NARAYANA IIT ACADEMY INDIA

40 YEARS OF EXCELLENCE

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1. An Hyperbola, is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distances from a fixed straight line (called directrix) is always constant which is always greater than unity. (fixed ratio is eccentricity which is denoted by "e")
2. General second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents hyperbola, if $abc+2fgh-af^2-bg^2-ch^2 \neq 0$ and $h^2 > ab$.
3.

	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	Conjugate hyperbola of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{-x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Equation of the transverse axis	Y = 0	X = 0
Equation of the conjugate axis	X = 0	Y = 0
Centre	(0,0)	(0,0)
Length of the transverse axis	2a	2b
Length of the conjugate axis	2b	2a
Coordinates of the foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of the directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{b}{e}$
Length of the latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Coordinates of the end points of the latus rectum	$\left(\pm ae, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{a^2}{b}, \pm be\right)$
Eccentricity	$e^2=1+\frac{b^2}{a^2}$	$e^2=1+\frac{a^2}{b^2}$
Coordinates of the vertices	$(\pm a, 0)$	$(0, \pm b)$
Focal distance	$ex \pm a$	$ey \pm b$
Parametric coordinates	$(a\sec¯, b\tan¯)$, ¯ $\in [0, 2®)$ (or) $x=\frac{a}{2}\left(t+\frac{1}{t}\right)$, $y=\frac{b}{2}\left(t-\frac{1}{t}\right)$	$(a\tan¯, b\sec¯)$, ¯ $\in [0, 2®)$

4. Equation of hyperbola, whose focus is at S(a, b) and the equation of directrix is $lx+my+n=0$ and its eccentricity "e" is given by $(x-a)^2+(y-b)^2=e^2\left(\frac{lx+my+n}{\sqrt{l^2+m^2}}\right)^{2.}$

$(SP = e\,PM \Rightarrow \quad SP^2 = e^2(PM)^{2.})$

5. Equation of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ can also be given by

$$\frac{(distance\ between\ any\ point\ P(x,y)\ \&\ conjugate\ axis)^2}{(length\ of\ the\ semi\ transverse\ axis)^2}-\frac{(distance\ between\ any\ point\ P(x,y)\ \&\ transverse\ axis)^2}{(length\ of\ the\ semi\ conjugate\ axis)^2}=1.$$

i.e. Equation of the conjugate axis, is $a_1x+b_1y+c_1=0$ and equation of the transverse axis is $b_1x-a_1y+c_2=0$ & Length of transverse axis 2p and length of conjugate axis is 2q then equation of the hyperbola, is $\frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{p^2}-\frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{a_1^2+b_1^2}}\right)^2}{q^2}=1$.

Some standard locus

1. A Hyperbola, is the locus of a point, which moves in a plane, such a way that the absolute difference of its distances from two fixed points A and B in the same plane, is constant (greater than, the distance between the two fixed points).
 (*i.e.* $|PA-PB|=k, \quad k<AB.$**, then locus of P is a Hyperbola**
 and if PA-PB = k and k < AB, then locus of P is a branch of Hyperbola)

 (If $|PA-PB|=k,\ k=AB$, then locus of P is a pair of ray AP and BP and if k >AB, no such P exists)
 (Here the two points A and B are foci SS' =2ae, $|SP-S'P|$ = 2a 2ae > 2a)

2. Locus of the centre of a circle which touches two given circles externally is a hyperbola.

3. A variable circle cuts two fixed perpendicular lines so that each intercept is of given length, then locus of the centre of the circle is a hyperbola.

4. If L_1 and L_2 are two variable lines which slides on coordinate axes, keeping constant difference between their x – intercept and y – intercept. These intercepts are "a" &"b". Then locus of the centre of the circle passing through these four points on coordinate axis is hyperbola. $\left(i.e.\ x^2-y^2=\frac{a^2}{4}-\frac{b^2}{4}\right)$.

5. Given base of a triangle and the ratio of the tangent of the base angles. Then the vertex moves on a hyperbola whose foci are the extremities of the base.
 (In a triangle ABC, BC=a and $\frac{\tan\left(\frac{B}{2}\right)}{\tan\left(\frac{C}{2}\right)}=a \Rightarrow \frac{s-a}{s-b}=a \Rightarrow \frac{a+b-c}{a-b+c}=a$ and

$b-c=\left(\frac{a-1}{a+1}\right)a$ which implies AC-AB = constant)

Example: In a triangle ABC, B(2, 2) and C (-2, -2) Then locus of A such that $\dfrac{\tan\left(\frac{B}{2}\right)}{\tan\left(\frac{C}{2}\right)} = 3+2\sqrt{2}$ is a hyperbola.

