
Section: SR **STRAIGHT LINES** **Date: 22-05-2020**

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a). Centroid of the triangle ABC $A(x_1,y_1)\ B(x_2,y_2)\ C(x_3,y_3)$ with vertices, is $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

b). Incentre of the triangle ABC $A(x_1,y_1)\ B(x_2,y_2)\ C(x_3,y_3)$ with vertices and AB = c, BC = a, CA = a, is $\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+ay_2+ay_3}{a+b+c}\right),$

c).Incentre of the triangle ABC $A(x_1,y_1)\ B(x_2,y_2)\ C(x_3,y_3)$ with vertices and AB = c, BC = a, CA = a, is I and AI:ID=b+c:a,

c). In any triangle, circumcentre, centroid & orthocenter will be collinear

d). In an isosceles triangle centroid, circumcentre, Incentre and orthocentre are collinear points.

e). In any equilateral triangle, centroid, circumcentre, Incentre and orthocenter all coincide.

f). The line segment, joining circumcentre and orthocenter is divided by centroid in the ratio 1 : 2 internally

Equation of line:

General form $ax+by+c=0, or\ ax+by+1=0$ (here a and b are the parametersand $|a|+|b|\neq 0,$)

If $a=0, b=0$ and $c=0$ then the equation represents entire two dimensional XY plane

If $a\neq 0, b=0$ and $c\neq 0$ then the equation represents a line, parallel to X-axis

If $a=0, b\neq 0$ and $c\neq 0$ then the equation represents a line, parallel to Y-axis

If $a\neq 0, b\neq 0$ and $c=0$ then the equation represents a line, passing through origin

If $a\neq 0, b=0$ and $c=0$ then the equation represents a line, Y- axis

If $a=0, b\neq 0$ and $c=0$ then the equation represents a line, X- axis

If $a=0, b=0$ and $c\neq 0$ there is no existence of such case.

If $a\neq 0, b\neq 0$ and $c\neq 0$ then the equation represents a line, which is neither vertical nor horizontal and not passing through origin.

a) $y=mx+c\ (m, slope,\ \ c, y-\text{intercept})$

b). $y-y_1=m(x-x_1)$ is equation of the line, passing through the point (x_1, y_1) and having slope m

c). $\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$ is the equation of the line passing through two given points$(x_1,y_1),(x_2,y_2)$

d). Equation of the line which makes a and b as intercepts on x-axis and y-axis is given by $\frac{x}{a}+\frac{y}{b}=1.$

e). Equation of the line in normal form is given by $x\cos\sim + y\sin\sim = p.$ where "p", distance from origin to the line, ~ – angle between the x- axis, & the line drawn from origin and perpendicular to the given line.

f). Equation of the line in symmetric form is given by $\frac{x-x_1}{\cos¯} = \frac{y-y_1}{\sin¯} = r.$

Any point on the line, which is at a distance of "r" units away from $(x_{1,} y_1)$ is$(x_1 + r\cos¯,\ y_1 + r\sin¯).$ $r = +ve$ if the point is on the right side of (x_1, y_1) and $r = -ve$ if the point is on the left side of (x_1, y_1)

g). Equation of a line which is parallel to the given line $ax + by + c = 0,\ is\ ax + by + d = 0$ and distance between the parallel lines $a_1x + b_1y + c_1 = 0 \,\&\, a_1x + b_1y + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a_1^2 + b_1^2}}$

h). Equation of a line which is perpendicular to the given line $ax + by + c = 0,\ is\ bx - ay + d = 0$

Position of two points, with respect to a given line $L_1:\ ax + by + c = 0.$

a). Suppose $P(x_1, y_1),\ Q(x_{2,}\ y_2)$ lie on the same side of the given line $\Rightarrow L_1(P).L_1(Q) > 0$

b). Suppose $P(x_1, y_1),\ Q(x_{2,}\ y_2)$ lie on the opposite side of the given line $\Rightarrow L_1(P).L_1(Q) < 0$

