IB Physics Internal Assessment

"How is the damping coefficient of a harmonic oscillator (spring) affected by the surface area of the disk attached to the spring?"

1 Introduction

Damped harmonic motion and air resistance have intrigued me since I first encountered them in Physics. I wanted to explore these topics in detail and more quantitatively than the standard IB Physics curriculum, because they provided solutions to the limitations of simplistic models that often do not represent real-world phenomena.

When studying projectile motion in my Physics class, I was discomforted by how objects always keep accelerating downwards according to this model, counterintuitive to my personal observations on how light objects fall. While "no air resistance" was mentioned as an assumption for this model, I immediately wondered about how this assumption could be factored into the model and how the force of air resistance might change depending on certain variables, such as the surface area.

Similarly, while we assumed perfect springs and pendulums without any damping when studying Simple Harmonic Motion, I immediately wondered how damping could be factored in. That day, I watched YouTube videos about the equation and graphs of a damped harmonic oscillator, which captivated me and resonated much better with my observations about periodic motion from daily life, such as swings or slinkies eventually slowing down rather than oscillating forever.

Moreover, the idea of constant change involved with both these scenarios (with force depending on velocity, and the velocity changing depending on force) and its translation into the mathematical language of differential equations was mind-boggling the first time I saw it.

Initially, I wanted to explore the impact of changing surface area on the drag experienced by an object. However, I realized that it would be difficult to collect accurate data for a free-falling object because of the object swinging side by side and the video analysis being inaccurate due to reasons such as parallax error.

Hence, I decided to combine air resistance with harmonic motion for my Physics Internal Assessment and explore how air resistance affects the damping behavior in a spring. Moreover, there was barely any existing literature on how surface area of something attached to a spring affects damping behavior. Most papers either looked at the impact of material properties, temperature, and friction on damping or explored air resistance in isolation. Therefore, after some research, I decided to explore how changing the surface area of a disk attached to a spring affects the damping coefficient.

2 Background Information

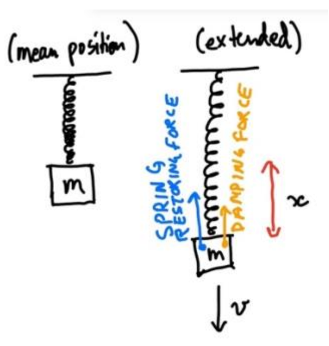

Figure 1: Mass-spring system (self-made using Notability)

This investigation looks at a mass-spring system shown in *Figure 1*. At a displacement of x from the mean position, the mass experiences two forces, the spring restoring force due to the extension of the spring, as well as a damping force opposing the motion of the mass.

For this paragraph, "damping force" and "drag force" are used interchangeably since damping is being caused by drag in our setup. For most scenarios, the damping force being proportional to velocity is a good model. In some scenarios (fast moving objects), the drag force due to air resistance being proportional to v^2 is more accurate because of the turbulent flow of air[1]. However, this does not result in an analytic solution to the differential equation and is not as problematic to our scenario as the spring moves at most at a speed of $0.3\ \mathrm{ms}^{-1}$. The drag force is then approximately given by $-cv$, where c is the *damping coefficient*, representing how quickly the oscillations die down. The negative sign signifies that the damping force opposes the direction of motion. The value of c depends on the drag coefficient (properties of the material, fluid, and temperature). According to literature, the value of c is also proportional to the surface area facing the direction of motion[2].

The spring restoring force is given by $-kx$, from Hooke's Law. The negative sign here signifies that it opposes the direction of displacement. Using the expressions for spring restoring force and damping force, we obtain:

$$F_{net} = -kx - cv \text{ --- (1)}$$

Rearranging, using Newton's Second law and writing velocity and acceleration in differential form, we get:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \text{ --- (2)}$$

This differential equation can be solved to obtain the following explicit expression for x in terms of t in the case where the system is underdamped (oscillating multiple times)[3]:

$$x(t) = Ae^{\frac{-c}{2m}t}\cos(\omega t + \phi) \text{ --- (3)}$$

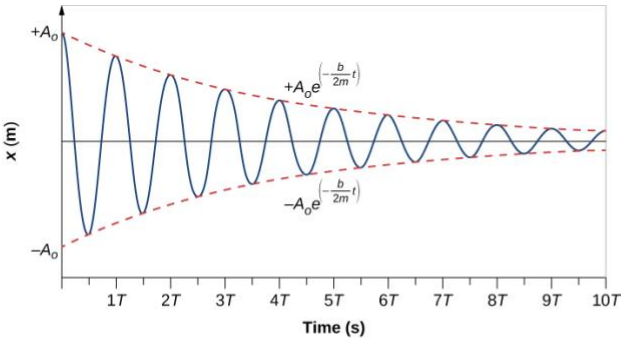

Figure 2: Position vs time for mass oscillating on a spring[4]

