ISYE6740-HW5

pkubsad

March 2021

1. Part One (Divorce classification/prediction). (30 points)

   (a) (15 points) Report testing accuracy for each of the three classifiers. Comment on their performance: which performs the best and make a guess why they perform the best in this setting.

   Answer: I have written the code and based on the output, I have the following accuracy for different classifiers. Gaussian Naive Bayes and KNN classifiers perform slightly better than logistic regression. One reason KNN might be performing better than LR is because KNN is not factor dependent. There might not be co-linearity among the factors and the number of train data is not that high, this makes NB perform better than LR.

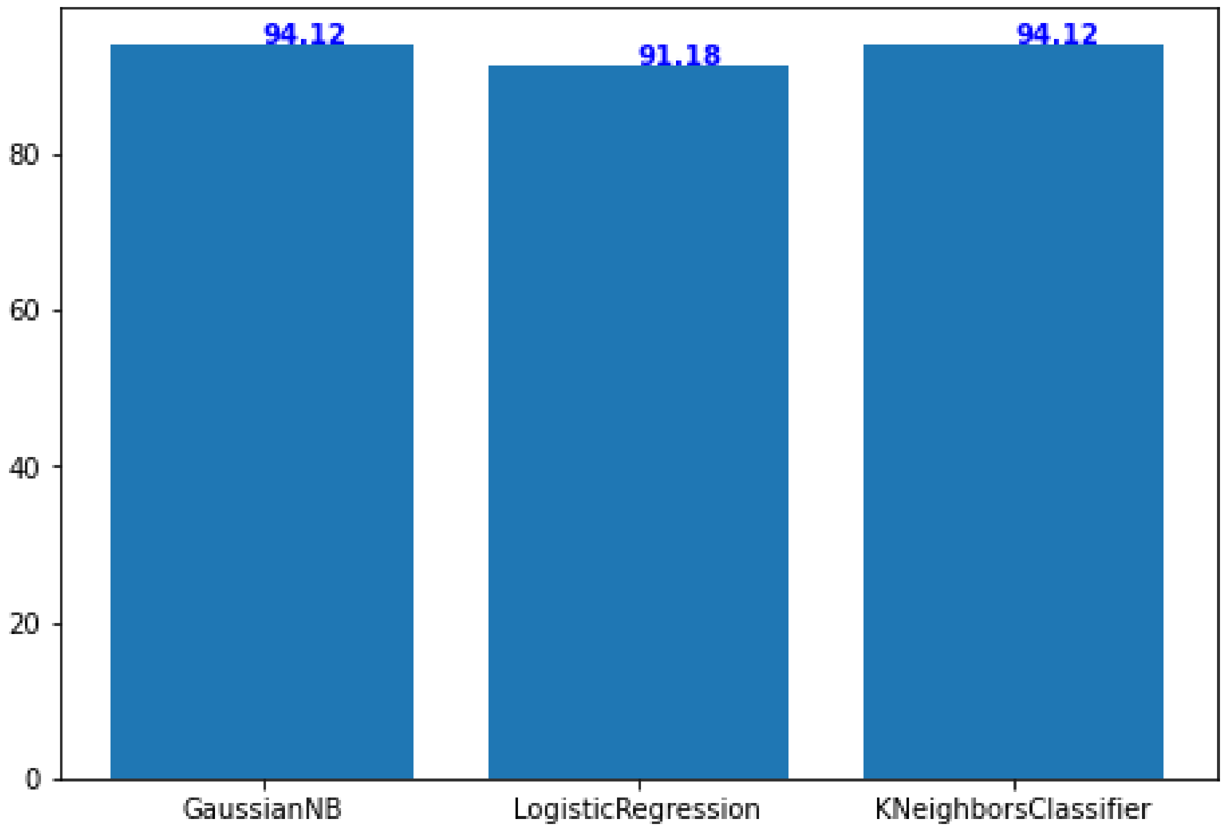


Figure 1: Accuracies of different classifiers

(b) (15 points) Now perform PCA to project the data into two-dimensional space. Build the classifiers (**Naive Bayes, Logistic Regression, and KNN**) using the two-dimensional PCA results. Plot the data points and decision boundary of each classifier in the two-dimensional space. Comment on the difference between the decision boundary for the three classifiers. Please clearly represent the data points with different labels using different colors.

