

HEAT & THERMODYNAMICS :-

①

1. THERMAL EXPANSION :-

On heating, matter expands.

SOLID EXPANSION

Linear expansion

$$l_0 \text{ at temp } t$$

$$l \text{ at temp } t$$

$$\text{change in length} \propto \Delta T \\ (l - l_0)$$

$$l - l_0 \propto l_0$$

$$l - l_0 = \alpha \cdot l_0 \cdot \Delta T$$

Linear expansion.

$$l = l_0 (1 + \alpha \Delta T)$$

$$\text{unit of } \alpha \rightarrow \alpha = \frac{l - l_0}{l_0 \Delta T}$$

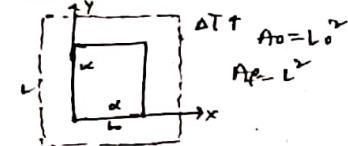
$$\alpha \rightarrow {}^\circ C^{-1}, K^{-1}, {}^\circ F^{-1}$$

These formulae are valid for small temp. rises

Areal expansion / superficial exp.

Sq. plate
at temp t

i) Isotropic material:



Sq. plate remains as square.

$$\Delta A = [l_0(1 + \alpha \Delta T)]^2$$

$$= l_0^2 (1 + \alpha \Delta T)^2 \\ = l_0^2 (1 + 2\alpha \Delta T)$$

$$\Delta A = A_0 (1 + \beta \Delta T) \quad \text{coeff. of superficial expansion.}$$

$\beta = 2\alpha$

$\beta = 2\alpha$ coeff. of superficial expansion.

$$V_f = V_0 (1 + (\alpha_1 + \alpha_2 + \alpha_3) \Delta T)$$

$$l_1 = l_0 (1 + \alpha_1 \Delta T)$$

$$l_2 = l_0 (1 + \alpha_2 \Delta T)$$

$$l_3 = l_0 (1 + \alpha_3 \Delta T)$$

$$V_f = V_0 l_1 l_2 l_3$$

$$V_f = V_0 (1 + \gamma \Delta T)$$

$\gamma = \alpha_1 + \alpha_2 + \alpha_3$
coeff. of cubical expansion.

$$\gamma = \alpha_1 + \alpha_2 + \alpha_3$$

If material is isotropic

$$\gamma = 3\alpha$$



Square becomes rectangular plate

$$A_0 = l_0^2$$

$$A_f = l_1 l_2$$

$$\Delta A = l_0(l_0 + \alpha_1 \Delta T) \cdot l_0(l_0 + \alpha_2 \Delta T) \\ = A_0 (1 + \alpha_1 \Delta T)(1 + \alpha_2 \Delta T)$$

$$\Delta A = A_0 (1 + \alpha \Delta T)$$

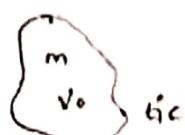
$$\alpha = \alpha_1 + \alpha_2$$

$$\beta = \alpha_1 + \alpha_2$$

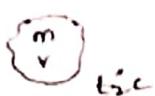
Anisotropic material: If material properties are different in diff. directions, then material is called as anisotropic.

Isotropic material: If material properties are same in all directions, material is called as isotropic.

Variation of density with temperature :-



initial density (ρ_0) = $\frac{m}{V_0}$ at temp. t_0



Final density (ρ) = $\frac{m}{V}$ at temp. t

$$\rho = \frac{m}{V_0(1+\alpha\Delta T)}$$

$$\rho = \frac{\rho_0}{1+\alpha\Delta T}$$

$$\rho = \rho_0(1-\alpha\Delta T)$$

ne/
exp

T^2
 \propto

cuboid

Variation of time period of simple pendulum:

$$T_0 = 2\pi \sqrt{\frac{l_0}{g}}$$

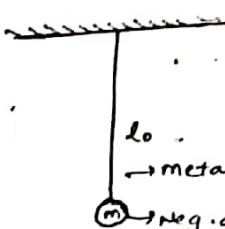
$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$= 2\pi \sqrt{\frac{l_0(1+\alpha\Delta T)}{g}} = T_0 (1+\alpha\Delta T)^{1/2}$$

$$T = T_0(1 + \frac{\alpha\Delta T}{2})$$

$$\frac{T - T_0}{T_0} = \left(\frac{\alpha}{2}\right) \Delta T$$

Fractional change in time period



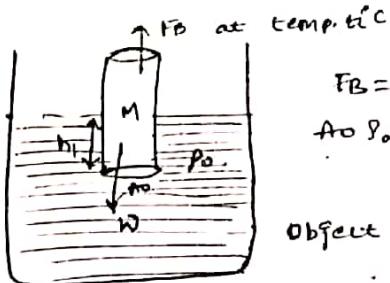
i) temp ↑, length ↑
Time period ↑
(one oscillation takes more than 2 seconds)

watch runs slow
(it loses time.)

ii) temp ↑, length ↓, Time period
(one oscillation takes less than 2 sec)
watch runs faster (it gains time.)

loss (or) gain of time per sec. = $\frac{1}{2} \times \alpha \times \Delta T$
loss (or) gain of time per day = $\frac{1}{2} \times \Delta T \times 86,400$

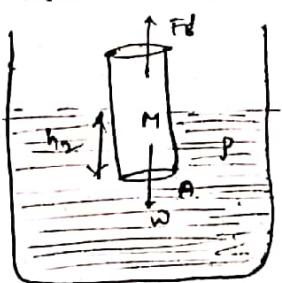
Variation of Force of Buoyancy with temperature:



$$F_B = W$$

$$\text{And } \rho_0 g h_1 g = Mg$$

Object floats.



at temp. t
still object floats

$$W = F_B$$

$$W = (\rho h_2) g l$$

$$W = A_0 (1 + \beta_S \Delta T) h_2$$

$$\Rightarrow W = A h_2 \rho g \left[\frac{1 + \beta_S \Delta T}{1 + \beta_L \Delta T} \right]$$

$$\Rightarrow W = A h_2 \rho g \left[\frac{1 + \beta_S \Delta T}{1 + \beta_L \Delta T} \right]$$

$$\rho_1 h_1 \gamma_1 g = \rho_2 h_2 \gamma_2 g \left[\frac{1 + \beta_s \Delta T}{1 + \gamma_s \Delta T} \right]$$

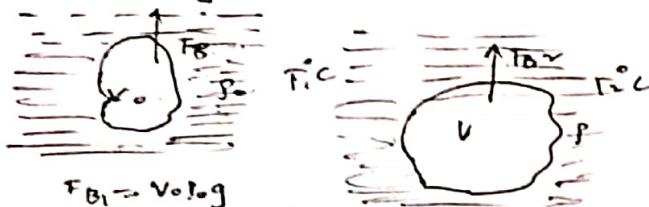
(3)

$$h_2 = h_1 \left[\frac{1 + \gamma_s \Delta T}{1 + \beta_s \Delta T} \right]$$

- 1F $\gamma_s - \beta_s = 2\ell \Rightarrow h_2 = h_1$
- 2F $\gamma_s > \beta_s \Rightarrow h_2 > h_1$
- 3C $\gamma_s < \beta_s \Rightarrow h_2 < h_1$

temp. T

\rightarrow If total object is there inside the fluid (liquid)



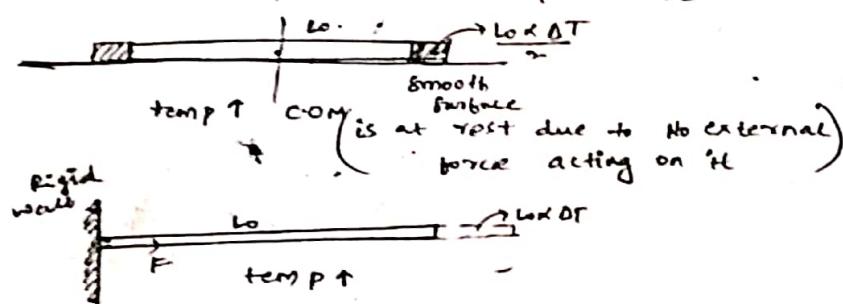
$$F_{B2} = \rho_f g = V_0 (1 + \beta_s \Delta T) \left(\frac{10}{1 + \gamma_s \Delta T} \right) g$$

$$F_{B2} = V_0 \rho_0 g \left(\frac{1 + \gamma_s \Delta T}{1 + \beta_s \Delta T} \right)$$

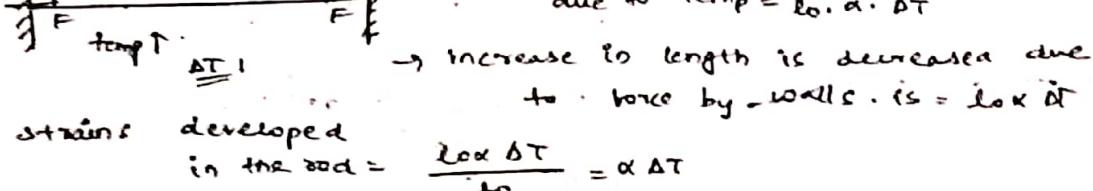
$$F_{B2} = F_{B1} \left(\frac{1 + \beta_s \Delta T}{1 + \gamma_s \Delta T} \right)$$

THERMAL STRESS :-

\rightarrow Stress developed due to the temp. changes.



increase (decrease) in length of rod
due to temp. = $L_0 \cdot \alpha \cdot \Delta T$



→ Increase in length is decreased due to force by walls. i.e. = $L_0 \alpha \Delta T$

strains developed

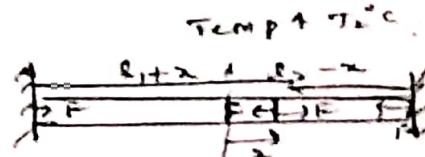
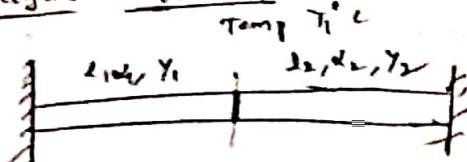
$$\text{in the rod} = \frac{L_0 \alpha \Delta T}{L_0} = \alpha \Delta T$$

Thermal stress developed in the rod = $\alpha \Delta T (\gamma)$

Thermal force acting on the rod by the walls = $F \times (\text{stress}) A$

$$F = YA \alpha (\Delta T)$$

Thermal stress when a node are there there in between 2 rigid supports:-



(3)

$$\epsilon_1 = \alpha_1 (1 + \alpha_1 \Delta T)$$

$$\epsilon_2 = \alpha_2 (1 + \alpha_2 \Delta T).$$

Strain developed in the 1st rod = $\frac{\epsilon_1 (L_1 \times 10^3) - (L_1 + x)}{L_1} = \frac{\alpha_1 \epsilon_1 \Delta T - x}{L_1}$

Strain developed in the 2nd rod = $\frac{\epsilon_2 (L_2 \times 10^3) - (L_2 - x)}{L_2} = \frac{\alpha_2 \epsilon_2 \Delta T + x}{L_2}$

Thermal force developed in the both rods should be equal.

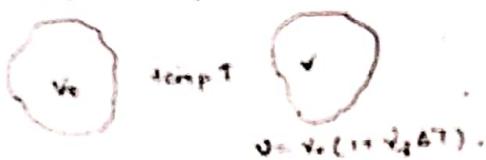
$$(\text{stress})_1 \cdot A_1 = (\text{stress})_2 \cdot A_2$$

$$\nu_1 (\text{strain}) = \nu_2 (\text{strain})$$

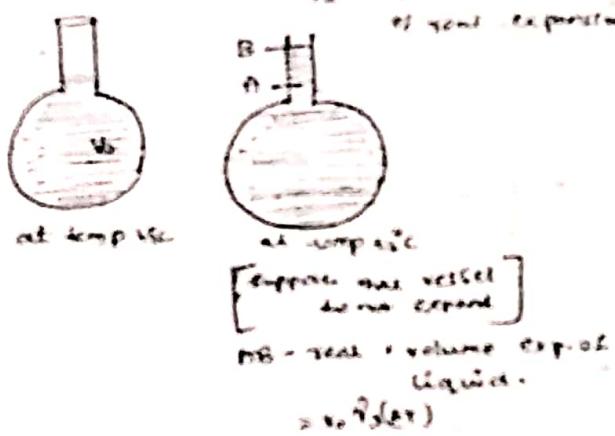
$$\nu_1 \left(\frac{\alpha_1 \epsilon_1 \Delta T - x}{L_1} \right) = \nu_2 \left(\frac{\alpha_2 \epsilon_2 \Delta T + x}{L_2} \right)$$

Apparent expansion of liquids:

Real exp. of liquid.

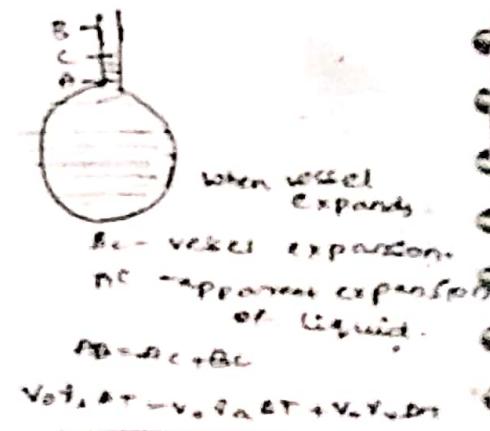


v_0 = real vessel
 v = apparent expansion



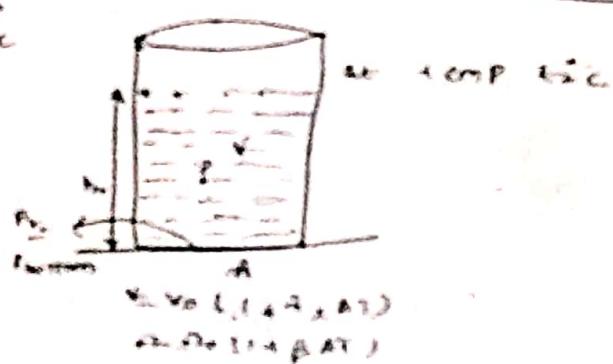
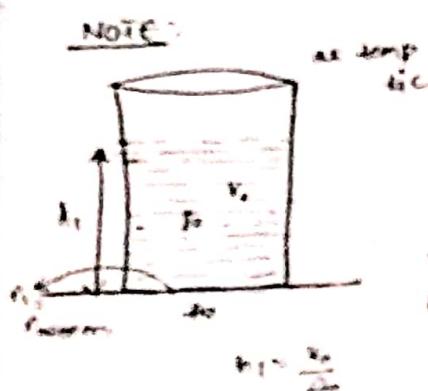
$$v_0 = \text{real} + \text{volume exp. of liquid.}$$

$$= v_0 + \alpha_1 \Delta T$$



$$v_0 = v_0 + \alpha_1 \Delta T$$

$$v_0 + v_0 + \alpha_1 \Delta T = v_0 + v_0$$



$$v_0 + \alpha_1 \Delta T = v_0 + \frac{v_0}{20} + \alpha_1 \Delta T$$

$$h_2 = \frac{V}{\rho} = \frac{\rho_0}{\rho_0} (1 + \alpha_1 \Delta T)$$

$$\Rightarrow h_2 = h_1 \frac{(1 + \alpha_1 \Delta T)}{(1 + \beta_0 \Delta T)}$$

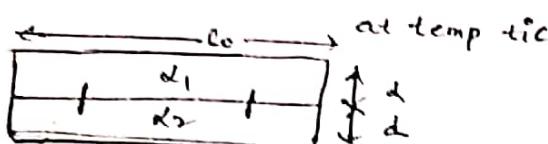
$P_1 = \rho_0 g h_1 \quad P_2 = \rho_0 g h_2 \quad \left(\frac{P_2}{P_1} \right) = \left(\frac{1 + \alpha_1 \Delta T}{1 + \beta_0 \Delta T} \right) \Rightarrow r = \left[\frac{P_2}{P_1} - \frac{1}{1 + \beta_0 \Delta T} \right]$

