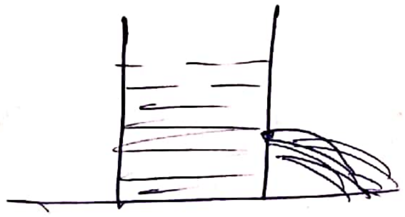
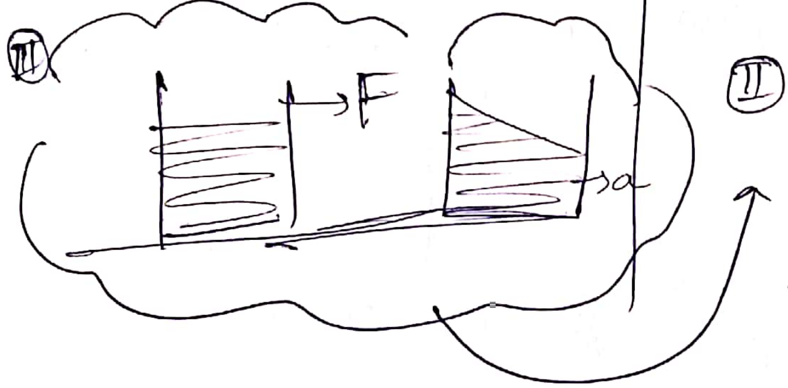
fluid Mechanics

fluid dynamics
(fluid under flow)

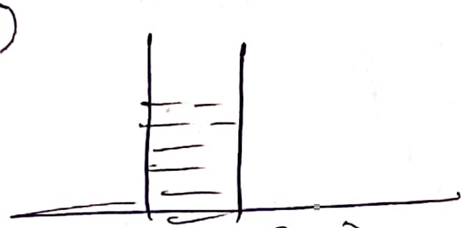


III



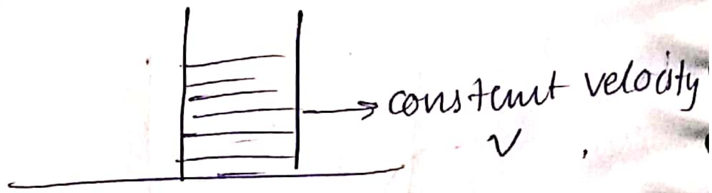
fluid statics
(fluids at rest wrt to container)

I



(OR)

II

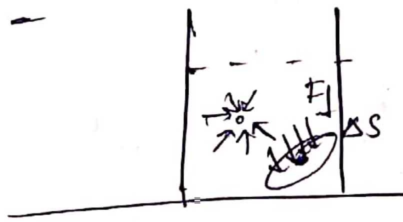


⇒ fluid Statics

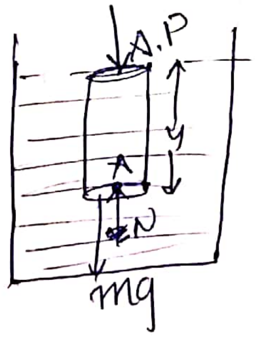
⇒ pressure at a point

- direction of pressure at a point is not defined

-



when small area is assumed at a point then direction of pressure is defined.

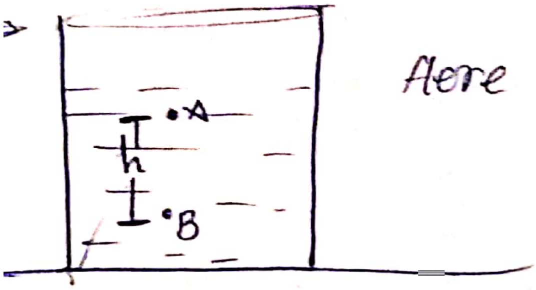


$$P_A \cancel{\Delta S} = P_0 \cdot \cancel{\Delta S} + \rho(\cancel{\Delta S} \cdot y) g$$

$$\boxed{P_A = P_0 + \rho g y}$$

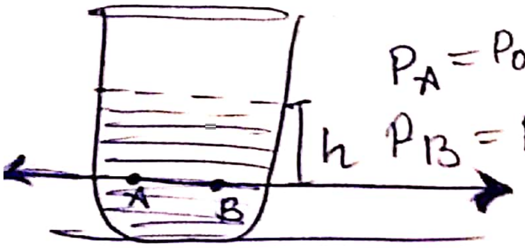
gauge pressure

absolute pressure



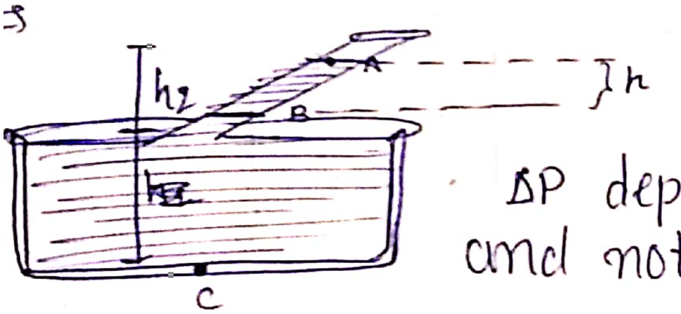
Here $P_B > P_A$

and $P_B - P_A = \rho g h$



$P_A = P_0 + \rho g h$
$P_B = P_0 + \rho g h$

At same horizontal leve[l] pressure difference is zero.

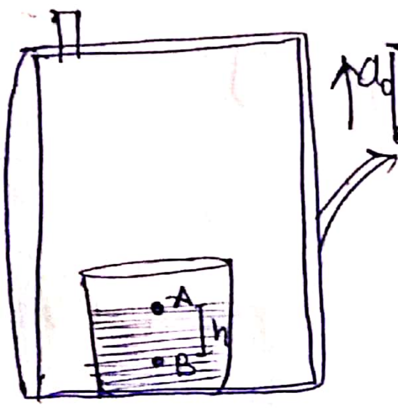


$|P_A - P_B| = \rho g h$

∆P depends on only difference in heigh[t] and not at incination or shape of conta[iner]

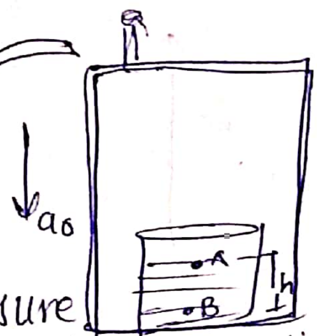
$P_C = \rho g(h_1 + h_2) + P_0$ } Absolute pressure at bottommos[t] point.

finding pressure difference when vessel is accelerating



$$\boxed{\Delta P_{AB} = \rho(a_0 + g)h}$$

$$\boxed{\Delta P_{AB} = \rho(g - a_0)h}$$



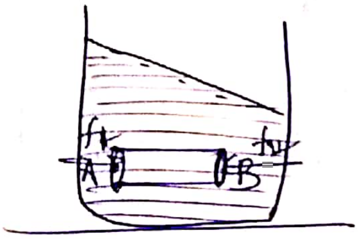
__Note__ :- Along horizontal line pressure remains same. if vessel is kept in lift accelerating upward.

If the lift is freely falling under gravity,

$a_0 = g$,

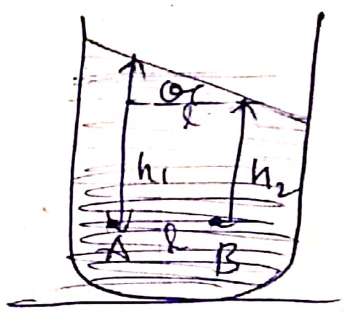
$\Rightarrow \Delta P_{AB} = 0 \Rightarrow$ Pressure at all points is same and equal to P_0 if vessel is in atmosphere orelse in vacuum its zero.

→ If vessel is accelerating horizontally



$f_1 - f_2 = ma_0$

$P_A - P_B = \rho l a_0$



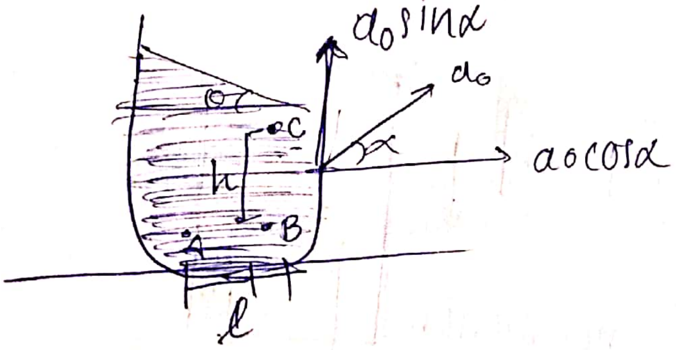
→ Along horizontal pressure will vary here.

$\Delta P_{AB} = \rho g(h_1 - h_2) = \rho l a_0$

$$\boxed{\frac{a_0}{g} = \frac{l}{h_1 - h_2}}$$

→ All isobaric lines will make an angle $\tan^{-1}(a_0/g)$ with horizontal.

$$\Rightarrow \boxed{a_0 = g\tan\theta}$$



$P_A - P_B = \rho a_0 \cos\alpha L$

$P_B - P_C = \rho(g + a_0\sin\alpha)h$

$\cancel{P_A - P_C = \rho L(g + a_0(\cos\alpha + \sin\alpha))}$

$$P_A - P_C = \rho a_0\cos\alpha L + \rho(g + a_0\sin\alpha)h$$

$$\tan\theta = \frac{a_0\cos\alpha}{g + a_0\sin\alpha} \qquad \theta = \tan^{-1}\left(\frac{a_0\cos\alpha}{g + a_0\sin\alpha}\right)$$

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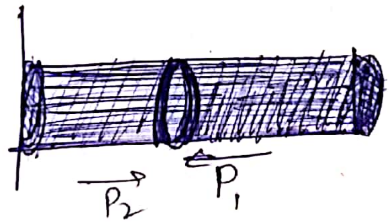
⇒ finding pressure difference in rotating vessel containing fluid.



pressure increases as we go away from axis.

