Electro Magnetic Induction

Magnetic flux :- No. of magnetic field lines passing through unit area and $\perp^{lar}$ to it.

Magnetic flux is defined as,

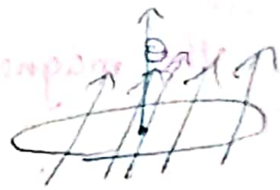
$$\phi = \vec{B} \cdot \vec{ds}$$

$$\phi = B(ds)\cos\theta$$

θ is angle between $\vec{B}$ and $\vec{ds}$

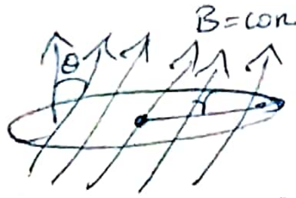
→ Total magnetic flux through closed loop is

B is variable



B is constant

$$\phi_{total} = B\,(\text{area})\cos\theta$$



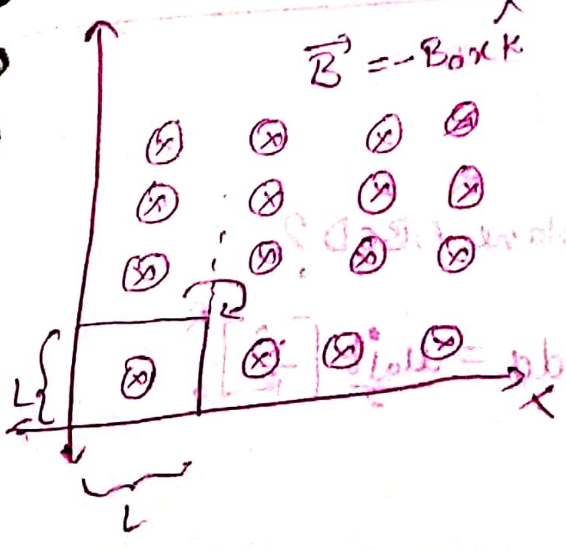
$$d\phi = B \cdot ds\cos\theta$$

$$\phi_{net} = \int d\phi = \int B \cdot ds\cos\theta$$

$$\phi_{total} = B \cdot \pi r^2 \cos\theta$$

Q :- If square loop ABCD is rotated by an angle of 180° about BC side then find magnitude of change in magnetic field.

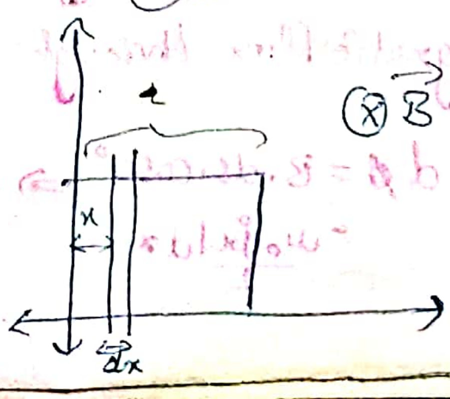
$$\vec{B} = -B_0 x \hat{K}$$



Ans :-

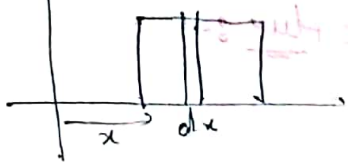
Area vector is inside the page.

$$\otimes \vec{B} = -B_0 x \hat{K}$$



$$\otimes \vec{B} = -B_0 x \hat{K}$$

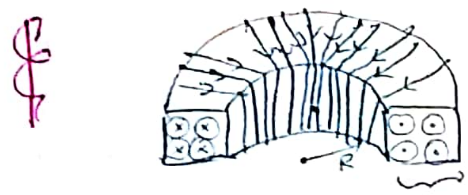
$d\phi = B_0 x (ds) \cos 0$
$= B_0 x\, l \cdot dx$
$d\phi = B_0 l\, x dx$
$\phi_{total} = \int d\phi \quad B_0 l \int_0^l x dx$
$\phi_1 = B_0 l \cdot \frac{L^2}{2} = \frac{B_0 l^3}{2}$



$d\phi = B_0 x\, ds \cos 180°$
$= -B_0 x\, l dx$
$\int d\phi = -B_0 l \int_L^{2l} x dx$
$\phi_{net_2} = -B_0 l \left(\frac{3L^2}{2}\right)$
$\phi_2 = -\frac{3 B_0 l^3}{2}$

change in $\phi = \phi_2 - \phi_1$
$= -\frac{B_0 l^3}{2} + \left(-\frac{3B_0 l^3}{2}\right)$
$= -2 B_0 L^3$.

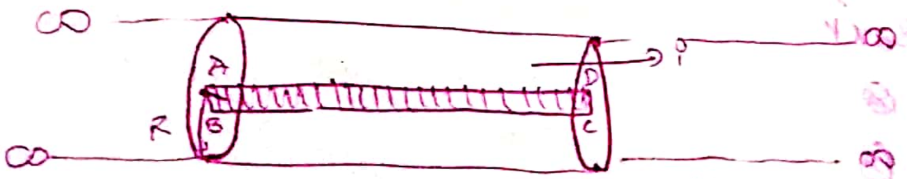
Q) Find the magnetic flux through cross section of toroid.



N - total no. of turns.

$B = \frac{\mu_0 N i}{2\pi x}$
$d\phi = B \cdot l \cdot dx \cos 0.$
$d\phi = \frac{\mu_0 N i l}{2\pi} \frac{dx}{x}$
$\phi = \int d\phi = \frac{\mu_0 N i L}{2\pi} \ln (x)\Big|_R^{R+l}$
$\phi = \frac{\mu_0 N i l}{2\pi} \ln\left(\frac{L+R}{R}\right)$

Q)



Find the magnetic flux through the plane ABCD?

Ans $B = \frac{\mu_0 j x}{2}$ $\quad d\phi = B \cdot ds \cdot \cos 0°$ $\Rightarrow \phi = \int d\phi = \frac{\mu_0 j l}{2}\left[\frac{x^2}{2}\right]_0^R$
$= \frac{\mu_0 j x l dx}{2}$

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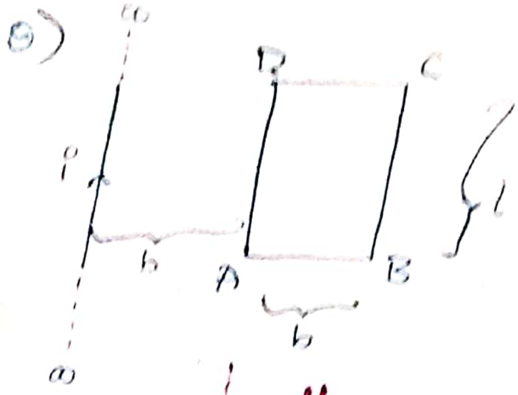
$\left[\phi = \frac{\mu_0 a i}{4\pi}\right]$

Q) Find the ϕ_{mag} through ABCD loop.

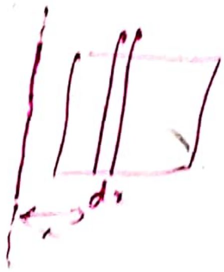


$\cancel{d\phi = B\,ds\cos 0}$

$\cancel{= B\,l}$

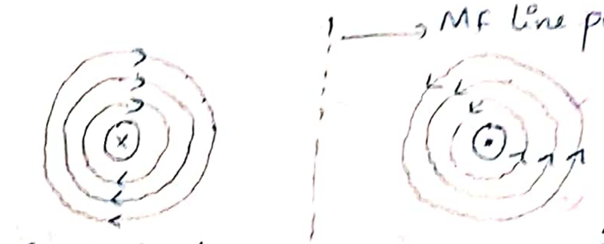
Ans $d\phi = B\,ds\cos 0°$

$d\phi = \frac{\mu_0 i}{2\pi x}\, l\,dx$



$\phi = \frac{\mu_0 i l}{2\pi}\int_b^{2b}\frac{dx}{x} \Rightarrow \phi = \frac{\mu_0 i l}{2\pi}\ln(x)_b^{2b}$

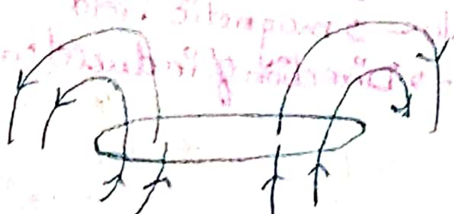
$\phi = \frac{\mu_0 i l}{2\pi}\ln(2)$

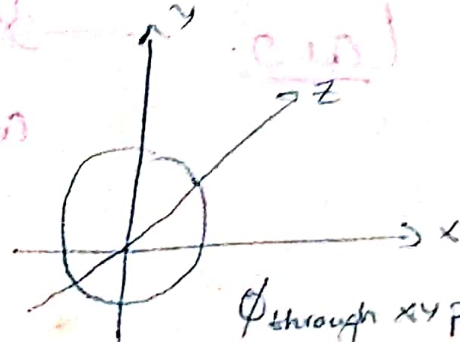


Semi circular loop out of plane.

C.S view of circular wire MF lines

* Total flux through XY plane is zero.





ϕ through loop $= \pi R^2$

ϕ total $= 0$

ϕ through XY plane except πR^2 $= -\pi R^2$

Note:- ϕ_{total} through the plane in which circular conducting coil is there is zero.

* Refer to NCERT Pg-166 12-Part-I.

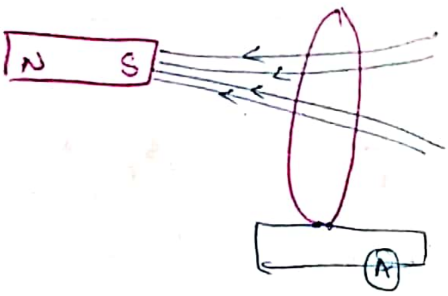
Faraday's Law

⟹ It states that whenever there is a change of ϕ_{mag} through closed loop, there exist emf in the loop.

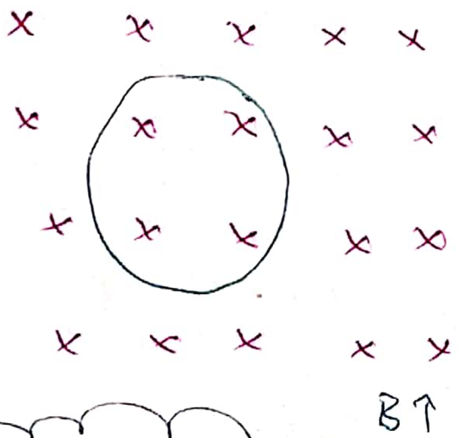
⟹ Emf developed in the loop is called as induced emf. Current developed is called induced current.

