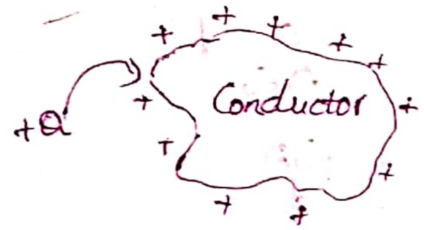
Capacitors

↳ device which stores the energy in form of electric field.

Capacitance :- ability to store the charge by the conductor (or) non-conductor objects

Capacitance of a conductor (isolated conductor) :-



Capacitance of conductor is defined as the ratio of charge to potential

charge ∝ potential

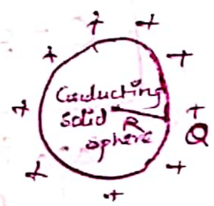
$Q \propto V$

$Q = CV$

$C = \frac{Q}{V}$

↳ capacitance

Example



$V\uparrow = \frac{Q\uparrow}{4\pi\epsilon_0 R}$

$V = \frac{Q}{4\pi\epsilon_0 R}$

$4\pi\epsilon_0 R = \frac{Q}{V}$

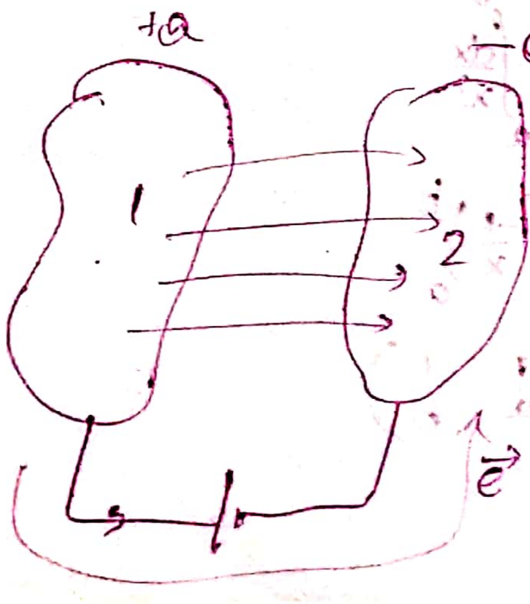
$\boxed{C = 4\pi\epsilon_0 R}$

Capacitor of spherical surface.

$C \propto R$

Capacitance depends only on physical dimension

Capacitance for combination of conductors :-



$C = \frac{\text{charge on any one conductor}}{\text{P. difference b/w conductors}}$

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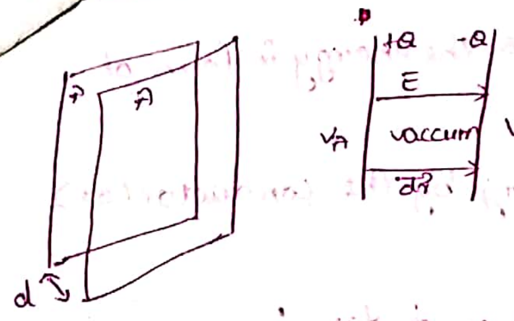
...el Plate conductor



$$|\vec{E}| = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$dv = -\vec{E}\cdot\vec{dr}$$

$$dv = -Ed\cos 0^\circ$$

$$V_B - V_A = -\left(\frac{Q}{A\epsilon_0}\right)d$$

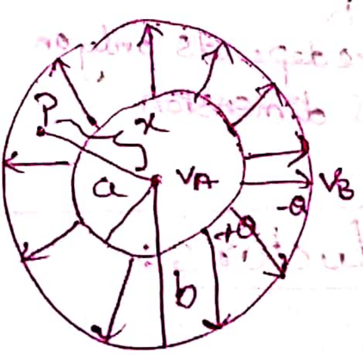
$$V_A - V_B = \frac{Q}{A\epsilon_0}d$$

$$\frac{Q}{V_A - V_B} = \frac{\epsilon_0 A}{d}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}} *$$

capacitance of parallel plate conductor.

Capacitance of spherical capacitor (hollow sphere)



Electric field intensity at point P is

$$E = \frac{Q}{4\pi\epsilon_0 x^2}$$

$$dv = -E(dx)\cos 0$$

$$\int_{V_A}^{V_B} dv = \frac{-EQ}{4\pi\epsilon_0}\int_a^b \frac{dx}{x^2}$$

$$V_B - V_A = \frac{-Q}{4\pi\epsilon_0}\left(\frac{-1}{x}\right)_a^b$$

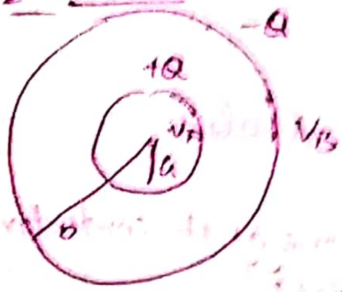
$$V_A - V_B = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$V_A - V_B = \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V_A - V_B} = 4\pi\epsilon_0\left(\frac{ba}{b-a}\right)$$

$$\boxed{C = 4\pi\epsilon_0\left(\frac{ba}{b-a}\right)}$$

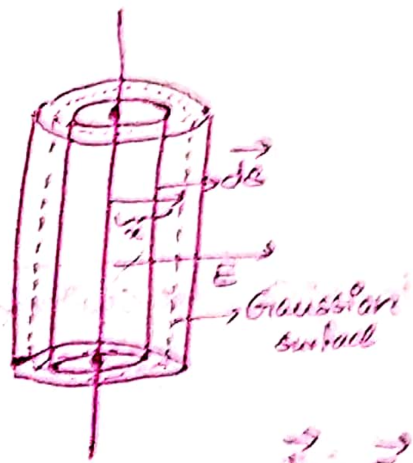
2nd Method



$$V_A - V_B = \left(\frac{KQ}{a} - \frac{KQ}{b}\right) - \left(\cancel{\frac{KQ}{b}} - \cancel{\frac{KQ}{b}}\right)$$

$$V_A - V_B = KQ\left(\frac{b-a}{ba}\right)$$

We know the potentials $\quad C = \frac{Q}{V_A - V_B} = \frac{4\pi\epsilon_0 (ba)}{b-a}$



$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{in}}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi r l \epsilon_0}$$

$$\vec{dv} = -\vec{E}\cdot\vec{dr}$$

$$\int_{V_A}^{V_B} \vec{dv} = -\int_a^b \frac{Q\,dr}{2\pi r l \epsilon_0}$$

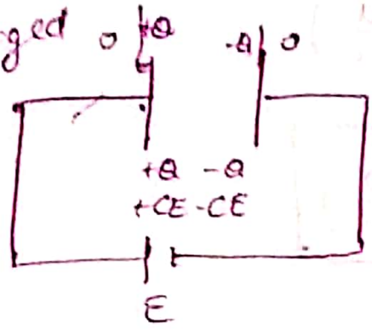
$$V_B - V_A = \frac{-Q}{2\pi l \epsilon_0} \ln(b/a)$$

$$V_A - V_B = \frac{Q}{2\pi\epsilon_0 l} \ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}}$$

Important points regarding parallel plate conductor

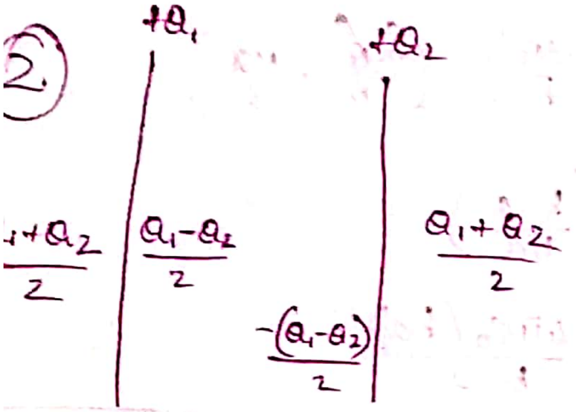
(1) initially uncharged



$$C = \frac{Q}{E}$$

$$Q = EC$$

(2)



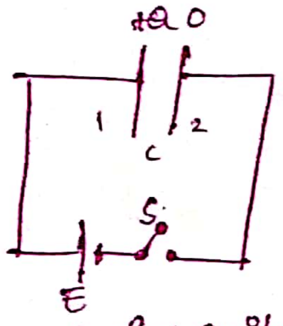
$C = \dfrac{\epsilon_0 A}{d}$ is fixed value

$$C = \frac{\text{charge on any one of the conductor which provides } \vec{E}}{\text{potential diff}}$$

$$C = \frac{\frac{Q_1 - Q_2}{2}}{\Delta V}$$

$$\Delta V = \frac{Q_1 - Q_2}{2C}$$

) Find the charge on the first plate after the switch is closed.



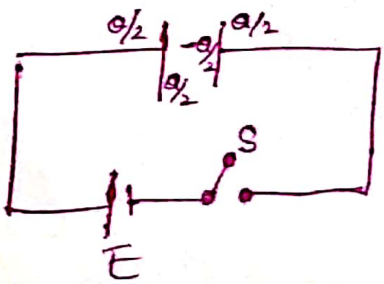
$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

$$Q = VC$$

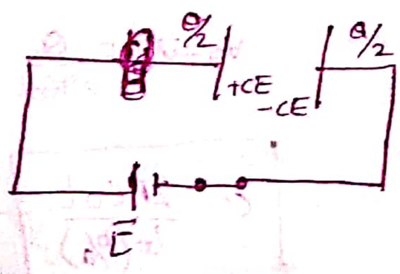
$$v = \frac{KQ}{R}$$

V

Before closing switch



After closing switch



If $\frac{Q}{2} < CE$ charge supplied by battery is $CE - \frac{Q}{2}$

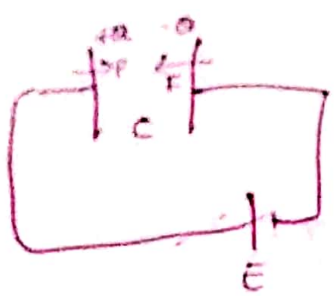
If $\frac{Q}{2} > CE$ charge ~~supplied~~ received by battery is $\frac{Q}{2} - CE$

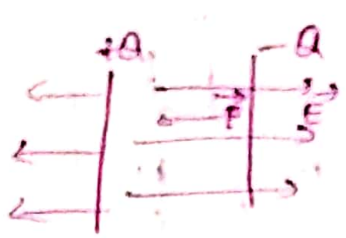
If $\frac{Q}{2} = CE$ charge supplied by battery is zero.

After closing switch P.d across capacitor becomes E.

