NARAYANA IIT ACADEMY INDIA

SRAMAYEVA JAYATHE

40 YEARS OF EXCELLENCE

BASARA SARASWATHI BHAVAN_MDP N-120

Sec: SIO-A **Properties of triangles** **Date:21/04/2020**

PROPERTIES OF TRIANGLE: SYNOPSIS

1. $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{simC}=$ 2R (Sine law)

2. $\cos A=\dfrac{b^2+c^2-a^2}{2bc},\cos B=\dfrac{a^2+c^2-b^2}{2ac},\cos C=\dfrac{a^2+b^2-c^2}{2ab}$ (Cosine formula)

3. a = b cos C + c cos B,
b = a cos C + c cos A,
c = a cos B + b cos A (projection formula)

4. In any ΔABC,

$\tan\left(\dfrac{A-B}{2}\right)=\dfrac{a-b}{a+b}\cot\dfrac{C}{2}$	$\sin\left(\dfrac{A-B}{2}\right)=\dfrac{a-b}{c}\cos\dfrac{C}{2}$	$\cos\left(\dfrac{A-B}{2}\right)=\dfrac{a+b}{c}\sin\dfrac{C}{2}$
$\tan\left(\dfrac{B-C}{2}\right)=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$	$\sin\left(\dfrac{B-C}{2}\right)=\dfrac{b-c}{a}\cos\dfrac{A}{2}$	$\cos\left(\dfrac{B-C}{2}\right)=\dfrac{b+c}{a}\sin\dfrac{A}{2}$
$\tan\left(\dfrac{C-A}{2}\right)=\dfrac{c-a}{c+a}\cot\dfrac{B}{2}$	$\sin\left(\dfrac{C-A}{2}\right)=\dfrac{c-a}{b}\cos\dfrac{B}{2}$	$\cos\left(\dfrac{C-A}{2}\right)=\dfrac{c+a}{b}\sin\dfrac{B}{2}$

$\sin\dfrac{A}{2}=\sqrt{\dfrac{(s-b)(s-c)}{bc}}$	$\cos\dfrac{A}{2}=\sqrt{\dfrac{s(s-a)}{bc}}$	$\tan\dfrac{A}{2}=\sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}=\dfrac{\Delta}{s(s-a)}$
$\sin\dfrac{B}{2}=\sqrt{\dfrac{(s-c)(s-a)}{ca}}$	$\cos\dfrac{B}{2}=\sqrt{\dfrac{s(s-b)}{ca}}$	$\tan\dfrac{B}{2}=\sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}=\dfrac{\Delta}{s(s-b)}$
$\sin\dfrac{C}{2}=\sqrt{\dfrac{(s-a)(s-b)}{ab}}$	$\cos\dfrac{C}{2}=\sqrt{\dfrac{s(s-c)}{ab}}$	$\tan\dfrac{C}{2}=\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}=\dfrac{\Delta}{s(s-c)}$

$r=\dfrac{\Delta}{s}$, $r=(s-a)\tan\dfrac{A}{2}=(s-b)\tan\dfrac{B}{2}=(s-c)\tan\dfrac{C}{2}$, $r=4R\sin\left(\dfrac{A}{2}\right)\sin\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right)$			
$r_1=\dfrac{\Delta}{s-a}$	$r_1=\dfrac{a\cos\left(\dfrac{B}{2}\right)\cos\left(\dfrac{C}{2}\right)}{\cos\left(\dfrac{A}{2}\right)}$	$r_1=s\tan\left(\dfrac{A}{2}\right)$	$r_1=4r\sin\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)\cos\left(\dfrac{C}{2}\right)$
$r_2=\dfrac{\Delta}{s-b}$	$r_2=\dfrac{c\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{C}{2}\right)}{\cos\left(\dfrac{B}{2}\right)}$	$r_2=s\tan\left(\dfrac{B}{2}\right)$	$r_2=4r\cos\left(\dfrac{A}{2}\right)\sin\left(\dfrac{B}{2}\right)\cos\left(\dfrac{C}{2}\right)$
$r_3=\dfrac{\Delta}{s-c}$	$r_3=\dfrac{c\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)}{\cos\left(\dfrac{C}{2}\right)}$	$r_3=s\tan\left(\dfrac{C}{2}\right)$	$r_3=4r\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right)$