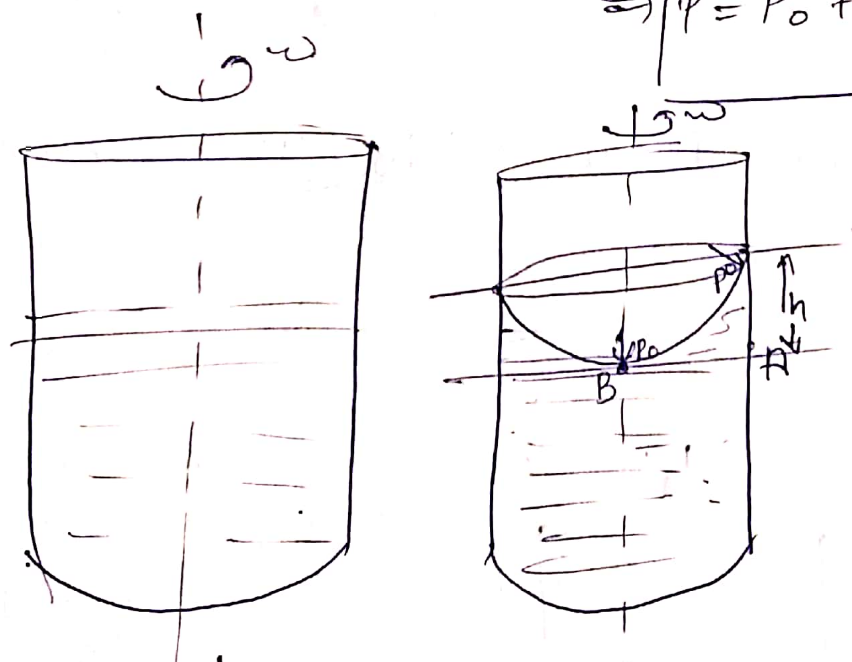
$$f_{net} = (P+dP)\Delta s - P.\Delta s$$
$$= dP.\Delta s.$$

$$\Rightarrow dm.x\omega^2 = dP.ds$$
$$\rho.ds.dx.x\omega^2 = dP.ds$$

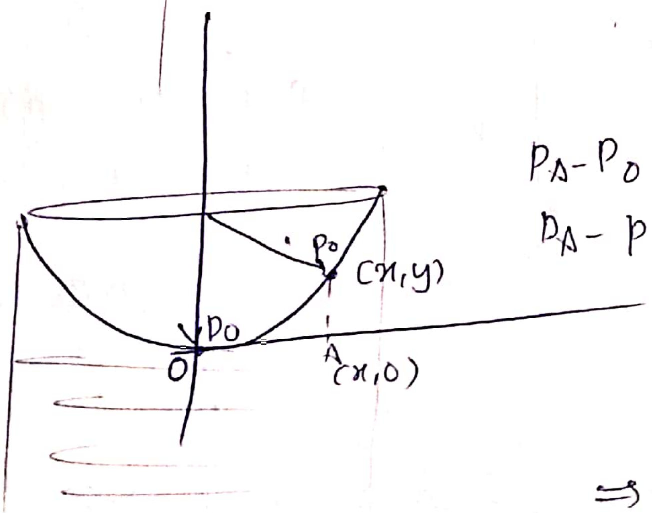
$$\Rightarrow \int_{P_0}^{P} dP = \rho\frac{x^2\omega^2}{2}$$

$$\Rightarrow \boxed{P = P_0 + \rho\frac{x^2\omega^2}{2}}$$

⇒



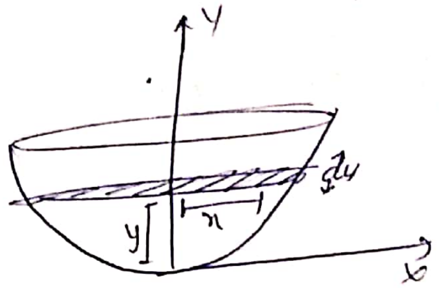
$$P_A - P_0 = \rho g h$$
$$P_A - P_0 = \rho\frac{R^2\omega^2}{2}$$
$$\Rightarrow \rho g h = \rho\frac{R^2\omega^2}{2}$$
$$\Rightarrow \boxed{h = \frac{R^2\omega^2}{2g}}$$



$$P_A - P_0 = \rho g y$$
$$P_A - P_0 = \frac{\rho\omega^2x^2}{2}$$
$$\Rightarrow \boxed{y = \frac{\omega^2x^2}{2g} = kx^2}$$ where $k = \frac{\omega^2}{2g}$

⇒ The free surface has parabolic shape.

Volume of parabolic bowl in previous case (paraboloid)



$$dV = \pi x^2 . dy$$

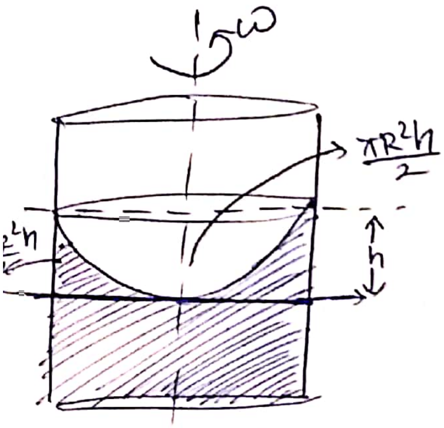
$$\int_0 dv = \int_0^h \pi \left(\frac{2gy}{\omega^2}\right) . dy$$

$$V = \frac{2g\pi}{\omega^2} . \left[\frac{h^2}{2}\right]$$

$$= \frac{g\pi}{\omega^2} . \left[\frac{R^2\omega^2}{2g}\right]^2$$

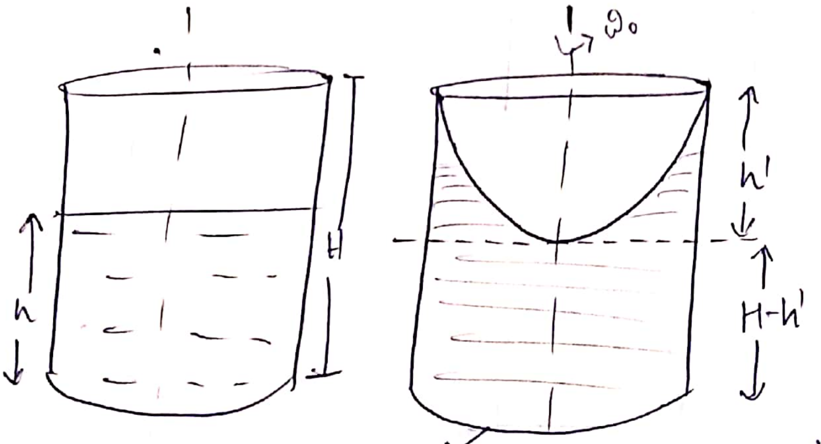
$$= \frac{\pi R^4 \omega^2}{4g}$$

$$= \frac{\pi R^2}{2} . \left[\frac{R^2\omega^2}{2g}\right] = \frac{\pi R^2 h}{2}$$



Volume of paraboloid is half of volume of cylinder in which it is present.

$\Rightarrow$ Spilling of liquid out of vessel



$$h' = \frac{R^2\omega_0^2}{2g}$$

volume of liquid is same, thus,

$$\pi R^2 h = \pi R^2 (H-h') + \frac{\pi R^2 h'}{2}$$

$$\Rightarrow h = H - \frac{h'}{2}$$

$$\Rightarrow h' = 2(H-h) = \frac{R^2\omega_0^2}{2g}$$

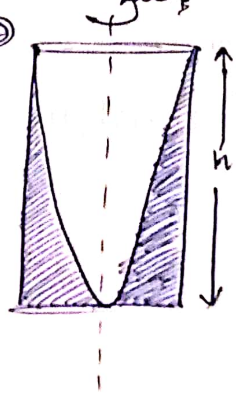
Here $\cancel{\boxed{h < H/2}}$

$\boxed{h > H/2}$

$$\Rightarrow \boxed{\omega_0 = \frac{2}{R}\sqrt{g(H-h)}}$$

→ If $\omega > \omega_0$ liquid will spill out of vessel.

→ If $\omega \leq \omega_0$ liquid remains inside the vessel.

⇒ 

$\omega'_f > \omega_0$

$$H = \frac{R^2 \omega_0'^2}{2g}$$

$$\omega' = \sqrt{\frac{2gH}{R^2}} = \frac{\sqrt{2gH}}{R}$$

Volume of liquid $= \frac{\pi R^2 H}{2}$

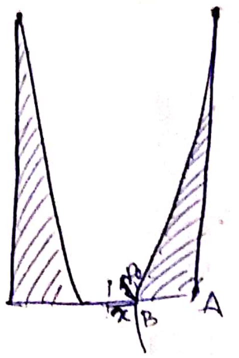
⇒ Initial amout of liquid $\pi R^2 h$ should be more that this.

$$\Rightarrow \pi R^2 h > \frac{\pi R^2 H}{2}$$

$$\Rightarrow \boxed{h > H/2}$$

Thus at this condition the bottom of paraboloid will touch the bottom of vessel.

⇒ Now on increasing ω' again bottom of vessel will be exposed to atmosphere



$$P_A - P_0 = \rho g H$$

$$P_A - P_0 = \frac{\rho \omega^2 \cdot (R^2 - x^2)}{2}$$

$$\cancel{\rho} g H = \frac{\cancel{\rho} \omega^2}{2} \cdot (R^2 - x^2)$$

$$R^2 - \frac{2gH}{\omega^2} = x^2$$

$$\boxed{\text{Area of surface} = \pi x^2 = \pi \left(R^2 - \frac{2gH}{\omega^2}\right)}$$

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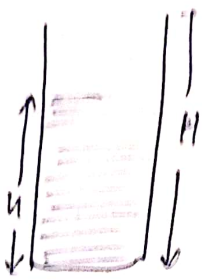
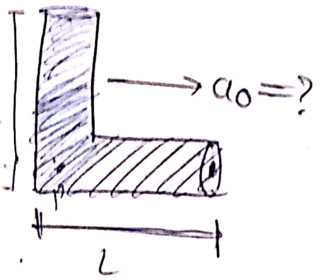


→ If $h > H/2$, spilling occurs first before exposure of bottom.

If $h = H/2$ both spilling and exposure ~~bot~~ happen simultaneously.

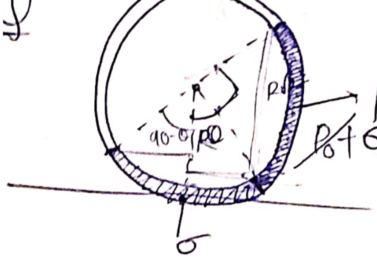
If $h < H/2$: exposure occurs first and then spilling takes place.