[1] *Air Friction*. hyperphysics.phy-astr.gsu.edu/hbase/airfri.html. Accessed 28 Oct. 2023.
[2] *Effect of Size on Drag*. www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/sized.html. Accessed 1 Oct. 2023.
[3] Libretexts. "5.3: General Solution for the Damped Harmonic Oscillator." *Physics LibreTexts*, 30 Apr. 2021, phys.libretexts.org/Bookshelves/Mathematical_Physics_and_Pedagogy/Complex_Methods_for_the_Sciences_(Chong)/05%3A_Complex_Oscillations/5.03%3A_General_Solution_for_the_Damped_Harmonic_Oscillator. Accessed 3 Nov. 2023.
[4] Image from OpenStax. 15.5 Damped Oscillations | University Physics Volume 1. courses.lumenlearning.com/suny-osuniversityphysics/chapter/15-5-damped-oscillations. Accessed 2 Jan. 2024.

Figure 2 shows a graph of Equation 3, clearly showing the damping behavior as the amplitude decreases over time.

At the peak or trough, the cos $(\omega t + \phi)$ term becomes either -1 or 1, and $t = nT$, where T is the time period of the oscillation. Therefore, the height of the nth peak is given by:

$$X_n = Ae^{\frac{-cnT}{2m}} \text{ --- (4)}$$

The value of c can be experimentally determined by measuring the peak-to-peak distance between the crest and trough initially and after n cycles[5]. This assumes that the effect of damping between crest and trough is negligible, which was seen to be true experimentally later. After performing some algebraic manipulation on Equation 4, we obtain:

$$c = \frac{2m}{nT} \ln\left(\frac{X_{0,peak\ to\ peak}}{X_{n,peak\ to\ peak}}\right) \text{ --- (5)}$$

Equation 5 agrees with the intuition that an increase the ratio between initial amplitude and final amplitude should mean an increase in the damping coefficient.

3 Hypothesis

As the surface area of the disks increase, more air molecules will hit the disk, slowing it down quicker. Hence, I hypothesize that there will be more damping present in the system, increasing the damping coefficient.

Furthermore, I hypothesize that the relationship between the damping coefficient and surface area will be linear. This is because the drag force is directly proportional to the surface area according to most literature. However, the relationship between the *damping coefficient* and surface area should not be directly proportional because there will be damping due to factors apart from air resistance (corresponding to the intercept of the graph). Hence, this relationship should be linear.

4 Experiment design

4.1 Variables

The **independent variable** for the experiment was the surface area of the MDF wood disks used. Eight different disks, with diameter ranging from 6 cm to 13 cm in $(1 \pm 0.01$[6]$)$cm increments were attached at the bottom of a mass on a spring. This range was used since the preliminary trial indicated that the 6cm disk experienced very little damping and the 13cm disk experienced a significant amount of damping. Hence, this range with eight increments was determined to be useful for analyzing a broad range of damping behavior.

The **dependent variable** for the experiment was the distance in meters between the bottom of the disk and the ultrasonic sensor over a period of 30 seconds. The frequency of the sensor was set to 25Hz (to ensure accuracy in the acoustically live room[7]), so position was recorded in increments of $\frac{1}{25} = 0.04$s. The uncertainty in position is ± 1mm[8]. A period of 30 seconds was chosen since preliminary trials indicated that 30 seconds was enough time for there to be a significant change in peak-to-peak displacement resulting in accurate data.

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[5] "MIT OpenCourseWare." *MIT OpenCourseWare*, ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/experimental-determination-of-damping-ratio. Accessed 14 Nov. 2023.
[6] Uncertainty from Vernier Caliper
[7] "Motion Detector - Vernier." *Vernier*, 5 Jan. 2024, www.vernier.com/product/motion-detector. Accessed 8 Jan 2024.
[8] IBID.

The temperature and material of the spring were **controlled** to ensure that drag force was not impacted by changes in the material properties of the spring. Temperature as well as material can affect material properties of the spring such as stiffness and internal friction, which can affect the damping of the system.

The shape of the wooden disks was **controlled** to ensure that the drag force was not impacted by changes in the drag coefficient (which is dependent on the shape of the object) and was only impacted by change in surface area throughout the experiment.