AREA

a). Area of triangle with vertices $(x_1, y_1)(x_{2,}\ y_2)\ (x_3, y_3)$ is $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{vmatrix}.$

b). Area of quadrilateral formed by the points

$(x_{i,} y_i);\ i = 1,2,3,4\ \ is \frac{1}{2}\left\{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}\right\}$

c). Area of a polygon of "n" sides whose vertices, are given by

$(x_{i,} y_i), i = 1, 2, ...n,\ is\ \frac{1}{2}\left\{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + ... + \begin{vmatrix} x_n & y_n \\ x_1 & y_3 \end{vmatrix}\right\}$

d). Area of a triangle, formed by the lines $a_1x + b_iy + c_i = 0\ \ i = 1,2,3.$

is $\frac{1}{2|c_1c_2c_3|}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$ where $\left.\begin{matrix} C_1 = a_1b_2 - a_2b_1 \\ C_2 = a_2b_3 - a_3b_2 \\ C_3 = a_3b_1 - a_1b_3. \end{matrix}\right\}$co factors of $c_{1,}\ c_{2,}\ c_3$ and $y = m_ix + c_i\ \ \ i = 1,2,3.$

e) Area of triangle formed by the lines $y = m_ix + c_i,\ i = 1,2,3.\ is \frac{1}{2}\left|\frac{(c_1 - c_2)^2}{m_1 - m_2} + \frac{(c_2 - c_3)}{m_2 - m_3} + \frac{(c_3 - c_1)^2}{m_3 - m_1}\right|.$

f). Four points will be collinear, if area of the quadrilateral formed by these points, is zero.

g). Three points will be collinear, if area of the triangle, formed by these points, is zero.

h). Area of a parallelogram, formed by $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_2 = 0$, $a_2 + b_2y + d_1 = 0$ and $a_2x + b_2y + d_2 = 0$ is $= \left|\frac{p_1p_2}{\sin¯}\right|.$ where p_1, p_2 are the distance between the pair of parallel lines and

$\sin^{-}$, angle between the lines. $= \left|\frac{p_1 p_2}{\sin^{-}}\right|. \quad = \frac{|c_1 - c_2||d_1 - d_2|}{|a_1 b_2 - b_1 a_2|}$

$$\left[\because \tan^{-} = \frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2},\ \sin^{-} = \frac{a_2 b_1 - a_1 b_2}{\sqrt{a_1^2 + b_2^2}\sqrt{a_2^2 + b_2^2}.}\right] \ \& P_1 = \frac{c_1 - c_2}{\sqrt{a_1^2 + b_1^2}}.\ P_2 = \frac{d_1 - d_2}{\sqrt{a_2^2 + b_2^2}}.$$

Examples: i). Area of a rhombus, formed by the lines $ax \pm by \pm c = 0$ is $\left|\frac{2c^2}{ab}\right|$

j). Area of the rhombus, formed by the lines

$$\frac{x}{a} + \frac{y}{b} = 1,\quad \frac{x}{b} + \frac{y}{a} = 1,\quad \frac{x}{a} + \frac{y}{b} = 2,\quad \frac{x}{b} + \frac{y}{a} = 2,\quad is\ \left|\frac{a^2 b^2}{b^2 - a^2}\right|$$

Intercepts on the axes:

Consider the line $ax \pm by \pm c = 0$ $(a, b, c > 0)$

i). Sum of the intercepts made by the line, on the axes $\left|\frac{c(a+b)}{ab}\right|$

ii).Length of the perpendicular, drawn from the origin, to the line is $\frac{|c|}{\sqrt{a^2 + b^2}}$

iii). Area of the triangle, formed by the line and the coordinate axes is, $\frac{c^2}{|2ab|}$.

iv). Length of the line segment intercepted between the coordinate axes is, $\frac{|c|\sqrt{a^2 + b^2}}{|ab|}$

Concurrency of three lines: Three or more lines, pass through a point; then the lines are called concurrent lines.

$$\left.\begin{matrix} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \\ a_3 x + b_2 y + c_3 = 0 \end{matrix}\right\} \text{are concurrent then } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

(i.e. point of intersection of two lines has to lie on the third line)

or $\alpha_1, \alpha_2, \alpha_3 \in R$ and not all zero such that $\alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3 = 0.$

Example: i) $$\left.\begin{matrix} (a-b)x + (b-c)y + (c-a) = 0 \\ (b-c)x + (c-a)y + (a-b) = 0 \\ (c-a)x + (a-b)y + (b-c) = 0 \end{matrix}\right\} \begin{matrix} are\,concurrent\,because \\ L_1 + L_2 + L_3 = 0 \\ here\ \alpha_1 = \alpha_2 = \alpha_3 = 1 \end{matrix} \quad (or) \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0.$$

ii) If the lines are $\left.\begin{matrix} ax + by + c = 0 \\ bx + cy + a = 0 \\ cx + ay + b = 0 \end{matrix}\right\}$ are concurrent,

