BASARA SARASWATHI BHAVAN_MDP N-120

Sec: Sr. **INDEFINITE INTEGRATION SYNAPSIS** **Date:12-06-2020**

SOME STANDARD FORMULAE:

(i) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\ell n|ax+b| + C$

(iii) $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + C$

(iv) $\int a^{px+q} dx = \frac{a^{px+q}}{p\ \ell na} + C; a > 0$

(v) $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$

(vi) $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$

(vii) $\int \tan(ax+b)dx = \frac{1}{a}\ell n|\sec(ax+b)| + C$

(viii) $\int \cot(ax+b)dx = \frac{1}{a}\ell n|\sin(ax+b)| + C$

(ix) $\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$

(x) $\int \operatorname{cosec}^2(ax+b)dx = -\frac{1}{a}\cot(ax+b) + C$

(xi) $\int \sec(ax+b).\tan(ax+b)dx = \frac{1}{a}\sec(ax+b) + C$

(xii) $\int \operatorname{cosec}(ax+b).\cot(ax+b)dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + C$

(xiii) $\int \sec x\ dx = \ell n|\sec x + \tan x| + C$ (or) $\ell n\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

(xiv) $\int \operatorname{cosec} x\ dx = \ell n|\operatorname{cosec} x - \cot x| + C$ (or) $\ell n\left|\tan\frac{x}{2}\right| + C$ (or) $-\ell n|\operatorname{cosec} x + \cot x| + C$

(xv) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(xvi) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(xvii) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$

(xviii) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ell n\left|x + \sqrt{x^2+a^2}\right| + C$ (or) $\sinh^{-1}\frac{x}{a} + C$

(xix) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ell n\left|x + \sqrt{x^2-a^2}\right| + C$ (or) $\cosh^{-1}\frac{x}{a} + C$

(xx) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ell n\left|\frac{a+x}{a-x}\right| + C$

(xxi) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ell n\left|\frac{x-a}{x+a}\right| + C$

(xxii) $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

(xxiii) $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ell n\left|x + \sqrt{x^2+a^2}\right| + C$

(xxiv) $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ell n\left|x + \sqrt{x^2-a^2}\right| + C$

(xxv) $\int e^{ax}.\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$

(xxvi) $\int e^{ax}.\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx + b\cos bx) + C$

THEOREMS ON INTEGRATION

i) $\int C\, f(x).dx = C\int f(x).dx$

ii) $\int \left(f(x) \pm g(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx$

iii) $\int f(x)\,dx = g(x) + C_1 \Rightarrow \int f(ax+b)\,dx = \dfrac{g(ax+b)}{a} + C_2$.

iv) $\int \left(f(x)\right)^n f'(x)dx = \dfrac{\left(f(x)\right)^{n+1}}{n+1} + c$

v) $\int \dfrac{f'(x)}{\left(f(x)\right)^n} dx = \dfrac{\left(f(x)\right)^{-n+1}}{-n+1} + c$ where $n \neq 1$

vi) $\int \dfrac{f'(x)}{f(x)} dx = \log_e\left(|f(x)|\right) + c$

vii) $\int \dfrac{f(x)g^1(x) - g(x)f^1(x)}{f(x)g(x)} dx = \ln\left(\dfrac{g(x)}{f(x)}\right) + c$

Examples:

1) $\int \dfrac{\sqrt{5+x^{10}}}{x^{16}} dx$ Is equal to $\dfrac{-1}{75}\left(1+\dfrac{5}{x^{10}}\right)^{\frac{1}{2}} + c$

2) $\int \dfrac{5x^4+4x^5}{\left(x+1+x^5\right)^2} dx = \ldots.\quad \dfrac{x^5}{x+1+x^5} + c$

3) $\int \dfrac{\left(x^2-1\right)}{x^3\sqrt{2x^4-2x^2+1}} dx = \dfrac{1}{2}\sqrt{2 - \dfrac{2}{x^2} + \dfrac{1}{x^4}} + c$

4) $\int \dfrac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3} dx = \ldots.\quad \dfrac{x^{10}}{2\left(x^5+x^3+1\right)^2} + c$

5) $\int \left(x^{7m}+x^{2m}+x^m\right)\left(2x^{6m}+7x^m+14\right)^{1/m} dx = \dfrac{1}{14(m+1)}\left(2x^{7m}+7x^{2m}+14x^m\right)^{\frac{m+1}{m}} + c$

6) $\int \left(x^2+x\right)\left(x^{-8}+2x^{-9}\right)^{\frac{1}{10}} dx = \dfrac{5}{11}\left(x^2+2x\right)^{\frac{11}{10}} + C$

