MINI PROJECT-II REPORT

PROJECT TOPIC:- Virtual Harmonics Analyser

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➢ Part Completed In MiniProject I :

In Miniproject1, our main aim was analysing harmonic components present in any distorted, non-sinusoidal waveform, in a system containing non-liner load (where the current isn't proportional to the voltage and it fluctuates based on the alternating load impedance) by using an analyser. By the analysis we get to know the amount or percentage of the harmonic components which is superimposed in the fundamental or the sinusoidal wave to get the final distorted wave.

For analysis we used the **FFT Analysis toolbox** inside **powergui** block in MATLAB Simulink. This toolbox acts as an inbuilt analyser which employs Fast Fourier Transform (**FFT**) algorithms to analyse the harmonics present in the input wave.

There, we have simulated two circuits in MATLAB Simulink environment, a **A) Single phase bridge Rectifier** and a **B) Three phase bridge Rectifier**, both of which are non-linear. Then, using the FFT toolbox of MATLAB we performed an analysis of the input waveform of one and the output waveform of another circuit to study the variation of the distortions in the waveforms by changing the Load Inductance.

➢ Limitations of FFT :

There is no doubt about how incredibly powerful Fourier analysis can be. However, its popularity and effectiveness have some downsides.

When the number of harmonics is very large and at the same time certain harmonics are distant from the other, the conventional frequency detecting method like FFT are not satisfactory as,

- It is not useful for analyzing time-variant, non-stationary signals.
- It cannot compute the **damping coefficients** of different frequencies.
- Fourier transforms **distribute time-domain noise uniformly throughout the frequency domain** which leads to limitation in the certainty with which peak frequencies, widths, magnitudes and phases could be computed.
- Discrete sampling of a time-domain continuous signal causes limitation in obtaining the spectral information content.
- It **cannot provide simultaneous time and frequency information**.

Thus, for an energy spectrum as the Fourier transform is essentially an integral over time, we lose all information that varies with time. In other words, we can comment on what happens a signal, not when it happens.

➢ Introduction - What is Prony?

Prony Analysis was first introduced into power system applications in 1990 . Prony's method is a signal processing technique for extraction of the sinusoid or exponential signals by solving a set of linear equations. Prony analysis basically tries to find out the spectrum of different damped sinusoids of distinct frequencies, phase and amplitudes which, when combined, exactly fit the sampled measured values of the signal over an interval of time. Also, unlike Fourier analysis, here we **can also calculate the damping coefficients**.

➢ The advantages of Prony Analysis over Fourier Transform :

To overcome the limitations of FFT, there are several algorithms available for analyses. One such advanced algorithm includes **Prony**.

- Harmonics with time varying magnitudes in power systems are called power system transient harmonics. The accuracy of Fourier transform is affected when these transient or time varying harmonics exist. Prony analysis is applied as an analysis method for harmonic supervisors and as a harmonic reference generation method as, being an autoregressive spectrum analysis method, it has some very valuable features.
- Prony analysis does not require frequency information prior to **filtering**. Due to the ability to identify the damping factors of transients, Prony analysis can accurately identify growing or decaying components of signals. Transient harmonics thus can be correctly identified from Prony analysis for the Prony-based harmonic supervision.
- FFT analysis requires at least one complete cycle of the signal to complete its analysis, whereas **Prony Analysis has such an algorithm that it can complete its entire analysis within even half a cycle**, if sufficient samples are taken.

➢ BASICS AND MATHEMATICS OF PRONY :

Prony analysis directly estimates the parameters of the eigen structure described in the signal by fitting a sum of complex damped sinusoids to evenly spaced sample (in time) values of the output:

$\hat{y}$ (t) = $\sum_{i=1}^{L} A_i \, e^{(\sigma_i t)} \cos(2\pi f_i t + \phi_i)$ ....(1)

Where,

A_i = Amplitude of the component i
σ_i = Damping co-efficient of component i
ϕ_i= Phase of component i
f_i = frequency of component i
L= total no of damped exponential components i
$\hat{y}$ (t) = Estimate of observed data y(t) consisting of N samples $y(t_k)$=y[k], k=0,1,2.....N-1 that are evenly spaced

Using Euler's theorem, $\cos(2\pi f_i\, t+ \phi_i)$ can be represented as a sum of exponentials:

$$cos(2\pi f_i t + \phi_i) = \frac{e^{j(2\pi f_i t+\phi_i)} + e^{-j(2\pi f_i t+\phi_i)}}{2} = \frac{e^{j2\pi f_i t} e^{j\phi_i}}{2} + \frac{e^{-j2\pi f_i t} e^{-j\phi_i}}{2}$$

.........................(2)

Inserting (3.2) into (3.1) and letting t=kT the samples of y(t) are written as

y[k]=$\sum_{i=1}^{L} C_i \, \mu_i^k$ ....(3)

where $C_i = \frac{A_i}{2} e^{j\phi_i}$
.........................(4)

$\mu_i = e^{(\sigma_i + j2\pi f_i)T}$, T is the sampling period

The original Prony analysis computes C_i and μ_i in three basic steps:

Step-1

Solve linear prediction model, which is constructed by the observed data set.