$\therefore$ ~~Ch~~ Final charge on 1st plate $= \frac{Q}{2} + EC$

Force acting b/w two plates of capacitor





$$|\vec{E}| = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

Force acting on 2nd plate is $|\vec{F}| = QE$

$$|\vec{F}| = Q\left(\frac{Q}{2A\epsilon_0}\right)$$

$$\boxed{|\vec{F}| = \frac{Q^2}{2A\epsilon_0}}$$

is independent of distance b/w plates

~~Force acti~~

Energy stored in capacitor

Work required to add 'dq' charge is $dw = (dq)\left(\frac{q}{C}\right)$

Total work required to get the charge of Q

is $W_{ext} = \int dw = \int_0^Q dq\,\frac{q}{C}$

$$W_{ext} = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^Q$$

$$W_{ext} = \frac{Q^2}{2C}$$

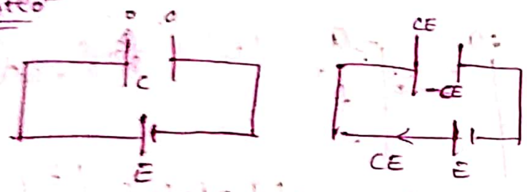
$\boxed{\text{Energy stored in the capacitor} = \frac{Q^2}{2C}}$ *

$U = \frac{Q^2}{2C}$

$= \frac{(CV)^2}{2C}$

$\boxed{U = \frac{1}{2}CV^2}$ *

Note:-



Energy stored in the capacitor is $= U = \frac{1}{2}CE^2$

Charge supplied by battery is $= CE$

Work done by battery is $W_b = (\text{Charge})(\text{P.d})$

$= CE \cdot E$

$= CE^2$

Heat generated in system $= W_{battery} - \left(\text{increase in energy of system}\right)$

Heat generated $= CE^2 - \frac{1}{2}CE^2$

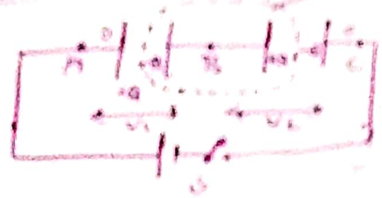
$= \frac{1}{2}CE^2$

Connection

Series co

Connection of Capacitors

Series connection



→ Isolated plates

[Charge on Isolated plates always remains same]

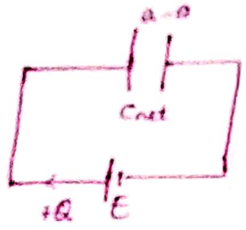
$V_1 = V_A - V_B$

$V_2 = V_B - V_C$

$V_1 + V_2 = V_A - \cancel{V_B} + \cancel{V_B} - V_C$

$= V_A - V_C$

$V_1 + V_2 = E$



$V_1 + V_2 = E$:

$Q = C_{net} E$

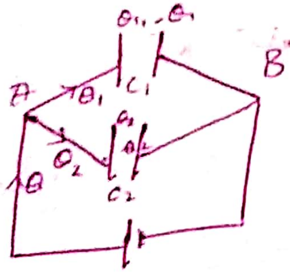
$\frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{net}}$ *

$Q = C_1 V_1$

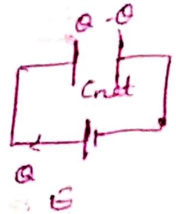
$Q = C_2 V_2$

$\boxed{\frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2}}$ [For series]

Parallel connection



→



$V_A - V_B = E$

$Q_1 = C_1 E$

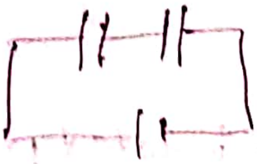
$Q_2 = C_2 E$

$Q = Q_1 + Q_2$

$C_{net} E = C_1 E + C_2 E$

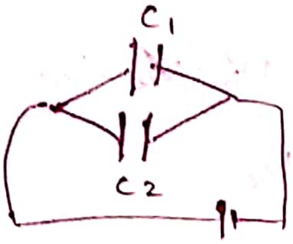
$\boxed{C_{net} = C_1 + C_2}$ [For parallel]

$V\left(\frac{C_1C_2}{C_1+C_2}\right) = \frac{Q}{\cancel{C}}$



$V_1 = \frac{Q}{C_1}$

$V_1 = V\left(\frac{C_2}{C_1+C_2}\right)$, $V_2 = V\left(\frac{C_1}{C_1+C_2}\right)$



$C_1+C_2 = \frac{Q}{V}$

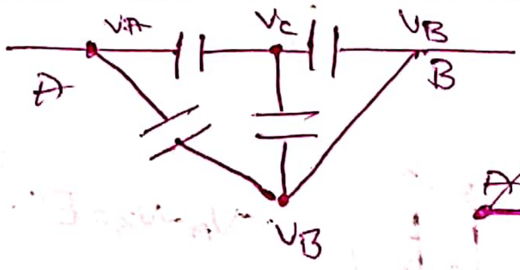
$Q_1 = C_1 V$

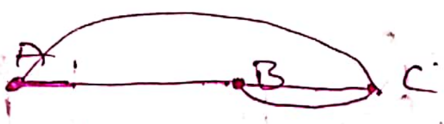
$Q_1 = \left(\frac{C_1+C_2}{Q}\right) C_1 = \frac{Q C_1}{C_1+C_2}$

$Q_2 = \left(\frac{C_1+C_2}{Q}\right) C_2 \quad \frac{Q C_2}{C_1+C_2}$

Finding net capacitance of circuit

a)





C 2C

$\frac{2C^2}{3C} + C$

$= \frac{5C}{3}$

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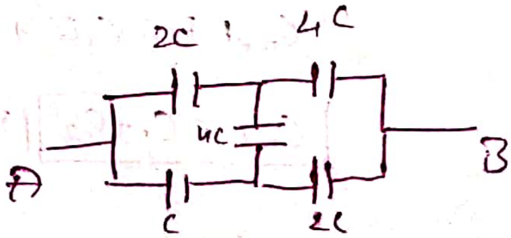
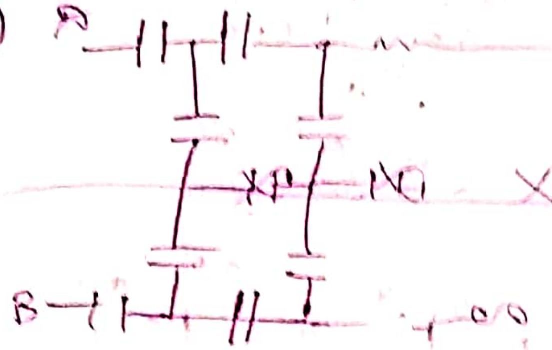
q)



$C_{net} = 2C$

9)



$$\frac{1}{x} = \frac{2}{\frac{c}{2}+x} + \frac{2}{c}$$

$c^2 + 2cx$

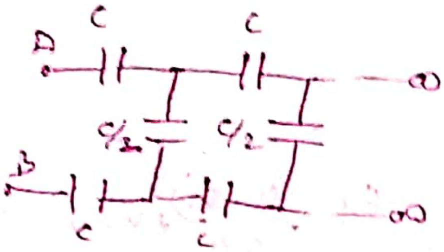
$$c^2 + 2cx = 2cx + 2(c+2x)x$$

$$c^2 + 2cx = 2cx + 2cx + 2x^2$$

$$2x^2 + 2cx - c^2 = 0$$

$$\frac{-2c \pm \sqrt{4c^2 + 12c^2}}{4}$$

$$\frac{-2c \pm 4c}{4} = \frac{c}{2}$$



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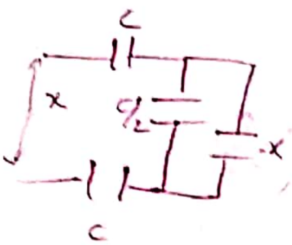


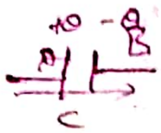
$$\frac{1}{x} = \frac{1}{c} + \frac{1}{c} + \frac{1}{\frac{c}{2}+x}$$

$$\frac{1}{2x} = \frac{2c + 2x}{c(c+2x)}$$

$$4x^2 + 2cx - c^2 = 0$$

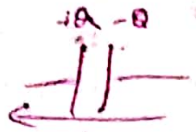
$$x = \left(\frac{\sqrt{5}-1}{4}\right)c$$

Q) Potential drop across capacitor



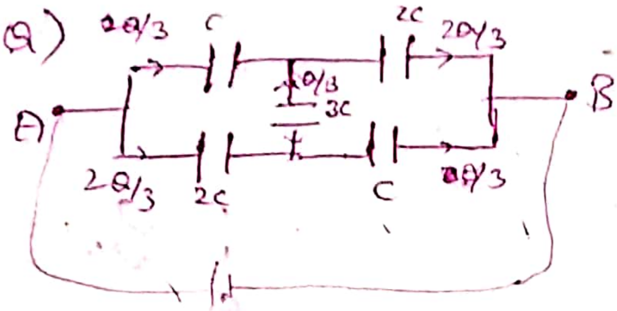
Potential drop $V_B - V_A = -(V_A - V_B)$

$$= \frac{-Q}{C}$$



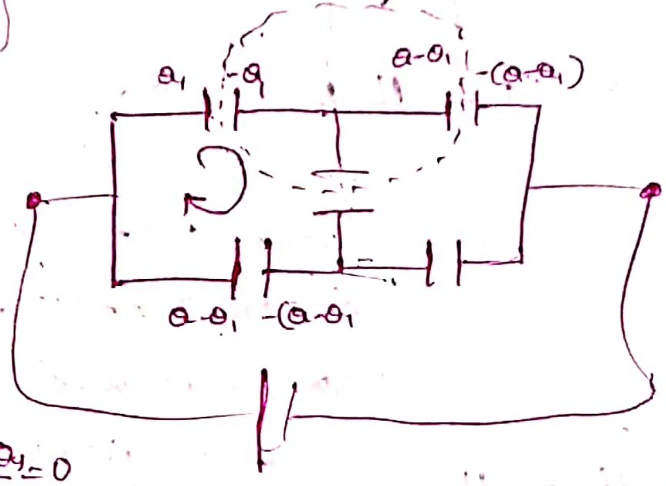
Potential drop $= V_A - V_B = \frac{Q}{C}$

Q)



As these are isolated total charge is 0.

$C_{net} = \frac{Q}{V}$



$$-\frac{Q_1}{C} + \frac{Q-2Q_1}{3C} + \frac{Q-Q_1}{2C} = 0$$

$$\left(\frac{Q}{3C} + \frac{Q}{2C}\right) - \left(\frac{Q_1}{C} + \frac{2Q_1}{3C} + \frac{Q_1}{2C}\right) = 0$$

$$\frac{5Q}{6C} - \frac{13Q_1}{6C} = 0$$

$$\boxed{5Q = 13Q_1}$$

$$-\frac{Q_1}{C} - \frac{(Q-Q_1)}{2C} + V = 0$$

$$V = \frac{Q_1}{C} + \frac{Q-Q_1}{2C}$$

$$V = \frac{Q_1}{C} + \frac{Q}{2C} - \frac{Q_1}{2C}$$

$$V = \frac{Q_1}{2C} + \frac{Q}{2C}$$

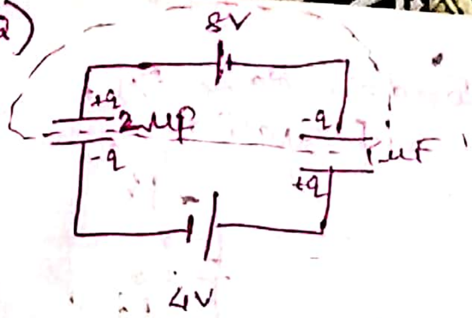
$$V = \frac{\frac{5Q}{13}}{2C} + \frac{Q}{2C}$$

$$V = \frac{9Q}{13C}$$

$$\frac{Q}{V} = \frac{13C}{9}$$

$$\boxed{C_{net} = \frac{13C}{9}}$$

a)



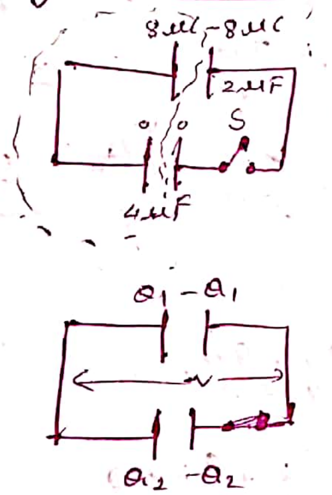
Find the charge on 1µF capacitor

$$-8 + \frac{q}{1\mu F} - 4 + \frac{q}{2\mu F} = 0$$

$$\frac{3q}{2\mu F} = 12 \text{ volts}$$

$$q = 8\mu C$$

Charge distribution between capacitors.



$$\cancel{\frac{8-x}{2} = \frac{x}{4}}$$

$$\frac{(8-x)}{\cancel{2}} = \frac{x}{\cancel{4}\,2}$$

$$16 - 2x = x$$

$$\boxed{x = 16/3}$$

$$Q_1 + Q_2 = 8$$

$$\frac{Q_1}{2} = \frac{Q_2}{4}$$

$$2Q_1 = Q_2$$

$$Q_1 + 2Q_1 = 8$$

$$Q_1 = \frac{8}{3}\mu C$$

$$Q_2 = \frac{16}{3}\mu C$$

Loss of energy $= U_i - U_f$

$$= \frac{1}{2}(2)(4)^2 - \left(\frac{1}{2}\times 2\times\left(\frac{4}{3}\right)^2 + \frac{1}{2}\times 4\times\left(\frac{4}{3}\right)^2\right)$$

$$= \frac{1}{\cancel{2}}\times\cancel{2}\times 16 - \frac{1}{\cancel{2}}\times\cancel{6}^{\,3}\times\frac{16}{9}$$

$$= \frac{9\times 16}{9} - \frac{3\times 16}{9}$$

Loss in energy $= \dfrac{16\times\cancel{6}\,2}{\cancel{9}\,3} = \dfrac{32}{3}\ \mu J$

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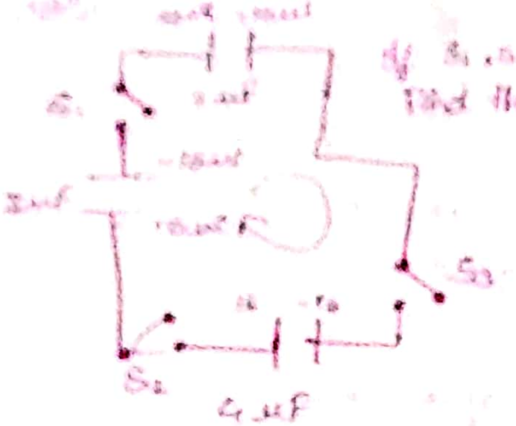

If S_1, S_2, S_3 are closed simultaneously, find the charge on $3\mu F$ capacitor.

