example: An ellipse whose length of major axis is 2a and that of minor axis is 2b, slides between twoperpendicular lines . Then

i). the locus of its centre is $x^2+y^2=a^2+b^2$

ii). the locus of its foci is, $\left(x^2+y^2\right)\left(x^2y^2+b^4\right)=4a^2x^2y^2$

7). Tangents at the extremities of latus rectum, of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect, on thecorrespondingdirectrix,

8). Locus of the foot of the perpendicular on a tangent to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from either of its foci isauxiliarycircle $x^2+y^2=a^2$

9). If the tangent at any point P, on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the major axis in T and minor axis in T' then

a). $CN.CT=a^2$

b). $CN'.CT'=b^2$

(Where N, N' are the feet of the perpendiculars from P on the respective axes & C-centre)

10. Locus of the point of intersection of the tangents, drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at two differentpointswhose eccentric angles are $\theta, \theta+\frac{\alpha}{2}$ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

11. Tangent at a point P $\left(a\cos\theta, b\sin\theta\right)$, on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, intersects the major axis in T. N,is thefoot of the perpendicular from P to same axis. The circle drawn on NT as diameter intersects theauxiliary circle,orthogonally,

12. The portion of the tangent at P$\left(a\cos\theta, b\sin\theta\right)$to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, intercepted between thecurve andthe directrix subtends, a right angle at the corresponding focus (S) (where Q is the point of intersection of tangent with corresponding directrix).

13. **Director circle:** Locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, which areperpendicular to each other, is director circle. i.e. $x^2+y^2=a^2+b^2$.

14. Locus of the foot of the perpendicular, drawn from the centre upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *is* $\left(x^2+y^2\right)^2=a^2x^2+b^2y^2$

15. A tangent is drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ and 'N" be the foot of the perpendiculardrawn from origin (0, 0) to tangent at P. Then

a) coordinates of the point N $\left(\frac{ab^2\cos\theta}{a^2\sin^2\theta+b^2\cos^2\theta}, \frac{a^2b\sin\theta}{a^2\sin^2\theta+b^2\cos^2\theta}\right)$

b) Length of the line segment NP = $\frac{(a^2-b^2)\sin\theta\cos\theta}{2\sqrt{(a^2\sin^2\theta+b^2\cos^2\theta)}}$ and OP =

$\frac{ab}{2\sqrt{(a^2\sin^2\theta+b^2\cos^2\theta)}}$

,

c) Area of the triangle ONP = $\frac{ab(a^2-b^2)\sin\theta\cos\theta}{2(a^2\sin^2\theta+b^2\cos^2\theta)} = \frac{1}{2}\frac{(a^2-b^2)}{\frac{a}{b}\tan\theta+\frac{b}{a}\cot\theta}$

and area of the triangle ONP is maximum when $\tan\theta=\frac{b}{a}$ and

$P\left(\pm\frac{a^2}{\sqrt{a^2+b^2}}, \pm\frac{b^2}{\sqrt{a^2+b^2}}\right)$

16) The tangent at any point "P" on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersects the tangents at the vertices A (a, 0), A' (-a, 0) in the points, V &V , then

a) $(AV)\times(A'V')=b^2$

b) $\angle V'SV=90^o$

c) $V'S'SV$ is a cyclic quadrilateral.

d) Circle, drawn on V V' as diameter, passes through foci S, S'

Normal:

1) Equation of normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_1, y_1) $is\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

2) Equation of normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $(a\cos\theta, b\sin\theta)$ is a$\sec\theta-by\,\mathrm{cosec}\,\theta=a^2-b^2$

3) Equation of normal drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(\pm\frac{a^2}{\sqrt{a^2+b^2m^2,}}\pm\frac{b^2m}{\sqrt{a^2+b^2m}}\right)$ is

$y=mx\pm\frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$

4) Condition for the line $y=mx+c$, is to be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$c^2=\frac{m^2(a^2-b^2)^2}{a^2+b^2m^2}$

5) The line $lx+my+n=0$ is normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$

6) Normal at an end of a latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, passes through one extremity of theminor axis,then the eccentricity "e" of the ellipse is given by $e^4+e^2-1=0$. and e = $\sqrt{\frac{\sqrt{5}-1}{2}}$

7) If the normal at any point "P" of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the major and minor axis in G and Hrespectively and CF is perpendicular upon this normal from centre "C" of the ellipse, then

a) $PF.PG=b^2$

b) PF. PH = a²

c) $a^2(CG)^2+b^2(CH)^2=(a^2-b^2)^2$

d) $(PG)^2=(SP)(S'P)(1-e^2)$.

