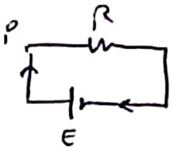
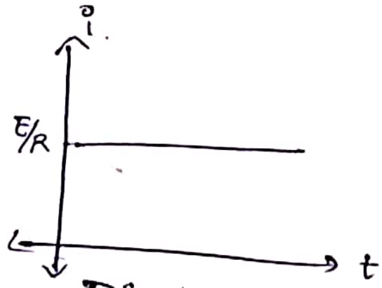
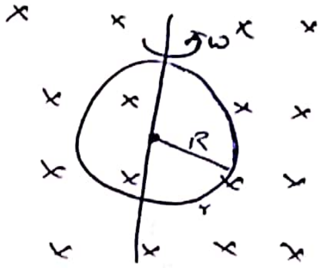
Alternating Current



$i = \frac{E}{R} = \text{constant}$



Direct current

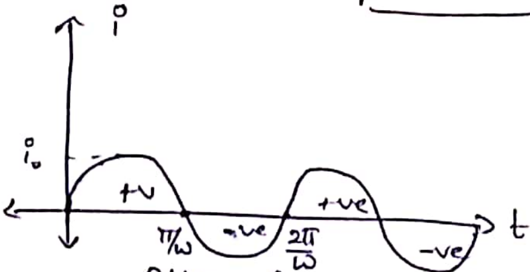


$e = B\omega\pi R^2 \sin\omega t$

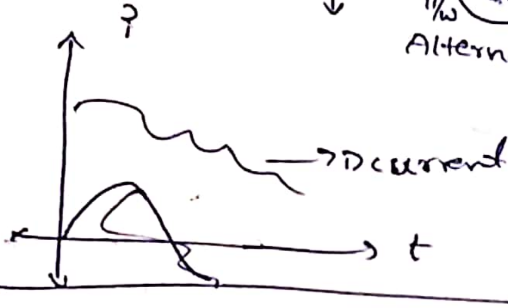
$e = e_0 \sin\omega t$

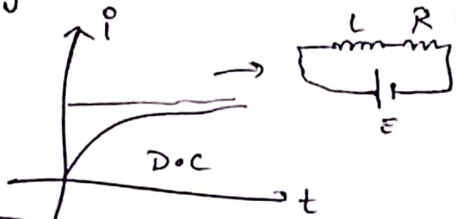
$i = \frac{e_0}{r} \sin\omega t$

$\boxed{i = i_0 \sin\omega t}$



Alternating current





AC Source

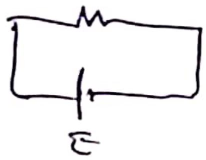
$V = V_0 \sin\omega t$

V_0 → peak voltage. (max voltage)

ω – angular frequency

frequency of source $= \frac{\omega}{2\pi}$

DC Source



$V = V_0 \sin\omega t$.

Current in the circuit

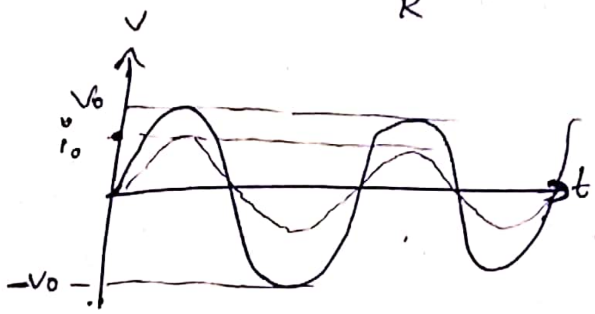
$i = \frac{V}{R} = \frac{V_0 \sin\omega t}{R}$

$i = \left(\frac{V_0}{R}\right) \sin\omega t$.

$\boxed{i = i_0 \sin \omega t}$

$\hookrightarrow$ peak current $= \frac{\text{peak voltage}}{R}$

★ In the resistor, voltage and current will be in same phase.



Measurement of A.C Values

$i = i_0 \sin \omega t$.