1) Integration of Type: $\int \dfrac{L_1(x)}{L_2(x)} dx$ where $L_1(x)$ and $L_2(x)$ are linear functions in x

To evaluate such integrals write $L_1(x)$ in terms of $L_2(x)$ (i.e. $L_1(x) = A.L_2(x) + B$) then

$\int \dfrac{L_1(x)}{L_2(x)} dx = \int \dfrac{A.L_2(x)+B}{L_2(x)} dx = Ax + B\int \dfrac{1}{L_2(x)} dx$ **OR** divide and proceed

2) Integration of Type: $\int \dfrac{L_1(x)}{\sqrt{L_2(x)}} dx$ where $L_1(x)$ and $L_2(x)$ are linear functions in x

To evaluate such integrals write $L_1(x)$ in terms of $L_2(x)$ (i.e. $L_1(x) = A.L_2(x) + B$) then

$\int \dfrac{L_1(x)}{\sqrt{L_2(x)}} dx = \int \dfrac{A.L_2(x)+B}{\sqrt{L_2(x)}} dx = A.2.\sqrt{L_2(x)} + B\int \dfrac{1}{\sqrt{L_2(x)}} dx$ **OR** take $t^2 = L_2(x)$ and proceed

3) Integration of Type: $\dfrac{1}{2}\sec^2 x dx = dt$ where $L_1(x)$ and $L_2(x)$ are linear functions in x

To evaluate such integrals write $L_1(x)$ in terms of $L_2(x)$ (i.e. $L_1(x) = A.L_2(x) + B$) then

$\int L_1(x)\sqrt{L_2(x)} dx = \int A.\left(L_2(x)\right)^{\frac{3}{2}} + B\sqrt{L_2(x)}\ dx = A.\dfrac{2}{5}.\left(L_2(x)\right)^{\frac{5}{2}} + B\int \sqrt{L_2(x)}\, dx$ **OR** take $t^2 = L_2(x)$ and proceed

4) Integration of Type: $\int \sqrt{\dfrac{L_1(x)}{L_2(x)}} dx$ where $L_1(x)$ and $L_2(x)$ are linear functions in xtake $t^2 = L_2(x)$ and proceed

5)Integration of Type: $\int \frac{1}{Quadratic}dx$ or $\int \frac{1}{\sqrt{Quadratic}}dx$ or $\int \sqrt{Quadratic}\,dx$

Examples: $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\ dx$

Express $ax^2 + bx + c$ in the form of perfect square & then apply the suitable formula

In case of $\int \frac{1}{Quadratic}$, Quadratic equation can be factorized, then partial fraction will help to integrate.

6)Integration of type: $\int \frac{linear}{quadratic}dx$ or $\int \frac{linear}{\sqrt{quadratic}}dx$ or $\int linear.\sqrt{quadratic}\,dx$

$\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx, \int (px+q)\sqrt{ax^2+bx+c}\ dx$

Express px + q = A (differential co – efficient of denominator) + B and
find the values of A and B and proceed.

In case of $\int \frac{linear}{quadratic}dx$, Quadratic equation can be factorized, then partial fraction will help to integrate.

7a)Integration of the type

$\int \frac{1}{x\left(x^n+1\right)}dx$ or $\int \frac{1}{x^n\left(x^n+1\right)^{\frac{1}{n}}}dx$ or $\int \frac{1}{x^2\left(x^n+1\right)^{\frac{n-1}{n}}}dx$

Take x^n common and let t = $\left(1+\frac{1}{x^n}\right)$ & proceed

Examples:

1) $\int \frac{dx}{x^{20}(1+x^{20})^{\frac{1}{20}}} = -\frac{1}{19}\left(1+\frac{1}{x^{20}}\right)^{\frac{19}{20}}+C$

2) $\int \frac{dx}{x^{22}\left(x^7-6\right)} = A\left\{\log u^6+9u^2-2u^3-18u\right\}+c \quad A=\frac{1}{54432}, u=\left(\frac{x^7-6}{x^7}\right)$

7b)Integration of the type

I = $\int \frac{x^m}{\left(ax+b\right)^n}dx$ where m,n are natural numbers

Put t = ax + b then I = $\frac{1}{a^{m-1}}\int \frac{\left(t-b\right)^m}{t^n}dt$

[example: $\int \frac{x^2}{\left(x+2\right)^3}dx$]

