ECON408: Assignment 1

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Question 1

Create the following variables:

- `D`: A floating point number with the value 10,000
- `r`: A floating point number with the value 0.025 (i.e., 2.5% net interest rate)

Compute the present discounted value of a payment (`D`) made in `t` years, assuming an interest rate of `r = 2.5%`. Save this value to a new variable called `PDV` and print your output.

Hint: The formula is

$$\text{PDV}(D, t) = \frac{D}{(1+r)^t}$$

For $t = 10$, calculate this PDV

Question 2

Now assume that you have an asset the pays D every year from $t = 0, \ldots T$. Write code which will price this as the PDV of all payoffs,

$$P_T(D) = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t D$$

Derive the analytic solution for the limit of $P_\infty(D) \equiv \lim_{T\to\infty} P_T(D)$ and plot the price as the horizon increases. That is:

- On the x-axis plot $T = 1, \ldots 30$
- On the y-axis plot $P_T(D)$ at that horizon
- Plot a horizontal line at the asymptotic P_∞ you calculated

Question 3

Now instead of having constant dividents, assume that dividends follow the process

$$\log D_{t+1} = \log D_t + \sigma w_{t+1}$$

Where

- w_{t+1} is a draw from a unit random normal (in Julia you can simulate with `randn()`)
- $D_0 = 1.0$
- $\sigma = 0.001$

For this,

1. Write code to simulate a sequence of dividends with the process and initial condition for $t = 0, \ldots T = 30$.
2. Plot three simulated sequences of dividends (i.e, the D_t for $t = 0, \ldots 30$) on the same graph with the shared x-axis.

Question 4

Using the simulated sequences of dividends from Question 3, calculate the P_0 assuming perfect foresight (i.e., they were able to know the sequence of w_{t+1} even for $t \geq 0$). The formula remains the same, except where $\{D_0, \ldots D_T\}$ is an argument which allows for time-dependent dividends

$$P_T(\{D_t\}_{t=0}^T) = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t D_t$$

All from the same $D_0 = 1.0$ initial condition calculate the $P_T(\{D_t^n\}_{t=0}^T)$ for $n = 1, \ldots N$ simulated sequences of dividends (i.e. see Question 3)

Plot a histogram of the prices for $N = 100$ simulations and compare to the deterministic case, which is nested if $\sigma = 0$. (Hint: see the Julia By Example lecture for more on histograms)

Question 5

Using the code in Keynesian Multiplier example from our lecture on Geometric Series.

Consider if the true government expenditures are $g + \sigma\epsilon$ where $\epsilon \sim N(0,1)$ i.e., a unit normal and with $\sigma = 0.01$. Consequently the law of motion for `y_t` becomes

$$y_t = by_{t-1} + i + (g + \epsilon)$$

Using this:

1. Take the code which generates the time-path of aggregate output from the initial condition (i.e., the `calculate_y` function) and change it code so that it implements the new process with the random ϵ
2. Redo the "Changing Consumption as a Fraction of Income" figure with the random simulations
3. Redo the "An Increase in Investment on Output" figure
4. Redo the "An Increase in Government Spending on Output" figure

You may be able to regenerate those figures by directly copy/pasting the plotting code with little to no modification after you write a replacement for `calculate_y`.