M-II

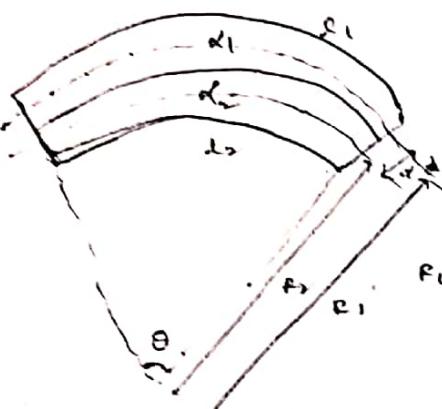
$$P_1 = \frac{mg}{A} \quad P_2 = \frac{mg}{A} \frac{(1 + \alpha_1 \Delta T)}{(1 + \beta_0 \Delta T)}$$

Bi-metallic

Bi-metallic strip:



If $\alpha_1 > \alpha_2$ and temp t



$$\Rightarrow R_{avg} = \frac{L_2 + (L_1 + \alpha_2 \Delta t) d}{2(\alpha_1 - \alpha_2) \Delta t}$$

$$R_{avg} = \frac{R \cdot d}{2(\alpha_1 - \alpha_2) \Delta t} \cdot \frac{d}{(\alpha_1 - \alpha_2) \Delta t}$$

$$L_1 = P_1 \cdot \theta = L_0 (1 + \alpha_1 \Delta T)$$

$$L_2 = P_2 \cdot \theta = L_0 (1 + \alpha_2 \Delta T)$$

$$(P_1 - P_2) \theta = \Delta \theta \Delta T (\alpha_1 - \alpha_2)$$

$$\theta = \frac{\Delta \theta \Delta T (\alpha_1 - \alpha_2)}{d}$$

Rad. of
bi-metallic strip

$$\begin{aligned} R_{avg} &= \frac{P_1 + P_2}{2} = \frac{(P_1 + P_2) \theta}{2 \Delta T} \\ &= \frac{L_0 (2 + (\alpha_1 + \alpha_2) \Delta t)}{2 \Delta T} \\ &= \frac{L_0 (2 + (\alpha_1 + \alpha_2) \Delta t)}{2 \cdot 9.8 (1 + \alpha_1 \Delta t) \Delta t} \cdot d \end{aligned}$$

Calorimetry:



ΔT (increase in
temp.)

dQ (heat energy supplied)

$$dQ \propto \Delta T \quad \text{Solid/Liquid.}$$

$$dQ \propto m \Rightarrow dQ = m \cdot c \cdot (\Delta T)$$

Specific heat capacity

(3)

$$\begin{aligned} \epsilon_1 &= \alpha_1(1 + \Delta T) \\ \epsilon_{12} &= \alpha_2(1 + \Delta_2 \Delta T). \end{aligned}$$

Strain developed in the 1st rod = $\frac{\epsilon_1(1 + \Delta_1 \Delta T) - (\epsilon_1 + \alpha)}{\epsilon_1} = \frac{\alpha_1 \Delta_1 \Delta T - \alpha}{\alpha_1}$

strain developed in the 2nd rod = $\frac{\epsilon_2(1 + \alpha_2 \Delta T) - (\epsilon_2 + \alpha)}{\epsilon_2} = \frac{\alpha_2 \Delta_2 \Delta T + \alpha}{\alpha_2}$

thermal stress developed in the both rods should be equal.

$$(\text{stress})_1 A_1 = (\text{stress})_2 A_2$$

$$\gamma_1 \cdot (\text{strain}) = \gamma_2 \cdot (\text{strain}) \alpha$$

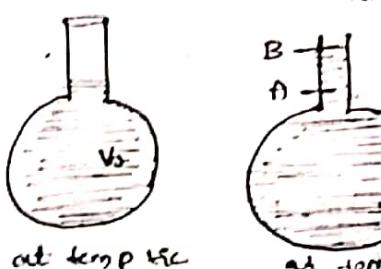
$$\boxed{\gamma_1 \left(\frac{\alpha_1 \Delta_1 \Delta T - \alpha}{\alpha_1} \right) = \gamma_2 \left(\frac{\alpha_2 \Delta_2 \Delta T + \alpha}{\alpha_2} \right)}$$

Apparent expansion of liquids:

Real exp. of liquid.

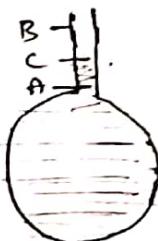
$$V = V_0(1 + \gamma_a \Delta T).$$

γ_a = real coeff. of real-expansion.



[Suppose that vessel do not expand]

$$\begin{aligned} \gamma_{AB} &= \text{real + volume exp. of liquid.} \\ &= V_0 \gamma_a (\Delta T) \end{aligned}$$



when vessel expands

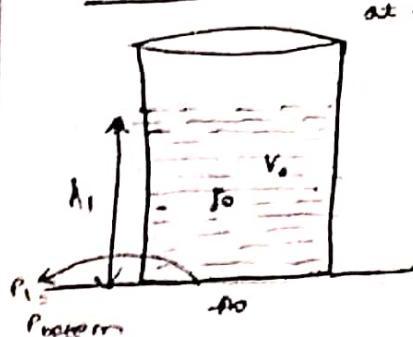
γ_{AC} = vessel expansion.
 γ_{BC} = apparent expansion of liquid.

$$\gamma_{AB} = \gamma_{AC} + \gamma_{BC}$$

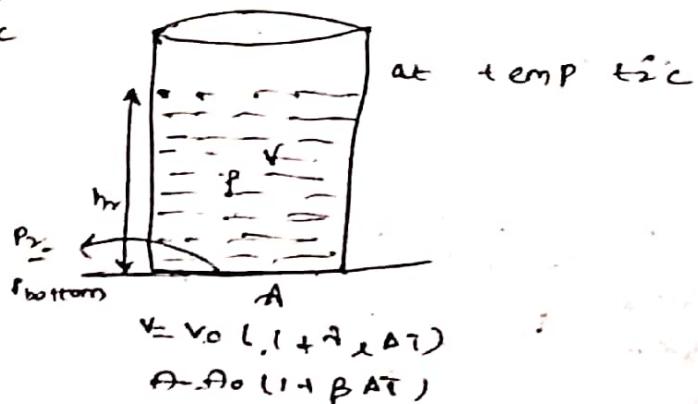
$$V_0 \gamma_a \Delta T = V_0 \gamma_a \Delta T + V_0 \gamma_{BC} \Delta T$$

$$\boxed{\gamma_{BC} = \gamma_a + \gamma_v = \gamma_a + \beta \Delta T}$$

NOTE:



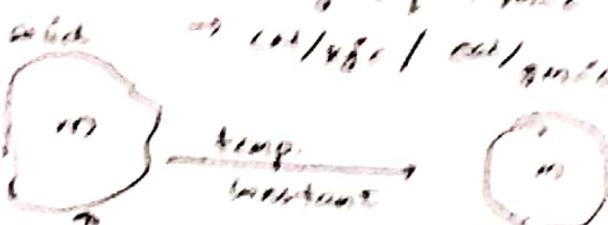
at temp t_1 C



SL (Heat) \rightarrow $\frac{10}{m \cdot T}$

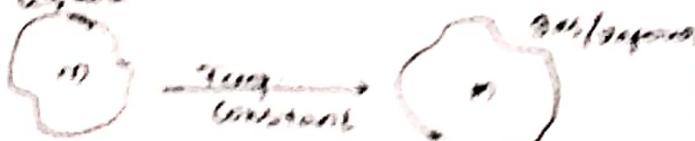
\Rightarrow $3/2 \text{ gms} / 3/2 \text{ molar}$

\Rightarrow $10/8/2 / 10/1 \text{ molar}$



ΔH \rightarrow at melting point

liquified



ΔH \rightarrow at boiling point

10 J/mole

$3/2 \times 10/1$

Measure heat of fusion

10 J/mole \rightarrow measure heat of vaporization

Principle of Calorimetry

\rightarrow When some objects at initial temperatures are kept in contact, then after some time, they attain same temperature
heat lost by heat gained by
hot objects \rightarrow cool objects

Example:



heat lost by \rightarrow heat gained by

$m_1 \cdot C_1 \cdot (T_1 - t) = m_2 \cdot C_2 \cdot (t - T_2)$

$$T = \frac{m_1 C_1 m_2 C_2 T_2 + m_2 C_2 m_1 T_1}{m_1 C_1 + m_2 C_2}$$

2 gm
 10°C

4 gm
 0°C

$$\begin{aligned} Q_1 &= m \cdot S \cdot \Delta T \\ &= (2)(\frac{1}{2})(10) \\ &= 10 \text{ cal.} \end{aligned}$$

$$\begin{aligned} Q_2 &= m \cdot S \cdot \Delta T \\ &= (4)(\frac{1}{2})(10) \\ &= 20 \text{ cal.} \end{aligned}$$

$$\Delta Q = m \cdot S \cdot \Delta T$$

$$\Delta Q = m \cdot S \cdot (T_f - T_i)$$

$$\Delta Q = m \cdot S \cdot (T_f - T_i)$$

$$\frac{\Delta Q}{m \cdot S} = T_f + 10$$

$$T_f = \frac{\Delta Q}{m \cdot S} - 10$$

$$T_f = m \cdot S + 10$$

$$T_f = m \cdot S + 10$$

2 gm
 10°C

$$\begin{aligned} Q_3 &= m \cdot S \cdot \Delta T \\ &= (2)(\frac{1}{2})(10) \\ &= 10 \text{ cal.} \end{aligned}$$

4 gm
 10°C

$$\begin{aligned} Q_4 &= m \cdot S \cdot \Delta T \\ &= (4)(\frac{1}{2})(10) \\ &= 20 \text{ cal.} \end{aligned}$$

$Q_5 = m \cdot S \cdot \Delta T$

$$\begin{aligned} Q_5 &= m \cdot S \cdot \Delta T \\ &= (2)(\frac{1}{2})(10) \\ &= 10 \text{ cal.} \end{aligned}$$

temp. T

10°C

0°C

-10°C

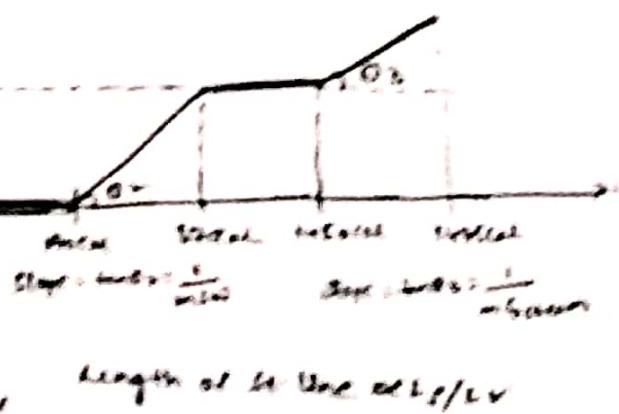
-20°C

-30°C

-40°C

-50°C

-60°C



Curious case of water (4th pt)

$m_1 \text{ steam} + \frac{1}{m_1} \text{ ice} \rightarrow \frac{1}{m_1} \text{ mixture}$
 $m_2 \text{ water} + m_3 \text{ ice} \rightarrow \text{mixture}$

$$\boxed{Q_1 = Q_2 = Q_3}$$

Water and Ice mix :-



Q_1 = heat required to take temp. to 0°C Q_2 = heat released when water comes to 0°C.

$Q_1 = m_1 s_w (T_2 - 0^\circ) = m_1 s_w T_2$

Q_3 = heat reqd to melt the ice comp. Q_3 = heat released when water changes to ice. $T_{\text{ice}} = 0^\circ$

\Rightarrow If $Q_1 = Q_3$, system attain equilibrium. temp of 0°C
 ice and water remains in same phase.

\Rightarrow If $Q_1 < Q_3$, final state, ice may exist

\Rightarrow If $Q_1 > Q_3$ & $Q_1 = Q_3 + Q_3$, system has total ice at 0°C.

\Rightarrow If $Q_1 > Q_3$ & $Q_1 > Q_3 + Q_3$, system has water+ice at 0°C.

\Rightarrow If $Q_1 > Q_3$ & $Q_1 > Q_3 + Q_3$ system will have complete ice at -ve temperature.

\Rightarrow If $Q_1 < Q_3$, $Q_3 = Q_1 + Q_4$, system will be complete water at 0°C.

\Rightarrow If $Q_1 < Q_3$, $Q_3 > Q_1 + Q_4$, system will have complete water at $T > 0^\circ$.

$$Q_3 = (Q_1 + Q_4) = (m_1 + m_2) s_w (T_2)$$

$$\Rightarrow \boxed{T = \frac{Q_3 (Q_1 + Q_4)}{(m_1 + m_2) s_w}}$$

\Rightarrow If $Q_1 < Q_3$ & $Q_3 < Q_1 + Q_4$, system will have ice+water at 0°C

$$Q_3 - Q_1 = [\text{Mixture}]_{\text{ice}}$$

$$\boxed{\text{Mass lost} = \frac{Q_3 - Q_1}{L_f}}$$

$$\text{mass of water left} = m_2 + M$$

$$\text{mass of ice left} = m_1 - M$$

Q) If 2gm of ice at 0°C is mixed with 2gms of water at 40°C, then find the amount of ice left in the after equilibrium.

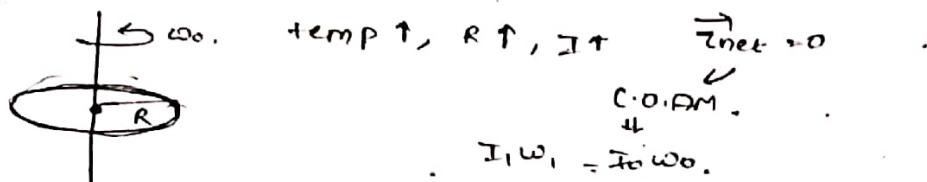
Sol.: The heat released when water \rightleftharpoons frozen water at 0°C .
is $(2)(17)(40) = 80 \text{ cal}$.

$$80 \text{ cal} = (\text{mice melted})(80) (\text{as})$$

$$\text{mice melted} = 1 \text{ gm}$$

\Rightarrow 3gm's of water & 1gm of ice left. at 0°C .