$Q_1+Q_2=-5$

$-Q_2+Q_3=31$

$Q_3+Q_1=-26$

$Q_3+\frac{21}{2}=31$

$Q_3=31-\frac{21}{2}$

$=\frac{62-21}{2}$

$Q_2-Q_1=-26$

$Q_1+Q_2=-5$

$2Q_2=21$

$Q_2=21/2$

$\boxed{Q_3=\frac{41}{2}}$

$Q_1+Q_2=-5 \rightarrow (1)$

$-Q_2+Q_3=31 \rightarrow (2)$

$-Q_3-Q_1=-26 \rightarrow (3)$

$Q_1+Q_2=-5$

$Q_2-Q_3=-31$

$Q_1+Q_3=26$

$q_1-q_3=-5$

$q_3-q_2=31$

$q_2-q_1=-26$

$-\frac{q_1}{2}-\frac{q_2}{4}-\frac{q_3}{3}=0$

$\frac{q_1}{2}+\frac{q_2}{4}+\frac{q_3}{3}=0$

$6q_1+3q_2+4q_3=0$

$(q_3-5)6+(q_3-31)3+4(q_3)=0$

$q_3=\frac{123}{13}\ \mu C$

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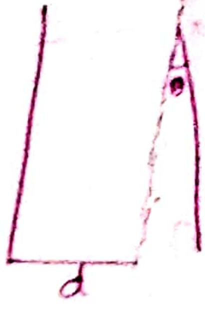
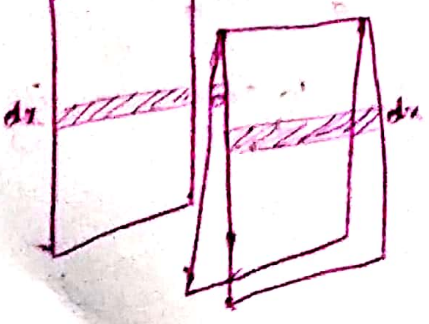
a)

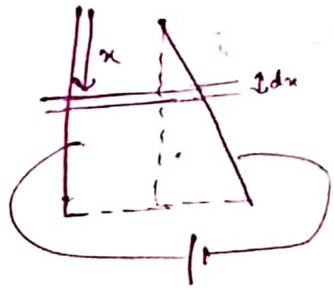


$\oint \vec{E}\cdot\vec{dr}=\frac{q}{\epsilon_0}$



$L \times dx$ $\quad dc \quad \xleftrightarrow{d + x\tan\theta}$

$$dc = \frac{\epsilon_0 (L\,dx)}{d + x\tan\theta}$$



Net capacitance of capacitors is

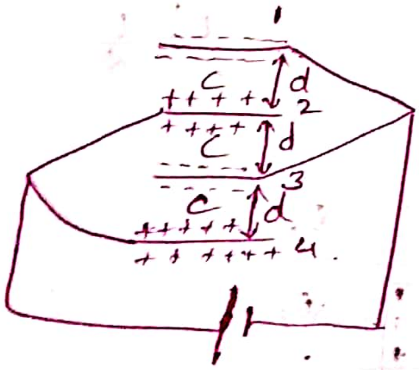
$$C_{net} = dC_1 + dC_2 + dC_3 + \dots + dC_n$$

$$= \int_0^L \frac{\epsilon_0 (L\,dx)}{d + x\tan\theta}$$

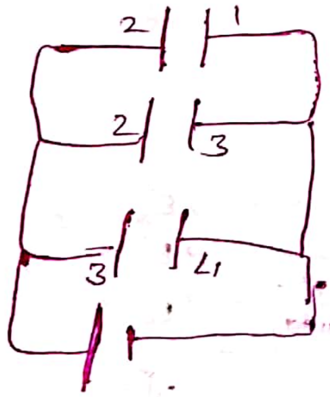
$$= \epsilon_0 L \frac{1}{\tan\theta} \Big(\ln(d + x\tan\theta)\Big)_0^L$$

$$\boxed{C_{net} = \frac{\epsilon_0 l}{\tan\theta} \left(\ln\left(\frac{d + L\tan\theta}{d}\right)\right)}$$

Finding Capacitance when plates combination is given :-

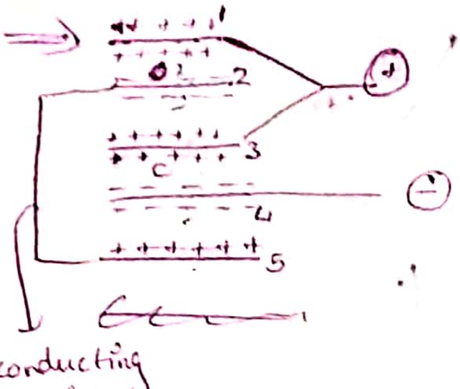


$\Longrightarrow$



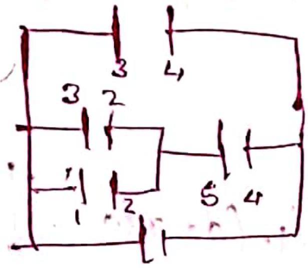
$$C_{net} = 3C$$

$$C_{net} = \frac{3A\epsilon_0}{d}$$



$\equiv$

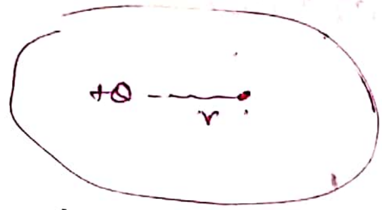
$\equiv$



$$C_{net} = \frac{5C}{3} = \frac{5\epsilon_0 A}{3d}$$

Capacitance with dielectric material

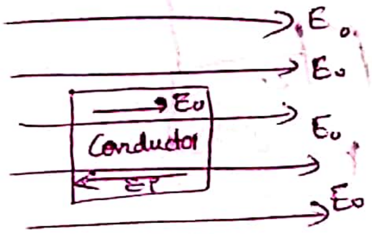
$\Rightarrow$ $+Q \xrightarrow[r]{\text{vaccum}}$ $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$

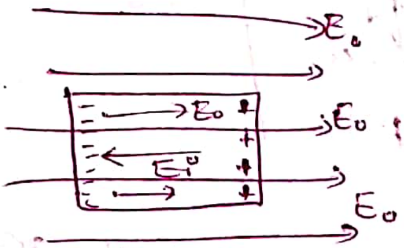


$$E' = \frac{Q}{4\pi K\epsilon_0 r^2}$$

$$E' = \frac{E}{K}$$

$\Rightarrow$





$$E_{net} = \frac{E_0}{K}$$

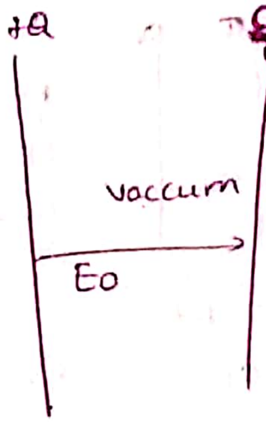
$$E_{net} = E_0 - E_i$$

$$\frac{E_0}{K} = E_0 - E_i$$

$$E_i = E_0 - \frac{E_0}{K}$$

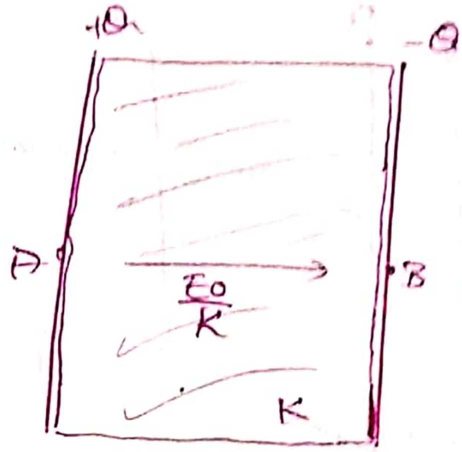
$$E_i = E_0\left(1 - \frac{1}{K}\right)$$

Capacitance of 2 Plates



$$E_0 = \frac{Q}{A\epsilon_0}$$

$$C = \frac{\epsilon_0 A}{d}$$



$$V_A - V_B = \left(\frac{E_0}{K}\right) d$$

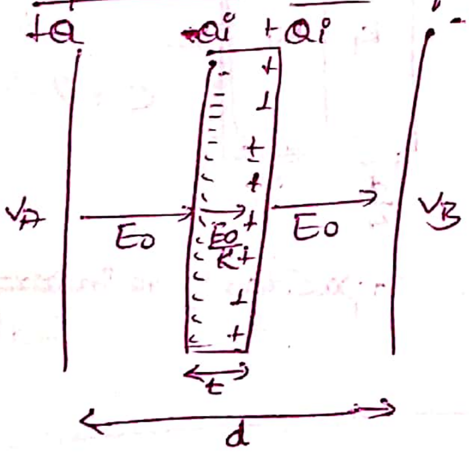
$$V_A - V_B = \frac{Q}{A\epsilon_0 K}(d)$$

$$\Delta V = \frac{Qd}{KA\epsilon_0}$$

$$\boxed{\frac{Q}{\Delta V} = \frac{K\epsilon_0 A}{d}}$$

$$\boxed{C_{\substack{\text{with}\\\text{dielectric}}} = K\, C_{\substack{\text{without}\\\text{dielectric}}}}$$

Capacitance with partially filled dielectric



$$V_A - V_B = E_0(d-t) + \frac{E_0}{K} t$$

$$\Delta V = E_0\left(d - t + \frac{t}{K}\right)$$

$$\Delta V = \frac{Q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$$\frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d - t + t/K}$$

$$\boxed{C = \frac{\epsilon_0 A}{d - t + t/K}}$$ → is independent of position of dielectric

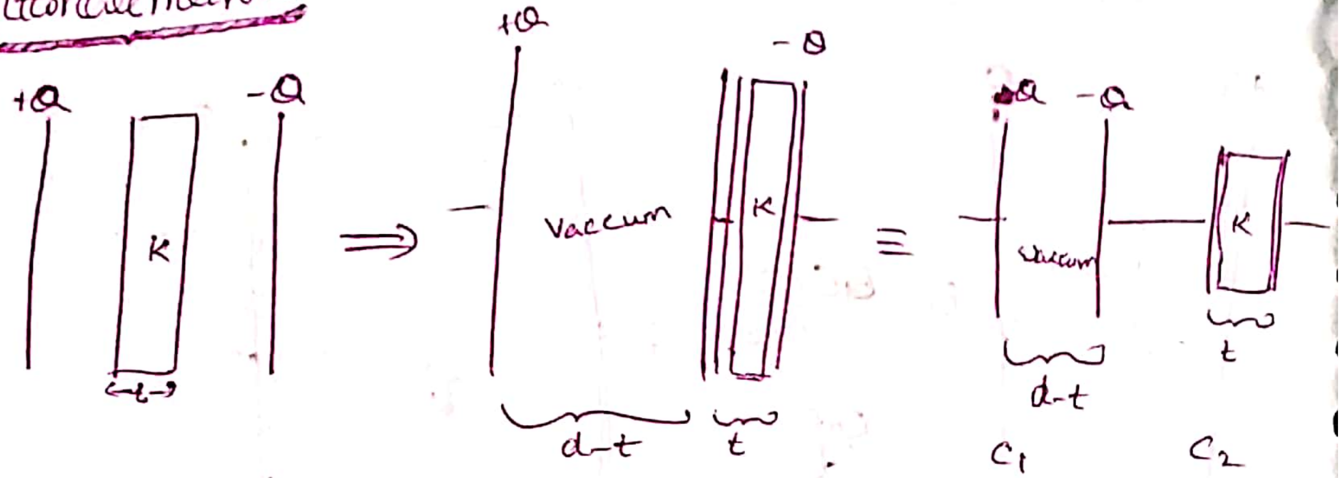
Induced electric field

$$E_i = E_0\left(1 - \frac{1}{K}\right)$$

$$\frac{Q_i}{A\epsilon_0} = \frac{Q}{A\epsilon_0}\left(1 - \frac{1}{K}\right) \rightarrow \boxed{Q_i = Q\left(1 - \frac{1}{K}\right)}$$ → Induced charge on surface of non conducting material.