Answer: Based on the output below, LR performs better on the reduced dataset. But, the classifier is in the proximity of 2 classes, it is prone to cause more classification errors on increased data sets.KNN classifier, even though it mis-classifies couple of data points, it is far from both classes and would perform even if data changes slightly.

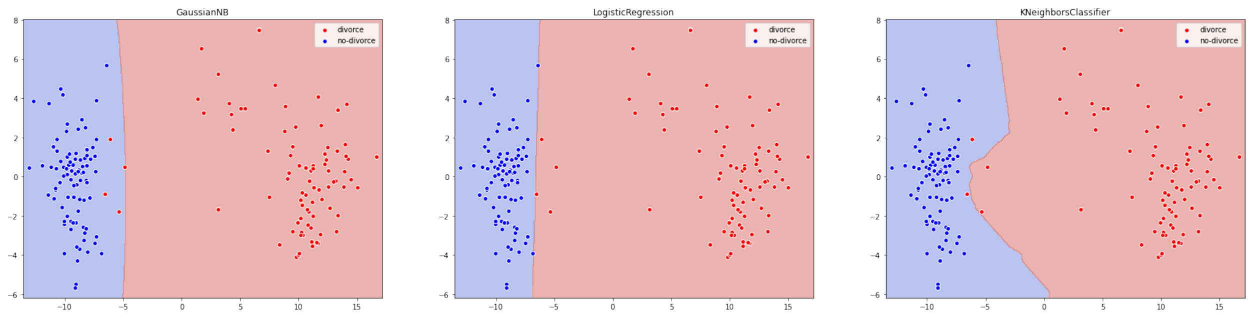


Figure 2: Classification using reduced features by PCA

2. Part Two (Handwritten digits classification). (35 points)

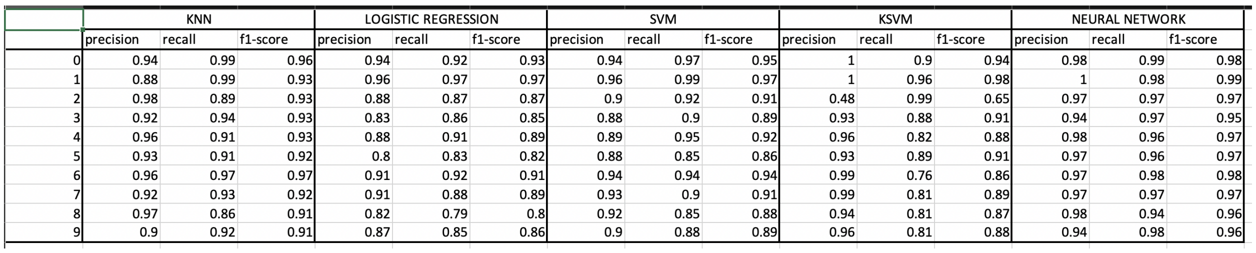
(a) (25 points) Report confusion matrix, precision, recall, and F-1 score for each of the classifiers. For precision, recall, and F-1 score of each classifier, we will need to report these for each of the digits. So you can create a table for this. For this question, each of the 5 classifier, **KNN, logistic regression, SVM, kernel SVM, and neural networks**, accounts for 10 points.

Answer: Please find below summary of classification report on the dataset by different classifier.
For KNN, I have run cross validation to determine optimal value of K. Best performance was observed at **k=6**. I have commented this part of code to avoid it running again and again.