Magnetic flux, $\phi = B(\text{area})\cos\theta$ can be changed by

- B is variable
- Area is Variable
- θ is variable (when loop rotates)



⟹ Whenever there is a relative motion b/w bar magnet and coil, current can be found in the coil.



Induced emf in the loop is given by rate of change of magnetic flux

$$\text{Induced emf} = \left|\frac{d\phi}{dt}\right|$$

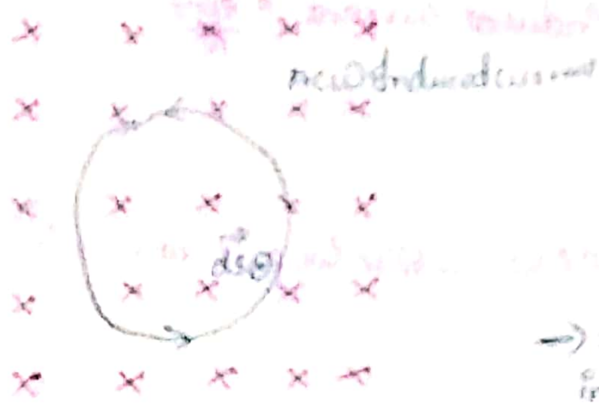
Lenz's Law

Left hand thumb rule

Thumb ⟶ magnetic field

Curl fingers ⟶ Direction of induced current

⟹ Always induced current produced in the loop such that "it opposes the cause of it's creation".



B is increasing

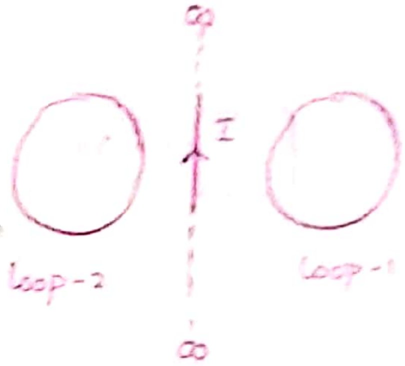
$\phi = B \cdot ds \cos 0^\circ$

$\phi = B \cdot ds$

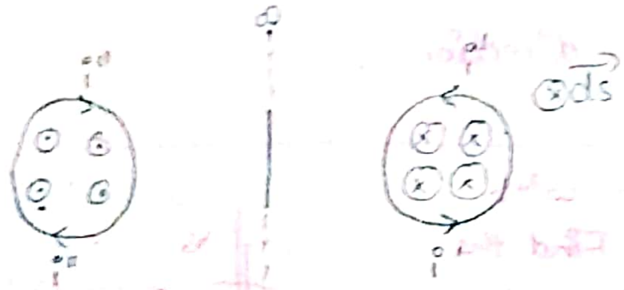
$B\uparrow \Rightarrow \phi\uparrow$

→ Induced current is in ACW in direction.

Q) Find the direction of current in loop 1 and loop 2 if "I" is increasing?



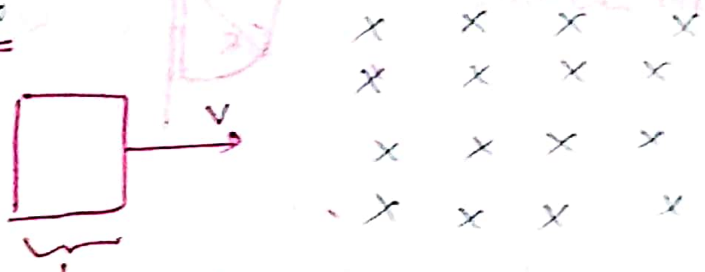
⇒ If 'I' is increasing the direction of induced current in loop-1 ACW
loop-2 CW



$\phi = B \cdot ds$

$B\uparrow \; \phi\uparrow$

Example



B is uniform
v is constant.

Resistance of loop is R

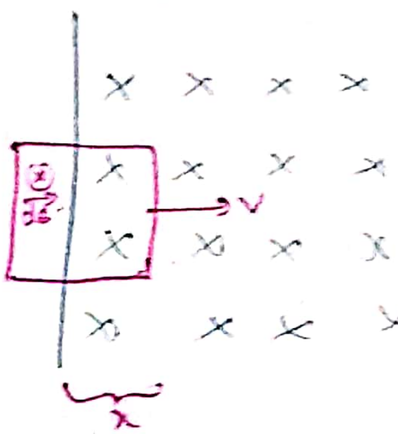
While entering loop :-

$\phi = B \cdot ds \cos 0^\circ$

$\phi = B \cdot lx$

Induced emf $= \left|\frac{d\phi}{dt}\right| = Bl\frac{dx}{dt}$

$E = Blv$

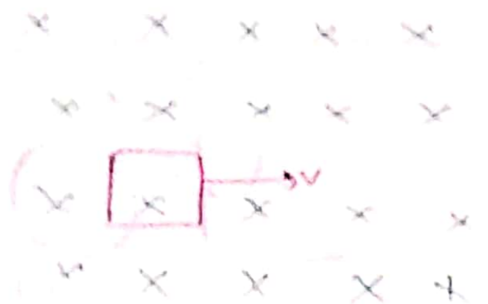


ii) If R is the resistance of loop, Induced current $= \frac{emf}{R}$

$$\boxed{i = \frac{Blv}{R}}$$

iii) Induced current direction is ACW while loop is entering $\vec{B}$

While loop is moving inside the $\vec{B}$:



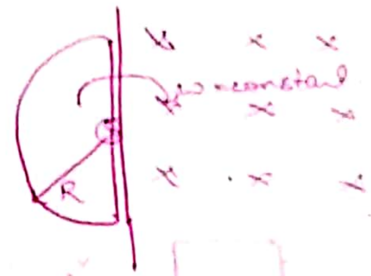
$\phi = Bl^2 \Rightarrow \frac{d\phi}{dt} = 0 \Rightarrow (emf = 0)$

$\Rightarrow$ Induced current $= 0$

While loop leaving the $\vec{B}$

Current is $\frac{Blv}{R}$ in CW direction.

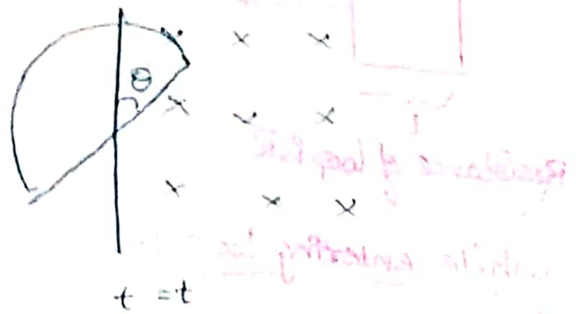
Q) If semi circular wire rotates with constant angular velocity ω. Find the induced emf as function of time and draw a graph b/w emf and time?



Ans:-

$$\omega = \frac{d\theta}{dt}$$

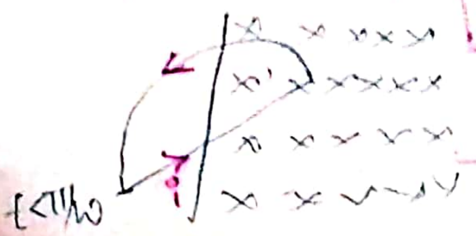
$$\text{Area} = \frac{\theta}{2\pi} \times \pi R^2 = \frac{1}{2}\theta R^2$$



$$\phi = B(\text{Area}) = B \cdot \frac{1}{2}\theta R^2$$

$$\phi = \frac{1}{2} B R^2 \theta$$

$$\left|\frac{d\phi}{dt}\right| = \frac{1}{2} B R^2 \cdot \frac{d\theta}{dt} \Rightarrow emf = \frac{1}{2} B R^2 \omega = \text{constant}$$



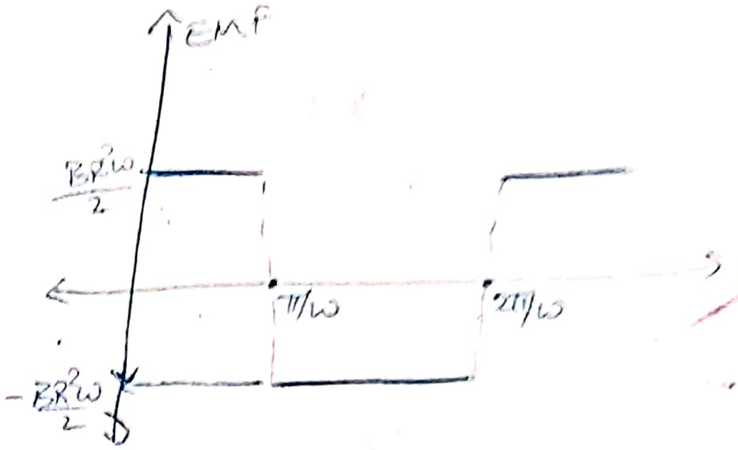
while entering ϕ is ⊙ ↑

i is in ACW

while leaving; $\phi \downarrow$

i is inflow

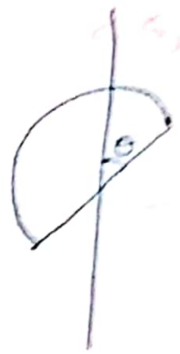
$t > \pi/\omega$



Q:- If loop is rotating with constant angular acceleration then find induced emf and draw graph b/w emf and time.