First we write y[k] as a linear prediction model as

$$\boldsymbol{y[k] = a_1 y[k-1] + a_2 y[k-2] \ldots \ldots \ldots \ldots . a_L y[k-L]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$\boldsymbol{y[k]}$ is computed for **k=**$\boldsymbol{(L+1), (L+2), \ldots \ldots \ldots \ldots N}$

For example, $\boldsymbol{y[L+1] = a_1 y[L] + a_2 y[L-1] \ldots \ldots \ldots \ldots . a_L y[1]}$

We can write $\boldsymbol{y[k]}$ in matrix form for various values of k as:

$$\begin{bmatrix} \boldsymbol{y[L+1]} \\ \boldsymbol{y[L+2]} \\ \cdot\cdot \\ \cdot\cdot \\ \boldsymbol{y[N]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{y[L]} & \boldsymbol{y[L-1]\ldots.y[1]} \\ \boldsymbol{y\{L+1]} & \boldsymbol{y[L]\ldots.y[2]} \\ \cdot\cdot \quad \cdot\cdot & \cdot\cdot \quad \cdot\cdot \\ \cdot\cdot \quad \cdot\cdot \quad \cdots & \cdot\cdot \quad \cdot\cdot \\ \boldsymbol{y[N-1]\ y[N-2]\ldots.y[N-L]} & \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \cdot\cdot \\ \cdot\cdot \\ \boldsymbol{a_L} \end{bmatrix}$$

Or,

$$d = Da \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

where

$$d = \begin{bmatrix} y[L+1] \\ y[L+2] \\ \cdot\cdot \\ \cdot\cdot \\ y[N] \end{bmatrix}, \ D = \begin{bmatrix} y[L] & y[L-1]\ldots\ \ldots\ y[1] \\ y[L+1] & y[L]\ldots\ \ldots\ y[2] \\ & \cdot\cdot \\ & \cdot\cdot \\ y[N-1]\ y[n-2]\ldots & \ldots y[N-L] \end{bmatrix}, \ a = \begin{bmatrix} a_1 \\ a_2 \\ \cdot\cdot \\ \cdot\cdot \\ a_L \end{bmatrix}$$

Assuming N>2L, the linear prediction coefficients vector a is estimated by solving the over-determined least square problem, which is computed using

$$a = \frac{d}{D} \text{ or in Matlab } \quad \mathbf{a = pinv(D)*d} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Step-2

We need to find roots of characteristic polynomial formed from the linear prediction coefficients

$$\mu^L \text{-} a_1\mu^{L-1} - \cdots .. \ -a_{L-1}\mu - \ a_L = (\mu-\mu_1)(\mu-\mu_2)\ldots\ldots(\mu-\mu_L) \ \ldots\ldots.. (8)$$

As vector a is known from (3.7), the roots μ_i of the polynomial (3.8) can be readily computed. In Matlab the roots can be computed as

$$\mu_i = roots([1; -a])$$

Step-3

Then we solve the original set of linear equations to yield the estimates of the exponential amplitude and sinusoidal phase

$$\begin{bmatrix} y[1] \\ y[2] \\ .. \\ .. \\ y[N] \end{bmatrix} = \begin{bmatrix} 1 & 1 \;.. & .. & 1 \\ \mu_1 & \mu_2 & .... & \mu_L \\ \mu_1^2 & \mu_2^2 & .. \;\; .. & \mu_L^2 \\ & .. \;.. \;.. & & \\ \mu_1^{n-1} & \mu_2^{n-1} & . \;\; .. & \mu_L^{n-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ .. \\ .. \\ C_L \end{bmatrix}$$

Or Y1=UC

Where

$$Y1=\begin{bmatrix} y[1] \\ y[2] \\ .. \\ .. \\ y[N] \end{bmatrix} \qquad U=\begin{bmatrix} 1 & 1 \;.. & .. & 1 \\ \mu_1 & \mu_2 & .... & \mu_L \\ \mu_1^2 & \mu_2^2 & .. \;\; .. & \mu_L^2 \\ & .. \;.. \;.. & & \\ \mu_1^{n-1} & \mu_2^{n-1} & . \;\; .. & \mu_L^{n-1} \end{bmatrix} \text{ and } \quad C=\begin{bmatrix} C_1 \\ C_2 \\ .. \\ .. \\ C_3 \end{bmatrix}$$

Now, C can be computed as

C= pinv(U)*Y1

As C and μ are known, the amplitude, frequency, phase and damping co efficient can be computed as follows

<u>Finding Frequency(f_i)</u>

$$\mu_i = e^{(\sigma_i + j2\pi f_i)T}$$

$$= e^{\sigma_i T} . e^{(j2\pi f_i)T}$$

$$= e^{\sigma_i T}(cos2\pi f_i T + jsin2\pi f_i T)$$

$$\text{Real}(\mu_i) = e^{\sigma_i T} . cos2\pi f_i T$$

$$\text{Imaginary}(\mu_i) = e^{\sigma_i T}.sin2\pi f_i T$$

$$\frac{\text{Imaginary}(\mu_i)}{\text{Real}(\mu_i)} = \frac{e^{\sigma_i T}.sin2\pi f_i T}{e^{\sigma_i T}.cos2\pi f_i T} = \tan 2\pi f_i T$$