7c)Integration of the type

$\int \frac{dx}{x^m\left(ax+b\right)^n}$ where m, n are natural numbers

Put $t=\frac{ax+b}{x}$

[example: $\int \frac{dx}{x^3\left(ax+b\right)^2}$]

8)Integration of the type: $\int \frac{dx}{L_1\sqrt{L_2}}$ OR $\int \frac{dx}{Quadratic\sqrt{Linear}}$ take Linear = t^2 and proceed

Example: $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ OR $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$ Put px + q = t^2. And proceed

9)Integration of the type: $\int \frac{dx}{\text{Linear}\sqrt{\text{Quadratic}}}$, take L = 1/t and proceed

Example: $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put ax + b = $\frac{1}{t}$;

10)Integration of the type: $\int \frac{dx}{Q_1\sqrt{Q_2}}$, take x = 1/t and proceed

Example: $\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$, put x= $\frac{1}{t}$

11)Integration of the type:

(i) $\int \frac{dx}{a \pm b\sin^2 x}$ OR $\int \frac{dx}{a \pm b\cos^2 x}$ OR $\int \frac{dx}{a\sin^2 x \pm b\sin x\cos x \pm c\cos^2 x}$ OR $\int \frac{dx}{a \pm b\sin 2x}$ OR $\int \frac{dx}{a \pm b\cos 2x}$

$\int \frac{1}{a\cos^2 x \pm b\sin^2 x}dx$ OR $\int \frac{1}{(a\cos x \pm b\sin x)^2}dx$ OR $\int \frac{1}{a\sin^2 x \pm b\cos^2 x \pm c}dx$

(Denominator is the expression in terms of sin2x or cos2x or $\sin^2$x or $\cos^2$x or $\sin^4$x or $\cos^4$x)
Multiply Numerator & Denominator by $\sec^2$ x and hence convert the question in the form of $f(\tan x), \sec^2 x$ & put tan x = t **or** Multiply Numerator & Denominator by cosec^2 x and hence convert the question in the form of $f(\cot x), \cos ec^2 x$ & put cotx = t

12)Integration of the type: $\int \frac{dx}{a \pm b\sin x}$ OR $\int \frac{dx}{a \pm b\cos x}$ OR $\int \frac{dx}{a \pm b\sin x \pm c\cos x}$

Convert sines & cosines into their respective tangents of half the angles and then, put $\tan\frac{x}{2} = t$,

$\frac{1}{2}\sec^2 x dx = dt$ and proceed

13)Integration of the type: $\int \frac{a.\cos x + b.\sin x}{\ell.\cos x + m.\sin x}dx.$, $\int \frac{1}{a+b\tan x}dx$, $\int \frac{1}{a+b\cot x}dx$, $\int \frac{a+b\cot x}{c+d\cot x}dx$,

$\int \frac{\tan x}{a+b\tan x}dx$, $\int \frac{1}{a\sin x + b\cos x}dx$

Express Numerator = A(Denominator) + B$\frac{d}{dx}$(Denominator). And find the value of the constants A and B by comparing the co oefficients of cosx and sinx and proceed.

(i.e. $\int \frac{N}{D}dx = \int \frac{A.(D)+Bd(D)}{D}dx = Ax + B\ln|D| + c$)

Examples: 1) $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx$ 2) $\int \frac{3\sin x+2\cos x}{2\sin x+3\cos x}dx$ 3) $\int \frac{1}{2+3\tan x}dx$

14)Integration of the type: $\int \frac{a.\cos x + b.\sin x + c}{\ell.\cos x + m.\sin x + n}dx.$

Express Numerator = A(Denominator) + B$\frac{d}{dx}$(Denominator) +K and find the values of A, Band K by comparing the co oefficients of cosx and sinx and proceed.

(i.e. $\int \frac{N}{D}dx = \int \frac{A.(D)+Bd(D)+K}{D}dx = Ax + B\ln|D| + K\int \frac{1}{D}$)

Example:

15). Integration of the type $\int \frac{1}{a\sin x + b\cos x}dx$ or $\int \frac{1}{\left(a\cos x \pm b\sin x\right)^2}dx$

convert $a\sin x + b\cos x$ as a single term = $\sqrt{a^2+b^2}\left(\sin(A+x)\right)$ OR $\sqrt{a^2+b^2}\left(\cos(x-A)\right)$ and proceed.