Q 

find acceleration required for liquid not to ~~fall~~ Spill of from tube.

$$\cancel{ma_0}.\ \cancel{\rho} a_0 . l = \cancel{\rho} g h$$

$$\boxed{a_0 = gh/l}$$

Q 

two of equal volume

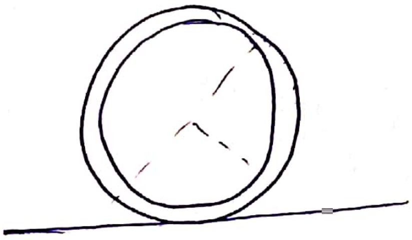
$$\cancel{P_0 + \sigma g R(1-\sin\theta) = \rho g R\sqrt{2} + \sigma g R(1-\cos\theta) + P_0}$$

$$\cancel{\sigma g R(\cos\theta - \sin\theta) = \rho g R\sqrt{2}}$$

$$\cancel{\sin\left(\frac{\pi}{4} - \theta\right) = \frac{2\rho}{\sigma}}$$

$$\cancel{\frac{\pi}{4} - \theta = \sin^{-1}\left(\frac{2\rho}{\sigma}\right)}$$

$$\cancel{\frac{\pi}{4} - \sin^{-1}\left(\frac{2\rho}{\sigma}\right) = \theta}$$



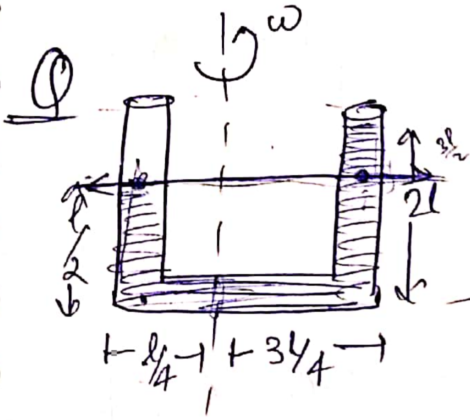
$$\sigma g R(\cos\theta - \sin\theta) = \rho g R(\sin\theta + \cos\theta)$$

$$(\sigma - \rho)\cos\theta = (\rho + \sigma)\sin\theta$$

$$\tan\theta = \frac{\sigma - \rho}{\rho + \sigma}$$

$$\theta = \tan^{-1}\left(\frac{\sigma - \rho}{\rho + \sigma}\right)$$

Q

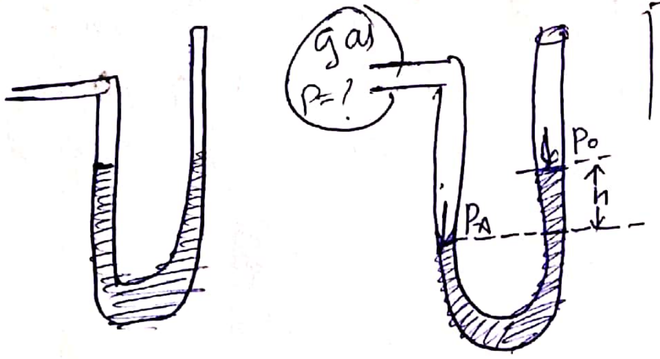


$$P_0 + \cancel{\rho g} \frac{\rho l^2\omega^2}{32} = \cancel{P_0} + \frac{9\rho l^2\omega^2}{32} + \frac{3\rho g l}{2}$$

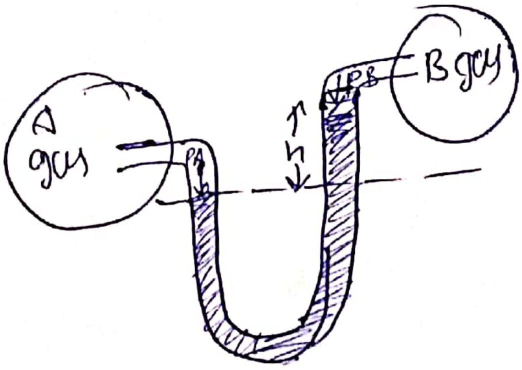
$$\frac{\rho l^2\omega^2}{4} = \frac{3\rho g l}{2}$$

$$\boxed{\omega = \sqrt{\frac{6g}{l}}}$$

Manometer

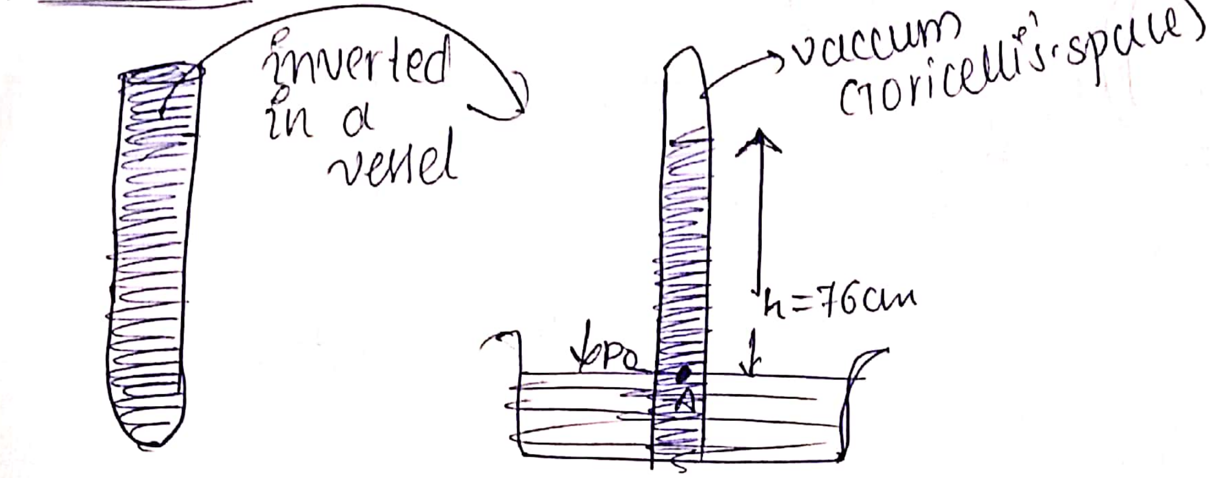


$$\boxed{P_{gas} = P_0 + \rho_m g h}$$



$$\boxed{\begin{aligned} P_A &= P_B + \rho_m g h \\ P_A - P_B &= \rho_m g h \end{aligned}}$$

Barometer

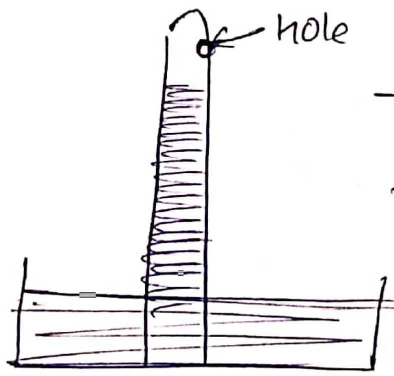


$$P_A = P_0 = \rho_m g h.$$

$$\Rightarrow P_0 = (13.6\ gm/cc)\ (980\ cm/s^2)\ (76\ cm)$$

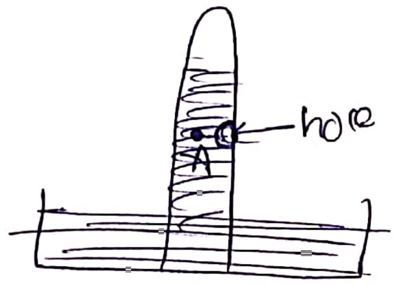
$$\boxed{P_0 = 1.013 \times 10^5\ N/m^2\ (SI)}$$

(1)



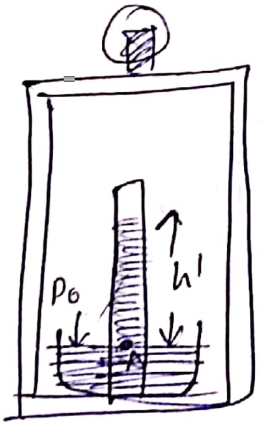
The total mercury column will fall down as value of pressure at top of column becomes P_0.

(2)



$P_A < P_0 \Rightarrow$ Mercury won't come out and reading remains unchanged.

(3)



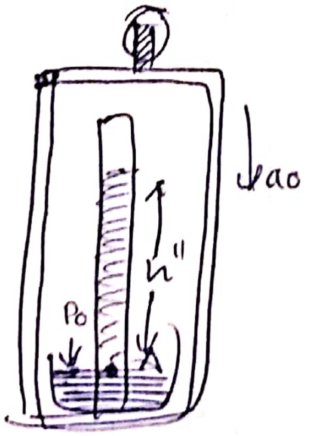
$P_A = P_0$

$P_0 = \rho g h = \rho_m (g + a_0)\, h'$

$\Rightarrow h' = \dfrac{gh}{g + a_0}$

$\Rightarrow h' < h$

(4)

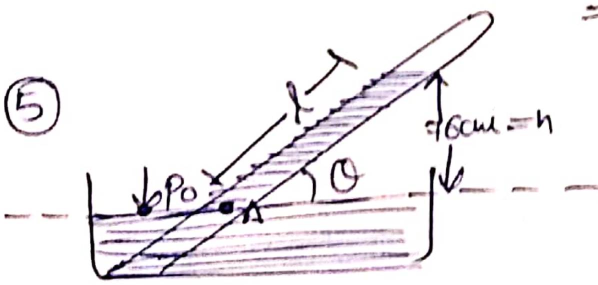


$P_0 = P_A = \rho g h = \rho (g - a_0) \cdot h''$

$\Rightarrow h'' = \dfrac{gh}{g - a_0}$

$\Rightarrow h'' > h$

(5)

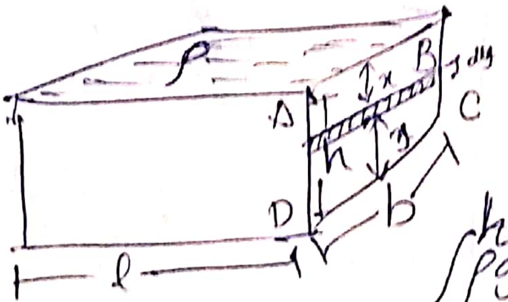


$\sin\theta = h/l$

$\boxed{l = h/\sin\theta}$

l = length of mercury column in the tube.

⇒ Force acting on a surface by fluid.



Neglect atmospheric pressure

find the force acting on ABCD? Also find to torque activity on it about axis CD?