$\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = 1/2$

$\langle \sin \omega t \rangle = \langle \cos \omega t \rangle = 0$

Average value :-

If AC is given by $i = i_0 \sin \omega t$

Average value of current in one time period (T)

$$\langle i \rangle = \frac{\int i\,dt}{\int dt} = \frac{\int_0^T i_0 \sin \omega t \, dt}{\int_0^T dt}$$

$$= \frac{i_0 \left(\frac{-\cos \omega t}{\omega}\right)_0^T}{T}$$

$$\frac{1 - \cos 2\omega t}{2}$$

$$= 1 -$$

$$\langle i_0 \rangle = \frac{i_0}{T}\left(\frac{-\cos 2\pi + \cos 0}{\omega}\right)$$

$$\langle i \rangle = \frac{i_0}{T}\left(\frac{-1+1}{\omega}\right) = 0$$

$$\langle i \rangle = 0.$$

Q) Find the average value of current. $i = i_0 \sin^2(\omega t)$

Ans

$$\langle i \rangle = \frac{i_0}{2} \int 1 - \cos(2\omega t)$$

$$\langle i \rangle = \frac{i_0}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T$$

$$\langle i \rangle = \frac{i_0}{2}\left(\frac{T}{T}\right)$$

$$\langle i \rangle = \frac{i_0}{2}.$$

If a voltage is given by $v = 3 + 4\sin^2\omega t$. Find the avg value of voltage

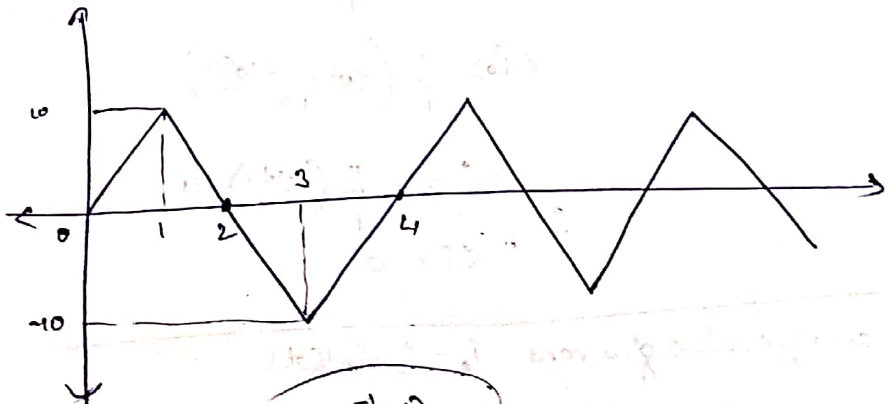
$$\langle v \rangle = \langle 3 \rangle + 4\langle \sin^2\omega t \rangle$$
$$= (3) + 4 \times \frac{1}{2}$$
$$\langle v \rangle = 5$$

RMS

· Root mean square

If current in the element is $i = i_0 \sin\omega t$.

$$i_{avg} = 0$$
$$i_{rms} = \sqrt{\langle i^2 \rangle}$$
$$i = \sqrt{\langle i_0^2 \sin^2\omega t \rangle}$$
$$i = \sqrt{i_0^2 \langle \sin^2\omega t \rangle}$$
$$i = \frac{i_0}{\sqrt{2}}$$
$$\boxed{i_{rms} = \frac{i_{peak}}{\sqrt{2}}}$$



$i = 10\sin^{-1}(\omega t)$ $T = 4$ sec

$\frac{10}{\sqrt{2}}$

$\frac{1}{\sqrt{1-}}$

$\sin^{-1}\omega t$

$x\sin^{-1}x -$

$\int \sin^{-1}x$

$\sin^{-1}x \cdot x - \int \frac{x}{\sqrt{1-x^2}}$

$\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\int \sin\theta$

$(-\cos\theta)$

$$i_{rms}^2 = \frac{\int_0^T i^2\,dt}{\int_0^T dt} = \frac{\int_0^4 i^2\,dt}{4} = \frac{\int_0^1 i^2\,dt + \int_1^3 i^2\,dt + \int_3^4 i^2\,dt}{4}$$

$$i_{rms} = \sqrt{\frac{\int_0^1 (10t)^2\,dt + \int_1^3 (10(2-t))^2\,dt + \int_3^4 (10t-40)^2\,dt}{4}}$$

$i_{rms} = 5$ amp.

AC Measuring Instruments

hotwire ammeter / hot wire voltmeter

★ hot wire ammeter (or) voltmeter will always show a the reading of rms value.

Q) If current in the element is $i = 3 + 4\sin\omega t$, Find the reading by hot wire ammeters?

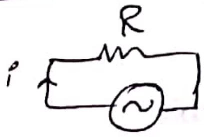
$$i_{rms} = \sqrt{\langle i^2 \rangle_{avg}} = \sqrt{\langle (3+4\sin\omega t)^2 \rangle}$$

$$= \sqrt{\langle 9 + 16\sin^2\omega t + 24\sin\omega t \rangle}$$

$$= \sqrt{9+8}$$

$$i_{rms} = \sqrt{17}$$

Behaviour of Resistance with AC Source.