So, $$f_i = \frac{1}{2\pi T}\arctan\left(\frac{\text{Imaginary}(\mu_i)}{\text{Real}(\mu_i)}\right)$$

Finding Damping Co-efficient(σ_i)

$$\text{abs}(\mu_i) = e^{\sigma_i T}.|(cos2\pi f_i T + jsin2\pi f_i T)|$$

$$= e^{\sigma_i T}.1$$

$$= e^{\sigma_i T}$$

$$\sigma_i = (\log(abs(\mu_i)))/T$$

Finding Amplitude(A_i)

$$C_i = \frac{A_i}{2}e^{j\phi_i}$$

$$\text{Abs}(C_i) = \frac{A_i}{2}$$

$$A_i = 2(\text{Abs}(C_i))$$

Finding Phase Angle(ϕ_i)

$$C_i = \frac{A_i}{2}e^{j\phi_i}$$

$$= \frac{A_i}{2}(cos\phi_i + sin\phi_i)$$

$$\text{Real}(C_i) = \frac{A_i}{2}cos\phi_i$$

$$\text{Imag}(C_i) = \frac{A_i}{2}sin\phi_i$$

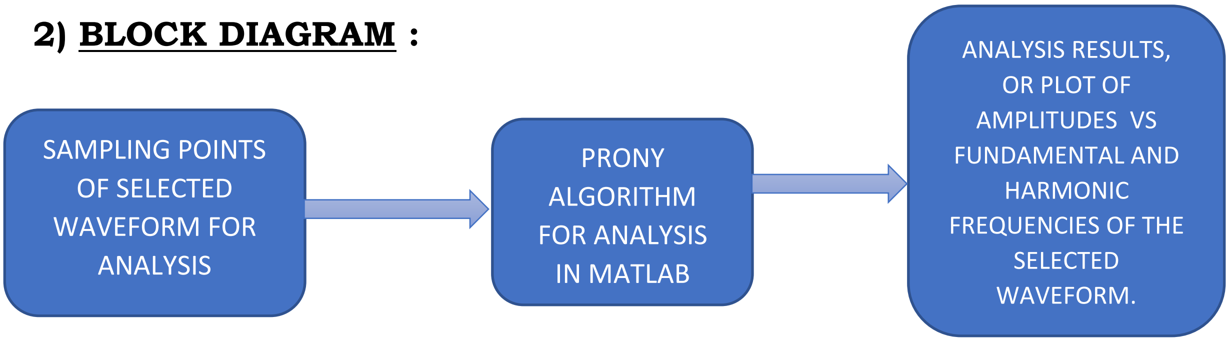
$$\frac{\text{Imag}(C_i)}{\text{Real}(C_i)} = \tan(\phi_i)$$

$$\phi_i = \arctan\left(\frac{Imag(C_i)}{Real(C_i)}\right) \qquad [in\ radian]$$

➢ <u>PROJECT DETAILS</u> :

1) **<u>AIM and OVERVIEW :</u>** In this project we are going to find the amount of content of different harmonics of a wave, which is related to a non-linear system, with respect to the fundamental frequency component of the wave. Here, we have used Prony's method for performing the analysis. For performing the analysis, the following steps were carried out:
    - The targeted wave is fed into the scope in Simulink and from the logging tab of the scope, the "load data to workspace" checkbox is checked.
    - There, the variable name is given and the saving format is selected as "Structure with Time".
    - From the Modelling tab of the main window, the starting and the ending time of the waveform, which we want to analyse, is selected. Then, the model is run.
    - Now, in MATLAB, the sampling values stored inside the named variable in a Structure in the workspace is initialized in a variable "y" in the main code by selecting the path of the named variable.
    - In the code, we performed all the steps of Prony analysis and found all the frequency components, corresponding amplitudes and damping coefficients of those frequencies. The negative frequencies were removed and the those with higher damping coefficients were eliminated. The final frequency and the normalized amplitude values were plotted against each other with respect to the fundamental (which is here 50Hz).
    - This resultant plot is compared with the result which we get from the FFT Toolbox in Simulink.

