Section: Senior **PARABOLA** **Date:29-05-2020**

EQUATION OF THE PARABOLA

1. Parabola is a conic section, which is the locus of a point which moves in such a way that its distance from a fixed point is always equal to its distance from a fixed line, in the same plane. The fixed point is **focus**, fixed line is **directrix**.
   A line, perpendicular to the directrix and passing through the focus is **axis** of the parabola.
   The point on the axis of the parabola which is also a point on the parabola is called **vertex** of the parabola.
2. General second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents parabola if $\Delta \neq 0$ and $h^2=ab$ where $\Delta = abc+2fgh-af^2-bg^2-ch^2$
3. Equation of parabola whose directrix is $ax+by+c=0$ & focus is at (m, n), is
   $$\left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right| = \sqrt{(x-m)^2+(y-n)^2}$$
4. Equation of the parabola, can be expressed as "square of the distance of any point P, on the parabola and the axis is, equal to product of the length of the latus rectum and distance between the point P and the tangent at the vertex.
   i.e. $\left(\frac{ax+by+c}{\sqrt{a^2+b^2}}\right)^2$ = (length of the latus rectum). $\left(\frac{bx-ay+d}{\sqrt{a^2+b^2}}\right)$
   here ax + by + c = 0 and bx – ay + d =0 are the equations of axis and tangent at the vertex
5. **Two parabolas** are said to be **equal** if they have the same latus rectum.

6a) Perpendicular distance from focus on directrix is equal to half the latus rectum

6b) Vertex is middle point of the focus and the point of intersection of directrix and axis.

7a) Equation of a parabola whose axis, is x-axis and tangent at the vertex is y axis, is $y^2=4ax$

7b) Equation of a parabola whose axis, is y-axis and tangent at the vertex is x axis, is $x^2=4by$

7c) Equation of a parabola whose axis, is parallel to x-axis (or) symmetrical about the line y = a, and tangent at the vertex is y axis, is $(y-a)^2=4bx$

7d) Equation of a parabola whose axis, is parallel to y-axis (or) symmetrical about the line x = a, and tangent at the vertex is x axis, is $(x-a)^2=4by$

7e) Equation of a parabola whose axis, is parallel to x-axis (or) symmetrical about the line y = a, is $(y-a)^2=4b(x-c)$

7f) Equation of a parabola whose axis, is parallel to y-axis (or) symmetrical about the line x = a, is $(x-a)^2=4b(y-c)$

7g) Equation $y^2+2ax+2by+c=0$ represents a parabola, whose axis is parallel to the axis of x, and its vertex is $\left(\frac{b^2-a}{2a}, -b\right)$ and equation of latus rectum is $x=\frac{b^2-a-c}{2a}$

LOCUS:

1a) Every member of a family of circles, passes through a fixed point and is tangential to a fixed line, then locus of the centre of circles is, a parabola.

1b) Locus of the centre of the circle which touches the line $x+y=0$ and passes through the point (a,a) is a parabola.

DOUBLE ORDINATE: Let P(x, y) be any point on the parabola $y^2=4ax$ Draw PN perpendicular to the axis of the parabola , produce it to the point N on the parabola, PN is double ordinate and is equal to 2y

1) A double ordinate of the parabola $y^2=4ax$,is of length 8a. Then the lines from the vertex to its two ends are At right angles.

EQUATION OF CHORD/FOCAL CHORD:

Chord joining the points $P\left(at_1^2, 2at_1\right) \& Q\left(at_2^2, 2at_2\right)$ of the parabola $y^2=4ax$

a) Slope of $PQ=\frac{2}{t_1+t_2}$

b) equation of the chord PQ is $y\left(t_1+t_2\right)=2x+2at_1t_2$

c) length of the chord PQ is $|a||t_1-t_2|\sqrt{\left(t_1+t_2\right)^2+4}$

d) PQ passes through focus (a, 0) then it is called focal chord and $\mathbf{t_1.t_2=-1}$

e) PQ is a focal chord and if $P\left(at_1^2,\ 2at_1\right)$ *then* $Q\left(\frac{a}{t_1^2}, -\frac{2a}{t_1}\right)$, because $t_1.t_2=-1$