Angular velocity:



$$\frac{2}{3} m k^2 (\omega_0) = m k^2 (1 + 2\alpha \Delta T) \omega_4$$

$$\boxed{\omega = \frac{\omega_0}{1 + 2\alpha \Delta T}}$$

f) If α = constant and length of rod at temperature T_1 is l_0 . find the length of rod at temp T_2 ($T_2 \gg T_1$)

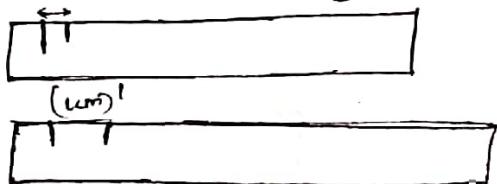
Sol: $de = \alpha \cdot k \cdot dT$

$$\int_{l_0}^l \frac{de}{e} = \alpha \int_{T_1}^{T_2} dT \Rightarrow \log\left(\frac{l}{l_0}\right) = \alpha \cdot (T_2 - T_1)$$

$$\therefore \boxed{l = l_0 \cdot e^{\alpha(T_2 - T_1)}}$$

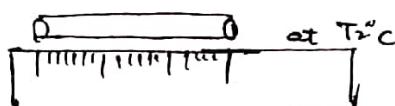
READING MY METAL SCALES:-

1cm at T_1 (calibration temperature)



$$(1\text{cm})' = 1\text{cm} (1 + \alpha_{scale} \Delta T)$$

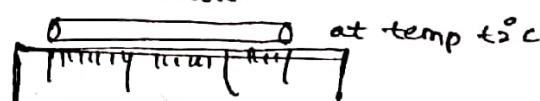
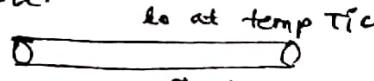
i) finding the length of wooden rod



$$\text{Reading} = \frac{l_0}{(1\text{cm})'} = \frac{l_0}{(1 + \alpha_{scale} \Delta T)}$$

$$\text{Reading} = \frac{l_0}{(1\text{cm})'(1 + \alpha_{scale} \Delta T)}$$

ii) find the reading of metal rod.



$$\text{Reading} = \frac{l}{(1\text{cm})'} = \frac{l_0(1 + \alpha_{rod}(T_2 - T_1))}{(1\text{cm})(1 + \alpha_{scale}(T_2 - T_1))}$$

$$\text{Reading} = \frac{\alpha_0 (H \times \gamma_0 (T_2 - T_1))}{(\text{length}) \times \alpha_{\text{scale}} (\gamma_2 - \gamma_0)}$$

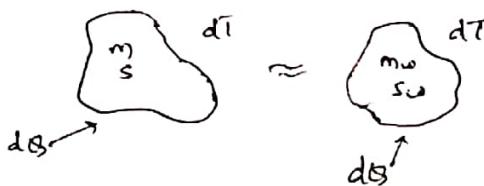
(9)

$T_2 \rightarrow$ temp. at which measurement is done

$T_1 \rightarrow$ temp. at which rod has length ℓ_0 .

$\gamma_0 \rightarrow$ calibration temperature.

Water Equivalent:-



$$Q_1 \quad m_s(dT) = m_w s_w (dT) \Rightarrow \boxed{m_w = \frac{m_s}{s_w}}$$

Temperature Scales:-

$$Q_1 \quad \begin{array}{ccc} \text{Melting point} & {}^{\circ}\text{C} & {}^{\circ}\text{F} \\ \text{of ice} & 0^{\circ}\text{C} & 32^{\circ}\text{F} \\ (\text{LFP}) & \text{100 parts} & \text{180 parts} \\ & R_1 & R_2 \end{array}$$

$$\begin{array}{ccc} & & 273 \text{ K} \\ & & x \\ & & \text{100 parts} \\ & & R_3 \\ \text{Boiling point} & 100^{\circ}\text{C} & 212^{\circ}\text{F} \\ \text{of water} & & \\ (\text{BFP}) & & 373 \text{ K} \\ & & y \end{array}$$

$$\delta^{\circ}\text{C} = \delta\text{K}$$

$$100 \text{ parts of } {}^{\circ}\text{C} = 180 \text{ parts of } {}^{\circ}\text{F}.$$

$$\frac{5}{9} \text{ part of } {}^{\circ}\text{C} = 1 \text{ part of } {}^{\circ}\text{F}$$

$$\Rightarrow \frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273} = \frac{R-x}{y-x}$$

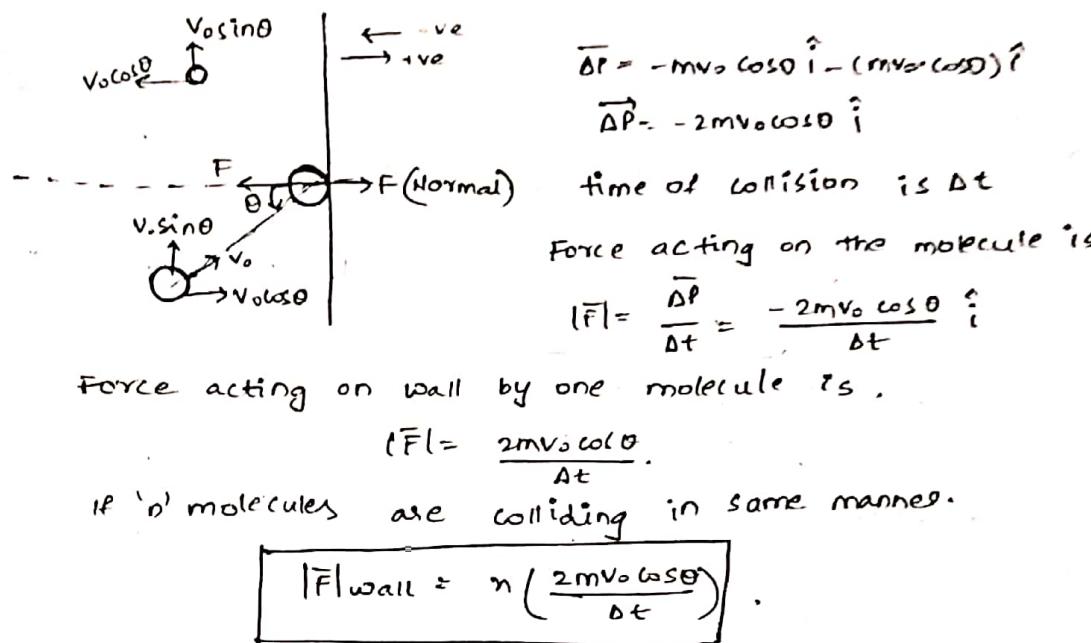
$$\boxed{\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}}$$

$$\frac{\text{Reading} - \text{LFP}}{\text{BFP} - \text{LFP}} = \text{constant}$$

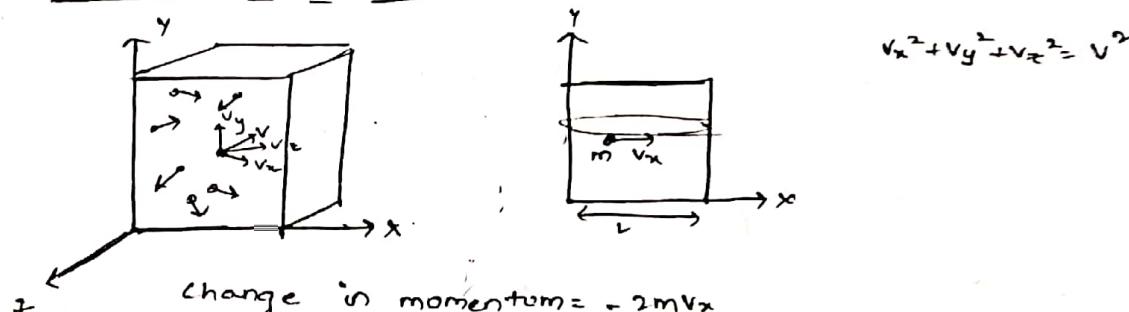
(9) KINETIC THEORY OF GASES:

Assumptions:

- 1) Molecule size is very small (negligible)
 - 2) No interaction b/w gas molecules.
(except at collision)
 - 3) All collisions (collisions b/w molecules or) with walls of container are elastic.
 - 4) No-of collisions in unit time per unit area
- Pressure \propto by the molecules on the wall.



PRESSURE OF A GAS:



Change in momentum = $-2m v_x$
time of collision (Δt) = $\frac{2l}{v_x}$

Force applied on the wall by the molecule = $\frac{\Delta P}{\Delta t} = \frac{2m v_x}{2l/v_x} = \frac{mv_x^2}{l}$

assuming

$$v_x \approx v_y \approx v_z$$

$$F = \frac{m}{l} \left(\frac{v^2}{3} \right)$$

Mole of gas,

$$F_{\text{net}} = N_A \times \frac{m}{l} \left(\frac{v^2}{3} \right)$$

[Assuming all the molecules to be having common (same) velocity]

Pressure acting on the wall is:

$$P = \frac{F_{\text{net}}}{\text{area}} = \frac{N a \left(\frac{mv^2}{3L} \right)}{1^2}$$

$$P = \frac{1}{3L} (m N A V^2)$$

↓ vol.

$$\boxed{PV = \frac{2}{3} (\text{KE of the gas})} *$$

→ If molecules are having diff. speeds.

$$F_{\text{net}} = F_1 + F_2 + \dots + F_n$$

$$F_{\text{net}} = \frac{mv_1^2}{3L} + \frac{mv_2^2}{3L} + \dots + \frac{mv_n^2}{3L}$$

$$= \frac{m \cdot N_A}{3L} \left(\frac{v_1^2 + v_2^2 + \dots + v_n^2}{N_A} \right)$$

$$V_{\text{rms}} \text{ mean square } (V_{\text{rms}}) = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

$$F_{\text{net}} = \frac{m \cdot N_A}{3L} (V_{\text{rms}})^2$$

$$\text{Pressure } (P) = \frac{F}{A} = \frac{m N A}{3 L^3} (V_{\text{rms}})^2$$

$$R = 8.314 \text{ J/mole-K}$$

$$PV = \frac{m \cdot N_A}{3} (V_{\text{rms}})^2$$

$$PV = \frac{M}{3} (V_{\text{rms}})^2$$

$$\rightarrow V_{\text{rms}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}}$$

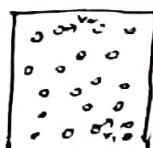
$$V_{\text{avg}} = \frac{v_1 + v_2 + \dots + v_n}{n} = \sqrt{\frac{RT}{\pi M}}$$

$$(V_{\text{mp}} < V_{\text{avg}} < V_{\text{rms}})$$

at same temp.

$$V_{\text{rms}} \text{ probable} = V_{\text{mp}} = \sqrt{\frac{2RT}{M}}$$

MAXWELL-BOLTZMANN DISTRIBUTION OF VELOCITIES:-



dN - No. of molecules are having velocity in b/w v and $v+dv$.

gas at some temperature

$f = \frac{dN}{N} = \text{fraction of molecules which are having velocity in b/w } v \text{ and } v+dv$

$$dN/f \propto T^{-3/2} e^{-mv^2/2KT} \cdot v^2 dv$$

$$dN(\text{or}) f \propto e^{-\alpha V} \cdot V^{\gamma} dV$$

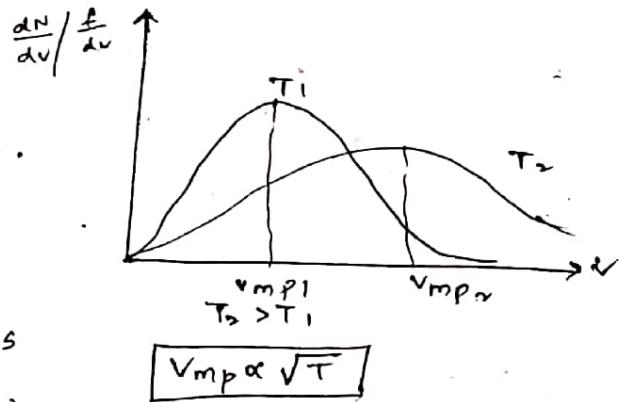
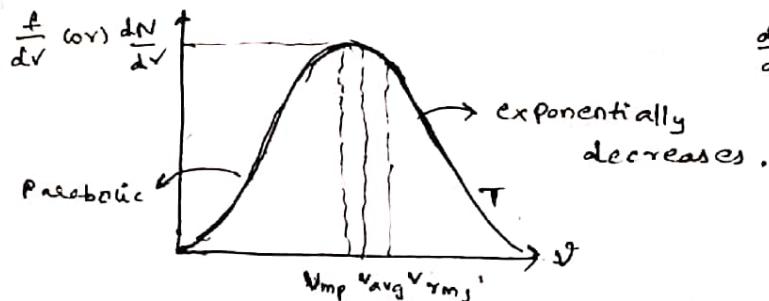
(3)

At a particular temp:

$$f(\text{or}) dN \propto e^{-\alpha V^2} \cdot V \cdot dV$$

If V is low, $f \propto V^2$ (parabola).

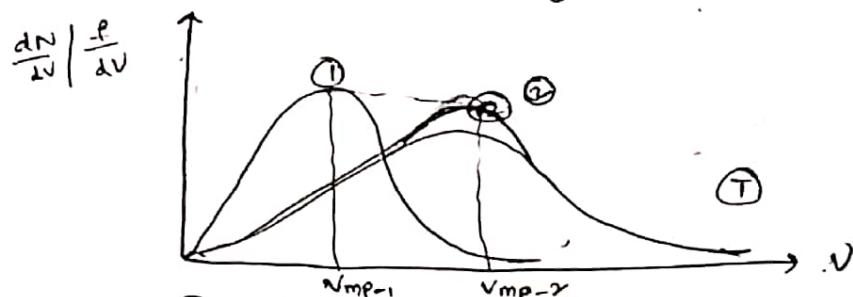
If V is high, $f \propto e^{-\alpha V^2}$ (exponentially decreasing)



* area - under the graph is always equal to 1 (If ' f ' is on Y-axis).

(or) equal to total No. of molecules. (If dN is on Y-axis)

→ With the mass, velocity distribution decreases.