Ans Area $= \frac{1}{2}\theta R^2$

$$\phi = B \times \frac{1}{2}\theta R^2 \cos 0$$

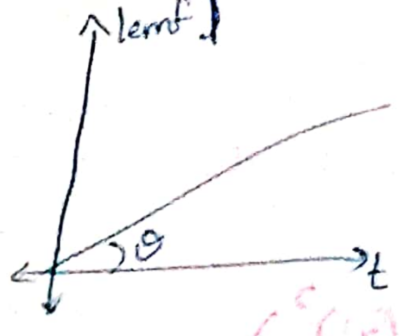


$$\frac{d\phi}{dt} = B \times \frac{1}{2}\theta R^2 \cos 0$$

$$\frac{d\phi}{dt} = \frac{BR^2}{2}\frac{d\theta}{dt}$$

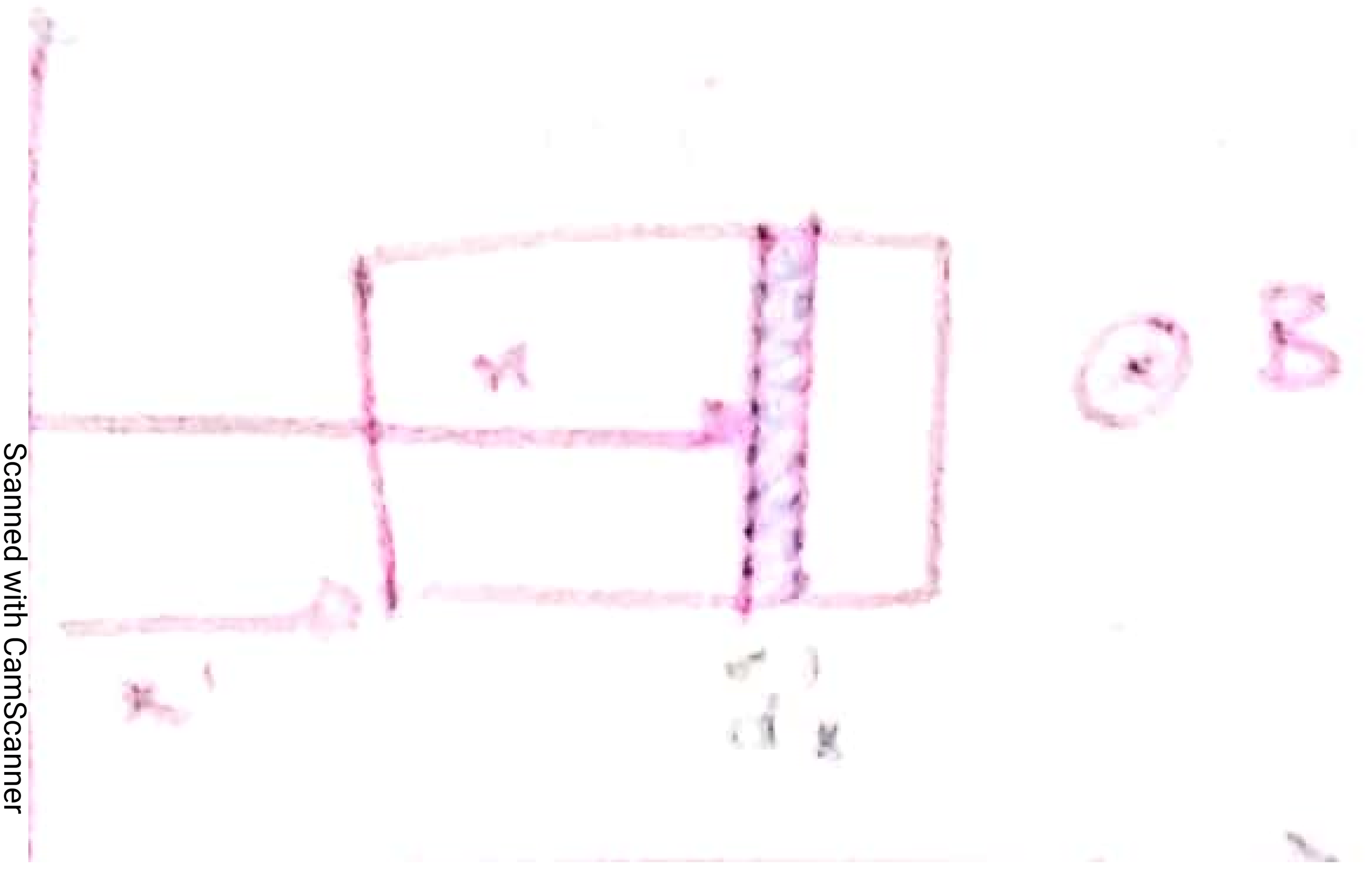
$$emf = \frac{BR^2}{2}\frac{d\theta}{dt} \qquad \boxed{emf = \frac{BR^2}{2}\alpha t}$$

Induced emf magnetic field $= \left(\frac{BR^2}{2}\alpha\right) t$



$$\tan\theta = \frac{BR^2\alpha}{2}$$

the induced emf in the loop?



$\phi = B_0\left(l^2 + \frac{l^2 + 2lx'}{2}\right)$

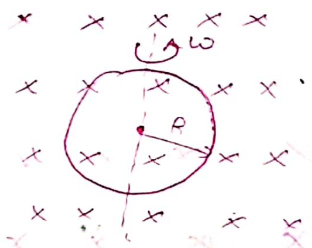
$\phi = B_0\left(\frac{3l^2}{2} + lx'\right)$

Emf $= \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}\left(B_0\left(\frac{3l^2}{2} + lx'\right)\right)$

$= B_0 l \frac{dx'}{dt}$

$\boxed{\text{Emf} = B_0 l v}$ $\boxed{\text{Induced current is in ACW}}$

Q) (i) Find the expression for induced emf
(ii) Find the charge flown through loop when it rotates by π?
(Resistance of wire is r)



$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t$

Ans

(i) $\phi = \oint B\,ds \cos\theta$

$= B\pi R^2 \cos\theta = B\pi R^2 \cos\omega t$

$\cancel{= B\pi R^2 \cos}$

Emf $= \left|\frac{d\phi}{dt}\right| = \left|B\pi R^2 \frac{d(\cos\omega t)}{dt}\right|$

$= \left|B\pi R^2 (-\sin\omega t)\cdot\omega\right|$

EMF $= B\pi R^2 \omega \sin\omega t \longrightarrow$ AC emf

(ii) Charge flown $= \left|\frac{-B\pi R^2 - B\pi R^2}{r}\right|$

$\boxed{\text{charge flown} = \frac{2B\pi R^2}{r}}$

Note: ① $EMF = \frac{d\phi}{dt} \Rightarrow$ Induced current $i = \frac{emf}{r}$

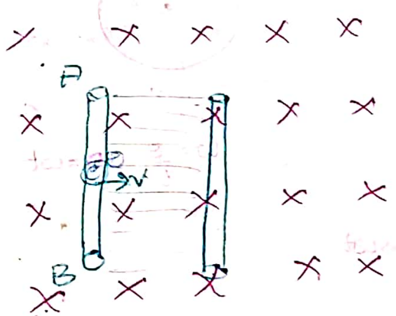
$$\frac{dq}{dt} = \frac{d\phi}{r\,dt}$$

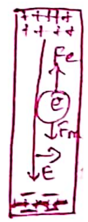
$$\boxed{dq = \frac{d\phi}{r}}$$

② Charge flow is independent of time

$$\boxed{\text{charge flow} = \frac{\phi_{final} - \phi_{initial}}{\text{Resistance}}}$$

Motional EMF





Dir
Higher potential

At steady state

$\vec{F_m} = -\vec{F_e}$, No more accumulation of charge.

$-e(\vec{v}\times\vec{B}) = -(-e\vec{E})$

$\vec{E} = -\vec{v}\times\vec{B}$

$\Delta V = \int \vec{E}\cdot\vec{dl}$

$|\Delta V| = \int(\vec{v}\times\vec{B})\cdot\vec{dl}$

$= vBl\cos 0$

$|\Delta V| = vBl$

Area covered by rod in time t

$= (vt)l$

$\phi = B(vt)l$

$\left|\frac{d\phi}{dt}\right| = Bvl$

Higher po
palm
Keep you
in the d
has high
potentia
Examp

Higher potential and lower potential is decided by right hand palm rule. Right hand ~~Thumb~~ Palm Rule

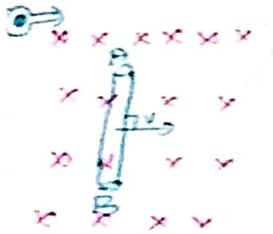
~~keep your thumb in the velocity of~~

Keep your thumb in the direction of velocity and all other fingers in the direction of magnetic field then the above surface of hand has higher potential and the lower side of hand have lower potential

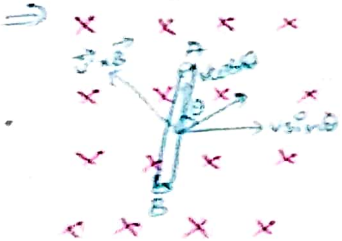
Example

$|\Delta V| = (\vec{v} \times \vec{B}) \cdot \vec{dl}$

$|\Delta V| = 0$



$|\Delta V| = V_A - V_B = BLV$

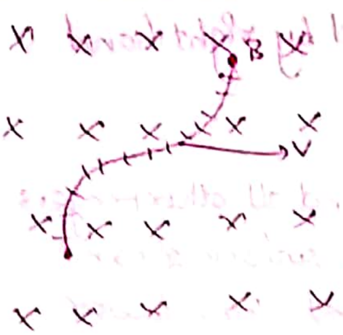


$\Delta V = V_A - V_B = Bl(v\sin\theta)$

$\Delta V = (\vec{v} \times \vec{B}) dl$

$= vB\sin 90 \cdot l\cos(90-\theta)$

$\Delta V = vBl\sin\theta$



$$\Delta V_1 = (\vec{v}\times\vec{B})\cdot\vec{dl_1}$$

$$\Delta V_2 = (\vec{v}\times\vec{B})\cdot\vec{dl_2}$$

$$\vdots$$

$$\Delta V_n = (\vec{v}\times\vec{B})\cdot\vec{dl_n}$$

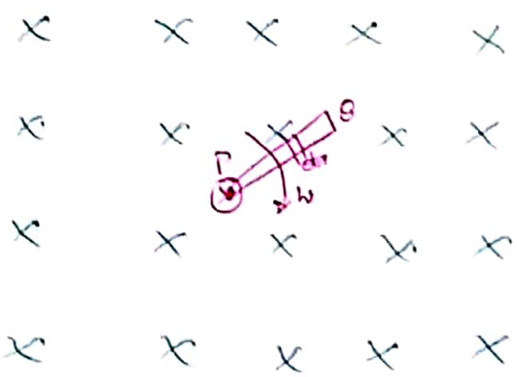
$$V_{AB} = \Delta V_1 + \Delta V_2 + \cdots + \Delta V_n$$

$$V_{AB} = (\vec{v}\times\vec{B})(\vec{dl_1}+\vec{dl_2}+\cdots+\vec{dl_n})$$

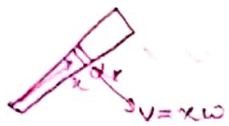
$$\boxed{V_{AB} = (\vec{v}\times\vec{B})\cdot\vec{AB}}$$

does not depend on shape
depends only on initial and final post of conductor.