f) length of the focal chord $PQ=|a|\left(t_1+\frac{1}{t_1}\right)^2 \geq 4a$

g) length of the smallest focal chord of $y^2=4ax$ *is* $4a$ (i.e. latus rectum of a parabola is the smallest focal chord)

h) Circle drawn on any focal chord, as diameter, touches its directrix

i) If focal chord PQ makes an angle $\sim$, with the axis of the parabola, then the length of the focal chord $PQ=4a\operatorname{cosec}^2\sim$

j) Let P(x, y) be any point on the parabola $y^{2,}=4ax$ whose focus is at S (a, o) then the focal distance of the point SP = a + at^2(i.e. a+ x) (SP is equal to perpendicular distance of P from the directrix x + a =0)

k) $\frac{2SP.SQ}{SP+PQ}=2a$

(i.e. harmonic mean of SP and SQ, is the semi latus rectum of the parabola $y^2=4ax$)

l) Through the vertex O of $y^{2,}=4ax$, chords OP and OQ are drawn $\left(O(0,0), P\left(at_1^2, 2at_1\right) \& Q\left(at_2^2, 2at_2\right)\right)$,right angles to each other, then $t_1.t_2=-4$ m).

For all position of P, PQ cuts the axis of the parabola at a fixed point (4a,0) ie chord which subtends right angle at the vertex of the parabola, then the parameters of the chord joining $t_1 \& t_2$ *is* $t_1.t_2=-4$.

9) $P(at_1^2, 2at_1) \& Q(at_2^2, 2at_2)$ is a focal chord of the parabola $y^2 = 4ax$. (O-vertex), then area of the triangle OPQ is $\Delta = a^2|t_1 - t_2|$ and difference of the distance of P and Q from the axis of the parabola is $\frac{2\Delta}{Q}$

Position of a point $P(x_1, y_1)$ with respect to $y^2 = 4ax$

$P(x_1, y_1)$ lies inside or on or outside as $y_1^2 - 4ax_1 < 0 (or) = 0 \ (or) > 0.$

INTERSECTION OF A LINE AND A PARABOLA:

1a) Intersection of a line $y = mx + c$ & $y^2 = 4ax. \Rightarrow mx^2 - 4ax + 4ac = 0,$

$D = 0 \Rightarrow c = \frac{a}{m} (m \neq 0) \Rightarrow$ the line touches the parabola and the equation of the tangent is $y = mx + \frac{a}{m}$

And the point of contact of the line $y = mx + c$ and the parabola $y^2 = 4ax$ is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

b) $D > 0 \Rightarrow a > mc \Rightarrow$ the line intersects parabola at two distinct points.

c) $D < 0 \Rightarrow a < mc$ the line does not touch (or) intersects the parabola.

EQUATION AND PROPERTIES OF TANGENT

1a) Equation of the tangent drawn to the parabola $y^2 = 4ax$ at the point $(at_1^2, 2at_1)$ is

$t_1 y = x + at^2$

1b) Equation of the tangent drawn to the parabola $y^2 = 4ax$ at the point $\left(\frac{a}{m^2}, \frac{-2a}{m}\right)$ is

$y = mx + \frac{a}{m}$

1c) Equation of the tangent drawn to the parabola $y^2 = 4ax$ at the point (x_1, y_1) is

$yy_1 = 2ax + 2ax_1$

1d) Equation of the tangent drawn to the parabola $(y-k)^2 = 4a(x-h)$ at $\left(h + \frac{a}{m^2}, k + \frac{2a}{m}\right)$ is

$(y-k) = m(x-h) + \frac{a}{m}$

1e) Equation of the tangent drawn to the parabola $(y-k)^2 = 4a(x-h)$ at (x_1, y_1) is

$(y-k)(y_1 - k) = 2a(x-h) + 2a(x_1 - h)$

1f) the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ then $\text{l.n} = a.m^2$.

1g) the line $x\cos\sim + y\sin\sim = p$ touches the parabola $y^2 = 4ax$ then $a\sin^2\sim + p\cos\sim = 0$ and the point of contact is $(a\tan^2\sim, -2a\tan\sim)$

2a) Tangents, drawn to the parabola $y^2 = 4ax$ at $P(at_1{}^2, 2at_1) \& Q(at_2{}^2, 2at_2)$ intersect at R $(at_1t_2,\ a(t_1 + t_2))$

(Here x-coordinate of the point of intersection R is the geometric mean of $at_1^2 \& at_2^2$ and y-coordinate of intersection R is the Arithmetic mean of $2at_1 \& 2at_2$)

2b) locus of point of intersection of perpendicular tangents to the parabola is, its directrix (or) tangents drawn to the parabola, at the extremities of any focal chord intersect at right angles on its directrix.