2) **BLOCK DIAGRAM** :



3) **MATLAB CODE** : Following is the code for performing Prony analysis on the samples values from the selected non-sinusoidal waveform in MATLAB.

```
%% Defining and initializing various parameters and variables
Ts= 0.0001; %sampling time
y=rectifier_source_current1(:,1); % values from simulink scope loaded in workspace and stored here
y=transpose(y); % to convert to row vector
N= length(y); %no. of samples
L= 20; % no. of frequency components chosen
disp(y)

% Implementing Step1--computing linear prediction coefficient vector
y1= y(L:-1:1); %first row of D
y2= y(L:(N-1)); %first column of D
D= toeplitz(y2,y1); %creating matrix D
d= transpose(y(L+1:N)); %creating vector d
a= pinv(D)*d;

% Implementing Step2--finding roots of polynomial
mu= roots([1;-a]); % roots return as a column vector
mu=transpose(mu); % converted to row vector
b=length(mu);

% Implementing step3--finding C matrix
for i=1:N %no. of rows in U
    for j=1:L %no. of columns in U
            u(i,j)= power(mu(j),i-1);   % U matrix is being created
    end
end
y3=transpose(y(1:N));
disp(u)
C=pinv(u)*y3;
disp(C);
```

```
C=pinv(u)*y3;
disp(C);
```

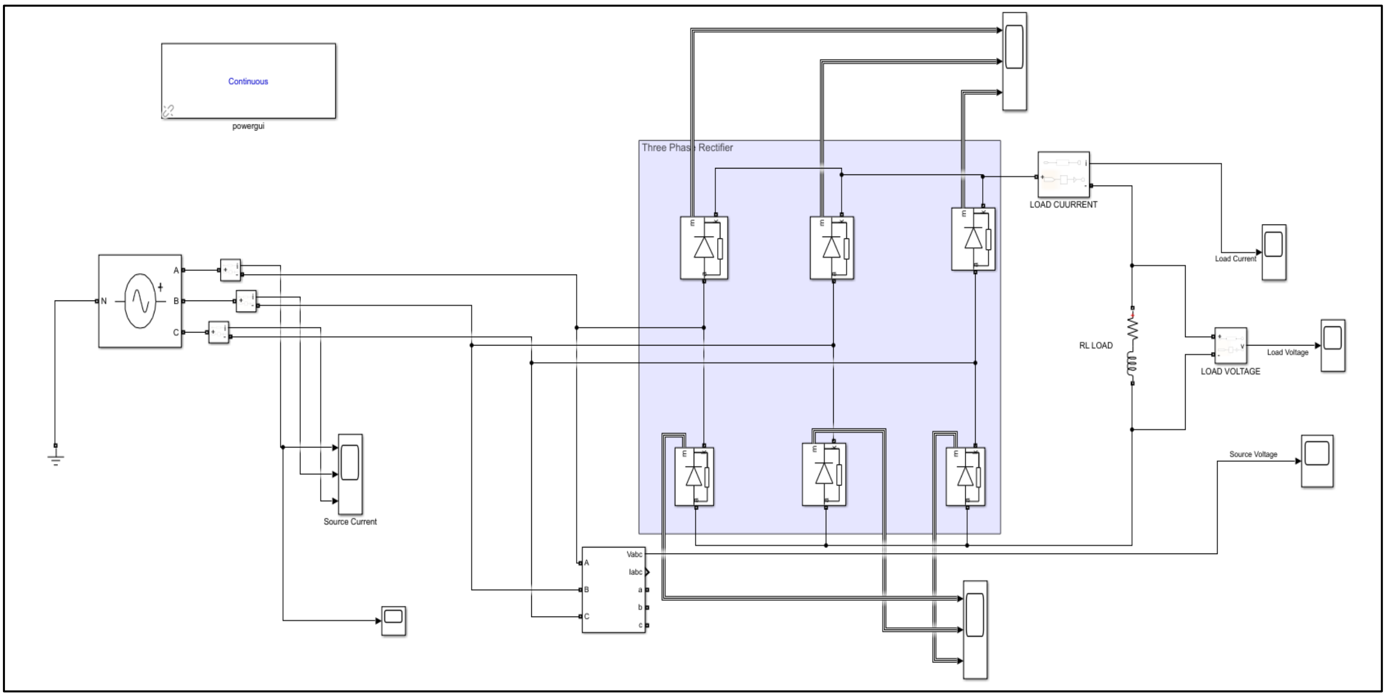
```
% Finding frequency, Amplitude and Damping Coefficients
C= transpose(C); % C converted to row vector
for i=1:L
    f(i)= ( atan(imag(mu(i))/real(mu(i))) )/(2*pi*Ts);
    amp(i)=2*abs(C(i));
    sigma(i)= ( log(abs(mu(i))) )/Ts;
end
disp(f)
disp(amp)
disp(sigma)
```

```
% We now remove the negative frequencies and find the corresponding
% amplitude and daming coefficient vectors.
j=1;
for i=1:L
    if f(i)>0
        f1(j)=f(i);
        amp1(j)=amp(i);
        sigma1(j)=sigma(i);
        j=j+1;
    end
end
sigma1
```

```
% We now remove the higher dampings and find the corresponding
% amplitude and frequency vectors.
j=1;l=length(sigma1);
for i=1:l
    if  sigma1(i)<500   %
```