16). Integration of the type a) $\int \cos mx.\cos nx.dx$, $\int \sin mx.\sin nx.dx$, $\int \cos mx.\sin nx.dx$ and

Write the integrant as a sum of two terms and proceed

b) $\int \tan(a+b)x.\tan ax.\tan bx.dx$ = $\int (\tan(a+b)x - \tan ax - \tan bx)dx$

c) $\int \frac{1}{\sin(x-a)\sin(x-b)}dx = \frac{1}{\cos(a-b)}\int \frac{\cos\left((x-b)-(x-a)\right)}{\sin(x-a)\sin(x-b)}dx$

d) $\int \frac{\sin(x+a)}{\sin(x+b)}dx = \int \frac{\sin\left((x+b)+(a-b)\right)}{\sin(x+b)}dx$

17)Integration of type $\int \sin^{m} x.\cos^{n} xdx$

Case: i If atleast one of m or n is odd natural number, then if m is odd put cos x = t and vice – versa.

Case: ii When m + n is a negative even integer (say **k**), multiply and divide by $\cos^{k} x$ hence convert the question in the form of $f(\tan x), \sec^2 x$ then put tan x = t, to evaluate the integration.

Case: iii If m and n are even natural number then converts higher power into higher angles.

Examples: 1) $\int \left(\frac{\sin^2 x}{\cos^{14} x}\right)^{\frac{1}{3}} dx$ 2) $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ 3) $\int \frac{\cos^4 x}{\sin^3 x\left(\sin^5 x + \cos^5 x\right)^{\frac{3}{5}}}xdx$

18)Integrals of the form

$\int_{\sim} \frac{dx}{\sqrt{(x-\sim)(\;\;-x)}}$, $\int_{\sim} \sqrt{\frac{x-\sim}{\;\;-x}}dx$, $\int_{\sim} \sqrt{(x-\sim)(\;\;-x)}dx$ can also be solved by using the substitution

$x = \sim\cos^{2-} + \;\; \sin^{2-}$ or x = $\alpha\sec^2\theta - \beta\tan^2\theta$

18a) Some standard substitution:

$a^2 + x^2 \Rightarrow x = a\tan^{-}$ (or) $a\cot^{-}$, $a^2 - x^2 \Rightarrow x = a\sin^{-}$ (or) $a\cos^{-}$,

$x^2 - a^2 \Rightarrow x = a\sec^{-}$ (or) $a\cos ec^{-}$

$\sqrt{\frac{a-x}{a+x}}$ (or) $\sqrt{\frac{a+x}{a-x}} \Rightarrow x = a\cos 2^{-}$

$\sqrt{\frac{x}{a-x}} \Rightarrow$ x = $a\sin^{2-}$

$\sqrt{\frac{x}{a+x}} \Rightarrow$ x = $a\tan^{2-}$

$\sqrt{(x-a)(b-x)}$ or $\sqrt{\frac{x-a}{b-x}} \Rightarrow$ x = $a\cos^{2-} + b\sin^{2-}$

$\sqrt{(x-a)(x-b)}$ or $\sqrt{\frac{x-a}{x-b}} \Rightarrow$ x = $a\sec^{2-} - b\tan^{2-}$

19) To evaluate theintegral of the form

$\int \frac{dx}{\sin x\left(a \pm b\cos x\right)}$, $\int \frac{dx}{\cos x\left(a \pm b\sin x\right)}$, $\int \frac{dx}{\sin x\left(\exp ression\ in\ terms\ of\ \cos x\right)}$ or

$\int \frac{dx}{\cos x\left(\exp ression\ in\ terms\ of\ \sin x\right)}$

Multiply and divide by sinx or cosx and let t = sinx or cosx and proceed

Examples: 1) $\int \frac{1}{\sin x+\sin 2x}dx$, 2) $\int \frac{1}{\sin x\left(2\cos^2 x-1\right)}dx$

20a) To evaluate the integral, of the form $\int \frac{dx}{(x-a)^{\frac{m}{n}}(x-b)^{2-\frac{m}{n}}}$

Take $t=\frac{x-a}{x-b}$ & $dx=\frac{a-b}{(x-b)^2}$ and proceed

Examples: 1) $\int \frac{dx}{(x-2)^{\frac{7}{8}}(x-1)^{\frac{9}{8}}}$ 2) $\int \frac{dx}{\left((x-2)^2(x-1)^4\right)^{\frac{1}{3}}}dx$ 3) $\int \frac{xdx}{\sqrt[2012]{(1+x^2)^{1012}(2+x^2)^{3012}}}$

