ELASTICITY :- (on app forces, solid deform)

bodies
↓
Non-rigid bodies

Elastic object:

→ If object is able to regain its shape & size after removal of deforming forces object is said to be elastic

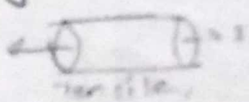
↳ external forces.

~~Inelastic Object~~ **Plastic Object:**

→ If object is unable to regain its shape & size after removal of deforming forces object is said to be plastic

STRESS: defined as force acting per unit area.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$



Tensile stress

Compressive stress.

STRAIN:

$$\text{Strain} = \frac{\text{change in dimension}}{\text{Original dimension}}$$

Stress $\propto$ strain (Generally but not always)

Stress = [Modulus] strain

$$* \boxed{\frac{\text{stress}}{\text{strain}} = [\text{Modulus}]}$$

(only for small deformation)

→ For large deformation $\frac{\text{stress}}{\text{strain}}$ ↓

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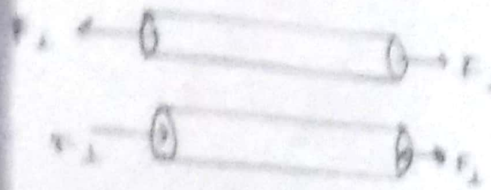
Yung's Modulus (length variation)



($\perp$ to area of C.S)

Longitudinal stress = $\dfrac{F_\perp (\perp \text{ to, area of C.S})}{\text{area of C.S}} = \dfrac{F_\perp}{A}$

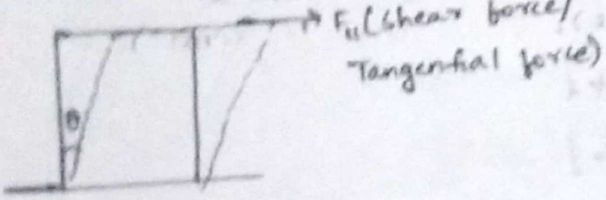
Longitudinal strain = $\dfrac{\text{Change in length}}{\text{original length}} = \dfrac{\Delta l}{l}$

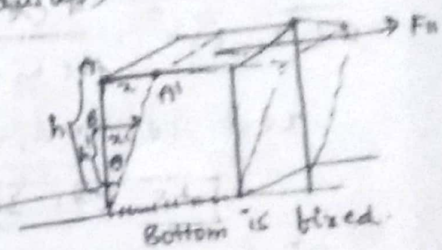
$\dfrac{\text{Long. stress}}{\text{long. strain}}$ = Yung's modulus.

→ Depends on material.

$$\boxed{Y = \frac{F_\perp / A}{\Delta l / l} = \frac{F(l)}{A(\Delta l)}}$$

2) Shear Modulus :- (Rigidity modulus)





$\tan\theta = \dfrac{x}{h} = \dfrac{x'}{h'}$.

$\theta = \dfrac{x}{h} = \dfrac{x'}{h'}$

Shear stress = $\dfrac{F_{\parallel} (\text{Parallel to area})}{\text{area}}$

Shear strain = θ = angle of tilt
$= \dfrac{x}{h} = \dfrac{x'}{h'}$

$\dfrac{\text{Shear stress}}{\text{Shear strain}}$ = Shear modulus (η)

$$\eta = \frac{F_{\parallel}/\text{area}}{\theta} = \frac{F_{\parallel}/\text{area}}{x/h}$$

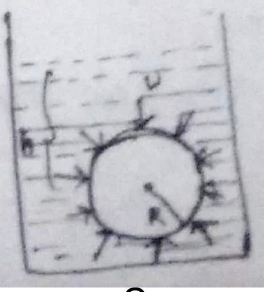
$$\boxed{\eta = \frac{(F_{\parallel})(h)}{(\text{area})(x)}}$$

→ For an ideal liquid,
Bulk modulus = ∞
(vol. strain) = 0.
Rigid modulus = 0
(tangential drag) = 0.

→ For a perfect rigid body,
~~Y, B~~ Y = 0, B = ∞
(∞)

3) Bulk Modulus :

Solid sphere



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Volume stress = Force acting per unit area
= Pressure

Volume strain = $\frac{\text{change in Volume}}{\text{Original Volume}} = \frac{\Delta V}{V}$

BULK MODULUS (B) = $\frac{\text{Volume stress}}{\text{Volume strain}}$

$$\boxed{B = \frac{\Delta P}{\frac{-\Delta V}{V}} = \frac{\Delta P \cdot V}{-\Delta V}}$$

Q) If a rod has length l_1 under app. of force 3N, and has length l_2 under app. of 4N, Find its orig. length.