The initial amplitude (distance from mean position) the spring was launched from was **controlled** at approximately 4 cm via a measuring tape. While changing the amplitude does not theoretically affect the damping coefficient (if the material properties of the spring do not change), it was observed from preliminary trials that a very small amplitude resulted in readings with large fractional uncertainties because of a very small peak to peak displacement. This was happening in the preliminary trials because the spring was stretched to a fixed position and released, but the distance from the mean position kept changing between trials as the spring would sag more due to the increase in mass.

The mass of the system was **not controlled** because it does not have any impact on the *damping coefficient* and so it does not justify the effort of somehow obtaining disks of varying surface area but similar (or negligible) mass[9]. While changing the mass does change the *damping ratio* or the speed at which the amplitude slows down, it does not change the *damping coefficient* since it accounts for this change of mass by multiplying by a factor of m in Equation 5.

4.2 Apparatus

- Clamp stand (1)
- Metal bar (1)
- Helical spring (1)
- Vernier Ultrasonic Motion Detector (1) ($\pm$0.04s, $\pm$1mm)
- Measuring Tape (1)
- (198 ± 0.1) g fixed aluminum mass with hook (1)
- (0.51 ± 0.1) g reusable adhesive (blue tack) (1)
- Weighing scale ($\pm$0.1g) (1)
- Vernier Caliper ($\pm$0.1mm) (1)
- Ruler (1)
- Power cable, display and pen for sensor
- Spirit level (1)

4.3 Diagrams and Images

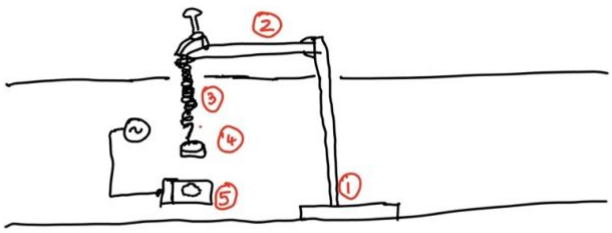

Figure 3: Diagram of experimental setup (self-made using Notability); (1): Clamp stand; (2): Metal bar; (3): Spring; (4): 200g mass; (5): Motion sensor

[9] (https://physics.stackexchange.com/users/127931/jmac), Jmac. How Does Damping Coefficient Vary with Mass? https://physics.stackexchange.com/q/320388. Accessed 15 Dec. 2023.

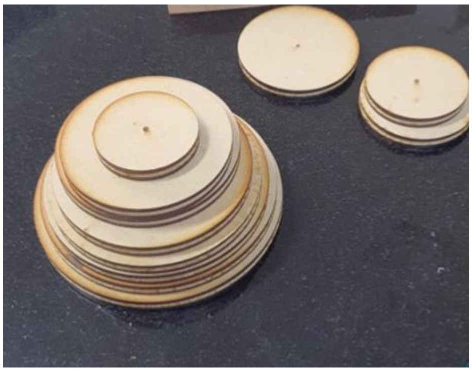

Figure 4: Images of experimental setup: Spring mass system (left); MDF disks (right)

4.4 Methodoıogy

I conducted a preliminary trial a few days prior to the experiment to ensure that the experiment was viable and to determine the values for the amplitude, mass, disk area, etc. that resulted in relevant data and the right amount of damping. Furthermore, I learnt how to use and export data from the sensor and looked up on how to interpret the CSV files. I also made decisions such as what kind of spring and adhesive to use. After the preliminary trial was successful, the following methodology was decided.

I. Setup the apparatus as depicted in *Figure 3*.
II. Weigh the disk on the weighing scale and note down the weight.
III. Measure the diameter of the disk using a vernier caliper and note it down.
IV. Attach adhesive to the center of the disk and stick it to the mass-spring system.
V. Stretch the mass down by pushing it down roughly 4cm from its spring mass equilibrium position using the ruler. Measure 4cm by extending the measuring tape parallel to the clamp stand.
VI. Ensure that the disk is completely level and not being pushed down unevenly using the Spirit level.
VII. Another person operates the sensor's display while the spring is being stretched down using the ruler by the first person.
VIII. Once the sensor starts recording, let go of the ruler swiftly to ensure there is no initial force, especially in another axis, other than the spring restoring force.
IX. Wait 30 seconds and stop the sensor from recording. Save the recorded run.
X. Repeat steps V-IX two more times to generate data for three trials.
XI. Repeat steps II-X seven more times to generate data for the eight disks.
XII. Weigh and note down the mass and adhesive to calculate the total mass of the system.
XIII. Transfer data for all the $3 \times 8 = 24$ runs from the sensor to laptop using Bluetooth.
XIV. Clean up the apparatus.