$IA=\dfrac{r}{\sin\left(\frac{A}{2}\right)}, IA=4R\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$ $8R^2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)=\dfrac{abc}{2r}$	$IB=\dfrac{r}{\sin\left(\frac{B}{2}\right)}=$ $IB=4R\sin\left(\frac{A}{2}\right)\sin\left(\frac{C}{2}\right)$	$IC=\dfrac{r}{\sin\left(\frac{C}{2}\right)}=$ $IC=4R\sin\left(\frac{A}{2}\right).\sin$
$I_1A=\dfrac{r_1}{\sin\left(\frac{A}{2}\right)}$	$I_1B=\dfrac{r_1}{\cos\left(\frac{B}{2}\right)}$	$I_1C=\dfrac{r_1}{\cos\left(\frac{C}{2}\right)}$
$I_2A=\dfrac{r_2}{\cos\left(\frac{A}{2}\right)}$	$I_2B=\dfrac{r_2}{\sin\left(\frac{B}{2}\right)}$	$I_2C=\dfrac{r_2}{\cos\left(\frac{C}{2}\right)}$
$I_3A=\dfrac{r_3}{\cos\left(\frac{A}{2}\right)}$	$I_3B=\dfrac{r_3}{\cos\left(\frac{B}{2}\right)}$	$I_3C=\dfrac{r_3}{\sin\left(\frac{C}{2}\right)}$
$II_1=a\sec\left(\frac{A}{2}\right)$	$II_2=b\sec\left(\frac{B}{2}\right)$	$II_3=c\sec\left(\frac{C}{2}\right)$
$I_1I_2=c\,\mathrm{cosec}\frac{C}{2}=$	$I_1I_3=b\,\mathrm{cosec}\frac{B}{2}$	$I_2I_3=a\,\mathrm{cosec}\frac{A}{2}$

I.Length of the median, bisector and altitude:

1. length of the bisector through the angle A, is $\dfrac{2bc}{b+c}\cos\left(\dfrac{A}{2}\right)$ or $\dfrac{abc}{2R(b+c)}\mathrm{cosec}\left(\dfrac{A}{2}\right)$ or $\dfrac{2\Delta}{(b+c)}\mathrm{cosec}\left(\dfrac{A}{2}\right)$

2. Length of external angular bisector of angle A is $\dfrac{2bc}{|b-c|}\sin\dfrac{A}{2}$ and the length of external angular bisector of angle C is $\dfrac{2ab}{|a-b|}\sin\dfrac{C}{2}$

3. length of the median through A, is $\dfrac{1}{2}\sqrt{2b^2+2c^2-a^2}$ or $\dfrac{1}{2}\sqrt{b^2+c^2+2bc\cos A}$

4. length of the aitutude through A, B and C are respectively $\dfrac{2\Delta}{a}$, $\dfrac{2\Delta}{b}$ and $\dfrac{2\Delta}{c}$

II) M-N theorem and some standard results

1) In a triangle ABC, D is any point on BC such that $\dfrac{BD}{DC}=\dfrac{m}{n}$, $\underline{|BAD}=\alpha$, $\underline{|CDA}=\theta$, $\underline{|CAD}=\beta$ then

i) $(m+n)\cot\theta=m\cot\alpha-n\cot\beta$

ii) $(m+n)\cot\theta=n\cot B-m\cot C$

2) Let O be any point inside a triangle ABC such that $\underline{|OAB}=\underline{|OBC}=\underline{|OCA}=w$ then

i) $\cot w=\cot A+\cot B+\cot C=\dfrac{a^2+b^2+c^2}{4\Delta^2}$

ii) $\mathrm{cosec}^2 w=\mathrm{coec}^2 A+\mathrm{cosec}^2 B+\mathrm{cosec}^2 C=\dfrac{a^2b^2+b^2c^2+c^2a^2}{4\Delta^2}$