① → heavier mass (Ex: O₂)

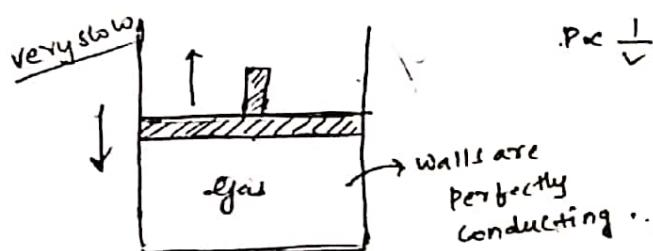
② → lighter mass (Ex: He)

$$V_{mp2} > V_{mp1} \Rightarrow V_{mp} \propto \frac{1}{\sqrt{M}}$$

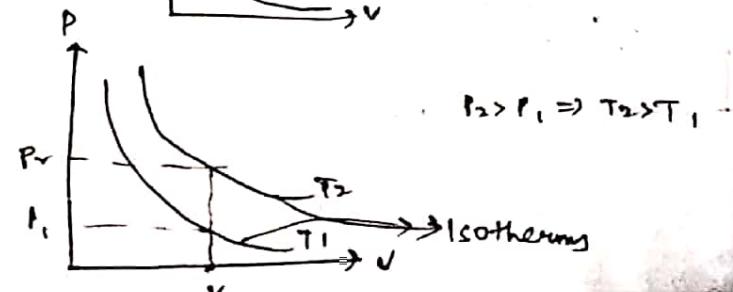
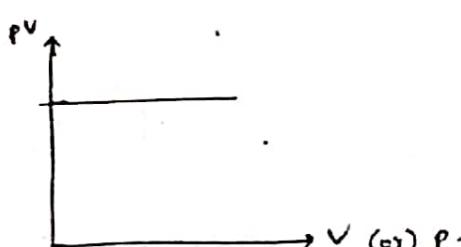
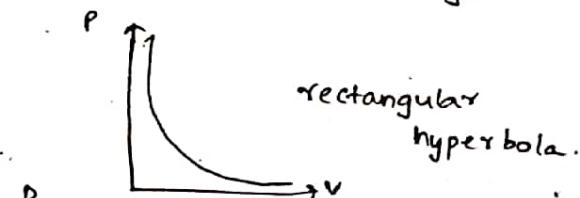
GAS LAWS:-

i) BOYLE'S LAW:-

If $T = \text{constant}$; $PV = \text{constant}$



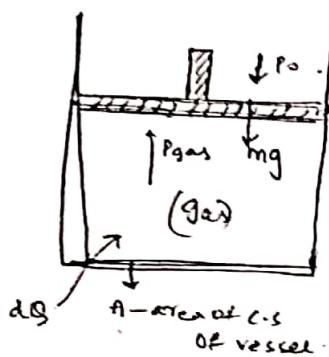
$P \propto \frac{1}{V}$ for given mass of gas.



CHARLE'S LAW:

(4)

$P \rightarrow \text{constant}$.



$$F_{\text{gas}} = mg + P_0 A \text{ from.}$$

Piston is free
to move

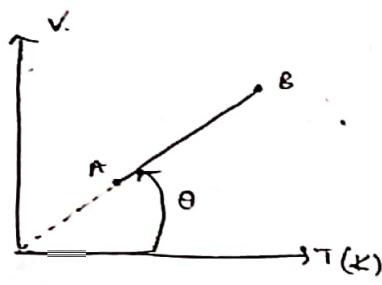
$$P_{\text{gas}} A = mg + P_0 A$$

$$P_{\text{gas}} = \frac{mg}{A} + P_0$$

$$P_{\text{gas}} = \text{constant.}$$

dQ A = area of c.s
of vessel.

temp \uparrow , \Rightarrow Vol \uparrow



$$\text{If } P \rightarrow \text{const.} \Rightarrow V \propto T$$

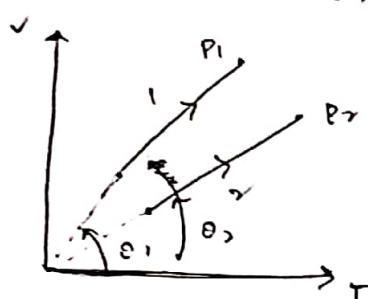
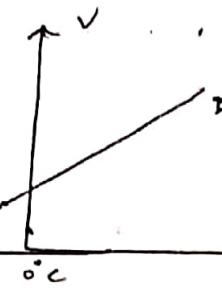
$$PV = nRT$$

$$V = \left(\frac{nR}{P}\right)T$$

$$V \propto T$$

$$\text{Slope (tan } \theta) = \frac{nR}{P}$$

$$\tan \theta \propto \frac{1}{P}$$

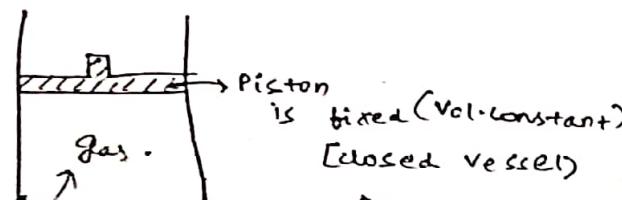


$$\tan \theta_1 > \tan \theta_2$$

$$P_1 < P_2$$

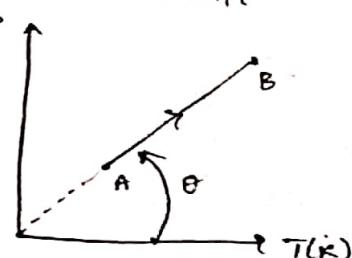
CHARLE'S LAW-II:-

$V = \text{constant.}$



dQ temp \uparrow , pressure \uparrow

If $V = \text{constant.}$



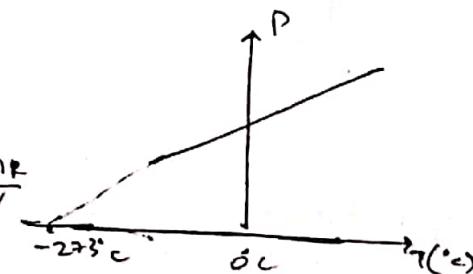
If $V = \text{const.}$, $P \propto T$

$$PV = nRT$$

$$P = \left(\frac{nR}{V}\right)T$$

$$\text{Slope} = \tan \theta = \frac{nR}{V}$$

$$\Delta P \propto \frac{1}{V}$$

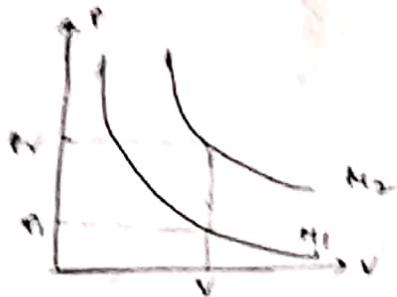




tan theta = $\frac{V_2 - V_1}{V_2}$

$$V_2 > V_1$$

For same Pres & Temp. of different gases i.e.
(different mol masses)



$$PV = nRT$$

$$PV = \frac{m}{M} RT$$

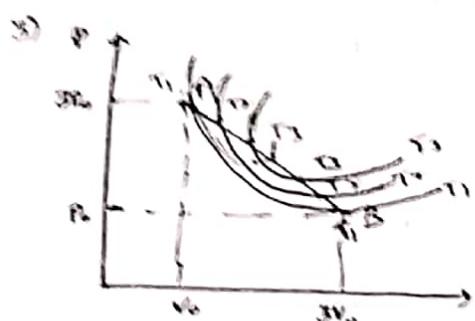
$$PV = \frac{1}{M} RT$$

$$PV = \frac{1}{M} RT$$

$$P_2 > P_1 \Rightarrow M_2 < M_1$$



$$\theta_{T_2 P_2} \rightarrow P_2 > P_1$$



What happens to temperature
as changes its state from
A to B.

$$T_3 > T_2 > T_1$$

Sol: $T_A = T_B$

∴ Temp. is first increasing and then decreasing.

What is maximum temp?

Line equation: $y = mx + c$

$$y = \left(-\frac{P_0}{2V_0} \right) x + 4P_0$$

$$P = -\frac{P_0}{V_0} \cdot (x) + 4P_0$$

$$\frac{nRT}{V} = -\left(\frac{P_0}{V_0}\right)(x) + 4P_0$$

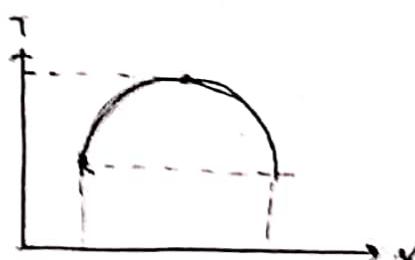
$$T = -\left(\frac{P_0}{V_0}\right)\left(\frac{V^2}{nR}\right) + \frac{4P_0 \cdot V}{nR}$$

$$y = mx^2 + c$$

$$\frac{dy}{dx} = 0 \Rightarrow -\frac{P_0}{V_0} \left(\frac{2V}{nR} \right) + \frac{4P_0 \cdot V}{nR} = 0$$

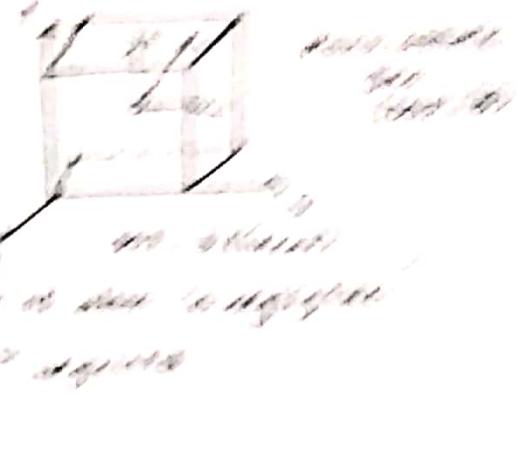
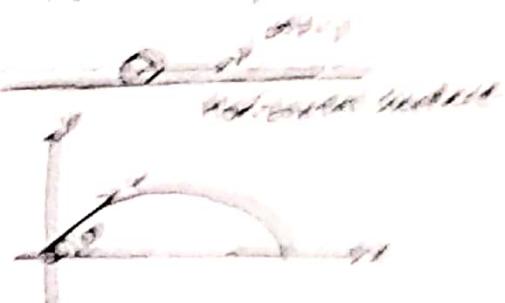
$$\Rightarrow \boxed{V = 2V_0}$$

$$\boxed{T_{max} = \frac{4P_0 \cdot V_0}{n \cdot R}}$$



Degrees of freedom to

→ first requirement says it does not provide for them



Bottom is rigid


bottom is rigid (bottom is fixed)
→ has no freedom to move
or rotate requirement
has no freedom to move horizontally.
(horizontal translation)

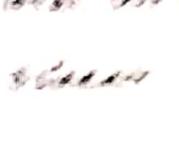
Bottom is hinged

→ bottom has 2 degrees
of freedom


bottom is hinged
→ has 2 degrees of freedom
(bottom is movable)
→ (horizontal) freedom not lost

Top is rigid

Top is rigid →


top is rigid
→ has 2 degrees of freedom
→ second part has 2 degrees

has 2 degrees of freedom (top part has 2 degrees of freedom)

has 2 degrees of freedom (bottom part has 2 degrees of freedom)

LAW OF EQUI-PARTITION THEORY.

→ Each degree of freedom will have $\frac{1}{2} kT$ energy

$k = \text{Boltzmann's constant}$.

If DOF of i^{th} gas = f

$$k = \frac{e}{f}$$

$$\text{Energy of the molecule} = f \left(\frac{1}{2} kT \right) + \frac{f}{2} kT$$

$$\text{Energy of the mole of gas} = n_A \left(\frac{f}{2} kT \right) + \frac{f}{2} (n_A kT) = \frac{f}{2} RT$$

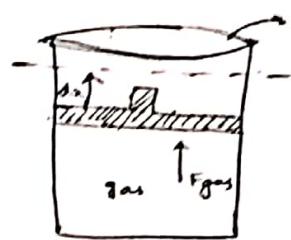
$$\text{Energy of } 'n' \text{ moles of gas} = n \left(\frac{f}{2} RT \right)$$

$$\text{Internal Energy of gas} = \frac{f}{2} nRT$$

$$U = \frac{f}{2} nRT \quad (\text{at Temp})$$

$$dU = \frac{f}{2} nR dT$$

Work done (dW)



area of C-S-A

work done by the gas = $dW = \bar{F} \cdot dx$

$$dW = F_{\text{gas}} (dx) \cos 0^\circ$$

$$dW = F_{\text{gas}} \cdot dx$$

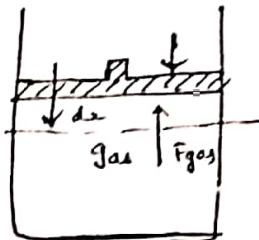
$$dW = (P_{\text{gas}} \cdot A) dx$$

$$= P_{\text{gas}} \cdot (A \cdot dx)$$

$$= P_{\text{gas}} \cdot dV$$

dW is +ve

(Vol. of the gas increases)



work done by the gas = $dW = \bar{F}_{\text{gas}} \cdot dx$

$$dW = F_{\text{gas}} (dx) \cos 180^\circ$$

$$dW = -F_{\text{gas}} (dx)$$

$$dW = -P_{\text{gas}} (dV)$$

change in volume
 $(V_f - V_i)$

In general work done by the gas $dW = P_{\text{gas}} (dV)$

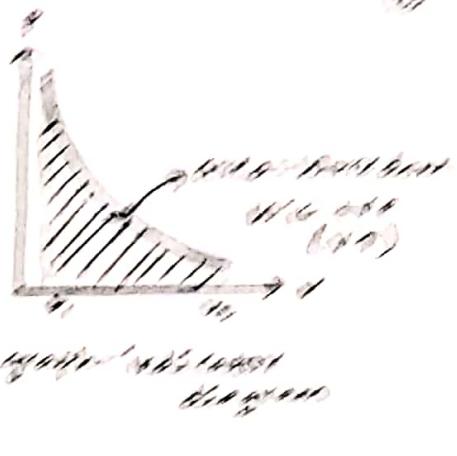
If Volume increases, dV is +ve, dW is +ve

If Volume decreases, dV is -ve, (dW) is -ve

↳ work done on the gas.

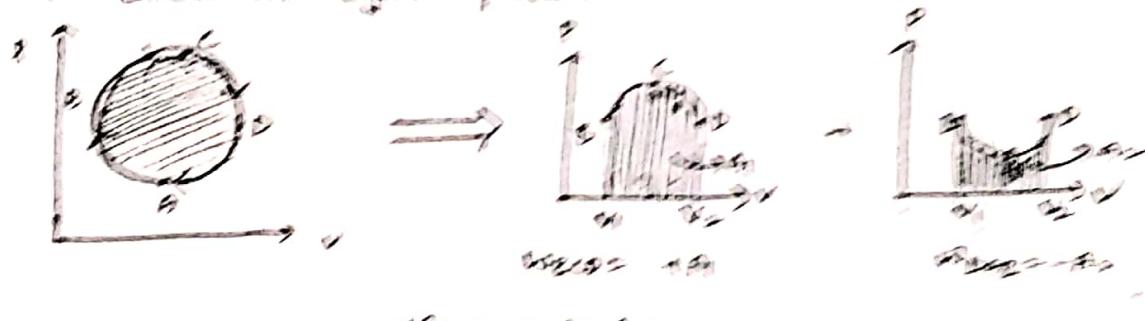
Part 6: Heat Transfer

200 - 200



Cyclic Process

- If initial and final states of the process are same, then process
is called as cyclic process



Temperature
area enclosed by the cycle.