4.5 Risk Assessment

There are no ethical issues associated with this experiment. There are no environmental issues either, since the wooden disks were being re-used from a previous experiment, so no new waste was generated. Moreover, electrical appliances including the sensor and the room lights were turned off when not in use.

Relevant safety precautions were taken to ensure that all the heavy parts of the apparatus were tightly and correctly screwed in and/or attached, including the clamp stand and metal bar. A step back was taken while the spring was swinging for extra caution in case the setup would break. The switch was turned off before plugging in and removing the sensor from the plug point to prevent injury.

5 Results and Analysis
5.1 Qualitative Data

For the first few disks, there was no visible damping for the 30 seconds data was recorded and the amplitude of the oscillations remained more or less the same. For the disks with larger surface area, the oscillations seemed to die down quicker by the end of the 30 seconds.

5.2 Raw Data

Disk	Disk Diameter in mm ($\pm$0.1mm)	Disk Mass in grams ($\pm$0.1g)
Disk 1	60.0	17.9
Disk 2	70.0	24.6
Disk 3	80.0	33.6
Disk 4	90.0	43.3
Disk 5	100.0	54.3
Disk 6	110.0	66.6
Disk 7	120.0	80.6
Disk 8	130.0	94.3

Table 1: Raw data for disk weight (step II)

Slotted weight mass in grams ($\pm$0.1g)	Glue tag mass in grams ($\pm$0.1g)	Total fixed mass in grams ($\pm$0.2g)
198.1	0.5	198.6

Table 2: Raw data from weighing fixed mass (step XII)

Disk	Time taken in seconds till 40th cycle ($\pm$0.08s)			
	Trial 1	Trial 2	Trial 3	Average
Disk 1	20.84	20.76	20.76	20.79
Disk 2	21.08	21.12	21.16	21.12
Disk 3	21.52	21.52	21.52	21.52
Disk 4	21.96	22.00	22.00	21.99
Disk 5	22.48	22.48	22.48	22.48
Disk 6	23.00	23.00	23.04	23.01
Disk 7	23.60	23.68	23.60	23.63
Disk 8	24.16	24.16	24.16	24.16

Table 3: Raw data for time period of oscillations (from raw sensor data)

Disk	Peak to peak amplitude in meters ($\pm$2mm)					
	Trial 1		Trial 2		Trial 3	
	Initial cycle	40th cycle	Initial cycle	40th cycle	Initial cycle	40th cycle
Disk 1	0.067	0.052	0.056	0.043	0.056	0.045
Disk 2	0.044	0.035	0.069	0.051	0.060	0.045
Disk 3	0.071	0.052	0.073	0.053	0.076	0.054
Disk 4	0.085	0.055	0.085	0.058	0.090	0.057
Disk 5	0.082	0.052	0.077	0.049	0.088	0.056
Disk 6	0.076	0.048	0.075	0.047	0.079	0.047
Disk 7	0.083	0.048	0.089	0.054	0.086	0.050
Disk 8	0.085	0.047	0.089	0.050	0.085	0.048

Table 4: Raw data for amplitudes (from raw sensor data)

5.3 Processed Data

Disk	Ratio of amplitudes ($\frac{X_0}{X_{40}}$)			
	Trial 1	Trial 2	Trial 3	Average
Disk 1	1.29	1.30	1.24	1.28 ± 0.03
Disk 2	1.26	1.35	1.33	1.31 ± 0.05
Disk 3	1.37	1.38	1.41	1.38 ± 0.02
Disk 4	1.55	1.47	1.58	1.53 ± 0.06
Disk 5	1.58	1.57	1.57	1.57 ± 0.01
Disk 6	1.58	1.60	1.68	1.62 ± 0.05
Disk 7	1.73	1.65	1.72	1.70 ± 0.04
Disk 8	1.81	1.78	1.77	1.79 ± 0.02

Table 5: Processed data for ratio of amplitudes

Surface area of disk ($10^{-3}\mathrm{m}^2$)	Damping coefficient ($10^{-3}\mathrm{kgs}^{-1}$)
2.83 ± 0.01	5.1 ± 0.5
3.85 ± 0.01	5.8 ± 0.8
5.03 ± 0.01	7.0 ± 0.3
6.36 ± 0.01	9.4 ± 0.8
7.85 ± 0.02	10.2 ± 0.3
9.50 ± 0.02	11.1 ± 0.7
11.31 ± 0.02	12.5 ± 0.6
13.27 ± 0.02	14.1 ± 0.3