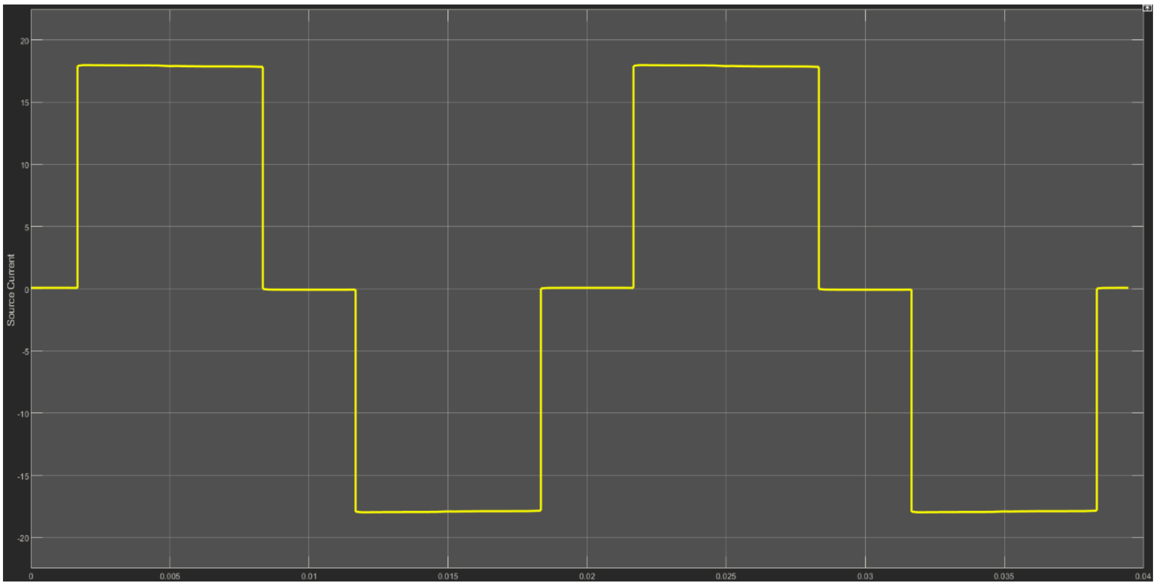
```
% We now remove the higher dampings and find the corresponding
% amplitude and frequency vectors.
j=1;l=length(sigma1);
for i=1:l
        if  sigma1(i)<500 && f1(i)<500   %
        sigma2(j)=sigma1(i);
        amp2(j)=amp1(i);
        f2(j)=f1(i);
        j=j+1;
    end

end

%Normalizing amplitude vector
amp_new =amp2./max(abs(amp2(:)))
stem(f2,amp_new); %% plotting normalised amplitude values w.r.t. its corresponding frequencies
 ylabel("Normalised amplitudes");xlabel("Frequency")
```

A) **CIRCUIT DIAGRAM OF A THREE PHASE BRIDGE RECTIFIER :**

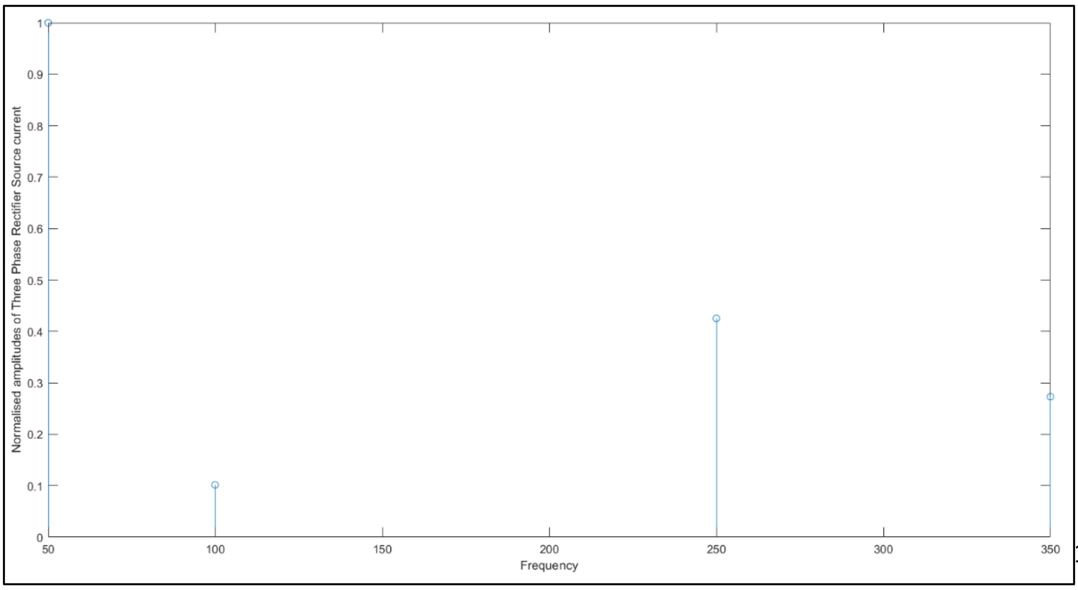


Here, a three phase AC supply drives the R-L load as shown in the figure, on passing through a Three Phase Bridge Rectifier. The waveform patterns are viewed using the Scope, to which the Ammeters and Voltmeters are connected, wherever required. The parameters chosen are: **R=30Ω , L=500 mH, V_{in}=230 V (A.C.) , f = 50 Hz.**