$\int_0^h \rho g x \cdot b\,dx = \int dF$

$\rho g b \cdot \left[\dfrac{x^2}{2}\right]_0^h = F$

$\boxed{F = \dfrac{\rho g b \cdot h^2}{2}}$

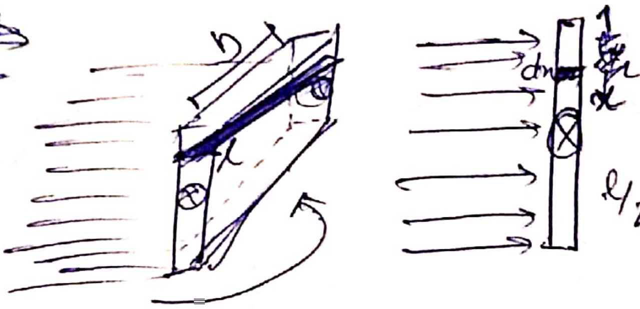
$d\tau = (dF)\, y$

$= b\,dy \cdot \rho g (h - y) \cdot y$

$= bd$

$= \rho g b \left[\dfrac{h y^2}{2} - \dfrac{y^3}{3}\right]_0^h$

$= \rho g b \left[\dfrac{h^3}{2} - \dfrac{h^3}{3}\right]$



~~dx. x~~

~~dx. db~~

$$\int dT = \int \rho g \left(\frac{l}{2} - x\right) \cdot dx \cdot b \cdot x$$

$$T_1 = \rho g b \left[\frac{l}{2}\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{l/2}$$

$$= \rho g b \left[\frac{l^2}{4} - \frac{l^2}{8}\right]$$

$$= \rho g b \left[\frac{l^2}{8}\right]$$

$$\int dT = \int \rho g \left(\frac{l}{2} + x\right) dx \cdot b$$

$$T_2 = \rho g b \left(\frac{lx}{2} + \frac{x^2}{2}\right)_0^{l/2}$$

$$= \frac{3 \rho g b l^2}{8}$$

$$T_{net} = \frac{\rho g b l^2}{\cancel{4}\, 2} = F \cdot \frac{l}{2}$$

$$F = \frac{\rho g b l}{2}$$

$$T_1 = \int \rho g \left(\frac{l}{2} - x\right) \cdot dx \cdot b \cdot x$$

$$= \rho \cdot g b \cdot \left[\frac{l}{2} \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_0^{l/2}$$

$$= \rho g b \left[\frac{l^3}{16} - \frac{l^3}{24}\right]$$

$$= \frac{\rho g b l^3}{8} \times \frac{1}{6}$$

$$T_2 = \int \rho g \left(\frac{l}{2} + x\right) \cdot dx \cdot b \cdot x$$

$$= \frac{\rho g b l^3}{8} \cdot \frac{5}{6}$$

$$T_{net} = \frac{4 \rho g b l^3}{8 \cdot 6}$$

$$F \cdot \frac{l}{2} = \frac{\rho g b \cancel{l^3}\, l^2}{\cancel{12}\, 6}$$

$$\boxed{F = \frac{\rho g b l^2}{6}}$$

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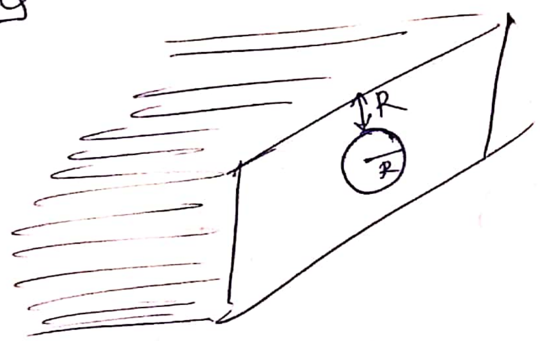
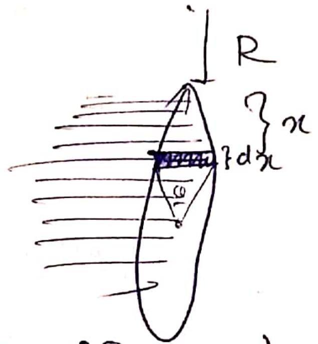
Q





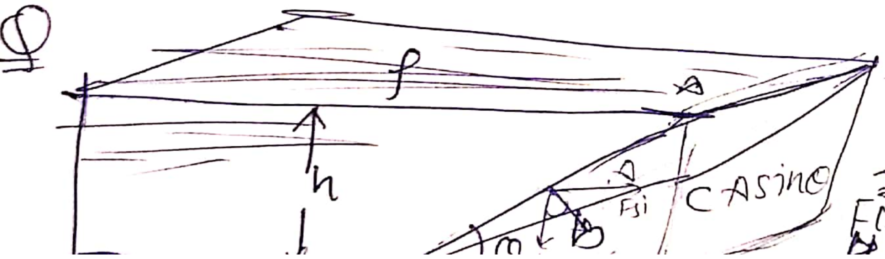
~~$dF = \rho g(h+x)$~~.

$F_H = \rho g \cdot (y_{cm})$ (Area ⊥lar to force)

$= \rho g (2R)(\pi R^2)$

$= 2\rho g \pi R^3$

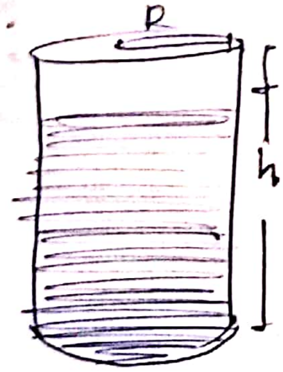
Q



B find Fnet on ABCD

$F_i = \rho g \cdot \frac{h}{2} \cdot bh$

$\Rightarrow$



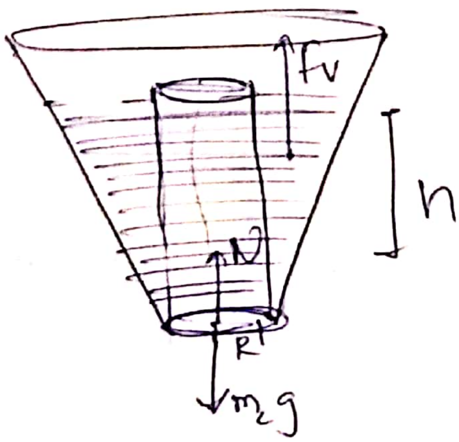
$m_1 = \rho \pi R^2 h$

$f_{bottom} = P(\text{area})$

$= \rho(\pi R^2 h) g$

$= m_1 g$

$=$ weight of liquid above the bottom.



$m_2 > m_1$.

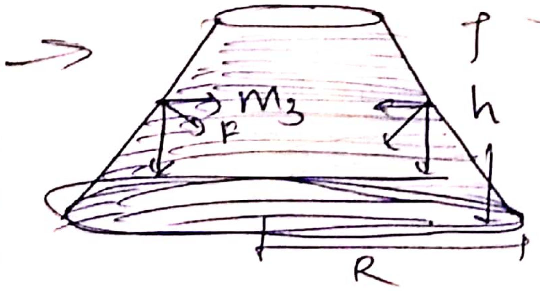
$f_{bottom} = \rho(\pi R'^2 h) g$

$= m_1 g < m_2 g$

$N = m_1 g$

$F_v + N = m_2 g$

$\boxed{F_v = (m_2 - m_1) g}$



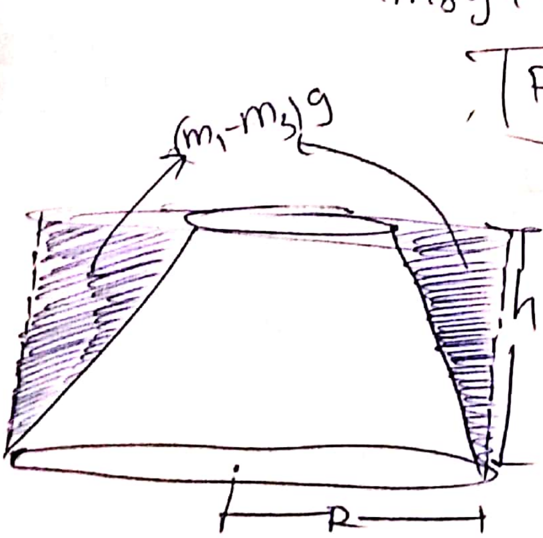
$F_{bottom} = \rho g h \cdot \pi R^2$

$= m_1 g$

$m_3 g < m_1 g$

$m_3 g + F_w = m_1 g$

$\boxed{F_w = (m_1 - m_3) g}$



Force ~~weigh~~ by walls on liquid inside vessel is weight of imaginary liquid remained after make arrange beside

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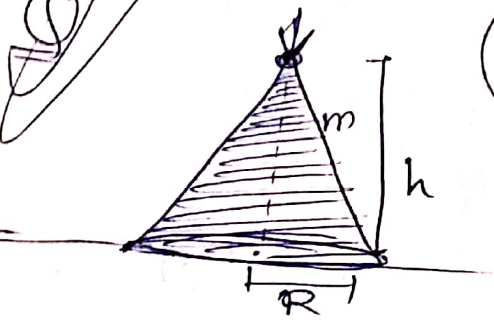
Q

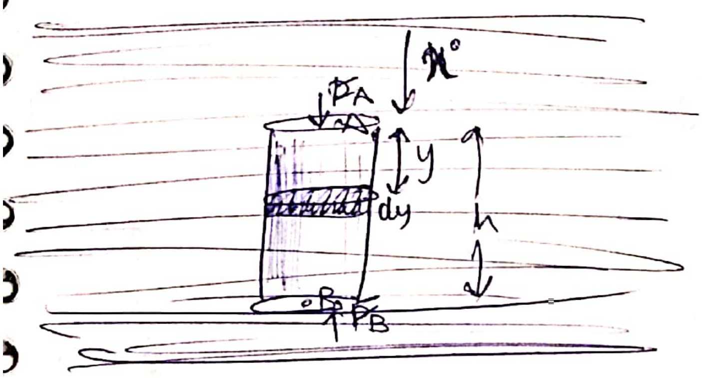


$$\left(\rho\pi R^2 h - \frac{\rho\pi R^2 h}{3}\right) g \geq mg$$

$$\frac{2\rho\pi R^2 h}{3} \geq mg$$

$$\boxed{m' = \frac{1}{3}\rho\pi R^3}$$

Force of buoyancy:-



~~$F_B = \rho g \pi R^2 h$~~

$$F_A = (\rho g x_0 + P_0)\pi R^2$$

$$F_B = (\rho g(x_0+h) + P_0)\pi R^2$$

$$F_B > F_A$$

$\Rightarrow$ Net force $= F_2 - F_1$

$= \rho g h \pi R^2$

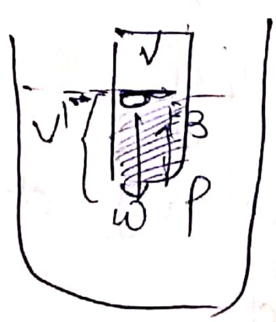
$= \rho g$ (Volume of cyl

$F_B =$ weight of liquid displaced by object.

$F_B =$ (volume of liquid displaced). $\rho_{liquid} \cdot g$

$$\boxed{F_B = (\text{volume of object immersed})\ \rho_{liquid} \cdot g}$$

Q



$$\sigma V g = \rho V' g$$

$$\boxed{V' = \frac{\sigma V}{\rho}}$$

if $\sigma < \rho$; $\frac{\sigma}{\rho} < 1$, $\frac{V'}{V} < 1$ (Flotation is possible)

floats with partially submersion.

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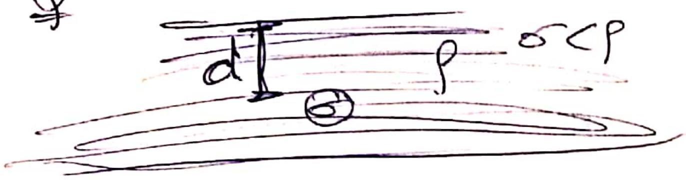
If $\sigma = \rho$, $\frac{\sigma}{\rho} = 1 \Rightarrow V' = V$ (floatation is possible)
complete submersion.