$i = \left(\frac{V_0}{R}\right) \sin \omega t$

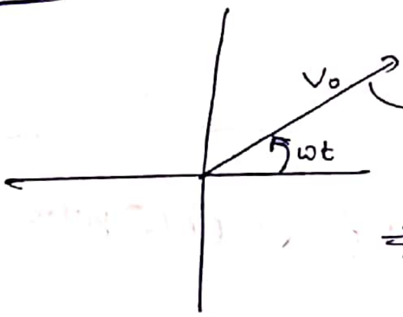
$i = i_0 \sin \omega t$

Current & Voltage are in same phase in resistor.

Phasors-

$V = V_0 \sin(\omega t)$ → with an angle ωt with horizontal line.

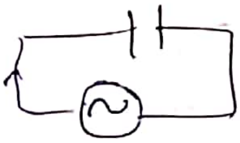
↓ length of magnitude V_0



→ voltage phasor.

⇒ Phasors rotates about origin

Behaviour of Capacitor with AC source



$V = V_0 \sin \omega t$

charge on the capacitor $= C(\Delta V)$

$q = CV_0 \sin \omega t$

Current in circuit $= i = CV_0 \omega \cos \omega t$.

$i = i_0 \cos \omega t$

$i = i_0(\sin(\omega t + \pi/2))$

Current and voltage will differ by phase angle $\pi/2$ in capacitor

Current leads voltage by $\pi/2$ phase.

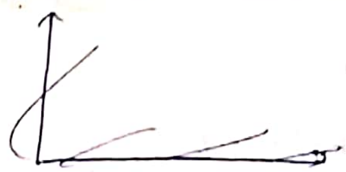
$i_0 = CV_0\omega$

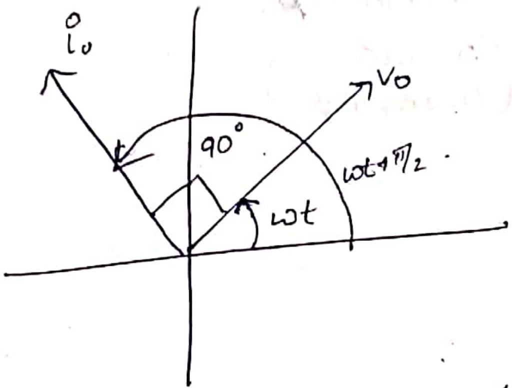
$i = \frac{V_0}{(1/\omega C)}$

$\frac{1}{\omega C}$ = Reactance of capacitor to AC current (X_C)

$X_c = \frac{1}{\omega C}$

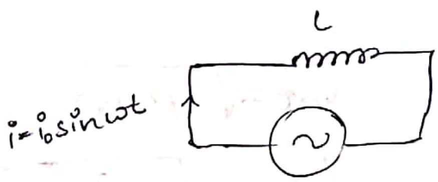
$\boxed{X_c = \frac{1}{2\pi f C}}$





Cur leads in Capa

Behaviour of Inductor with AC Source

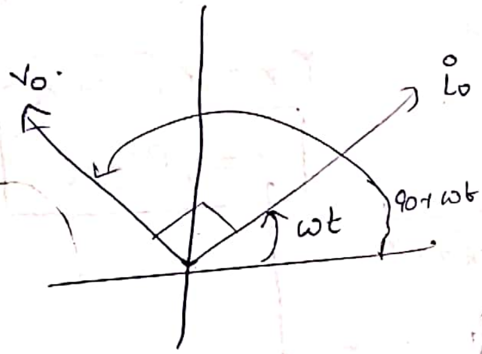


P.d across inductor $= L\frac{di}{dt}$

$V_0 = L\, i_0 \omega \cos\omega t$.

$V = V_0 \cos\omega t$.

$V = V_0 \sin(\omega t + \pi/2)$



Voltage leads current in ~~in~~ inductor by phase angle $\pi/2$.

$V_0 = i_0(\omega L)$

Reactance of Inductor $\boxed{X_L = \omega L = 2\pi f L}$

Past Knowlege

$y_1 = A_1 \sin(\omega t)$

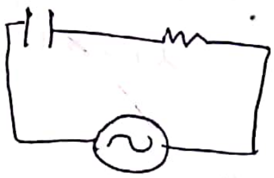
$y_2 = A_1 \sin(\omega t + \phi)$

$y = A \sin(\omega t + \theta)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right)$$