Q) Find the P.d across the ends of rod.



Ans



~~$dB = B\,dx$~~

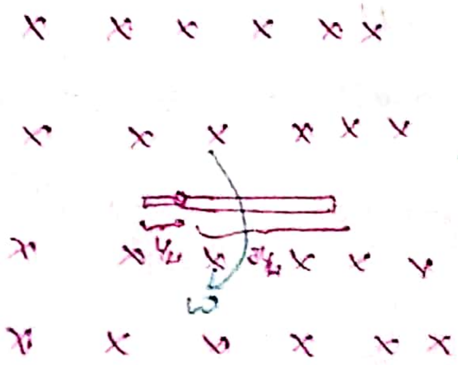
$$dv = B\,dx\,x\omega$$

$$V_{net} = \int dv = B\omega\int x\,dx$$

Q is higher potential
P is lower potential

$$\boxed{V_{net} = \frac{B\omega l^2}{2}}$$

Q)



Ans

$$\frac{x^2}{2} \quad \frac{9l^2}{16} - \frac{l^2}{16} \div 2$$

$$\div 2$$

~~Pot across~~

$$V_Q - V_0 = \frac{B\omega}{2}\left(\frac{3l}{4}\right)^2 = \frac{9B\omega l^2}{32}$$

$$V_P - V_0 = \frac{B\omega}{2}\left(\frac{l}{4}\right)^2 = \frac{B\omega l^2}{32}$$

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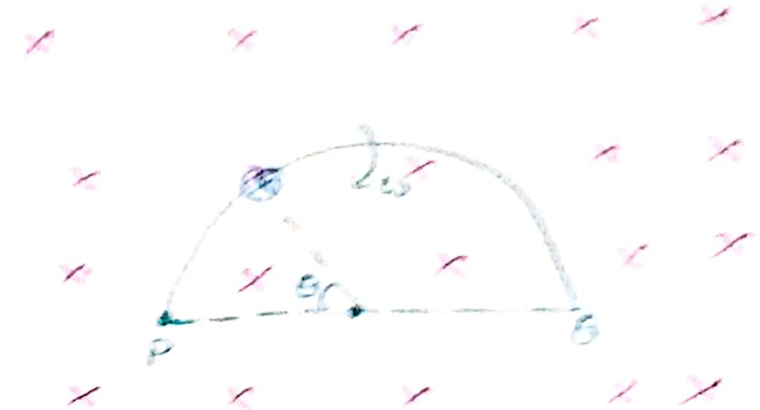
$V_Q - V_P = \frac{6 B \omega l^2}{4 \times 2} = \frac{B \omega l^2}{4}$

Q)



Ans

$$V_P - V_O = \frac{B\omega(2R\sin\theta/2)^2}{2}$$

$$V_P - V_O = 2B\omega R^2 \sin^2(\theta/2)$$

$$V_Q - V_O = \frac{B\omega(2R\cos\theta/2)^2}{2}$$

$$V_Q - V_O = 2B\omega R^2 \cos^2\theta/2$$

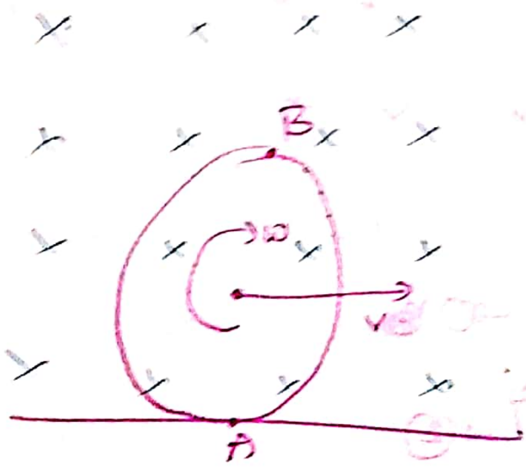
$$V_P - V_Q = 2B\omega R^2(\sin^2\theta/2 - \cos^2\theta/2)$$

$$V_P - V_Q = -2B\omega R^2 \cos\theta$$

$$V_Q - V_P = 2B\omega R^2 \cos\theta$$

Level - 1
Motional emf
Pg-44 3-10
Pg-49 3-14

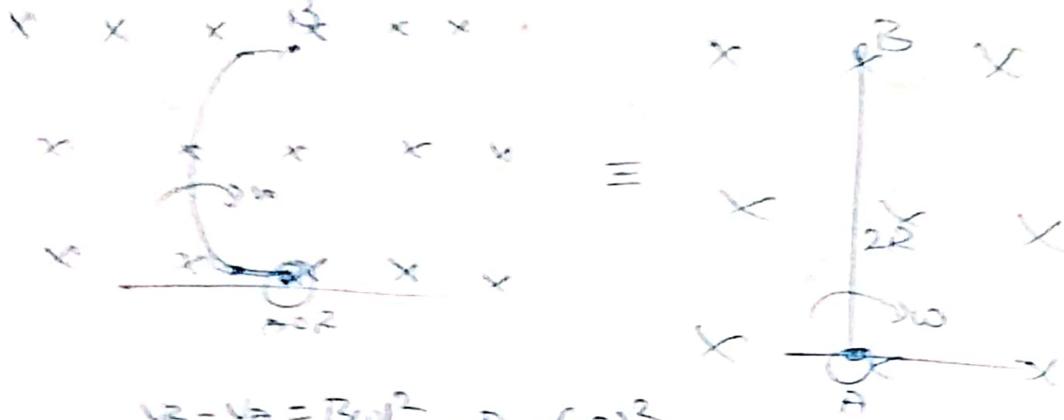
Q)



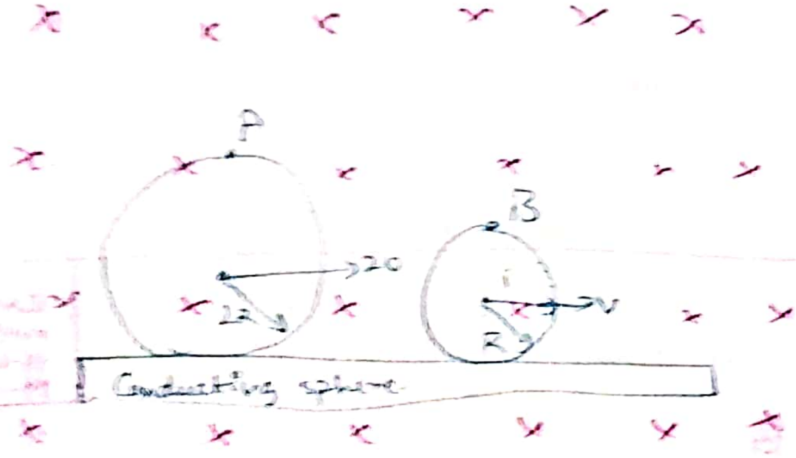
Find the potential difference across A and B?

within and in the ring is zero.

pure rotation



$$V_B - V_A = \frac{B\omega l^2}{2} = \frac{B\omega(2R)^2}{2} = 2B\omega R^2$$

Q) 

Find the p.d b/w A and B?

Ans

$$V_A - V_0 = \frac{B\omega(4R)^2}{2} \rightarrow ①$$

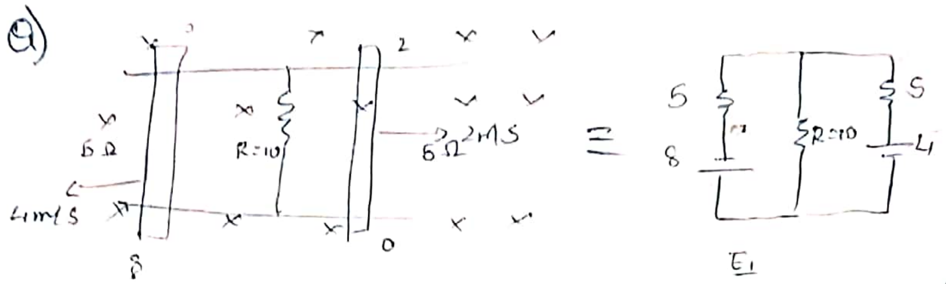
$$V_B - V_0 = \frac{B\omega(2R)^2}{2} \rightarrow ②$$

① − ②

$$V_A - V_B = 6B\omega R^2$$

zero.

Q)

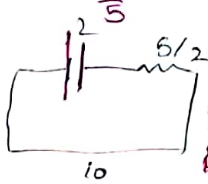


BLv

$2\times2\times1 = 4$

$4\times2\times1 = 8$

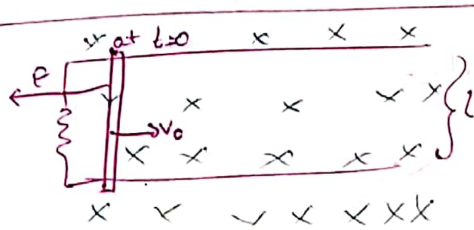
$$E_{net} = \frac{\frac{4}{5}-\frac{8}{5}}{\frac{2}{5}} = \frac{-\frac{4}{5}}{\frac{2}{5}} = 2$$



$R_{net} = 10 + 5/2$

$$i = \frac{2\times2}{25} = \frac{4}{25}$$

Q)



Find the velocity of rod as a function of time
m is mass of rod.

$$iR = B v_0 l$$

$$m\frac{dv}{dt} = -\frac{Bv_0 l}{R}\cdot Bl$$

Force and velocity are opposite.

$$m\int_{v_0}^{v}\frac{dv}{v} = -\frac{B^2L^2}{R}\int_0^t dt$$

$$m\ln\left(\frac{v}{v_0}\right) = \frac{-B^2L^2t}{R}$$

$$\ln\left(\frac{v}{v_0}\right) = \frac{-B^2L^2t}{mR}$$

$$\boxed{v = v_0 e^{-\frac{B^2L^2t}{mR}}}$$

Find the total heat lost through resistor

$$H = \int i^2 R\,dt$$

$$i = \frac{BL}{R} v_0 e^{-\frac{B^2L^2t}{mR}}$$

$$H = \int \frac{B^2L^2}{R^2} v_0^2 e^{-\frac{B^2L^2t\times2}{mR}} \cdot R\,dt = \int \frac{B^2L^2v_0^2}{R} e^{-\frac{B^2L^2t\times2}{mR}}\,dt$$

Scanned with CamScanner

$H = \frac{1}{2} m v_0^2$

Q)

If rod is released from rest find acceleration of rod.