2c) A chord of $y^2 = 4ax$ which subtends a right angle at the vertex, then locus of point of intersection of tangents drawn to the parabola, at the extremities of the chord is $x = -4a.\ (use\ t_1.t_2 = -4)$

3a) Area of a triangle formed by 3 points $P\left(at_1^2, 2at_1\right)\ Q\left(at_2^2, 2at_2\right) \& R(at_3^{\ 2}\ 2at_3)$ on a parabola $y^2 = 4ax$ is twice the area of the triangle P'Q'R', formed by tangents at these points (P',Q'& R' are the points of intersection of the tangents)

3b) Ortho centre of the $P'Q'R'$ is $\left(-a,\ a\left(t_1 + t_2 + t_3 + t_1t_2t_3\right)\right)$

(i.e. orthocentre of a triangle, formed by 3 tangents drawn to a parabola, lies on its directrix).

3c) Tangents to the parabola $y^2 = 4ax$ at any three points $P\left(at_1^2, 2at_1\right)\ Q\left(at_2^2, 2at_2\right) \& R(at_3^{\ 2}\ 2at_3)$ enclose a triangle ABC, (A,B,C are the points of intersection of these tangents), then circum circle of the $triangle\,ABC$ passesthrough the focus of the parabola, and the equation of the circle is $x^2 + y^2 - a(t_1t_2 + t_2t_3 + t_3t_1 + 1)x - a(t_1 + t_2 + t_3 - t_1t_2t_3)y + a^2\left(t_1t_2 + t_2t_3 + t_3t_1\right) = 0$

4) TP and TQ are any two tangents drawn, to a parabola and the tangent at a third point R cuts them in P' and Q' then $\frac{TP'}{TP} + \frac{TQ'}{RQ} = 1.$

5) PQ be any chord of the parabola and its mid point is R. The point of intersection of the tangents drawn to the parabola at the extremities of the chord PQ is T. Then the line RT is parallel to the axis of the parabola.

6) L, M, N are 3 points on the parabola $y^2 = 4ax$ whose ordinates are in AP. Then tangents drawn to the parabola atL and N, will meet on the ordinate of M.

7) If the tangent at P and Q on meet at T then

i) TP and TQ subtend equal angles at the focus S

ii) ST^2 = (SP).(SQ)

iii) The triangles SPT and STQ are similar

8) Tangent at any point $P\left(at^2, 2at\right)$ on the parabola $y^2 = 4ax,$ intersects its axis at $T\left(-at^2, 0\right)$. A (a, 0) is vertex.

$Z\left(0, at\right)$ is the point of intersection tangent at the vertex and tangent at P.

S(a, 0) be the focus.

$R\left(-a, 2at\right)$ is the point of intersection directrix and the line SZ produced. $Q\left(-a, at - \frac{a}{t}\right)$ is the point of intersection directrix and the tangent line at P.

8a) Tangent at any point $P\left(at^2, 2at\right)$ on the parabola $y^2 = 4ax,$ bisects the angle between the focal distance of the point and perpendicular on the directrix from the point

8b) Portion of the tangent to a parabola cut off between the directrix and the curve, (i.e. PQ) subtends a right angle at the focus S.

(slope of SQ).(slope of PS) = - 1

8c) If SZ is perpendicular to any tangent at P, then "Z" lies on the tangent at the vertex and $SZ^2 = AS.SP.$ (where A is vertex, and T is the point of intersection of tangent with axis of the parabola)

8d) Perpendicular drawn from the focus on any tangent to a parabola intersect it at the point where it cuts the tangent at the vertex (Z)
And also , circle,drawn on SP as diameter touches the tangent at the vertex, at Z. and the circle, may not touch the axis.

8e) Tis the image of P with respect to the line SZ, and R is the image of S with respect to the line PT.
(image of focus with respect to any tangent lies on its directrix)

8f) Z is the mid point of PT as well SR

8g) Mid point of SP is X (say) and XZ is parallel to the axis.