For solid & liquid we
use $A = L \times B = \pi r^2$

All Calculations

For solid and liquid

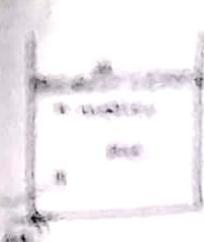
$d = \text{const}$)

- heat capacity per unit mass
(specific heat capacity)
- depends on nature of material

For gases:

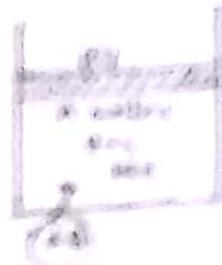
$$\begin{aligned} d &= n \\ d &= \frac{m}{M} \end{aligned} \quad \left[\text{constant} \right]$$

- more specific heat capacity
- expansion ratio of gas
- depends on process



Moving sphere
of radius
 r
(x, y, z constant)
 $\Delta V = \pi r^3$

Volume
change
 $\Delta V = \pi r^3$



Volume change
 $\Delta V = \pi r^3$
 $\Delta V = \frac{4}{3} \pi r^3$

$\Delta U = \frac{1}{2} k_B T \Delta V$

$$\Delta U = \frac{1}{2} k_B T \Delta V$$

$$\Delta U = n C_v \Delta T$$

Volume change is not depending
on initial volume

$$\Delta U = \frac{1}{2} k_B T \Delta V$$

$$\Delta U = \frac{1}{2} k_B T$$

$$\text{Mean volume } (\bar{V}) = \frac{3V}{4}$$

At const. Volume,

$$\Delta U = \Delta H$$

$$\Delta U = n C_v \Delta T$$

$$\boxed{\Delta U = n C_v \Delta T} \quad (\text{internal energy depends on temp. only})$$

At const. pressure,

$$\Delta U = \Delta H + \Delta PV$$

$$\Delta U = n C_v \Delta T + PV \Delta T$$

$$\Delta U = n C_v \Delta T + n R \Delta T$$

$$\boxed{C_p - C_v = R}$$

$$\text{Adiabatic exponent } (\gamma) = \frac{C_p}{C_v} = \frac{(C_v + R)^{\frac{1}{2}}}{(C_v)^{\frac{1}{2}}} \Rightarrow \boxed{\gamma = 1 + \frac{R}{C_v}}$$

$$C_p = C_v + R$$

$$C_p = \frac{C_v}{\gamma} + R = \left(\frac{C_v}{\gamma - 1}\right) R$$

Mean free path λ = mean free distance

Distance travelled by molecule from time t to
surface reflection with time t .



$$\text{Mean free path } (\lambda) = \frac{s}{N \tau \text{ m}^{-3}}$$

s = mean free path per unit volume

N = density of molecules \Rightarrow molecules $\#$

$$\text{No. of molecules} = \frac{\text{No. of molecules}}{N_A}$$

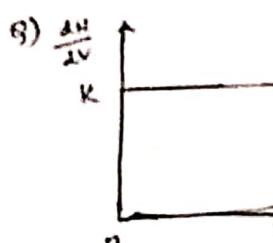
$$PV = nRT$$

$$PV = \left(\frac{\text{no. of molecules}}{N_A} \right) RT$$

$$\frac{PV}{N_A} = \left(\frac{\text{no. of molecules}}{V} \right) \frac{RT}{N_A}$$

$$\frac{PV}{N_A} = \text{no. of molecules per unit volume} \Rightarrow \lambda = \frac{1}{\sqrt{2 \cdot \pi L^2} \left(\frac{PV}{N_A} \right)}$$

$$\Rightarrow \lambda = \frac{PT}{\sqrt{2 \cdot \pi L^2} \cdot P \cdot N_A}$$



Given graph is hypothetic hypothetical velocity distribution of gas

$$\frac{dN}{dv} = \frac{K}{2v_0} \cdot v_0 \cdot \frac{3k}{2}$$

Find the avg. velocity of the gas?

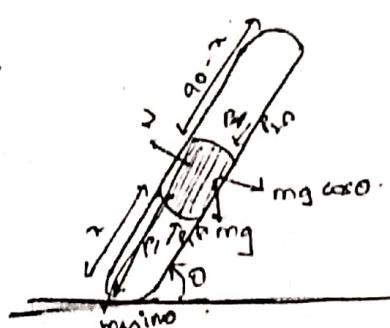
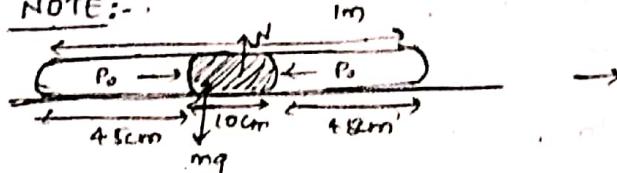
$$\text{Soln: avg. velocity over number} = \frac{\int v \cdot dN}{\int dN} = \frac{\int v \cdot \frac{dN}{dv} dv}{\int dN} = \langle v \rangle$$

$$\langle v \rangle = \frac{\int v_0 \frac{dN}{dv} dv + \int_{2v_0}^{3v_0} v \cdot \frac{dN}{dv} dv}{\int_{2v_0}^{3v_0} \left(\frac{dN}{dv} \right) dv}$$

$$\langle v \rangle = \frac{K \left(\frac{v_0^2}{2} \right) + \int_{v_0}^{3v_0} -\frac{kv^2}{2} dv + \int_{v_0}^{3v_0} \frac{3kv}{2} dv}{K \cdot v_0 + KV_0}$$

$$= \frac{\frac{Kv_0^3}{2} + \frac{5kv_0^3}{3}}{2kv_0} = \frac{13v_0^2}{12}$$

NOTE:-



$$P_2 \sin \theta + mg \sin \theta = P_1 \cos \theta$$

$$P_1 = P_2 + \frac{mg \sin \theta}{A}$$

$$P_1 = P_2 + \frac{P_m g A (10 \text{ cm}) (9.8 \sin \theta)}{A}$$

$$P_1 = P_2 + P_m g (10 \text{ cm}) \sin \theta$$

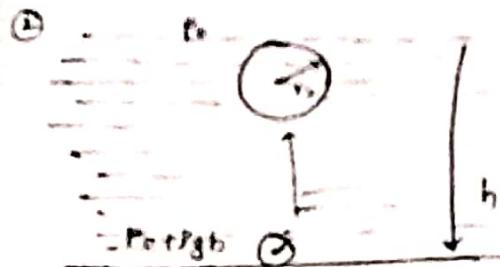
θ = const. in process of tilt of tube.

$$P_0(45 \text{ cm})/T_0 = P_1(x) / T_1$$

$$\left[P_1 = \frac{P_0(45)}{x} \right]$$

$$P_0(45 \text{ cm})/T_0 = P_2(90-x) / T_2$$

$$P_2 = \frac{P_0(90-x)}{T_2} \quad \boxed{T_2 = \frac{P_0(45x)}{90-x}}$$



Neglecting surface tension

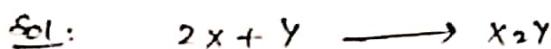
\rightarrow If T constant when the bubble rises.

$$\begin{aligned} P_1 V_1 &= P_2 V_2 \\ (P_0 + \rho_0 g h) \frac{4}{3} \pi r_1^3 &= (P_0) \left(\frac{4}{3} \pi r_2^3 \right) \\ \boxed{(P_0 + \rho_0 g h) r_1^3 &= P_0 r_2^3} \end{aligned}$$

\rightarrow If T variable

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \boxed{\frac{(P_0 + \rho_0 g h) r_1^3}{T_1} = \frac{P_0 r_2^3}{T_2}}$$

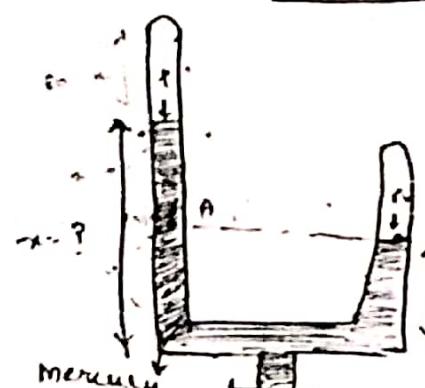
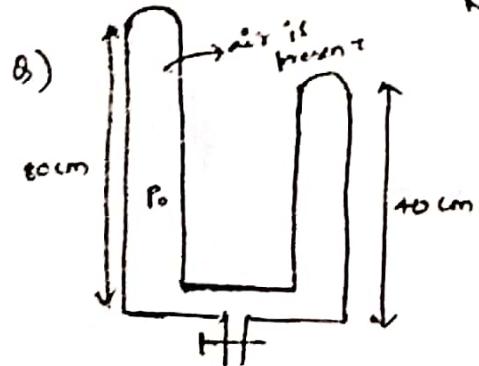
Q) In a container of volume V , ~~one~~ atoms of 'x' gas is present. If ' n ' atoms of 'y' gas is inserted into the container, (Temp. of container T is maintained const.) and both the gases react and give X_2Y molecule. Find pressure of the gas after rxn completed.



$$2\left(\frac{N}{2}\right) + \frac{N}{2} \longrightarrow \frac{N}{2}$$

$\Rightarrow \frac{N}{2}$ atoms of X_2Y + $\frac{3N}{2}$ atoms of y will be left in beaker at last.

$$P(V) = \frac{\left(\frac{N}{2} + \frac{3N}{2}\right) RT}{N_A} \Rightarrow \boxed{P = \frac{2N}{N_A} \left(\frac{RT}{V} \right)}$$



Find the value of x ?

$P_0 = 80 \text{ cm of mercury}$

$$T_0(80) = T_0(x=0)$$

$$T_0 = \frac{T_0(80)}{80-x}$$

$$P_0 = P_0$$

$$\frac{4P_0}{3} = \left(\frac{80T_0}{80-x}\right)^2 (m+2)(k-10)$$

$$T_0(40) = T_0(30)$$

$$T_0 = \frac{4P_0}{3}$$

(6)

NOTE
→ T_{270}

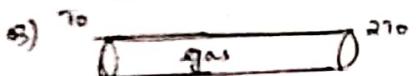
$$\log\left(\frac{4x80}{3}\right) = \log\left(\frac{80x80}{80-x}\right) + \log(m-10)$$

$$\frac{320}{3} = \frac{6400 - 800x^2 + 960x}{80-x}$$

$$25600 - 320x^2 = 19200 - 2400x - 3x^2 + 270x$$

$$590x^2 = 8800 + 32x^2$$

$$\Rightarrow 3x^2 - 590x + 8800 = 0$$



→ cylinder of length 'L' and area of C.S = A.

→ Temp of the gas changes linearly from T_0 to 270 .

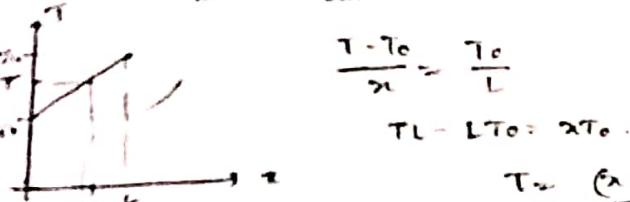
→ assume pressure of the gas is same throughout the tube and value 'P' is p.

→ Find the total no. of moles of gas in cylinder?



$$PV = nRT \Rightarrow P \cdot A \cdot dx = dn \cdot R \cdot \frac{(x+L)T_0}{L}$$

$$\Rightarrow n = \frac{PV}{RT} \Rightarrow dn = \int dx = \int \frac{PA \cdot dx}{R \cdot (L+x)T_0}$$

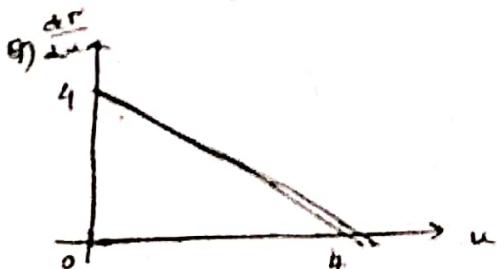


$$\frac{T-T_0}{x} = \frac{T_0}{L}$$

$$TL - LT_0 = xT_0$$

$$T_x = \frac{(x+L)T_0}{L}$$

$$\Rightarrow n = \frac{PAL}{RT_0} \left[\ln(L+x) \right]_0^L = \left[\frac{PAL}{RT_0} \cdot \ln 2 \right]$$



graph of hypothetical velocity distribution
is given

Find the rms velocity of gas.

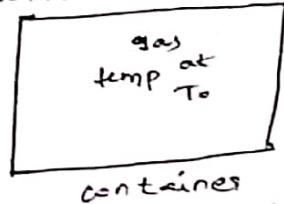
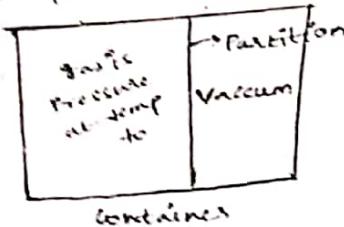
$$\text{Sol: } \langle u^2 \rangle = \frac{\int u^2 \cdot dN}{\int dN} = \frac{\int u^2 (4-u) \cdot du}{\int \frac{dN}{du} \cdot du} = \frac{\frac{dN}{du} = 4-u}{\int 4u^2 - u^3 \cdot du} = \frac{4}{8}$$

$$\left[\frac{4u^3}{3} - \frac{u^4}{4} \right]_0^4 = \frac{8u^4}{8 \times 3} = \frac{8}{3}$$

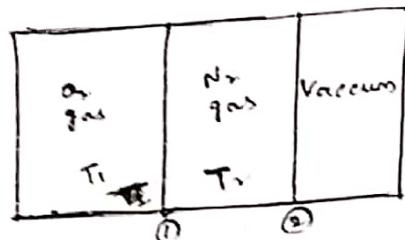
$$\Rightarrow V_{rms} = \sqrt{\frac{8}{3}}$$

NOTE:

→ Temp. of the gas in free expansion does not change.
If partition is removed



8)



then partition ① is removed and allowed the gases to occupy the container. Find the avg. molecular speed of O2 gas? 25

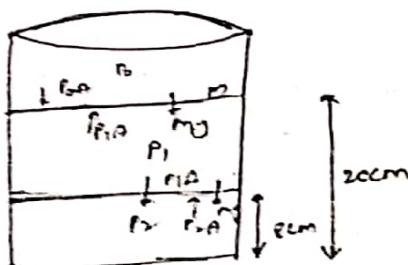
$$\text{Soln:- avg. velocity of O}_2 = \sqrt{\frac{8RT_1}{M_1}} = V_0$$

$$N_2 = \sqrt{\frac{8RT_2}{M_2}} = \sqrt{\frac{8}{7}} V_0.$$

$$\sqrt{\frac{T_1 \cdot M_2}{T_2 \cdot M_1}} = \sqrt{\frac{7}{8}} \Rightarrow T_1 = T_2$$

→ Temp. is same even after partition 1, 2 are removed. Avg. velocity of O₂ molecule remains same (V₀).