20b) To evaluate the integral, of the form $\int \frac{dx}{(x-a)^m(x-b)^n}$

Take $t=\frac{x-a}{x-b}$ $if\ m<n$ & $x-a=\frac{(a-b)t}{1-t}$, $x-b=\frac{(a-b)}{1-t}$ and I = $\int \frac{(1-t)^{m+n-2}}{(a-b)^{m+n-1}t^m}dt$

21) Integration of the type

$\int \frac{x^2\pm 1}{x^4\pm Kx^2+1}dx$ or $\int \frac{x^2\pm a}{x^4\pm Kx^2+a^2}dx$ where K is any constant

Divide Numerator & Denominator by x²& put $x\mp\frac{1}{x}$ or $x\mp\frac{a}{x}=t$, and hence convert the question in the form of $\int f\left(x\pm\frac{1}{x}\right),1\mp\frac{1}{x^2}\ dx$

Examples: 1) $\int \sqrt{\tan x}dx$, 2) $\int \frac{1}{1+x^4}dx$, 3) $\int \frac{x^2}{1-x^2+x^4}dx$, 4) $\int \frac{x^2-3}{x^4+5x^2+9}dx$ 5) $\int \frac{1}{\sin^6 x+\cos^6 x}dx$

22) Integration of the type

$\int \frac{ax^2+b}{cx^4+dx^2+e}dx$ wherea,b,c,d and e are constants & cx^4+dx^2+e is factorable then use partial fraction to split the integration and integrate

Examples: 1) $\int \frac{1}{x^4-1}dx$, 2) $\int \frac{x^2}{x^4-1}dx$, 3) $\int \frac{x^2-3}{x^4+5x^2+4}dx$

23) To evaluate the integral of the form:

$\int \frac{\sin x\pm\cos x}{\exp ression\ in\ terms\ of\ \sin 2x}dx$ **or** $\int \sin x\pm\cos x).(\exp ression\ in\ terms\ of\ \sin 2x)dx$,

take $t=\sin x\pm\cos x$ & convert sin2x in the form of

$(\sin x\pm\cos x)$ using $\sin 2x=\begin{cases}(\sin x+\cos x)^2-1\\1-(\sin x-\cos x)^2\end{cases}$ and proceed

Examples:

1). $\int \frac{\sin x+\cos x}{9+16\sin 2x}dx$ = 2). $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ 3) $\int \frac{dx}{(\tan x+\cot x+\sec x+\cos ecx)}$

4). $\int \frac{\sin x-\cos x}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}}dx$ 5) $\int \frac{1}{\sin x+\sec x}dx$

24) To evaluate the integrals of the form ,

$\int \sqrt{\sec^2 x\pm a}\,dx$ or $\int \sqrt{co\sec^2 x\pm a}\,dx$

$\int \sqrt{\sec^2 x\pm a}\,dx = \int \frac{\sec^2 x\pm a}{\sqrt{\sec^2 x\pm a}}dx = \int \frac{\sec^2 x}{\sqrt{\sec^2 x\pm a}}dx\pm a\int \frac{\cos x}{\sqrt{1\pm\cos^2 x}}dx$

In the first part take u = tanx and in the econd part take v = sinx and proceed.

25) To evaluate the integral of the form:

a) $\int e^{ax}\sin bx\,dx=\frac{e^{ax}}{a^2+b^2}\left(a\sin bx-b\cos bx\right)+c$

b) $\int e^{ax}\sin\left(bx+c\right)dx=\frac{e^{ax}}{a^2+b^2}\left(a\sin\left(bx+c\right)-b\cos\left(bx+c\right)\right)+d$

c) $\int e^{ax}\cos bx\,dx=\frac{e^{ax}}{a^2+b^2}\left(a\cos bx+b\sin bx\right)+c$

d) $\int e^{ax}\cos\left(bx+c\right)dx=\frac{e^{ax}}{a^2+b^2}\left(a\cos\left(bx+c\right)+b\sin\left(bx+c\right)\right)+d$

Examples:

1) $\int e^{4x}\sin^2 x dx$ $=$ $\frac{e^{4x}}{40}\left(5-4\cos 2x-2\sin 2x\right)+c$

2) $\int e^{4x}\sin x\left(4\cos^2 x-1\right)dx$ $=$ $\frac{e^{4x}}{25}\left(4\sin 3x-3\cos 3x\right)+c$

3) $\int e^{3x}x\cos 4x dx$ 4) $\int\sin(\ln x)dx$

26). Integration by parts:

a) Product of two functions f(x) and g(x) can be integrated, using formula:

$\int\left(f\left(x\right)g\left(x\right)\right)dx=f\left(x\right)\int\left(g\left(x\right)\right)dx-\int\left(\frac{d}{dx}\left(f\left(x\right)\right)\int\left(g\left(x\right)\right)dx\right)dx$ **or**

$\int u(x).d\left(v(x)\right)=u(x).v(x)-\int v(x)d\left(u(x)\right)$

(i) When you find integral $\int g\left(x\right)dx$ then it will not contain arbitrary constant.