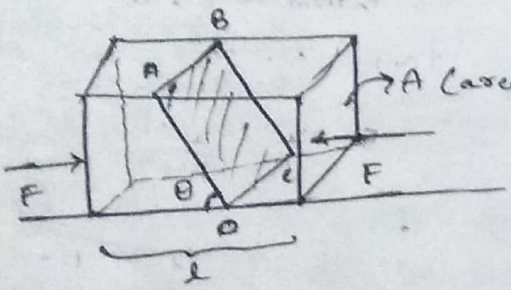
Sol: Let orig. leng = l

$$\frac{3(l)}{A(l_1-l)} = \frac{4(l)}{A(l_2-l)} \quad \text{(Yung's modulus is const)}$$

$$3l_2 - 3l = 4l_1 - 4l$$

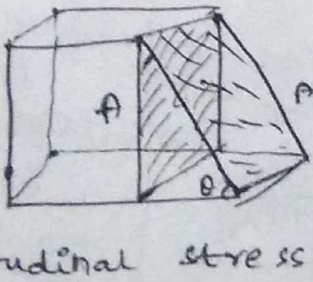
$$\boxed{l = 4l_1 - 3l_2}$$

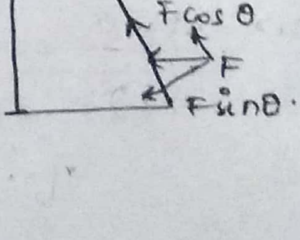
Q)



Find the longitudinal and shear stress on surface ABCD?

Sol:





longitudinal stress = $\frac{F_\perp}{\text{area}}$

$= \frac{F\sin\theta}{A/\sin\theta} = \frac{F\sin^2\theta}{A}$

If $\theta = 90°$, longitudinal stress is max.

Shear stress = $\frac{F_\parallel}{A}$

$= \frac{F\cos\theta}{A/\sin\theta}$

$= \frac{F\sin 2\theta}{2A}$

If $\theta = 45°$, shear stress is maximum.

ELASTIC POTENTIAL ENERGY STORED IN THE WIRE: (STRAIN ENERGY)

Y of A $\quad Y = \dfrac{(F)(L_0)}{(A)(x)}$

$$Y = \frac{FL_0}{Ax} \Rightarrow F = \frac{YAx}{L_0} = \boxed{\left(\frac{YA}{L_0}\right)(x) = Kx} \left(K = \frac{YA}{L_0}\right).$$

elastic P.E stored in the wire $= U = \frac{1}{2}Kx^2$

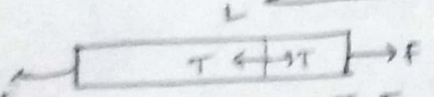
$$= \frac{1}{2}\left(\frac{YA}{L_0}\right)(x)^2 = \frac{1}{2}YA\left(\frac{x}{L_0}\right)^2 (L_0)$$

$$\Rightarrow U = \frac{1}{2}(Y)(\text{strain})^2(\text{Volume})$$

$$\boxed{U = \frac{(\text{stress})(\text{strain})(\text{Volume})}{2}}$$

$$\frac{U}{\text{Volume}} = \text{energy density} = u$$

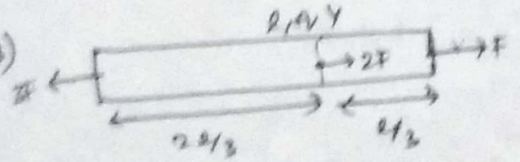
$$\boxed{u = \frac{1}{2}(\text{stress})(\text{strain})}$$



T = F

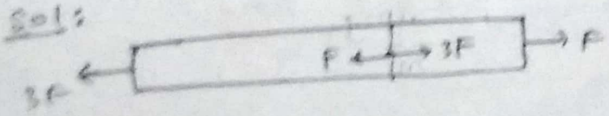
$e = \dfrac{FL}{AY}$ (e - extension)

8)

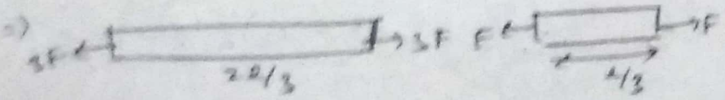


Find the extension of the rod.