Table 6: Processed data for calculated damping coefficient

The peak-to-peak amplitudes were determined for the first and 40th cycle using the data collected from the sensor, so that their ratio could be determined. Moreover, the time elapsed between the first and 40th cycle was also determined using the sensor data. The 40th cycle was used since there was a large enough gap between the initial and 40th amplitude to reduce the uncertainty and all the data points had at least 40 cycles. This was done by loading the data into Google Sheets. This was done to be able to insert these values into the equation $c = \frac{2m}{nT}\ln\left(\frac{X_{0,peak\ to\ peak}}{X_{n,peak\ to\ peak}}\right)$ to determine the damping coefficient for each disk. An example for the first trial for Disk 8 is shown below:

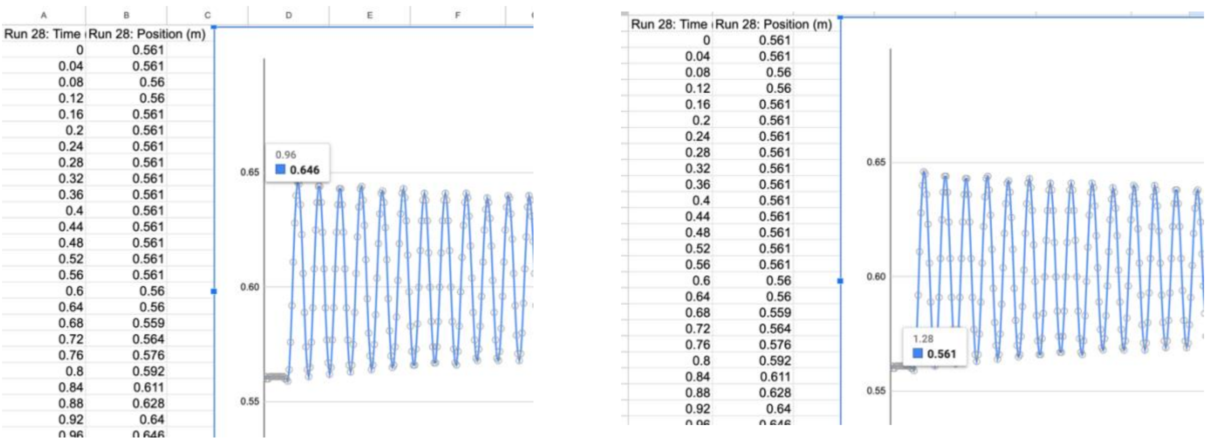

Figure 5: Determination of peak and trough for initial cycle using pos vs time graph from sensor data

Peak to peak amplitudes can be determined by subtracting the distance from the sensor for crests with the distance for troughs. From figure 5, we get $X_{0,p2p} = X_{0,top} - X_{0,bottom} = 0.646 - 0.561 = 0.085$. This corresponds to the initial amplitude for Disk 8 in *Table 4*. A similar calculation was performed for the 40th cycle to obtain the peak-to-peak amplitude of 0.047. Using the uncertainty rule for subtraction, when $X = \Delta X_{top} - \Delta X_{bottom}$, $\Delta X = \Delta X_{top} + \Delta X_{bottom} = 0.001 + 0.001 = 0.002$. The 0.001 is from the 1mm precision of the sensor. This 0.002 uncertainty applies for the peak-to-peak displacement of both the initial and the 40th cycle.

The time taken till the 40th cycle was calculated to be able to substitute in the nT term. This was done by taking the difference in the time the sensor measured the 40th crest and the time the sensor measured the first crest. For Disk 8, these times were 25.12s and 0.96s respectively (the time for the first crest is visible in Figure 5). So, $nT = 25.12 - 0.96 = 24.16$, which is visible in *Table 3*. Since two values are being subtracted again, there uncertainties are added, so the uncertainty in the time is 0.04+0.04 = 0.08. The 0.04 is from the frequency of the sensor.

The calculations for amplitude and time were repeated for the three trials. The average time was calculated and entered into Table 3. For the amplitudes, their ratio was calculated for each trial and then averaged (since each trial had a different initial amplitude), shown in Table 5. The uncertainty in the ratio was determined by taking the half range. The calculation for Disk 8 is as follows: Uncertainty in ratio = $\frac{max-min}{2} = \frac{1.81-1.77}{2} = 0.02$.

The mass of the eighth disk was measured to be 0.0943 kg. The total mass was calculated as $m = \text{Mass of disk} + \text{Mass of weight} + \text{Mass of adhesive} = 0.0943 + 0.1981 + 0.0005 = 0.2929$ kg. Using the uncertainty rule for addition, $\Delta m = 0.0001 + 0.0001 + 0.0001 = 0.0003$ kg.