Scanned with CamScanner

Alternate method



C_1 & C_2 are in series

$$C_1 = \frac{\epsilon_0 A}{d-t}$$

$$C_2 = \frac{K\epsilon_0 A}{t}$$

$$C_{net} = \frac{C_1 C_2}{C_1 + C_2}$$

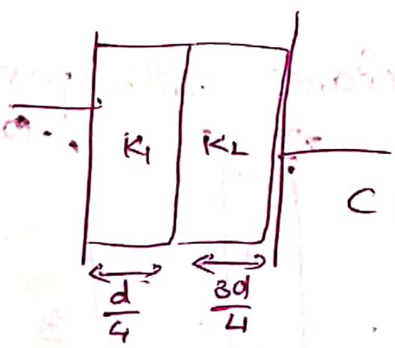
$$= \frac{\frac{\epsilon_0 A}{d-t}\,\frac{K\epsilon_0 A}{t}}{\frac{\epsilon_0 A}{d-t} + \frac{K\epsilon_0 A}{t}}$$

$$C_{net} = \frac{\epsilon_0 A}{d-t+t/K}$$

$$C_1 = \frac{4K_1\epsilon_0 A}{d} \qquad C_2 = \frac{4K_2\epsilon_0 A}{3d}$$

$$C_{net} \; \frac{\frac{4K_1\epsilon_0 A}{d} \times \frac{4K_2\epsilon_0 A_2}{3d}}{\frac{4(3K_1\epsilon_0 A + K_2\epsilon_0 A_2)}{3d}}$$

$$C_{net} = \left(\frac{4K_1K_2}{3K_1+K_2}\right)\frac{A\epsilon_0}{d}$$

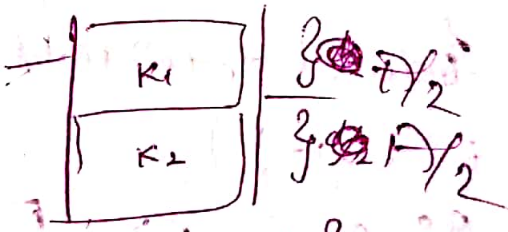


C = ?

Capacitors are in ~~parallel~~ series

$$C_1 = \frac{K_1\epsilon_0 A}{d\,2} \qquad C_2 = \frac{K_2\epsilon_0 A}{d\,2}$$

$$C_{net} = C_1 + C_2 = \frac{\epsilon_0 A}{d\,2}(K_1+K_2)$$



C = ?

Capacitors are in ~~ser~~ parallel

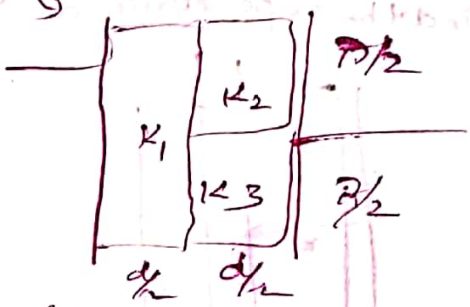
$C_1 = \frac{2K_1 A\epsilon_0}{d}$ $C_2 = \frac{K_2 A \epsilon_0 \cancel{2}}{\cancel{2} d}$ $C_3 = \frac{K_3 A \epsilon_0 \cancel{2}}{\cancel{2} d}$

C_2, C_3 in paralle

$$C_{23} = (K_2+K_3)\frac{A\epsilon_0}{d}$$

C_1, C_{23} is in series

(a)

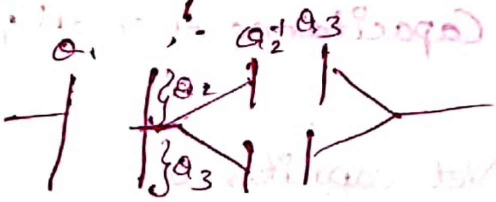


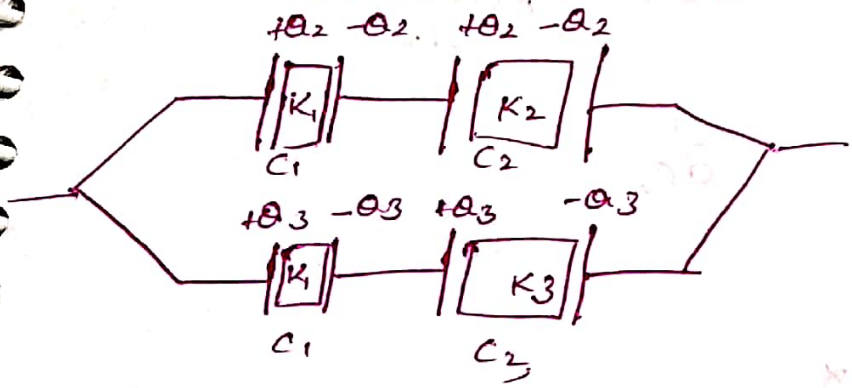
$$\frac{1}{C_{net}} = \frac{d}{2K_1A\epsilon_0} + \frac{d}{(K_2+K_3)A\epsilon_0}$$

$$\cancel{\frac{1}{C_{net}} = \frac{d}{A\epsilon_0}\left(\frac{K_2+K_3+2K_1}{2K_1(K_2+K_3)}\right)}$$

$$\cancel{\boxed{C_{net} = \frac{A\epsilon_0}{d}\left[\frac{2K_1(K_2+K_3)}{2K_1+K_2+K_3}\right]}}$$

$\neq$



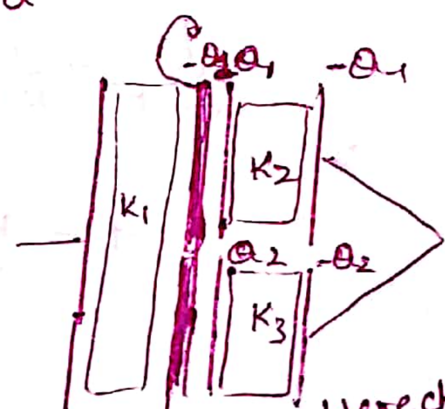


charge density must ~~be~~ remain constant. so it is diveded into 2 capacitors

$C_{up} = \left(\frac{K_1K_2}{K_1+K_2}\right)\frac{A\epsilon_0}{d}$ $C_{do} = \left(\frac{K_1K_3}{K_1+K_3}\right)\frac{A\epsilon_0}{d}$

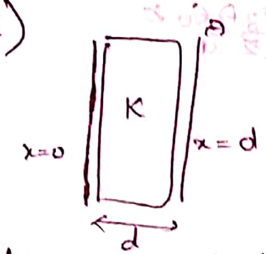
$C_{net} = C_{up} + C_{down}$

$$\boxed{C_{net} = \left(\frac{K_1K_2}{K_1+K_2} + \frac{K_1K_3}{K_1+K_3}\right)\frac{A\epsilon_0}{d}}$$

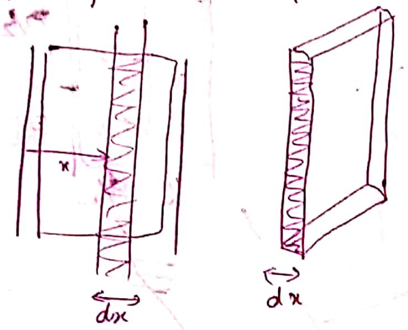


Here charge density is not same so it should be broke into two

Q)



If dielectric constant is variable which is $K = K_0(1+\alpha x)$ where K_0 and α are constants. Find the capacitance of capacitors



Capacitance of small part $dc = \frac{K\epsilon_0 (A)}{dx}$

$dc = K_0 \frac{(1+\alpha x)\epsilon_0 A}{dx}$

Net capacitance,

$$\frac{1}{C_{net}} = \frac{1}{dc_1} + \frac{1}{dc_2} + \dots + \frac{1}{dc_n}$$

$$\frac{1}{C_{net}} = \int \frac{1}{dc}$$

$$\frac{1}{C_{net}} = \int_0^d \frac{dx}{K_0(1+\alpha x)\epsilon_0 A}$$

$$\frac{1}{C_{net}} = \frac{1}{K_0 \epsilon_0 A \alpha} \left(\ln(1+\alpha x)\right)_0^d$$

$$C_{net} = \frac{K_0 \epsilon_0 A \alpha}{\ln(1+\alpha d)}$$

$dV = -E\,dr$

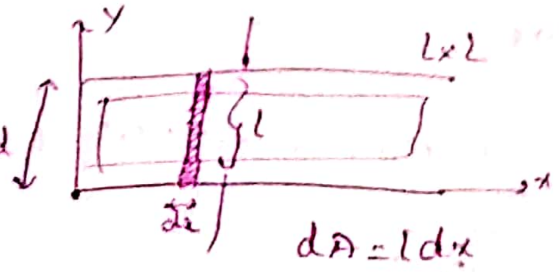
$$dV = \frac{E_0\,dx}{K_0(1+\alpha x)}$$

$$\frac{E_0}{K_0}\left[\frac{\ln(1+\alpha x)}{\alpha}\right]_0^d$$

$$dV = \frac{E_0}{K_0}\left[\frac{\ln(1+\alpha d)}{\alpha}\right]$$

$$V = \frac{Q}{A\epsilon_0 K_0}\left(\frac{\ln(1+\alpha d)}{\alpha}\right)$$

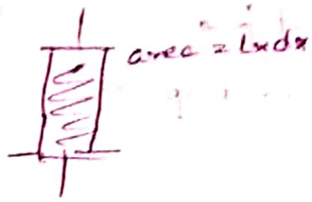
$$C = \frac{K_0 \epsilon_0 A \alpha}{\ln(1+\alpha d)}$$