Sol:



⇒



$$e = \frac{\cancel{3}F(2\ell)}{\cancel{3}AY} \qquad e = \frac{F\ell}{3AY} \qquad \boxed{e_{tot} = \frac{7FL}{3AY}}$$

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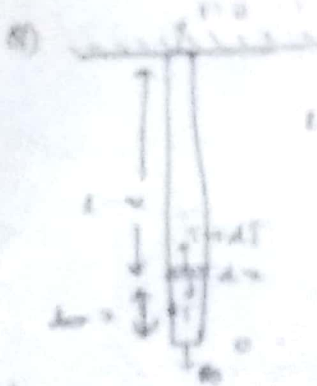
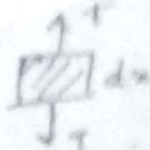
8) 

L, A, Y, M

Find the extension in the rod due to its own weight?

Sol:

$T = \frac{(l-x)w}{l}$; $T = \left(\frac{Mx}{L}\right)(g)$



$de = \frac{T\,dx}{A\cdot Y}$

$x = \left(\frac{Mx}{L}\right)g$

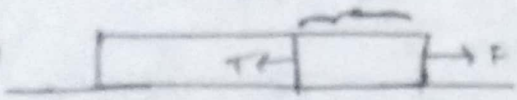
$e = \int de = \frac{Mg}{LAY}\int x\,dx$

$= \frac{MgL^2}{2LAY} = \boxed{\frac{MgL}{2AY}}$

NOTE: Since, tension is varying linearly, we can get directly by using average method

$\frac{\left(\frac{0+Mg}{2}\right)(L)}{AY} = \frac{MgL}{2AY}$

L, A, Y

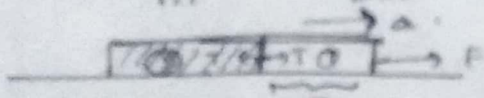
6) 

Smooth surface

Find the extension of rod?

Sol: $T = F\left(1 - \frac{x}{L}\right)$

$e = \frac{F_{avg}\cdot L}{AY} = \frac{FL}{2AY}$

8) 

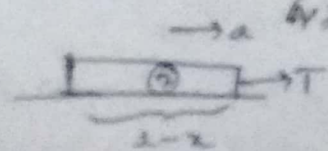
L, A, Y

Rough (μ)

Find the extension in the Rod.

Sol:

$a = \frac{F - \mu mg}{m}$



$T = \left(\frac{l-x}{l}\right)ma - \mu\left(\frac{l-x}{l}\right)mg$

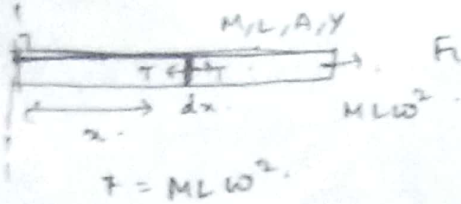
$T = \left(\frac{l-x}{l}\right)(F - \mu mg)$

$$T = \left(\frac{L-x}{L}\right) m \left(y/g + F/m - y/g\right)$$

$$\Rightarrow \frac{F(L-x)}{L}$$

$$e = \frac{\left(0 + \frac{F}{2}\right)L}{AY} = \frac{FL}{2AY}$$

Q) ω



Find the extension in the rod.

$$T = ML\omega^2$$

$$e = \frac{\left(\frac{0+F}{2}\right)L}{AY} = \frac{FL}{2AY} = \frac{ML\omega^2(L)}{2AY}$$

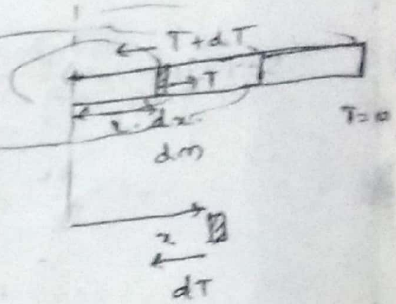
$$= M$$

$$T = \left(\frac{Mx}{L}\right) \cdot x \cdot \omega^2$$

$$de = \frac{\frac{Mx^2\omega^2}{L} \cdot dx}{AY} \Rightarrow \int de = e = \frac{M\omega^2}{LAY} \int_0^L x^2\,dx$$

$$\Rightarrow \frac{M\omega^2 L^3}{3LAY} = \frac{M\omega^2 L^2}{3AY}$$

Method;