then $\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$ or

$$\Rightarrow (a+b+c)(a + bw + cw^2)(a + bw^2 + cw) = 0$$

iii) If the lines $\left.\begin{matrix} ax + y + 1 - 0 \\ x + by + 1 = 0 \\ x + y + c = 0 \end{matrix}\right\}$ are concurrent $\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$

iv) If the lines $\left.\begin{array}{l} a_1x+b_1y+1=0 \\ a_2x+b_2y+1=0 \\ a_3x+b_3y+1=0 \end{array}\right\}$ are concurrent $\Rightarrow \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{array}{c} (a_1,b_1),(a_2,b_2),(a_3,b_3) \\ \text{are collinear points.} \end{array}$

v) If $(a_1,b_1)\,(a_2,b_2)$ *and* (a_3,b_3) are collinear points then $\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$ but its converse is not always true.

Example: $(0,0),(2,3),(2,-3)$ are not collinear points but $\begin{vmatrix} 0 & 0 & 1 \\ 2 & 3 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 0.$

Distance between a point and the line:

Equation of a line, passing through the point (x_1, y_1) and making an angle "¯" with positive direction of X – axis, is $\frac{x-x_1}{\cos ¯} = \frac{y-y_1}{\sin ¯} = r.$

Any point on the line, which is at a distance of "r" units away from (x_1, y_1) is $(x_1 + r\cos ¯,\ y_1 + r\sin ¯)$.

Consider the line $ax+by+c=0$, and the point P(x_1, y_1) not on the line. Equation of the line, PQ, perpendicular to the given line, is $\frac{x-x_1}{\cos ¯} = \frac{y-y_1}{\sin ¯}$, $\left[\text{where } \tan ¯ = \frac{b}{a} \right]$.

Where Q is the foot of the perpendicular drawn from P to the given line $ax+by+c=0$,

a). Point on the line PQ, at a distance of "r" units from (x_1, y_1) is (x_1, y_1) which lies on $ax+by+c=0 \Rightarrow r = \frac{-(ax_1+by_1+c)}{a\cos ¯ + b\sin ¯} \Rightarrow r = \frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}.}$

Distance from (x_1, y_1) to the line $ax+by+c=0$, is $\frac{|ax_1+by_1+c_1|}{\sqrt{a^2+b^2}.}$

and distance of the line $ax+by+c=0$, from the origin is $\frac{|c|}{\sqrt{a^2+b^2}}$

b).Foot of the perpendicular from (x_1, y_1) to the line is $(\sim,\ \ )$ given, by

$$\frac{\sim - x_1}{a} = \frac{\ \ - y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}.$$

c). image of the point (x_1, y_1) with respect to the line $ax+by+c=0$, is $(\sim,\ \ )$ & it is given by

$$\frac{\sim - x_1}{a} = \frac{\ \ - y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}.$$

d). Image of the point (x_1, y_1) with respect to the x-axis is $(x_1, -y_1)$

Image of the point (x_1, y_1) with respect to the y-axis is $(-x_1, y_1)$.

Image of the point (x_1, y_1) with respect to the line y=x is (y_1, x_1).

Image of (x_1, y_1) with respect to the line $y = x\tan ¯$ *is* $(x_1\cos 2¯ + y_1\sin 2¯,\ x_1\sin 2¯ - y_1\cos 2¯)$.

Image of (x_1, y_1) with respect to the point origin is $(-x_1, -y_1)$.

Angle between the lines:

a). If is ¯ the angle, which the line segment, joining the points $\left(x_{1,}\ y_1\right)\left(x_2, y_2\right)$, subtends at the origin then $\tan^{-} = \dfrac{\dfrac{y_1}{x_1} - \dfrac{y_2}{x_2}}{1 + \dfrac{y_1 y_2}{x_1 x_2}} = \dfrac{y_1 x_2 - x_1 y_{2.}}{x_1 x_2 + y_1 y_{2.}}$

$\cos^{-} = \dfrac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + x_2^2}\sqrt{y_1^2 + y_2^2}}.\ :\ \sin^{-} = \dfrac{x_2 y_1 - y_2 x_1}{\sqrt{x_1^2 + x_2^2}\sqrt{y_1^2 + y_2^2}}$

b). Angle between the two lines $a_1 x + b_1 y + c_1 = 0,\ a_2 x + b_2 y + c_2 = 0$ is $\tan^{-} = \left|\dfrac{a_2 b_1 - a_1 b_2}{a_1 b_2 + b_1 a_2}\right|$