9)



$$P_1 A + mg = P_0 A$$

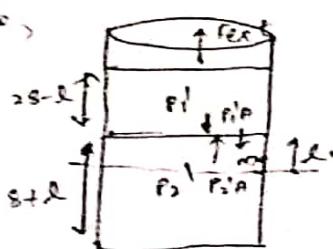
$$P_1 = \frac{mg}{A} + P_0 = 2P_0$$

$$P_0 A = P_1 A + Mg$$

$$P_2 = 2P_0 + P_0 = 3P_0$$

$$\text{Given } \frac{mg}{A} = P_0$$

Now,



$$3P_0 (20\pi A) = P_2' = (20+l)\pi A$$

$$P_2' = \frac{24P_0}{20+l}$$

$$2P_0 (l\pi A) = P_1' (20-l)\pi A$$

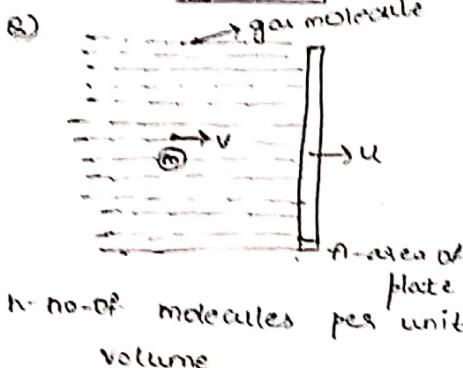
$$P_1' = \frac{24P_0}{20-l}$$

$$P_1 = n_1 \cdot P_0$$

$$P_1' = n_1' \cdot P_0'$$

$$\frac{n_1}{n_1 + 1} = 1 + \frac{1}{2 \cdot 10^{-2}}$$

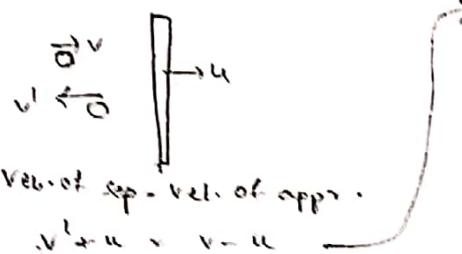
\Rightarrow $x_1 = 1$



If F_1 is force required to move plate with const. velocity u towards right and F_2 is force required to move left (collisions are elastic).

Then find $F_2 - F_1$ value?

(b)



$$v' = v - 2 \cdot u$$

$$\Delta P = -mv' - mu$$

$$\Delta P = -m(v' + u)$$

$$\Rightarrow \Delta P = -2m(v - u) \Rightarrow F \cdot \frac{\Delta P}{\Delta x} = 1$$

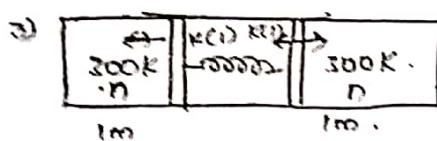
$$\text{Volume} = A(N - u)$$

$$\text{No. of molec.} = nA(N - u)$$

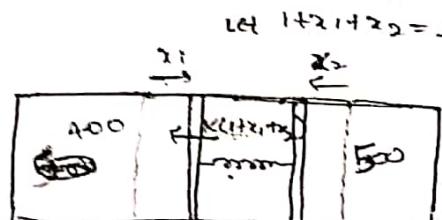
$$F_1 = nA(v - u) (\rho m)(v - u) \quad \rho_m = 2mnR(v + u)^2 \\ = 2mn \rho (v - u)^2$$

$$= F_2 - F_1 = 2mn \rho (4uv) = 8mn \rho vu$$

Q1-12-2018 Questions:-



\Rightarrow



$$\text{Let } 1 + 2 + 4 + x_2 = 12$$

$$\frac{K_0 \cdot x_1}{T_1} = \frac{K_0 \cdot x_1 (1 + x_1)}{T_1} = \frac{P_1 V}{T_1} = \text{const.}$$

$$\frac{K_0 \cdot x_2}{T_2} = \frac{K_0 \cdot x_2 (1 + x_2)}{T_2} = \frac{P_2 V}{T_2}$$

$$3 + 3x_2 = 4 + 4x_2$$

$$4x_2 - 3x_1 = 1 \quad \therefore 3x_1 - 4x_2 = 1$$

$$x_1 = 1 + 4x_2$$

$$\frac{3x_1 - 1}{4} = x_2$$

No. of molec. const.:

~~$$\frac{K_0 \cdot x_1}{200} = \frac{K_0 \cdot (1 + 4x_2)}{400}$$~~

~~$$4 = 3(1 + x_1)(1 + 2x_1 + x_2)$$~~

~~$$4 = 3(1 + x_1)(1 + 2x_1 + x_2) \approx 3(1 + x_1) \left(\frac{3x_1 + 4x_2 + 1}{2} \right)$$~~

$$P_1 = P_2$$

$$W(n+2) = k(1+2+3+4)$$

$$k_2 = \frac{1+2+4}{3}$$

$$k_2 = 3 + \frac{1+2+4}{3}$$

$$k_2 = 3 - \left(\frac{1+2+4}{3}\right) \Rightarrow k_2 = \frac{4+5}{3}$$

$$P_A = k_2$$

$$2000\text{N}^2 \cdot 1 = \frac{\mu E(400)}{9}$$

$$4000\text{N}^2 \cdot 1 = \frac{25}{(1+2)}$$

$$1^2 + 2 - 3 = 0 \Rightarrow 1 = \frac{12\sqrt{402}}{2} = \frac{\sqrt{402}-1}{2} = 1.3$$

$$\therefore x = 130\text{cm}$$

$$PV = nRT$$

$$P = \text{const.}$$

$$\frac{PdV}{PV} = \frac{nRdT}{nRT} \Rightarrow \frac{dV}{V} = \frac{dT}{T}$$

$$dV = \left(\frac{1}{T}\right)dT$$

$$\boxed{T = \frac{1}{k}V}$$

$$PV = nRT$$

$$V = \text{const.}$$

$$\frac{dP}{P} = \frac{nRdT}{nRT}$$

$$\frac{dP}{P} = \frac{dT}{T} \Rightarrow dP = P\left(\frac{1}{T}\right)dT$$

pressure const.

THERMO-DYNAMICS:

1st Law of Thermo-dynamics:-

↳ Conservation of Energy

$dQ:$

$$U = \frac{f}{2} nRdT \rightarrow \text{Total K.E. of gas molecules.}$$

$\Delta U:$

$$dU = \frac{f}{2} nRdT = nC_V dT \Rightarrow \boxed{C_V = \frac{f}{2} R}$$

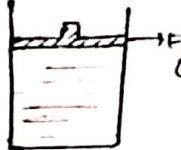
$$dW = P \cdot dV$$

→ Total Work done in changing volume from V_1 to V_2 is:

$$W = \int dW = \int_{V_1}^{V_2} P \cdot dV$$

$$dQ = n \cdot C \cdot dT$$

↳ depends on nature of gas
↳ depends on process.



$$dQ = n \cdot C_V dT$$



$$dQ = n \cdot C_P dT$$

If $T = \text{const.}$ (Isothermal process)

$$dT = 0$$

$$dQ = nC_V dT$$

$$\left[C_V = \frac{\partial Q}{\partial T} = \frac{n \cdot dQ}{n \cdot dT} \right]$$

$$C_V = \frac{dQ}{dT} = \infty$$

→ If process is adiabatic

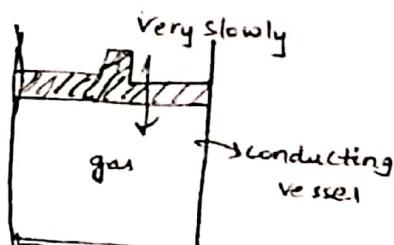
$$dQ = 0; nC \cdot dT = 0 \Rightarrow \boxed{C = 0}$$

ISO-THERMAL PROCESS:-

$T = \text{constant}$

$$PV = nRT = \text{constant}$$

$$\boxed{P_1 V_1 = P_2 V_2 = nRT_1}$$



$$P_1 V_1 = P_2 V_2 = nRT_1$$

$$T = \text{constant} \Rightarrow U = \frac{f}{2} nRdT = \text{constant}$$

$$\boxed{dU = 0}$$

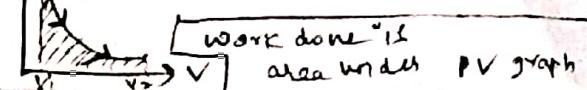
⇒ By 1st law of FD: $dQ = dU + dW$

$$\Rightarrow \boxed{dQ = dW}$$

Work done in Iso-thermal process in changing vol. from V_1 to V_2 is

$$W = \int dW = \int_{V_1}^{V_2} P \cdot dV = \int_{V_1}^{V_2} \frac{nRT}{V} \cdot dV =$$

$PV = \text{const}$



Work done is

area under PV graph

$$\boxed{nRT \ln \frac{V_2}{V_1}} \rightarrow \begin{array}{l} \text{heat absorbed} \\ (\text{or) rejected by} \\ \text{the gas.} \end{array}$$

heat absorbed when gas expands
rejected " gas compressed

ADIASTATIC PROCESS:-

$\delta = \text{constant}$

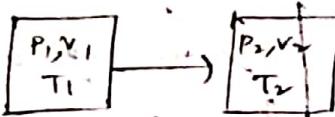
$d\delta = 0$

P, V, T are variables

Ideal gas eqn:

$$PV = nRT$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$



vessel walls
are
adiabatic/
non-conductive/
insulated
 $\alpha = \text{const.}$

(2)

$\text{L} \rightarrow \text{NO}_2$,
change
in heat
with
curr.

Equation of process:-

1st Law of T.D: $d\delta = dU + dW$

$$dU = -dW \Rightarrow dW = -dU$$

$$P \cdot dV = -nC_V dT$$

$$P \cdot dV = -nC_V \left[\frac{P \cdot dV + V dp}{MR} \right]$$

$$(P \cdot dV)R = -C_V(P \cdot dV) - C_V(V \cdot dp)$$

$$C_P(P \cdot dV) = -C_V(V \cdot dp)$$

$$\int \frac{dV}{V} = - \int \frac{dp}{P} \Rightarrow \ln V = -\ln P + C$$

$$\ln V + \ln P = C \Rightarrow \ln P \cdot V = C$$

$$PV^{\gamma} = e^C = \text{const ant}$$

$$\begin{aligned} TV^{\gamma-1} &= \text{const.} \\ P^{\gamma-1} \cdot T^{\gamma} &= \text{const.} \end{aligned}$$

Work done in adiabatic process:

$$dW = -dU$$

$$= -nC_V(\delta T) = \frac{mR\delta T}{1-\gamma} = \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$$

$$= -nC_V \left(\frac{P_2 V_2}{MR} - \frac{P_1 V_1}{MR} \right) = -C_V \left(\frac{P_2 V_2}{P} - \frac{P_1 V_1}{P} \right)$$

$$W = \int_{V_1}^{V_2} P \cdot dV \quad [P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K]$$

$$= \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} \cdot dV$$

$$\Rightarrow P = K \left[\frac{V_1^{-\gamma+1} - V_2^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2} = K \left[\frac{V_2^{\gamma-1} - V_1^{\gamma-1}}{1-\gamma} \right] = \frac{P_2 V_2^{\gamma-1} - P_1 V_1^{\gamma-1}}{1-\gamma}$$

$$\Rightarrow W = \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$$

$$\Rightarrow W = \frac{nR(T_2 - T_1)}{1-\gamma}$$

→ All sudden
processes are
adiabatic.

Note:

→ If. eqn of process is $PV^x = \text{const}$.

$$W = \frac{P_2 V_2 - P_1 V_1}{1-x} = \frac{nR(T_2 - T_1)}{1-x} \quad (\text{Polytropic process})$$

ISOBARIC PROCESS:

$P = \text{constant}$.

$$PV = nRT \Rightarrow V = \frac{(nR)}{P}T \quad [V \propto T]$$

$$dQ = nC_p dT$$

$$dV = nC_v dT \Rightarrow dW = P \cdot dV = n \cdot R \cdot dT$$

Work done in changing vol. from V_1 to V_2 :

$$W = \int_{V_1}^{V_2} P \cdot dV = P \cdot (V) \Big|_{V_1}^{V_2} = P(V_2 - V_1)$$

$$\boxed{W = nR(T_2 - T_1)}$$

$$\frac{dW}{dQ} = \frac{nR(T_2 - T_1)}{nC_p(T_2 - T_1)} = \frac{P}{C_p} \Rightarrow \frac{dW}{dQ} = \frac{P}{C_p}$$

For mono-atomic,

$$\frac{dW}{dQ} = \frac{P}{\frac{5}{2}R/2} = \frac{2}{5} \Rightarrow \boxed{dW = \frac{2}{5}(dQ)}$$

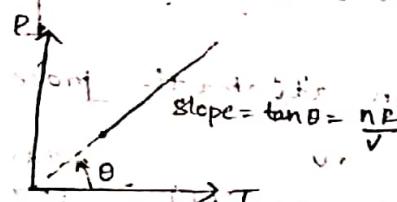
For di-atomic,

$$\frac{dW}{dQ} = \frac{P}{\frac{7}{5}R/2} = \frac{5}{7} \Rightarrow \boxed{\frac{dW}{dQ} = \frac{5}{7}}$$

ISO-CHORIC PROCESS:-

$V = \text{constant}$; $dW = 0$

$$dQ = dU = nC_v dT$$



POLYTROPIC PROCESS:-

Equation of process is $PV^x = \text{constant}$

Work done by the gas $dW = \frac{nRdT}{1-x}$

$$W = \int_{T_1}^{T_2} dW = \frac{nR}{1-x} \int_{T_1}^{T_2} dT = \frac{nR(T_2 - T_1)}{1-x}$$

Internal energy change (ΔU) = $nC_v dT$

1st law of T.D; $dQ = dU + dW$

$$nC_v dT = nC_v dT + \frac{nRdT}{1-x}$$

$$\Rightarrow \boxed{C_v = \frac{R}{1-x}}$$

Slope of P-V graph for isothermal, adiabatic:- (4)

$$\text{Slope (isothermal)} = \frac{dP}{dV}$$

Isothermal:

$$PV = \text{constant}$$

$$P \cdot dV + V \cdot dP = 0$$

$$\therefore \frac{dP}{dV} = -\frac{P}{V} = \text{slope}$$

→ Slope of isothermal at point A is $-\frac{P_1}{V_1}$

Adiabatic:

$$PV^\gamma = \text{constant}$$

$$P \cdot (\gamma \cdot V^{\gamma-1} dV) + dP (V^\gamma) = 0$$

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

→ Slope of adiabatic = γ (slope of isothermal)

→ Adiabatic curves are more steeper than isothermal

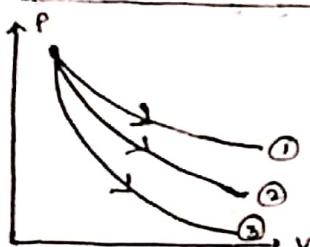
Isothermal:

$$P \cdot dV = -V \cdot dP \Rightarrow P = \frac{dP}{-(dV)} = \text{Bicothalma}$$

Adiabatic:

$$\gamma P dV + (dP) \cdot V = 0 \Rightarrow \gamma \cdot P = \frac{dP}{-(dV)} = \text{Badiahtic}$$

Badiahtic = γ (B isothermal)



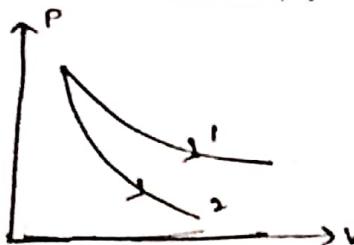
$$PV^\gamma = \text{constant}$$

$$\textcircled{1} PV^{x_1} = \text{constant}$$

$$\textcircled{2} PV^{x_2} = \text{constant}$$

$$\textcircled{3} PV^{x_3} = \text{constant}$$

$$(x_3 > x_2 > x_1)$$



If $\textcircled{1} \rightarrow$ Isothermal

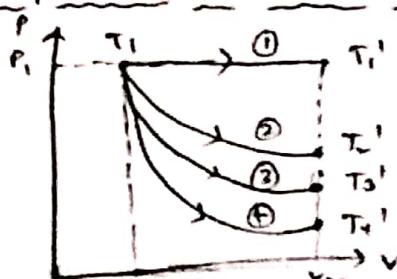
→ No intersection if both are isothermal

$\textcircled{2} \rightarrow$ may be Isothermal/adiabatic

If $\textcircled{1} \rightarrow$ Adiabatic

$\textcircled{2} \rightarrow$ must be adiabatic

Comparison of work done:-



$\textcircled{1}$ Isobaric

$\textcircled{2}$ Isothermal

$\textcircled{3}$ Adiabatic (diatomic)

$\textcircled{4}$ Adiabatic (mono atomic)

$$\begin{aligned} \gamma_{\text{mono}} &= 5/3 \approx 1.66 \\ \gamma_{\text{di}} &= \gamma_{\text{tri}} = 1.4 \end{aligned}$$

Widabaric \rightarrow Wadi-isothermal \rightarrow Wadi-dia \rightarrow Wadi-mono [area] (5)

$$\begin{array}{l} PV = nRT \\ P \propto T_F \end{array} \quad \left| \begin{array}{l} T_1' > T_2' > T_3' > T_4' \\ (dT)_4 < dT_3 < (dT)_2 < (dT)_1 \\ \downarrow \\ dU_4 < dU_3 < dU_2 < dU_1 \end{array} \right. \quad (\Delta T = T_F - T_i)$$

(b) If one fourth of heat supplied to the gas is converted into work, then find the equation of process and also molar specific heat capacity of gas?