Hyperbola and Conjugate Hyperbola:

1. If e and e' are the eccentricities of a hyperbola and its conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{(e')^2}=1.$
2. Two hyperbolas are said to be equal if they have same eccentricity and same latus rectum.
3. The foci of a hyperbola and foci of its conjugate hyperbola are con cyclic and form the vertices of a square.

Position of a point with respect to a Hyperbola:

(x_1, y_1) Lies inside, outside, on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1>0, <0, =0$ respectively.

Equation of the chord

1. General point on hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $(a\sec\theta, b\tan\theta)$, $\theta \in [0, 2\pi)$ (or) $x=\frac{a}{2}\left(t+\frac{1}{t}\right)$, $y=\frac{b}{2}\left(t-\frac{1}{t}\right)$
2. Equation of the chord, joining the points $(a\sec\theta_1, b\tan\theta_1)$ and $(a\sec\theta_2, b\tan\theta_2)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, is $\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right).$
3. Focal chord, joining the points on the Hyperbola $(a\sec\theta_1, b\tan\theta_1)$ and $(a\sec\theta_2, b\tan\theta_2)$ then $\tan\frac{\theta_1}{2}, \tan\frac{\theta_2}{2}=\frac{1-e}{1+e}(or)\frac{1+e}{1-e}$
4. If the eccentric angles of the extremities of a focal chord of a Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are $\theta_1 \& \theta_2$

   Then eccentricity of the Hyperbola e= $\dfrac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}=\dfrac{\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)}$

Equation of Tangent:

1a) The line $y=mx+c$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then $c=\sqrt{a^2m^2-b^2}$

1b) Condition for the line $y=mx+c$, to intersect the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, at two distinct points is $c^2>a^2m^2+b^2$

1c) Condition for the line $y=mx+c$ does not intersect or touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $c^2<a^2m^2+b^2$

1d) The line $y = mx + c$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the points

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2 - b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2 - b^2}}\right)$$

1e) Condition for the line $lx + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $n^2 = a^2l^2 - b^2m^2$

1f) The line $lx + my = n$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\frac{a^2l}{n}, \frac{-b^2m}{n}\right)$

(compare $lx + my = n$ and $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$)

1g) The line $x\cos\alpha + y\sin\alpha = p$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is, $a^2\cos^2 a - b^2\sin^2\alpha = p^2$

2a) Equation of the tangent at drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

2b) Equation of the tangent drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a\sec\theta, b\tan\theta)$ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1.$

2c) Equation of the tangent drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{am^2 - b^2}$

3a) Slope of the tangent, drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) *is* $\frac{b^2x_1}{a^2y_1}$ and at $(a\sec\theta, b\tan\theta)$ is $\frac{b}{a}\operatorname{cosec}\theta$

4) Point of intersection of tangents, drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the points $(a\sec\theta_1, b\tan\theta_1)$ & $(a\sec\theta_2, b\tan\theta_2)$

$$\left(\frac{a\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}, \frac{b\sin\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}\right)$$

5) **Locus of the feet of the perpendiculars, drawn from foci to a tangent drawn to the hyperbola** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **at any point, is auxiliary circle** $x^2 + y^2 = a^2$.

6) **Product of the perpendicular distances, calculated from foci, to any tangent, drawn to the hyperbola** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ *is* b^2.

7) Tangents drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at two points P and Q whose eccentric angles are θ_1, θ_2 are parallel if $|\theta_1 + \theta_2| = \pi$

8) **Locus of the point of intersection of the perpendicular tangents drawn to the hyperbola** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

is director circle. i.e. $x^2 + y^2 = a^2 - b^2$ **if a² > b²**

The equation of the director circle for the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ does not exist, if $a^2 < b^2$

The equation of the director circle for the hyperbola $x^2-y^2=a^2$ is $x^2+y^2=0$ i.e. point circle.

9) The foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are S and S' . P and Q are the points of intersection of tangents at the vertices and tangent at any point R on the hyperbola. Then the points S, S', P and Q are con cyclic.