(ii) $\int g\left(x\right)dx$ should be taken as same at both places.

(iii) The choice of f(x) and g(x) can be decided by **ILATE** guideline.
The function will come later is taken an integral function (g(x)).
[I$\rightarrow$Inverse function, L$\rightarrow$Logarithmic function A$\rightarrow$Algebraic function T$\rightarrow$Trigonometric function E$\rightarrow$Exponential function.]

b) To evaluate $\int f$ (x) g(x) dx, particularly in the form

$\int$ (Algebraic function) (Exponential function)

$\int$ (Algebraic function) (Trigonometric function)

$\int$ (Algebraic function) (Algebraic function – particularly linear function), can use the following formula

$\int f(x)g(x)dx = f(x).\int g(x)dx-f'(x)\int\int g(x)dx+f''(x)\int\int\int g(x)dx-f'''(x)\int\int\int\int g(x)dx+......$

Examples:

i). $\int e^{2x}\left(2x^3+3x^2-8x+1\right)dx$

ii**).** $\int\left(3x^2+x-2\right)\sin^2(3x+1)dx$

iii). $\int\left(\ln x\right)^4 dx$

iv) $\int\frac{x^2-7x+1}{\left(2x+1\right)^{\frac{1}{3}}}dx$

c)Integration of the form $\int\frac{f(x)g'(x)}{\left(g(x)\right)^2}dx = f(x)\left(\frac{-1}{g(x)}\right)-\int\frac{-1}{g(x)}.f'(x)dx$

Examples:

i) $\int\frac{x^2}{\left(xCosx-Sinx\right)^2}dx=-\cot x+\frac{x\cos ecx}{x\cos x-\sin x}+c$

ii) $\int \frac{x^2+20}{(x\sin x+5\cos x)^2}dx = ... \frac{-x\sec x}{(x\sin x+5\cos x)} + \tan x + c$

d) Integrations of the form

i) $\int e^x\left(f(x)+f^1(x)\right)dx \;=\; e^x f(x)+c$

ii) $\int e^{kx}\left(f(kx)+f^1(kx)\right)dx \;=\; e^{kx} f(kx)+c$

iii) $\int e^x\left(f(x)-f^{11}(x)\right)dx \;=\; e^x\left(f(x)-f^1(x)\right)+c$

In general $\int e^x\left(f(x)-(-1)^n f^n(x)\right)dx \;=\; e^x\sum_{r=0}^{n-1}(-1)^r f^r(x)$ where $f^n(x)=\frac{d^n\left(f(x)\right)}{dx^n}$

vi) $\int\left(f(\log x) + f^1(\log x)\right)dx = x f(\log x) + c$

Examples:

a). $\int \frac{e^x\left(2-x^2\right)}{(1-x)\sqrt{1-x^2}}dx = ... \frac{e^x(1+x)}{\sqrt{1-x^2}}+c$

b). $\int\left(\ln(\ln x)+\frac{1}{(\ln x)^2}\right)dx = ... x\ln(\ln x)-\frac{x}{\ln x}+c$

c). $\int \frac{e^{\sin x}\left(x\cos^3 x-\sin x\right)}{\cos^2 x}dx = \ldots e^{\sin x}\left(x-\sec x\right)+c$

d). $\int e^x\left(x\cos x-\sec x\tan x\right)dx = ... e^{\sin x}\left(x-\sec x\right)+c$

e). $\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x-x\sin x+\cos x}{x^2\cos^2 x}\right)dx = ... xe^{x\sin x+\cos x}-\frac{e^{x\sin x+\cos x}}{x\cos x}+c$

f). $\int e^{f(x)}\left(xf'(x)+\frac{f''(x)}{\left(f'(x)\right)^2}\right)dx = \ldots\; e^x\left(x-\frac{1}{f'(x)}\right)+c$

e) To evaluate the integral of the form

v) $\int\left(f(x)g'(x)+f'(x)g(x)\right)dx = f(x)g(x)+c$

iv) $\int\left(xf'(x)+f(x)\right)dx \;= xf(x)+c$

Examples:

i) $\int \sin 51x(\sin x)^{49}\,dx = \frac{\sin 50x\sin^{50}x}{50}+c$

ii) $\int\left(\ln(1+\cos x)-x\tan\left(\frac{x}{2}\right)\right)dx = \ldots\; x\ln(1+\cos x)+c$

iii) $\int\left(\frac{\cos x}{x}-\sin x\ln x\right)dx \;=\; \cos x.\ln x+c$

iv) $\int \frac{\sec^2 x-2010}{\sin^{2010}x}dx = \sec x\,\mathrm{cosec}^{2010}x+c,$