For KSVM, I have used the following formula to start fine tuning the gamma parameter:

$$\gamma = \frac{1}{2\sigma^2}$$

src: https://stats.stackexchange.com/questions/317391/gamma-as-inverse-of-the-variance-of-rbf-kernelhyperref



	KNN			LOGISTIC REGRESSION			SVM			KSVM			NEURAL NETWORK		
	precision	recall	f1-score	precision	recall	f1-score	precision	recall	f1-score	precision	recall	f1-score	precision	recall	f1-score
0	0.94	0.99	0.96	0.94	0.92	0.93	0.94	0.97	0.95	1	0.9	0.94	0.98	0.99	0.98
1	0.88	0.99	0.93	0.96	0.97	0.97	0.96	0.99	0.97	1	0.96	0.98	1	0.98	0.99
2	0.98	0.89	0.93	0.88	0.87	0.87	0.9	0.92	0.91	0.48	0.99	0.65	0.97	0.97	0.97
3	0.92	0.94	0.93	0.83	0.86	0.85	0.88	0.9	0.89	0.93	0.88	0.91	0.94	0.97	0.95
4	0.96	0.91	0.93	0.88	0.91	0.89	0.89	0.95	0.92	0.96	0.82	0.88	0.98	0.96	0.97
5	0.93	0.91	0.92	0.8	0.83	0.82	0.88	0.85	0.86	0.93	0.89	0.91	0.97	0.96	0.97
6	0.96	0.97	0.97	0.91	0.92	0.91	0.94	0.94	0.94	0.99	0.76	0.86	0.97	0.98	0.98
7	0.92	0.93	0.92	0.91	0.88	0.89	0.93	0.9	0.91	0.99	0.81	0.89	0.97	0.97	0.97
8	0.97	0.86	0.91	0.82	0.79	0.8	0.92	0.85	0.88	0.94	0.81	0.87	0.98	0.94	0.96
9	0.9	0.92	0.91	0.87	0.85	0.86	0.9	0.88	0.89	0.96	0.81	0.88	0.94	0.98	0.96

Figure 3: Classification report for all classifiers

(b) (10 points) Comment on the performance of the classifier and give your explanation why some of them perform better than the others.

Answer: From the above table, we see the best performance is neural network classifier. We can attribute this to gradient descent at each node. With 10 nodes, and just 1 layer, we see such an improved performance.If we increase the number of layers, the accuracy will improve further. Also, KNN and SVM are not performing well with full data and requires us to down sample the data.

(2.1) (5 points) Calculate class prior $\mathbb{P}(y = 0)$ and $\mathbb{P}(y = 1)$ from the training data, where $y = 0$ corresponds to spam messages, and $y = 1$ corresponds to non-spam messages. Note that these class prior essentially corresponds to the frequency of each class in the training sample. Write down the feature vectors for each spam and non-spam messages.

Answer:
There are 3 spam messages out of 7 so $\mathbb{P}(y = 0) = 3/7 = 0.429$
There are 4 non-spam messages out of 7 so $\mathbb{P}(y = 1) = 4/7 = 0.571$

Feature Vector for spam:

	secret	offer	low	price	valued	customer	today	dollar	million	sports	is	for	play	healthy	pizza
1	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
2	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
3	2	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Feature Vector for non-spam:

	secret	offer	low	price	valued	customer	today	dollar	million	sports	is	for	play	healthy	pizza
4	0	0	1	1	1	1	0	0	0	0	0	1	0	0	0
5	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0
7	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1

(2.2) Calculate the maximum likelihood estimates of $\theta_{0,1}$, $\theta_{0,7}$, $\theta_{1,1}$, $\theta_{1,15}$ by maximizing the log-likelihood function above.

Answer: Source:https://www.cs.cornell.edu/courses/cs5740/2017sp/res/nb-prior.pdf

$$\ell(\theta_{0,1}, \ldots, \theta_{0,d}, \theta_{1,1}, \ldots, \theta_{1,d}) = \sum_{i=1}^{m} \sum_{k=1}^{d} x_k^{(i)} \log \theta_{y^{(i)},k}$$

We can use the method of Langrangian multipliers to solve the problem, which makes us to maximize the following langrangian function:

$$J = \sum_i \sum_k x_k^{(i)} log\theta_{y^{(i)},k} + \lambda(\sum_k \theta_{y^{(i)},k} - 1)$$