$$dT = (-dm)(x)(\omega^2)$$

$$= -\left(\frac{M}{L}dx\right)(x)(\omega^2)$$

$$\int_0^T dT = -\frac{M\omega^2}{L}\int_L^x x \cdot dx$$

$$T = -\frac{M\omega^2}{L}\left[\frac{x^2}{2}\right]_L^x$$

$$\boxed{T = \frac{M\omega^2}{2L}(L^2 - x^2)}$$

$$de = \frac{T \cdot dx}{A \cdot Y} = \frac{M\omega^2}{2LAY}(L^2 - x^2) \cdot dx$$

$$e = \frac{M\omega^2}{2LAY}\int^L L^2 \cdot dx - x^2 \cdot dx$$

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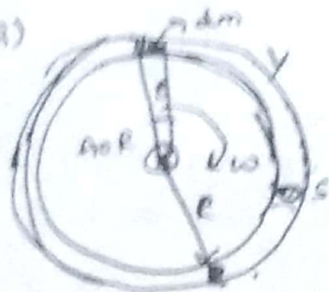
$$e = \frac{M\omega^2}{2LAY}\left(L^3 - \frac{L^3}{3}\right) = \frac{M\omega^2}{2LAY}\left(\frac{2L^3}{3}\right).$$

$$\boxed{e = \frac{M\omega^2 L^2}{3AY}}$$

Q)



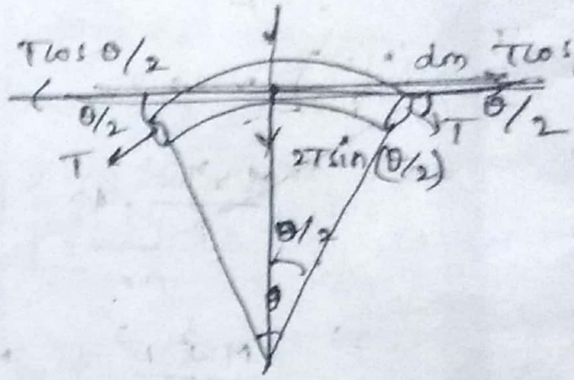
If Ring rotaes wi th angular velocity 'ω' about an AOR passing through centre and ⊥r to ring,

λ – mass per unit length of ring   Y – Young's modulus.

S – area of CS

What is the increase in radius of the ring?

Sol.



$$2T\sin(\theta/2) = dm\,R\omega^2.$$

$$\cancel{2}T(\theta/\cancel{2})\ \cancel{2T\sin(\theta/2)} = \lambda(R\cancel{\theta})\cdot R\omega^2$$

$$\boxed{T = \lambda R^2\omega^2}$$

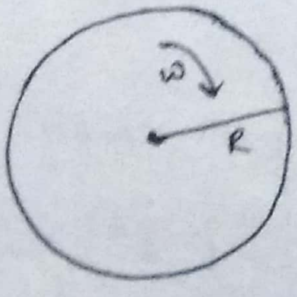
$$\text{Stress} = \frac{T}{S} = \frac{\lambda R^2\omega^2}{S}$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2\pi(R+dR) - 2\pi R}{2\pi R} = \frac{dR}{R}$$

$$\frac{dR}{R} = \frac{\lambda R^2\omega^2}{SY}$$

$$\boxed{dR = \frac{\lambda R^3\omega^2}{SY}}$$

Q)



If Breaking stress of material is x, find the max. angular velocity of the ring allowed not to rupture it.

Sol.

$$x \geq \frac{\lambda R^2\omega^2}{S} \Rightarrow \frac{xS}{\lambda R^2} \geq \omega^2 \Rightarrow \boxed{\omega \leq \sqrt{\frac{xS}{\lambda R^2}}}$$

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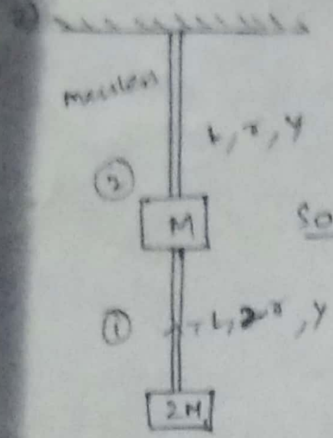
2) 

Find the total elastic P.E stored in the system.

Sol. ① T = 2Mg.