8h) Quadrilateral TSPR is a rhombus, because TS = SP and TR is parallel to SP, ST is parallel to PR

PAIR OF TANGENTS

Pair of tangents, are drawn to the parabola, from the point $R(x_{1,}y_1)$ is $T^2 = SS_1$

$$\left[\left(yy_1 - 2a\left(x + x_1\right)\right)^2 = \left(y^2 - 4ax\right)\left(y_1^2 - 4ax_1\right)\right]$$

CHORD OF CONTACT

1a) Equation of the chord of contact of tangents, drawn from a point $R\left(x_1, y_1\right)$ to the parabola $y^2 = 4ax$ *is* $yy_1 = 2a\left(x + x_1\right)$.

1b) Length of the chord of contact PQ is, $PQ = \dfrac{\sqrt{y_1^2 - 4ax_1}\sqrt{y_1^2 + 4a^2}}{a}$

1c) Area of the triangle RPQ, formed by pair of tangents RP, RQ drawn to the parabola $y^2 = 4ax$, from the point R(x_1, y_1) and the chord of contact *is* $\dfrac{\left(y_1^2 - 4ax_1\right)^{3/2}}{2a}\left(a > 0\right)$.

2) Tangents are drawn to the parabola $y^2 = 4ax$, at the points where the line $lx + my + n = 0$ meets the Parabola then the point of intersection of these tangents is $\left(\dfrac{n}{l}, \dfrac{-2am}{\ell}\right)$.

EQUATION OF CHORD WHOSE MID POINT IS GIVEN

1) Equation of chord, drawn to the parabola $y^2 = 4ax$, whose mid-point $\left(x_1, y_1\right)$ is $yy_1 - 2a\left(x + x_1\right) = y_1^2 - 4ax_1$

2a) locus of mid points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex, is $y^2 = 2a\left(x - 4a\right)$

2b) locus of mid points of the chords of the parabola $y^2 = 4ax$ which pass through the vertex is $y^2 = 2ax$.

LENGTH OF THE SUBTANGENT AND SUB NORMAL

1) Tangent drawn to the parabola $y^2 = 4ax$ at P(at^2, 2at) meets its axis at T. Normal drawn to the parabola Meets its axis at G. Foot of the perpendicular dropped from P to the x-axis is N. Then

a) PT = length of the tangent = $2at\sqrt{1+t^2}$ b) PG = length of normal = $2a\sqrt{1+t^2}$

c) TN = length of sub tangent = $2at^2$ d) NG = length of subnormal = 2a.

2) Length of subtangent at any point P(x,y) on the parabola equals twice the abscissa of the point P.

3) Length of subnormal is constant for all points on the parabola and is equal to the semi latus rectum.

EQUATION OF NORMAL

1a) Equation of normal to the parabola $y^2 = 4ax$ at (x_1, y_1) is $y - y_1 = \frac{-y_1}{2a}(x - x_1)$

1b) Equation of normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $y + tx = 2at + at^3$

1c) Equation of normal to the parabola $y^2 = 4ax$ at $(am^2, 2am)$ is $y = mx - 2am - am^3$

$\left(m = \frac{-y_1}{2a} \Rightarrow \ y_1 = -2am \ \& \ x_1 = am^2\right)$ and slope of the normal is '-t' or 'm'

1d) Equation of normal to the parabola $(y-k)^2 = 4a(x-h)$ at $(am^2 + h, -2am + k)$ is

$(y-k) = m(x-h) - 2am - am^3$

PROPERTIES OF NORMAL

1a) Normal drawn to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$, meets the parabola again at

$Q(at_2^2, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1}$

1b) The point of intersection of normals, drawn to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$ &

$Q(at_2^2, 2at_2)$ mee $R\left(2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2)\right)$

1c) Normals at $P(at_1^2, 2at_1)$ & $Q(at_2^2, 2at_2)$ drawn to the parabola $y^2 = 4ax$,meet on the parabola, then $t_1.t_2 = 2$

1d) Normals, drawn to the parabola $y^2 = 4ax$, at $P(at_1^2, 2at_1)$ & $Q(at_2^2, 2at_2)$, meet on the parabola then Product of the ordinates of P & Q is $8a^2$ $\quad (2at_1 \times 2at_2 = 4a^2t_1t_2 = 8a^2)$

1e) PQ is a focal chord of the parabola $y^2 = 4ax$. Tangent drawn at P and normal drawn at Q are parallel

1f) Tangent and normal drawn to the parabola $y^2 = 4ax$, at meets its axis at M and N respectively .Circle drawn on MN, as diameter has its centre at the focus.