$ma = mg - \frac{Bvl \cdot Bl}{R}$

$CE = Bvl$

$E = \frac{Bvl}{C}$

$ma = mg - ilB$

$q = C(Bvl)$

$ma = mg - CBlaBl$

$\frac{dq}{dt} = i = CBl\frac{dv}{dt}$

$a(m + CB^2l^2) = gm$

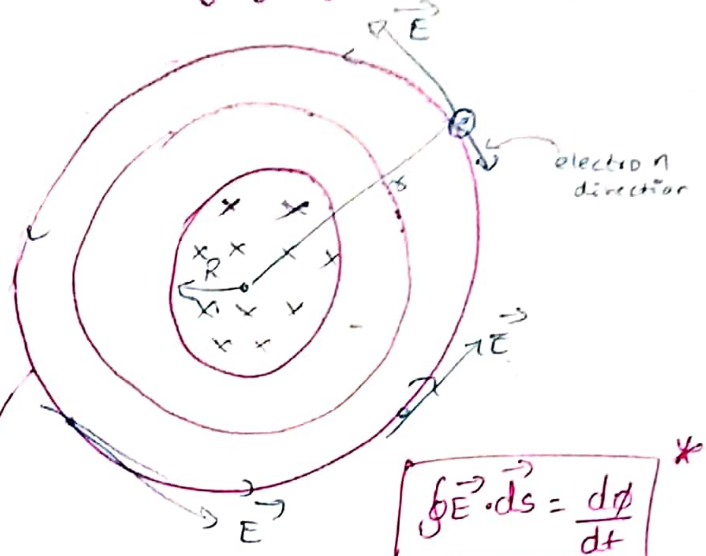
$i = CBla$

$$\boxed{a = \frac{mg}{m + CB^2l^2}}$$

Q)

Induced Electric field

⇒ Time varying magnetic field will produce electric field.



$B = f(t) \uparrow \Rightarrow \phi \uparrow$

Induced emf $= \dfrac{-d\phi}{dt}$

$-\oint \vec{E}\cdot\vec{ds} = -\dfrac{d\phi}{dt}$

$\oint E \cdot ds \cos 0 = \dfrac{d}{dt}(B\pi R^2)$

$$\boxed{\oint \vec{E}\cdot\vec{ds} = \frac{d\phi}{dt}}$$ *

$E \oint ds = \pi R^2 \dfrac{dB}{dt}$

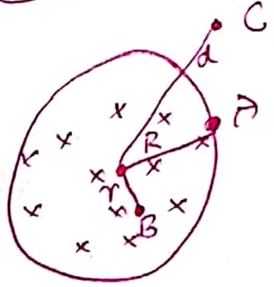
$E \cdot 2\pi r = \pi R^2 \dfrac{dB}{dt}$

$$\boxed{E = \frac{R^2}{2r}\left(\frac{dB}{dt}\right)}$$ *

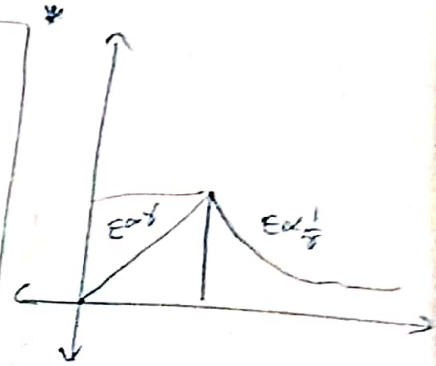
These are induced electric field line

Note:- Induced electric field line are concentric circles (they are closed loop)

* Induced electric field are non-conservative. Workdone $\neq 0$ in closed loop.

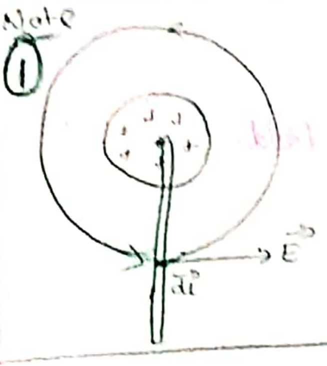


$$\boxed{\begin{aligned} E_C &= \frac{R^2}{2d}\left(\frac{dB}{dt}\right) \\ E_A &= \frac{R}{2}\left(\frac{dB}{dt}\right) \\ E_B &= \frac{r}{2}\left(\frac{dB}{dt}\right) \end{aligned}}$$ *



Note

1



As rod is kept along the radial direction

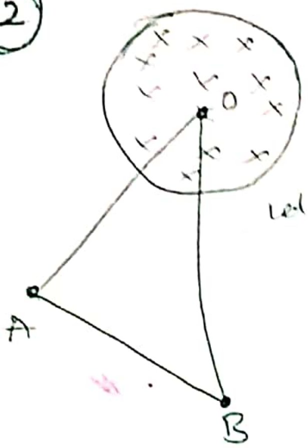
$\Delta V = -\int \vec{E} \cdot \vec{dl}$

$\Delta V = 0$

$\Delta V = 0$

If a rod is kept along radial directions P.d across the rod is zero

2



α - angle made by rod at centre

$B = f(t) \uparrow$

OA & OB are imaginary rods

Let OA and OB be imaginary rods.

Total induced emf in the loop $= \frac{d\phi}{dt}$

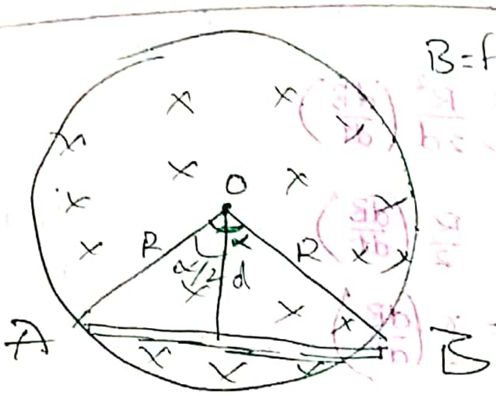
$= \frac{d}{dt}\left(B \cdot \frac{1}{2}R^2\alpha\right)$

$\vec{\mathcal{E}}_{OA} + \vec{\mathcal{E}}_{AB} + \vec{\mathcal{E}}_{BO} = \frac{1}{2}R^2\alpha \frac{dB}{dt}$

$\mathcal{E}_{AB} = \frac{1}{2}R^2\alpha \frac{dB}{dt}$

$$\boxed{V_B - V_A = \frac{1}{2}R^2\alpha \frac{dB}{dt}}$$

3



$2d \sin\theta/2 \times d$

$2d^2 \sin\theta/2$

$B = f(t) \uparrow$

$V_B - V_A = \frac{1}{2} d \times 2\sqrt{R^2 - d^2} \cdot \frac{dB}{dt}$

$V_B - V_A = d\sqrt{R^2 - d^2} \cdot \frac{dB}{dt}$

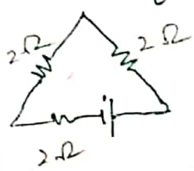
$d = R\cos\alpha/2$ $\quad V_B - V_A = R\cos\alpha/2 \; R\sin\alpha/2 \frac{dB}{dt}$

$$\boxed{V_B - V_A = \frac{1}{2}R^2 \sin\alpha \frac{dB}{dt}}$$

Q)

A — equilateral tri

OAB equilat having resis Find the change of



Q)

Em

Em

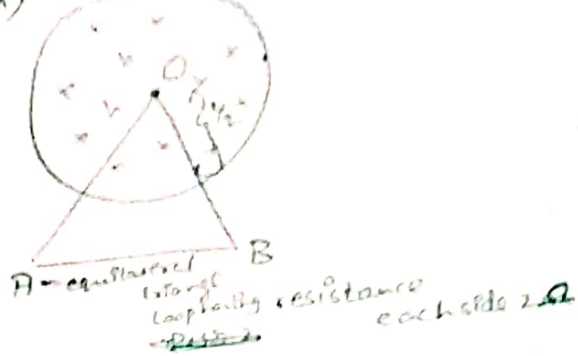
E

ction

tions

loop $= \frac{d\phi}{dt}$

Q)



$E_{ef} = \frac{1}{2} R^2 \frac{\pi}{3} \frac{dB}{dt}$

$Emf = \frac{1}{2} \times \frac{1}{4} \times \frac{\pi}{3} \times 2$

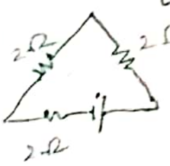
$Em = \frac{\pi}{12}$

$\cancel{\frac{\pi}{12} - \frac{\pi \times 2}{18\,9}}$

$\cancel{\pi\left(\frac{1}{9} + \frac{1}{12}\right)}$

OAB equilateral triangle loop having resistance of each side 2Ω. Find the P.d across AB if rate of change of field is 2 T/sec.

$6 \quad \frac{\pi}{18}$



$R_{net} = 6\,\Omega$

$i = \frac{\pi}{12 \times 6}$

$V_{AB} = \frac{\pi}{12} - \frac{\pi}{36} = \frac{\pi}{18}$

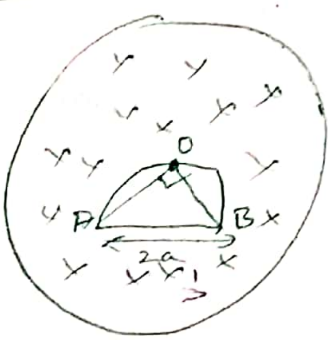
$V_B - V_A = \frac{\pi}{18}\ V$

$i \times 2 = \frac{\pi}{12 \times 6} \times 2 = \frac{\pi}{36}$

V across 2 Ω resistor.