By taking first derivative wrt $\theta_{y^{(i)},k}$ and set it 0, we have

$$\nabla_{\theta_y^{(i)},k} J = \sum_i \frac{x_k^{(i)}}{\theta_y^{(i)}, k} + \lambda = 0$$

We can get then get $-\lambda = \sum_i \sum_k x_k^{(i)} = \sum_i = |D|$, the total number of objects in datapoints.By plugging in lambda we get:

$$\theta_y^{(i)}, k = \frac{\sum_i x^{(i)}}{|D|}$$

This denotes the total number of objects in D that belong to class C_k.
Using the above calculation, calculations for

$$\theta_{0,1} = \theta_{spam,secret} = \frac{\text{number of times 'secret' shows in spam}}{\text{number of words in spam messages}} = \frac{3}{9} = 0.333$$

$$\theta_{0,7} = \theta_{spam,today} = \frac{1}{9} = 0.111$$

$$\theta_{1,1} = \theta_{non-spam,secret} = \frac{1}{15} = 0.0667$$

$$\theta_{1,15} = \theta_{non-spam,pizza} = \frac{1}{15} = 0.0667$$

(2.3) (15 points) Given a test message "today is secret", using the Naive Bayes classier that you have trained in Part (a)-(b), to calculate the posterior and decide whether it is spam or not spam.

Answer: To figure out if the test string is spam or non-spam, we can run the probabilities calculated in the previous step.

$$\theta_{spam,\text{today is secret}} = \theta_{spam,today} * \theta_{spam,is} * \theta_{spam,secret}$$

$$\theta_{spam,\text{today is secret}} = 0.111 * 0.111 * 0.333 = 0.00410$$

$$\theta_{non\_spam,\text{today is secret}} = \theta_{non\_spam,today} * \theta_{non\_spam,is} * \theta_{non\_spam,secret}$$

$$\theta_{non\_spam,\text{today is secret}} = 0.0667 * 0.0667 * 0.0667 = 0.000297$$

The probability that the test message is spam is higher than non_span. We can conclude that this test message is more likely a spam.