The damping coefficient can be calculated using the ratio R, time till 40th cycle T_{40} and total mass m using (by substituting into Equation 5):

$$c = \frac{2m}{T_{40}} \ln (\mathrm{R})$$

For disk 8:

$$c = \frac{2 \times 0.2929}{24.16} \ln(1.79) = 14.1 \times 10^{-3} \mathrm{kg\ s^{-1}}$$

The uncertainty in the damping coefficient depends on the uncertainties of the mass, time and amplitude ratio. The error in the damping coefficient can be approximated using the following equation[10]:

$$\Delta c = \sqrt{\left(\frac{\partial c}{\partial m}\Delta m\right)^2 + \left(\frac{\partial c}{\partial T_{40}}\Delta T_{40}\right)^2 + \left(\frac{\partial c}{\partial R}\Delta R\right)^2}$$

After calculating the partial derivatives, the uncertainty becomes:

$$\Delta c = \sqrt{\left(\frac{c}{m}\Delta m\right)^2 + \left(\frac{c}{T_{40}}\Delta T_{40}\right)^2 + \left(\frac{2m}{RT_{40}}\Delta R\right)^2}$$

We can substitute the calculated value for c and m and the average values of T_{40} and R for Disk 8, as well as the uncertainties in m, T_{40} and R calculated:

[10] *Propagation of Errors*. courses.cit.cornell.edu/virtual_lab/LabZero/Propagation_of_Error.shtml. Accessed 20 Dec. 2023.

$$\Delta c = \sqrt{\left(\frac{0.0141}{0.2929}0.0003\right)^2 + \left(\frac{0.0141}{24.16}0.08\right)^2 + \left(\frac{2 \times 0.2929}{1.79 \times 24.16}0.02\right)^2}$$

The uncertainty for the damping coefficient is $3 \times 10^{-4}\text{kg s}^{-1}$. This uncertainty and value was populated in the last row and second column of *Table 6*. The surface area of the disk was calculated:

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{130 \times 10^{-3}}{2}\right)^2 = 0.01327\text{m}^2$$

Using the uncertainty rule for powers, $\frac{\Delta A}{A} = 2\frac{\Delta d}{d}$ and so $\Delta A = \pi\left(\frac{d}{2}\right)^2 \times 2\frac{\Delta d}{d} = \frac{\pi}{2} \times d \times \Delta d$. By using the uncertainty from the Vernier Caliper the diameter, $\Delta d = \frac{\pi}{2} \times\ 130 \times 10^{-3} \times 0.0001 = 2.04 \times 10^{-5}$. This results in the surface area of Disk 8 as $(13.27 \pm 0.02) \times 10^{-3}\text{m}^2$ (populated in the last row and first column of *Table 6*). This process was repeated for the other disks.

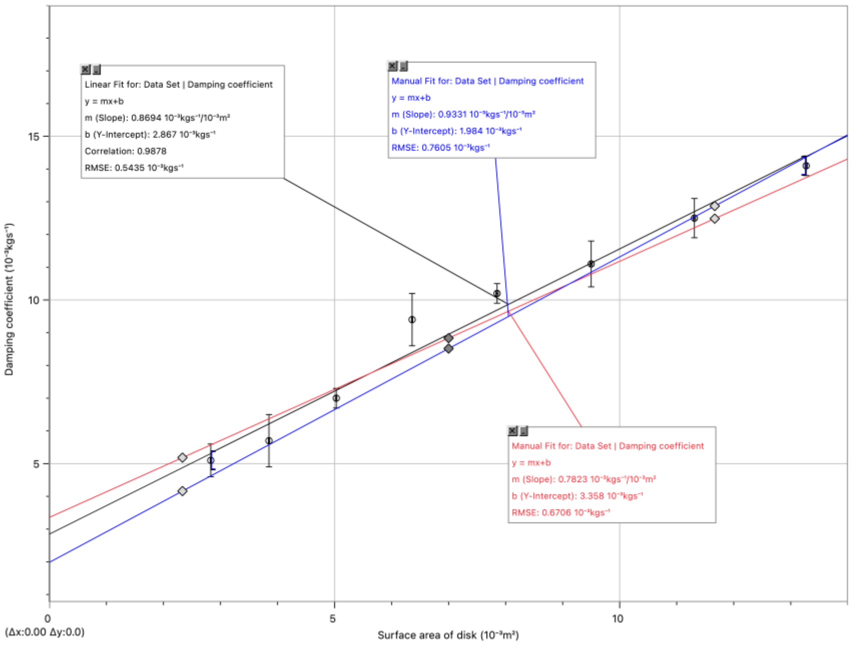

Figure 6: Damping coefficient against surface area

The damping coefficient was plotted against the surface area in *Figure 6* using Logger Pro and the data from *Table 6*. Vertical error bars correspond to uncertainty in the damping coefficient. Horizontal error bars for uncertainty in the surface area have also been included but are too small to be visible. Linear regression was performed (black line) and a maximum slope line (blue) and minimum slope line (red) were drawn, since the relationship appeared to be linear. Max and min lines were used to calculate the uncertainty in gradient and intercept.