3) Three circles touch one another externally. c_1, c_2, c_3 are the centres r_1, r_2, r_3 are radii respectively then i) "s" of the triangle $c_1c_2c_3$, is $r_1 + r_2 + r_3$

ii) area of the triangle $\sqrt{r_1r_2r_3(r_1 + r_2 + r_3)}$

iii) "r" of the triangle $\sqrt{\dfrac{r_1r_2r_3}{r_1 + r_2 + r_3}}$

4) a) If cosA +2 cosB +cosC = 2 then sides are in A P

b) a^2, b^2, c^2 are in A P then cotA, cotB and cotC are in AP

c) If a, b and c are in AP then $\cot\left(\frac{A}{2}\right), \cot\left(\frac{B}{2}\right), \cot\left(\frac{C}{2}\right)$ are in AP

d) If in a triangle ABC, $\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in A P

e) If r_1, r_2, r_3 are in H P then the sides are in A P

III. Distance between the centres and the sides and vertices:

1. Distance between **Orthocentre** and the **vertices** A, B and C are 2R cos A, 2RcosB and 2RcosC

2. Distance between **Orthocentre** and the **sides** BC, CA and AB are respectively $2R\cos B\cos C$, $2R\cos C\cos A$ and $2R\cos A\cos B$

3, Distance between the **Incentre** and the **vertices** A,B and C are respectively $\dfrac{r}{\sin\left(\frac{A}{2}\right)}$ = 4R sin $\frac{B}{2}\sin\frac{C}{2}$, $\dfrac{r}{\sin\left(\frac{C}{2}\right)}$ = 4R sin $\frac{B}{2}\sin\frac{A}{2}$ and $\dfrac{r}{\sin\left(\frac{B}{2}\right)}$ = 4R sin $\frac{A}{2}\sin\frac{C}{2}$

4. Distance between **Incentre** and the **sides** BC, CA and AB are respectively, r, r, r

5. Distance between **circumcentre** and the **vertices** A, B and C are R, R, R

6. Distance between **circumcentre** and the **sides** BC, CA and AB are respectively R cos A, RcosB and RcosC

7. Distance between **centroid** and the **vertices** A, B and C are two- third of the corresponding length of the median

8. Distance between **centroid** and the **sides** BC, CA and AB are respectively $\frac{2\Delta}{3a}$, $\frac{2\Delta}{3b}$ and $\frac{2\Delta}{3c}$

IV. Distance between the centres:

1. Distance between its incentre and circumcentre is = $\sqrt{R^2 - 2rR} = R\sqrt{1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

2. The distance between the circum – centre and the orthocentre of a triangle ABC if R $\sqrt{1 - 8\cos A\cos B\cos C}$

3. Distance between its incentre and orthocenter, is = $\sqrt{2r^2 - 4R^2\cos A\cos B\cos C}$

Example:

i) In a triangle, the line joining the circumcentre to the incentre, is parallel to the side BC then r = RcosA and cosB + cosC = 1

ii) In a triangle, the line joining the circumcentre to the orthocentre, is parallel to the side AC then tanA.tanC = 3 (because RcosA = 2RcosAcosC)

V.Types of triangle:

1. In an equilateral triangle ABC,

i) $r : R : r_1 = 1 : 2 : 3$

ii) $r_1 = r_2 = r_3 = 3r = \dfrac{3R}{2}$

iii) $R = 2r$

2. If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ then the triangle ABC is an equilateral triangle.

3. If $\tan A + \tan B + \tan C = 3\sqrt{3}$ then the triangle is an equilateral triangle.

4. If $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle is an equilateral triangle.

5. If $\cot\frac{A}{2} = \frac{b+c}{a}$, then the ΔABC is right angled triangle

6. If $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$ then the triangle is right triangle

7. If $\sin^2 A + \sin^2 B + \sin^2 C = 2$ or $\cos^2 A + \cos^2 B + \cos^2 C = 1$ or $a^2 + b^2 + c^2 = 8R^2$ then the triangle is right angled triangle

8. If in a triangle ABC, $(r_2 - r_1)(r_3 - r_1) = 2r_2 r_3$, then the triangle is right triangle

9. If in a triangle $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$ then the triangle is right triangle

10. If in a triangle $r_1 = s$, then the triangle is right triangle.

11. If $r = r_1 - r_2 - r_3$, then the triangle is right triangle and right angle at A

12. If $rr_1 = r_2 r_3$, then the triangle is right triangle and right triangle at A

13. $2R + r = r_1$ then the triangle is right triangle and right angle at A.

