

Extended Appendix: Ultimate Compliance Scenario Engine Tests (Advanced)

A. Non-Gaussian Marginals via t-Copulas

$$C(u_1, \dots, u_d; \Sigma, \nu) = t_{\Sigma, \nu}(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_d))$$

Enhances tail dependence modeling beyond Gaussian assumptions.

B. Skewness Stressing

Perturbing skewness using skew-normal or transformation of moments:

$$\text{Skewness} = \frac{\mathbb{E}[(X - \mu)^3]}{\sigma^3}$$

Allows explicit stress on asymmetry.

C. Historical Scenario Backtests

Application of realized crisis shocks (e.g., 2008 GFC):

$$R_P = w^\top F_{hist}$$

Validates real-world scenario responses.

D. Conditional Expected Shortfall

$$\text{CoES}_\alpha(L \mid F > q) = \mathbb{E}[L \mid L \leq \text{VaR}_\alpha(L), F > q]$$

Captures systemic risk conditional on factor thresholds.

E. Dynamic Stress Paths

Multi-step stress progression:

$$F_t = \phi F_{t-1} + \epsilon_t$$

Reveals path-dependent vulnerabilities.

F. Extreme Value Theory (EVT)

Fitting Generalized Pareto Distribution to tails:

$$P(X > x \mid X > u) \approx (1 + \xi \frac{x - u}{\beta})^{-1/\xi}$$

Robust extrapolation for rare losses.

G. Joint Adversarial Scenarios

Formal optimization:

$$\min_F w^\top F \quad \text{s.t.} \quad F \in \mathcal{S}$$

Identifies worst-case systemic configurations.

H. Additional Future Tests

- Cross-asset contagion simulations
- Liquidity stress (execution cost modeling)
- Covariance uncertainty perturbations
- Counterparty default cascades

I. Reproducibility

Python code implements each advanced module as functions, ensuring reproducibility for regulators or academic reviewers.