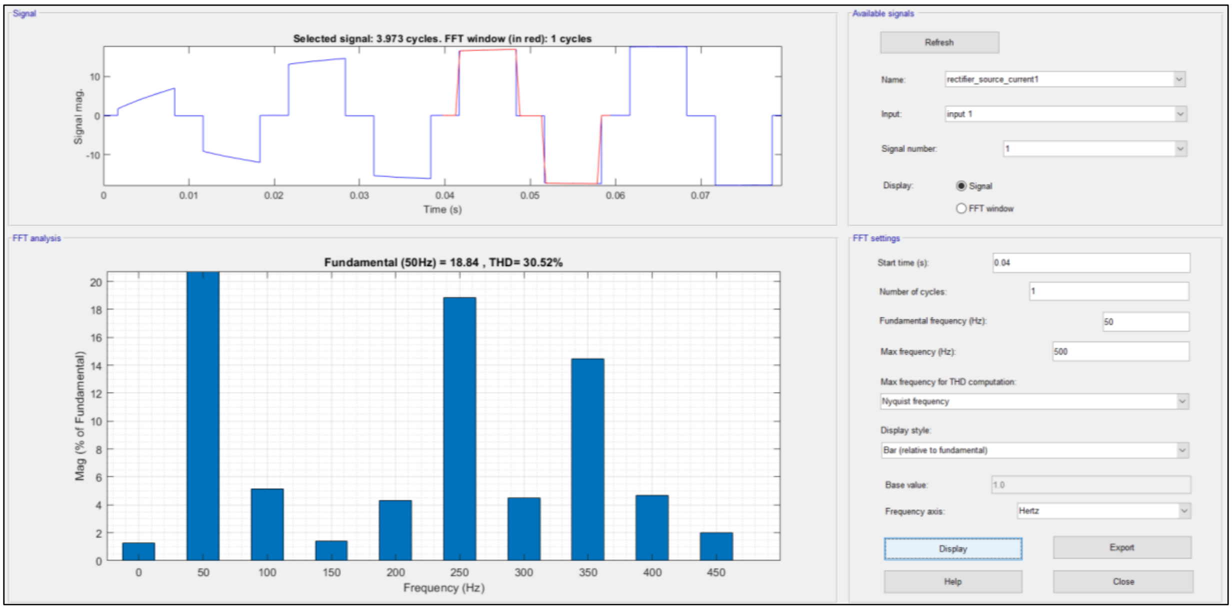
- **<u>SIGNAL FOR ANALYSIS :</u>** The line current or the input source current is taken as the signal for analysis, which when viewed in scope looks as follows:



- **<u>PRONY RESULTS :</u>** The results obtained from analysing the above signal by sampling its amplitude values at the time interval of 0.0001s and running the sample values through the lines of code which was shown above, gives the following plot of normalized amplitude vs corresponding frequency.