1g) PQ islatus rectum of the parabola $y^2 = 4ax$, Tangents at P (a, 2a) , Q (a,-2a) drawn to Parabola $y^2 = 4ax$ intersect at the point $(-a, 0)$ at right angle. And the normal drawn at

P (a, 2a) , Q (a,-2a) intersect at T $(3a, 0)$ on the axis, at right angle PTQR is a square. Equation of normal drawn to the parabola at P is $x + 3y - 3a = 0$.

1h) Normal chord of a parabola $y^2 = 4ax$, at a point whose ordinate is equal to the abscissa subtends, a right angleat the focus

1i) Normal at any point P of a parabola is equally inclined to the focal distance of the point and the axis of the parabola (S-focus, G-point of intersection of normal with x-axis) i.e. $\underline{|SGP} = \underline{|SPG} \Rightarrow SP = SG$.

NORMAL CHORD

1a) Normal, drawn to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$, meets the parabola again at

$Q(at_2^2, 2at_2)$ then PQ is called normal chord

1b) $PQ^2 = \frac{16a^2(1 + t_1^2)^3}{t_1^4}$ (square of the length of normal chord).

1c) Length of normal chord PQ, is minimum when $t_1 = \sqrt{2}.\left[\left(\frac{d(PQ)}{dt_1}\right) = 0 \Rightarrow t_1 = \sqrt{2}\right]$

1d) Length of the shortest normal chord, drawn to the parabola $y^2 = 4ax$ *is* $6\sqrt{3}a$.

1e) If PQ, a normal chord subtends right angle at vertex of the parabola, then $t^2 = 2$.

2a) Normal drawn to the parabola $y^2 = 4ax$ *at* $P(t)$ other than vertex, meets the parabola again at $Q(t_1)$

Then a) least distance between $Q(t_1)$ and its vertex is $4\sqrt{6}a$

2b) Least distance between $Q(t_1)$ and the axis of the parabola is, $4\sqrt{2}$.

$\left[QR = |2at_1| = 2a|t_1| \quad OQ > 4\sqrt{2}a\right]$.

2c) Normal drawn to the parabola $y^2 = 4ax$ at $P(am^2, -2am)$ meets the parabola again at Q.

If tangents at P & Qmeet at a point R, then area triangle PQR *is* $\frac{4a^2}{m^3}(1+m^2)^3$

2d) If a normal drawn, to the parabola $y^2 = 4ax$ at a point $P(at^2, 2at)$ makes an angle "¯" with the axis of the parabola, will cut the curve again at $Q(at_1^2, 2at_1)$ an angle

¤ $= \tan^{-1}\left(\frac{\tan¯}{2}\right)$.

CONORMAL POINTS

For the parabola $y^2 = 4ax$. three normals are drawn from T(h, k)

$P(am_1^2, -2am_1) Q(am_2^2, -2am_2)\ R(am_3^2, -2am_3)$ are the feet of the normal / co normal points

1a) equation of the normal drawn to the parabola y2=4ax, at $(am^2, -2am)$ is,

$y = mx - 2am - am^3$ It passes through $(h,k) \Rightarrow am^3 + m(2a-h) + k = 0$

a) $\sum m_i = 0 \quad i = 1,2,3$ b) $\frac{2a-h}{k}$ c) $m_1.m_2.m_3 = \frac{-k}{a}$

1b) Algebraic sum of the slope of the normals at co normal points is zero

$(i.e.\ \ m_1 + m_2 + m_3 = 0)$

1c) Algebraic sum of the ordinates of the co normal points is zero

$(i.e. -2am_1 - 2am_2 - 2am_3 = 0)$

1d) Centroid of the triangle PQR is $\left(\frac{2}{3}(h-2a), 0\right)$ and at a distance of $\frac{2}{3}.|h-2a|$ from its vertex

(i.e. centroid of the triangle, formed by joining the co-normal points, lies on its axis)