Q1

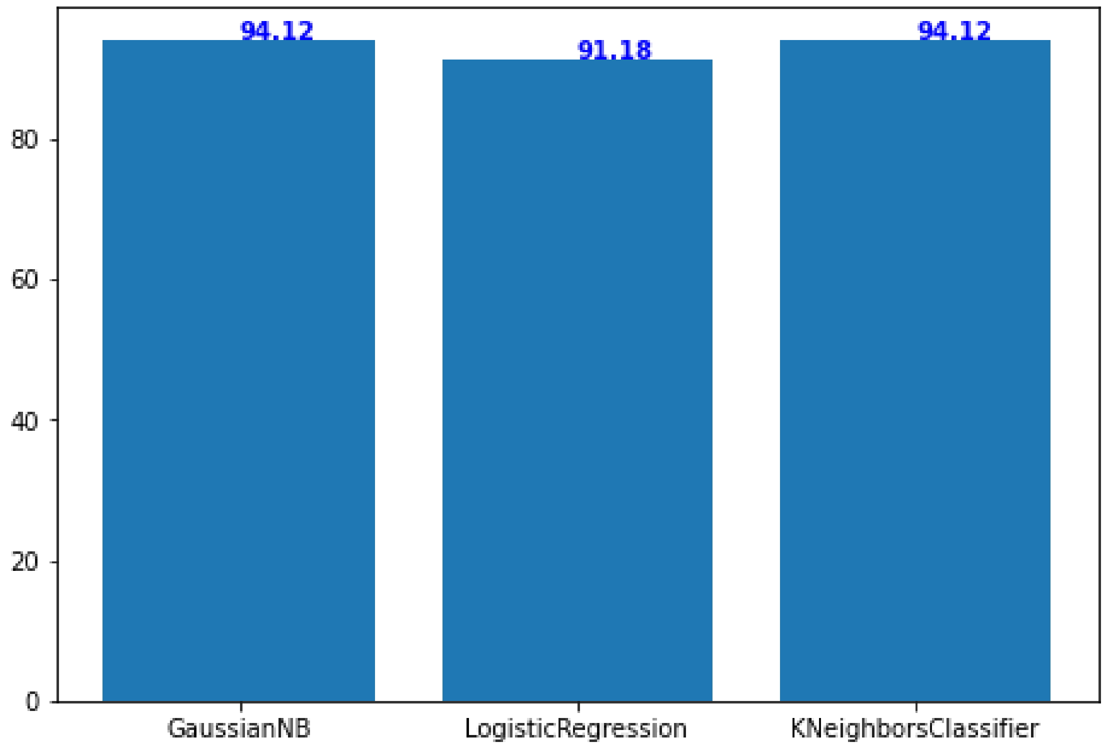
March 22, 2021

[1]:
```
import pandas as pd
import numpy as np
import scipy.io
from sklearn.model_selection import train_test_split
from sklearn.naive_bayes import GaussianNB
from sklearn.metrics import accuracy_score
import math
from sklearn.linear_model import LogisticRegression
from sklearn.neighbors import KNeighborsClassifier
from sklearn import svm
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from sklearn import preprocessing
from sklearn.metrics import confusion_matrix
from sklearn.metrics import classification_report
from sklearn.model_selection import GridSearchCV
from scipy.spatial.distance import pdist,squareform
from sklearn.datasets import fetch_openml
from sklearn.neural_network import MLPClassifier
from sklearn.exceptions import ConvergenceWarning
import warnings
```

[2]:
```
def perform_classification(model,factorsTrain, factorsTest, labelTrain,␣
 ↪labelTest):
    model.fit(factorsTrain,labelTrain)
    accuracy=accuracy_score(labelTest,model.predict(factorsTest))
    return float("{:.2f}".format(accuracy*100))
```

[3]:
```
data = pd.read_csv('./data/marriage.csv', header=None)
accuracies=[]
models=[GaussianNB(),LogisticRegression(penalty='none'),KNeighborsClassifier()]
factorsTrain, factorsTest, labelTrain, labelTest = train_test_split(data[data.
 ↪columns[:-1]], data[54], test_size=0.2, random_state=1,shuffle=True)
print(type(factorsTrain))
print(factorsTrain.shape)
print(type(labelTrain))
print(labelTrain.shape)
for model in models:
    accuracies.
 ↪append(perform_classification(model,factorsTrain,factorsTest,labelTrain,labelTest))

fig = plt.figure()
ax = fig.add_axes([0,0,1,1])
modelNames=[type(model).__name__ for model in models]
ax.bar(modelNames,accuracies)
for i, v in enumerate(accuracies):
    ax.text(i,v, str(v), color='blue', fontweight='bold')
plt.show()
```

```
<class 'pandas.core.frame.DataFrame'>
(136, 54)
<class 'pandas.core.series.Series'>
(136,)
```