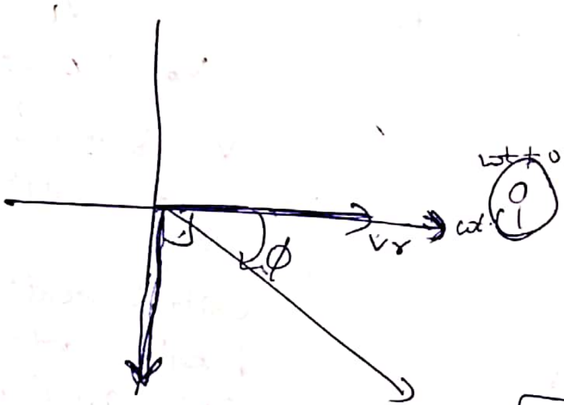
CR circuit in AC



~~$i = i_1 \sin \omega t$~~

$i_1 = i_1' \sin \omega t$

$i_2 = i_2' \sin(\omega t + \pi/2)$



$V_{net} = \sqrt{V_R^2 + V_C^2}$

$iZ = \sqrt{(iR)^2 + (iX_C)^2}$

$\cancel{i} Z = \cancel{i}\sqrt{R^2 + X_C^2}$

$$\boxed{Z = \sqrt{R^2 + X_C^2}}$$

impedence of circuit.

$V_{net} = \sqrt{V_R^2 + V_C^2}$

~~$V_{net} = \sqrt{V_R^2}$~~

$$\boxed{i_0 = \frac{\cancel{\sqrt{V_R^2 + V_C^2}}}{\sqrt{R^2 + X_C^2}}}$$

$$\tan\phi = \frac{V_C}{V_R} = \frac{iX_C}{iR}$$

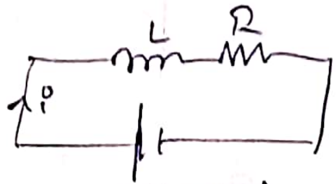
$$\tan\phi = \left(\frac{X_C}{R}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_C}{R}\right)}$$

; Current in circuit $\Rightarrow$ $i = i_0 \sin(\omega t + \phi)$

$$i_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$$

LR circuit in AC



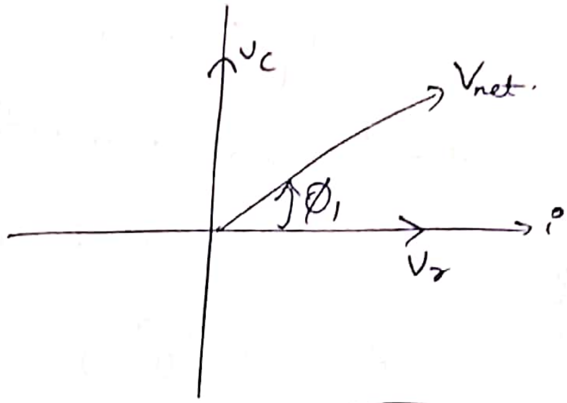
$V = V_0 \cancel{\cos} \sin \omega t$.

~~Vnet=~~

$i = i_0 \sin(\omega t - \phi)$

~~Vnet~~

$$i_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$$



$$V_{net} = \sqrt{V_c^2 + V_R^2}$$

$$iZ = \sqrt{(X_L i)^2 + (iR)^2}$$

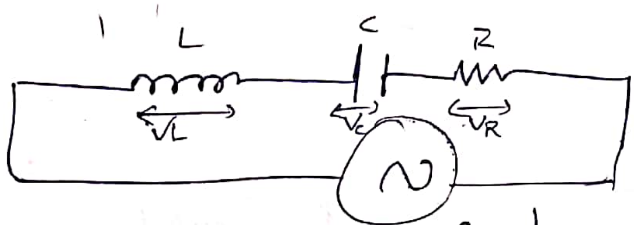
$$iZ = i\sqrt{(X_L)^2 + R^2}$$

$$Z = \sqrt{(X_L)^2 + R^2}$$

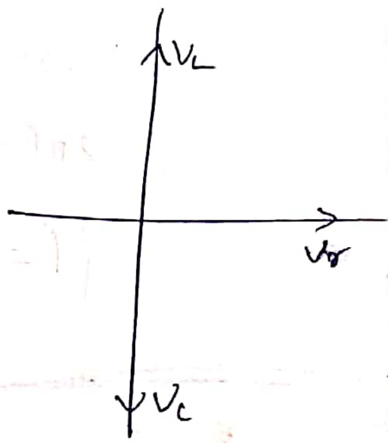
$$\cancel{\tan}\ \tan\theta = \frac{V_c}{V_r}$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