$$\text{Uncertainty in slope} = \frac{\max - \min}{2} = \frac{0.93 - 0.78}{2} = 0.08\text{kgs}^{-1}\text{m}^{-2}$$

$$\text{Uncertainty in intercept} = \frac{\max - \min}{2} = \frac{3.36 - 1.98}{2} = 0.7 \times 10^{-3}\text{kgs}^{-1}$$

6 Conclusion

From *Figure 6*, it is clearly visible that the damping coefficient is positively correlated with the surface area of the disk, confirming the qualitative aspect of the hypothesis. Furthermore, the line of best fit has a correlation coefficient of 0.988, which is high enough to establish that there is a strong linear correlation between the variables, confirming the quantitative aspect of the hypothesis. The RMSE of $0.54 \times 10^{-3}\text{kgs}^{-1}$ means that data points are on average 0.54 off from the linear fit, making the model fairly accurate in predicting the damping coefficient.

The line of best fit passes through most of the error bars, indicating that a linear fit is appropriate for the data since the predicted value largely falls within the margin of error. The line does not go through the error bars for the fourth and the fifth readings, however. For the fourth point this can be explained by the large error due to the deviation in the amplitude ratios between the trials. Looking at the graph, I realized this deviation was due to irregularities in the amplitudes for these readings *(Figure 7)* compared to the ideal smooth graph *(Figure 2)*, resulting in variations in the 40th amplitude among the trials. This error, likely due to the mass being stretched or provided an initial velocity sideways due to shaky hands during this reading, is further evaluated in the first three limitations discussed in Section 7.2. Nevertheless, since these outliers were ultimately likely caused by experimental errors and limited data collection and processing (as detailed in section 7.2), they do not undermine the hypothesis of a linear relationship between the variables.

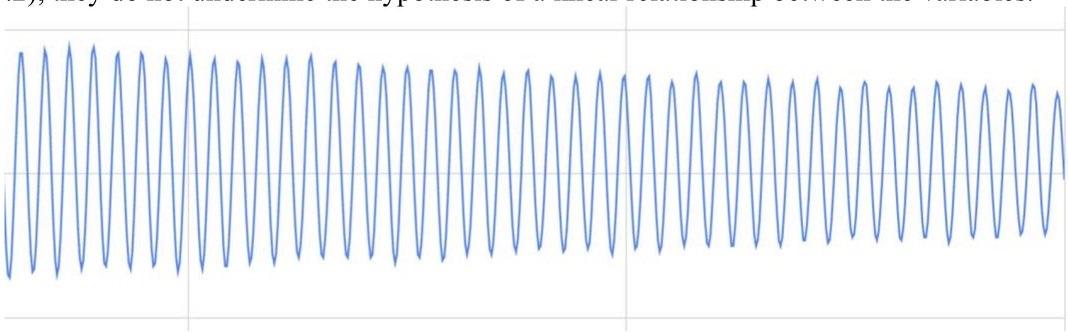

Figure 7: Position vs time, fourth disk, third trial

As outlined in the hypothesis, the linear but not proportional relationship between the surface area and damping coefficient indicates that there are two components of damping involved, one related to air resistance and the other related to the spring itself. That is, $F_{damping} = F_{air} + F_{other} = mAv + bv = v(mA + b)$. The damping coefficient is the coefficient of v, which is $mA + b$. This explains the linear relationship between the variables. Extrapolating the data (looking at the intercepts of the three lines) gives us the y-intercept of $(2.9 \pm 0.7) \times 10^{-3}\text{kgs}^{-1}$. This can be interpreted as the damping in the spring without any air resistance. This inference is only true if the linear relationship continues to hold for smaller disk sizes. The slope of $(0.87 \pm 0.07)\ \text{kgs}^{-1}\text{m}^{-2}$ represents the increase in the damping coefficient for every m^2 increase in the surface area in our setup.