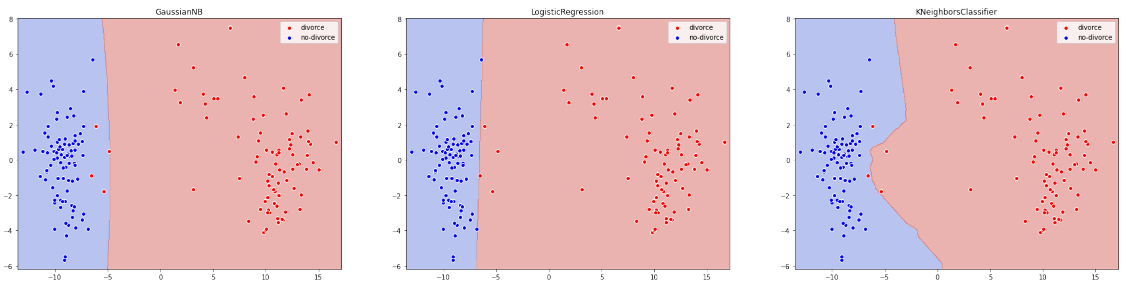


[4]:
```
pca=PCA(n_components=2)
reducedData=pca.fit_transform(data[data.columns[:-1]])
reducedData= pd.DataFrame(reducedData)
display(reducedData)
labels=data[54]
```

```
nbModel=GaussianNB()
nbModel.fit(reducedData,labels)

lr_two=LogisticRegression(penalty='none')
lr_two.fit(reducedData,labels)

knn_two=KNeighborsClassifier()
knn_two.fit(reducedData,labels)


x1_min = reducedData.iloc[:,0].min()
x2_min = reducedData.iloc[:,1].min()
x1_max = reducedData.iloc[:,0].max()
x2_max = reducedData.iloc[:,1].max()

x1,x2 = np.meshgrid(np.arange(x1_min-0.5,x1_max+0.5,0.05), np.arange(x2_min-0.
 ↪5,x2_max+0.5,0.05))
fig,axes = plt.subplots(1,3,figsize=(30,7))

i=0
for model in models:
    model.fit(reducedData,labels)
    grid_model= model.predict(np.array([x1.ravel(),x2.ravel()]).T)
    axes[i].contourf(x1,x2,grid_model.reshape(x2.shape),cmap='coolwarm',␣
 ↪alpha=0.4)
    axes[i].scatter(reducedData.loc[labels==1,0],reducedData.loc[labels==1,1],␣
 ↪c='red',edgecolors='white', label='divorce')
    axes[i].scatter(reducedData.loc[labels==0,0],reducedData.loc[labels==0,1],␣
 ↪c='blue',edgecolors='white', label='no-divorce')
    axes[i].legend()
    axes[i].set_title(type(model).__name__)
    i+=1
```

```
            0         1
0   -5.357120 -1.777214
1    7.342823  1.329477
2    1.351507  3.955616
3    5.439529  3.493741
4   -6.131208  1.909016
..        ...       ...
165 -8.328628 -2.876844
166 -9.111102 -5.675709
167 -8.875186  0.260375
168 -7.274430 -3.066975
169 -8.748430 -1.143669
```

```
[170 rows x 2 columns]
```