$B = B_0 + \alpha t$

Q)



$\therefore \frac{dB}{dt} = \alpha$

$\int \vec{E} \cdot d\vec{l}$ along BOA but not AB

$Emf = \frac{1}{2} R^2_{\theta\,in} \frac{\pi}{2} \frac{dB}{dt}$

$\cancel{\frac{\pi R^2}{4} \frac{dB}{dt}}$

$Emf = \frac{\pi R^2}{4} \frac{dB}{dt}$

$\cancel{\frac{\pi R^2}{4} \frac{dB}{dt}}$

$\cancel{Emf} = \frac{1}{2} R^2 \frac{d(B_0 + \alpha t)}{dt}$

$= \frac{1}{2} R^2 \boxed{\alpha}$

$Emf = \frac{1}{2} a \cdot 2a \frac{dB}{dt}$

$Emf_{AB} = a^2 \alpha$

Emf of semicircular loop $= \frac{\pi a^2}{2} \frac{dB}{dt}$

$= \frac{\pi a^2}{2} \alpha$

Emf without AB $= a^2 \alpha \left(\frac{\pi}{2} - 1\right)$

Self Inductance

No external field

$B = \frac{\mu_0 i}{2R}$

$\phi = B(\pi R^2) = \frac{\mu_0 i}{2R}(\pi R^2)$

$\phi_{self} = \frac{\mu_0 i \pi R}{2}$

$i\uparrow, B\uparrow, \phi\uparrow$

{every loop opposes changes in currents by itself}

$\downarrow$

self Induction

$\phi \propto i$

$\phi = Li$

$L = \frac{\phi}{i}$

$\downarrow$

Self inductance $= \frac{\text{Tesla m}^2}{\text{amp}}$ = Henry.

$L_{ring} = \frac{\phi}{i} = \frac{\frac{\mu_0 i \pi R}{2}}{i} = \frac{\mu_0 \pi R}{2}.$

Q) $n \to$ no. of turns per unit length

Find the inductance of solenoid?

$B = \mu_0 n i$

$\phi = \mu_0 n i \pi R^2 \longrightarrow$ Flux through each turn

$L = \mu_0 n \pi R^2 \longrightarrow$ inductance through each turn

$\phi_{total} = (nl)(\mu_0 n \pi R^2 i)$

$\phi_{total} = n^2 \mu_0 l \pi R^2 i$

$L_{total} = n^2 \mu_0 l \pi R^2$

$L_{of\ solenoid} = \mu_0 n^2 \pi R^2 l$

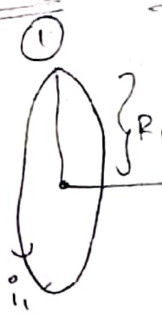
$L \propto l$

$\propto R^2$

$\propto n^2$

Q)

Mutual I



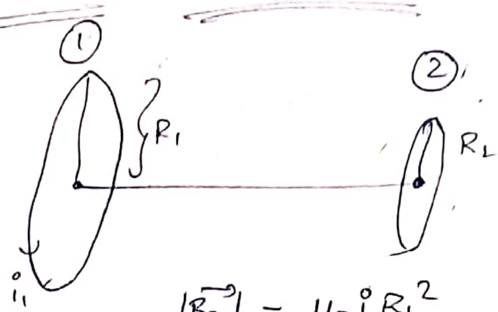
flux th

i_1

Mut

Q)

Mutual Inductance



$$|\vec{B_2}| = \frac{\mu_0 i R_1^2}{2(R_1^2+d^2)^{3/2}}$$

Flux through second coil $\phi_2 = B_2(\pi R_2^2)\cos 0°$.

$$\phi_2 = \frac{\mu_0 i_1 R_1^2}{2(R_1^2+d^2)^{3/2}}(\pi R_2^2)$$

$$\boxed{\phi_2 = \frac{\mu_0 \pi R_1^2 R_2^2 i_1}{2(R_1^2+d^2)^{3/2}}}$$

$$M_{21} = M_{12} = \frac{\mu_0 \pi R_1^2 R_2^2}{2(R_1^2+d^2)^{3/2}}.$$

$i_1 \uparrow \quad \phi \uparrow \quad i_2 \neq 0$

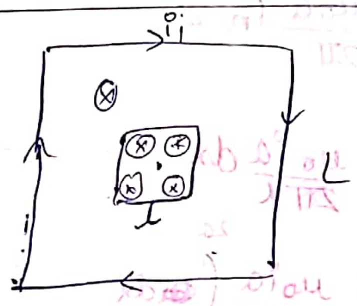
Mutual Conductance

$\phi_2 \propto i_1$ (or) $\phi_1 \propto i_2$

$\phi_2 = M_{21} i_1 \qquad \phi_1 = M_{12} i_2$

$M_{12} = M_{21} = \frac{\phi_2}{i_1}$ (or) $\frac{\phi_1}{i_2}$

Q)



$l <<< L$

Find the M between two coil.

$B = 2\sqrt{2}\frac{\mu_0 i_1}{\pi L}$.

$$\boxed{\phi_2 = \frac{2\sqrt{2}\mu_0}{\pi}\frac{l^2}{L} i_1}$$

$$M = \frac{2\sqrt{2}\mu_0}{\pi}\frac{l^2}{L}$$

Note:- Self Inductance (or) Mutual Inductance depends only on physical dimensions not on the current

Q)

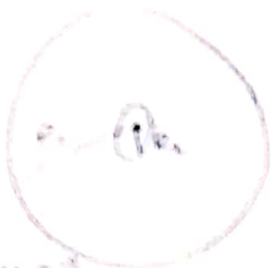
$m = ?$

Rings are concentric and coplanar

$$M = \frac{\mu_0 B}{2R_1}\pi R_2^2$$

$$M = \frac{\mu_0 \pi}{2}\frac{R_2^2}{R_1}$$

Q)



$m = ?$

Rings are concentric but not coplanar

They are kept $\perp^{r}$

$\vec{B} \perp d\vec{s}$

$\phi = 0$

$M = 0$

Q)

$\frac{\mu_0 i}{2\pi x}$

$$\phi = \frac{\mu_0 a}{2\pi}\ln 2$$

Ans
$$d\phi = \frac{\mu_0 i a}{2\pi x}dx$$

$$\int d\phi = \frac{\mu_0 i a}{2\pi}\int_a^{2a}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 i a}{2\pi}\ln(2)$$

$$m = \frac{\mu_0 a}{2\pi}\ln(2)$$

Induced EMF ac...

Symbol of inductor...

Consider a inductor...

Flux through...

Induced em...

Induced emf ...

Energy stored...

Induced EMF across Inductor

→ will show Inductance.

Symbol of Inductor → ⌇

Consider a Inductor with Inductance L.

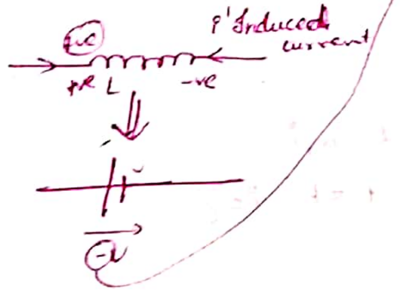
Flux through Inductor $= \phi = Li$

Induced emf $= e = -\frac{d\phi}{dt}$

$$e = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

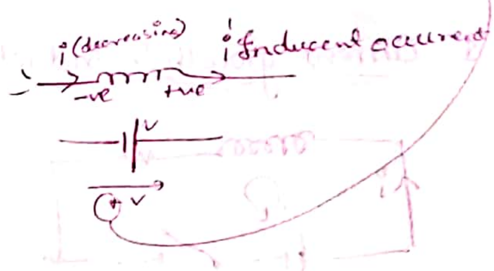
$\frac{di}{dt} > 0$

Induced emf $e = -L\frac{di}{dt}$ is $-ve$



$\frac{di}{dt} < 0$

Induced emf $e = -L\frac{di}{dt}$ is $+ve$



Energy stored in Inductor

i from 0 to i_0.

at an Instant -

di/dt

P.d across inductor $= L\frac{di}{dt}$

Power delivered by external agent at that Instant

$$P = Vi$$

$$P_{ext} = \left(L\frac{di}{dt}\right) i$$

→ instantaneous power

$$\frac{dW_{ext}}{dt} = L\frac{di}{dt} \cdot i$$

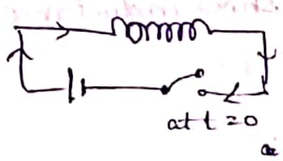
$$\int dW_{ext} = L\int_0^{i_0} di\, i$$

$$W_{ext} = L\left[\frac{i^2}{2}\right]_0^{i_0}$$

$$\boxed{W_{ext} = L\frac{i_0^2}{2} = \frac{1}{2}Li_0^2}$$

Scanned with CamScanner

Note:- If Inductor alone is connected to battery.

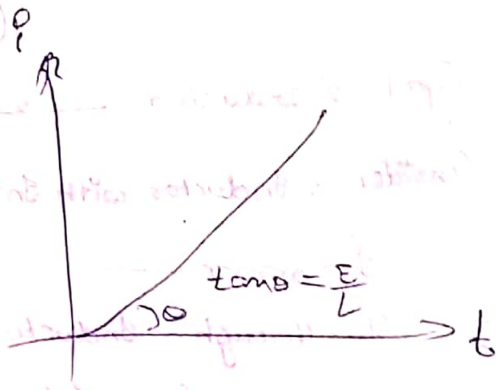


$$L\frac{di}{dt} = E$$

$$E\int_0^t dt = L\int di$$

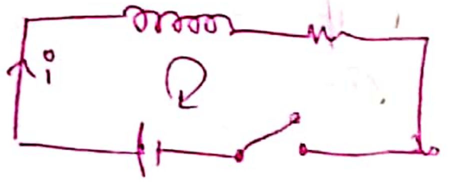
$$Et = Li$$

$$i = \left(\frac{E}{L}\right)t \qquad i \propto t.$$



LR circuits

I Current growth in inductor



at t = 0, i = 0

at t = t i = ?

$$-L\frac{di}{dt} - iR + E = 0$$

$$Ldi + iRdt = Edt$$

$$Ldi = dt(E - iR)$$

$$\frac{L\,di}{E - iR} = dt$$

$$L\left[\ln(E - iR)\right]_0^i = [t]_0^t$$

$$\frac{-1}{R}L\ln\left(\frac{E - iR}{E}\right) = t$$

$$\frac{E - iR}{E} = e^{-t/LR}$$

$$\boxed{i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

at $t = 0$

$i = \frac{E}{R}(1-1) = 0$

$\downarrow$

acts as insulator

at $t = \infty$

$i = \frac{E}{R}(1-0)$

$\boxed{i_{max} = \frac{E}{R}}$ → acts as conducting wire

Time constant

time taken to increase the current by 63% of maximum value.