(i.e. The circle drawn on PQ as diameter, passes through these four points and equation of the circle is $x^2+y^2-b\cot\left(\frac{\theta}{2}\right).\left(\tan^2\frac{\theta}{2}-1\right)y-a^2e^2=0$)

10). Equation of common tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ is $y=\pm x\pm\sqrt{a^2-b^2}$

and their point of contacts P $\left(\frac{a^2}{\sqrt{a^2-b^2},}\ \frac{b^2}{\sqrt{a^2-b^2}}\right)$ & Q $\left(\frac{-b^2}{\sqrt{a^2-b^2}},\frac{-a^2}{\sqrt{a^2-2}}\right)$

length of the common tangent (i.e. distance between the points PQ) is $\sqrt{2}\left(\frac{a^2+b^2}{\sqrt{a^2-b^2}}\right)$

Normal:

1). Equation of normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (x_1, y_1) is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

2). Equation of normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $(a\sec\theta, b\tan\theta)$ is $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

(or) $ax\cos\theta+by\cot\theta=a^2+b^2$

3). Equation of normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ *is* $y=mx\pm\frac{(a^2+b^2)m}{\sqrt{a^2-b^2m^2}}$ and the point of contact is $\left(\frac{\pm a^2}{\sqrt{a^2-b^2m^2}},\mp\frac{b^2m}{\sqrt{a^2-b^2m^2}}\right)$

4). If the line $lx+my+n=0$ is normal to the Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

5). Normal at any point P$(a\sec\theta, b\tan\theta)$ on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the transverse axis and conjugate axes at G and g and CF be perpendicular to the normal from the centre "C" A and A' be the vertices, then

a). PF. PG.=b^2

b). $PF.Pg=a^2$.

c). Locus of middle points of G and g is also a hyperbola with eccentricity $\frac{e}{\sqrt{e^2-1}}$. (e- eccentricity of given Hyperbola)

d). $AG.A'G=a^2\left(e^4\sec^2\theta-1\right)$

e). Minimum length of PG is $\frac{b^2}{a}$.

f) Locus of the foot of the perpendicular from the centre upon any normal to the Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\left(x^2+y^2\right)^2\left(a^2y^2-b^2x^2\right)=\left(a^2+b^2\right)^2x^2y^2$

Reflection:

1) Tangent and normal at any point P on the Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, are the bisectors of the external and internal angles $\angle S'PS$. $\left(i.e.\ SG=e.SP\ \&\ S'G=eS'P\right)$ (G is the point of intersection normal and the transverse axis)

Image of S' with respect to the tangent drawn at P, lies on the line SP extended.

2). If an incident ray passing through, the focus (S) strikes the Hyperbola, then the reflected ray will pass through the other focus (S').

Co normal Points:

1). Points on the Hyperbola, the normal at which, to the Hyperbola, pass through a given point are called co normal points.

2). A maximum of four normal can be drawn from a given point to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

3). Sum of the eccentric angles of the co normal points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

is an odd multiple of π (i.e. $\Rightarrow \theta_1+\theta_2+\theta_3+\theta_4=(2n+1)\pi$)

4). If $\theta_1,\theta_2,\theta_3,\theta_4$ are eccentric angles of four feet of the normals, drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

from a point (h, k) then

a). $\sum\cos\left(\theta_1+\theta_2\right)=0$

b). $\sum\sin\left(\theta_1+\theta_2\right)=0$

c). $\sin\left(\theta_1+\theta_2\right)+\sin\left(\theta_2+\theta_3\right)+\sin\left(\theta_3+\theta_1\right)=0.$ $\left(use\sum z_1z_2=0\Rightarrow\sum\left(\cos\left(\theta_1+\theta_2\right)+i\sin\left(\theta_1+\theta_2\right)\right)=0\right)$

5). If the normal, drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, at four points $P\left(x_1,y_1\right)$, $Q\left(x_2,y_2\right)$, $R\left(x_3,y_3\right)$ and $S\left(x_4,y_4\right)$ are concurrent, then. $\left(x_1+x_2+x_3+x_4\right)\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\right)=4.$ (or) $\sum_{i=1}^{4}\sec\theta_i.\sum_{i=1}^{4}\cos\theta_i=4$

$\left(y_1+y_2+y_3+y_4\right)\left(\frac{1}{y_1}+\frac{1}{y_2}+\frac{1}{y_3}+\frac{1}{y_4}\right)=4$

Equation of the chord whose mid point is given

a) Equation of a chord, bisected at a given point $(x_{1,}\,y_1)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1=\frac{x_1^{\,2}}{a^2}-\frac{y_1^{\,2}}{b^2}-1$$

Some Examples:

1). Locus of mid point of the chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which subtends a right angle at the origin

is $\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)=\frac{x^2}{a^4}+\frac{y^2}{b^4}$

2). Locus of mid points of the chord of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which pass through a fixed point $(\sim,\ \ )$ is a hyperbola

whose centre is $\left(\frac{\sim}{2},\frac{\ }{2}\right)$.