If $\sigma > \rho$, $\frac{\sigma}{\rho} > 1$ $\underbrace{\frac{V'}{V} > 1 \Rightarrow V' > V}_{\text{not possible}}$ (body sinks).

Q



$$\rho \cancel{V} g - \sigma \cancel{V} g = \sigma \cancel{V} a$$

$$\frac{(\rho - \sigma) g}{\sigma} = a$$

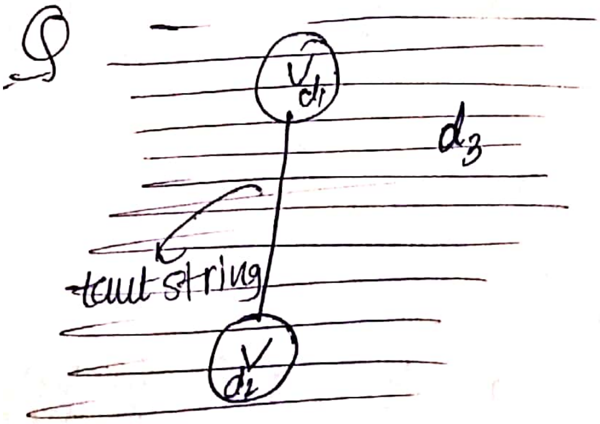
$$v = \sqrt{2 \times \frac{\rho - \sigma}{\sigma} g \times d}$$

$$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h$$

$$\frac{1}{\cancel{2}} \cdot \cancel{2} \left(\frac{\rho - \sigma}{\sigma}\right) \cancel{g} d = \cancel{m}\cancel{g} h$$

$$\boxed{h = \frac{d(\rho - \sigma)}{\sigma}}$$

Q



$$V d_1 g + T = V d_3 g$$

$$T + V d_3 g = V d_2 g$$

$$\cancel{V} d_3 \cancel{g} - \cancel{V} d_1 \cancel{g} = \cancel{V} d_2 \cancel{g} - d_3 \cancel{V} \cancel{g}$$

$$2 d_3 = d_1 + d_2$$

$$\boxed{d_3 = \frac{d_1 + d_2}{2}}$$

$F_B > 0$ | $\omega_2 > F_B$

$\Rightarrow d_3 > d_1$ | $\Rightarrow d_2 > d_3$

$\Rightarrow d_1 < d_3 < d_2$

Q. If 6 - square plate (iron) of side 'a' arranged in form of cube. If density of iron is 4 times of ρ_w find minimum value of t

$4\rho_w \cdot a^2 g = \rho_w x \cdot t \times \rho g$

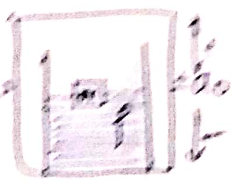
$4\rho_w \times t \times 6 a \times g = \rho_w \cdot a^3 g$

$\boxed{a = 24t}$

$4\rho_w \cdot a^2 g = (\rho_w \times t \times \rho g) \times g$

$a - \frac{t}{2}$

Q. Note



$f_B' = \left[\begin{matrix}\text{volume of object} \\ \text{inside fluid}\end{matrix}\right](\rho_{fluid})(g \pm a_0)$

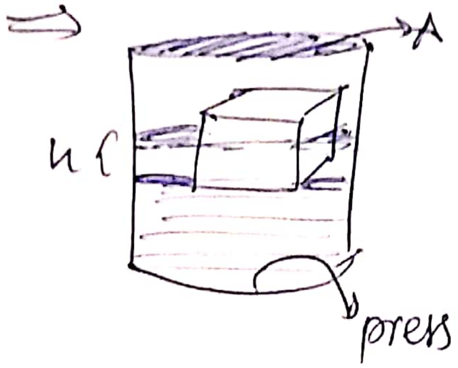
$g + a_0$ upward $g - a_0$ downward

$\boxed{V_1 = \frac{d V_{total}}{\rho}}$

$\rho V_1' \cdot (g + a_0) = d (g + a_0) V_{total}$

$\boxed{V_1' = \frac{d V_{total}}{\rho}}$

both are equal



m $\quad mg = \rho . L^2 . x \cancel{g}$

$$\boxed{x = \frac{m}{\rho L^2}}$$

$$\boxed{xL^2 = \frac{m}{\rho}} = \cancel{Ah}\,(A-L^2)h$$

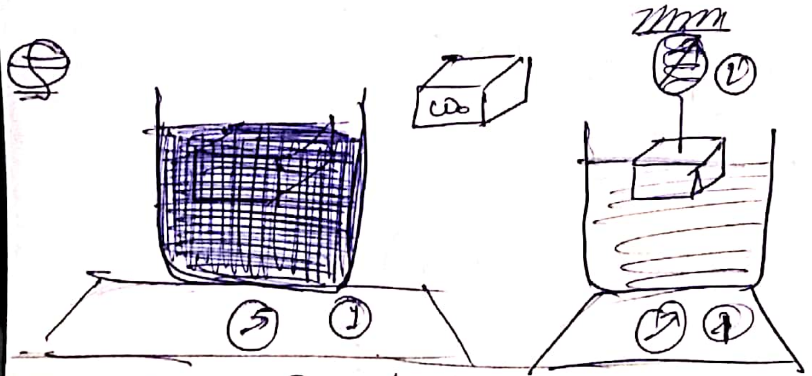
$$\boxed{h = \frac{m}{\rho(A-L^2)}}$$

Increase in pressure at bottom.

$$\Delta p = \rho_w g h$$

$$\Delta p = \cancel{\rho_w} g \cdot \frac{m}{\cancel{\rho_w}(A-L^2)}$$

$$\boxed{\Delta p = \frac{mg}{A-L^2}}$$



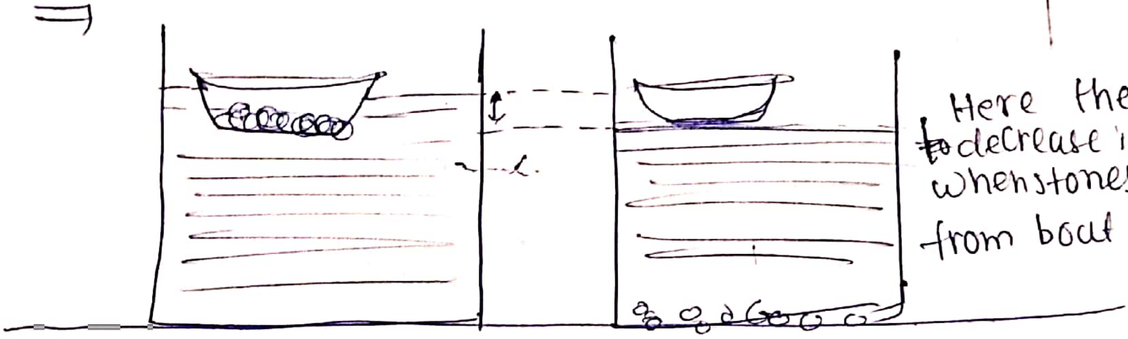
Reading (1) = wt. of water+Beaker

Reading (2) = $w_0 - F_B$

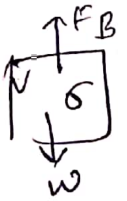
= apparent wt.

Reading (1) = wt of (water+beaker) + F_B.

⇒



Here there will be an decrease in level of water when stones are thrown from boat into the water



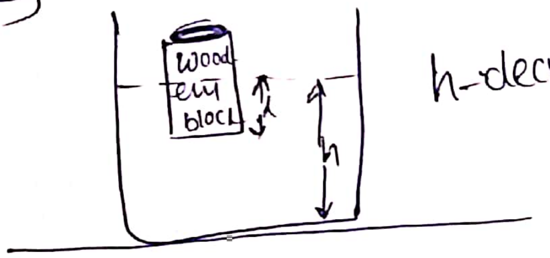
$$F_B = w$$

$$V_m \cdot \rho_w \cdot g = \sigma V g$$

$$V_m = \frac{V \cdot \sigma}{\rho_w}$$

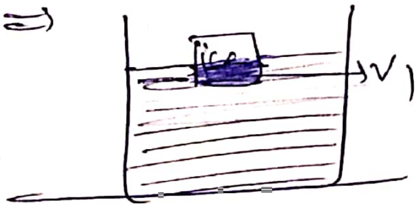
if $\sigma > \rho_w$, object sinks

⇒



h-decreases as well as l-decreases

⇒



if ice melts completely,
volume produced $= V_2$

$$m = V_{ice}\,\rho_{ice} = V_2\,\rho_w$$

$$\boxed{V_1 = \frac{V_{ice}\,\rho_{ice}}{\rho_w}}$$

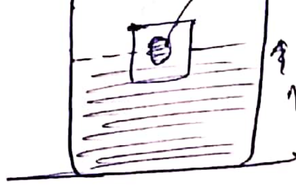
$$\boxed{V_2 = \frac{V_{ice}\,\rho_{ice}}{\rho_w}}$$

Thus water level remains same.

⇒

trapped solid object trapped in $\sigma > \rho_w$

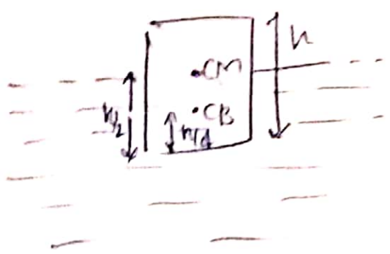
After ice melts, water level decreases

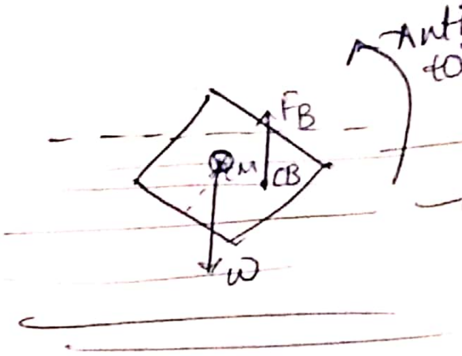


solid object $\sigma < \rho_w$

water level remains same.

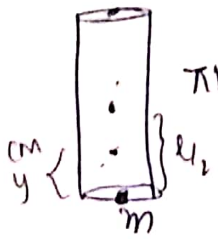
Centre of buoyancy
⇒ point where force of buoyancy will act.
⇒ centre of immersed volume





block tries to regain its equilibrium and thus is in stable equilibrium.