No existing literature on this exact topic (relationship between damping coefficient and surface area) could be found. However, this experiment confirms the view that the drag force is proportional to the surface area facing the direction of motion (since the total damping force turned out to be linear)[11]. This also confirms that assuming drag force as proportional to v instead of v^2 was a good approximation in this scenario since a linear relationship would not have been observed if it had not been a good approximation. This relationship would not have been observed because the differential equation with a v^2 term would have a different (non-analytic) solution and the damping coefficient calculations carried out in the data analysis section would have been incorrect.

7 Evaluation

7.1 Strengths

[11] *Effect of Size on Drag*. www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/sized.html. Accessed 1 Oct. 2023.

The experiment had a wide range of data for the independent variable as well as conducted enough trials to establish a precise relationship between the variables. Moreover, aspects apart from the area and weight of the disks were controlled successfully (material, build, etc.) due to the precise manufacturing process. The experiment is also simple to conduct and requires only basic apparatus.

7.2 Limitations

Problem	Potential Solution	Impact on results
Despite measures taken to ensure that all movement was in the vertical axis, the spring was observed to swing left and right in some trials.	More care can be taken when releasing the spring, or sophisticated equipment can be used rather than shaky hands. Equipment can be more securely attached, since the clamp stand was wobbling which was also causing these movements.	Movement in the horizontal plane resulted in amplitudes deviating from the true damping behavior of the system. This random error significantly impacted the motion, seen qualitatively while performing the experiment and when observing the large deviations between the consecutive peaks in *Figure 7* (which should not exist ideally). Since the inaccurate amplitudes were the largest source of error, this problem is likely a big source of error in the experiment and the cause of the outliers.
The sampling rate of the sensor was too slow, resulting in jagged measurements and inaccurate amplitudes calculated.	The sampling rate can be increased to 50Hz, experiment can be conducted in a more acoustically suitable location to reduce any errors due to this.	The low sampling rate also could have led to the disparity in amplitude ratios measured among the three trials, since it resulted in a lower precision of the position of the mass when it was near its maximum position (the measured positions weren't exactly at the amplitude). This is shown by the jagged peaks and troughs evident in *Figures 5 and 7*. Consequently, inaccuracies in amplitude determination, being a primary source of error, likely contributed to the presence of outliers in the data.
Only one data point was used for the amplitudes, making the random errors discussed in the two problems above affect the data since one or two inaccurate measurements affected the data to a large extent.	Instead of using the 40^{th} amplitude and the initial amplitude to calculate the damping coefficient using Equation 5, an exponential curve can be fit on all the measured amplitudes (as shown in Figure 2). This would ensure that inaccurate sensor measurements for one amplitude did not significantly affect the calculated damping coefficient. Furthermore, more trials can be taken to reduce the random errors.	This issue exacerbated the two issues discussed above. For example, for the disk 4 outlier, the damping ratio was likely overestimated for Trials 1 and 3 (see *Table 5)*. This may have been due to two inaccurate amplitudes considered in these trials, which significantly affected the calculated value of the damping coefficient. Fitting a curve and taking more trials would have reduced the probability of these random errors affecting the processed results.
The mass was not released at exactly 4cm from the mean	The initial amplitude can be more carefully measured from	This likely did not have a major impact on the result. Having the amplitude too

position. While I did ensure that the amplitude was enough for the change in amplitude to be visible in the data, the initial amplitude wasn't kept completely constant throughout the trials and was only roughly measured. Raw data shows that initial amplitude varied somewhat across trials.	the center of mass by having another person hold a measuring tape properly.	low would have resulted in the error being extremely high (what happened in the preliminary trial). However, since the amplitude was large enough to where this wasn't a problem, the initial amplitude changing between trials would not have affected the result since the damping coefficient does not depend on the initial amplitude, only the ratio.
The peak-to-peak amplitude was measured and the assumption that no damping occurred in the middle was made to simplify calculations.	The data can be normalized (displacement centered around mean position) and then a single amplitude can be directly measured rather than the peak-to-peak amplitude.	This likely did not have a major impact on the result. This is because damping was occurring fairly slowly in the experiment, and so this assumption would not have caused any significant error.

Table 7: Limitations of experiment

7.3 Extensions

Impact of changing material properties and shape on the damping coefficient can be explored, since these are the other factors that impact drag apart from surface area. Fluids apart from air can be used, such as oil or water. The impact of damping due to the viscosity or density of these fluids can also be explored.

The same experiment can also be repeated with a spring with a greater spring constant and a higher initial amplitude, to see how the results fare with a very large velocity. It's likely that the approximation of being proportional to velocity would not be accurate due to the High Reynolds number and turbulent flow of air.

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