14. If $r : R : r_1 = 2 : 5 : 12$ then the triangle is right triangle

15. $b\cos B = a\cos A$ then the triangle is isosceles or right triangle

16. If $b\cos A = a\cos B$, then the triangle is isosceles

17. If $\cos A = \frac{\sin B}{2\sin C}$ then the triangle is isosceles.

VI. Height (altitude)

$h_1, h_2 \,\&\, h_3$ are the lengths of the altitudes drawn from the vertices A, B and C, and $h_1 = \frac{2\Delta}{a}$, $h_2 = \frac{2\Delta}{b}$ and $h_3 = \frac{2\Delta}{b}$ then

a) $\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} = \frac{1}{r}$

b) $\frac{-1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} = \frac{1}{r_1}$

c) $\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r^2} = \frac{r_1 r_2 r_3}{r}$

d) $\frac{1}{h_1} + \frac{1}{h_2} + \frac{-1}{h_3} = \frac{1}{r_3} = \frac{1}{h_1} + \frac{1}{h_2} - \frac{1}{h_3} = \frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$

e) $h_1 h_2 h_3 = \frac{a^2 b^2 c^2}{8R^3}$

f) $\frac{\cos A}{h_1} + \frac{\cos B}{h_2} + \frac{\cos C}{h_3} = \frac{1}{R}$

g) $\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{1}{h_3^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$

h) $\frac{bh_1}{c} + \frac{ch_2}{a} + \frac{ah_3}{b} = \frac{a^2 + b^2 + c^2}{2R}$

i) minimum value of $\frac{b^2 h_1}{c} + \frac{c^2 h_2}{a} + \frac{a^2 h_3}{b}$ is 6Δ

j) If $\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}=2$ then the least value of $h_1h_2h_3=216$

k) $(\Sigma h_1)\left(\Sigma\frac{1}{h_1}\right)=(\Sigma a)\left(\Sigma\frac{1}{a}\right)$

l) $(\Sigma h_1)(\Sigma h_1h_2)(\Pi a)=(\Sigma a)(\Sigma ab)(\Pi h_1)$

m) $\left(\Sigma\frac{1}{h_1}\right)\Pi\left(\frac{1}{h_1}+\frac{1}{h_2}-\frac{1}{h_3}\right).(\Pi a^2)=16R^2$

VII. Area:

Area of the triangle ABC,

1. $\Delta=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\sqrt{s(s-a)(s-b)(s-c)}=\frac{abc}{4R}=2R^2\sin A\sin B\sin C=$
$\sqrt{rr_1r_2r_3}=rs=s(s-a)\tan\frac{A}{2}=4Rr\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=r^2\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

$R=\frac{abc}{4\Delta}$, $S=\frac{a+b+c}{2}$

VIII.

Properties of Triangle:

1 The reflection of the orthocenter with respect to any side of the triangle lies on the circum circle

2. If H is the orthocenter of a triangle ABC then C, A, B are the orthocenter of triangles HAB, HBC, HCA respectively

3. The circum centre of a triangle is same as the orthocenter of its midpoint triangle

4. The orthocenter of a triangle is same as the incenter of its pedal triangle.

5. If D,E,F are the feet of the altitudes of a triangle ABC then A, B, C are the excentres of the triangle DEF. and the circumradius of triangle DEF is $\frac{R}{2}$

6. If AD is the altitude of a right angled triangle ABC with A = $\frac{\pi}{2}$ then AD^2 = BD. DC

7. **Ptolemy's theorem**: In a cyclic quadrilateral ABCD, AC.BD = AB.CD + BC.AD i.e., in a cyclic quadrilateral the product of diagonals is equal to the sum of the producted of the lengths of the opposite sides,