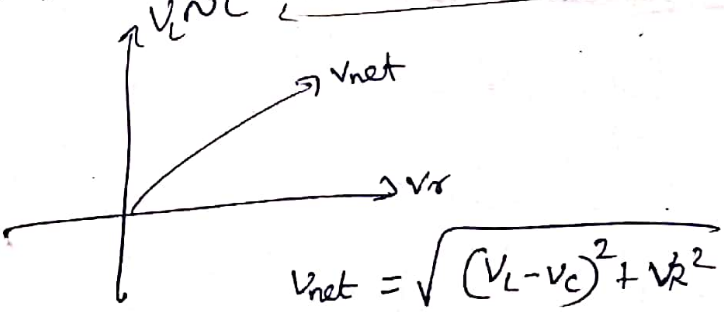
LCR series circuit



$V = V_0 \sin \omega t$.



difference two higher − lower



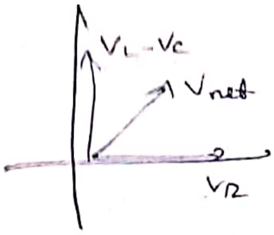
$$V_{net} = \sqrt{(V_L - V_c)^2 + V_R^2}$$

$$Z = \sqrt{(X_L - X_c)^2 + R^2}$$

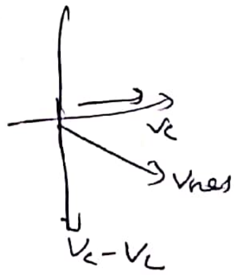
$$\tan\phi = \frac{V_L \sim V_c}{V_R}$$

$$\tan\phi = \frac{X_L \sim X_c}{R}$$

- If $X_L > X_C$ voltage leads current by ϕ
  $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
  circuit behaves as inductive.



- If $X_L < X_C$ current leads voltage by ϕ
  $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$
  circuit acts as capacitive.



- If $X_L = X_C$, $V_L = V_C$
  $Z_{min} = R$
  Current is maximum
  Resonanble condition.
  Circuit nature is resistive.

At resonance

$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$ → resonance frequency.

Powe
Suppos
Instan
Average

Power in AC

Suppose current in circuit is $i = i_0 \sin \omega t$

$v = V_0 \sin(\omega t + \phi)$

Instantaneous power delivered is $P_{inst} = vi$

$P_{inst} = V_0 i_0 \sin \omega t \sin(\omega t + \phi)$

Average power in one time period is

$$\langle P \rangle = \frac{\int_0^T P\,dt}{\int_0^T dt}$$

$$\langle P \rangle = \langle V_0 I_0 \sin(\omega t) \sin(\omega t + \phi) \rangle$$

$$= V_0 I_0 \langle \sin \omega t \sin(\omega t + \phi) \rangle$$

$$= V_0 I_0 \langle \sin(\omega t) [\sin \omega t \cos \phi + \cos \omega t \sin \phi] \rangle$$

$$= V_0 I_0 [\langle \sin^2(\omega t) \cos \phi \rangle + \langle \sin \omega t \cos \omega t \sin \phi \rangle]$$

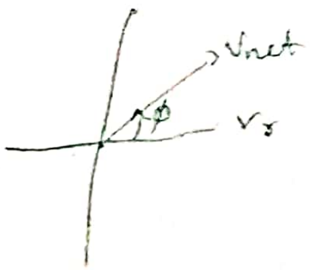
$$= V_0 I_0 \left[\cos \phi \times \frac{1}{2} + \sin \phi \times 0\right]$$

$$\boxed{\langle P \rangle = \frac{V_0 I_0 \cos \phi}{2}}$$

$$P_{avg} = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos \phi$$

$$\boxed{P_{avg} = V_{rms} \cdot I_{rms} \cos \phi}$$

$\cos \phi \longrightarrow$ power factor



$$\boxed{\cos \phi = \frac{V_R}{V_{net}} = \frac{iR}{iZ} = \frac{R}{Z}}$$

$P_{avg} = \frac{V_{rms}}{} \frac{V_{rms}}{Z} \cdot \frac{R}{Z} = \frac{(V_{rms})^2 R}{Z^2}$

$$\boxed{P_{avg} = \frac{(V_{rms})^2 R}{Z^2}}$$

$P_{avg} = V_{rms} I_{rms} \cos\phi$

$= (I_{rms} Z)(I_{rms}) \frac{R}{Z}$

$$\boxed{P_{avg} = (I_{rms})^2 R}$$

Note:-

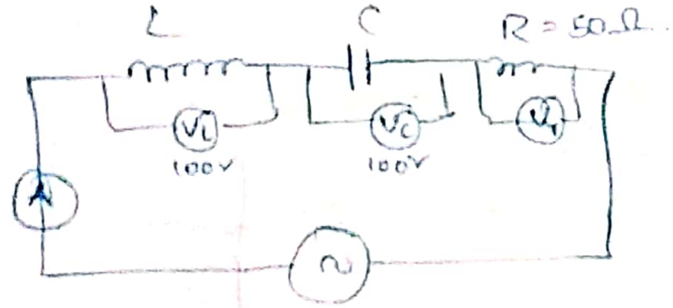
In LCR series circuit, at resonance

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

$\cos\phi = 1$

$\phi = 0^\circ$

Q)