$dA = l\,dx$

If dielectric constant of material changes as $K_0 = K_0 x$. Find the capacitance of capacitor

$$dC = \frac{K_0 \epsilon_0 x \, l\, dx}{d}$$



$$C_{net} = dC_1 + dC_2 + dC_3 + \cdots + C_{nd}$$

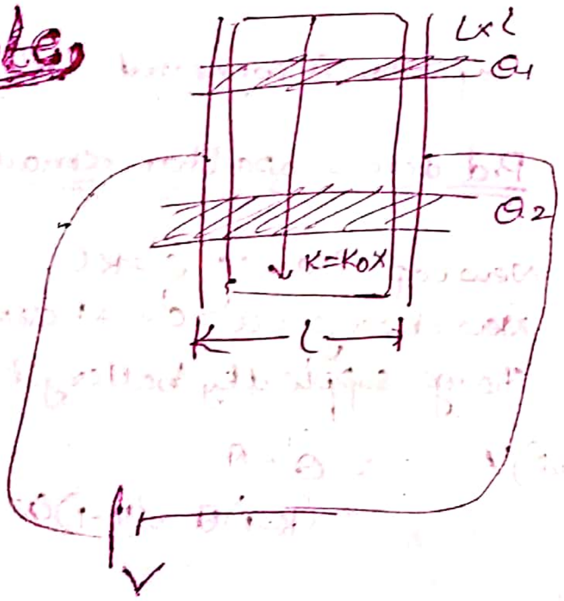
$$C_{net} = \int_0^l \frac{K_0 \epsilon_0 l}{d} \, x \, dx$$

$$= \frac{K_0 \epsilon_0 l}{d} \left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{K_0 \epsilon_0 l \cdot l^2}{2d}$$

$$\boxed{C_{net} = \frac{K_0 \epsilon_0 L^3}{2d}}$$

Note



K↑ as we go down

$dC_1 < dC_2$

P.d across dC_1 and dC_2 are same

$Q_1 < Q_2$

$\sigma_1 < \sigma_2$

Effect of dielectric in capacitors

Case i If battery is disconnected and dielectric is kept inside capacitor.

isolated capacitor

Q remains same in an isolated capacitor

New capacitance = KC

New $V = V' = \frac{V}{K}$

initial energy stored in capacitor $= \frac{1}{2}CV^2$

final energy stored in capacitor $= \frac{1}{2}C'V'^2$

$= \frac{1}{2}KC\frac{V^2}{K^2}$

final energy $= U_f = \frac{1}{K}U_i$

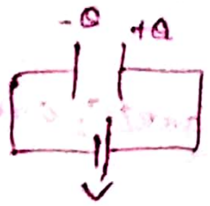
Energy lost $= U_i - U_f$

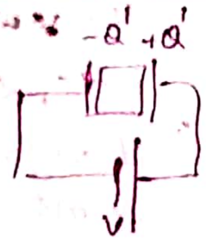
$= U_i - \frac{U_i}{K}$

$= \frac{1}{2}CV^2\left(1 - \frac{1}{K}\right)$

Case ii

Dielectric is inserted when capacitor is connected to battery





P.d across capacitor remains same

New capacitance $C' = KC$

New charge $= Q' = C'V = KCV = KQ$

Charge supplied by battery is

$= Q' - Q$

$= (K-1)Q = (K-1)CV$

Workdone by battery $W_b = (Q_{sup})V$

$W_b = (K-1)CV \cdot V$

$= (K-1)CV^2$

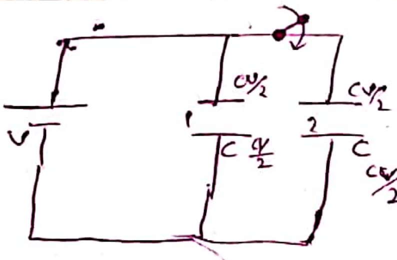
Increase in energy of capacitor $\Delta U = U_f - U_i$

$= \frac{1}{2}(KC)V^2 - \frac{1}{2}CV^2$

$= \frac{1}{2}(K-1)CV^2$

energy lost $= \frac{1}{2}(K-1)CV^2$

Q)



Intially switch is closed position Now switch is open and dielectric of dielectric constant $K=3$ is kept inside capacitor 2. Find the ration of initial and final energies.

$U_i = \frac{(CV)^2}{2C\times 2} + \frac{(CV)^2}{2C\times 2} = CV^2$

$U_{F_1} = \frac{K^2C^2V^2}{4\times 2C}$ $\quad Q' = \frac{KCV}{2}$

$= \frac{K^2V^2C}{8}$

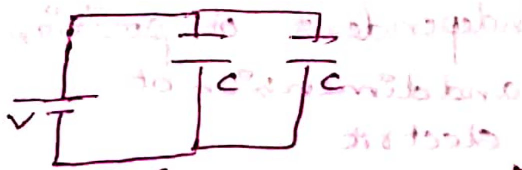
$= \frac{9V^2}{8}$

$U_{F_2} = \left(\frac{CV}{2}\right)^2 \frac{1}{2KC}$

$= \frac{CV^2}{8K}$

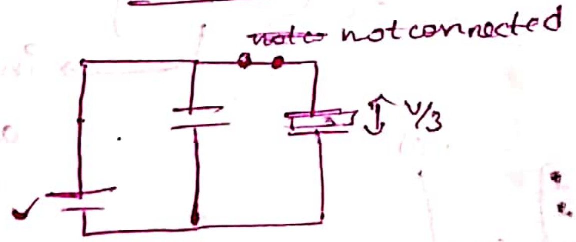
$= \frac{CV^2}{24}$

intially



$U_i = \left(\frac{1}{2}CV^2\right)2 = CV^2$

finally

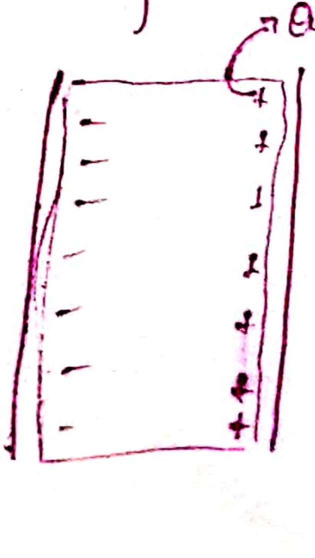


$U_F = \frac{1}{2}CV^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2$

$= \frac{1}{2}CV^2 + \frac{CV^2}{6} = \frac{2CV^2}{3}$

$\frac{U_F}{U_i} = \frac{2}{3}$

Q) Force and Stress acting on dielectric in capacitor



$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$Q_i = Q\left(1 - \frac{1}{K}\right)$

Net force acting on dielectric effect is zero

Force acting on one surface of dielectric

ie $F = QE$

$F = Q_i E_0$

$$F = Q\left(1-\frac{1}{K}\right)\left(\frac{Q}{A\epsilon_0}\right)$$

$$\boxed{F = \frac{Q^2}{A\epsilon_0}\left(1-\frac{1}{K}\right)}$$

Stress acting on the dielectric is

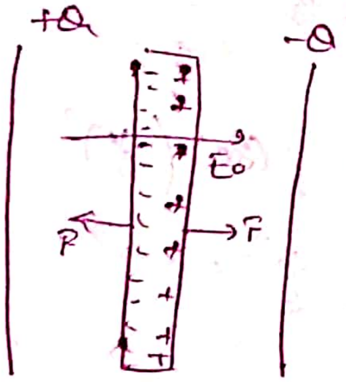
$$= \frac{F}{\text{Area}}$$

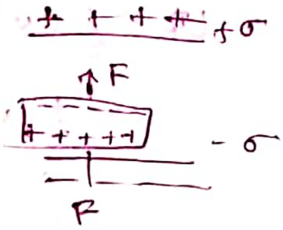
$$= \frac{\frac{Q^2}{A\epsilon_0}\left(1-\frac{1}{K}\right)}{A}$$

$$= \frac{Q^2}{A^2\epsilon_0}\left(1-\frac{1}{K}\right)$$

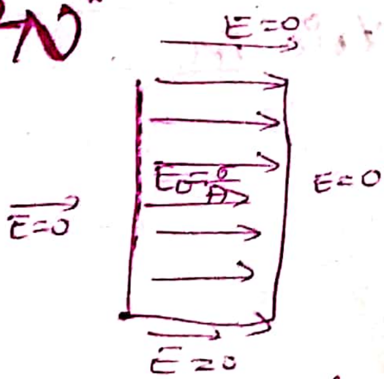
$$\boxed{\text{stress} = \frac{\sigma^2}{\epsilon_0}\left(1-\frac{1}{K}\right)}$$

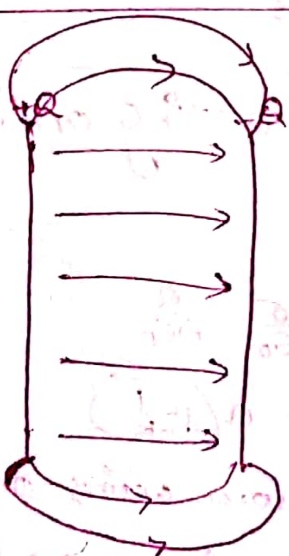
→ independent of position and dimension of electric





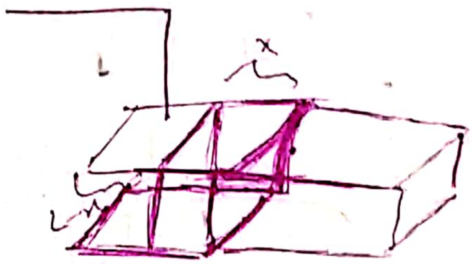
½N*

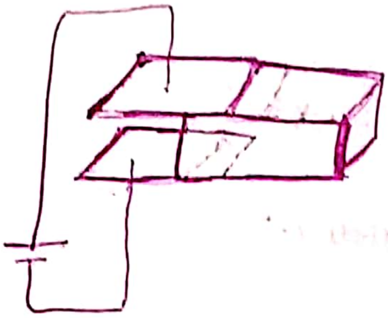




endeffect/fringing effect

Assumption





$F = -\frac{du}{dx}$

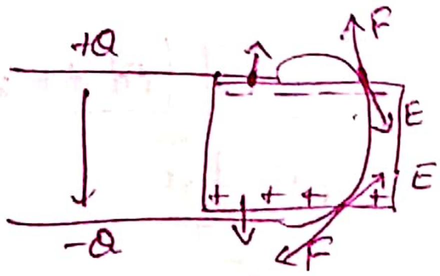
Capacitance of capacitor

$c = c_1 + c_2$

$C = \frac{\epsilon_0 (l-x) b}{d} + \frac{K\epsilon_0 (bx)}{d}$

$C = \frac{\epsilon_0 b}{d} (l - x + Kx)$

$$\boxed{C = \frac{\epsilon_0 b}{d} (l + (k-1)x)}$$



$F = \left|\frac{dU}{dx}\right|$

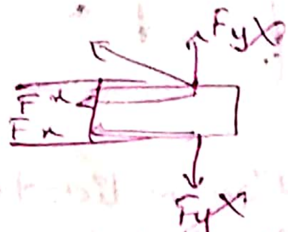
$F = \frac{d}{dx}\left(\frac{1}{2} CV^2\right)$