at $t = \frac{L}{R}$

$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}\cdot\frac{L}{R}}\right)$

$i = i_{max}(1 - e^{-1})$

$= i_{max}\left(1 - \frac{1}{2.7}\right)$

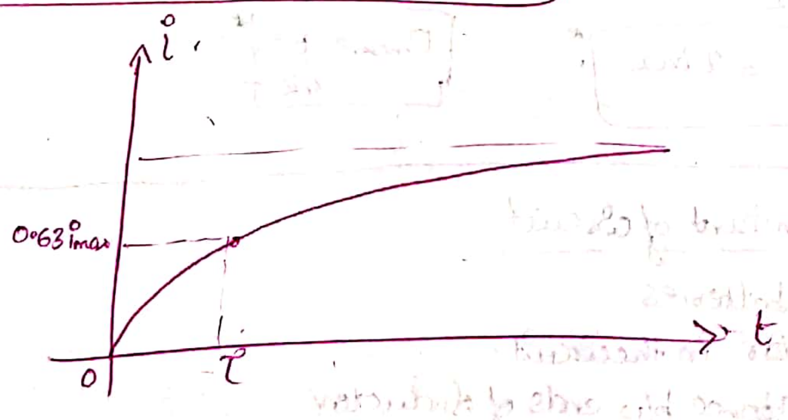
$i = 0.63\, i_{max}$

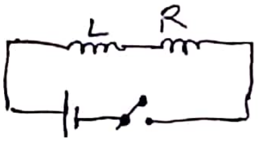
$\boxed{t = \frac{L}{R} = \tau}$ → time constant = τ.

$i = 63\%$ of max current

$$\boxed{\text{Finally } i = i_{max}\left(1 - e^{-t/\tau}\right)}$$

$$\boxed{\tau = \frac{L}{R}}$$





Final energy stored in the inductor

$$U_F = \frac{1}{2} L i_0^2$$

$$U_F = \frac{1}{2} L \left(\frac{E}{R}\right)^2$$

$$U_F = \frac{LE^2}{2R^2}$$

Rate of energy stored in the Inductor

$$E = \frac{1}{2} L i^2$$

$$= \frac{1}{2} L \left[\frac{E}{R}\left(1 - e^{-tR/L}\right)\right]^2$$

$$E = \frac{LE^2}{2R^2}\left[1 - e^{-tR/L}\right]^2$$

$$U = \frac{dE}{dt} = \frac{\cancel{L}E^2}{\cancel{2}R^{\cancel{2}}} \cancel{2} \left(1 - e^{-tR/L}\right) \cdot e^{-tR/L} \cdot \frac{\cancel{R}}{\cancel{L}}$$

$$\boxed{U = \frac{E^2}{R}\left(1 - e^{-\frac{R}{L}t}\right)\left(e^{-tR/L}\right)} *$$

~~f(x)~~ $f(x) = (1-x)x$

$f(x)$ is max at $x = 1/2$

at $e^{-R/L\,t} = \frac{1}{2}$, u is maximum

$$e^{-R/L\,t} = \frac{1}{2}$$

$$\cancel{-}\frac{R}{L}t = \cancel{-}\ln 2$$

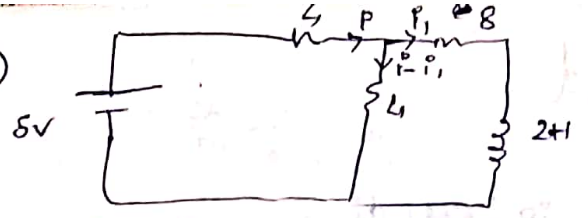
$$\boxed{t = \frac{L \ln 2}{R} = \tau \ln 2} *$$

$$U_{max} = \frac{E^2}{R}\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\boxed{U_{max} = \frac{E^2}{4R}} *$$

Find the time constant of circuit

1. Shortcircuit the batteries
2. Open the inductor in the circuit.
3. Find the net Resistance b/w ends of Inductor
4. Time constant $\tau = \frac{L}{R_{net}}$

o) 

Find the current through inductor and battery as a function of time.

$5 - 4i - 4i + 4i_1 = 0$

$-8i_1 - 2\frac{di_1}{dt} + 4i - 4i_1 = 0$

$\frac{5+4i_1}{8} = i$

$-8i_1 - 2\frac{di}{dt} + \frac{5}{2} + i_1 - 4i_1 = 0$

$2\frac{di}{dt} = -11i_1 + \frac{5}{2}$

$\frac{di_1}{-22i_1+5} = \frac{1}{4L}$

$\ln(-22i_1+5) = \frac{t}{4L}$

$\tau = \frac{L}{R_{net}} = \frac{L}{5}$

$i_0 = i_{max}\left(1 - e^{-\frac{t5}{L}}\right)$

at $t = \infty$

$R_{net} = 4 + \frac{8\times4}{8+4}$

$R_{net} = 4 + \frac{8}{3} = \frac{20}{3}\,\Omega.$

$i = \frac{5}{20/3} = \frac{3}{4}$ amp

$i_1 = \left(\frac{4}{4+8}\right)i = \frac{4}{12}\left(\frac{3}{4}\right) = \frac{1}{4}$ amp

$i_1 = \frac{1}{4}\left(1 - e^{-8t}\right)$

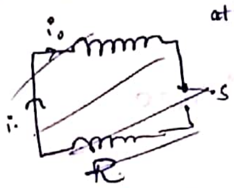
Current through inductor

$i = \frac{5}{8} + \frac{1}{8}\left(1 - e^{-8t}\right)$

$i = \frac{1}{8}\left[6 - e^{-8t}\right]$ current from battery.

Decay of Current

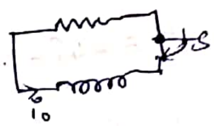
at $t=0, i=i_0$



decreasing current $\frac{di}{dt} < 0$

$$-iR - L\frac{di}{dt} = 0$$

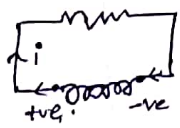
at $t=0$



$$-L\frac{di}{dt} = iR$$

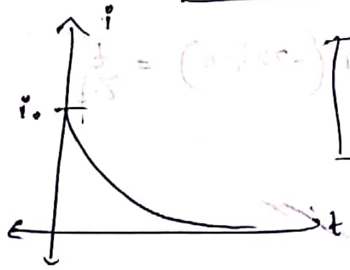
$$\int_{i_0}^{i} \frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

at $= t = t$



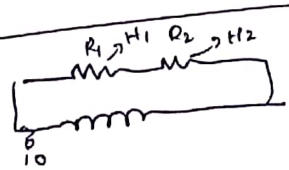
$$\ln\left(\frac{i}{i_0}\right) = -\frac{Rt}{L}$$

$$\boxed{i = i_0 \times e^{-Rt/L}}$$



$\boxed{\text{Total Heat produced in the reston} = \frac{1}{2}Li_0^2}$

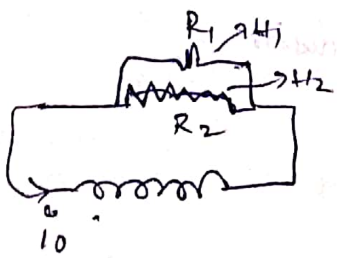
Notes



$$H_1 + H_2 = \frac{1}{2}Li_0^2$$

$$H_1 = \left(\frac{R_1}{R_1+R_2}\right)\left(\frac{1}{2}Li_0^2\right)$$

$$H_2 = \left(\frac{R_2}{R_1+R_2}\right)\left(\frac{1}{2}Li_0^2\right)$$



$$H_1 + H_2 = \frac{1}{2}Li_0^2$$

$$H_1 = \left(\frac{R_2}{R_1+R_2}\right)\left(\frac{1}{2}Li_0^2\right)$$

$$H_2 = \left(\frac{R_1}{R_1+R_2}\right)\left(\frac{1}{2}Li_0^2\right)$$

$$\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q \;\equiv\; \boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$q = A\sin(\omega t \pm \phi)$

at $t=0$, $q = Q_0$

$Q_0 = A\sin(\phi)$

$\boxed{Q_0 = A}$

at $t=0$, $i=0$

$i = -\frac{dq}{dt} = -A\omega\cos(\omega t+\phi)$

$0 = -A\omega\cos\phi$

$\cos\phi = 0$

$\boxed{\phi = \pi/2}$

$q = Q_0\sin(\omega t + \pi/2)$

$\boxed{q = Q_0\cos(\omega t)}$

in the current in $i = -\frac{dq}{dt}$

$i = Q_0\omega\sin\omega t$

$i_{max} = Q\omega \;\equiv\; \boxed{v_{max} = A\omega}$

U_C

$\frac{(Q_0\cos\omega t)^2}{2C} = \frac{Q_0^2}{2C}\cos^2\omega t$

inductor U_L

$\frac{1}{2}Li^2 = \frac{1}{2}LQ_0^2\omega^2\sin^2\omega t$

$= \frac{1}{2}Q_0^2\left(\frac{1}{C}\right)\sin^2\omega t$

$= \frac{Q_0^2}{2C}\sin^2\omega t$

$U_{total} = U_C + U_L$

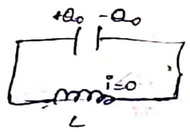
$= \frac{Q_0^2}{2C}$

Total energy of system is conserved in LC oscillation.

at $t=0$

$q = Q_0$ $\quad U_C = \frac{Q_0^2}{2C}$

$i=0$ $\quad U_L = 0$.