$V_{rms} = 100V$

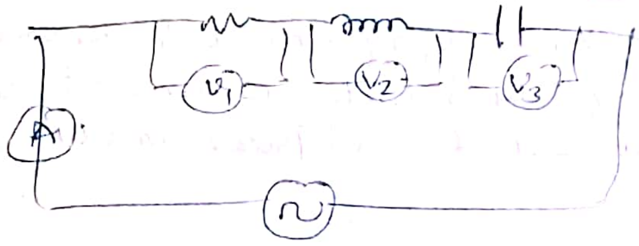
$2\pi \quad 50$

$\omega = 100\pi$

$\boxed{V_r = 200 \cdot \text{volts}}$

$i = 4 \text{ Amp}$

a)



V_{net}

$E = E_0 \sin\omega t$

$V_1 = 100V$ $V_2 = 125V$ $V_3 = 150$

$25^2 + 100^2$

$\sqrt{625 + 10000} = \sqrt{1625}$

Find (a) Net impedance

(b) Power factor.

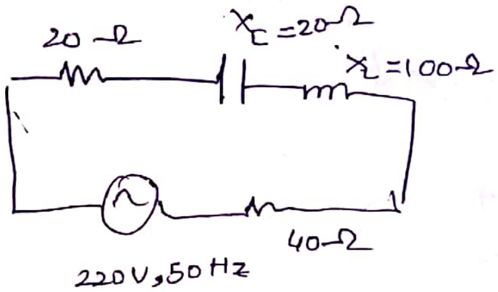
$V_{net} = \sqrt{1625}$

$\cos\phi = \frac{R}{Z} = \frac{\cancel{20}\,4}{\sqrt{17}\times 5} = \frac{4}{\sqrt{17}}$

$Z = \frac{V_{net}}{i} = \frac{\sqrt{1625}}{5} = \sqrt{\frac{\cancel{1625}}{\cancel{25}}} = \sqrt{\frac{10625}{25}} = \cancel{\sqrt{425}}$

$\boxed{Z = 5\sqrt{17}}$

Q)



$\frac{1}{\omega C} = 20$ $\omega L = 100$

$\frac{60}{\sqrt{6400+3600}}$

10000

$\frac{\cancel{60}\,3}{\cancel{100}\,5}$ 60

$Z = \sqrt{(100-20)^2 + 60^2}$

$= \sqrt{80^2 + 60^2} = 100Ω$

$\cos\phi = \frac{R}{Z} = \frac{60}{100} = 0.6$

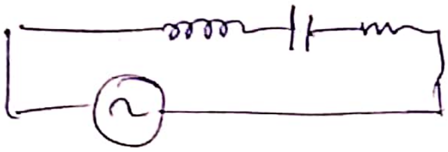
$-NH_2$ N_2.

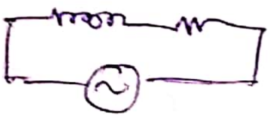
Q) In a LCR series circuit $100\,\Omega$ is connected to AC source of 200V, $\omega = 300$ rad/s. When only capacitance is removed current laggs behind voltage by $60°$. If only inductor is removed current lead voltage by $60°$. Find current and Power in LCR circuit

Ans:

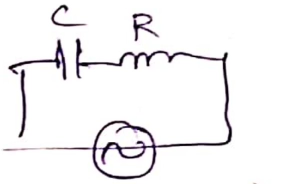
$R = 100$

$V_0 = 200$ $\quad \omega = 300$ rad/s.





$\phi = 60°$



$\tan\phi = \frac{X_L}{R}$

$X_L = \sqrt{3}R$

$\tan\phi = \frac{X_C}{R}$

$X_C = \sqrt{3}R$

$Z = R$

$i_{rms} = \frac{200}{100} = 2$ amp.

$P = V_{rms}\, i_{rms} \cancel{\cos\phi}^{1}$

$= 200 \times 2$

$P = 400$ Watts

Q) In LCR series circuit. $\boxed{R = 100\,\Omega \quad L = 180 \times 10^{-3}\ H \quad C = 10 \times 10^{-6}\ F}$ are connected in series with source of 20V & 750Hz. Find the ~~at~~ time in which Resistance having thermal capacity 2J/°C will get heated by 10°C.