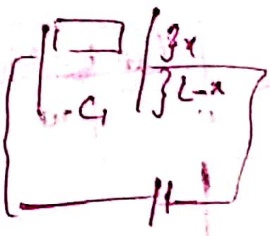
$F = \frac{1}{2} V^2 \frac{dC}{dx}$

$F = \frac{1}{2} V^2 \left(\frac{\epsilon_0 b}{d} (k-1)\right)$

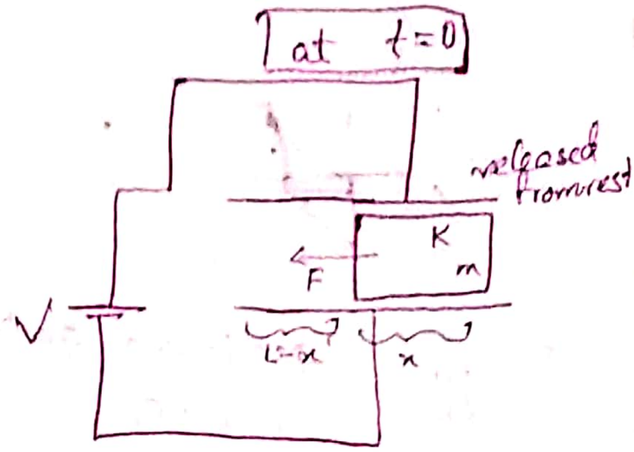
$$\boxed{F = \frac{\epsilon_0 b}{2d} (k-1) V^2}$$

constant force





$$F = \frac{\varepsilon_0 b}{2d}(k-1)V^2$$



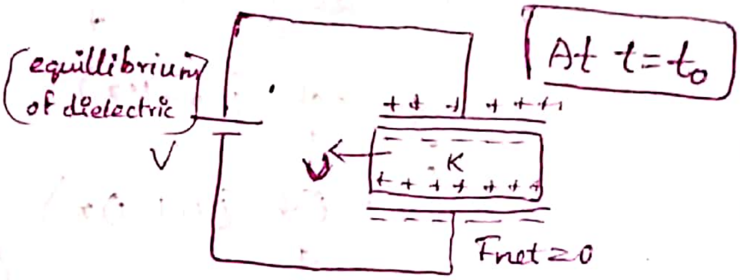
Acceleration of dielectric

$$a = \frac{F}{m}$$

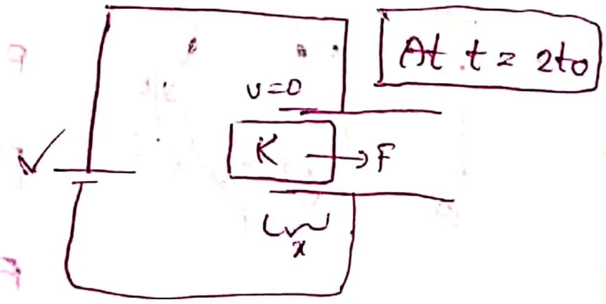
$$\boxed{a = \frac{\varepsilon_0 b}{2md}(k-1)V^2}$$

$$s = ut + \frac{1}{2}at^2$$

$$L-x = \frac{1}{2}at_0^2$$



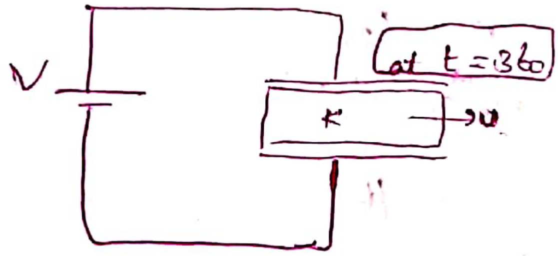
$$\boxed{t_0 = \sqrt{\frac{2(L-x)}{a}}}$$

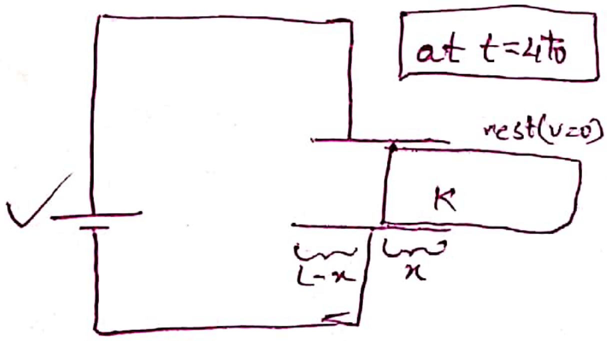


Motion of dielectric is oscillating but it is periodic (Not SHM)

Time period of revolution

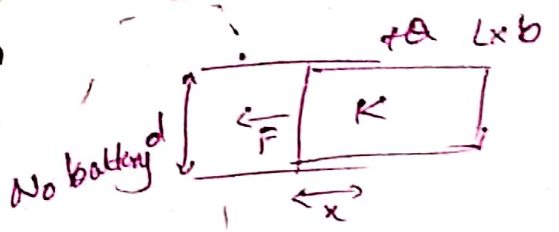
$$T = 4t_0 = 4\sqrt{\frac{2(L-x)}{a}}$$





Scanned with CamScanner

Force acting on dielectric if it is not connected to battery



$C = C_1 + C_2$

$= \frac{\epsilon_0 b}{d}\left(l + (K-1)x\right)$

$\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)$

$F = -\frac{dU}{dx}$

$F = \left|\frac{dU}{dx}\right|$

$F = \left|\frac{d}{dx}(U)\right|$

$F = \left|\frac{d}{dx}\left(\frac{1}{2}CV^2\right)\right|$

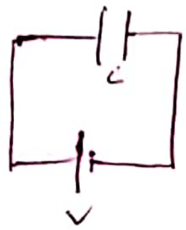
$F = \left(\frac{d}{dx}\left(\frac{Q^2}{2C}\right)\right)$

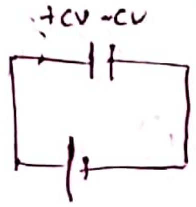
$F = \frac{Q^2}{2\cancel{C}}\left(\frac{1}{C^2}\frac{dC}{dx}\right)$

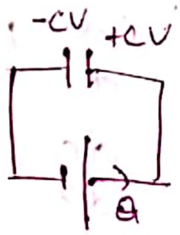
$$\boxed{F = \frac{Q^2}{2C^2}\left[\frac{\epsilon_0 b(K-1)}{d}\right]}$$

$F \propto \frac{1}{C^2}$ is variable

Q) Find the heat produced in the circuit if battery is reverse connected to capacitor







$$Q = CV - (-CV)$$

$$Q = 2CV$$

$$W_b = Q \cdot V$$

$$W_b = 2CV^2$$

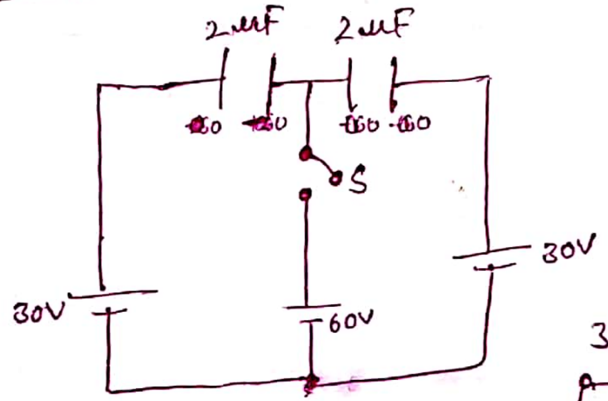
$$W_b = \Delta U + \text{heat lost}$$

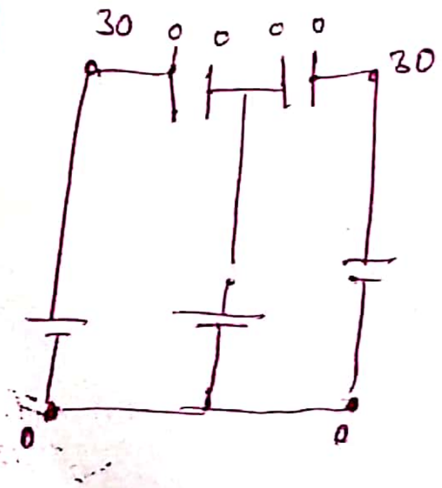
$$\text{heat lost} = W_b - \Delta U$$

$$= 2CV^2 - \left(\tfrac{1}{2}CV^2 - \tfrac{1}{2}CV^2\right)$$

$$\text{heatlost} = 2CV^2$$

Q) Find the heat produced in the circuit if switch is closed.

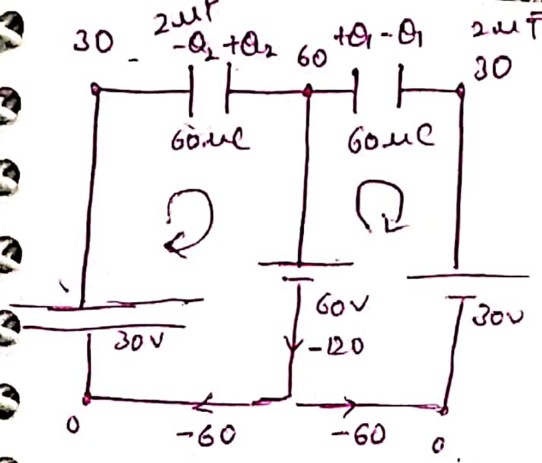




$$\tfrac{1}{2}CV^2$$

$$H = \cancel{2} \times \tfrac{1}{\cancel{2}} \times 2 \times 30 \times 30$$

$$= 1800$$



$$-\frac{Q_1}{2} - 30 + 60 = 0$$

$$\frac{Q_1}{2} = 30$$

$$Q_1 = 60\mu C$$

$$+\frac{Q_2}{2} - 60 + 30 = 0$$

$$Q_2 = 60\mu C$$

$$U_f = \frac{1}{2} \times$$

Work done by 60V battery is $= 120 \times 60$
$= 7200\mu J = 7.2 mJ$

Energy absorbed by the battery of 30V, is $= 60(30)$
$= 1800\mu J$
$= 1.8 mJ$

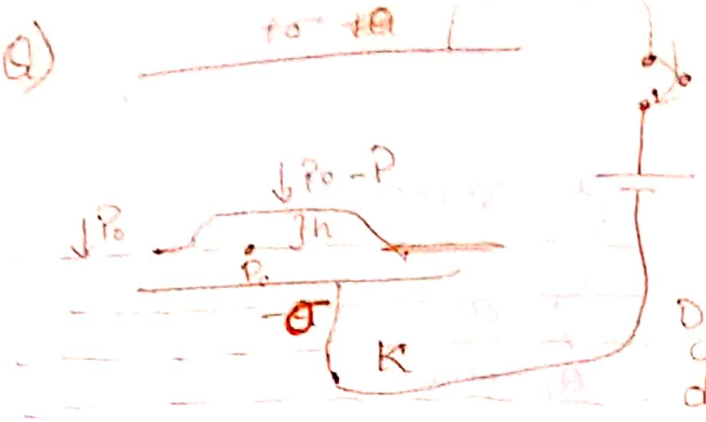
Energy stored in the capacitor $= \left(\frac{Q^2}{2C}\right)2 = \frac{60 \times 60}{2\mu F} = \frac{3600}{2} = 1800\mu J$
$= 1.8 mJ$

W_b by 60 V battery = energy absorbed by 30V battery + energy stored in capacitor + Heat lost

$7.2 = 2(1.8) mJ + 1.8 mJ + Heat$

Heat $= 1.8 mJ$

Q)

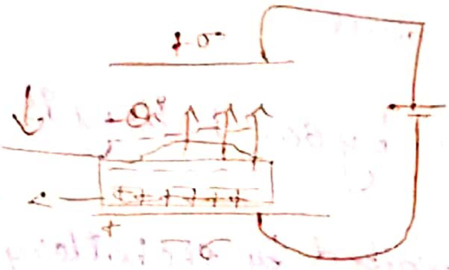


Find the height raised by the dielectric liquid surface.

Dielectric constant-K
density-ρ

$$F = \frac{Q^2}{2A\epsilon_0}\left(1-\frac{1}{K}\right)$$

$\rho g h =$



$$P = \frac{F}{A} = \frac{Q^2}{2A^2\epsilon_0}\left(1-\frac{1}{K}\right)$$

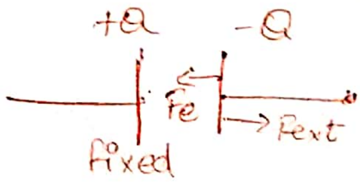
$$P = \frac{\sigma^2}{2\epsilon_0}\left(1-\frac{1}{K}\right)$$

$$\cancel{P_0} = \cancel{P_0} - P_0 + \rho g h$$

$$P = \rho g h$$

$$\rho g h = \frac{\sigma^2}{2\epsilon_0}\left(1-\frac{1}{K}\right) \Rightarrow \boxed{h = \frac{\sigma^2}{2\epsilon_0 \rho g}\left(1-\frac{1}{K}\right)}$$

Q)



Find the work done by external agent in the moving the plate slowly by $\frac{d}{2}$ distance.