[5]:
```python
data = scipy.io.loadmat('./data/mnist_10digits.mat')


data_train=pd.DataFrame(data=np.column_stack((data['xtrain']))).T

data_train=pd.DataFrame(preprocessing.minmax_scale(data_train))

rindex =  np.random.randint(0, len(data_train), 5000)
data_train=data_train.loc[rindex]
display(data_train.shape)

label_train=pd.DataFrame(data=np.column_stack((data['ytrain'])))
label_train=label_train.loc[rindex]


data_test=pd.DataFrame(data=np.column_stack((data['xtest']))).T
data_test=pd.DataFrame(preprocessing.minmax_scale(data_test))

label_test=pd.DataFrame(data=np.column_stack((data['ytest'])))




# This is the code to determine optimum K value for knn, since it takes a long␣
 ↪time
# I have run it once and got the value of k
# and have commented the code to avoid running this again and again.
# I have also copied down the output I got for optimal value of k=6

# Source: https://stackoverflow.com/questions/62003285/
 ↪how-can-we-find-the-optimum-k-value-in-k-nearest-neighbor
# k_range = list(range(5,50))
# weight_options = ["uniform", "distance"]
# param_grid = dict(n_neighbors = k_range, weights = weight_options)
# knn = KNeighborsClassifier()
# grid = GridSearchCV(knn, param_grid, cv = 10, scoring = 'accuracy')
# grid.fit(data_train,label_train.values.ravel())

# print (grid.best_score_)
# print (grid.best_params_)
# print (grid.best_estimator_)

# output of the above code:
# 0.9366000000000001
# {'n_neighbors': 6, 'weights': 'distance'}
# KNeighborsClassifier(n_neighbors=6, weights='distance')


knn_model=KNeighborsClassifier(n_neighbors=6)
knn_model.fit(data_train,label_train.values.ravel())
knn_confusion=classification_report(label_test,knn_model.predict(data_test))
print(type(knn_confusion))
print(knn_confusion)

logistic_model=LogisticRegression(penalty='none')
logistic_model.fit(data_train,label_train.values.ravel())
logistic_confusion=classification_report(label_test,logistic_model.
 ↪predict(data_test))
print(logistic_confusion)

svm_model = svm.SVC(kernel='linear')
svm_model.fit(data_train,label_train.values.ravel())
svm_model_confusion=classification_report(label_test,svm_model.
 ↪predict(data_test))
print(svm_model_confusion)
```

```
(5000, 784)


<class 'str'>
              precision    recall  f1-score   support

           0       0.94      0.99      0.96       980
           1       0.89      1.00      0.94      1135
           2       0.98      0.89      0.93      1032
           3       0.91      0.95      0.93      1010
           4       0.95      0.93      0.94       982
           5       0.94      0.91      0.92       892
           6       0.96      0.97      0.96       958
           7       0.92      0.92      0.92      1028
           8       0.97      0.85      0.91       974
           9       0.91      0.91      0.91      1009

    accuracy                           0.93     10000
   macro avg       0.94      0.93      0.93     10000
weighted avg       0.93      0.93      0.93     10000

/Library/Frameworks/Python.framework/Versions/3.8/lib/python3.8/site-
packages/sklearn/linear_model/_logistic.py:763: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
              precision    recall  f1-score   support

           0       0.92      0.96      0.94       980
           1       0.94      0.95      0.95      1135
           2       0.88      0.81      0.84      1032
           3       0.81      0.82      0.82      1010
           4       0.86      0.89      0.88       982
           5       0.81      0.82      0.81       892
           6       0.92      0.89      0.90       958
           7       0.87      0.89      0.88      1028
           8       0.78      0.80      0.79       974
           9       0.87      0.83      0.85      1009

    accuracy                           0.87     10000
   macro avg       0.87      0.87      0.87     10000
weighted avg       0.87      0.87      0.87     10000

              precision    recall  f1-score   support

           0       0.93      0.98      0.95       980
           1       0.96      0.98      0.97      1135
           2       0.91      0.88      0.89      1032
           3       0.85      0.89      0.87      1010
           4       0.90      0.94      0.92       982
           5       0.87      0.87      0.87       892
           6       0.95      0.93      0.94       958
           7       0.92      0.91      0.91      1028
           8       0.89      0.84      0.86       974
           9       0.92      0.86      0.89      1009
    accuracy                           0.91     10000
   macro avg       0.91      0.91      0.91     10000
weighted avg       0.91      0.91      0.91     10000
```