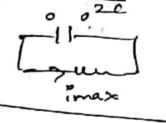
at $t = \frac{\pi}{2\omega} = \frac{T}{4}$

$q = Q_0 \cos \pi/2 = 0$

$U_C = 0$

$i = Q_0 \omega \sin \omega t = Q_0 \omega$

$U_L = \frac{Q_0^2}{2C}$



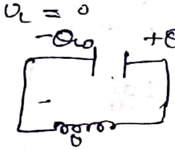
at $t = \pi/\omega \Rightarrow \omega t = \pi$

$q = Q_0 \cos \omega t = -Q_0$

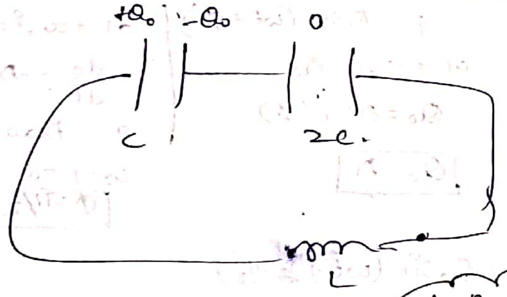
$U_C = \frac{Q_0^2}{2C}$

$i = Q_0 \omega \sin \omega t = 0$

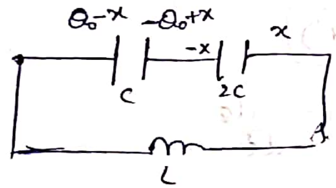
$U_L = 0$



Q)



If switch is closed at time $=0$ find maxcurrent in curreind.



Logic

$i_{max} \to di/dt = 0 \Rightarrow L\, di/dt = 0$,

P.d across Inductor at max current instant is zero.

$$-L\frac{di}{dt} - \frac{x}{2C} + \frac{Q_0 - x}{C} = 0$$

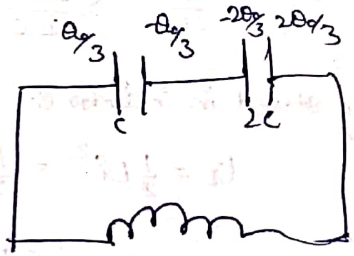
$$\frac{Q_0 - x}{C} = \cancel{L\frac{di}{dt}} + \frac{x}{2C}$$

$$\frac{2(Q_0 - x) - x}{2C} = \cancel{L\frac{di}{dt}}$$

$$\frac{2(Q_0 - x)}{\cancel{2C}} = \frac{x}{\cancel{2C}}$$

$$2Q_0 - 2x = x$$

$$x = \frac{2Q_0}{3}$$



By C

$\frac{Q_0^2}{2C} =$

$\frac{Q_0^2}{2C} -$

Q) Fin

Ans:

$\omega t = \pi$

Q_0

By C.O.Energy.

$$\frac{Q_0^2}{2C} = \frac{(Q_0/3)^2}{2C} + \frac{(\frac{2Q_0}{3})^2}{2(2C)} + \frac{1}{2} L i_{max}^2$$

$$\frac{Q_0^2}{2C} - \frac{Q_0^2}{18C} - \frac{4Q_0^2}{36C} = \frac{1}{2} L i_{max}^2$$

$$\frac{\cancel{12}}{\cancel{36}\,3} \frac{Q_0^2}{C} = \frac{1}{2} L i_{max}^2$$

$$\frac{2Q_0^2}{C3L} = i_{max}^2$$

$$i_{max} = Q_0 \sqrt{\frac{2}{3LC}}$$

Q) Find the charge as a function of time on 2C Capacitor.

<u>Ans:-</u>

$$\frac{Q_0 - q}{C} - L\frac{di}{dt} - \frac{q}{2C} = 0$$

$$\frac{Q_0}{C} - \frac{q}{C}\frac{3}{2} = L\frac{di}{dt}$$

$$\frac{3}{2C}\left(\frac{2Q}{3} - q\right) = L \cdot \frac{d^2q}{dt^2}$$

$$\frac{d^2q}{dt^2} = \frac{3}{2CL}\left(\frac{2Q}{3} - q\right)$$

$$\frac{2Q_0}{3} - q = x \Rightarrow \frac{-dq}{dt} = \frac{dx}{dt}$$

$$\frac{-d^2q}{dt^2} = \frac{d^2x}{dt^2}$$

$$-\frac{d^2x}{dt^2} = \frac{3}{2LC}(x)$$

$$\frac{d^2x}{dt^2} = \frac{3}{2LC}(-x)$$

$$\omega = \sqrt{\frac{3}{2LC}}$$

$$x = A\sin(\omega t \pm \phi)$$

at $t=0$, $x = \frac{2Q_0}{3}$

$$\frac{2Q_0}{3} = A\sin(\phi)$$

at $t=0$ $\quad i = \frac{dq}{dt} = 0$

$$\frac{dx}{dt} = A\omega\cos(\omega t \pm \phi) = 0$$

$$\omega t + \phi = \pi/2$$

$$0 + \phi = \pi/2 \Rightarrow \boxed{\phi = \pi/2}$$

$$\frac{2Q_0}{3} = A\sin(\pi/2)$$

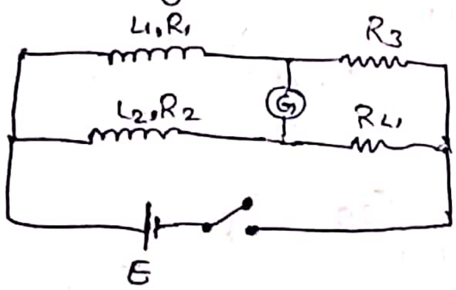
$$\boxed{A = \frac{2Q_0}{3}}$$

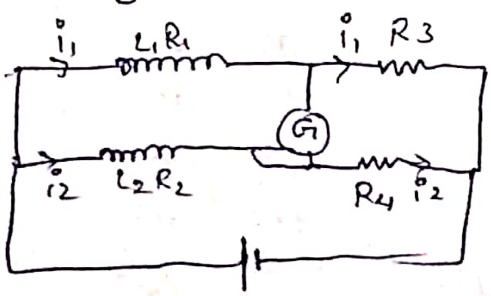
$$x = \frac{2Q_0}{3}(\cos\omega t)$$

$$\frac{2Q_0}{3} - q = \frac{2Q_0}{3}(\cos\omega t)$$

$$\boxed{q = \frac{2Q_0}{3}(1 - \cos\omega t)}$$

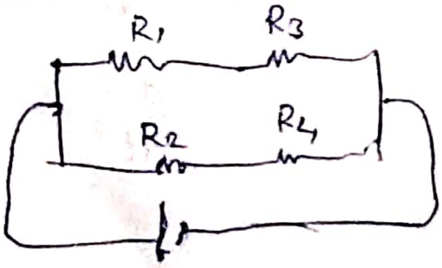
Q:- If switch is closed, find the condn for the reading of galvonometer is always zero?





At $t=\infty$, current in galvanometer is zero.

$$\Rightarrow \boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}} \Rightarrow$$



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$$\cancel{-}L_1\frac{di_1}{dt} \cancel{-} i_1R_1 = \cancel{-}L_2\frac{di_2}{dt} \cancel{-} i_2R_2$$

$$L_1\frac{di_1}{dt} + i_1R_1 = L_2\frac{di_2}{dt} + i_2R_2 \longrightarrow (1)$$

$$i_1R_3 = i_2R_4$$

$$\frac{di_1}{dt}R_3 = \frac{di_2}{dt}R_4$$

$$L_1\frac{di_1}{dt} + i_1R_1 = L_2\frac{R_3}{R_4}\frac{di_1}{dt} + \frac{i_1R_2R_3}{R_4}$$

$$\boxed{\frac{di_1}{dt}\left(L_1 - L_2\frac{R_3}{R_4}\right) = i_1\left(\frac{R_2R_3}{R_4} - R_1\right)}$$

at $t=0 \Rightarrow \boxed{i_1 = 0}$

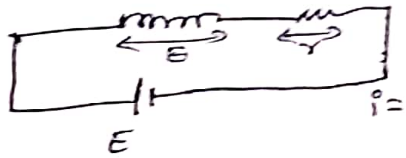
$$\frac{di_1}{dt}\left(L_1 - L_2\frac{R_3}{R_4}\right) = 0$$

$\downarrow$

0

$$L_1 - L_2\frac{R_3}{R_4} = 0$$

$$\boxed{\frac{L_1}{L_2} = \frac{R_3}{R_4}}$$

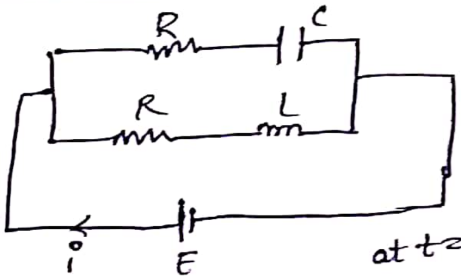


$V_R = 0$

$$L\frac{di}{dt} = E$$

$$\frac{di}{dt} = \frac{E}{L} \neq 0$$

Q)



at $t=0$

Find the current from battery as a function of time.

take $R = \sqrt{\frac{L}{C}}$

$$q = CE\left(1 - e^{-t/RC}\right)$$

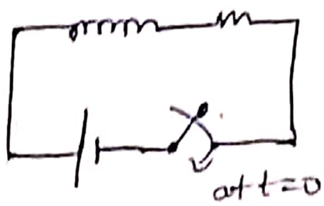
$$i_1 = \frac{dq}{dt} = CE\, e^{-t/RC} \cdot \frac{1}{RC} \Rightarrow \boxed{i_1 = \frac{E}{R}e^{-t/RC}}$$

$$i_2 = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$i = i_1 + i_2 = \cancel{\frac{E}{R}\left(1 - e^{-\frac{t}{\sqrt{LC}}}\right) + \frac{E}{R}\left(e^{-\frac{t}{\sqrt{LC}}}\right)}$$

$$i = \frac{E}{R}.$$

Q) Find the heat released in the circuit in one time constant?



$$H = \int i^2 R\, dt$$

$$H = \int \left(\frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\right)^2 \cdot R dt$$

$$H = \frac{E^2}{R} \int \left(1 - e^{-\frac{Rt}{L}}\right)^2 dt.$$

$$H = \frac{E^2}{R} \int \left[1 + e^{\frac{-2Rt}{L}} - 2e^{\frac{-Rt}{L}}\right] dt$$

$$H = \frac{E^2}{R}\left[t + \frac{e^{\frac{-2Rt}{L}}}{\frac{-2R}{L}} - \frac{2e^{\frac{-Rt}{L}}}{\frac{-R}{L}}\right]_0^{\tau}$$

$$H = \frac{E^2}{R}\left[\tau - \frac{L}{2R}\left(e^{-2} - 1\right) + \frac{2L}{R}\left(e^{-1} - 1\right)\right]$$