$\text{stress} = \frac{2Mg}{\pi(4r^2)}$ $\text{strain} = \frac{2Mg}{\pi(4r^2)(Y)}$

$\text{Volume} = \pi(L)(4r^2)$

$(EPE)_1 = \frac{1}{2} \times \frac{\cancel{4}M^2g^2}{\pi^2 \times 16r^4(Y)} \cancel{\pi}(L)(\cancel{4r^2})$

$= \frac{M^2g^2(L)}{2\pi r^2 Y}$

② $(EPE)_2$

T =

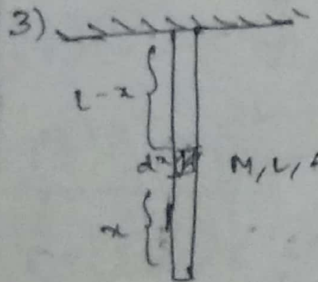
Sol. $U = \frac{1}{2}(\text{stress})(\text{strain})(\text{vol.})$

$= \frac{1}{2} \times \frac{F^2 L}{AY}$

$F_2 = 3mg \Rightarrow U_2 = \frac{9m^2g^2L}{2 \times \pi r^2 \times Y} = \frac{9m^2g^2L}{2\pi r^2 Y}$

$F_1 = 2mg \Rightarrow U_1 = \frac{4m^2g^2L}{2 \times \pi \times 4r^2 Y} = \frac{m^2g^2L}{2\pi r^2 Y}$

$U_{Tot} = U_1 + U_2 = \frac{5m^2g^2L}{\pi r^2 y}$

3) 

Find E.P.E stored in the wire.

$e = \frac{MgL}{2AY}$

Sol^n: $U = \frac{1}{2}(Y)(\text{strain})^2(\text{vol.})$

$= \frac{1}{2} \times \cancel{Y} \times \frac{M^2g^2}{4A^2Y^2} \times \cancel{A} \times L. = \cancel{\frac{M^2g^2L}{8A}}$

$T = \left(\frac{Mg}{L}\right)(x)$ $\text{stress} = \left(\frac{Mg}{AL}\right)(x)$.

energy stored in elemental part -

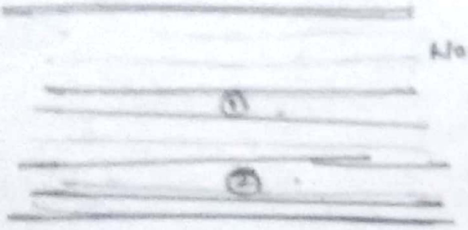
$$dU = \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{vol}$$

$$= \frac{1}{2} \times \left(\frac{M^2 g^2}{A^2 L^2} \times \frac{x^2}{Y} \right) (A\,dx)$$

$$\Rightarrow \frac{M^2 g^2}{2AL^2 Y} x^2\,dx$$

$$\Rightarrow \int_0^U dU = \int_0^L x^2\,dx \left(\frac{M^2 g^2}{2AL^2 Y} \right)$$

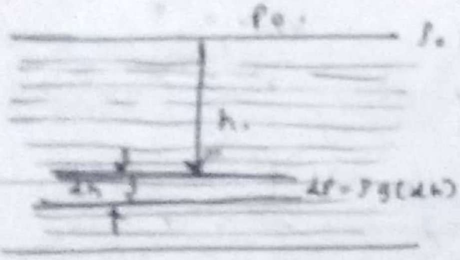
$$U = \frac{M^2 g^2 \times L^3}{6AL^2 Y} = \frac{M^2 g^2 L}{6AY}$$



Non ideal fluid
(compressible)

$\rho_2 > \rho_1$

Q)



ρ_0 (density at surface)

Find the pressure as function of depth? Bulk Modulus of liquid-B.

Sol:

$$B = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

$$\rho = m/V$$

$$\frac{d\rho}{dV} = \frac{-m}{V^2}$$

$$\frac{d\rho}{dV} = \left(\frac{-m}{V}\right)\left(\frac{1}{V}\right)$$

$$\frac{d\rho}{dV} = -\rho\left(\frac{1}{V}\right)$$

$$\boxed{\left(\frac{-dV}{V}\right) = \frac{d\rho}{\rho}} \quad — ①$$

$$B = \frac{dP}{d\rho} \times \rho$$

$$\boxed{B\left(\frac{d\rho}{\rho}\right) = dP}$$

$$B\left(\frac{d\rho}{\rho}\right) = \rho \cdot g \cdot dh$$

$$B\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = g\int_0^h dh$$

$$B\left[\frac{-1}{\rho}\right]_{\rho_0}^{\rho} = gh \Rightarrow \boxed{B\left(-\frac{1}{\rho} + \frac{1}{\rho_0}\right) = gh} \quad — ②$$