3). Locus of the mid points of normal chords of the hyperbola $x^2-y^2=a^2$ is $\left(x^2-y^2\right)^3=4a^2x^2y^2$

Equation of chord of contact of tangents,

1). Equation of chord of contact of tangents, drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from (x_1,y_1) is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1.$

2).The point which bisects every chord of the conic drawn through, it is called the centre of the conic. $\left(i.e.\ (0,0)\ for\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=1\right)$.

3). The chord of contact of tangents through P to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ subtends a right angle at the centre then

the locus of the point P is an ellipse $b^4x^2+a^4y^2=a^2b^2(b^2-a^2)$

4). Locus of the point of intersection of the tangents drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the extremities of a

normal chord of the hyperbola, is $\frac{a^6}{x^2}-\frac{b^6}{y^2}=\frac{1}{\left(a^2+b^2\right)^2}$

Pair of tangents

Pair of tangent drawn from (x_1,y_1) to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$is

$$\left(\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1\right)^2=\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}-1\right)\left(\frac{x_1^{\,2}}{a^2}-\frac{y_1^{\,2}}{b^2}\right)\ \left(T^2=SS_1\right)$$

Example:

1). Point of intersection of tangents drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the points where it is intersected by the line $lx+my+n=0.$ is $\left(\frac{-a^2l}{n},\frac{b^2m}{n}\right).$

Asymptotes.

1a). **Equation of the two asymptotes of the hyperbola** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ **are** $y=\pm\frac{b}{a}x$

1b). Equation of the two asymptotes of the hyperbola $x^2-y^2=a^2$ are $y=\pm x$

1c). Equation of the two asymptotes of the hyperbola $xy=c^2$ are $x=0$ and $y=0$

1d). Equation of the two asymptotes of the hyperbola $(x-\sim)(y-\quad)=c^2$ are $x=\sim$ and $y=$

2). A hyperbola and its conjugate hyperbola, have the same asymptotes.

3). Equation of the pair of asymptotes differ the hyperbola and the conjugate hyperbola by the same constant only. (i.e. Hyperbola + conjugate hyperbola = 2 Asymptotes).

4). Angle bisectors of the asymptotes are the transverse and conjugate axes of the hyperbola

5). Asymptotes, pass through the centre of the hyperbola

6). The asymptotes of a hyperbola are the diagonals of the rectangle formed by the lines drawn through the extremities of each axis parallel to the other axis.

7). Asymptotes are the tangent to the hyperbola from the centre.

8). Asymptotes of the Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are equally inclined to transverse axis of hyperbola

9). Angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ whose eccentricity is e is ¯ , then

a). $\cos\left(\frac{¯}{2}\right)=\frac{1}{e}$ (or) $\tan ¯=\frac{2ab}{a^2-b^2}$ (or) $¯=2\tan^{-1}\left(b/a\right)$

10). Equation of the asymptotes of the hyperbola $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $ax^2+2hxy+by^2+2gx+2fy+ª=0.$

(for finding ª use $\Delta=abc+2fgh-af^2-bg^2-ch^2=0$ (or) use that asymptotes pass through centre of the Hyperbola & centre can be obtained by solving the equations $\frac{\partial s}{\partial x}=0$ & $\frac{\partial s}{\partial y}=0$)

11). When asymptotes of a Hyperbola are given as $L_1=0$ & $L_2=0$, then the equation of the Hyperbola is $L_1L_2+ª=0.$

12). The product of the perpendiculars from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2+b^2}$, which is always constant

13a). Tangent, drawn at any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its asymptotes form a triangle whose area "ab" square units, which is always constant

13b). Tangent, drawn at any point on the hyperbola $x^2-y^2=a^2$ and its asymptotes form a triangle whose area "a^2" square units, which is always constant

13c). Tangent, drawn at any point on the hyperbola $xy=c^2$ and its asymptotes form a triangle whose area "$2c^2$" square units, which is always constant

14). The coordinates of point of intersection of two tangents to a rectangular hyperbola, referred to its asymptote as axes, are harmonic between the co ordinates of the point of contact

15). Portion of the tangent at $P\left(ct,\frac{c}{t}\right)$, drawn to the rectangular hyperbola $xy=c^2$ intercepted between the asymptotes (i.e. A and B) is bisected at the point of contact (i.e. PA = PB)

16). The chord PP' of the hyperbola $xy=c^2$. meets the asymptotes in Q and Q' then QP =Q'P'

17). The tangent at a point "P" of rectangular hyperbola $xy=c^2$. meets the asymptotes at "L" and "M" and "C" is the centre of the hyperbola, then $PL=PM=PC$.