$v = 0$

$$W_{ext} = F \times S = \frac{Q^2}{2A\epsilon_0} \cdot \frac{d}{2}$$

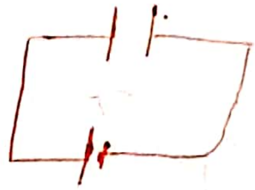
$$\boxed{W_{ext} = \vec{F}\cdot\vec{S} = \frac{Q^2}{4C}}$$

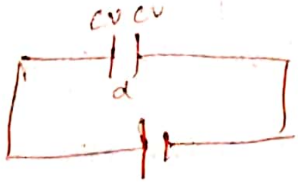
$F_e = F_Q$

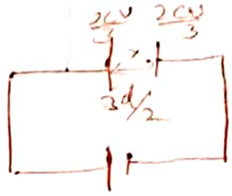
$$F_e = F_Q = \frac{Q^2}{2A\epsilon_0}$$

$F_{ext} = F_e$ (electrostatic force)

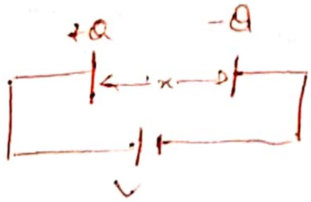
Q) Find the work done by external agent in moving the plate slowly by $\frac{d}{2}$ distance







$C = \frac{A\epsilon_0}{3d}$



$C = \frac{\epsilon_0 A}{x}$

charge on capacitor $= \frac{\epsilon_0 A V}{x}$

$$F = \frac{\epsilon_0^2 A^2 V}{x^2\, 2\epsilon_0 A}$$

$$F = \frac{\epsilon_0 A V}{2x^2} \Rightarrow dW = F_{ext}\, dx = \frac{\epsilon_0 A V^2 dx}{2x^2}$$

$$W_{total} = \int dW_{ext}$$

$$= \int_d^{3d/2} \frac{\epsilon_0 A V^2 dx}{2x^2}$$

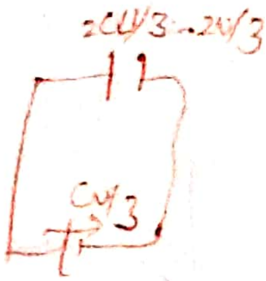
$$= \frac{\epsilon_0 A V^2}{2}\left(-\frac{1}{x}\right)_d^{3d/2}$$

$$= \frac{\epsilon_0 A V^2}{2}\left(\frac{-2}{3d} + \frac{1}{d}\right)$$

$$W_{total} = \frac{\epsilon_0 A V^2}{6d} = \frac{CV^2}{6}$$

Method ②

$W_B + W_{ext} = \Delta U$

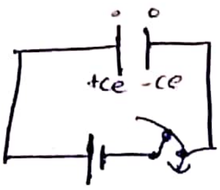


$$-\left(\frac{CV}{3}\right)V + W_{ext} = \frac{1}{2}\frac{2C}{3}V^2 - \frac{1}{2}CV^2$$

$$-\frac{CV^2}{3} + W_{ext} = -\frac{1}{6}CV^2$$

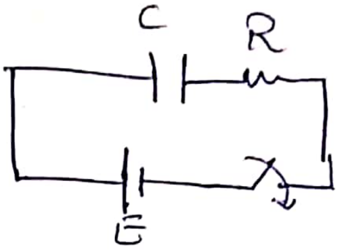
$$W_{ext} = \frac{1}{6}CV^2$$

RC Circuits



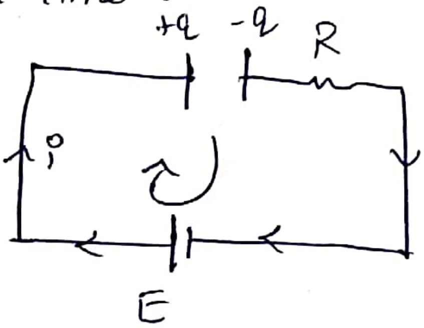
Charge on the capacitor increased from 0 to CE in no time

(time taken is zero)



Time will be taken for the capacitor to gain max. charge

⇒ at time t



$$\frac{-q}{C} - iR + E = 0$$

$$iR = \frac{-q}{C} + E$$

$$\frac{dq}{dt} = \frac{-q}{C} + E$$

$$\frac{CE - q}{C} = \frac{dq}{dt} R$$

$$\int_0^q \frac{dq}{CE - q} = \int_0^t \frac{dt}{RC}$$

$$\left[-\ln(CE - q)\right]_0^q = \frac{t}{RC}$$

$$\ln\left(\frac{CE - q}{CE}\right) = \frac{-t}{RC}$$

$$1 - \frac{q}{CE} = e^{-t/RC}$$

$$\star \boxed{q = CE\left(1 - e^{-t/RC}\right)}$$

$$\frac{q}{CE} = 1 - e^{-t/RC}$$

$$q = q_{max}\left(1 - e^{-t/RC}\right)$$

at $t = \infty$, $q = CE\left(1 - e^{-\infty}\right)$

$$\boxed{q = CE}$$

* Find the current in the circuit.

$$i = \frac{dq}{dt} = CE \cdot e^{-t/RC}$$

$$i = \frac{E}{R} e^{-t/RC}$$

$$\Rightarrow \boxed{i = i_{max}\, e^{-t/RC}}$$

⇒ Voltage across the capacitor (P.D)

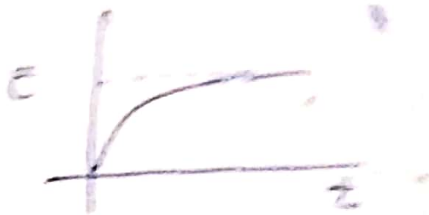
$$V_c = \frac{Q}{C}$$

$$V_c = \frac{CE\left(1 - e^{-t/RC}\right)}{C} \Rightarrow V_c = E\left(1 - e^{-t/RC}\right)$$

⇒ Voltage across the resistor is $V_R = iR$.

$$V = i_{max}\, e^{-t/RC} \cdot R$$

$$\boxed{V_R = E\, e^{-t/RC}}$$

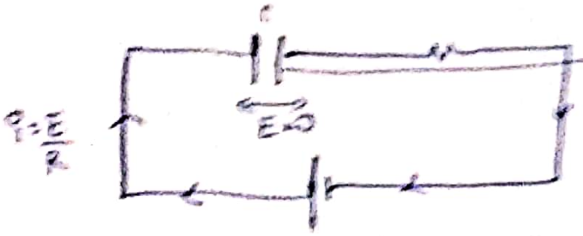


<u>Notes:-</u>

* at $t = 0$ (just after switch is closed)

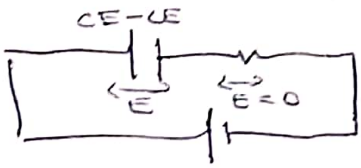
$q = 0$ $\quad$ $i_{max} = E/R$

$V_c = 0$ $\quad$ $V_R = E$



→ capacitor acts as conducting wire. (capacitor can be removed)

* at $t=\infty$ (after long time switch is closed)
(stedystate)



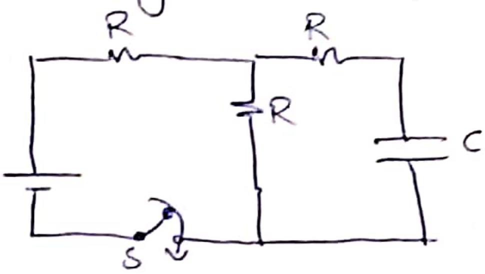
$q = CE$ $i = 0$

$V_c = E$ $V_R = 0$

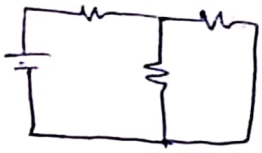
Capacitor acts as insulator as complete charge is filled

* Current in the branch of capacitor is zero.

Q) Find the current supplied by the battery at $t=0$ and $t=\infty$



at $t=0$

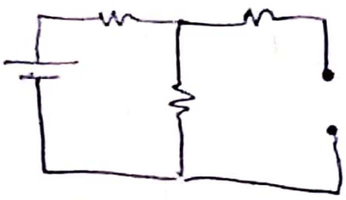


$R_{eff} = 3R/2$

$E = i R_{eff}$

$i = \frac{2E}{3R}$

at $t=\infty$



$R_{eff} = 2R$

$E = i(2R)$

$i = \frac{E}{2R}$

Time constant (τ)

$$q = q_{max}(1 - e^{-t/RC})$$

at $t = RC$

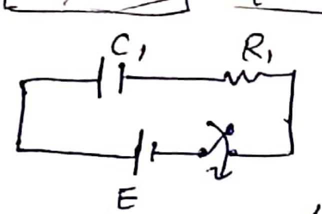
$$q = q_{max}\left(1 - \frac{1}{e}\right)$$

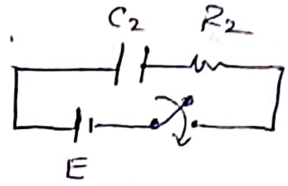
$$q = q_{max}\left(1 - \frac{1}{2.7}\right)$$

$$q = 0.63 q_{max}$$

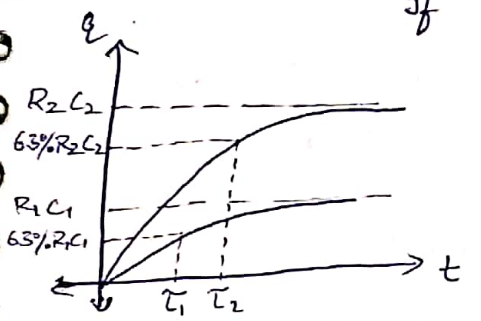
i.e Time constant is the time at which a capacitor achieves 63% of its maximum achievable charge

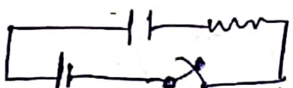
$\cancel{\boxed{t = RC}}$ $\boxed{\tau = RC}$





If $R_1C_1 < R_2C_2$



a)  If switch is closed at $t=0$, find the total heat generated (or) lost through resistor.

$$i = \frac{E}{R} e^{-t/RC}$$

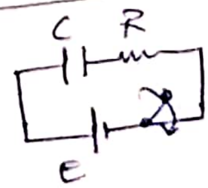
$$H = \int i^2 R \, dt$$

$$= \frac{E^2}{R} \int e^{-2t/RC} dt$$

$$= \frac{RCE^2}{R^2}\left[\frac{e^{-2t/RC}}{-1}\right]_0^{\infty}$$

$$= \frac{RCE^2}{2R}[0+1] = \frac{RCE^2}{R2} = \frac{CE^2}{2}$$

Work done by battery $= (CE)\,E = CE^2$

Q) 

If the switch is closed, find the max rate of energy stored in capacitor

Energy in capacitor $= \frac{1}{2}CU^2 = \frac{Q^2}{2C}$

$$= \frac{(CE)^2(1-e^{-t/RC})^2}{2C}$$

$$U = \frac{1}{2}CE^2(1-e^{-t/RC})^2$$

$$\boxed{\frac{dU}{dt} = \frac{E^2}{R}(1-e^{-t/RC})e^{-t/RC}}$$

$$\frac{dU}{dt} = \frac{1}{2}CE^2\,2(1-e^{-t/RC})\frac{e^{-t/RC}}{RC} = 0$$

$\therefore$ $f(x) = (1-x)x$ will be max at $x = 1/2$

$$\frac{dU}{dt} = \frac{E^2}{R}(e^{-t/RC} - e^{-2t/RC})$$

$$\frac{d^2U}{dt^2} = \frac{E^2}{R}\left(\frac{-e^{-t/RC}}{RC} + \frac{2e^{-2t/RC}}{RC}\right)$$