[6]:
```
# Create the SVM
# source: https://www.machinecurve.com/index.php/2020/11/25/
 ↪using-radial-basis-functions-for-svms-with-python-and-scikit-learn/
samples_mean=data_train.sample(n=1000)
display(samples_mean)

#squared node-wise 2-norm distance
m=squareform(pdist(samples_mean.values, 'euclid'))

#median of squared node-wise p2 norm
med=np.median(m)
sigma=np.sqrt(med/2)

# source:https://stats.stackexchange.com/questions/317391/
 ↪gamma-as-inverse-of-the-variance-of-rbf-kernel
gamma=(1/(2*(sigma**2)))
display(m)
display(med)
display(sigma)
display(gamma)


ksvm_model = svm.SVC(kernel='rbf',gamma=gamma)
ksvm_model.fit(data_train,label_train.values.ravel())
ksvm_model_confusion=classification_report(label_test,ksvm_model.
 ↪predict(data_test))
print(ksvm_model_confusion)
```

```
       0    1    2    3    4    5    6    7    8    9    ...  774  775  776  \
17295  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
50117  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
5971   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
37668  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
46974  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
...    ...  ...  ...  ...  ...  ...  ...  ...  ...  ...  ...  ...  ...  ...
44801  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
39284  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
20330  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
43819  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0
19196  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0

       777  778  779  780  781  782  783
```

```
17295  0.0  0.0  0.0  0.0  0.0  0.0  0.0
50117  0.0  0.0  0.0  0.0  0.0  0.0  0.0
5971   0.0  0.0  0.0  0.0  0.0  0.0  0.0
37668  0.0  0.0  0.0  0.0  0.0  0.0  0.0
46974  0.0  0.0  0.0  0.0  0.0  0.0  0.0
...    ...  ...  ...  ...  ...  ...  ...
44801  0.0  0.0  0.0  0.0  0.0  0.0  0.0
39284  0.0  0.0  0.0  0.0  0.0  0.0  0.0
20330  0.0  0.0  0.0  0.0  0.0  0.0  0.0
43819  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19196  0.0  0.0  0.0  0.0  0.0  0.0  0.0

[1000 rows x 784 columns]
```

```
array([[ 0.        , 10.00560704, 12.4474036 , ..., 12.00378449,
        10.9722645 , 10.65536021],
       [10.00560704,  0.        ,  9.67173324, ...,  9.51102802,
         8.77745881,  9.68547429],
       [12.4474036 ,  9.67173324,  0.        , ...,  9.3030776 ,
         8.55728159,  9.51546475],
       ...,
       [12.00378449,  9.51102802,  9.3030776 , ...,  0.        ,
         8.46518074,  9.32116112],
       [10.9722645 ,  8.77745881,  8.55728159, ...,  8.46518074,
         0.        ,  9.06734851],
       [10.65536021,  9.68547429,  9.51546475, ...,  9.32116112,
         9.06734851,  0.        ]])
```

```
10.285350515038212
```

```
2.2677467357531587
```

```
0.09722566076264488
```

```
              precision    recall  f1-score   support

           0       0.99      0.89      0.94       980
           1       1.00      0.97      0.98      1135
           2       0.49      0.98      0.65      1032
           3       0.90      0.90      0.90      1010
           4       0.95      0.85      0.90       982
           5       0.96      0.81      0.88       892
           6       0.99      0.77      0.87       958
           7       0.98      0.82      0.89      1028
           8       0.93      0.81      0.86       974
           9       0.96      0.79      0.87      1009

    accuracy                           0.86     10000
   macro avg       0.91      0.86      0.87     10000
weighted avg       0.91      0.86      0.88     10000
```

[7]:
```python
data = scipy.io.loadmat('./data/mnist_10digits.mat')



data_train=pd.DataFrame(data=np.column_stack((data['xtrain']))).T
label_train=pd.DataFrame(data=np.column_stack((data['ytrain'])))

data_train=pd.DataFrame(preprocessing.minmax_scale(data_train))

mlp = MLPClassifier(hidden_layer_sizes=(50,), max_iter=10, alpha=1e-4,
                    solver='sgd', verbose=10, random_state=1,
                    learning_rate_init=.1)
with warnings.catch_warnings():
    warnings.filterwarnings("ignore", category=ConvergenceWarning,
                            module="sklearn")
    mlp.fit(data_train, label_train)
    nn_confusion=classification_report(label_test,mlp.predict(data_test))
    print(nn_confusion)
```

```
/Library/Frameworks/Python.framework/Versions/3.8/lib/python3.8/site-
packages/sklearn/utils/validation.py:63: DataConversionWarning: A column-vector
y was passed when a 1d array was expected. Please change the shape of y to
(n_samples, ), for example using ravel().
  return f(*args, **kwargs)

Iteration 1, loss = 0.32062846
Iteration 2, loss = 0.15380246
Iteration 3, loss = 0.11584796
Iteration 4, loss = 0.09353969
Iteration 5, loss = 0.07927847
Iteration 6, loss = 0.07134286
Iteration 7, loss = 0.06220003
Iteration 8, loss = 0.05481447
Iteration 9, loss = 0.04937818
Iteration 10, loss = 0.04596632
              precision    recall  f1-score   support

           0       0.98      0.99      0.98       980
           1       1.00      0.98      0.99      1135
           2       0.97      0.97      0.97      1032
           3       0.94      0.97      0.95      1010
           4       0.98      0.96      0.97       982
           5       0.97      0.96      0.97       892
           6       0.97      0.98      0.98       958
           7       0.97      0.97      0.97      1028
           8       0.98      0.94      0.96       974
           9       0.94      0.98      0.96      1009

    accuracy                           0.97     10000
   macro avg       0.97      0.97      0.97     10000
weighted avg       0.97      0.97      0.97     10000
```