Ortho centre triangle PQR is $\left(h-6a, \frac{-k}{2}\right)$ and at a distance of $\frac{2}{3}.|h-2a|$ from its vertex

2) For the parabola $y^2 = 4ax$. three normals are drawn from T(h, k)

$P(am_1^2, -2am_1) Q(am_2^2, -2am_2)\ R(am_3^2, -2am_3)$ are the feet of the normal / co normal points

Then, equation of circle, passing through P, Q, R is $x^2 + y^2 - (h+2a)x + \frac{k}{2}y = 0$

2a) This circle always passes through vertex of the parabola $y^2 = 4ax$. (i.e. (0, 0)

Circumcentre of the circle is $\left(\frac{(h+2a)}{2}, \frac{-h}{4}\right)$

3) Condition for three real & distinct normals to be drawn from T(h, k) to the parabola

$y^2 = 4ax \quad h > 2a \text{ and } 27ak^2 < 4(h-2a)^3$

4) From an external point, only one normal can be drawn to a parabola.

REFLECTION PROPERTY

All rays of light, coming from the positive direction of $x-axis$ and parallel to the axis of the parabola $y^2 = 4ax$, after reflection, passes through the focus of the parabola. $y^2 = 4ax$

SOME LOCUS QUESTIONS / RESULTS RELATED TO CO NORMAL POINTS

a) Two of the three normals drawn to the parabola $y^2 = 4ax$ from T(h, k) are at right angles,then the locus of T (h, k) is $y\left(y^2+(3a-h)a\right)=0$

b) Two of the three normals drawn to the parabola $y^2 = 4ax$ from T(h, k) are coinciding, then the locus of T (h, k) is $27ay^2 = 4(x-2a)^3$

c) Three normals are drawn to the parabola $y^2 = 4ax$ from T(h, k). Then the locus $T(h,k)$ when one normal bisects the angle between the other two normal is 27 $ay^2 = (x-5a)^2(2x-a)$

d) Three normals are drawn to the parabola $y^2 = 4x$ from T(h, k). Then the locus $T(h,k)$ such that two of the normal make angles with the axis, the product of whose tangents is 2. Then the locus of T is a part of $y^2 = 4x$

e) Equation of normal, drawn to the parabola $y^2 = 4ax$ *is* $y = mx - 2am - am^3$. and it passes through $P(h,k)$Then locus of $P(h,k)$ such that two of the three normal, make complementary angles with the axis of parabola, is $y^2 = a(x-a)$

f) locus of $P(h,k)$, if the sum of the angles made by the normals, drawn to the parabola $y^2 = 4ax$ from $P(h,k)$with the axis is constant (a) is $y = (x-a)\tan a$

$$\left(\bar{\ }_1+\bar{\ }_2+\bar{\ }_3 = a \Rightarrow \tan\left(\bar{\ }_1+\bar{\ }_2+\bar{\ }_3\right) = a \Rightarrow \frac{s_1-s_3}{1-s_2} = a\right)$$

COMMON TANGENTS

1a) Equation of the common tangent to $y^2 = 4ax \,\&\, x^2 = 4by$ is $a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$ with slope $-\left(\frac{a}{b}\right)^{1/3}$

1b) Two equal parabolas having same vertex and their axes are at right angles are $y^2 = 4ax \,\&\, x^2 = 4by$ The common tangent drawn to the parabolas touch them at P(a,-2a) and Q(-2a,a). Then length of common tangent PQ is $3\sqrt{2}.\left(\frac{1}{4}\textit{latus rectum}\right) = \left(i.e.\ 3\sqrt{2}a\right)$ and slope of the common tangent -1

1c) Equation of the common tangent to $y^2 = 4x \,\&\, x^2 = 4y$ *is* $x + y + 1 = 0$

2) Consider the curves $c_1 : y = x^2 + 1$ & $C_2 : x = y^2 + 1$. If PQ is the shortest distance of the two curves. & $R\left(\frac{-1}{2},\frac{5}{4}\right)$ $S\left(\frac{5}{4},\frac{-1}{2}\right)$ are the points of contact of common tangents and the equation of the common tangent is $4x + 4y = 3$. and the shortest distance $PQ = \frac{3}{2\sqrt{2}}$ and the area of quadrilateral $PQRS$ *is* $\frac{5}{4}$

(both the curves are symmetrical about the line y = x)