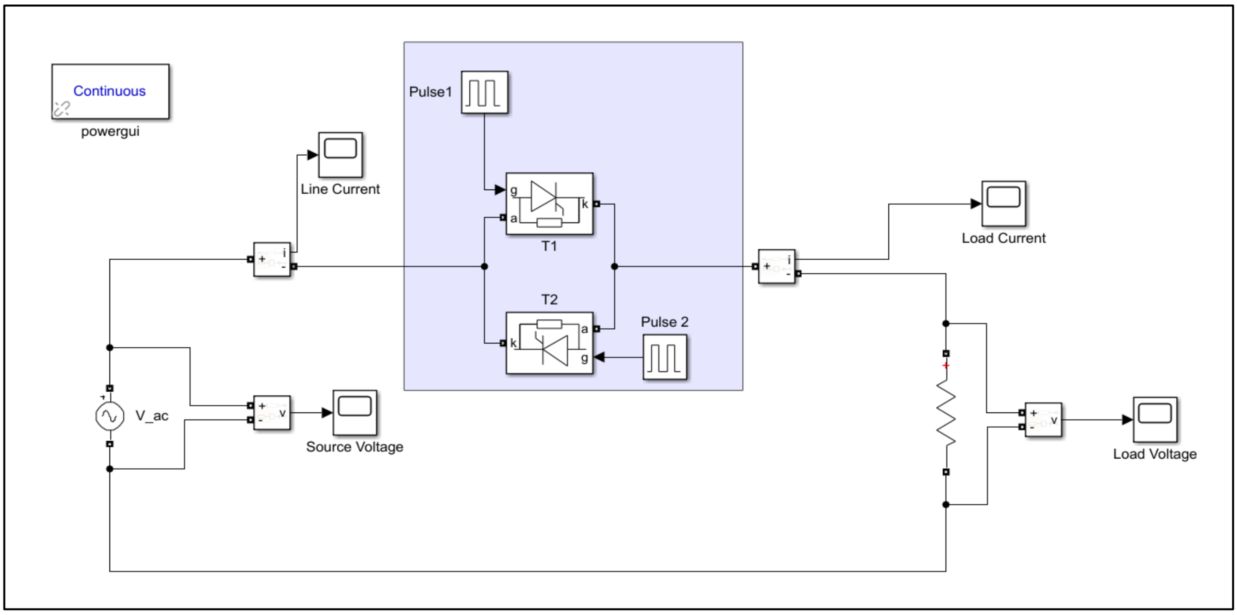


- <u>**COMPARING RESULTS FROM FFT :**</u>



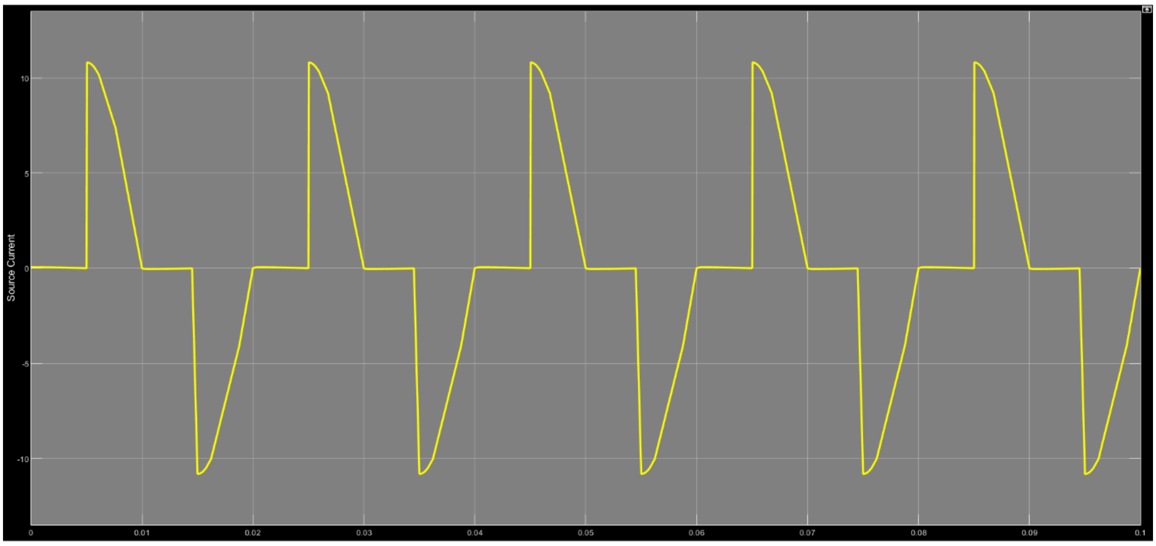
The distribution of fundamental and harmonic frequencies as obtained from **Prony Analysis** is **almost entirely similar** as those obtained by Fourier Analysis using the FFT toolbox**. The little difference in the distribution that occurs is because, unlike FFT, here, in Prony, the maximum damping coefficients are eliminated.** However, as expected, the dominant component is 50 Hz (that is the fundamental) and the **maximum harmonic component** is **250 Hz**, **followed by 350 Hz**, in both Prony and FFT analysis.

B) CIRCUIT DIAGRAM OF SINGLE PHASE AC TO AC VOLTAGE CONVERTER :

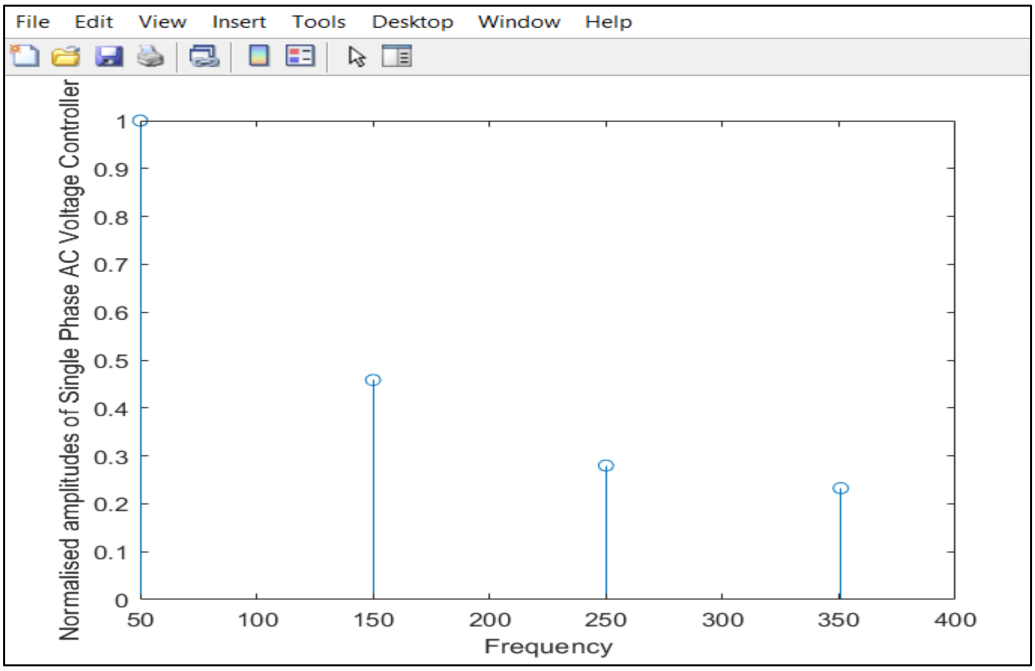


Here, a sinusoidal AC voltage source drives the R load as shown in the figure, on passing through a AC to AC voltage converter circuit. The waveform patterns are viewed using the Scope, to which the Ammeters and Voltmeters are connected, wherever required. The parameters chosen are: **R=30Ω , V_{in}=230 V (A.C.), f = 50 Hz.**