27) To evaluate the integral of the form $\int \frac{b+a\cos x}{(a+b\cos x)^2}dx$ divide both numerator and denominator by $\sin^2 x$ and take t = $a\,\mathrm{cosec}x+b\cot x$ and proceed $\int \frac{b+a\sin x}{(a+b\sin x)^2}dx$ divide both numerator and denominator by $\cos^2 x$ and take t = $a\sec x+b\tan x$ and proceed (or) the same can also be evaluated by by parts.

28) To evaluate the integral of the form $\int \frac{P(x)}{\sqrt{ax^2+bx+c}}dx$, where P(x) is a polynomial of degree n, (n > 2)

$\int \frac{P(x)}{\sqrt{ax^2+bx+c}}dx = Q(x)\sqrt{ax^2+bx+c}+$ ª $\int \frac{1}{\sqrt{ax^2+bx+c}}dx$, where Q(x) is a polynomial of degree (n-1).....(1)

i.e. $\int \frac{P(x)}{\sqrt{ax^2+bx+c}}dx$ = $\left(b_0x^{n-1}+b_1x^{n-2}+b_2x^{n-3}+....+b_{n-1}\right)\sqrt{ax^2+bx+c}+$ ª $\int \frac{1}{\sqrt{ax^2+bx+c}}dx$

for finding the values of $b_0, b_1, b_2, ...b_{n-1}$, ª i) first differentiate equation (1) both sides with respect to x and multiply by $\sqrt{ax^2+bx+c}$ and compare the corresponding coefficients

ii) now substitute the values in the required equation (1) which is the required result.

Examples:

1) $\int \frac{x^2+x+1}{\sqrt{x^2+2x+3}}dx$ = $\left(\frac{x-1}{3}\right)\sqrt{x^2+2x+3}+c$

2) $\int \frac{x^3+2x^2+x-7}{\sqrt{x^2+2x-3}}dx = \frac{1}{6}\left(2x^2+x-9\right)\sqrt{x^2+2x-3}-6\ln\left|(x+1)+\sqrt{x^2 2x-3}\right|+c$

29) To evaluate the integrals of the form

$\int f\left(x,(ax+b)^{\frac{m_1}{n_1}},(ax+b)^{\frac{m_2}{n_2}}\right)dx$ where m_1, n_1, m_2, n_2 are integers.

find the L C M of n_1, n_2 (say = n) and take $t^n = (ax+b)$ and proceed

Examples: 1) $\int \frac{x+x^{2/3}+x^{1/6}}{x\left(1+x^{1/3}\right)}dx$ = $\frac{3}{2}x^{2/3}+6\tan^{-1}\left(x^{1/6}\right)+c$

30) To evaluate the integrals of the form $\int x^m f(a+bx^n)^p\,dx$ where m,n and p are rational numbers

Case: 1 If p is an integer then take $x = t^s$ where s is the LCM of the denominators of m and n

Case: 2 If p is not an integer but $\sqrt{\frac{a-x}{a+x}}$ (or) $\sqrt{\frac{a+x}{a-x}} \Rightarrow x = a\cos 2$‾ = s, is an integer then take $t^s = a+bx^n$ and proceed

Case : 3 If $\frac{m+1}{n}$ is an integer but $\frac{m+1}{n}+p$ = s, is an integer, then take $t^s = a+bx^{-n}$ and proceed (where s is the denominator of the number p)

Examples: 1) $\int x^{\frac{1}{3}}\left(1+x^{\frac{4}{3}}\right)^{\frac{1}{3}}dx$ = $\frac{9}{16}\left(1+x^{\frac{4}{3}}\right)^{\frac{4}{3}}+c$ 2) $\int x^{\frac{-2}{3}}\left(1+x^{\frac{1}{3}}\right)^{\frac{1}{2}}dx$ = $2\left(1+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+c$

31) Some standard Reduction formula

1) $I_n = \int \sin^n x dx$ = $\frac{-\sin^{n-1}x.\cos x}{n}+\frac{n-1}{n}I_{n-2}$