Rectangular hyperbola.

A Hyperbola whose asymptotes are at right angles to each other is called rectangular Hyperbola $x^2-y^2=a^2$

Rectangular hyperbola	$x^2-y^2=a^2$		$xy=c^2$
Eccentricity	$\sqrt{2}$	**Eccentricity**	$\sqrt{2}$
Centre	**(0,0)**	Centre	**(0,0)**
Vertex	$(\pm a,0)$	Vertex	$(\pm a,0)$
Focus	$(\pm a\sqrt{2},0)$	Focus	$(\pm a\sqrt{2},\pm a\sqrt{2})$
Equation of transverse axis	Y = 0	Equation of transverse axis	**Y = x**
Equation of conjugate axis	X = 0	Equation of conjugate axis	**Y = -x**
Equation of directrix	$x=\pm\frac{a}{\sqrt{2}}$	Equation of directrix	$x+y=\pm\sqrt{2}a$
End points of the latus rectum is	$(\pm a\sqrt{2},\pm a)$	End points of the latus rectum is	
Equation of the tangent to the hyperbola at (x_1,y_1) is	$xx_1-yy_1=a^2$	Equation of the tangent, drawn to the Hyperbola at (x_1,y_1) is	$xy_1+x_1y=2c^2$ $(or)\frac{x}{x_1}+\frac{y}{y_1}=2.$
Equation of the tangent to the hyperbola at $(a\sec\text{¯},a\tan\text{¯})$ is	$x\sec\text{¯}-y\tan\text{¯}=a$	Equation of the tangent drawn to the Hyperbola at $\left(ct,\frac{c}{t}\right)$ is	$\frac{x}{t}+yt=2c.$

Equation of the normal to the hyperbola at (x_1, y_1) is	$xx_1 + yy_1 = 2x_1y_1$	Equation of the normal, to the Hyperbola at (x_1, y_1) is	$xx_1 - yy_1 = x_1^{\,2} - y_1^{\,2.}$
Equation of the normal to the hyperbola at $(a\sec^{-}, a\tan^{-})$ is	$x\sec^{-} + y\tan^{-} =$ $2a\sec^{-}\tan^{-}$	Equation of the normal, drawn to the Hyperbola at $\left(ct, \frac{c}{t}\right)$ is	$xt^3 - yt - ct^4 + c = 0$

Rectangular Hyperbola $x^2 - y^2 = a^2$ is converted in the form, by rotating the axes at an angle $\frac{\circledR}{4}$, in clock wise direction

$(x = X\cos\left(\frac{-\circledR}{4}\right) - Y\sin\left(\frac{-\circledR}{4}\right)$ and $y = X\cos\left(\frac{-\circledR}{4}\right) + Y\sin\left(\frac{-\circledR}{4}\right)$ (i.e. $x = \frac{X+Y}{\sqrt{2}}$ and $y = \frac{-X+Y}{\sqrt{2}}$)

1). Tangents, drawn to the Hyperbola $xy = c^2$ at the points $\left(ct_1, \frac{c}{t_1}\right) \& \left(ct_2, \frac{c}{t_2}\right)$ intersect at $\left(\frac{2ct_1t_2}{t_1+t_2}, \frac{2c}{t_1+t_2}\right)$.

2). If normal at t_1 drawn to the rectangular hyperbola $xy = c^2$. meets again the hyperbola at t_2, then $t_2 = -\frac{1}{t^3}$

3). If a triangle ABC, with the vertices A$\left(ct_{1,} \frac{c}{t_1}\right)$, B$\left(ct_{2,} \frac{c}{t_2}\right)$, C $\left(ct_{3,} \frac{c}{t_3}\right)$, is inscribed in a rectangular hyperbola $xy = c^2$, then the orthocenter of the triangle ABC, is $\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$ which lies, on the hyperbola.