8) If the normal at the points, whose eccentric angles are ~, ,¥ on the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ areconcurrent,then $\begin{vmatrix} \sec\text{~} & \operatorname{cos}ec\text{~} & 1 \\ \sec & \operatorname{cos}ec & 1 \\ \sec\text{¥} & \operatorname{cos}ec\text{ª} & 1 \end{vmatrix}=0$

9) If the normal at $P(x_{1,}y_1)\ Q(x_{2,}y_2)$ and $R(x_{3,}y_3)$, of the ellipse, are concurrent then

$$\Rightarrow \begin{vmatrix} \frac{a^2}{x_1} & -\frac{b^2}{y_1} & a^2-b^2 \\ \frac{a^2}{x_2} & -\frac{b^2}{y_2} & a^2-b^2 \\ \frac{a^2}{x_3} & -\frac{b^2}{y_3} & a^2-b^2 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} x_1 & y_1 & x_1y_1 \\ x_2 & y_2 & x_2y_2 \\ x_3 & y_3 & x_3y_3 \end{vmatrix}=0$$

Conormal points:

1) Points on the ellipse, the normal at which, to the ellipse, pass through a given point are called co normal points.

2) A maximum of four normal can be drawn from a given point to an ellipse.

3) Sum of the eccentric angles of the co normal points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an odd multiple of® (i.e. ⇒ ¯$_1$ + ¯$_2$ + ¯$_3$ + ¯$_4$ = $(2n+1)$®)

4) If ¯$_1$, ¯$_2$, ¯$_3$, ¯$_4$ are eccentric angles of four points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

the normal, at which are concurrent, then

a) $\sum\cos($¯$_1+$¯$_2)=0$

b) $\sum\sin($¯$_1+$¯$_2)=0$

c) $\sin\left(\bar{\ }_1+\bar{\ }_2\right)+\sin\left(\bar{\ }_2+\bar{\ }_3\right)+\sin\left(\bar{\ }_3+\bar{\ }_1\right)=0.$

$\left(use\sum z_1 z_2=0\Rightarrow\sum\left(\cos\left(\bar{\ }_1+\bar{\ }_2\right)+i\sin\left(\bar{\ }_1+\bar{\ }_2\right)\right)=0\right)$

5) If the normal, drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at four points $P\left(x_{1,}\ y_1\right)$, $Q\left(x_{2,} y_2\right)$, $R\left(x_{3,}\ y_3\right)$ and $S\left(x_{4,}\ y_4\right)$ are concurrent, then. $\left(x_1+x_2+x_3+x_4\right)\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\right)=4.$

Reflection:

1) Tangent and normal at any point P$\left(x_{1,} y_1\right)$ of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bisect the external and internalAnglebetween the focal radii to the point.
($\Rightarrow PG$ Bisects angle $\angle SPS'$ (i.e. normal PG at P bisects the internal angle between the focal radii SP
and S'P)**Image of S' with respect to the tangent at P, lies on the line SP extended**)
(example: Equation of a tangent, drawn to an ellipse at P is x + 4y = 9 = 0 and whose foci are]
$\left(\frac{\pm 3}{\sqrt{2}},0\right)$ then thepoint P (1,2))

2) If an incident ray passing through, the focus (S) strikes the ellipse, then the reflected ray will pass throughthe other focus (S').

3) If SM and S'M' are perpendiculars from the foci S, S' respectively, upon a tangent to the ellipse, then CM andCM' are parallel to S'P & SP respectively.

Equation of chord whose midpoint is given:

1) Equation of a chord which is bisected at $(x_{1,}\ y_1)$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=\frac{{x_1}^2}{a^2}+\frac{{y_1}^2}{b^2}$

Some examples:a). Mid point of the chord, intercepted on the line $2x-y+3=0$ by the ellips $\frac{x^2}{10}+\frac{y^2}{6}=1is\left(\frac{-30}{23},\frac{9}{23}\right)$

b) Locus of the mid point of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$\left(\frac{a^6}{x^2}+\frac{b^6}{y^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\left(a^2-b^2\right)^{2.}$

Pair of tangents:

1) Equation of pair of tangents drawn from $(x_{1,}\ y_1)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$\left(\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1\right)^2=\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1\right)$(Equation of PA & PB is given by. $T^2=SS_1$)

Equation of chord of contact:

1) Equation of chord of contact of tangents drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from the point (x_1,y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

2) Locus of the point, the chord of contact of tangents from which, to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the circle $x^2+y^2=c^2$ is $b^4c^2x^2+a^4c^2y^2=a^4b^4$

3) Locus of the point, the chord of contact of tangents from which, to the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtend 90^0 at the centre, is $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$

4) Tangents are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at points where it is intersected by the line $lx+my+n=0.$ then, the point of intersection of tangents at these points, is. $\left(-\frac{a^2l}{n},\frac{-b^2m}{n}\right)$

5) From $P(h,k), PA, PB$ tangents are drawn to the ellipse $\frac{x^2}{y^2}+\frac{y^2}{b^2}=1$ and given that area of the triangle formed by, chord of contact AB, and the co-ordinate axes, is constant, then locus of P is hyperbola.