$\omega =$

$X_L = \omega L = 180 \times 10^{-3} \times 750 \times 2\pi$

$= \frac{\cancel{18}^{9} \times \cancel{75}^{25}}{\cancel{10}_{2}} \times$

$X_C = \frac{1}{\omega C} = \frac{1}{10 \times 10^{-6} \times 750}$

$i^2 R$

$P = 20$ $\quad X_L = 225 \times 2\pi$

V_{rms} $\quad = 450 \times 3$

$X_C = \frac{10^4}{75}$

$= \frac{\cancel{100}^{4} \times 10^2}{\cancel{75}\,3}$

$X_C = \frac{\cancel{400}^{2}}{\cancel{2} \times 3 \times 3} = 22.$ $\quad = \frac{133.3}{2\pi \times 3}$

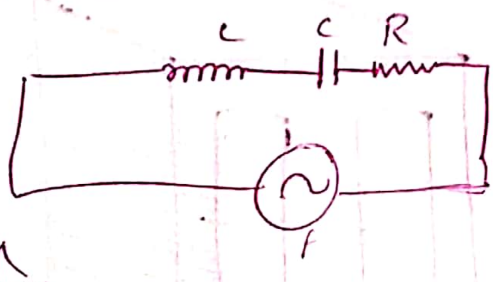
$\langle P \rangle = \frac{20 \times 20 \times 100}{100^2 + (X_L - X_C)^2}$

6450π

heat generated $= \langle P \rangle t$.

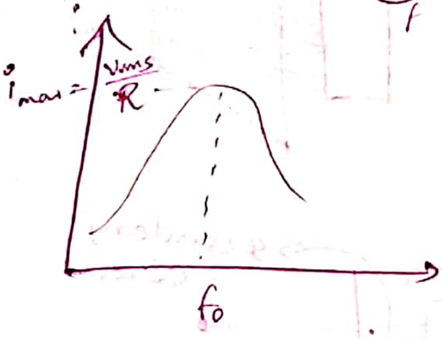
$20 = \langle P \rangle t$. $\quad t = \frac{20}{\langle P \rangle} \approx 345$ sec

Note:- In LCR series, at resonance



$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$Z = \sqrt{\left(2\pi f L - \frac{1}{2\pi f C}\right)^2 + R^2}$$

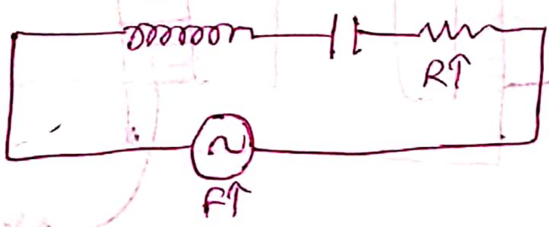


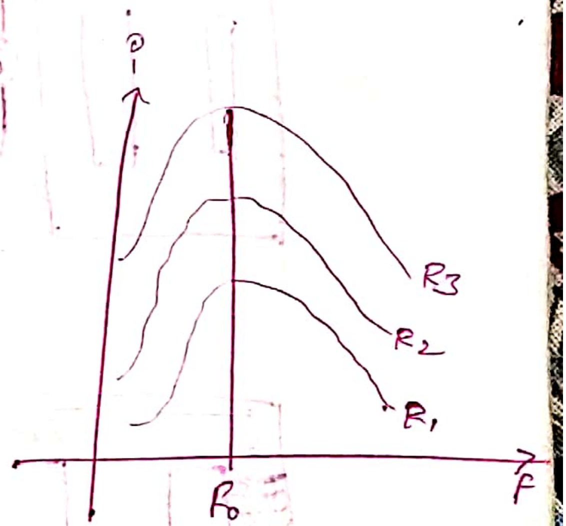
at $f = f_0 = \frac{1}{2\pi\sqrt{LC}}$ $\quad X_L = X_C$

at $f < f_0$, $X_L < X_C$ (Capacitive)

at $f > f_0$, $X_L > X_C$ (Inductive)

→ L and C are fixed.