(33)



$\pi R^2 l \rho$ $\quad \left[\frac{\pi R^2 l\rho}{m+\pi R^2 l\rho}\cdot\frac{l}{2}\right] = y_{cm}$

$$(m+\pi R^2 l\rho)\cancel{g} = \pi R^2\cdot x\cdot\sigma\cdot\cancel{g}$$

$$\frac{\pi R^2 l\rho}{m+\pi R^2\rho l}\cdot\frac{l}{\cancel{2}} = \frac{m+\pi R^2 l\rho}{\cancel{2}\sigma\pi R^2} = \frac{x}{2}$$

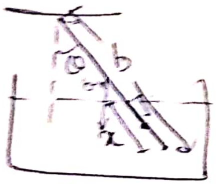
$$\sigma\rho\cdot(\pi R^2)^2\cdot l^2 = (m+\pi R^2\rho l)^2$$

$$\pi R^2 l\sqrt{\sigma\rho} = m+\pi R^2\rho l$$

$$\pi R^2 l\left[\sqrt{\sigma\rho}-\rho\right] = m$$

$$\boxed{\pi R^2 l\rho\left[\sqrt{\frac{\sigma}{\rho}}-1\right] = m}$$

(18)



$$P \cdot \frac{mg}{b}\,x\,\frac{b}{2} = \left(\frac{m}{b}\,x\right)\frac{g}{2}\left(b + \frac{x}{2}\right)$$

$$\frac{5}{9} \times \frac{b^2}{2} = \left(\frac{2b - x}{2}\right)x$$

$$5b^2 = 18bx - 9x^2$$

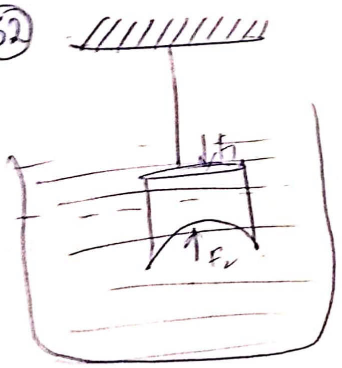
$$9x^2 - 18bx + 5b^2 = 0$$

$$9x^2 - 15bx - 3bx + 5b^2 = 0$$

$$3x[3x - 5b] - b[3x - 5b] = 0$$

$$\boxed{x = \frac{b}{3}}$$

(32)

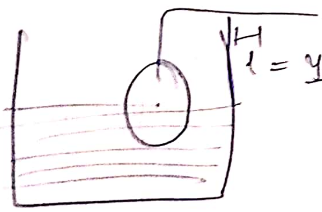


$$F_2 - F_1 = F_B = \rho g V$$

$$F_2 = \rho g V + \rho g h \pi R^2$$

$$= \rho g(\pi R^2 h + V)$$

(51)



$$\frac{y - x}{2}\left(\rho_{Al} V g - \rho_w \frac{V}{2} g\right)x + \frac{mg}{L}\frac{x^2}{2} = \frac{mg}{L}\frac{(l-x)^2}{2}$$

$$Vg\left[2.7\times10^3 - 0.5\times10^3\right]x + \frac{mg}{2}x^2$$

$$2Vg\left[2.2\times10^3\right]x + \frac{mg}{2}x^2 = \frac{mg}{2}\left(1 - \frac{x}{2}\right)^2$$

$$Vg\left(\rho_{Al} - \frac{\rho_w}{2}\right)x = mg \times \left(\frac{y - x}{2}\right)$$

$$V\left[2.2\times10^3\right] = \frac{4.4\times10^{-3}}{2} \times \left[\frac{y/x - 1}{2}\right]$$

$$\frac{4}{3}\left(\frac{22}{7}\right)\times 125\times10^{-3} \times \frac{10^6}{2} \quad 500\times 11$$

25

$$w_1 = \rho V g - d_1 V g$$

$$w_1 = (\rho - d_1) V g$$

$$w_2 = (\rho - d_2) V g$$

$$w_3 = (\rho - d_3) V g$$

$$w =$$

$$w_1 - w_2 = (d_2 - d_1) V g$$

$$\left(\frac{w_1 - w_2}{d_2 - d_1}\right)\frac{1}{g} = V$$

$$w_1(\rho - d_2) = w_2(\rho - d_1)$$

$$\cancel{w_2}(w_1 - w_2)\rho = w_1 d_2 - w_2 d_1$$

$$\rho = \frac{w_1 d_2 - w_2 d_1}{w_1 w_2}$$

Pascal law

If at any point of fluid pressure increase, that increase is distributed equally to every point without diminishing its value

$$P_A = P_0 + F/A$$

$$P_B = P_0 + F/A + \rho g h$$

fluid Dynamics

→ fluid is ideal . which means :- non compressible liquid

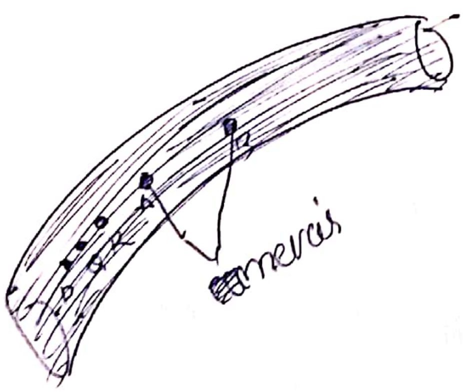
:- non-viscous fluid

:- irrotational (fluid doesn't acquire any angular veloc at any point of flui ~~flow~~ flow.)

~~Ideal fluid flow~~

~~i) Equal~~

Stream line flow :-



if all particles are having same velo (dir, mag) at a point in the fluid flow then flow in called stream lined flow

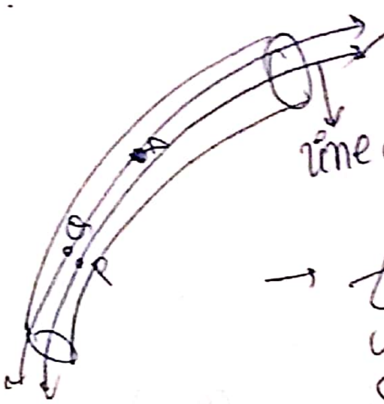
velocity at (A) may be different from that at B . But at a point all particles must have same velocity at a given point.

Turbulent flow :-

At a point in fluid flow different particles will have different velocities the flow is turbulent one.

$$\boxed{V_{P\ at\ A} \neq V_{Q\ at\ A}}$$

⇒ Line of flow : Path followed by a particle in fluid flow.

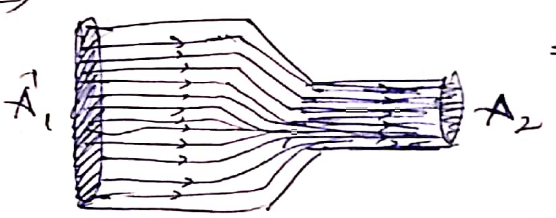


→ Generally lines of flow will be parallel to shape of pipe.

→ At any point if we draw a tangent it'll give us direction of velocity at that point.

→ If lines of flow are ||lel then it's streamline flow otherwise turbulent.

⇒

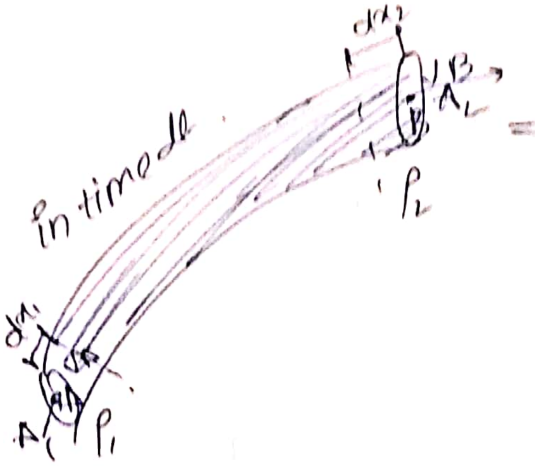


⇒ line of flow density = lines of flow per unit Area.

⇒ line flow density ∝ velocity of that point.

⇒ Ideal fluid always follows Equation of continuity.

→ Rate of mass of liquid entering the pipe is same as rate of mass of fluid leaving the pipe.



$\Rightarrow$ conservation of mass

$$\left(\frac{dm}{dt}\right)_{\text{into pipe}} = \left(\frac{dm}{dt}\right)_{\text{coming out of pipe}}$$

$$\rho_1 \frac{A_1 \cdot dx_1}{dt} = \frac{\rho_2 \cdot A_2 \cdot dx_2}{dt}$$

$$\Rightarrow \boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

for ~~incompressible~~ compressible fluid, and incompressible fluids also.

for incompressible fluids,

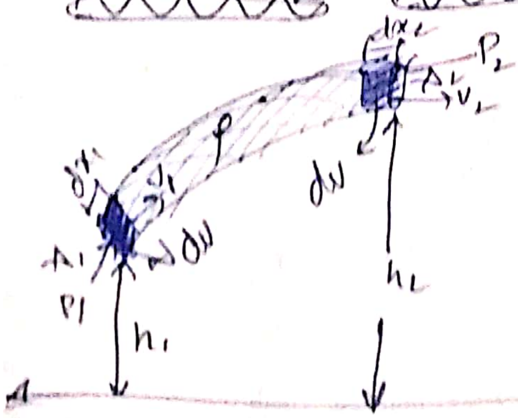
$$\Rightarrow \rho_1 = \rho_2$$

$$\Rightarrow A_1V_1 = V_2A_2 \Rightarrow \underbrace{AV}_{\text{rate of volume flow}} = \text{constant}$$

$\Rightarrow$ Incompressible fluids have same rate of volume as well as mass flow. ~~at~~ every

$\Rightarrow$ But compressible fluid rates of volume and mass flow are not equal.

Bernoulli's Theorem (consequence of conservation of energy).



$\Delta U = \cancel{\rho g}\, dm \cdot g(h_2 - h_1)$

$\Delta KE = \frac{1}{2} dm \cdot (V_2^2 - V_1^2)$

$dW_{\text{by } P_1} = P_1 A_1 \cdot dx_1$

$dW_{\text{by } P_2} = P_2 A_2 \cdot dx_2$

WE Theorem says:

Work done $= \Delta KE$

$$\Rightarrow (P_1 A_1 dx_1 - P_2 A_2 dx_2) - dmg(h_2 - h_1) = \frac{1}{2} dm.(v_2^2 - v_1^2)$$

$$\Rightarrow (P_1 dV - P_2 dV) - \rho.dV.g(h_2 - h_1) = \frac{1}{2}.\rho.dV.(v_2^2 - v_1^2)$$

$$\Rightarrow \boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

$$\Rightarrow \boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

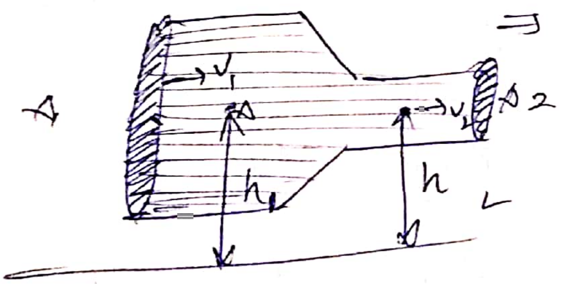
$$\Rightarrow \boxed{\frac{P}{\rho} + gh + \frac{v^2}{2} = \text{constant}}$$

P/ρ = pressure energy head

gh = 'U' head

$\frac{v^2}{2}$ = KE head

Bernoulli's Theorem along horizontal level in fluid flow.