8. In any right angled triangle, the orthocenter coincides with the vertex containing the right angled.

9. The mid – point of the hypotenuse of a right angled triangle is equidistant from the 3 vertices of the triangle.

10. The mid point of the hypotenuse of the right angled triangle is the circum – centre of the triangle.

11. **Pedal triangle**:

Let ABC be any triangle, and let AK, BL and CM be the perpendicular from A,B and C upon the opposite sides of the triangle. These three perpendiculars meet at a point O' which is called the orthocenter of the triangle ABC. The triangle KLM, formed by joining the feet of these perpendiculars is called the pedal triangle of ABC.

(i) Orthocentre of the triangle is the incetre of the pedal triangle.

(ii) If I_1, I_2, and I_3 be the centres of escribed circles which are opposite to A,B and C respectively and I is the centre of incircle then triangle ABC is the pedal triangle of the triangle $I_1 I_2 I_3$ and I is the orthocenter of the Δ triangle $I_1 I_2 I_3$.

(iii) Circum – centre of the pedal triangle of a given triangle bisects the line joining the circumcentre of the triangle to the orthocenter

12. The centroid of the triangle lies on the line joining the circumcentre to the orthocenter and divides it in the ratio 1 : 2

13.Circle circumscribing the pedal triangle of a given triangle bisects the sides of the given triangle and also the lines joining the vertices of the given triangle to the othocentre of the given triangle. This circle is known as nine – point circle

14. In a triangle ABC, bisector of $\lfloor C$ meets the side AB at D and circumcircle of E. Then the maximum value of CD. DE is $c^2/4$

15. let ABC be an acute angled triangle whose orthocentre is at H.
Altitude from A is produced to meet the circumcircle of triangle ABC at D then HD =4R cosB cosC, because image of orthocentre H in any side of an acute triangle, lie on its circumcircle.

IX. Formulae based on r and R

1. $a\cot A+b\cot B+c\cot C=2(R+r)$

2. cos A + cos B + cos C = 1 + $\frac{r}{R}\left(\frac{R}{r}\geq 2\right)$

3. $a\cos A+b\cos B+c\cos C=\frac{8\Delta^2}{abc}=4R\sin A\sin B\sin C$

4. $a\cot A+b\cot B+c\cot C=2(R+r)$

5. $\sin A+\sin B+\sin C=\frac{s}{R}=\frac{\Delta}{Rr}$

6. $\Delta\leq\frac{s^2}{4}$ and $\Delta\leq\frac{s^2}{3\sqrt{3}}$

7. In a triangle ABC, a cos(B – C) + b cos(C – A) + c cos(A – B)= $\frac{abc}{R^2}$

8. In $\Delta ABC \cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}$ is equal to $\frac{\Delta}{r^2}$

X. Results related to ex-radii

1. $r_1r_2r_3r=\Delta^2$

2. $r_1r_2+r_2r_3+r_3r_1=s^2$

3. $r_1+r_2+r_3=4R+r$

4. $-r_1+r_2+r_3+r=4R\cos A$

5. $r_1-r_2+r_3+r=4R\cos B$

6. $r_1+r_2-r_3+r=4R\cos C$

7. $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$

8. $\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

9. Minimum value of $\frac{r_1r_2r_3}{r}$ is 27

10. If $d_1, d_2 \& d_3$ are the diameters of three escribed circles of a triangles then

$d_1d_2+d_2d_3+d_3d_1=\frac{4\Delta^2}{r^2}$

XI. Solving some of the standard triangles

1. AL, BM, CN are the altitudes of triangles ABC then (where L,M and N are the feet of the perpendiculars drawn from A, B and C respectively O' – orthocenter of ABC)
AM = c cos A
AN = b cos A
BL=c cos B
O^1LCM, O^1LBN, O^1MAN are cyclic quadrilaterals, which implies