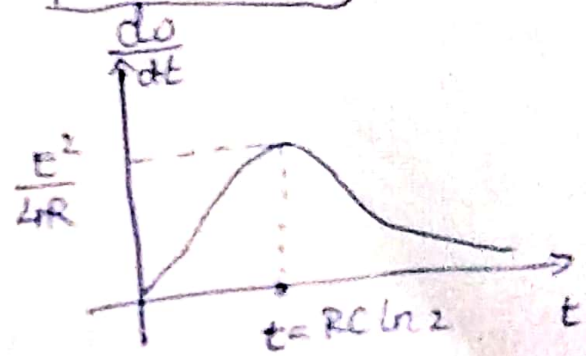
$$= \frac{E^2}{R^2C}(2e^{-2t/RC} - e^{-t/RC}) = 0$$

$$2e^{-2t/RC} = e^{-t/RC}$$

$$e^{-t/RC} = 1/2$$

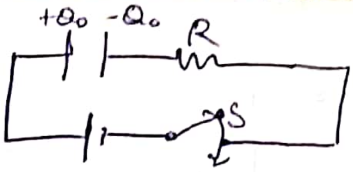
$$\left(\frac{dU}{dt}\right)_{max} = \frac{E^2}{R}\left(1-\frac{1}{2}\right)\frac{1}{2}$$

$$\boxed{\left(\frac{dU}{dt}\right)_{max} = \frac{E^2}{4R}}$$

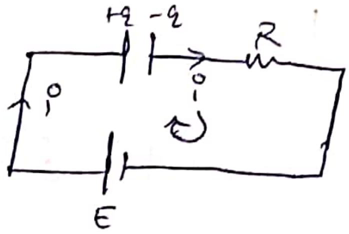


max

Q)



If switch is closed at $t=0$, find the charge on the capacitor as a function of time.



$$\frac{-q}{C} - iR + E = 0$$

$$\frac{CE-q}{C} = \frac{dq}{dt}R$$

$$\int \frac{dq}{CE-q} = \int \frac{dt}{RC}$$

$$\left[\frac{\ln(CE-q)}{-1}\right]_{Q_0}^{q} = \frac{t}{RC}$$

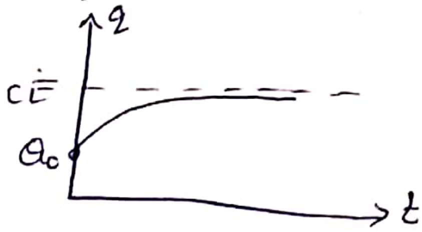
$$\ln\left(\frac{CE-q}{CE-Q_0}\right) = \frac{t}{RC}$$

$$CE-q = (CE-Q_0)e^{-t/RC}$$

$$q = CE-(CE-Q_0)e^{-t/RC}$$

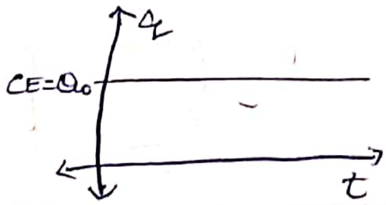
$$\boxed{q = CE-(CE-Q_0)e^{-t/RC}}$$

If $Q_0 < CE$

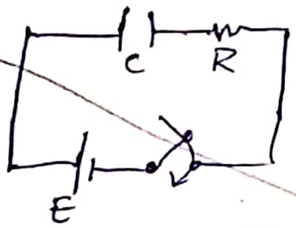


If $Q_0 > CE$

if $Q = CE$
q is constant

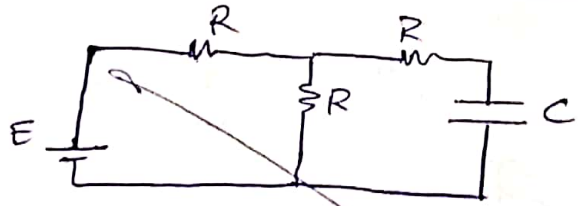


Note:



Time constant $= \tau = RC$

$$q = q_{max}(1-e^{-t/RC})$$



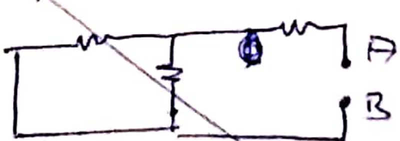
Time constant of circuit

(1) Short circuit the battery

$$R_{net} = \frac{3R}{2}$$

(2) Open the capacitor

$$\tau = \frac{C3R}{2}$$



(3) Find R_{net} across capacitor terminal

(4) $\tau = CR_{net}$

Time constant = $\tau = RC$

$q = q_{max}\left(1 - e^{-t/RC}\right)$

$$\boxed{q = q_{max}\left(1 - e^{-t/\tau}\right)}$$

Time constant of circuit

(1) Short circuit the battery

(2) Open the capacitor

(3) Find the R_{net} across the capacitor terminal

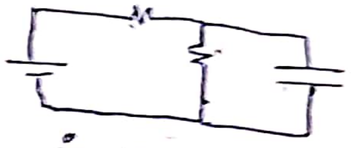
$R_{net} = \frac{3R}{2}$

(4) $\tau = C R_{net}$

$\tau = \frac{C3R}{2}$

$q = q_{max}\left(1 - e^{-t/\tau}\right) = q_{max}\left(1 - e^{\frac{-2t}{3RC}}\right)$

For finding q_{max} on capacitor i.e at $t = \infty$

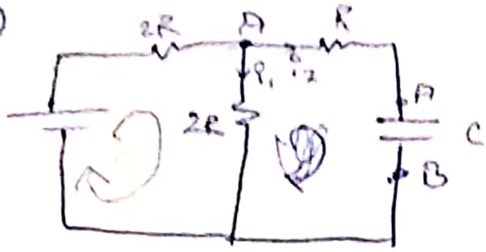


$\therefore\ q_{max} = \frac{CE}{2}$

$$\boxed{q = \frac{CE}{2}\left(1 - e^{\frac{-2t}{3RC}}\right)}$$

Finding max charge on capacitor

Q)



$R_{net\,AB} = 2R$

$$q = q_{max}\left(1 - e^{-t/2RC}\right)$$

$$i_2 = i_{max}\left(e^{-t/2RC}\right)$$

$$i_2 = \frac{E}{4R}\left(e^{-t/2RC}\right)$$

$$-i_1 \times 2R + E - i_1 2R - i_2 2R = 0$$

$$E = i_1 4R + i_2 2R$$

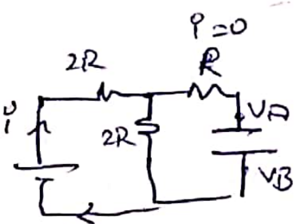
$$i_1 = \frac{E}{4R} + \frac{i_2}{2}$$

$$= \frac{E}{4R} + \frac{E}{8R}\left(e^{-t/2RC}\right)$$

$$\boxed{i_1 = \frac{E}{8R}\left(2 + e^{-t/2RC}\right)}$$

[(or)]

charge on capacitor $q = q_{max}\left(1 - e^{-t/2RC}\right)$



$$i_2 = \frac{dq}{dt} = \frac{CE}{2}\left(-\left(\frac{-1}{2RC}\right)e^{-t/2RC}\right)$$

$$i_2 = \frac{E}{4R} e^{-t/2RC}$$

$i = \frac{E}{4R}$

$V_A - V_B = i(2R)$

$= \frac{E(2R)}{4R}$

$V_A - V_B = \frac{E}{2}$

$$-i_1 R - \frac{q}{C} + i_2(2R) = 0$$

$$i_2(2R) = i_1 R + \frac{q}{C}$$

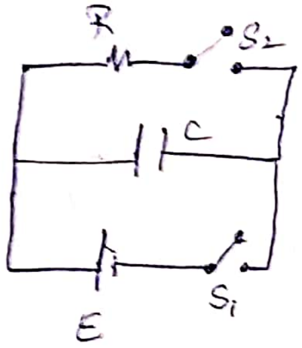
$$i_2(2R) = \frac{E}{4} e^{-t/2RC} + \frac{E}{2}\left(1 - e^{-t/2RC}\right)$$

$$i_2 = \frac{E}{8R}\left(2 - e^{-t/RC}\right)$$

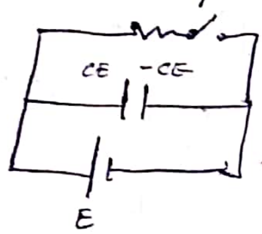
Current from battery $i = i_1 + i_2$

$$= \frac{E}{4R} e^{-t/2RC} + \frac{E}{8R}\left(2 - e^{-t/2RC}\right) = \frac{E}{8R}\left(2 + e^{-t/2RC}\right)$$

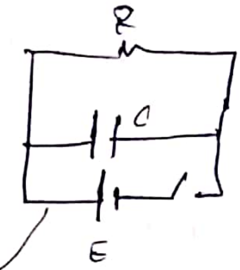
Q) Discharging of capacitor

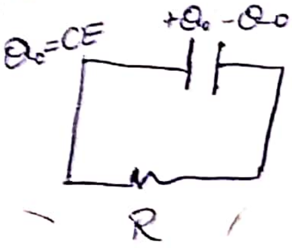


If S_1 is closed and S_2 is open

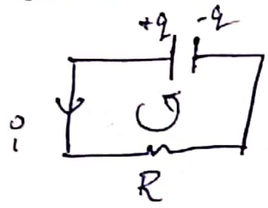


If S_2 is closed and S_1 is open





at $t = t$



$$-iR + \frac{q}{C} = 0$$

$$\frac{q}{C} = iR$$

$$\frac{q}{C} = -\frac{dq}{dt} R$$

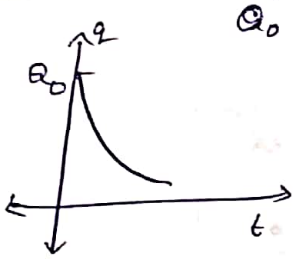
$$\int_{Q_0}^{q} \frac{dq}{q} = \int_0^t \frac{-dt}{RC}$$

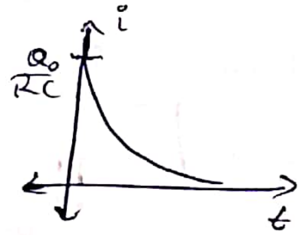
$$(\ln q)_{Q_0}^{q} = \frac{-t}{RC}$$

$$\ln\left(\frac{q}{Q_0}\right) = \frac{-t}{RC}$$

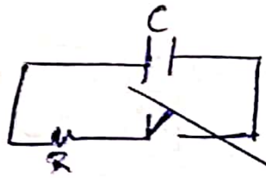
$$\boxed{q = Q_0 e^{-t/RC}}$$

$$i = \frac{dq}{dt} \Rightarrow \boxed{i = \frac{Q_0}{RC} e^{-t/RC}}$$





Q)



$$dH = \int i^2 R\, dt$$

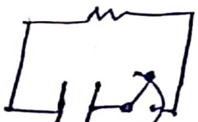
$$\int_0^H dH = \int_0^t \frac{Q_0}{RC} R e^{-t/RC}\, dt$$

$$= \left[-\frac{Q_0}{C} \cdot e^{-t/RC} \cdot RC\right]$$

$$H = Q_0 R e^{-t/RC} - Q_0 R$$

$$\boxed{H = Q_0 R\left(e^{-t/RC} - 1\right)}$$

Q) Find the heat lost through resistor.



Current $= i = \frac{Q}{RC} e^{-t/RC}$

~~$dH = i$~~ $H = \int i^2 R \, dt$

$$H = \int \frac{Q^2 R}{R^2 C^2} e^{-2t/RC} \, dt$$

$$H = \frac{Q^2}{RC^2} \int_0^\infty e^{-2t/RC} \cdot dt$$

$$H = \frac{Q^2}{RC^2} \left(\frac{e^{-2t/RC}}{-2/RC} \right)_0^\infty$$

$$= \frac{Q^2}{RC^2} \left(-\frac{RC}{2} \right) \left(e^{-\infty} - e^0 \right)$$

$$H = \frac{-Q_0^2}{2C} (0 - 1)$$

$$\boxed{H = \frac{Q_0^2}{2C}}$$

Energy stored in capacitor would be lost through resistor in discharging circuit

Heat lost = change in energy of capacitor

$$= \frac{Q_0^2}{2C} - 0$$

$$= \frac{Q_0^2}{2C}$$

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