$\Rightarrow$lines are parallel$\Rightarrow a_1.b_2 = a_2.b_1$ and lines are perpendicular $\Rightarrow$ $a_1.a_2 + b_1.b_2 = 0$

c). Equation of the lines, which pass through the point $P\left(x_1, y_1\right)$ and making, a given angle ~ with the given line $y = mx + c$ is $\therefore y - y_1 = \dfrac{m \pm \tan\sim}{1 \mp m\tan\sim}\left(x - x_1\right).$

(Where m_1 is slope of the required line)

d). Equation of line, passing through $P\left(x_1, y_1\right)$ Which lies on the given line $y = mx + c,$ and making equal angles with the given lines "¯" (given) is, obtained by solving (where m_1, $m_2$are slopes of the required line)

$\dfrac{m_1 - m}{1 + m_1 m} = \dfrac{m - m_2}{1 + m m_2} = \tan^{-}.$

Example: i). Equation of two sides of an equilateral triangle, whose one of the vertex is (2,3) and its opposite side is $x + y = 2$ is $y - 3 = \dfrac{-1 \pm \tan 60^0}{1 - \mp(-1)\tan 60^0}\left(x - 2\right).$ (or) use rotation.

ii). Ray of light is sent along the line $x - 2y - 3 = 0.$ upon reaching the time $3x - 2y - 5 = 0,$ The ray is reflected from it then equation of the reflected ray is, given by

$$\frac{\frac{1}{2} - \left(\frac{-2}{3}\right)}{1 + \left(\frac{1}{2}\right)\left(\frac{-2}{3}\right)} = \frac{\frac{-2}{3} - m}{1 + \left(\frac{-2}{3}\right)m} \Rightarrow m$$

Equation of angle bisectors

Angle bisector is the collection of all the points, which are equal distance from the given lines.

$L_1 \equiv a_1 x + b_1 y + c_1 = 0,\ L_2 \equiv o_2 x + b_2 y + c_2 = 0$are the two lines.

a). Equations of the angle bisectors of the lines are$B_1 = 0\, and\, B_2 = 0$ i.e.

$$\frac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2 x + b_2 y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

b). Bisector of the angle between the two lines $L_1 = 0,\ L_2 = 0$ which contains a given point A $If\ L_1\left(A\right) = positive\ and\ L_2\left(A\right) = positive$ or $L_1\left(A\right) = negative\ and\ L_2\left(A\right) = negative$ is the required bisector $then\ \dfrac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_2^2}} = +\left(\dfrac{a_2 x + b_2 y + c_2}{\sqrt{a_2^2 + b_2^2}}\right)$is the required bisector

(i.e. bisector of the region in which the given point A lies)

$If\ L_1(A) = positive\ and\ L_2(A) = negative\ then\ \frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_2^2}} = -\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the required bisector.

(i.e. bisector of the region in which the given point A lies)

c). Identification of acute / obtuse angle bisector of two given lines

$L_1 \equiv a_1x+b_1y+c_1=0,\ L_2 \equiv o_2x+b_2y+c_2=0$

If $c_{1,}\,c_2>0$ and if $a_1.a_2+b_1.b_2<0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = +\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the acute bisector of the lines

If $c_{1,}\,c_2>0$ and if $a_1.a_2+b_1.b_2<0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = -\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the obtuse bisector of the lines

If $c_{1,}\,c_2>0$ and if $a_1.a_2+b_1.b_2>0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = +\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the obtuse bisector of the lines

If $c_{1,}\,c_2>0$ and if $a_1.a_2+b_1.b_2>0$ then $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = -\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$ is the acute bisector of the lines

d). Another way of identifying an acute and obtuse bisectors, is

Let $L_1 \equiv a_1x+b_1y+c_1=0,\ L_2 \equiv o_2x+b_2y+c_2=0$ and $B_1=0\, and\, B_2=0$ are the bisectors of the given lines.

Take a point P on any one of the given lines $L_1=0, or\, L_2=0$ and find the distance between the point P and the bisector $B_1=0$ (say p) and that of the other bisectors $B_2=0$ is (say q).

If $|p|<|q| \Rightarrow B_1=0$ is the acute angle bisector.

If $|q|<|p| \Rightarrow B_2=0$ is the acute angle bisector.