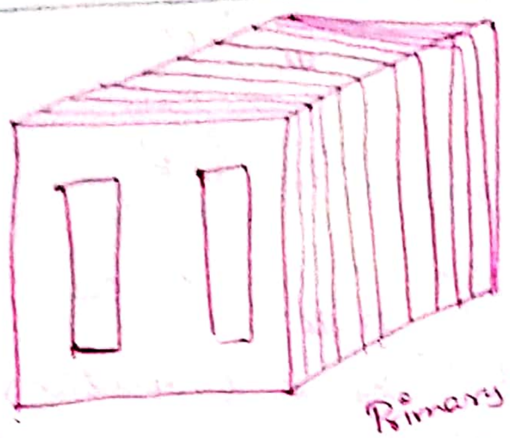


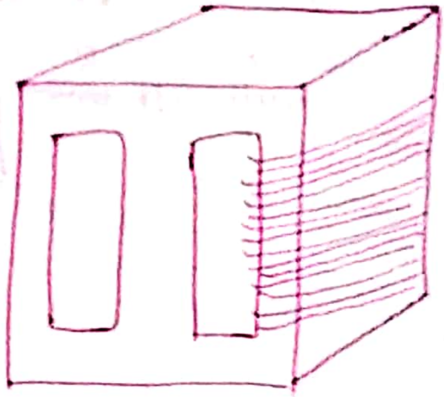


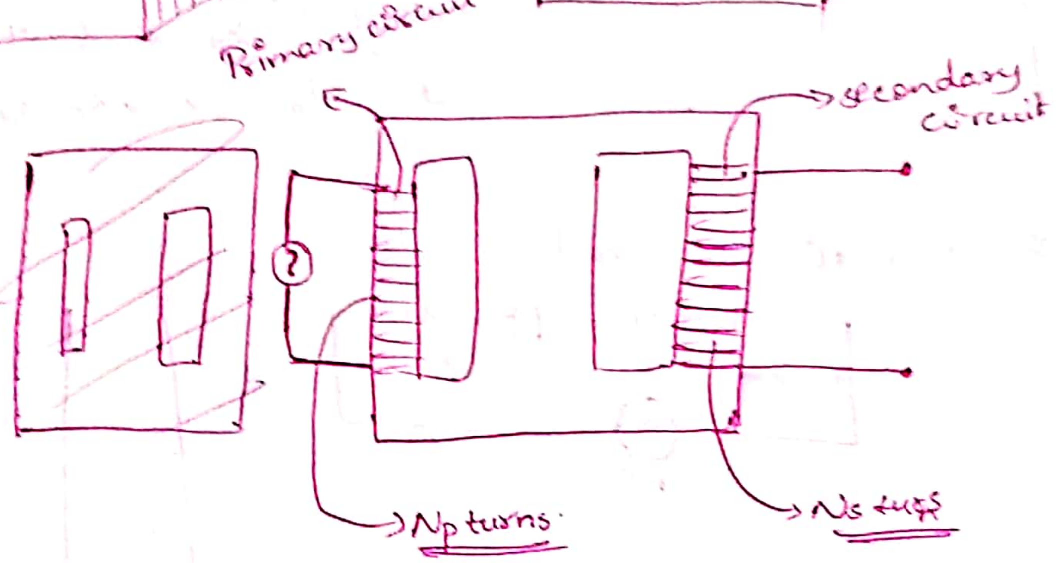
$R_3 < R_2 < R_1$

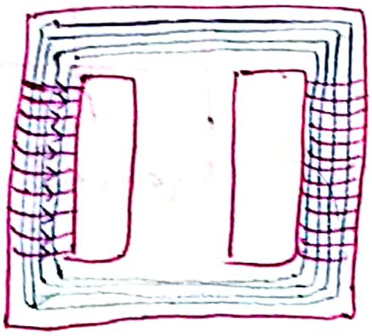
Transf

Transformer:









total flux in primary circuit

$\phi_p = N_p \phi$ (ϕ :- flux through each loop)

emf in the primary circuit

$$E_p = -\frac{d\phi_p}{dt} = -N_p \frac{d\phi}{dt}$$

total flux in secondary circuit

$\phi = N_s \phi$

emf in the secondary circuit is

$$E_s = -\frac{d\phi_s}{dt} = -N_s \frac{d(\phi)}{dt}$$

If $N_s = N_p$
$\Rightarrow E_s = E_p$

If $N_s > N_p$
$E_s > E_p$ Step up transformer.

If $N_s < N_p$
$E_s < E_p$ step down transformer.

$$\boxed{\frac{E_p}{E_s} = \frac{N_p}{N_s}}^{*} \quad E_s = \left(\frac{N_s}{N_p}\right) E_p$$

Efficiency of transformer $= \eta = \frac{\text{out put power}}{\text{input powers}}$.

$$\eta = \frac{P_s}{P_p} = \frac{E_s I_s}{E_p I_p} =$$

$$\boxed{\eta = \left(\frac{E_s}{E_p}\right)\left(\frac{I_s}{I_p}\right)}$$

For ideal transformer $\eta = 1$

$\eta < 1$

$$\frac{E_s}{E_p} \frac{I_s}{I_p} = 1$$

$$\boxed{\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}}$$

Quality Factor

Quality Factor $= \frac{X_C}{R} = \frac{X_L}{R} = \cancel{\frac{1}{R}} \ \frac{1}{R}\sqrt{\frac{L}{C}}$.