(n = no. of moles, gas is monoatomic)

Sol: $\frac{dQ}{t} = dW$

or $dQ = dU + dW$

$$3dW = dU \Rightarrow \frac{dU}{dW} = 3 \Rightarrow \frac{nC_v(dT)}{nR(dT)} = 3 \Rightarrow C_v = 3R$$

$\Rightarrow C_p = nC_v(dT) = 3P(dV)$

$$\frac{3nR}{2}(dT) = 3P(dV) \Rightarrow nR(dT) = 2P(dV)$$

$\Rightarrow PdV + VdP = 2P(dV)$.

$PdV = VdP$

$$\Rightarrow C = C_V + \frac{P}{1-\alpha}$$

$$\frac{dP}{dV} = \frac{P}{V} \text{ in } \cancel{\frac{2}{1-\alpha}} \Rightarrow \alpha = 1 \Rightarrow C = C_V + \frac{\alpha P}{2} = 2R$$

(c) If molar specific heat capacity is changing according to

(i) $C = C_V + \alpha T$

(ii) $C = C_V + \beta T$ (α, β, α are constants).

(iii) $C = C_V + \alpha P$

Find the eqn. of process for each relation.

Sol: if $dQ = dU + dW$

$nC(dT) = nC_V(dT) + \alpha P(dV)$

$$C_V(dT) + \frac{P(dV)}{n\alpha T} = \frac{\alpha P(dV)}{n\alpha T} = \frac{dT}{T}$$

$$\frac{n\alpha T \cdot dV}{V \cdot n\alpha T} = \frac{dT}{T} \Rightarrow R \left(\frac{dV}{V} \right) = \frac{dT}{T}$$

$R \alpha V = dT + C$

$$dV = \frac{dT - C}{R\alpha} \Rightarrow V = C + \frac{(dT + C)}{R\alpha}$$

$$(i) C = C_V + \frac{P(dV)}{n\alpha T} \Rightarrow \frac{P(dV)}{n\alpha T} = \beta \cdot V$$

$$\frac{R T \cdot dV}{V \cdot dT} = \beta \cdot V$$

$$R \int \frac{dV}{V^2} = \beta \int \frac{dT}{T}$$

$$\frac{C - R}{V} = \beta \rightarrow (cont)$$

$$cont = \frac{1}{\beta} (C - \frac{R}{V})$$

$$T = \frac{1}{\beta} (C - \frac{R}{V})$$

$$(ii) \frac{P \cdot dV}{n \cdot dT} = \alpha \cdot P$$

$$\int dV = \alpha n dT$$

$$V = \alpha n T + C$$

$$\left(\frac{V - C}{\alpha n} \right) = T$$

8)

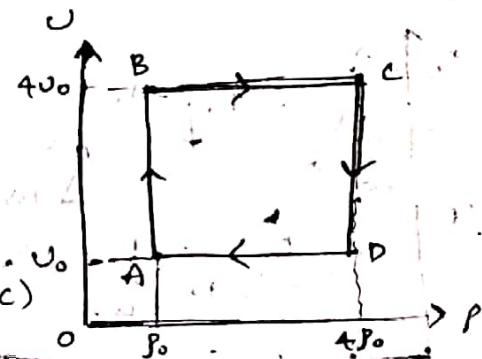
(i) Find the work done in process of BC?

(ii) Find total heat rejected/absorbed by the gas in one cycle?

Sol: (i) BC \rightarrow Isothermal.

$$(dV=0) \quad v_1 = \frac{m}{p_0} \quad v_2 = \frac{m}{4p_0}$$

(gas-monoatomic)



$$W_{BC} = nRT \ln\left(\frac{V_2}{V_1}\right) = nR \ln\left(\frac{1}{4}\right)(T) \quad (AB \rightarrow \text{isochoric})$$

$$W_{BC} = nR \ln\left(\frac{1}{4}\right) \left(\frac{U}{nR}\right)$$

$$= R \cdot \ln\left(\frac{1}{4}\right) \left(\frac{4U_0}{3R}\right)^{\times 2}$$

$$= -\frac{16U_0}{3}(\ln 2)$$

\rightarrow For complete cyclic process; $\Delta U=0$.

$$\Rightarrow dQ_{\text{cycle}} = dW_{\text{cycle}}$$

$$= W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= -\frac{16U_0}{3} \ln 2 + (\ln 2) \cdot \frac{4U_0}{3}$$

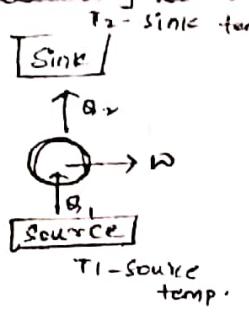
$$= -\frac{4U_0}{3}(\ln 2) \cdot -4U_0(\ln 2)$$

$$(C_V)_{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$$

$$(C_P)_{\text{mix}} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2}$$

$$\gamma_{\text{mix}} = \frac{n_1 \gamma_1 + n_2 \gamma_2}{n_1 + n_2}$$

Efficiency of Carnot Engine



By com. of energy

$$Q_1 = Q_2 + W \\ \Rightarrow W = Q_1 - Q_2$$

Efficiency of engine

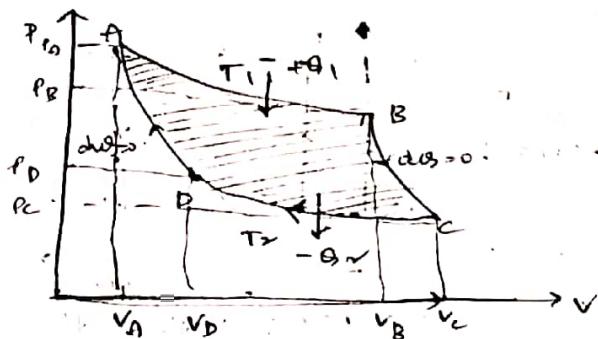
$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\star \quad \eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}} \quad \text{In Kelvin.}$$

→ 100% efficient engine is not possible

CARNOT'S CYCLE



AB - isothermal exp.

BC - adiabatic exp.

CD - isothermal comp.

DA - adiabatic comp.

$dQ_{\text{cycle}} = dW_{\text{cycle}}$

$dW_{\text{cycle}} = Q_1 - Q_2$

$$\text{Efficiency of cycle cycle } (\eta) = \frac{dW}{dQ} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{nRT_1 \ln \left(\frac{V_C}{V_D} \right)}{nRT_2 \ln \left(\frac{V_B}{V_A} \right)} = 1 - \frac{T_2 \ln \left(\frac{V_C}{V_D} \right)}{T_1 \ln \left(\frac{V_B}{V_A} \right)}$$

$$P_A V_A = P_B V_B \quad \text{--- 1} \quad P_C V_C = P_D V_D \quad \text{--- 2}$$

$$P_B V_B^{\gamma} = P_C V_C^{\gamma} \quad \text{--- 3} \quad P_D V_D^{\gamma} = P_A V_A^{\gamma} \quad \text{--- 4} \quad \text{--- 1} \times \text{--- 2} \times \text{--- 3} \times \text{--- 4} \quad \left(1 - \frac{T_2}{T_1} \right)$$

$$\Rightarrow V_A V_B^{\gamma} \cdot V_C \cdot V_D^{\gamma} = V_A^{\gamma} \cdot V_C^{\gamma} \cdot V_B \cdot V_D$$

$$\Rightarrow V_B^{\gamma-1} V_D^{\gamma-1} = V_C^{\gamma-1} V_A^{\gamma-1}$$

$$V_B V_D = V_C V_A$$

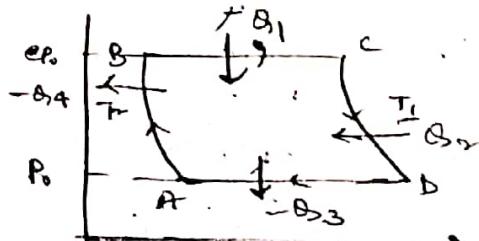
$$\frac{V_A}{V_B} = \frac{V_D}{V_C}$$

Q) If a mono-atomic ideal gas is taken through a cycle which consists of two isotherms at temp T_1 & T_2 ($T_2 < T_1$), and 2 isobars at pressures P_1 , P_2 . Find efficiency of cycle?

$$\text{Sol: } \eta = \frac{\text{work done}}{\text{Heat Supplied}} = \frac{dQ_{\text{cycle}}}{Q_1 + Q_2}$$

$$= \frac{Q_1 + Q_2 - Q_3 - Q_4}{Q_1 + Q_2}$$

$$= 1 - \left(\frac{Q_3 + Q_4}{Q_1 + Q_2} \right)$$



$$q_1 = mcp \Delta T = mcp(T_1 - T_2)$$

$$q_2 = mRT_1 \ln \frac{P_1}{P_0} = mRT_1$$

$$q_3 = mcp \Delta T = -mcp(T_2 - T_1) = mcp(T_1 - T_2)$$

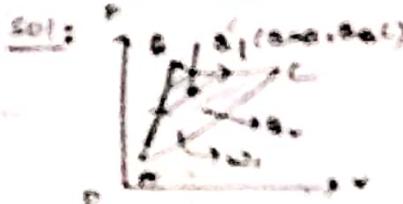
$$q_4 = mRT_2 \ln \frac{P_0}{P_1} = (mRT_2) = mRT_2$$

∴ $\eta = \frac{q_2 + q_3}{q_1 + q_2 + q_3 + q_4} = \frac{\cancel{mcp}(T_1 - T_2) + \cancel{mRT}_2}{\cancel{mcp}(T_1 - T_2) + \cancel{mRT}_1}$

$$\therefore 1 - \frac{\frac{1}{2}(T_1 - T_2) + T_2}{\frac{1}{2}(T_1 - T_2) + T_1} = 1 - \left(\frac{T_1 - 3T_2}{2T_1 + 5T_2} \right) = \frac{2(T_1 - T_2)}{2T_1 + 5T_2}$$



If efficiency of cycle ABCD is 20% and efficiency of cycle ACDE is 10%. Find the efficiency of ACBA?



$\Delta w_{AC} = \Delta w_{ABC}$

$$q_{AC} = \Delta Q = \Delta w_{AC}$$

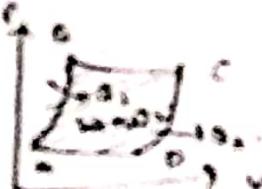
$$q_{AC} = \frac{w_{AC}}{\eta_{AC}} = \frac{1}{5} \Rightarrow q_{AC} = 0.2 \text{ kJ}$$



$\Delta w_{AC} = \Delta w_{ACDE}$

$$q_{AC} = \Delta Q = \Delta w_{AC}$$

$$q_{AC} = \frac{w_{AC}}{\eta_{AC}} = \frac{1}{10} \Rightarrow q_{AC} = 0.1 \text{ kJ}$$



$$q_{AC} = \frac{w_{AC}}{\eta_{AC}} = \frac{0.2 + 0.1}{0.2} = \frac{3}{2} \times 100$$

= 250%

$$\Delta w_{AC} = 10$$

$$q_{AC} = 10 \times 2 = 20$$

$$q_{AB} = 10$$

$$q_{BC} = 10$$

$q_{p-0} = 0$	$q_{0-1} = mcp \Delta T$
$q_{1-2} = mcp \Delta T$	$q_{2-3} = 0$
$q_{3-0} = mcp \Delta T$	$q_{0-p} = mcp \Delta T$

→ When some valve piston is connected via spring.

∴



With spring, i.e., in relaxed state.

∴ $\Delta w_{AC} = 0$

∴ Some torque.