If $|q|=|p| \Rightarrow L_1=0,\ L_2=0$ are perpendicular lines.

e). Equations of two straight lines $L_1 \equiv a_1x+b_1y+c_1=0,\ L_2 \equiv a_2x+b_2y+c_2=0$

Then, condition for origin to lie in one of the acute angles between the lines is $(a_1a_2+b_1b_2)c_1c_2<0.$

f). Condition for origin to lie in one of the obtuse angles between the lines is $(a_1a_2+b_1b_2)c_1c_2>0.$

g). Equation of the lines passing through $P(x_1, y_1)$ and equally inclined with the lines $a_1x+b_1y+c_1=0,\ a_2x+b_2y+c_2=0$ are those are parallel to the bisectors between these two lines and passing through the point $P(x_1, y_1)$

h). Condition for given points A, B to lie in the same, opposite and adjacent angles formed by two given lines $L_1=0$, and $L_2=0$

i). A and B lie in the same angle between lines $L_1=0$, and $L_2=0$, then

$L_1(A).L_1(B)>0\ \ \&\ \ L_2(A).L_2(B)>0$

ii). A and B lie in the opposite angles, between lines $L_1=0$, and $L_2=0$, then

$L_1(A).L_1(B) < 0$ & $L_2(A).L_2(B) < 0$

iii). A and B lie in the adjacent angles, between lines $L_1=0$, and $L_2=0$, then

$L_1(A).L_1(B) < 0$ & $L_2(A).L_2(B) > 0$ *or* $L_1(A).L_1(B) > 0$ & $L_2(A).L_2(B) < 0$

Position of a point with respect to a triangle

If $L_1 = 0,\ L_2 = 0,\ L_3 = 0$ are the equations of three sides of a triangle ABC, as in figure

The given $P(\sim,\ \ )$ lies inside the triangle, then

$L_1(P).L_1(A) > 0$ & $L_2(P).L_2(B) > 0$ & $L_3(P).L_3(C) > 0$

Equations of median, angular bisector

If $A(x_1, y_1),\ B(x_2, y_2)$ *and* $C(x_3, y_3)$ are the vertices of a triangle ABC, D,E,F are the mid points of the sides BC, CA and AB

then equation of median AD, drawn through A is, $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

then equation of median BE, drawn through B is, $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

then equation of median CF, drawn through C is, $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Equation of the angular bisector of $\underline{|A}$ *is*, $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

Equation of the angular bisector of $\underline{|B}$ *is*, $c\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + a\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0.$

Equation of the angular bisector of $\underline{|C}$ *is*, $a\begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$

Translation and Rotation:

If P(x, y) are coordinates of a point, referred to the co ordinate axes x and y with origin (0,0) and (X, Y) coordinates of the same point P referred to the new axes X and Y

a). If origin is shifted to (h, k) then

$x = X + h$ and

$y = Y + k$

b). If the axes are rotated t an angle "¯" in anticlock wise direction, then the point P (x, y) changes to (X, Y) and $x = X\cos^{-} - Y\sin^{-}$ & $y = X\sin^{-} + Y\cos^{-}$.

$X = x\cos^{-} + y\sin^{-}$ & $Y = -x\sin^{-} + y\cos^{-}$.

c). If the origin is shifted at O' (h, k) and the axes are rotated about the new origin O' by an angle "¯" in anticlock wise direction then

$x = h + X\cos^{-} - Y\sin^{-}$ and

$y = k + X\sin\theta + Y\cos\theta$

d). Through what angle the axes, must be rotated without translation, in anticlock wise sense so that the expression $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ does not contain the mixed product "xy" term is $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$.

Family of lines:

1). Family of lines, passing through the point A intersection of two lines $a_2x + b_2y + c_2 = 0.\ a_1x + b_1y + c_1 = 0$, is $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$

For different values of λ, we get different lines, passing through the point "A".

2). Equation of a line, passing through the point 'A' intersection $of\ L_1 = 0\ \&\ L_2 = 0$ and farthestfrom $B(x_1, y_1)$, is the line, passing through A, & perpendicular to AB.

Equation of a line, passing through the point 'A' intersection $of\ L_1 = 0\ \&\ L_2 = 0$ and nearestfrom $B(x_1, y_1)$, is the line, passing through A, &B itself.