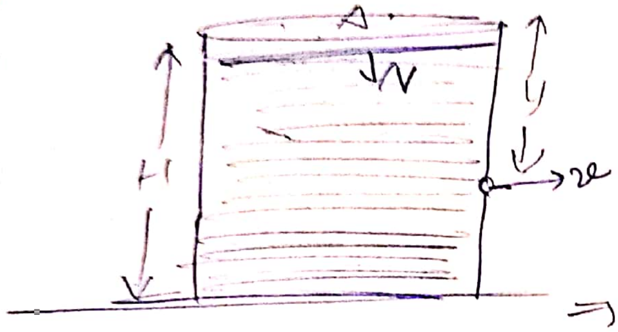
$$\Rightarrow A_1 v_1 = A_2 v_2$$

$$\Rightarrow P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \boxed{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$$

Application of BT and EC :-

① Velocity of Efflux.



$A \Rightarrow$ Area of cross section of vessel

$a \Rightarrow$ area of crossection of ...

$a << H$

$\Rightarrow$ By EC,

$AV = a, v$

$\Rightarrow V = \left(\frac{av}{A}\right)$

$\boxed{V << v}$

By BT,

$$\cancel{P_0} + \frac{1}{2}\rho V^2 + \cancel{\rho g H} = \cancel{P_0} + \frac{1}{2}\rho v^2 + \rho g(\cancel{H} - y)$$

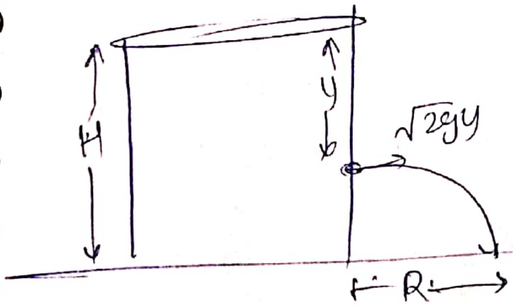
$$\cancel{\rho} g y = \frac{1}{2}\cancel{\rho}(v^2 - V^2)$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2gy}{1 - a^2/A^2}}}$$

As $a << A$

$\Rightarrow v = \sqrt{2gy}$

similar to velocity acquired by a part by freely falling body through a distance 'y'.



$$R = \sqrt{2gy} \cdot \sqrt{\frac{2(H-y)}{g}}$$

$$= 2\sqrt{y(H-y)}$$

$$R^2 = 4y(H-y)$$

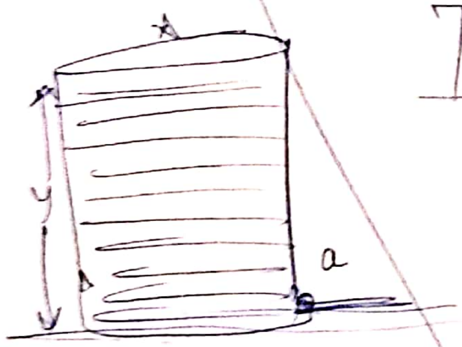
$$2R\left(\frac{dR}{dy}\right) = 4y(-1) + 4(H-y) = 0$$

$$4H - 8y = 0$$

$$\Rightarrow \boxed{y = H/2}$$

→ thus $\dot{R}$ will be maximum at $\boxed{y = H/2}$.

$\boxed{R_{max} = H}$



$$\boxed{A(a_T) = a \cdot (a_\omega)}$$

$$v = \sqrt{\frac{2gy}{1 - a^2/A^2}}$$

$$\frac{dv}{dt} = \left(\sqrt{\frac{2g}{1-a^2/A^2}}\right)\left(\frac{1}{2\sqrt{y}}\right) \cdot \sqrt{\frac{2gy}{1-a^2/A^2}}$$

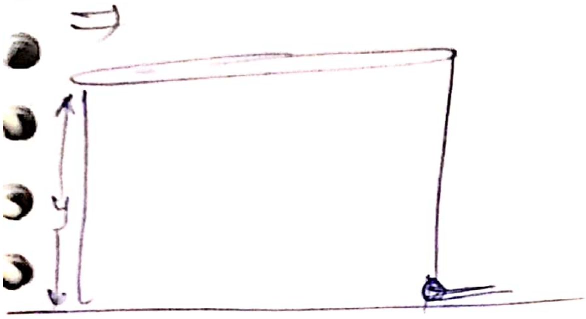
⇒

$$\boxed{a_\omega = \frac{g}{1 - a^2/A^2}} \approx g$$

$$\Rightarrow A(a_T) \approx a \cdot g$$

$$\boxed{a_T \approx ag/A}$$

⇒



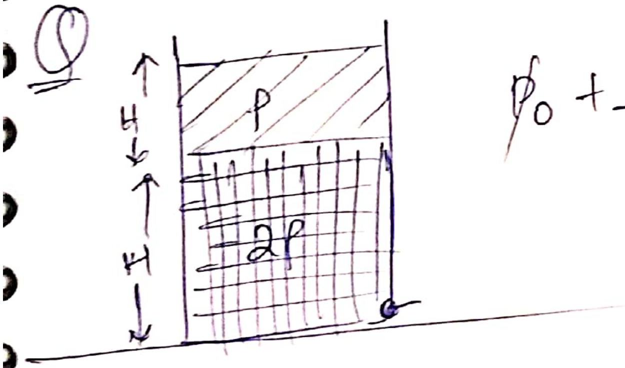
$$A \cdot V = a \cdot \sqrt{2gy}$$

$$A \cdot \frac{dv}{dt} = a \cdot \frac{\sqrt{2g}}{2\sqrt{y}} \cdot \frac{dy}{dt}$$

$$A \cdot \frac{dV}{dt} = a \frac{\sqrt{2g}}{\cancel{2}\cancel{\sqrt{y}}} \frac{a\sqrt{2g\cancel{y}}}{A}$$

$$\frac{dv}{dt} = \left(\frac{a^2}{A^2} g\right)$$

Q



$$\cancel{p_0} + \frac{1}{2}\rho V^2 + \cancel{\tfrac{1}{2}}\rho g(2H) = \cancel{p_0} + \frac{1}{2} \cdot 2\rho \cdot v^2 + \cancel{2\rho}$$

$$2\rho g H = \frac{1}{2} 2\rho v^2 - \frac{1}{2}\rho V^2$$

$$= \rho\left[v^2 - \frac{a^2 v^2}{2A^2}\right]$$

$$2\cancel{\rho} g H = \cancel{\rho}[v^2]\left[1 - \frac{a^2}{2A^2}\right]$$

$$\sqrt{\frac{2gH}{1 - \frac{a^2}{2A^2}}} = v$$

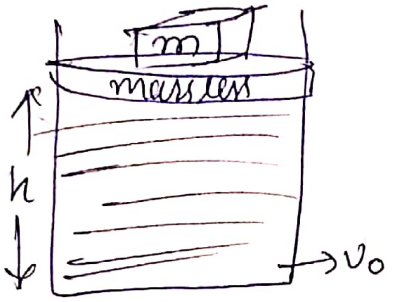
$$P_0 + \frac{1}{2} 2\rho \cdot v^2 + 0$$

$$= \cancel{p_0} + 3\rho g h + \cancel{\frac{1}{2} 2\rho \cdot v^2}$$

$$\cancel{\rho} v^2 = 3\cancel{\rho} g h$$

$$\boxed{v = \sqrt{3gh}}$$

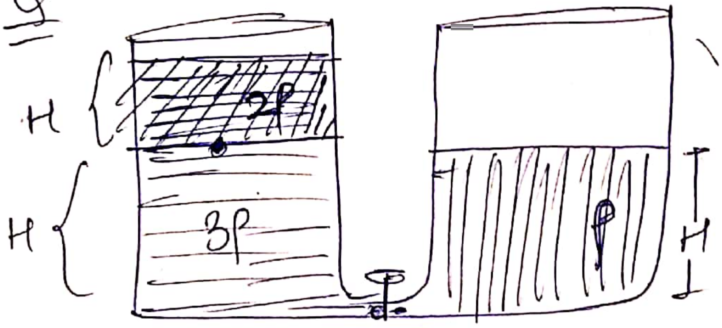
Q



$$\cancel{P_0} + \left(\frac{mg}{A}\right) + \frac{1}{2}\rho v^2 + \rho g h = \cancel{P_0} + \frac{1}{2}\rho v_0^2 + 0$$

$$\sqrt{2\left(\frac{mg}{\rho A} + gh\right)} = v_0$$

Q

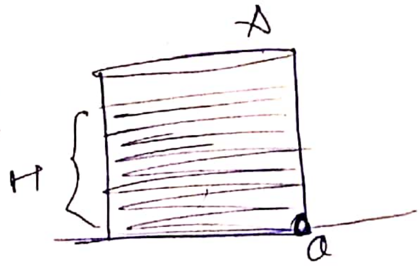


$$\cancel{P_0} + 2\rho g H + \cancel{\frac{1}{2}(2\rho)v^2} + 3\rho g H = \cancel{P_0} + \rho g H + \frac{1}{2}(3\rho)v^2$$

$$4\rho g h = \frac{1}{2} 3\cancel{\rho} v^2$$

$$\boxed{\sqrt{\frac{8gh}{3}} = v}$$

Q



time taken to empty is $\propto A$
$\propto H$
$\propto 1/a$

By, EC.

$$A \cdot \frac{-dy}{dt} = a \cdot \sqrt{2g} \cdot \sqrt{y}$$

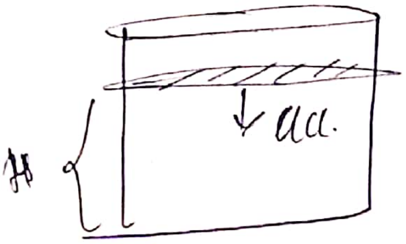
$$A \cdot \int_H^0 \frac{-dy}{\sqrt{y}} = a\sqrt{2g} \cdot \int dt$$

$$A\,[2\sqrt{y}]_H^0 = -(a\sqrt{2g})\,t$$

$$A\,[2\sqrt{H}] = a\sqrt{2g}\,(t) \Rightarrow \boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$

In general to empty tank from H_1 to H_2 $(H_1 > H_2)$.

