

Ideological Population Dynamics

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1 Characteristic Equations

Parameters:

1. $n_A \rightarrow$ fraction of A s in the organization
2. $n_{A'} \rightarrow$ fraction of A zealots in the organization
3. $n_B \rightarrow$ fraction of B s in the organization
4. $n_{B'} \rightarrow$ fraction of B zealots in the organization
5. $h_A \rightarrow$ fraction of A s in the hiring pool
6. $h_{A'} \rightarrow$ fraction of A zealots in the hiring pool
7. $h_B \rightarrow$ fraction of B s in the hiring pool
8. $h_{B'} \rightarrow$ fraction of B zealots in the hiring pool
9. $r_A \rightarrow$ fraction of A s resigning
10. $r_{A'} \rightarrow$ fraction of A zealots resigning
11. $r_B \rightarrow$ fraction of B s resigning
12. $r_{B'} \rightarrow$ fraction of B zealots resigning
13. $\rho_A \rightarrow$ probability that of $A \rightarrow A'$
14. $\rho_B \rightarrow$ probability that $B \rightarrow B'$

For later simplicity, we say that

$$r = r_A + r_{A'} + r_{AB} + r_B + r_{B'} \quad (1)$$

and, since TOPP is uniformly distributed among individuals in the organization,

$$r_w = \int_{n_w}^{\infty} f_{TOPP}(u) du \quad (2)$$

\dot{n}_A represents how n_A changes over time, measured here in interactions. We take into account, first, all those AB s that are converted to A through interactions with either As or A zealots. We also consider the B s who are converted to ideology A through interaction with A zealots. This conversion is only possible due to our assumption about preference falsification. if an B is interacting with an A zealot, then, during his interaction, he will falsify his worldview in the direction of A , thus acting as though he were an AB . There is an argument, therefore, that B s converted directly to A are not true As , but fraudulent ones who are unable to walk back their new public position. Next, we consider those As that are converted to zealots with probability ρ_A . Strictly speaking, however, ρ_A is not constant. Rather, it is a bijective mapping $\rho_A : S_{A+A'} \rightarrow P$, where $S_{A+A'} = \{n_A + n_{A'} \in [0, 1] : (n_A + n_{A'}) \pmod{5} = 0\}$ represents how homogeneous in worldview A the organization is, $P = \{p_1, p_2, \dots, p_{20}\}$ such that each $p_i \in [0, 1]$ is a set of ordered probabilities that A will be converted to zealotry, and ρ_A is defined by,

$$\rho_A(s_{A+A'_i}) = p_i \text{ where, } s_{A+A'_{i-1}} < n_A + n_{A'} \leq s_{A+A'_i}. \quad (3)$$

This will be incorporated later into the characteristic equations for a more accurate description of the system. Moving on, we consider all the resignations and the subsequent new hirings. Notice that r_A includes individuals who resign both as a result of their T_{OPP} and T_{HOM} tolerances being exceeded. This complexity should later be incorporated. Finally, $F n_A$ represents the constant number of people fired every epoch. This parameter should be thought about a bit more carefully. In the coded model, someone is fired every 5 interactions. But, perhaps that isn't the best way to do things.

$$\dot{n}_A = n_A n_{AB} + n_{A'}(n_{AB} + n_B) - \rho_A n_A n_{A'} - r_A n_A + r n_A h_A - F n_A + F h_A \quad (4)$$

$$\dot{n}_B = n_B n_{AB} + n_{B'}(n_{AB} + n_A) - \rho_B n_B n_{B'} - r_B n_B + r n_B h_B - F n_B + F h_B \quad (5)$$

$$\dot{n}_{A'} = \rho_A n_A n_{A'} - r_{A'} n_{A'} + r n_{A'} h_{A'} - F n_{A'} + F h_{A'} \quad (6)$$

$$\dot{n}_{B'} = \rho_B n_B n_{B'} - r_{B'} n_{B'} + r n_{B'} h_{B'} - F n_{B'} + F h_{B'} \quad (7)$$