- **SIGNAL FOR ANALYSIS :** The line current or the input source current (of AC Voltage Controller)is taken as the signal for analysis, which when viewed in scope looks as follows:



- **PRONY RESULTS :** The results obtained from analysing the above signal by sampling its amplitude values at the time interval of 0.0001s and running the sample values through the lines of code which was shown above, gives the following plot of normalized amplitude vs corresponding frequency .



- **COMPARING WITH RESULTS FROM FFT :**

The distribution here is also **almost entirely similar** as those obtained by Fourier Analysis using the FFT toolbox**. And the little difference that can be seen is due to elimination of the maximum damping coefficients.** However, in both analyses, the dominant component is 50 Hz (that is the fundamental) and the **maximum harmonic component** is **150 Hz**, **followed by 250 Hz**.

➢ CALCULATING THD :

The **total harmonic distortion** (**THD**) is a measurement of the harmonic distortion present in a signal and is defined as the ratio of the sum of the powers of all harmonic components to the power of the fundamental frequency.

So, if V be a distorted voltage waveform, then it consists of a D.C. component, as well as n number of harmonic components represented as,

$$\mathbf{V(t)} = V_0 + V_1 + V_2 + \ldots V_n, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(a)}$$

Where, V_0 denotes the D.C. or average component of voltage, V_1 represents the fundamental harmonic component (here, 50 Hz), V_2, the second harmonic component and so on upto V_n.

Taking r.m.s. values on both sides of equation (a), we get:

$$\mathbf{V_{rms}}^2\mathbf{(t)} = V_{0rms}{}^2 + V_{1rms}{}^2 + V_{2rms}{}^2 + \ldots V_{nrms}{}^2, \ldots\ldots\ldots\ldots\ldots\text{(b)}$$

$$\text{T.H.D} = \frac{\sqrt{V_{0rms}2 + V_{2rms}2 + \cdots V_{n\,rms}2}}{V1\,rms} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(c)}$$

i. For the **Three Phase Bridge Rectifier (Circuit A)**, from the results of Prony, the magnitude of the 50Hz component is 15.7, 250Hz component is 2.4016, 350Hz component is 1.001 and 100Hz component is 0.601, approximately. Hence, the **Total Harmonic Distortion from equation (c) is 17%.** But the **THD from the FFT analysis is found to be 30.52 %**

ii. Now, for the **Single Phase AC Voltage Controller (Circuit B)**, from the results of Prony, the magnitude of the 50Hz component is 9.07, 150Hz component is 2.16, 250Hz component is 1.54 and 350Hz component is 1.11, approximately. Hence, the **Total Harmonic Distortion from equation (c) is 32%.** But the **THD from the FFT analysis is found to be 60.50 %**

The difference in THD (lower in Prony for both of the signals) is because our code is a rough one and because in Prony, the higher dampings are eliminated unlike that in FFT. Also, the Prony's Technique is sensitive to external signals and noises. Therefore, technical corrections in the code can be made anytime to achieve perfection in the future.

Applications and Utility of analyser :

- **Power Quality Analysis**: It helps to carry out a detailed power quality analysis in the facility in order to determine the wave shapes of the current and voltage on their frequency spectrums.
- **Improving Energy Efficiency :** As harmonics can **cause energy loss** in several components, early detection of harmonic voltage industries can significantly improve their energy efficiency by adopting necessary measures, like providing **appropriate filters**.
- **Prevents Damage :** The high harmonic voltage often causes damage and failure of various machinery. Analysing the signal before use with a harmonic analyser, can prevent this.
- **Helps to determine possible source of harmonic voltage**: As nonlinear loads have the potential of distorting the supply voltage waveform, determining the possible nonlinear load which causes the distortion and applying appropriate filters can prevent problems to other loads connected to the same source too.

➢ CONCLUSION :

This was our overall project on analysing harmonic content in non-sinusoidal signals using two approaches, i.e. the mathematical cum programming approach using **PRONY** and the analysis using **FFT** (Fast Fourier Transform). We got to understand a lot of important concepts while working on it and also got to know its significance in real life. It is to be mentioned here that the Prony's technique is very sensitive to external noises and the code being rough, shows a bit difference with respect to the FFT analysis results. Further technical modifications can be made any time to yield better results in the future.

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- http://article.nadiapub.com/IJSIP/vol7_no4/33.pdf
- MATLAB and Simulink software.