Ideological Population Dynamics

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1 Characteristic Equations

Parameters:

1. $n_A \rightarrow$ fraction of As in the organization
2. $n_{A'} \rightarrow$ fraction of A zealots in the organization
3. $n_B \rightarrow$ fraction of Bs in the organization
4. $n_{B'} \rightarrow$ fraction of B zealots in the organization
5. $h_A \rightarrow$ fraction of As in the hiring pool
6. $h_{A'} \rightarrow$ fraction of A zealots in the hiring pool
7. $h_B \rightarrow$ fraction of Bs in the hiring pool
8. $h_{B'} \rightarrow$ fraction of B zealots in the hiring pool
9. $r_A \rightarrow$ fraction of As resigning
10. $r_{A'} \rightarrow$ fraction of A zealots resigning
11. $r_B \rightarrow$ fraction of Bs resigning
12. $r_{B'} \rightarrow$ fraction of B zealots resigning
13. $\rho_A \rightarrow$ probability that of $A \rightarrow A'$
14. $\rho_B \rightarrow$ probability that $B \rightarrow B'$

$\dot{n}_A$ represents how n_A changes over time, measured here in interactions. We take into account, first, all those ABs that are converted to A through interactions with either As or A zealots. We also consider the Bs who are converted to ideology A through interaction with A zealots. This conversion is only possible due to our assumption about preference falsification. if an B is interacting with an A zealot, then, during his interaction, he will falsify his worldview in the direction of A, thus acting as though he were an AB. There is an argument, therefore, that Bs converted directly to A are not true As, but fraudulent ones who are unable to walk back their new public position. Next, we consider those As that are converted to zealots with probability ρ_A. Strictly speaking, however, ρ_A is not constant. Rather, it is a bijective mapping $\rho_A : S_{A+A'} \rightarrow P$, where $S_{A+A'} = \{n_A + n_{A'} \in [0,1] : (n_A + n_{A'}) \pmod 5 = 0\}$ represents how homogeneous in worldview A the organization is, $P = \{p_1, p_2, ..., p_{20}\}$ such that each $p_i \in [0.1]$ is a set of ordered probabilities that A will be converted to zealotry, and ρ_A is defined by,

$$\rho_A(s_{A+A'_i}) = p_i \text{ where, } s_{A+A'_{i-1}} < n_A + n_{A'} \leq s_{A+A'_i}. \tag{1}$$

This will be incorporated later into the characteristic equations for a more accurate description of the system. Moving on, we consider all the resignations and the subsequent new hirings. Notice that r_A includes individuals who resign both as a result of their T_{OPP} and T_{HOM} tolerances being exceeded. This complexity should later be incorporated. Finally, Fn_A represents the constant number of people fired every epoch. This parameter should be thought about a bit more carefully. In the coded model, someone is fired every 5 interactions. But, perhaps that isn't the best way to do things.

$$\dot{n}_A = n_A n_{AB} + n_{A'}(n_{AB} + n_B) - \rho_A n_A n_{A'} - r_A n_A + r_A n_A h_A - F n_A \tag{2}$$

$$\dot{n}_B = n_B n_{AB} + n_{B'}(n_{AB} + n_A) - \rho_B n_B n_{B'} - r_B n_B + r_B n_B h_B - F n_B \tag{3}$$

$$\dot{n}_{A'} = \rho_A n_A n_{A'} - r_{A'} n_{A'} + r_{A'} n_{A'} h_{A'} - F n_{A'} \tag{4}$$

$$\dot{n}_{B'} = \rho_B n_B n_{B'} - r_{B'} n_{B'} + r_{B'} n_{B'} h_{B'} - F n_B \tag{5}$$