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$$\int_{\rho_0}^{\rho'} \frac{d\rho}{\rho} = \int_{P_0}^{P} dP$$

$$P - P_0 = B(\log \rho/\rho_0)$$

$$\boxed{P = P_0 + B\ln(\rho/\rho_0)} \quad -③$$

From Eq - ⑥

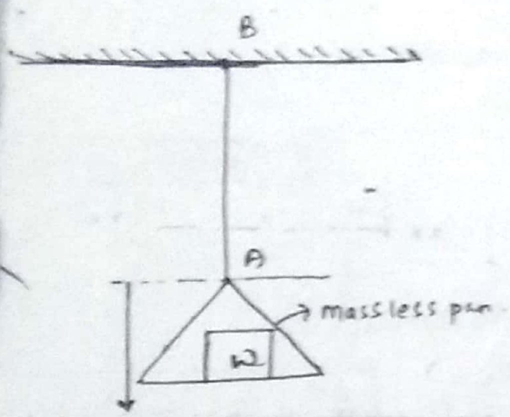
$$\frac{1}{\rho} = \frac{1}{\rho_0} - \frac{gh}{B}$$

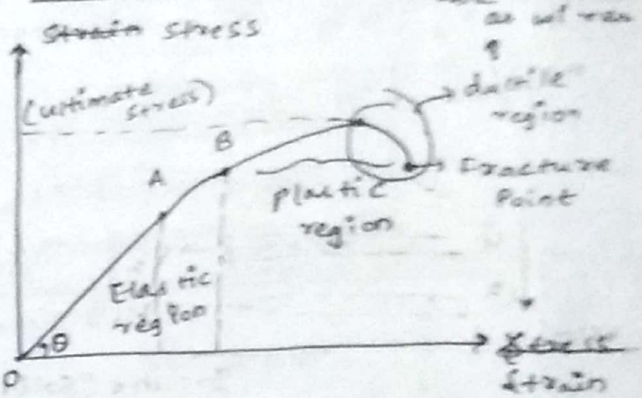
$$\rho = \frac{B\rho_0}{B - gh\rho_0}$$

$$\Rightarrow \frac{\rho}{\rho_0} = \frac{B}{B - gh\rho_0}$$

$$\Rightarrow \boxed{P = P_0 + B\ln\left(\frac{B}{B - gh\rho_0}\right)}$$

Behaviour of metal wire under increasing Load:





able to draw as wire

→ A material is said to be more brittle if its Fracture Point is more closer to ultimate stress point.

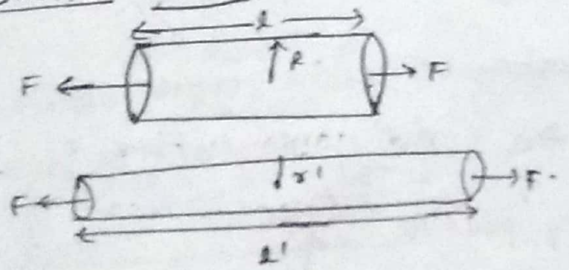
Stress ∝ Strain (Hooke's Law)

A - Propotionality limit

B - Elastic limit

$$\text{Slope} = \tan\theta = \frac{\text{stress}}{\text{strain}} = Y$$

Poisson's Ratio:



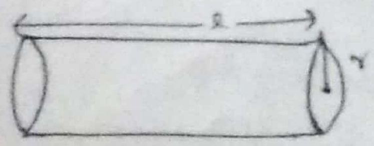
$\frac{\Delta l}{l}$ = longitudinal strain

$\frac{\Delta r}{r}$ = lateral strain (Transverse strain)

$l' > l$, $r' < r$

$$\text{Poisson's Ratio} = (\sigma) = \boxed{\frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$$

$$\sigma = \frac{-\Delta r/r}{\Delta l/l} \qquad \frac{1}{\sigma} = \frac{\Delta l/l}{-\Delta r/r} \Rightarrow \boxed{\frac{-\Delta r}{r} = \sigma\left(\frac{\Delta l}{l}\right)}$$



Volume(V) = $\pi r^2 l$

$V = \pi r^2 l$

$$\frac{1}{V}dV = 0 + 2\cdot\frac{dr}{r} + \frac{dl}{l}$$

$$\frac{dV}{V} = 2\cdot\left(-\sigma\cdot\frac{dl}{l}\right) + \frac{dl}{l}$$

$$\boxed{\frac{dV}{V} = (1-2\sigma)\,\frac{dl}{l}}$$

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