Circle and rectangular hyperbola:

1). Circle $x^2 + y^2 = a^2$ intersects $xy = c^2$ in four points $(x_{1,} y_1)$, (x_2, y_2), $(x_{3,} x_3)$ and (x_4, y_4), then

a). $\sum x_i = 0$

b). $x_1x_2x_3x_4 = c^4$

c). $\sum y_i = 0$

d). $y_1y_2y_3y_4 = c^4$

2). The circle $x^2 + y^2 + 2gx + 2fy + d = 0$ intersects $xy = c^2$ in four points P, Q, R, S and "C" is the centre of the rectangular hyperbola then $CP^2 + CQ^2 + CR^2 + CS^2 = 4r^2$ (r = radius of the circle).

3). The circle $x^2 + y^2 + 2gx + 2fy + d = 0$ intersects $xy = 1$ in four points $(x_i, y_i), i = 1, 2, 3, 4$ then,

a). $x_1x_2x_3x_4 = 1$

b). $y_1y_2y_3y_4 = 1 = \frac{1}{x_1x_2x_3x_4}$

4). The circle $x^2+y^2+2gx+2fy+d=0$ intersects $xy=c^2$ in four points $\left(ct_i,\frac{c}{t_i}\right)$ $i=1,2,3,4$ then $t_1t_2t_3t_4=1$

5). The centre of mean position of four points, bisects the distance between the centres of

the circle and rectangular hyperbola. *ie. mid point of* $(-g,-f)$ *and* $(0,0)$)

6). Three points A, B, C $\left(ct_i,\frac{c}{t_i}\right)$ $i=1,2,3$ are taken on the Hyperbola $xy=c^2$. Then centre of the circle passing through these points is $\left(\frac{c}{2}(t_1+t_2+t_3+\frac{1}{t_1t_2t_3}\right),\frac{c}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+t_1t_2t_3\right)$

7). A circle and a rectangular hyperbola meet in A,B,C,D. If the line AB passes through the centre of the circle then centre of the hyperbola lie at midpoint of CD

Normals to rectangular hyperbola:

1). Normals, drawn to the Hyperbola $xy=c^2$ at P, Q, R, S meet at A (h, k).then $AP^2+PQ^2+AR^2+AS^2=3AC^{2.}$
(C – centre of hyperbola)

2). Normals at three points P, Q, R on a rectangular hyperbola. $xy=c^2$ intersect at S on the curve. Then centroid of the triangle PQR is centre of the hyperbola $(i.e.(0,0))$.

3). If the tangent and normal drawn to a rectangular hyperbola, cuts off intercepts a_1 and a_2 on one axis and b_1 and b_2 on the other axis then $a_1a_2+b_1b_2=0$.

4). If the normal drawn to the rectangular hyperbola $xy=c^2$ at the point $P\left(ct,\frac{c}{t}\right)$, meets the curve again at Q and the circle on PQ as diameter cuts the rectangular hyperbola again at $P\left(-ct,\frac{-c}{t}\right)$
(i.e. the other end of the diameter through P.)

5). Locus of the foot of the perpendicular from the centre, upon any normal drawn to the hyperbola $x^2-y^2=a^2$ is $\left(x^2+y^2\right)^2\left(y^2-x^2\right)=4a^2x^2y^2$.

6). Equation of the normal drawn to the hyperbola at $\left(ct,\frac{c}{t}\right)$ is $xt^3-yt-ct^4+c=0$

This passes through the point P(h, k), then $ct^4-ht^3+kt+c=0$.
(i.e. a maximum of four normal, can be drawn to $xy=c^2$ from a given point)

a). Sum of the abscissas of feet of the normals = h. $\left(\sum xi=h\right)$

b). Sum of the ordinates of feet of the normals = k. $\left(\sum yi=k\right)$

c). Product of the abscissas of feet of the normals $=-c^4\left(\prod x_i=-c^4\right)$

d). Product of the ordinates of feet of the normals $=c^4.\left(\prod y_i=c^4\right)$.

e). $\sum_{i=1}^{4} x_i^{\,2} = h^2$ & $\sum_{i=1}^{4} y_i^{\,2} = k^2$ where $(x_{1,}y_1), (x_{2,}y_2), (x_3y_3), (x_{4,}y_4)$ are the feet of the normal

Normal, drawn to the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(\bar{\ }_1)$ *and* $Q(\bar{\ }_2)$ meeting the conjugate axes at G_1 and G_2 and if $\bar{\ }_1 + \bar{\ }_2 = \frac{\circledR}{2}$ then $(CG_1)\times(CG_2) = \frac{a^2e^2}{e^2-1}$ (C- centre of the hyperbola).