3). Parallelogram ABCD with equations of its side AB as $L_1 : a_1x + b_1y + c_1 = 0$, CD as $L_2 : a_1x + b_1y + c_2 = 0$,BC as $L_3 : a_2x + b_2y + d_1 = 0$ and AD as $L_4 : a_2x + b_2y + d_2 = 0$ Then equation of the diagonal BD is given by $L_2L_3 - L_1L_4 = 0$ and equation of the diagonal AC is given by $L_1L_2 - L_3L_4 = 0$

4). A line $ax + by + 1 = 0,$ is such that the algebraic sum of the perpendiculars on it, from a number of points $(x_i, y_i),\ (i = 1, 2, \ldots\ldots, n).$ is zero, then the line always passes through a fixed point is $(\bar{x}, \bar{y}) = \left(\bar{x} = \frac{\sum x_i}{n}, \bar{y} = \frac{\sum y_i}{n}\right)$. $\left[according\ to\ given\ \frac{\sum_{i=1}^{n}(ax_i + by_i + 1)}{\sqrt{a^2+b^2}} = \frac{a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i + 1}{\sqrt{a^2+b^2}} = 0\right]$

d). Examples:

i) if $4a^2 + 9b^2 - c^2 + 12ab = 0$ and a, b, c are parameters then the line $ax + by + c = 0$ passes through the points (2, 3) and (-2, -3).

$[\because 4a^2 + 9b^2 - c^2 + 12ab = 0 \Rightarrow (2a + 3b + c)(-2a - 3b + c) = 0.]$ (or) consider as a quadratic equation in terms of either a or b or c and solve

ii). $(2\cos\theta + 3\sin\theta)x + (3\cos\theta - 5\sin\theta)y - (5\cos\theta - 2\sin\theta) = 0,\ \forall\theta,$

then lines passes through a fixed point (1, 1).

iii). Consider the family $(x + y - 1) + \lambda(2x + 3y - 5) = 0\ \&\ (3x + 2y - 4) + \mu(x + 2y - 6) = 0$ then equation of the line, which belongs to both the families is, "line passing through both fixed points, (i.e. $int\,r\,section\ of\ L_1 \& L_2\ \&\ intersection\,of\ L_3 \& L_4).\ i.e.\ x - 2y + 8 = 0$

Some mislaneous examples

a). Rational points: A point (x,y) in a plane is said to be rational if both x and y are rational numbers. If both x and y are integers then the point (x,y) is called an integer point.

i). If co efficients in the equations of two non parallel lines are rational numbers then the point of intersection of two lines, will be a rational point

ii). The slope of a line, through two rational points, is also a rational.
iii). The centroid, circumcentre and orthocentre of a triangle whose vertices are rational points,are also rational points.
iv). An equilateral triangle with rational points as vertices, do not exist.
b). In a triangle ABC, image of the vertex A, with respect to the angular bisector of $\underline{|B}$ lies, on the line BC.
Image of the vertex B, with respect to the angular bisector of $\underline{|A}$ lies, on the line AC.
Similarly, image of the vertex C, with respect to the angular bisector of $\underline{|B}$, lies, on the line AB.
c). In a triangle ABC, image of the vertex A, with respect to the perpendicular bisectors of the sides AB and AC are B
and C respectively
d). Equation of two equal sides of an isosceles triangle ABC, AB, AC are given, then the third side BC, is parallel to the angular bisectors of AB and AC.
e). A line "L" has intercepts a and b on the coordinate axes. When the axes are rotated through an angle "Q" keeping the origin fixed, the same line "L" has intercepts p and q, then $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$ because Distance from origin to the lines, same.i.e.

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}.$$

f). i). A, B are two given points, "p" is any point on the line $ax+by+c=0.$
Now, maximum of $|PA-PB|$ is AB. And the point "P" for which $|PA-PB|$ is maximum, is, when obtained when P, A, B are collinear, and in the form

ii). A, B are two given points, "p" is any point on the line $ax+by+c=0.$
Now, minimum $PA+PB$ is AB. And the point "P" for which $PA+PB$ is minimum, is, when obtained when P, A, B are collinear, and in the form
g). In $triangle\, ABC,$ all the three vertices are given, then, to find out the equation of the internal angle bisector of $\underline{|A},$ Find the point D on BC, which divides BC, in the ratio c:b. now write the equation of AD. (or) angular bisectors AB, and AC whivh contains centroid of the triangle or mid point of BC. (or) use slope form

h). A line cuts x – axis at A, y – axis at B such that AB = l (given), (O-being origin) then the locus of circumcentre of the triangle OAB is $x^2+y^2=\frac{l^2}{4}$
locus of orthocentre of the triangle OAB is $x^2+y^2=0 \quad (i.e.\ x=y=o\ only\ origin)$
locus of incentre of the triangle OAB is $x^2+y^2=\frac{l^2}{9}$
i). If the point (a, a) falls between the lines $|x+y|=2.$ then $|a|<1$