∴ $q_{p-0} \neq mcp \Delta T$



$$P_{gas} - \frac{F_{gas}}{A} = P_0 + \frac{mg + kx}{A}$$

$$PV - nRT = \frac{PV}{T} = \text{const.}$$

$$\frac{P(V_0)}{T_0} = \left[P_0 + \left(\frac{mg + kx}{A} \right) \right] [V_0, A, x].$$

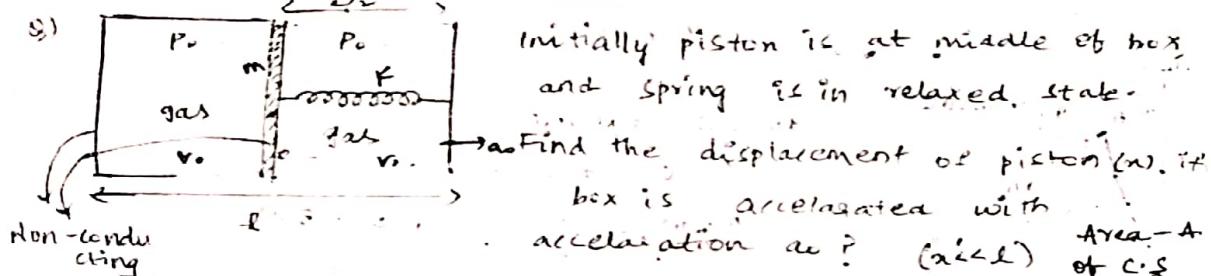
Change in Internal Energy of gas

$$dU = nC_V (\varphi - T^2)$$

Work done by gas = $w = \int dU = \int P.dV$

$$= \int \left(P_0 + \frac{mg + kx}{A} \right) . (Adx)$$

$$W_d = P_0 A x_0 + mgx_0 + \frac{kx_0^2}{2}$$



Soln:

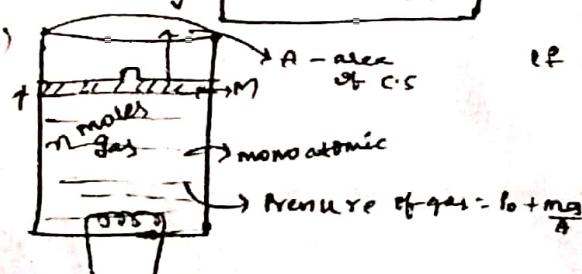
$P_1 A + kx = P_2 A + mao$
 $(P_2 - P_1)A = mao - kx.$

$P(V_0 - Ax)^2 = P_0 V_0^2$
 $P_1 = P_0 \left(\frac{V_0}{V_0 - Ax} \right)^2 = P_0 \left(1 + \frac{Ax}{V_0} \right)^2$

$P_1 - P_2 = \frac{P_0 x^2 A x}{V_0} \Rightarrow \frac{P_0 x^2 A x}{\frac{2\pi l^2}{4} / 2} = \frac{4P_0 Ax}{l}$

$\Rightarrow P_0 \frac{4Ax}{l} = mao - kx$

$$x = \frac{mao}{k + 4P_0 A/l}$$



$$\text{Q: } \frac{dQ}{dt} = P \quad V_{01} = A$$

$$dW = nRdT$$

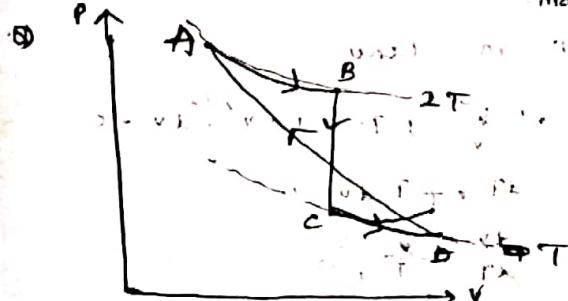
$$dQ = nCPdT$$

$$\frac{dW}{dQ} = \frac{1}{nC} = \gamma_s \Rightarrow dW = \frac{1}{\gamma_s} dQ \Rightarrow PdV = \frac{2}{\gamma_s} \times dQ \Rightarrow PdV = \frac{2}{\gamma_s} \times dQ$$

$$\Rightarrow \frac{2}{\gamma_s} dQ = dU$$

$$\Rightarrow V = \frac{2P}{\gamma_s(P_0A + mg)}$$

$$\text{Acceleration of piston is } \frac{dV}{dt} = \frac{2 \cdot (dP/dt)}{\gamma_s(P_0A + mg)}$$



AB, CD = Isothermal

DA = Adiabatic (gas is monoatomic)

$$\text{If } \frac{V_B}{V_A} = \frac{V_D}{V_C} = K$$

Find the value of $\ln K$ if the work done in ABCDA is zero.

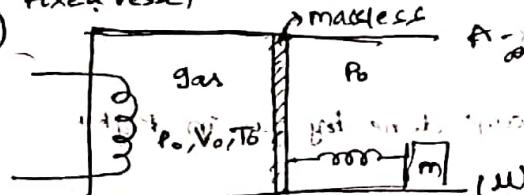
Sol:

$$W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0$$

$$nR(2T) \ln K + nRT \ln K + nRT \frac{V}{1 - \frac{1}{\gamma_s}} = 0$$

$$\Rightarrow \frac{nRT \ln K}{1 - \frac{1}{\gamma_s}} = \frac{nRT}{1 - \frac{1}{\gamma_s}} \Rightarrow \ln K = \frac{1}{\gamma_s}$$

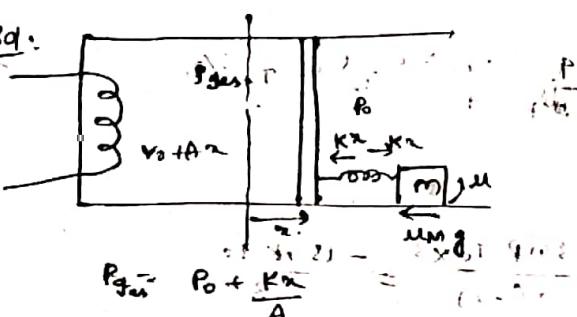
Q)



(a) Find the temp. of gas at the time of block's slide on the surface

Initially, spring is in relaxed state. (b) Find the heat supplied to the gas.

Sol:



$$P_{gas} = P_0 + \frac{kx}{A}$$

$$\frac{P_0 + kx}{V_0 + A \cdot 2} = \frac{P_0}{V_0}$$

$$T = \frac{(P_0 + kx)(V_0 + A \cdot 2)}{A \cdot P_0 V_0} \cdot T_0$$

$$kx = umg$$

$$x = \frac{umg}{k}$$

$$\text{At the time } x = umg$$

$$T = \frac{(P_0 + umg)(V_0 + A \cdot umg)}{A \cdot P_0 V_0} \cdot T_0$$

$$T = \frac{(P_0 + umg)(kV_0 + umg)}{A \cdot P_0 V_0} \cdot T_0$$

$$b) dQ = dU + dW$$

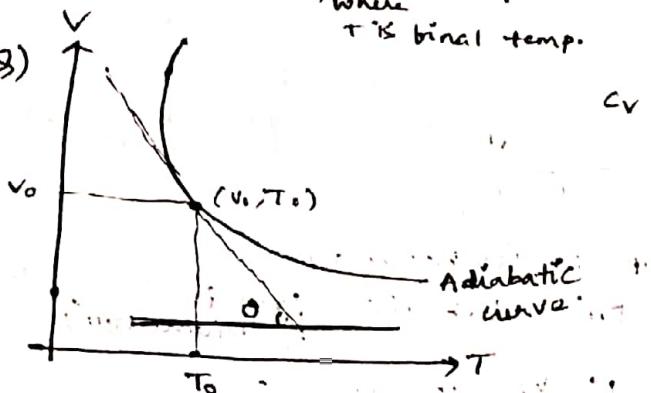
$$= nC_v \frac{3nR(\Delta T)}{2} + \int_0^{\mu mg} P_0 A dx + Kx \cdot dx.$$

$$= \frac{3nR\Delta T}{2} + \frac{P_0 A \cdot \mu mg}{K} + \frac{1}{2} \times K \times \frac{\mu^2 m^2 g^2}{K^2}$$

$$= \frac{3nR}{2} (T - T_0) + \frac{P_0 A \mu mg}{K} + \frac{\mu^2 m^2 g^2}{2K}$$

where T is final temp.

8)



C_V of the gas = ?

$$TV^{2-1} = \text{constant}$$

$$m =$$

$$\text{Sol: } m = -\tan \theta.$$

$$\Delta T \cdot V^{2-1} \neq T \cdot (2-1) \cdot \frac{dV}{V} \Rightarrow \Delta T \cdot V = 0.$$

$$\Delta T \cdot V + T \cdot dV = 0$$

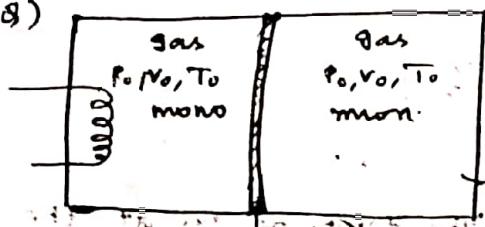
$$\frac{dV}{dT} = \frac{V - V_0}{T(T-1)}$$

$$\tan \theta = \frac{V_0}{T_0(2-1)}$$

$$-(2-1) = \frac{V_0}{T_0 + m \cdot 0}$$

$$C_V = \frac{R}{2-1} \cdot \frac{1}{T_0} \cdot \frac{1}{1-m} = \frac{R \cdot T_0 \cdot m}{K \cdot V_0 \cdot (2-1)}$$

9)



a) Find the temperature of each chamber when pressure of gas become

$$\left(\frac{243}{32}\right) P_0.$$

b) Find the work done by gas in Right chamber.

$$\text{Sol: a) } P_0 T_0 = \left(\frac{243}{32}\right)^{1-2} \cdot T_2$$

$$T_2^2 = \left(\frac{243}{32}\right)^{1-2} \cdot T_0^2$$

$$T_2 = \sqrt[3]{\frac{243}{32}} \cdot T_0$$

$$\frac{9}{4} \cdot \left(\frac{9}{4}\right)^{5/3} - T_0^{5/3} = T_2^{5/3}$$

b) $dQ = dU + dW$

$$= n \left(\frac{5R}{2}\right) \left(\frac{5}{4}\right) T_0^2 + \frac{dN \cdot n \cdot R \cdot T}{1-2} = \frac{5nR T_0^2 \cdot 3}{4(-2)} = \left(-\frac{15nR T_0}{8}\right)$$

$$a) PV^2 = \text{constant}$$

$$P_0 V_0^2 = \left(\frac{243}{32}\right) P_1 V_1^2$$

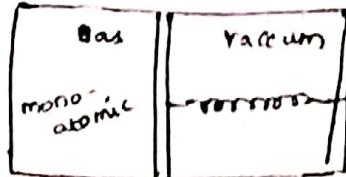
or $P_0 V_0^2 = \left(\frac{243}{32}\right) P_1 V_1^2$

$$\therefore V_1 = \frac{8V_0}{27}$$

$$\frac{P_0 \cdot V_0}{T_0} = \frac{\left(\frac{243}{32}\right) P_1 \cdot \frac{4V_0}{27}}{T_0}$$

$$T_1 = \frac{20 + T_0}{16}$$

Q)



A) area of C.S.

(12)

B) natural length

If natural length of spring is equal to the length of container
Find the molar specific heat capacity (C) of the gas in
the given situation?

Sol:

$$C = Cv + \frac{P}{T - x}$$

$$P_{gas} = \frac{kx}{A}$$

$$V_{gas} = A \cdot x$$

$$\frac{P_{gas} \cdot n}{k} = \frac{v_{gas}}{A}$$

$$\frac{P_{gas}}{P \cdot V^{-1}} = k/A^2 \Rightarrow [n = -1]$$

$$C = Cv + \frac{P}{2} = 2R$$

Q) A gas ~~vacuum~~ undergoes process such that $P \propto \frac{1}{T}$. If molar heat capacity for this process $\Rightarrow C = 33.24 \text{ J/mol-K}$. Find the DOF of gas.

Sol: $P \cdot T = \text{const}$

$$P^{1/2} \cdot T^{1/2} = \text{const}$$

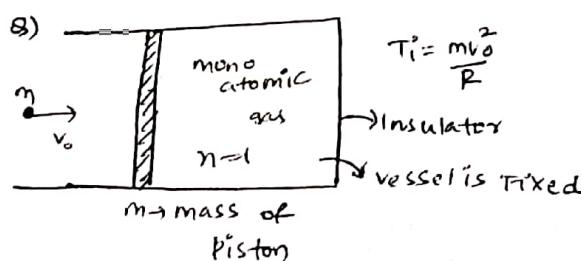
$$P \left(\frac{P \cdot V}{nR} \right)^{1/2} = \text{const} \Rightarrow P \cdot V^{1/2} = \text{const}$$

$$C = 4R$$

$$4R = \frac{fR}{2} + 2R$$

$$f = 4$$

$$C = Cv + \frac{P}{1-n}$$



A) If collision is perfectly ~~inelastic~~
Find the ratio of the max,
min temperature.

B) What if collision is perfectly elastic?

Soln:

$$mv_0 = (m+m)v$$

$$dW = \Delta KE$$

$$v = v_0/2$$

$$= \frac{1}{2}(2m) \left(\frac{v_0}{2} \right)^2 = 2mv_0^2 = \frac{mv_0^2}{4}$$

$$dQ = 0 \Rightarrow -dU = dW$$

$$\int dU = Cv(T_i - T_f) = -\frac{mv_0^2}{4}$$

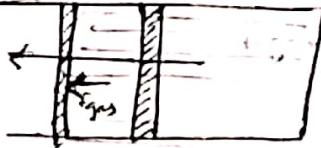
$$\frac{3R}{2} \left(\frac{mv_0^2}{R} - T_f \right) = \frac{mv_0^2}{4}$$

$$\frac{mv_0^2}{R} - T_f = -\frac{mv_0^2}{6R}$$

$$\Rightarrow T_f = \frac{7mv_0^2}{6R} = T_{\max}$$

$$T_f = T_{\max} = \frac{mv_0^2}{12}$$

(a) $m(v_0) \rightarrow$



$$\frac{T_{\max}}{T_{\min}} = \frac{-4T_i/6}{5T_i/6} = \frac{4}{5}$$

ω is +ve

$$dU = -dW$$

$$\frac{n(3P)}{2} (T_f - T_i) = -\frac{1}{2}(2m) \left(\frac{v_0}{2}\right)^2$$

$$T_f - T_i = \frac{-mv_0^2}{6R}$$

$$T_f - T_i = -\frac{mv_0^2}{6R} \approx \frac{5T_i}{6}$$

$$(b) m(v_0) = m(v_0) + m(v)$$

$$\Rightarrow v = v_0$$

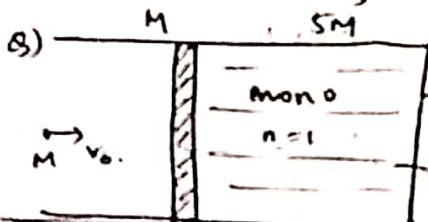
$$dW = \Delta KE$$

$$= \frac{1}{2}m(v_0)^2 - \frac{1}{2}mv_0^2$$

$$dU = -dW$$

$$\frac{3nR}{2} (T_f - T_i) = n + \frac{1}{2}mv_0^2$$

$$T_f - T_i = \frac{T_i}{3} \Rightarrow T_f = \frac{4T_i}{3}$$



$$dU = -dW$$

$$\frac{3nR}{2} (T_f - T_i) = -\frac{1}{2}mv_0^2$$

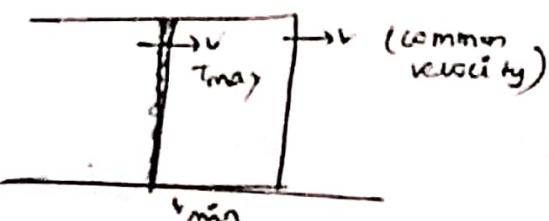
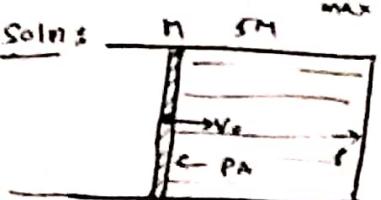
$$T_f - T_i = T_i/3$$

$$T_{\max} : T_{\min} = 2 : 1$$

$$T_i = \frac{Mv_0^2}{R}$$

Find the temp of gas attained?

Solns



$$m(v_0) : (5m)(v) \Rightarrow v = v_0/6$$

$$dU = -dW = -(\Delta KE)$$

$$= -(\Delta KE) \Rightarrow \Delta KE = \frac{1}{2}MV_0^2 - \frac{1}{2}(6m) \left(\frac{v_0}{6}\right)^2$$

$$\frac{3P}{2} (T_{\max} - T_i) = \frac{5MV_0^2}{12}$$

$$T_{\max} = T_i + \frac{5T_i}{12} = \frac{23T_i}{12}$$