$\angle MLN = \pi - 2A$

$\angle LMN = \pi - 2B$

$\angle MNL = \pi - 2C$

MN= a cos A LM = c cos C NL = b cos B

area AMN = $\Delta \cos^2 A$

area BNL = $\Delta \cos^2 B$

area CLM = $\Delta \cos^2 C$

area (LMN) = $2\Delta \cos A \cos B \cos C$

in radius (LMN)= 2R cosA cosB cosC

circum radius (LMN) = R/2

$$\frac{\text{perimeter}(LMN)}{\text{perimeter}(ABC)} = \frac{r}{R}$$

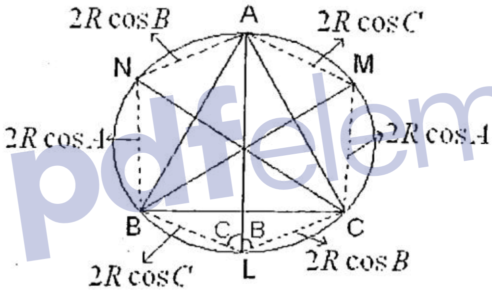
2. The internal angle bisector of triangle ABC, drawn from A,B and C meet its circumcircle at D, E, F respectively , then

(a) $\angle D = \frac{\pi}{2} - \frac{A}{2}$ $\angle E = \frac{\pi}{2} - \frac{B}{2}$ $\angle F = \frac{\pi}{2} - \frac{C}{2}$

(b) $DE = 2R\cos\frac{C}{2}$ $DF = 2R\cos\frac{B}{2}$ $DF = 2R\cos\frac{A}{2}$

(c) area $(DEF) = \frac{R\Delta}{2r}$ (d) $\frac{\text{area}(DEF)}{\text{area } ABC} = \frac{R}{2r}$

3.AL, BM, CN are the diameters of the circumcircle of a triangle ABC, then



(a) Area (triangle BLC) = $2R^2 \sin A \cos B \cos C$

(b) Area (triangle CMA) = $2R^2 \sin B \cos A \cos C$

(c) Area (triangle ANB) = $2R^2 \sin C \cos A \cos B$

(d) Area (ABC) = area (BLC) + area (CMA) + area (ANB)

4. DEF is triangle formed by joining the point of contacts of in circle with sides of ΔABC

a) sides of DEF are, $2r\cos\frac{A}{2}, 2r\cos\frac{B}{2}, 2r\cos\frac{C}{2}$

b) angles of DEF are $\frac{\pi}{2} - \frac{A}{2}, \frac{\pi}{2} - \frac{B}{2}, \frac{\pi}{2} - \frac{C}{2}$

c) area DEF is $\frac{r\Delta}{2R}$.

5. In the triangle $I_1I_2I_3$

a. Area = $8R^2 \cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right) = \frac{abc}{2r}$

b. Radius of the circle inscribed in the triangle $I_1I_2I_3$ is = $\frac{4R\cos A\cos B\cos C}{\cos\left(\frac{A}{2}\right)+\cos\left(\frac{B}{2}\right)+\cos\left(\frac{C}{2}\right)}$

c. $II_1.II_2.II_3 = 16R^2 r$

d. $(II_1)^2 + (I_2I_3)^2 = (II_2)^2 + (I_1I_3)^2 = (II_3)^2 + (I_2I_1)^2$

6. "I" is the incentre of triangle ABC, R_1, R_2, R_3 are radii of the circumcentre of triangle IBC, ICA, IAB then (a) $R_1 = \frac{a}{2\cos A/2}$ (b) $\frac{b}{2\cos B/2} = R_2$ (c) $R_3 = \frac{c}{2\cos C/2}$ (d) $R_1R_2R_3 = 2R^2r$

7. "O" is the circumcentre of triangle ABC, R_1, R_2, R_3 are the radii of the circumcircles of triangle OBC, OCA, OAB. Then $R_1 = \frac{R^2a}{4\Delta_1}$; $R_2 = \frac{R^2b}{4\Delta_2}$; $R_3 = \frac{R^2c}{4\Delta_3}$ & $\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3} = \frac{abc}{R^3}$

($\Delta_1, \Delta_2, \Delta_3$ are area of triangle OBC, OCA, OAB)