2) $I_n = \int \cos^n x dx$ = $\frac{\cos^{n-1}x.\sin x}{n}+\frac{n-1}{n}I_{n-2}$

3) $\mathrm{I_n} = \int \tan^n \mathrm{x dx} = \frac{\tan^{n-1}\mathrm{x}}{\mathrm{n}-1}-\mathrm{I}_{\mathrm{n}-2}, \mathrm{n}\geq 2$

4) $\mathrm{I_n} = \int \cot^n \mathrm{x\, dx} = -\frac{\cot^{n-1}\mathrm{x}}{\mathrm{n}-1}-\mathrm{I}_{\mathrm{n}-2}, \mathrm{n}\geq 2$

5) $\mathrm{I_n} = \int \sec^n \mathrm{x\, dx} = \frac{\tan \mathrm{x} \sec^{n-2}\mathrm{x}}{\mathrm{n}-1}+\frac{\mathrm{n}-2}{\mathrm{n}-1}\mathrm{I}_{\mathrm{n}-2}$

6) $\mathrm{I_n} = \int \mathrm{cosec}^n \mathrm{x\, dx} = \frac{\cot \mathrm{x} \cos\mathrm{ec}^{n-2}\mathrm{x}}{-(\mathrm{n}-1)}+\frac{\mathrm{n}-2}{\mathrm{n}-1}\mathrm{I}_{\mathrm{n}-2}$.

7) If $I_{(m,n)} = \int x^m(\log_e x)^n dx$, then $I_{(m,n)}$ $= \frac{x^{m+1}}{m+1}(\ln x)^n - \frac{n}{m+1}I_{(m,n-1)}$

8) If $I_{(m,n)} = \int \frac{x^n}{(\log_e x)^m} dx$, then $I_{(n,m)} = \frac{-x^{n+1}}{(m-1)(lnx)^{m-1}} + \frac{n-1}{m-1} I_{(n,m-1)}$

9) If $I_n = \int \frac{dx}{\left(1+x^2\right)^n}$, then $I_n = \frac{x}{(2n-2)(1+x^2)^{n-1}} + \frac{2n-3}{2n-2} I_{n-1}$

10) Let $I_{(m,n)} = \int \frac{\sin^m x}{\cos^n x} dx \ (n \neq 1)$ then $I_{(m,n)} = \frac{1}{(n-1)} . \frac{\sin^{m-1} x}{\cos^{n-1} x} + \frac{m-1}{n-1} I_{(m-2,n-2)}$

11) If $I_n = \int \frac{x^n}{\sqrt{ax^2+2bx+c}} dx$ then $(n+1)a\, I_{n+1} + (2n+1)b\, I_n + nc\ I_{n-1} = x^n \sqrt{ax^2+2bx+c}$

32) Some standard substitution:

1) $\int f\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right) dx$ take t = $x+\frac{1}{x}$

2) $\int f\left(x-\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right) dx$ take t = $x-\frac{1}{x}$

3) $\int f\left(x^2+\frac{1}{x^2}\right)\left(x-\frac{1}{x^3}\right) dx$ take t = $x^2+\frac{1}{x^2}$

4) $\int f\left(x^2-\frac{1}{x^2}\right)\left(x+\frac{1}{x^3}\right) dx$ take t = $x^2-\frac{1}{x^2}$

5) $\int f\left(e^{ax}\right) dx$ take t = e^{ax}

Examples:

1) $\int \left(\frac{x-1}{x+1}\right) \frac{dx}{\sqrt{x^3+x^2+x}} = 2\tan^{-1}\sqrt{x+\frac{1}{x}+1} + C$

2) $\int \frac{(x^2+1)}{(x^4-x^2+1)\cot^{-1}\left(x-\frac{1}{x}\right)} dx = -\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right| + C$

3) $\int \frac{\sin^3 x dx}{\left(\cos^4 x + 3\cos^2 x + 1\right)\tan^{-1}\left(\sec x + \cos x\right)} = \ln\left|\tan^{-1}(\sec x + \cos x)\right| + c$

33. The following integrals are elementary:

a) $\int \frac{e^x}{x} dx$ b) $\int \sin\left(x^2\right) dx$ c) $\int \sin\left(e^x\right) dx$ d) $\int \sqrt{x^3+1} dx$ e) $\int \frac{1}{\ln x} dx$ f) $\int \frac{\sin x}{x} dx$ g) $\int \frac{\cos x}{x} dx$ h) $\int \sqrt{1-k^2 \sin^2 x} dx$ where $\left(0 < k^2 < 1\right)$