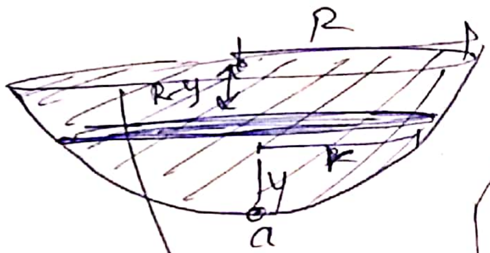
$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$



$$H = \frac{1}{2} a t^2$$

$$\frac{2H}{\frac{a^2}{A^2} \cdot g} = t^2 \Rightarrow \boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$

Q



$$A \cdot V = a \cdot v$$

$$P_0 + \rho g R + \frac{1}{2}\rho V^2 = \frac{1}{2}\rho v^2 + P_0$$

$$v = \sqrt{2gR}$$

$$A; \frac{dr}{dt} = a \cdot \sqrt{2gr}$$

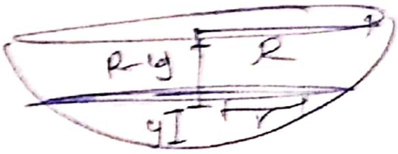
$$\frac{A}{a\sqrt{2g}} \cdot \left[2\sqrt{r}\right]_R^0 = t$$

$$A \cdot V = av$$

$$\frac{dA}{dt} V + A \cdot \frac{dV}{dt} = a \cdot \frac{d}{dt}\left(\sqrt{2gr}\right)$$

$$\frac{d}{dt}\left(\pi R^2\right) + A\frac{dv}{dt} = a \cdot \frac{d}{dt}\left(\sqrt{2gr}\right)$$

$$2\pi R\left(\frac{dr}{dt}\right) + A\,(acc) = a\sqrt{2g} \cdot \frac{1}{2\sqrt{r}} \cdot \frac{dr}{dt}$$



$$\cancel{R^2} = r^2 + \cancel{R^2} + y^2 - 2Ry$$

$$\boxed{2Ry - y^2 = r^2}$$

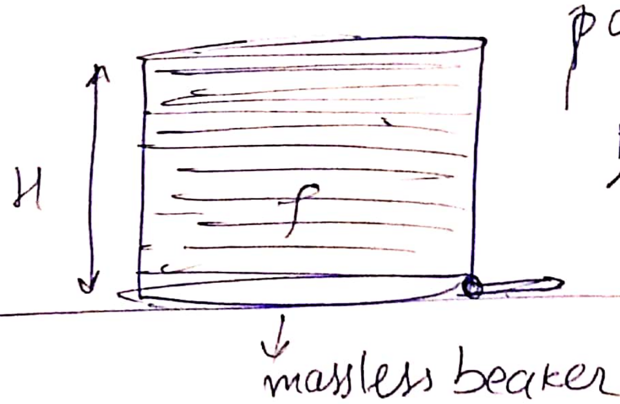
$$\int_R^0 \pi(2Ry - y^2)\,\frac{dy}{\cancel{dt}} = -a \cdot \sqrt{2g} \cdot \sqrt{y} \int_0^t dt$$

$$\int_R^0 \pi\left(2R\sqrt{y} - y^{3/2}\right) dy = -a\sqrt{2g}\,(t)$$

$$\left[\frac{\pi(2R)(y)^{3/2}}{3/2} - \frac{2\pi\, y^{5/2}}{5}\right]_R^0 = -a\sqrt{2g}\,(t)$$

$$-\frac{4R\pi}{3} \cdot R^{3/2} + \frac{2\pi}{5} \cdot R^{5/2}$$

force due to liquid flow



$$\rho a \cdot \left(\sqrt{\frac{2gH}{1-a^2/A^2}}\right)^2 = \rho AH \cdot [acc]$$

$$\cancel{H}\,[acc] = \frac{2g\cancel{H}a}{A^2-a^2} \cdot \frac{A^2}{\cancel{A}}$$

$$\boxed{acc \approx \frac{2gHa}{HA} = \frac{2ga}{A}}$$

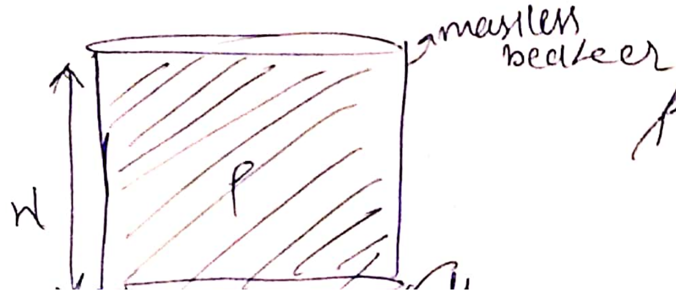
independent of height of fluid.

A beaker with mass,

$$\rho a\left(\frac{2gH}{A^2-a^2}\right)A^2 = (\rho AH + m)\,a$$

$$\boxed{a = \frac{2\rho g a H}{\rho AH + m}}$$

$\Rightarrow$



$$\rho a \cdot 2gy = \mu \rho AHg$$

$$\boxed{y = \frac{\mu AH}{2a}}$$

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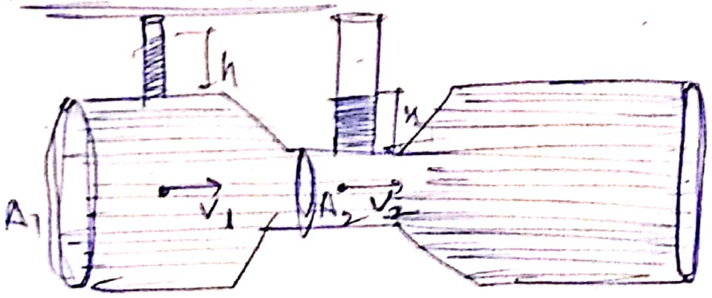
VENTURIMETER :-



: A device to mesure velocity of flow.

By E.C.

$$A_1 V_1 = A_2 V_2$$

By B.T;

$$\cancel{P_0} + \rho g(h+x) + \frac{1}{2}\rho v_1^2 = \cancel{P_0} + \rho g x + \frac{1}{2}\rho v_2^2$$

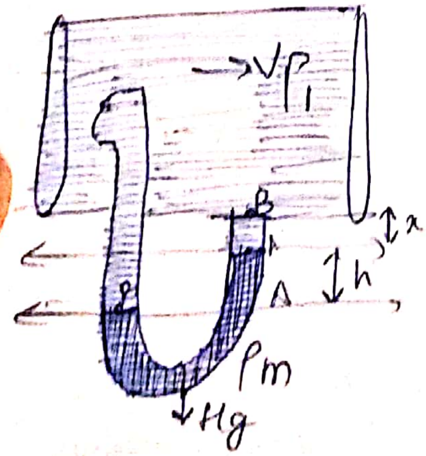
$$\rho g h = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

$$\boxed{v_1 = A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

$$\boxed{V_2 = A_1 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

$$\boxed{\text{Rate flow} = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

pitot tube :- A device used to mesure speed of fluids



$$P_A - P_B = \rho_1 g x + \rho_m g h$$

$$TE_A = TE_B$$

$$P_A + 0 + 0 = P_B + \frac{1}{2}\rho_1 v^2 + \rho_1 g(h+x)$$

$$\rho_1 g x + \rho_m g h = \frac{1}{2}\rho_1 v^2 + \rho_1 g(h+x)$$

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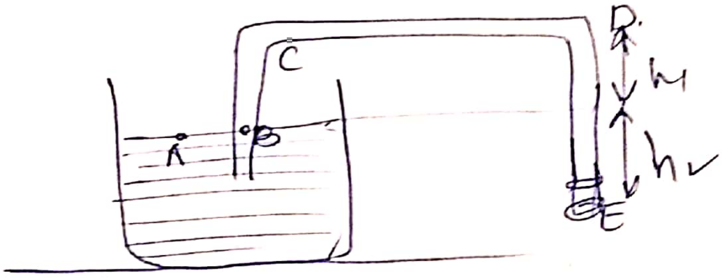
$$\Rightarrow \sqrt{\frac{2(\rho_m - \rho_1)gh}{\rho_1}} = v$$

if $\rho_1 << \rho_m \Rightarrow \boxed{v = \sqrt{\frac{2\rho_m gh}{\rho_1}}}$

$\Longrightarrow$ Siphon tube:-



$TE_E = TE_D$

$P_0 - \rho g h_2 + \cancel{\frac{1}{2}\rho v^2} = P_D + \cancel{\frac{1}{2}\rho v^2} + \rho g h_1$

$\boxed{P_D = P_0 - \rho g(h_1 + h_2)}$

$< P_0$

$\boxed{P_C = P_D}$

$TE_B = TE_C$

$P_B + \cancel{\frac{1}{2}\rho v^2} + 0 = P_C + \cancel{\frac{1}{2}\rho v^2} + \rho g h_1$

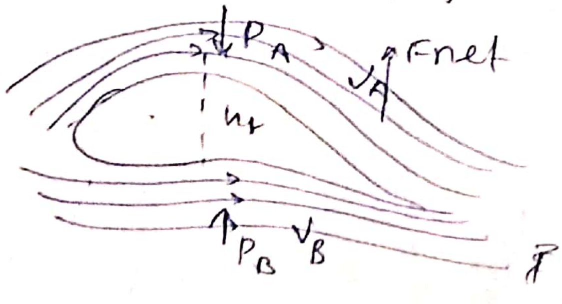
<u>Note</u> :- To make siphon tube work end of tube should be below fluid level.

$\boxed{P_B = P_C + \rho g h_1}$

$\Rightarrow \boxed{P_B = P_0 - \rho g h_2}$

Condition for working of siphon tube is $h_2 > 0$.

$\Longrightarrow$ <u>Aerodynamic lift</u>



$TE_A = TE_B$

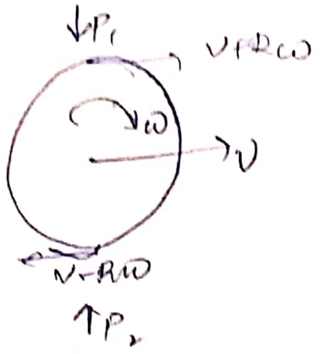
$P_A + \frac{1}{2}\rho v_1^2 = P_B + \frac{1}{2}\rho v_2^2 + 0$

$\Rightarrow P_A - P_B = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$\Rightarrow F_{net} = (P_A - P_B)$ Area of wing

$= \frac{1}{2}\rho(v_2^2 - v_1^2)$(Area of wing)

Magnus Effect



$$P_2 + \frac{1}{2}\rho(v-R\omega)^2 = P_1 + \frac{1}{2}\rho(v+R\omega)^2 + \rho g(2R)$$

$$(f_{net})_{upward} = (P_2 - P_1)(\text{Area effective})$$
$$= (P_2 - P_1)\pi R^2$$

Q If velocity of particle at point in stream flow is $\vec{v} = (V_0 + bx)\hat{i} + by\hat{j}$, then the eqn of streamline is?

$\Rightarrow$

$$V_x = V_0 + bx = \frac{dx}{dt}$$
$$V_y = -by = \frac{dy}{dt}$$

$$\frac{dy}{dx} = \frac{-by}{V_0 + bx}$$

$$\int \frac{dy}{y} = -b\int \frac{dx}{V_0 + bx}$$

$$\ln y = -1\ \ln|\quad)$$

y ____